Title: Conserved momenta of a ferromagnetic soliton

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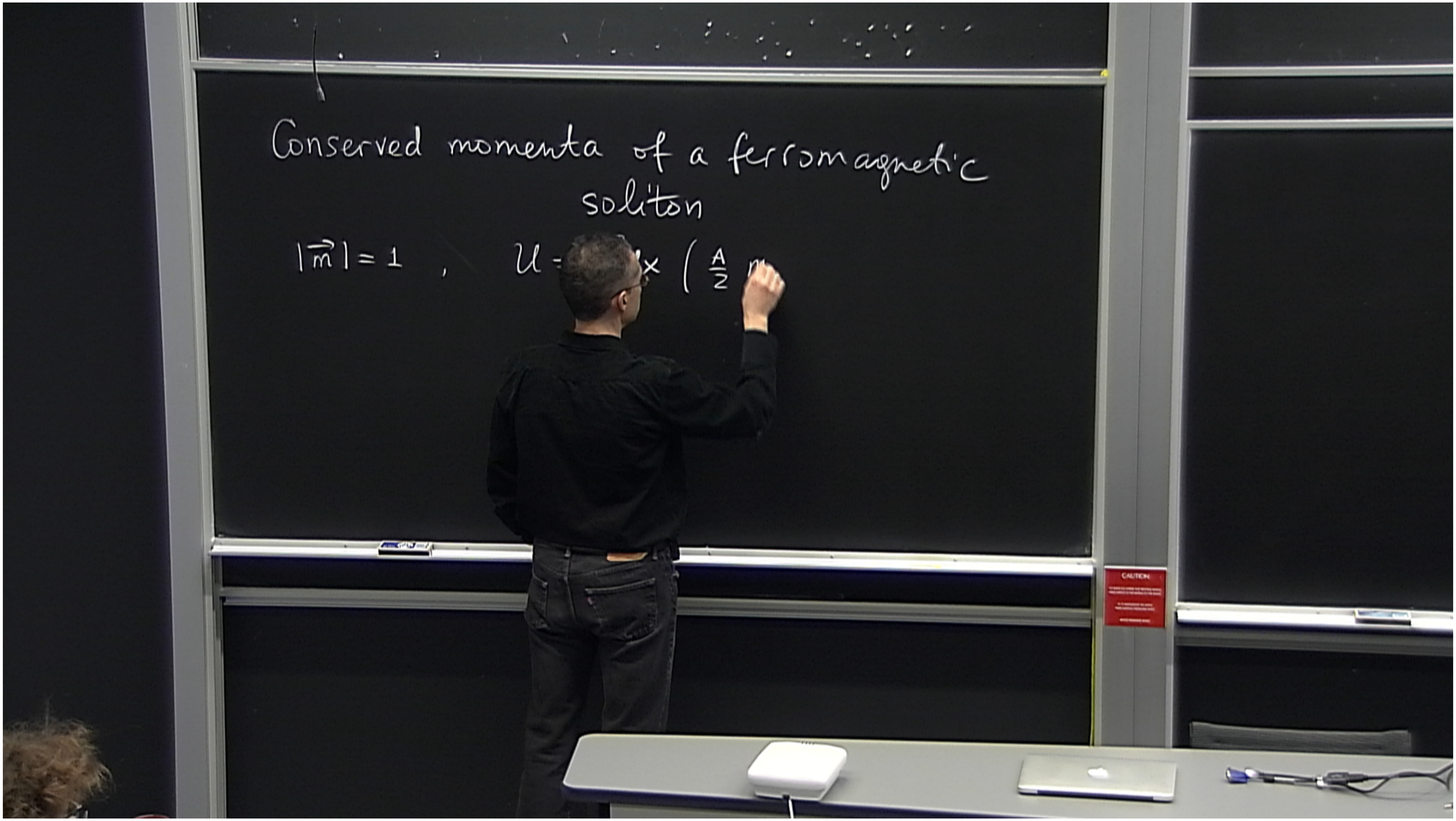
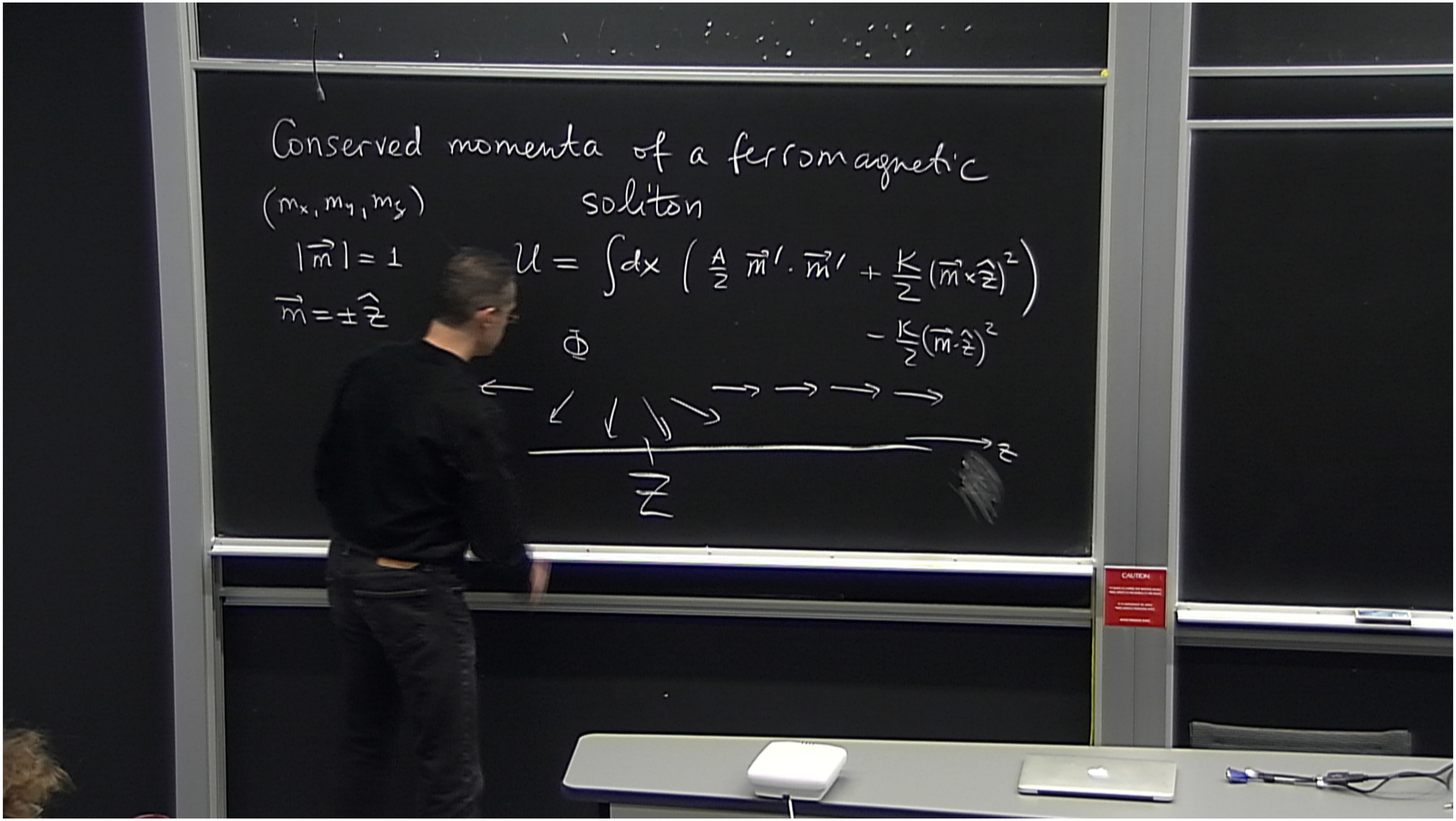
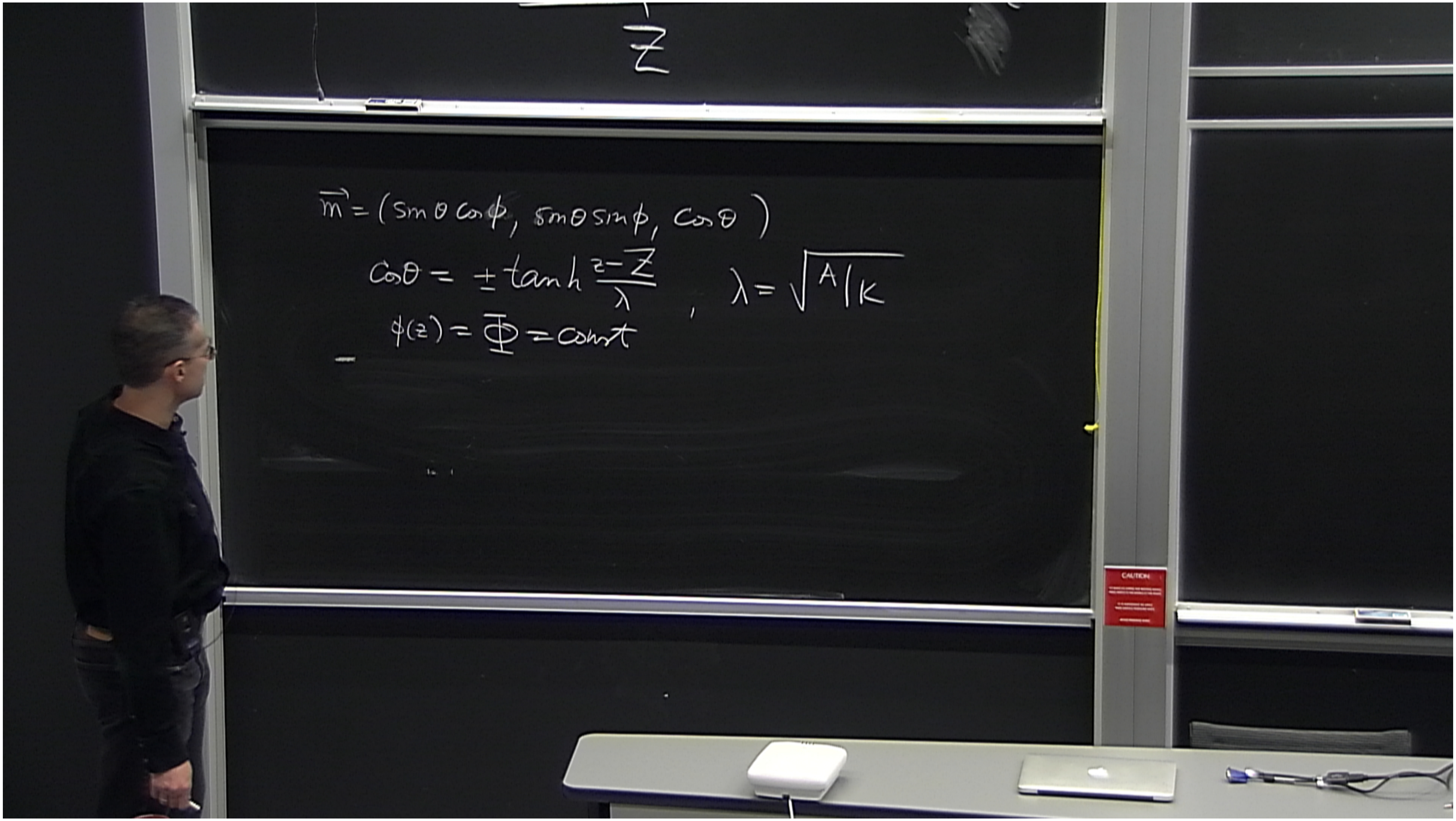
Abstract: <p>Solitons in a ferromagnet have interesting dynamics because atomic magnetic moments behave like little gyroscopes. A domain wall in a magnetic wire can be modeled as a bead on a string: it has two soft modes, position and orientation. This "bead" rotates when it is pushed and moves when twisted.</p>

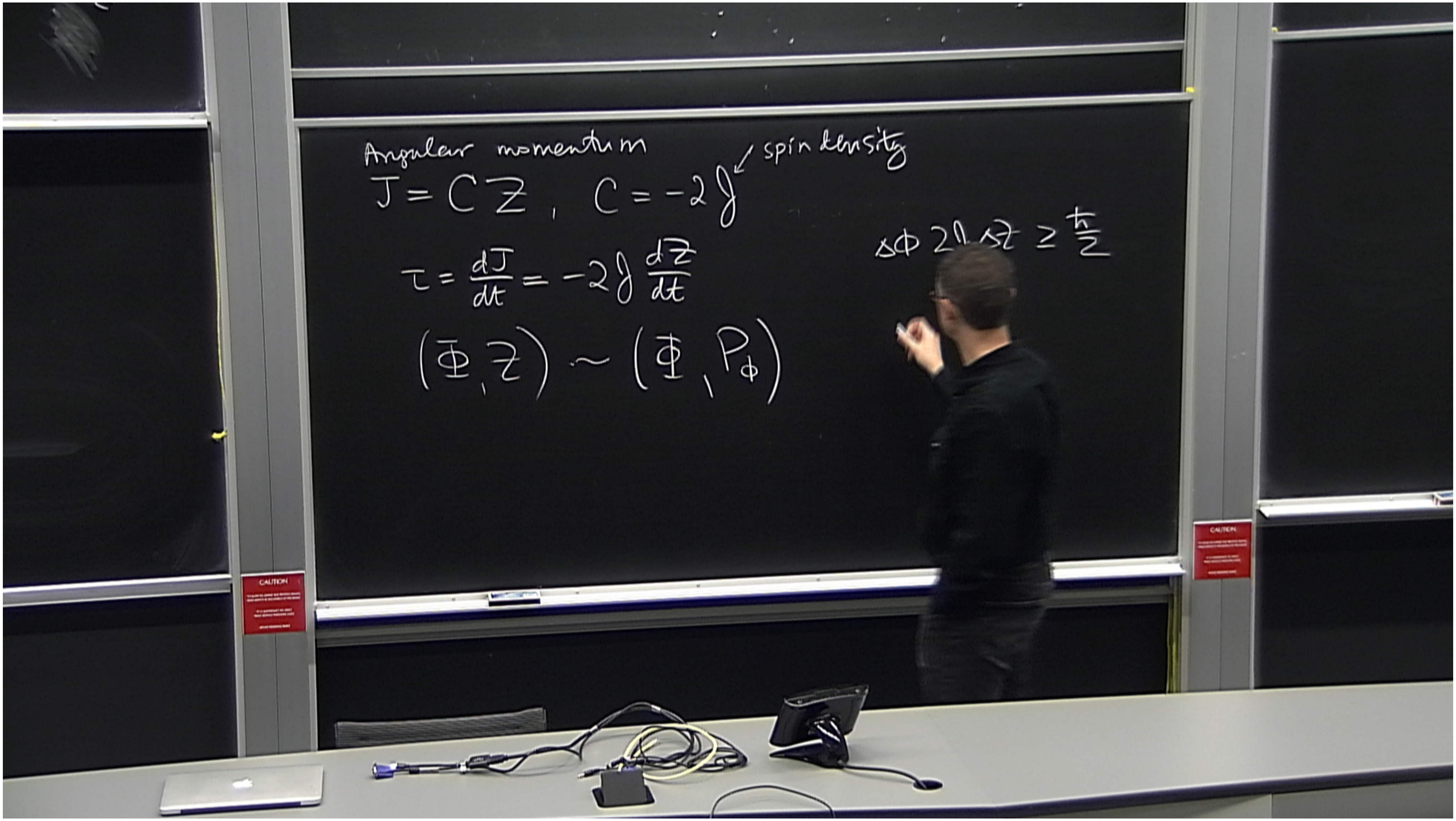
<p>From that one can deduce that the angular momentum is proportional to the domain wall's coordinate and the linear momentum to its azimuthal angle. In fact, the definition of conserved momenta for magnetic solitons has been discussed in the scientific literature for 40 years. A naive attempt to derive them from the application of Noether's theorem yields unphysical, gauge-dependent answers. To resolve this problem, we exploit a similarity between the dynamics of a ferromagnetic soliton and that of a charged particle in a magnetic field. For the latter, canonical momentum is also gauge-dependent and thus unphysical; the physical momentum is the generator of magnetic translations, a symmetry combining physical translations with gauge transformations. We use this analogy to unambiguously define conserved momenta for ferromagnetic solitons.</p>

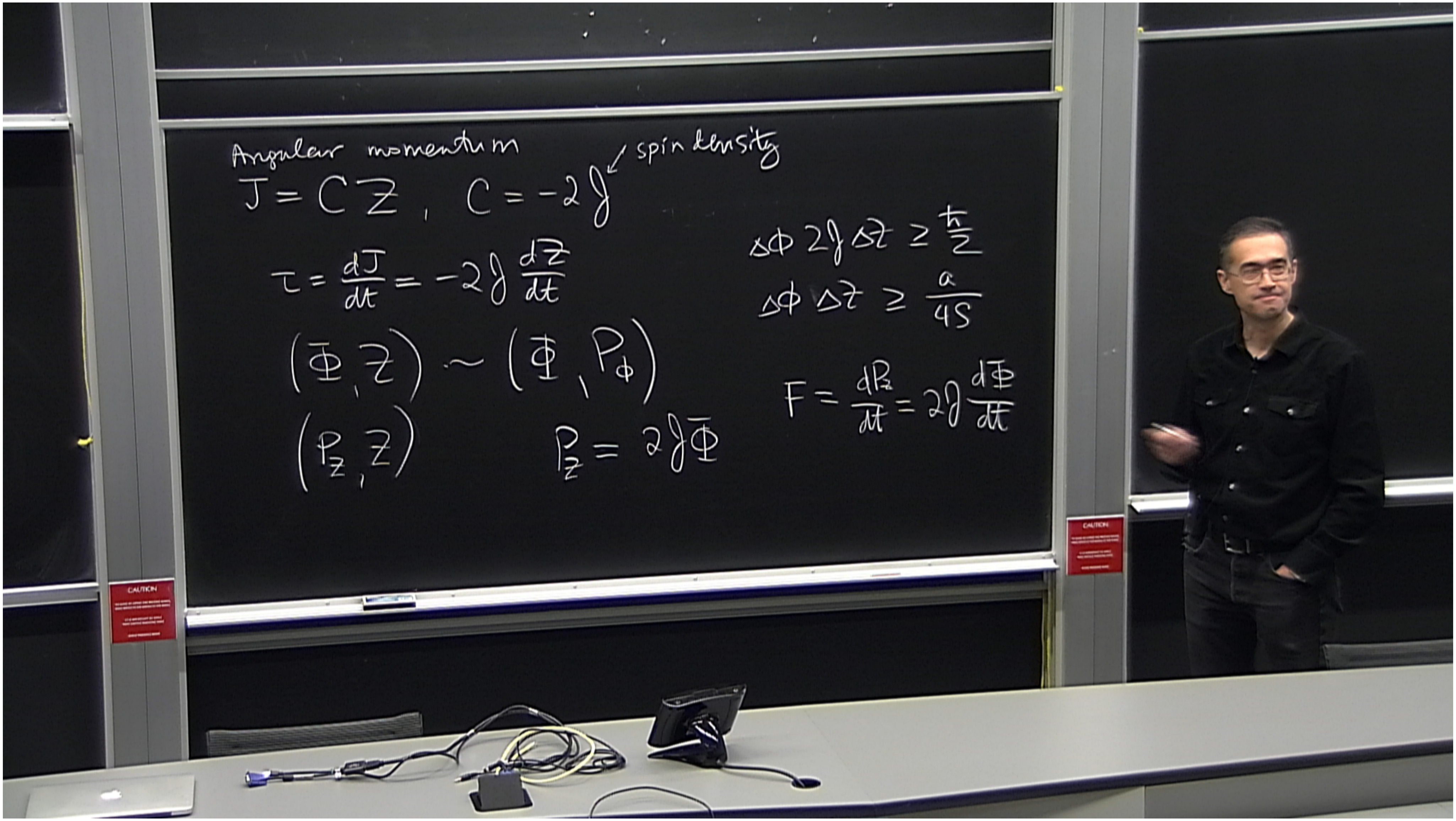

Conserved momenta of a ferromagnetic soliton

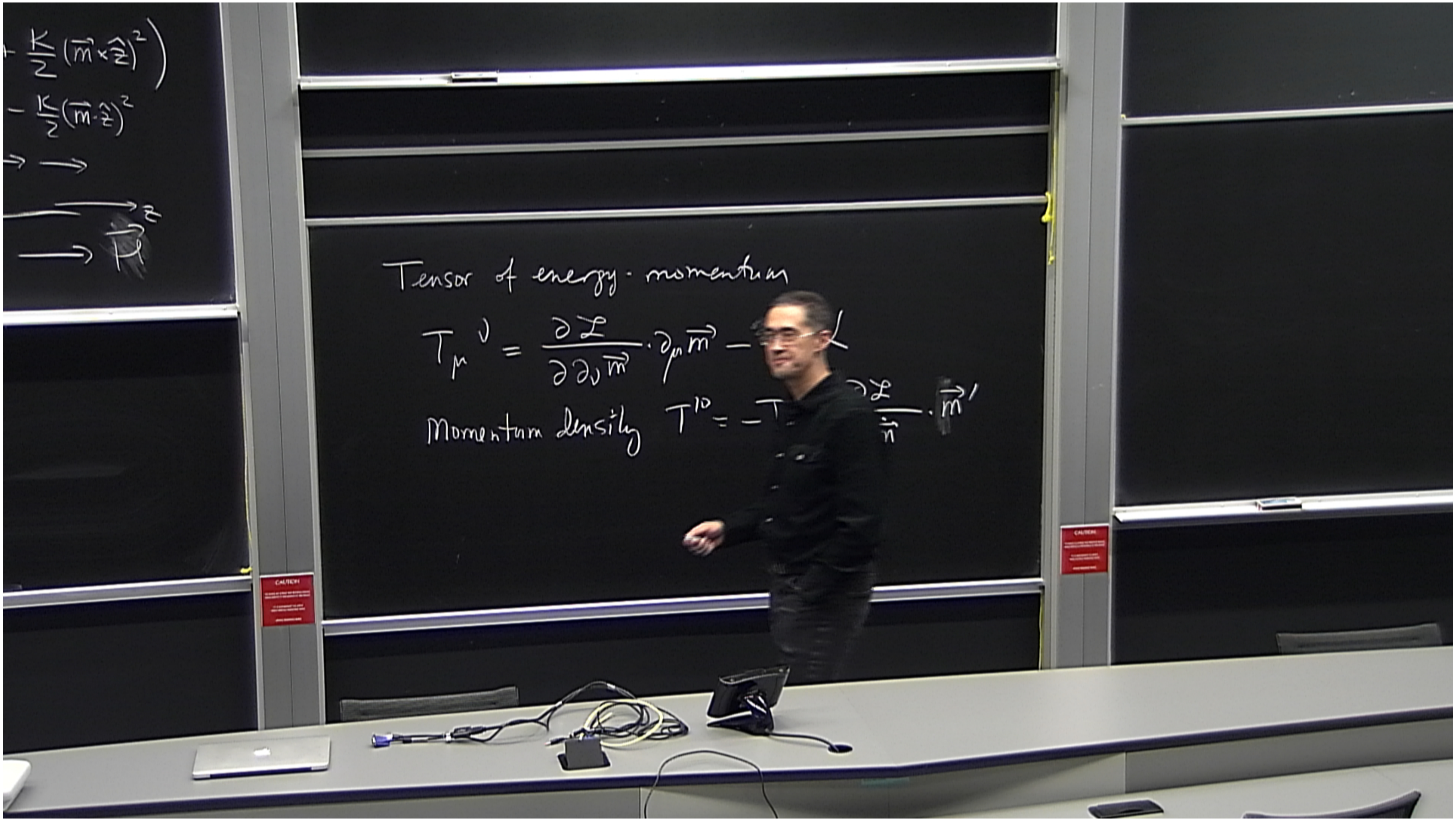

Conserved momenta of a ferromagnetic
soliton
$|\vec{m}| = 1$,

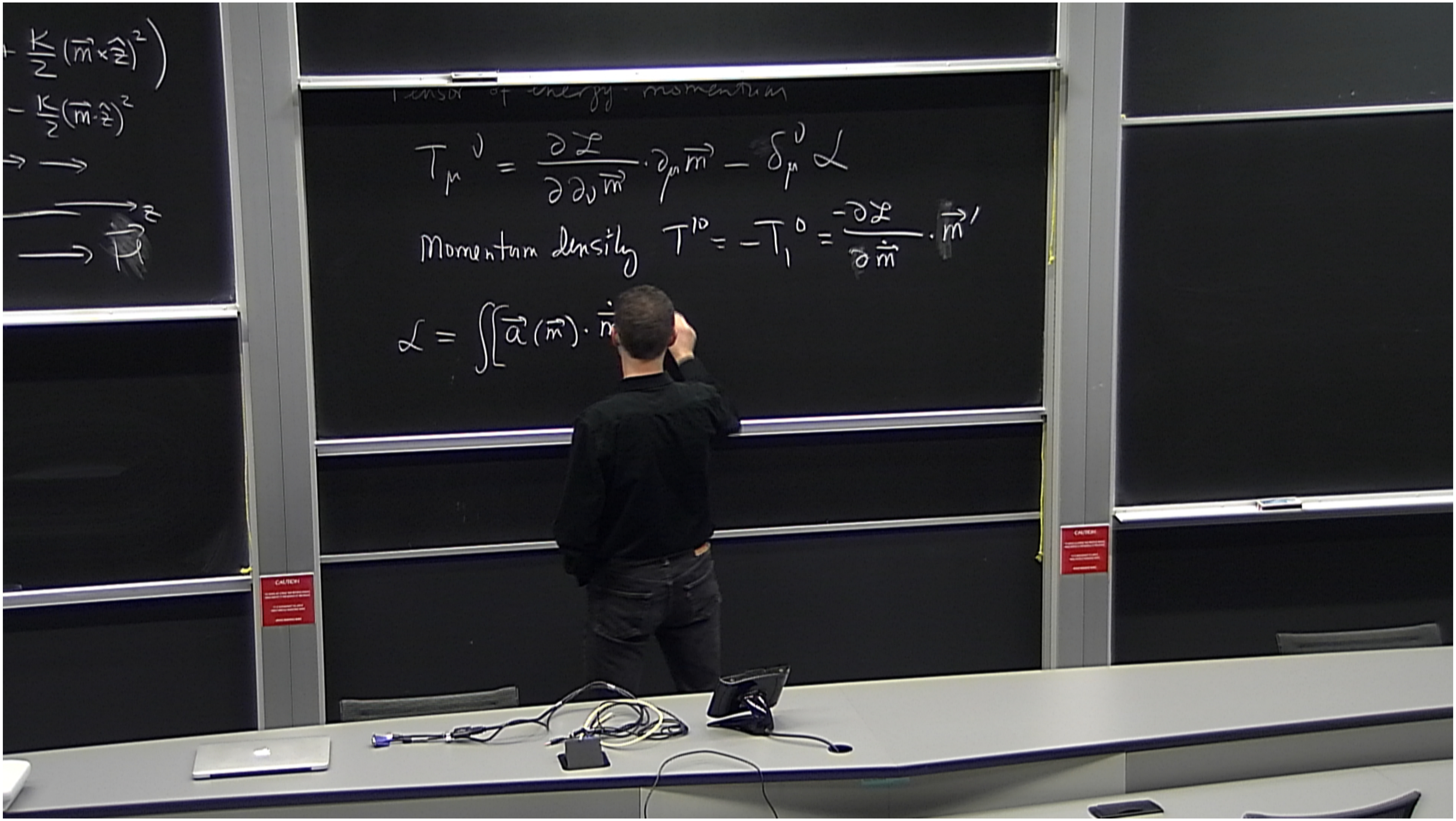

Conserved momenta of a ferromagnetic soliton
(m_x, m_y, m_z)
$|\vec{m}| = 1$
$\vec{m} = \pm\hat{z}$
$U = \int dx \left(\frac{A}{2} \vec{m}' \cdot \vec{m}' + \frac{K}{2} (\vec{m} \times \hat{z})^2 \right)$
$- \frac{K}{2} (\vec{m} \cdot \hat{z})^2$
Φ
Z
z

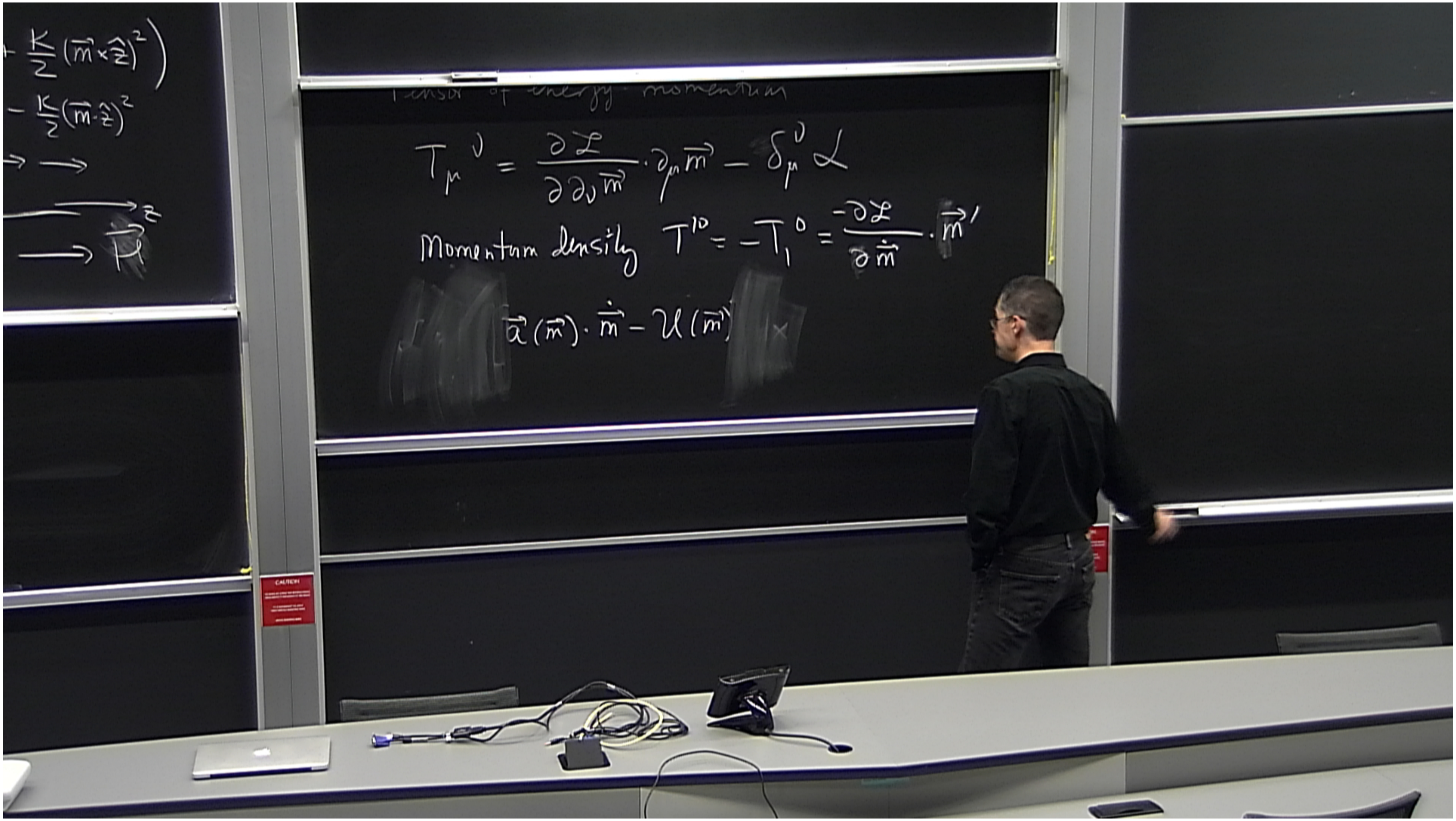

Z
$\vec{m} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$
$\cos\theta = \pm\tanh\frac{z-Z}{\lambda}$, $\lambda = \sqrt{A/K}$
$\phi(z) = \Phi = \text{const}$
CAUTION

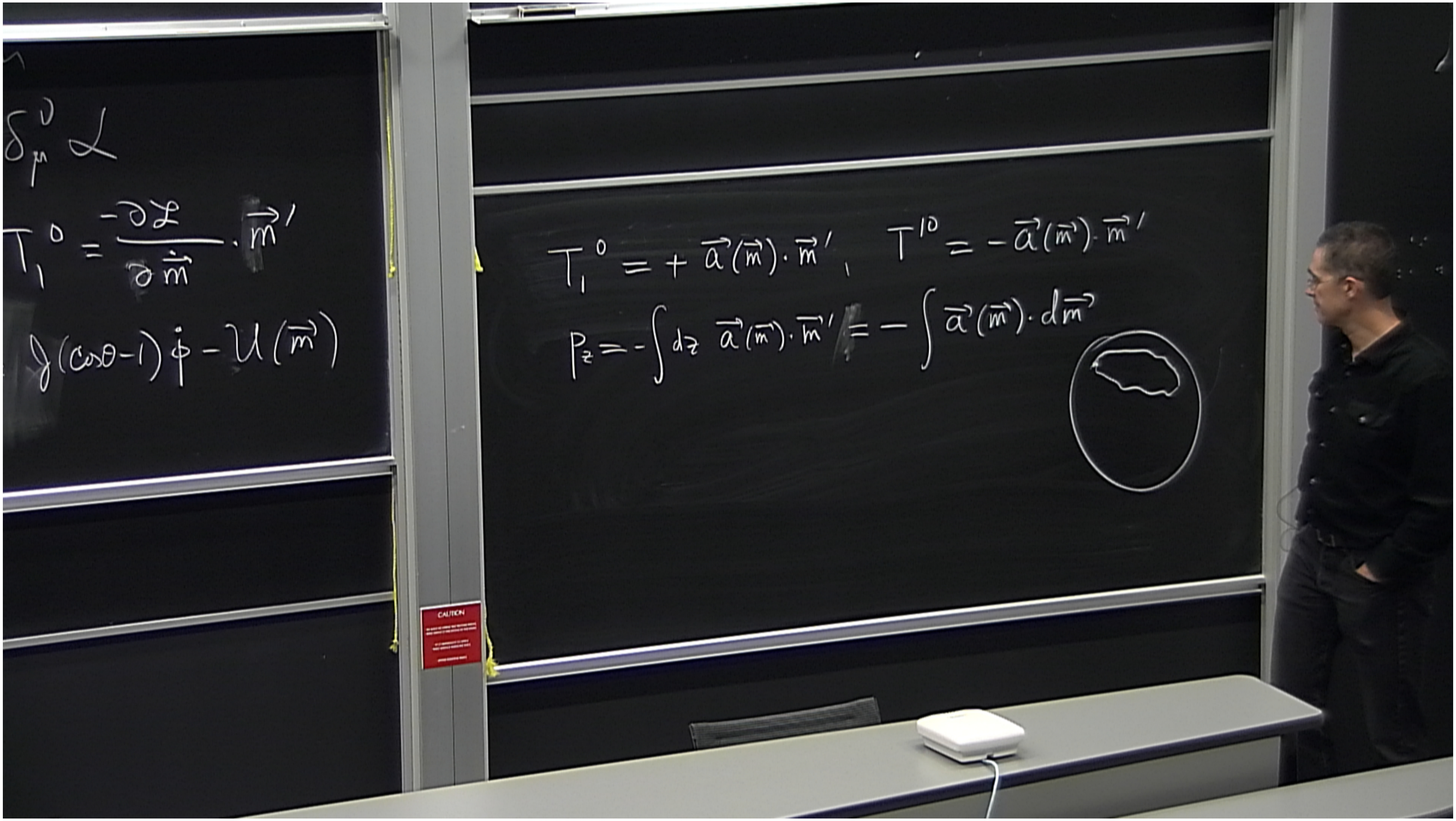

Angular momentum
spin density
J = C Z , C = −2g
τ = dJ/dt = −2g dZ/dt
(Φ, Z) ~ (Φ, Pφ)
CAUTION

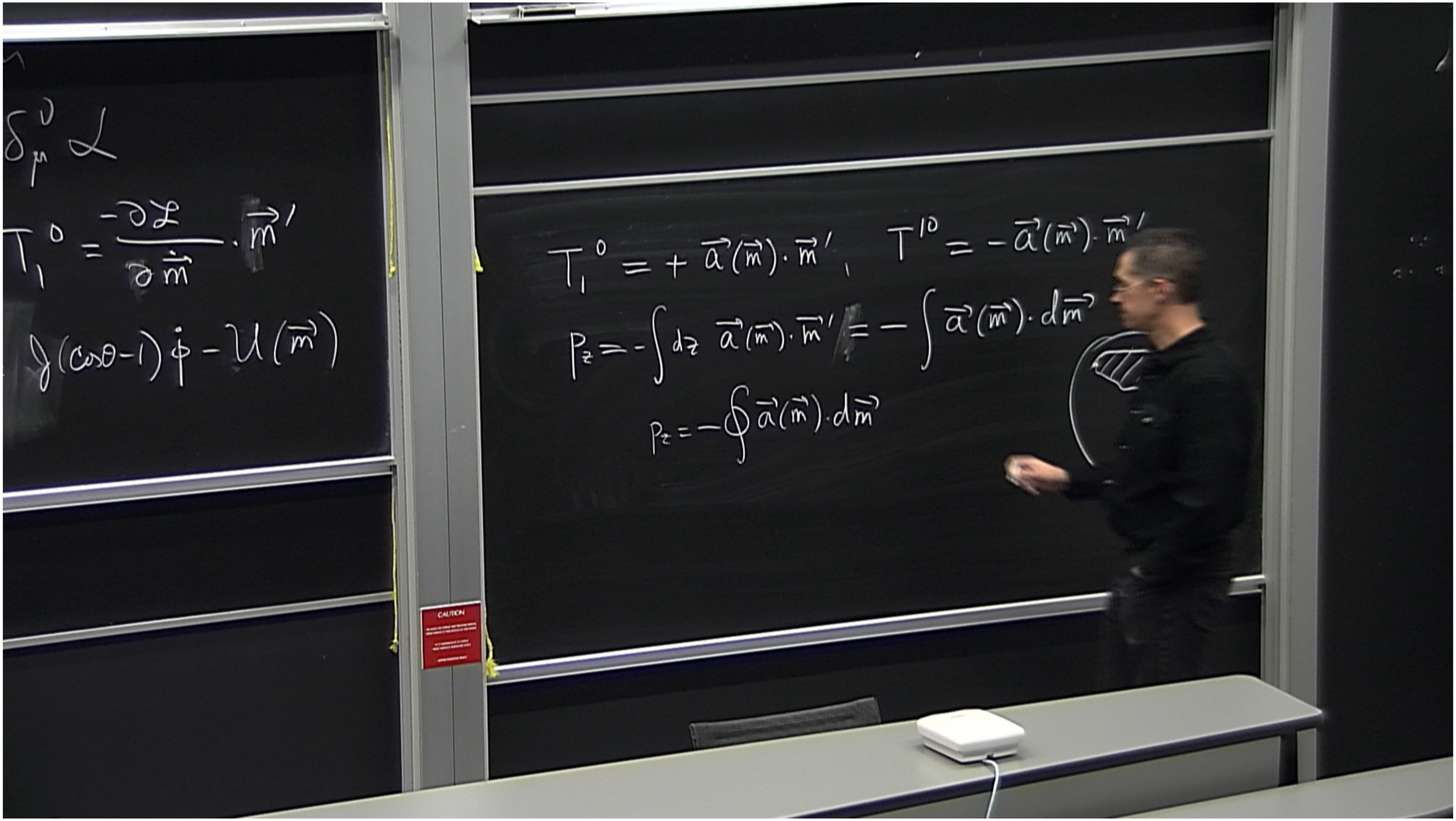

Angular momentum
spin density
J = CZ , C = -2J
τ = dJ/dt = -2J dZ/dt
(Φ, Z) ~ (Φ, P_Φ)
(P_Z, Z)
P_Z = 2JΦ
ΔΦ 2J ΔZ ≥ ħ/2
ΔΦ ΔZ ≥ a/4S
F = dP_Z/dt = 2J dΦ/dt


Tensor of energy-momentum
Momentum density

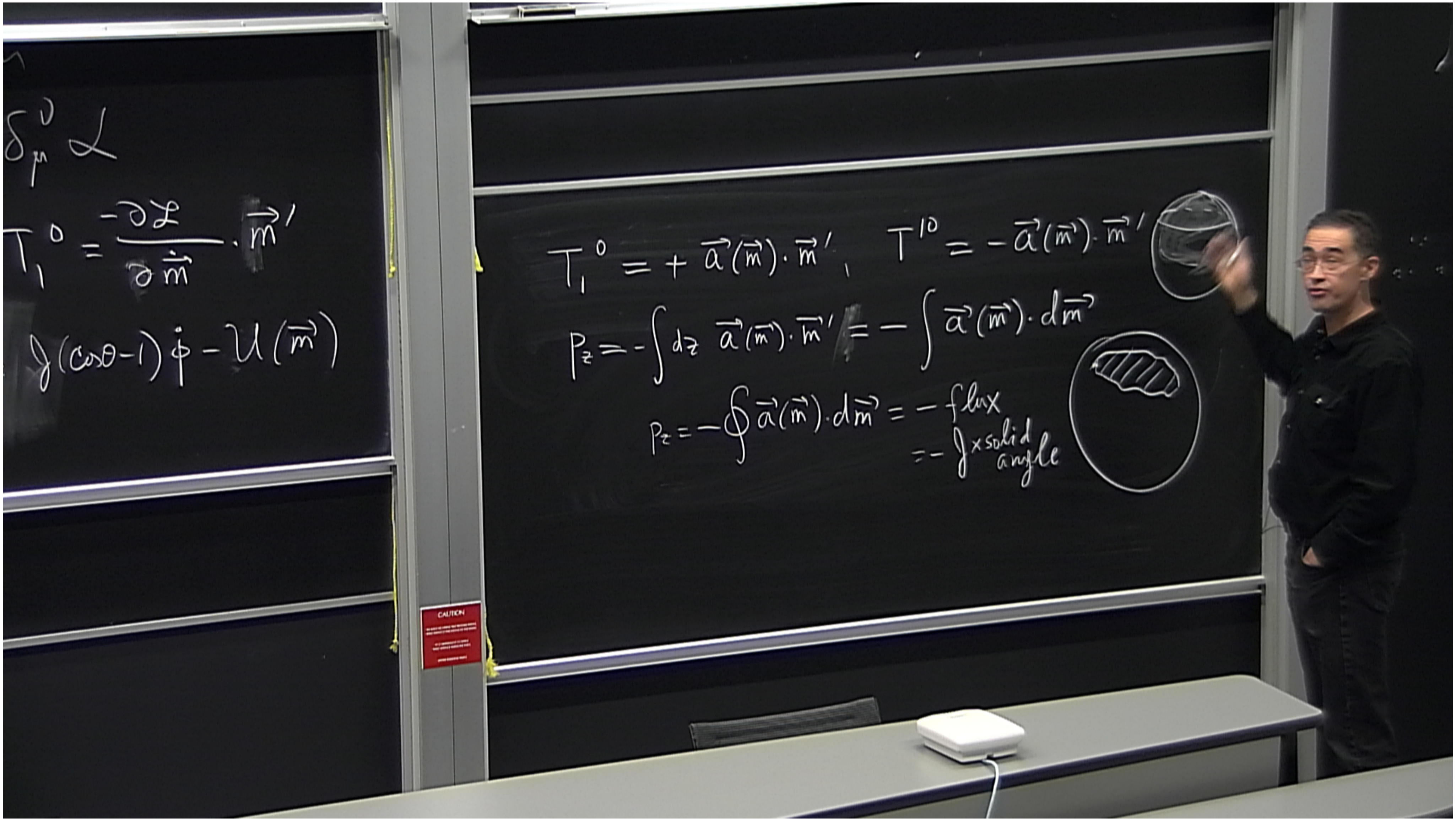

Tensor of energy-momentum
Momentum density
CAUTION
CAUTION


$+\frac{K}{2}(\vec{m}\times\hat{z})^2$
$-\frac{K}{2}(\vec{m}\cdot\hat{z})^2$
Tensor of energy-momentum
$T_\mu{}^\nu = \frac{\partial \mathcal{L}}{\partial \partial_\nu \vec{m}} \cdot \partial_\mu \vec{m} - \delta_\mu^\nu \mathcal{L}$
Momentum density $T^{10} = -T_1{}^0 = \frac{-\partial \mathcal{L}}{\partial \dot{\vec{m}}} \cdot \vec{m}'$
$\vec{a}(\vec{m}) \cdot \dot{\vec{m}} - \mathcal{U}(\vec{m})$

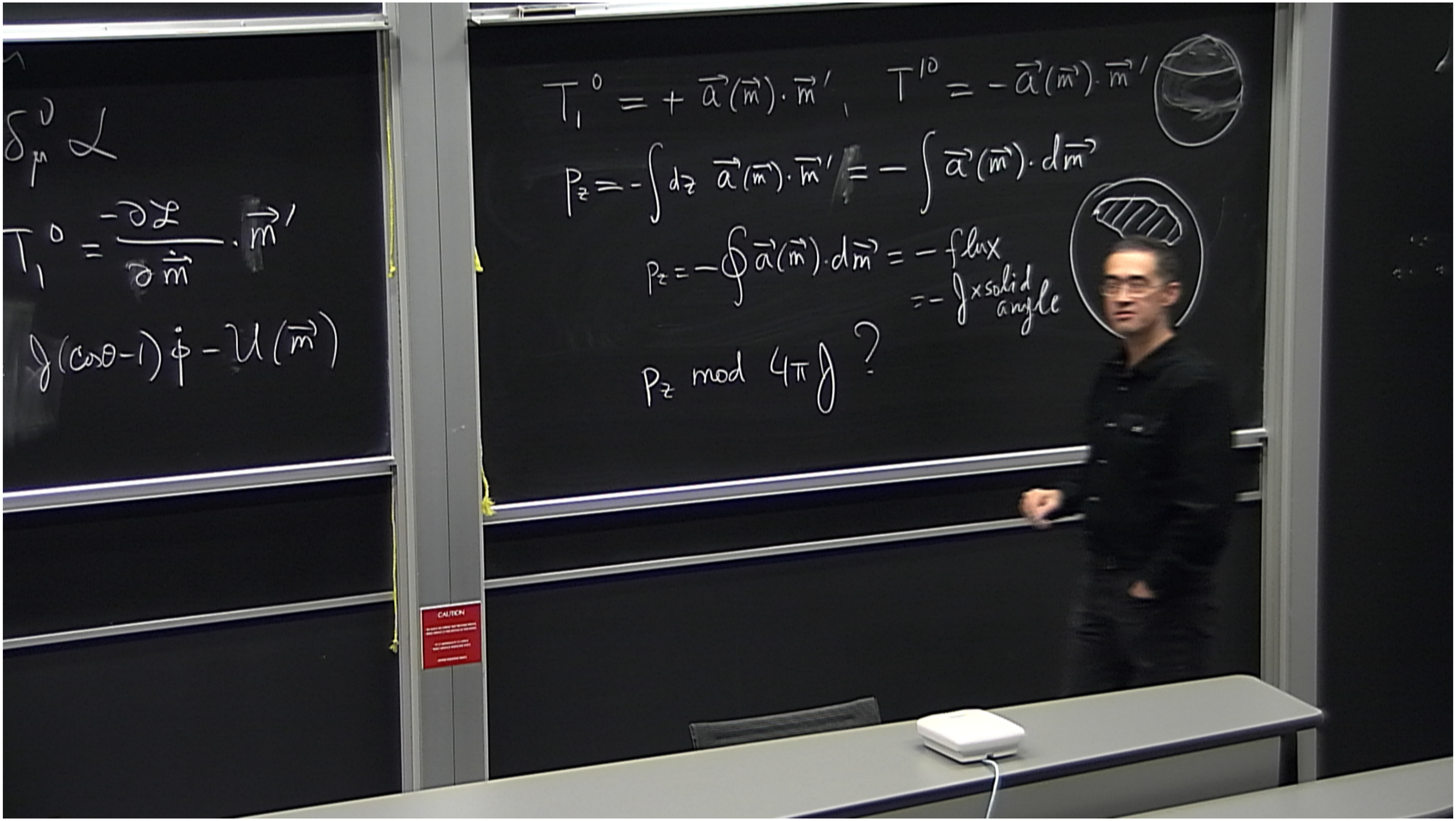

δ_μ^ν L
T_1^0 = -∂L/∂m · m'
(cosθ-1) φ - U(m)
T_1^0 = + a(m)·m', T^10 = - a(m)·m'
P_z = -∫dz a(m)·m' = -∫ a(m)·dm



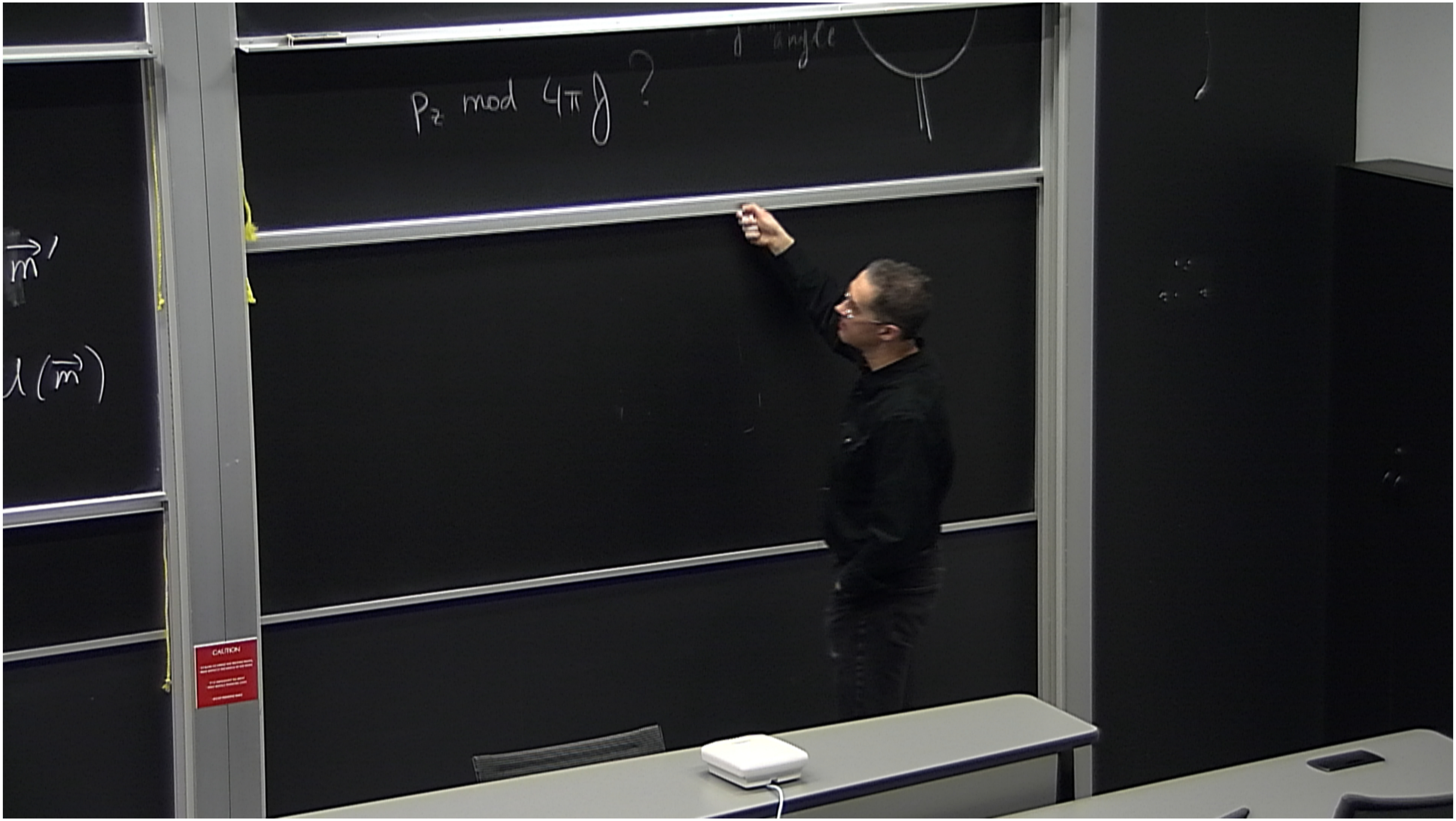
$\delta_\mu^\nu \mathcal{L}$
$T_1^{\ 0} = \frac{-\partial \mathcal{L}}{\partial \dot{\vec{m}}} \cdot \vec{m}'$
$\mathcal{J}(\cos\theta - 1)\dot{\phi} - \mathcal{U}(\vec{m})$
$T_1^{\ 0} = + \vec{a}(\vec{m}) \cdot \vec{m}' , \quad T^{10} = - \vec{a}(\vec{m}) \cdot \vec{m}'$
$p_z = -\int dz \ \vec{a}(\vec{m}) \cdot \vec{m}' = -\int \vec{a}(\vec{m}) \cdot d\vec{m}$
$p_z = -\oint \vec{a}(\vec{m}) \cdot d\vec{m}$
CAUTION

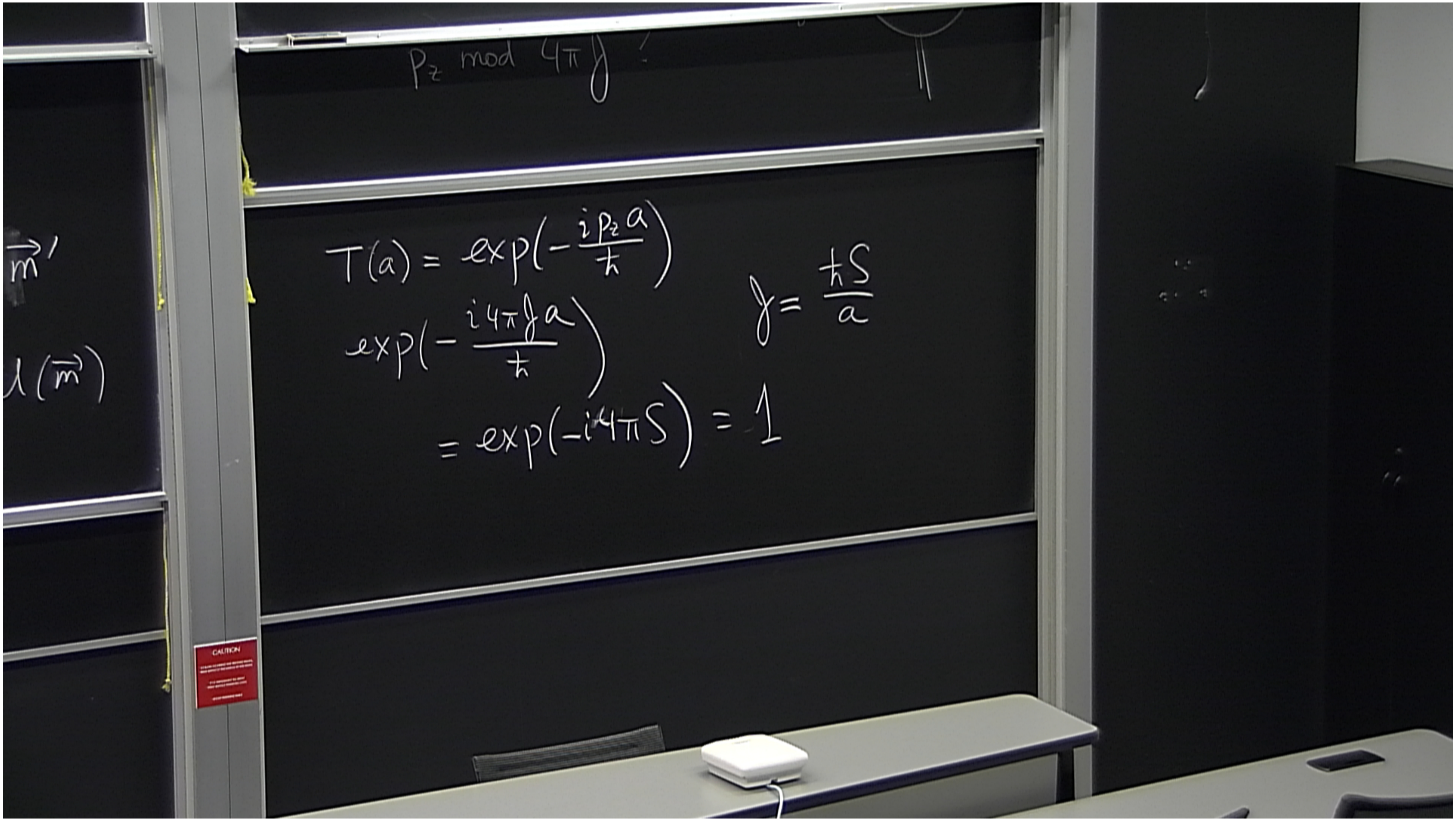


T1 0 = -∂L/∂ṁ · m'
J(cosθ-1)φ̇ - U(m)
T1 0 = + a(m)·m' , T 10 = - a(m)·m'
Pz = -∫dz a(m)·m' = -∫a(m)·dm
Pz = -∮a(m)·dm = - flux
= - J × solid angle
CAUTION

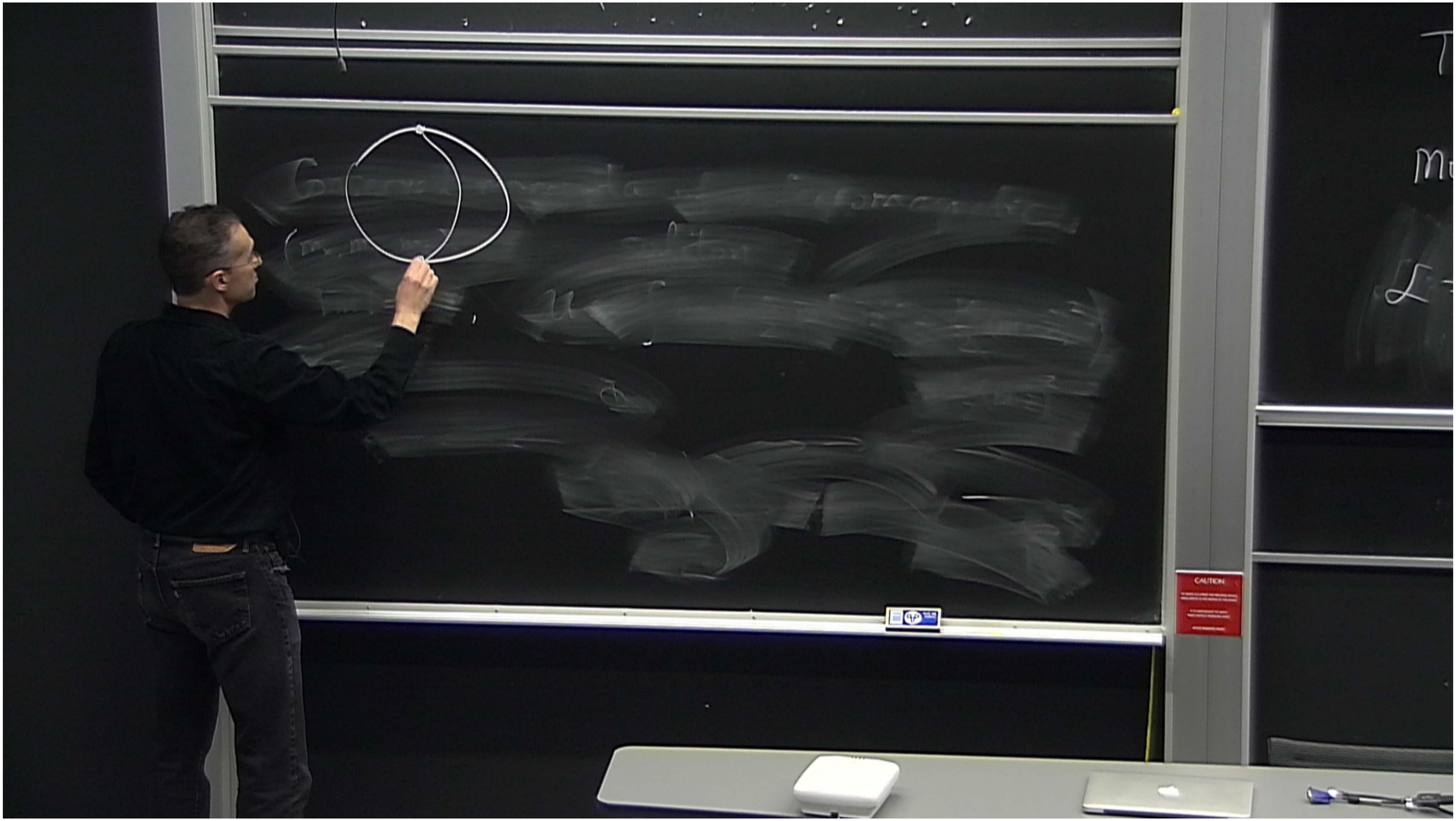


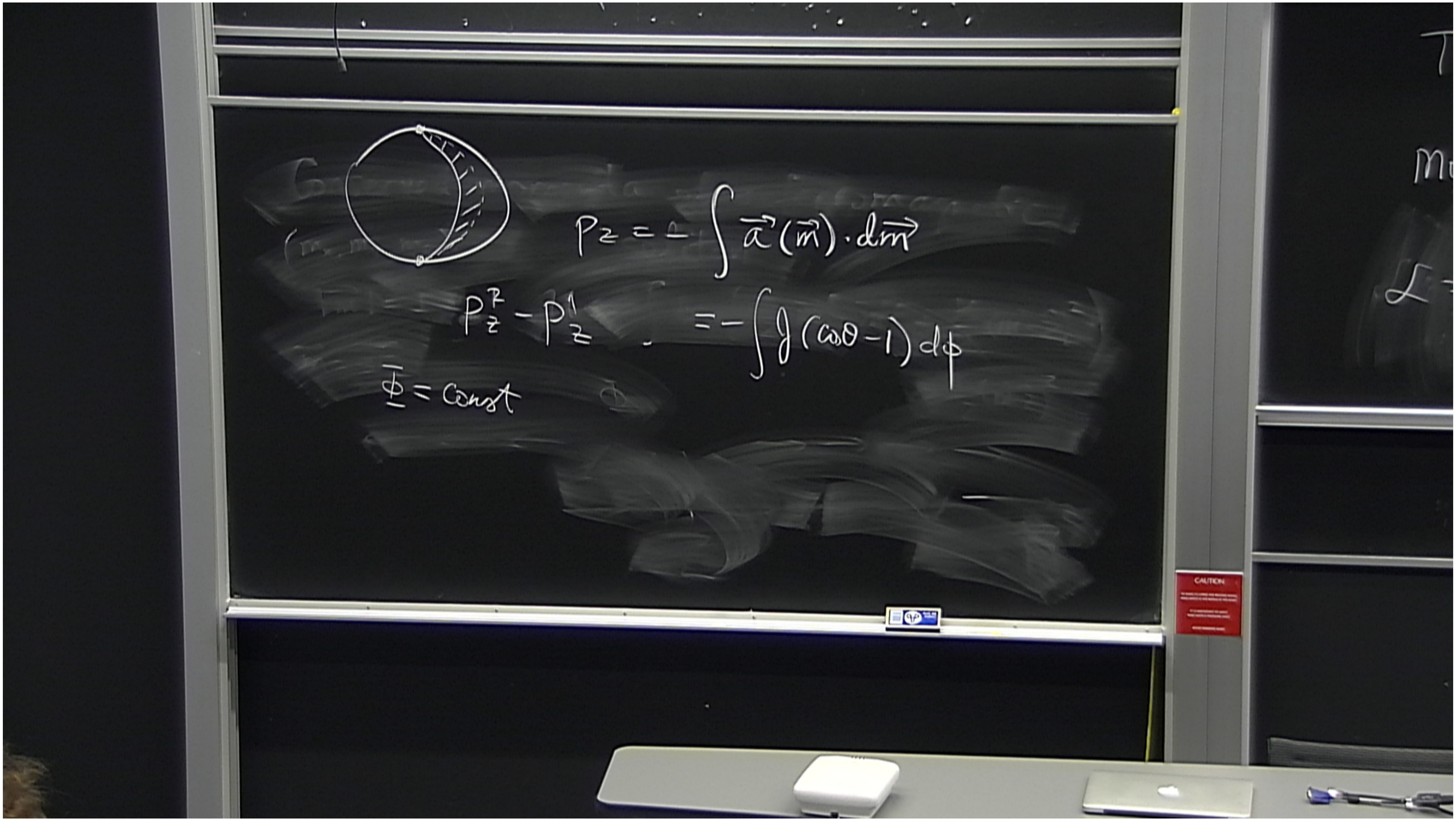
$\delta_\mu^\nu \mathcal{L}$
$T_1^{\,0} = \frac{-\partial \mathcal{L}}{\partial \dot{\vec{m}}} \cdot \vec{m}'$
$J(\cos\theta - 1)\dot{\phi} - U(\vec{m})$
$T_1^{\,0} = + \vec{a}(\vec{m}) \cdot \vec{m}'$, $T^{10} = - \vec{a}(\vec{m}) \cdot \vec{m}'$
$P_z = -\int dz\, \vec{a}(\vec{m}) \cdot \vec{m}' = -\int \vec{a}(\vec{m}) \cdot d\vec{m}$
$P_z = -\oint \vec{a}(\vec{m}) \cdot d\vec{m} = -$ flux
$= -J \times$ solid angle
$P_z \bmod 4\pi J$?
CAUTION

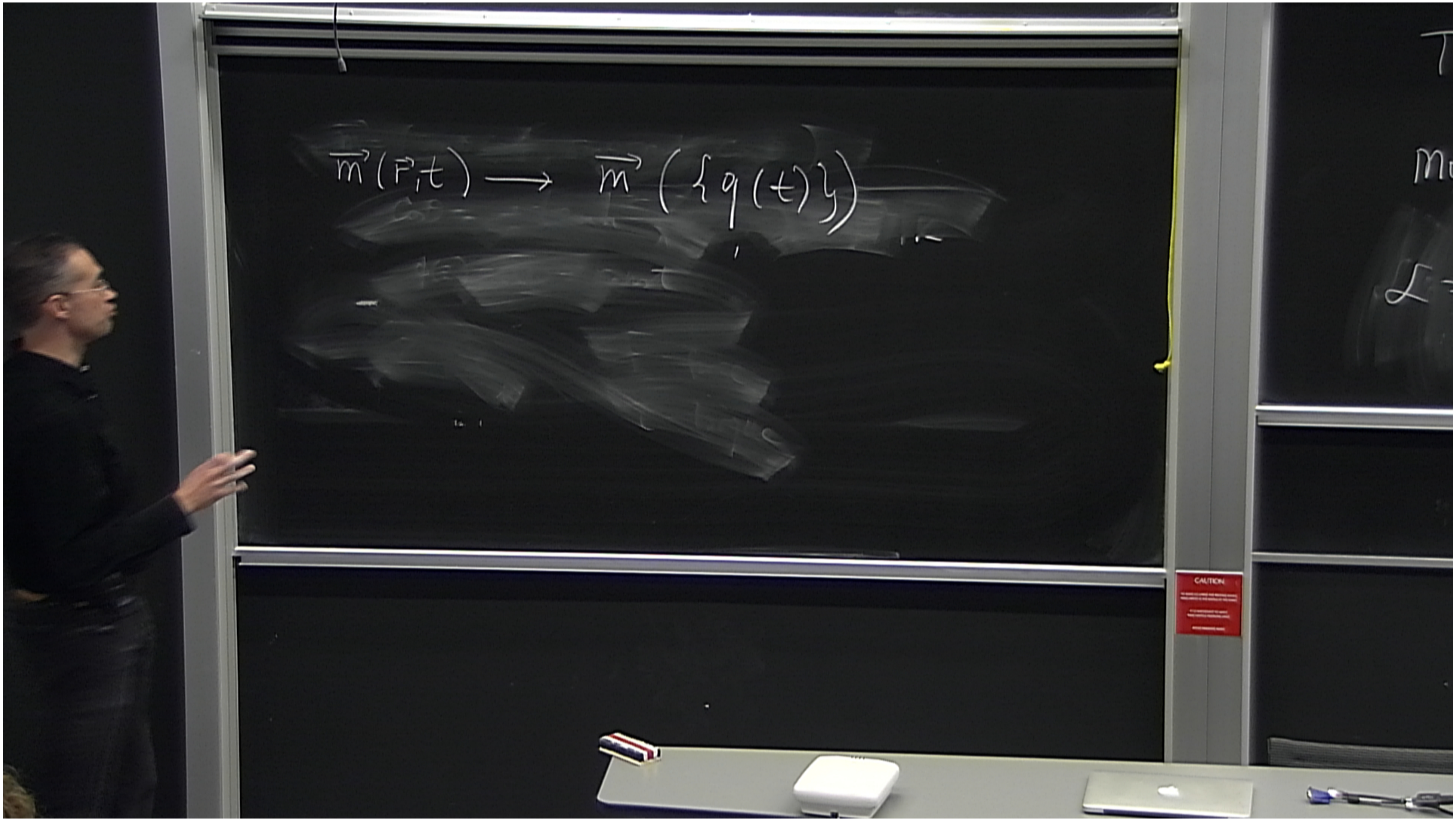

p_z mod $4\pi j$?
angle
CAUTION

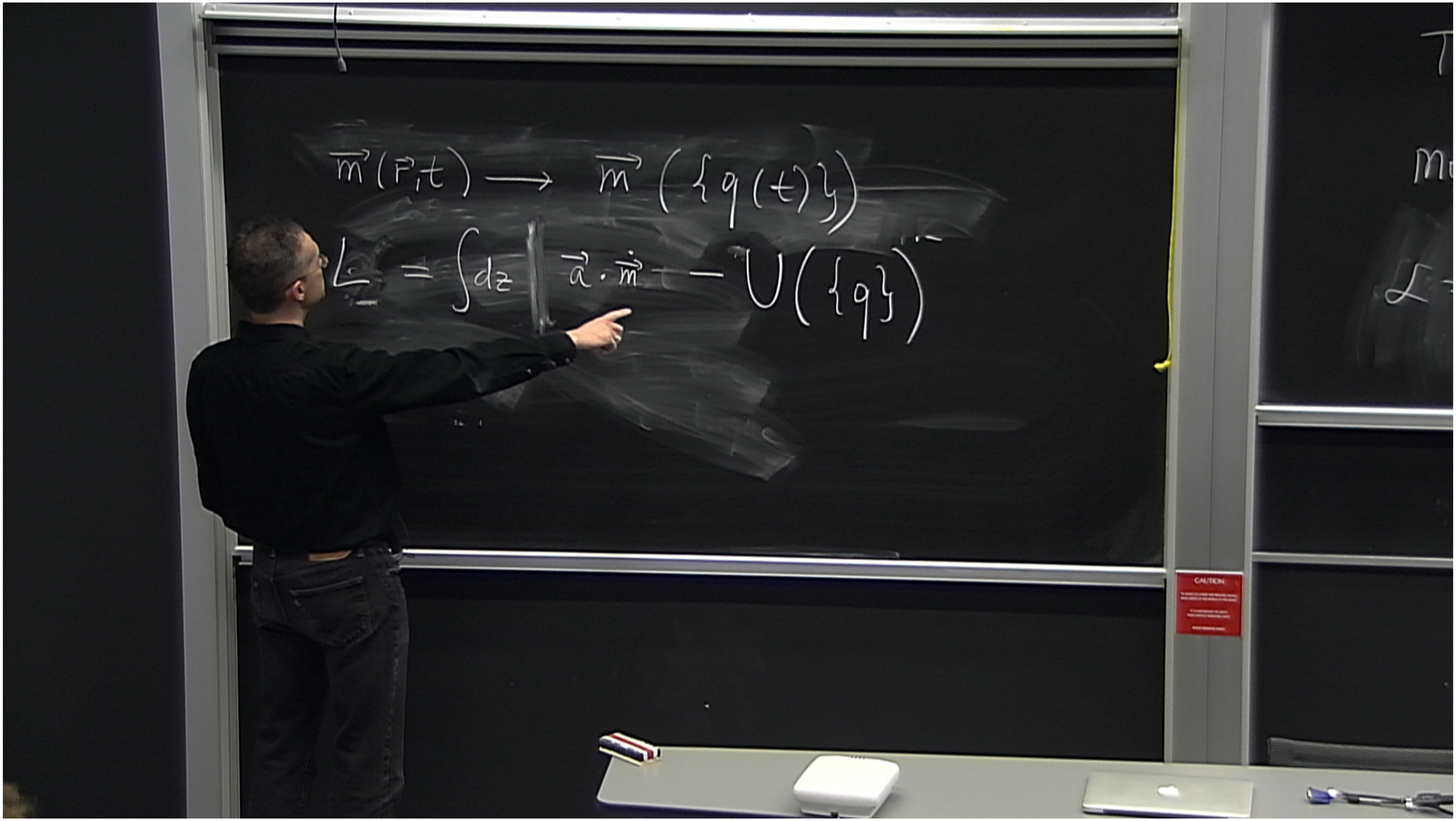


$p_z \bmod 4\pi J$?
$T(a) = \exp\left(-\frac{i p_z a}{\hbar}\right)$
$\exp\left(-\frac{i 4\pi J a}{\hbar}\right)$
$= \exp(-i4\pi S) = 1$
$J = \frac{\hbar S}{a}$

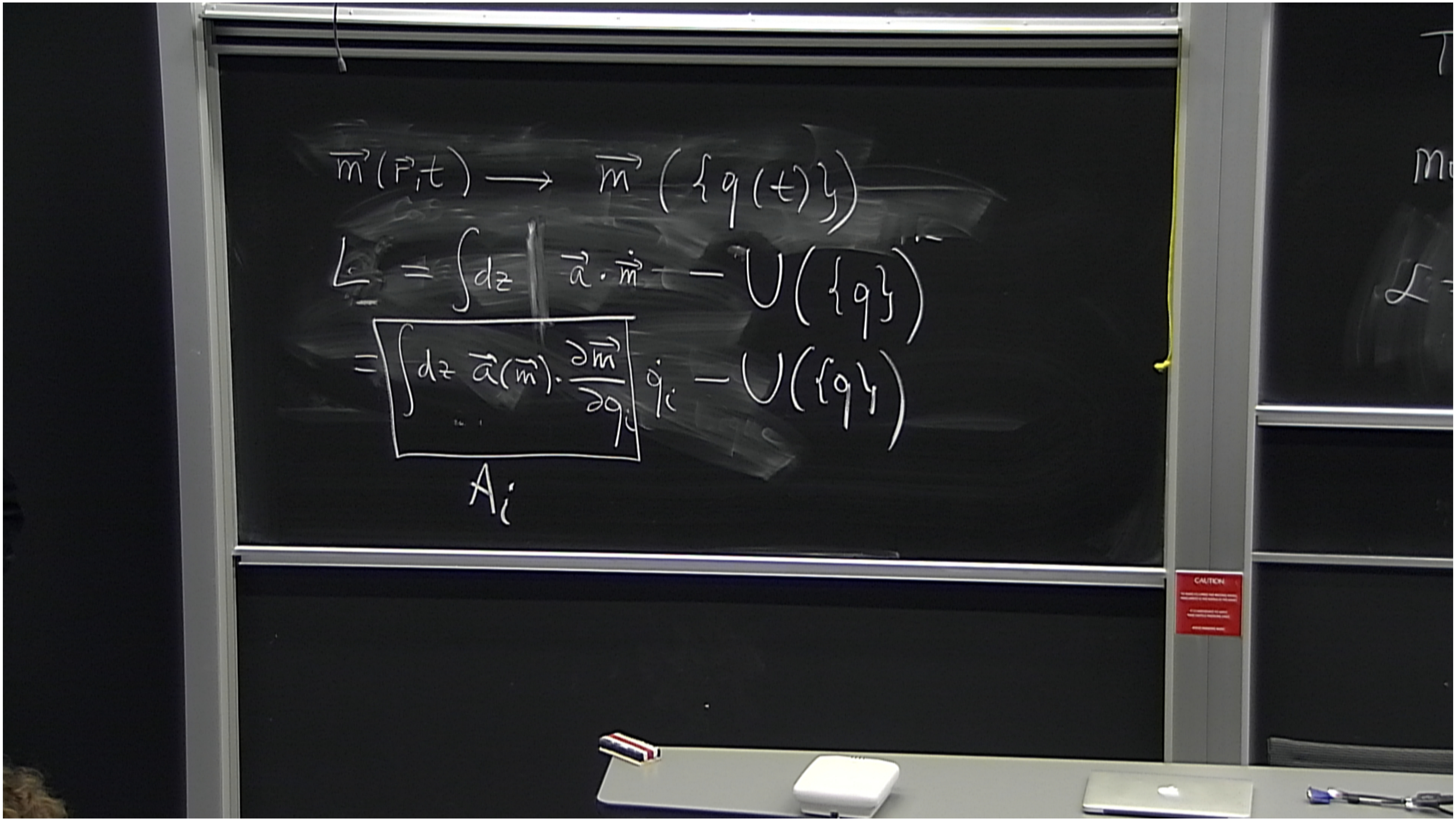


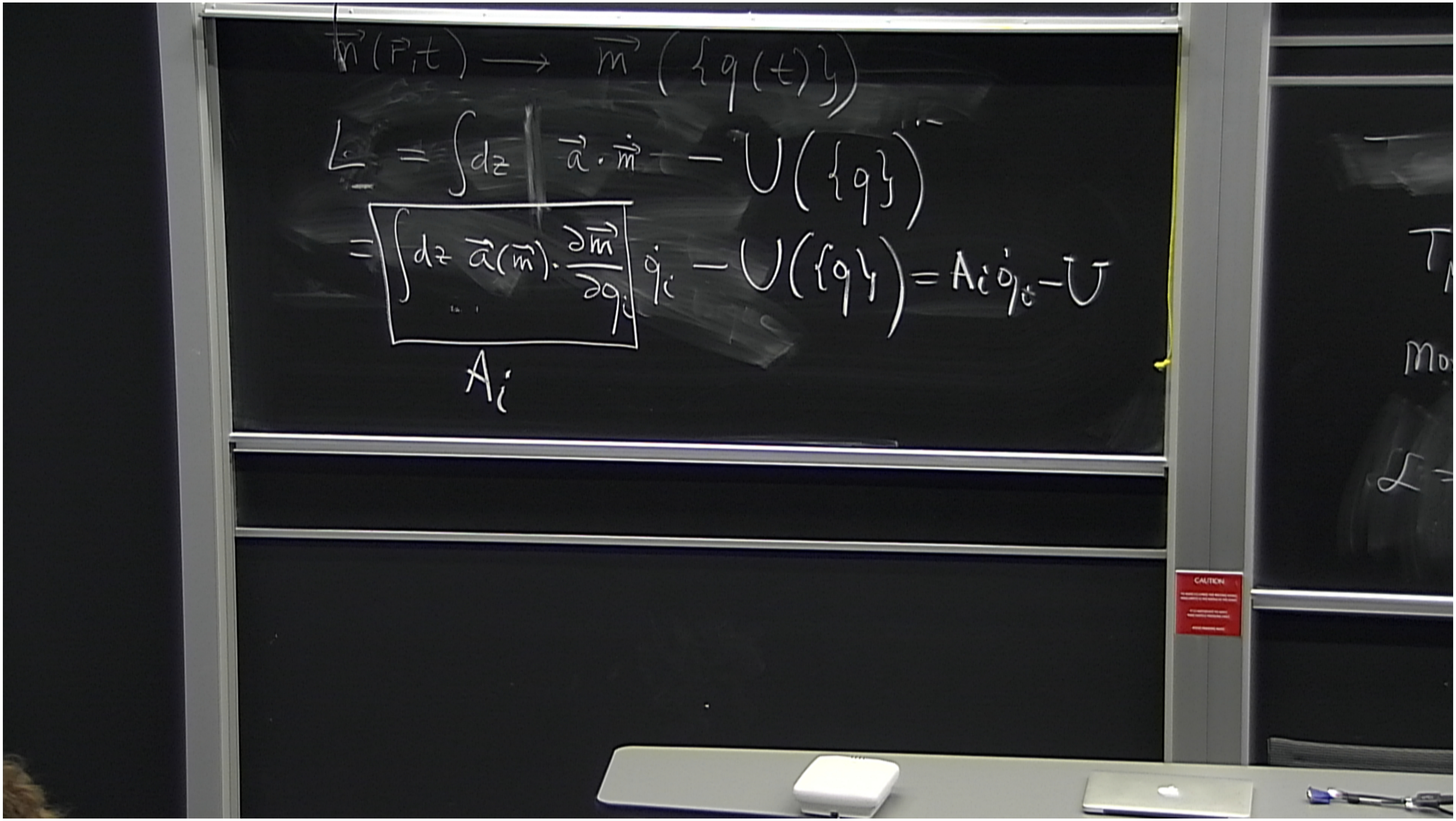

Pz = − ∫ a⃗(m⃗) · dm⃗
P^2_z − P^1_z = − ∫ g (cosθ − 1) dφ
Φ = Const

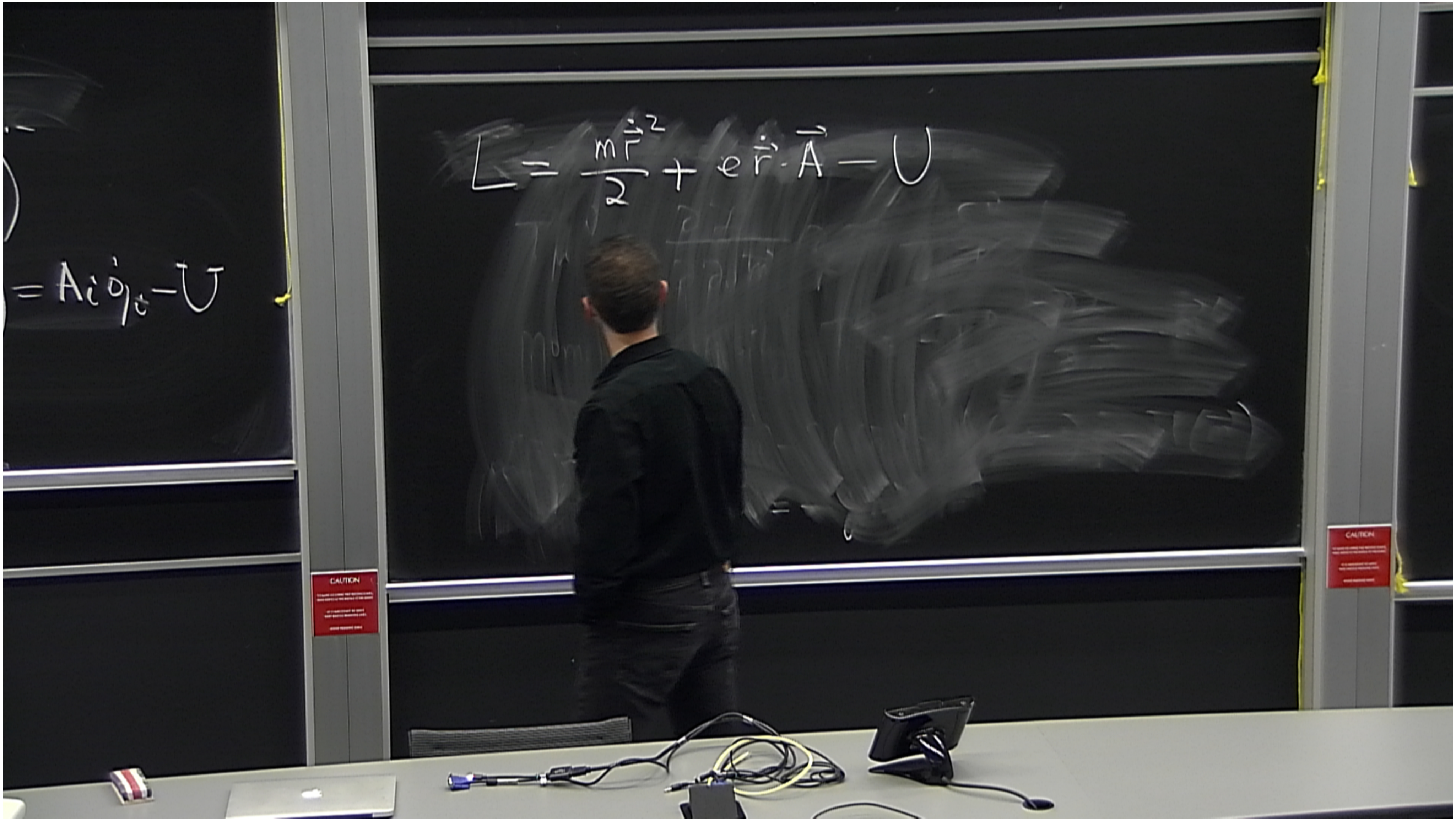

$\vec{m}(\vec{r},t) \longrightarrow \vec{m}(\{q(t)\})$

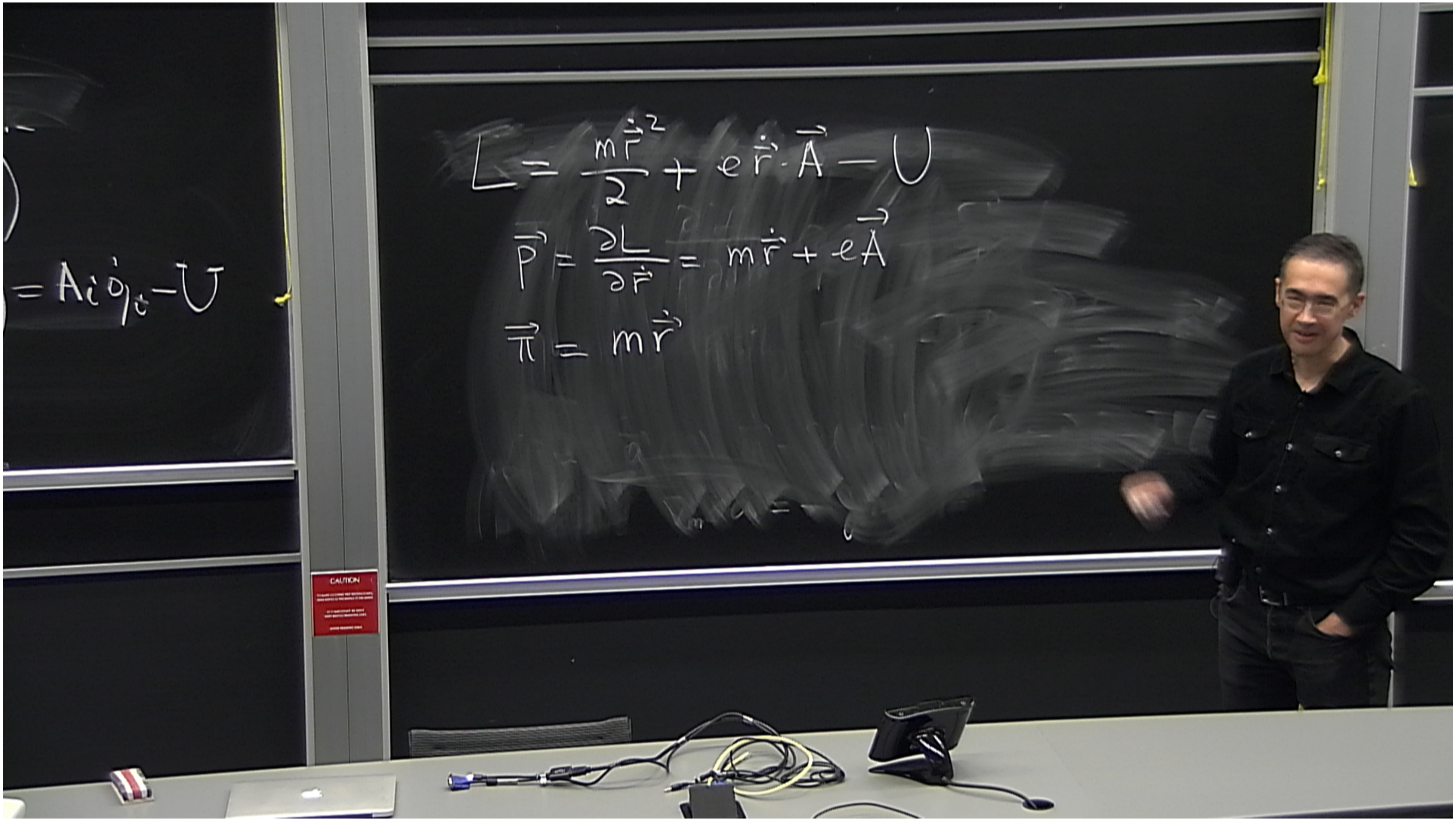


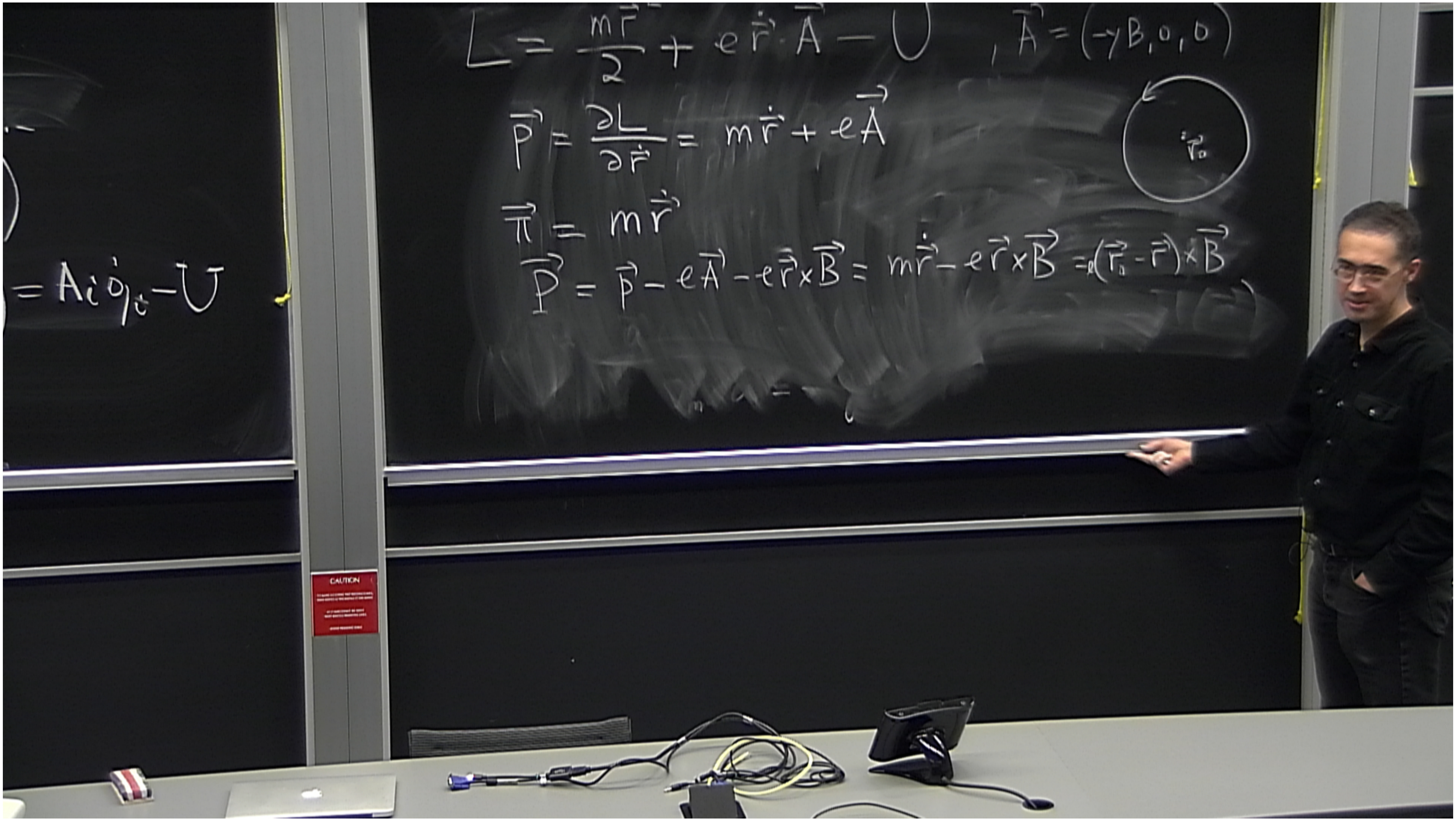
$\vec{m}(\vec{r},t) \longrightarrow \vec{m}(\{q(t)\})$
$L = \int dz \; \vec{a} \cdot \dot{\vec{m}} - U(\{q\})$
CAUTION

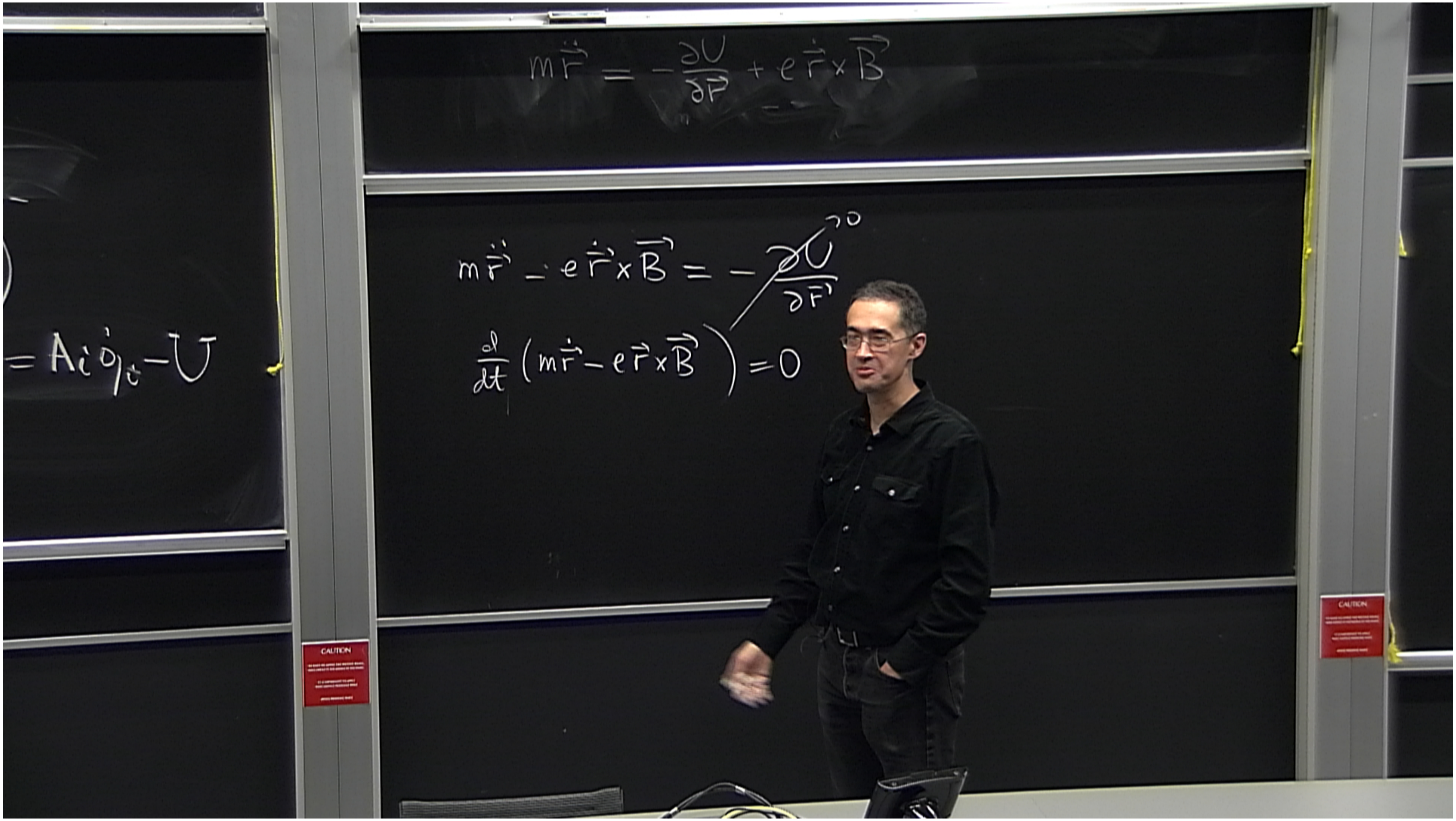

m⃗(r⃗, t) ⟶ m⃗({q(t)})
L = ∫dz a⃗ · m⃗̇ − U({q})
= ∫dz a⃗(m⃗) · ∂m⃗/∂q_i q̇_i − U({q})
A_i

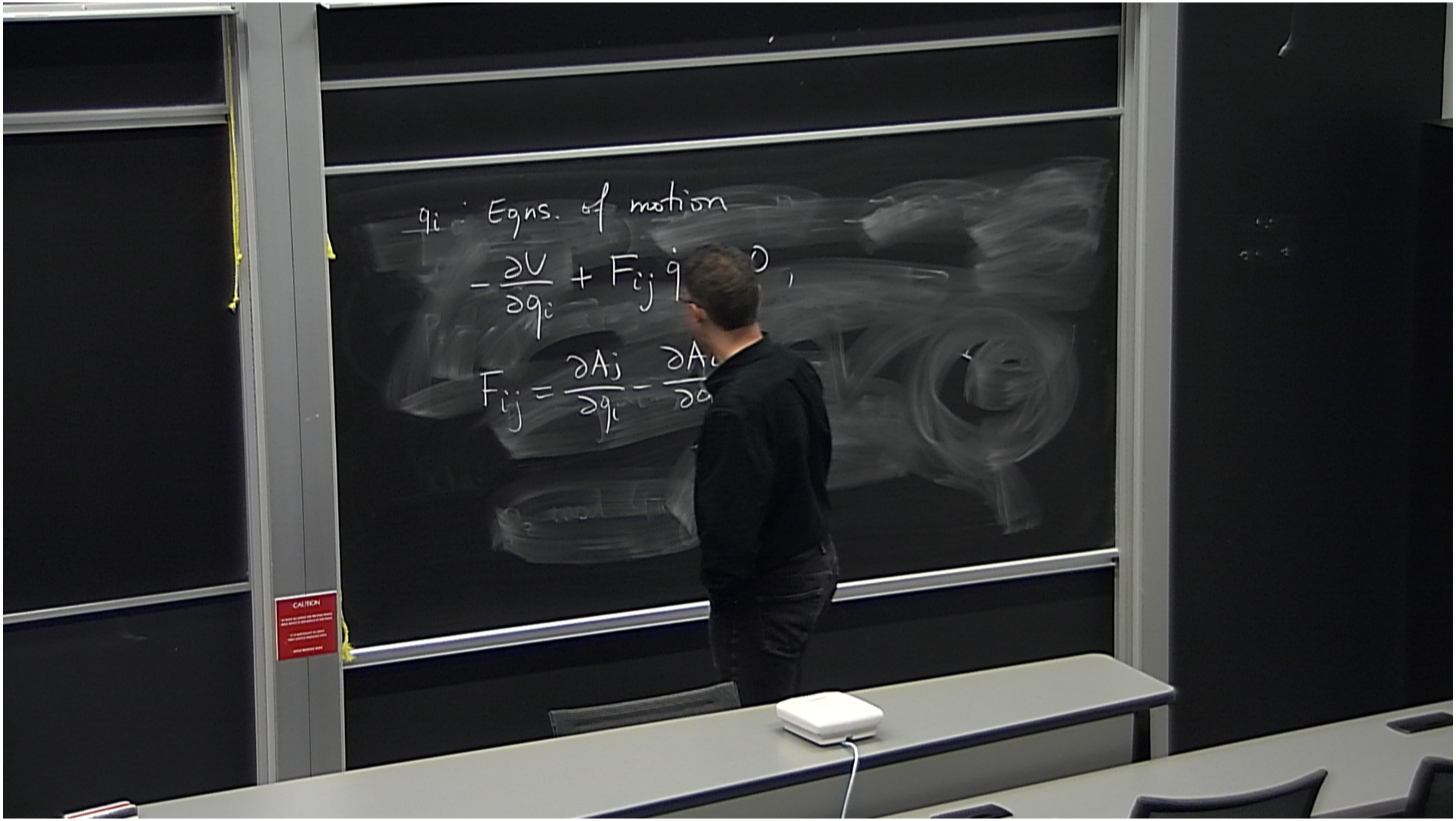

$\vec{m}(\vec{r},t) \longrightarrow \vec{m}(\{q(t)\})$
$L = \int dz \; \vec{a}\cdot\dot{\vec{m}} - U(\{q\})$
$= \int dz \; \vec{a}(\vec{m})\cdot\frac{\partial \vec{m}}{\partial q_i}\dot{q}_i - U(\{q\}) = A_i \dot{q}_i - U$
A_i
CAUTION

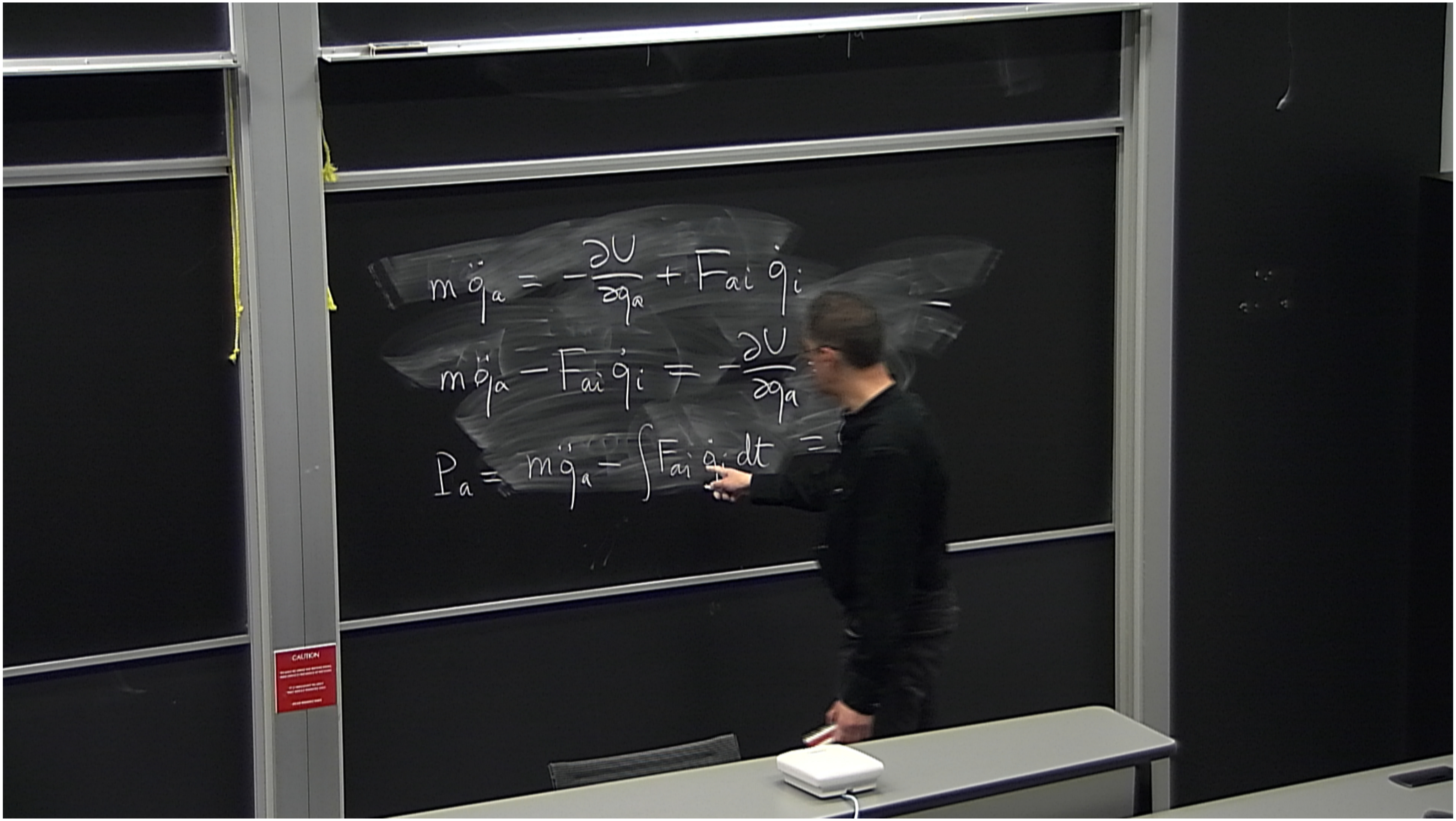

$L = \frac{m\dot{\vec{r}}^2}{2} + e\dot{\vec{r}}\cdot\vec{A} - U$

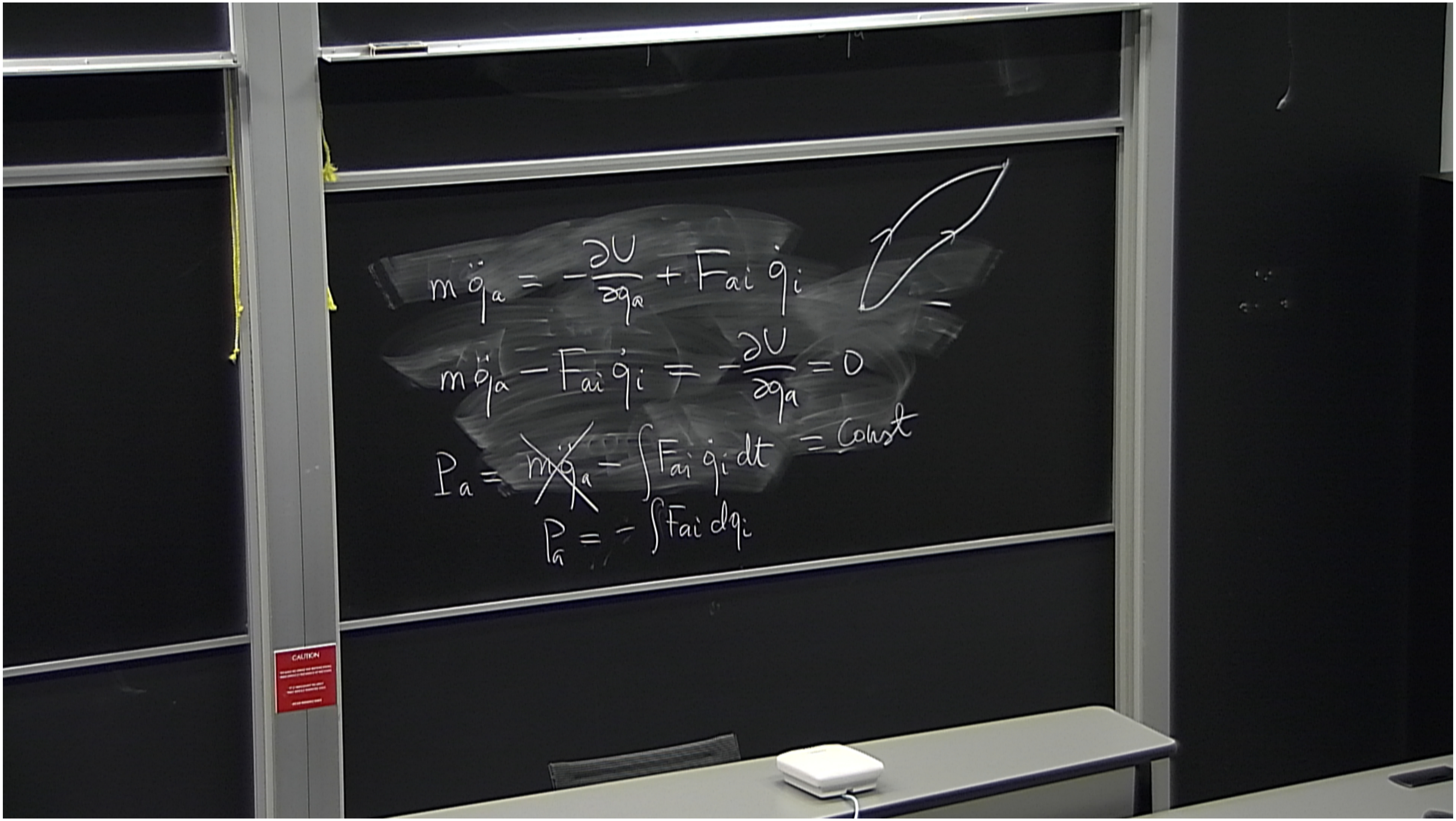

$L = \frac{m\dot{\vec{r}}^2}{2} + e\dot{\vec{r}}\cdot\vec{A} - U$
$\vec{p} = \frac{\partial L}{\partial \dot{\vec{r}}} = m\dot{\vec{r}} + e\vec{A}$
$\vec{\pi} = m\dot{\vec{r}}$
CAUTION

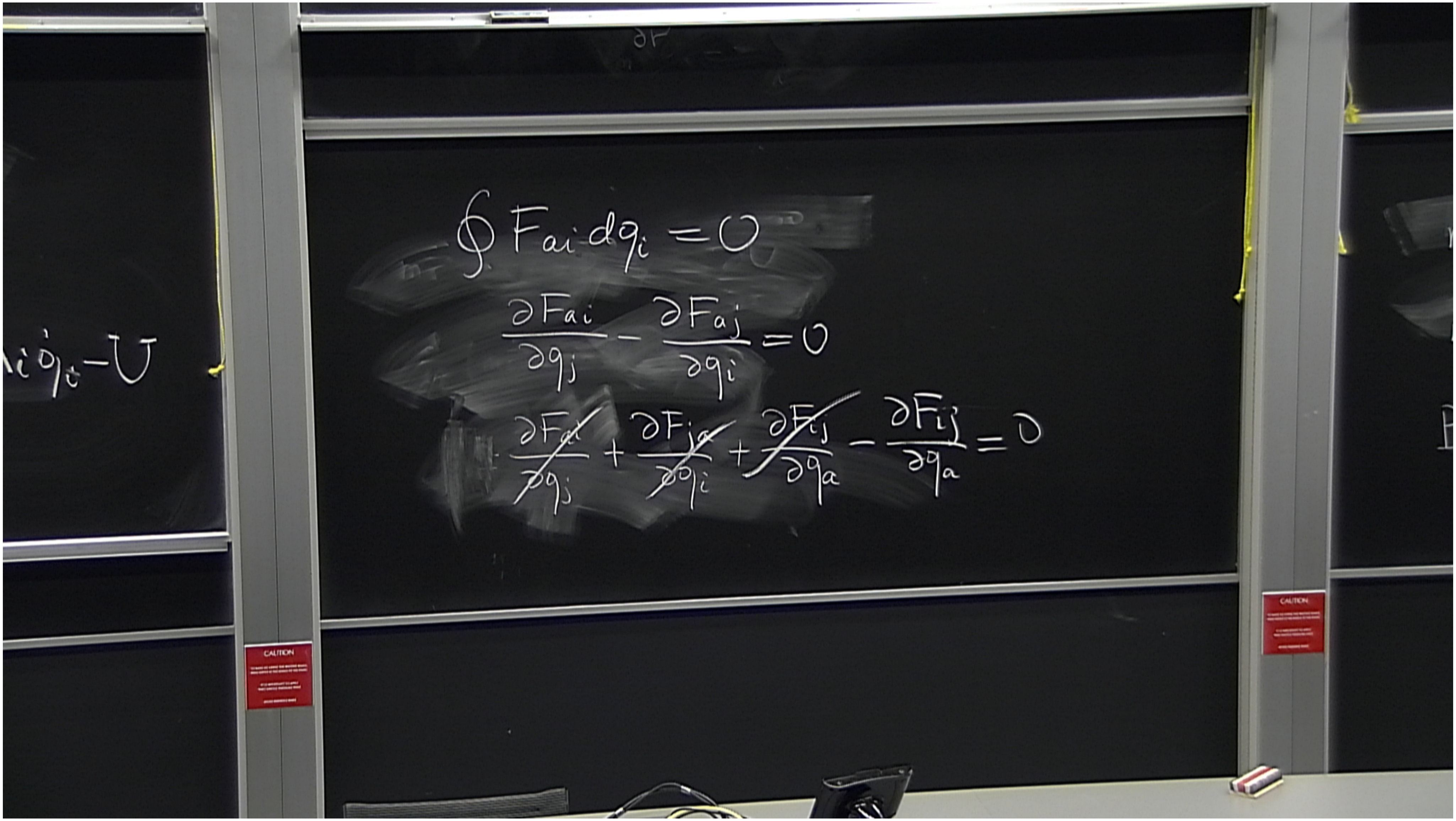

$L = \frac{m\dot{\vec{r}}^2}{2} + e\dot{\vec{r}}\cdot\vec{A} - U$, $\vec{A} = (-yB, 0, 0)$
$\vec{p} = \frac{\partial L}{\partial \dot{\vec{r}}} = m\dot{\vec{r}} + e\vec{A}$
$\vec{\pi} = m\dot{\vec{r}}$
$\vec{P} = \vec{p} - e\vec{A} - e\vec{r}\times\vec{B} = m\dot{\vec{r}} - e\vec{r}\times\vec{B} = e(\vec{r}_0 - \vec{r})\times\vec{B}$
$\vec{r}_0$
$= A_i \dot{q}_i - U$

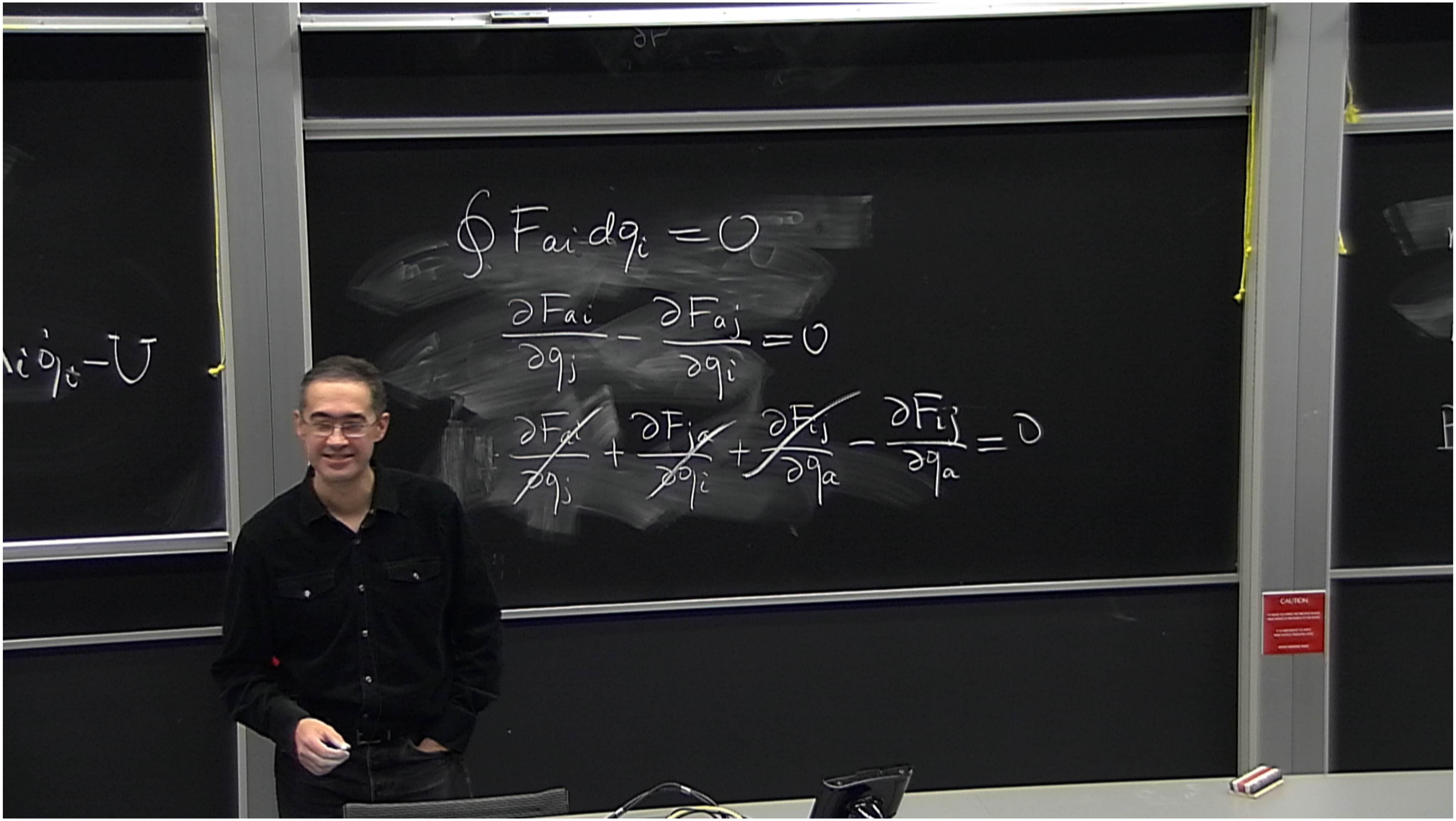

$m\ddot{\vec{r}} = -\frac{\partial U}{\partial \vec{r}} + e\dot{\vec{r}}\times\vec{B}$
$m\ddot{\vec{r}} - e\dot{\vec{r}}\times\vec{B} = -\frac{\partial U}{\partial \vec{r}}$
$\frac{d}{dt}\left(m\dot{\vec{r}} - e\vec{r}\times\vec{B}\right) = 0$
CAUTION

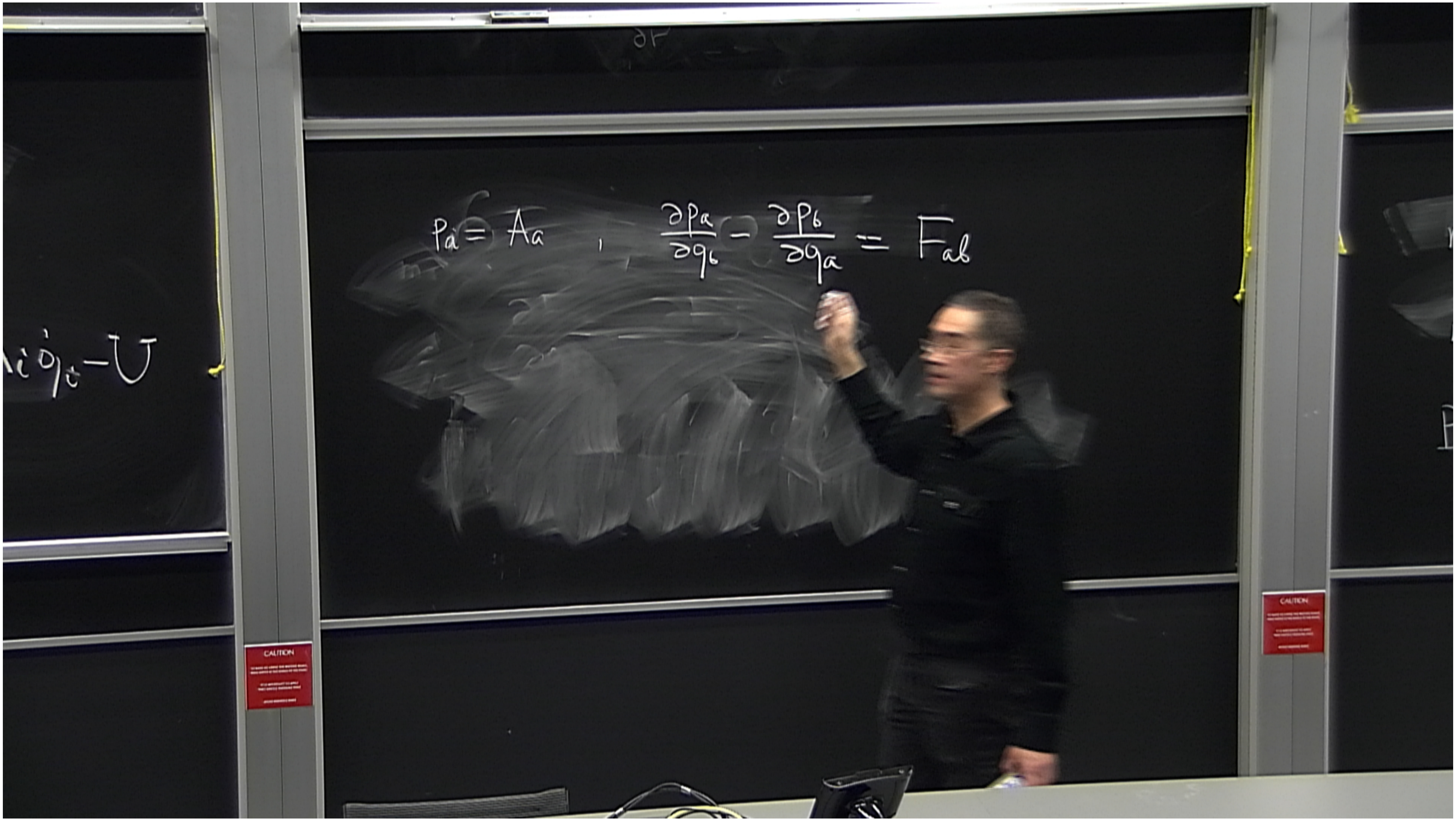

qi : Eqns. of motion

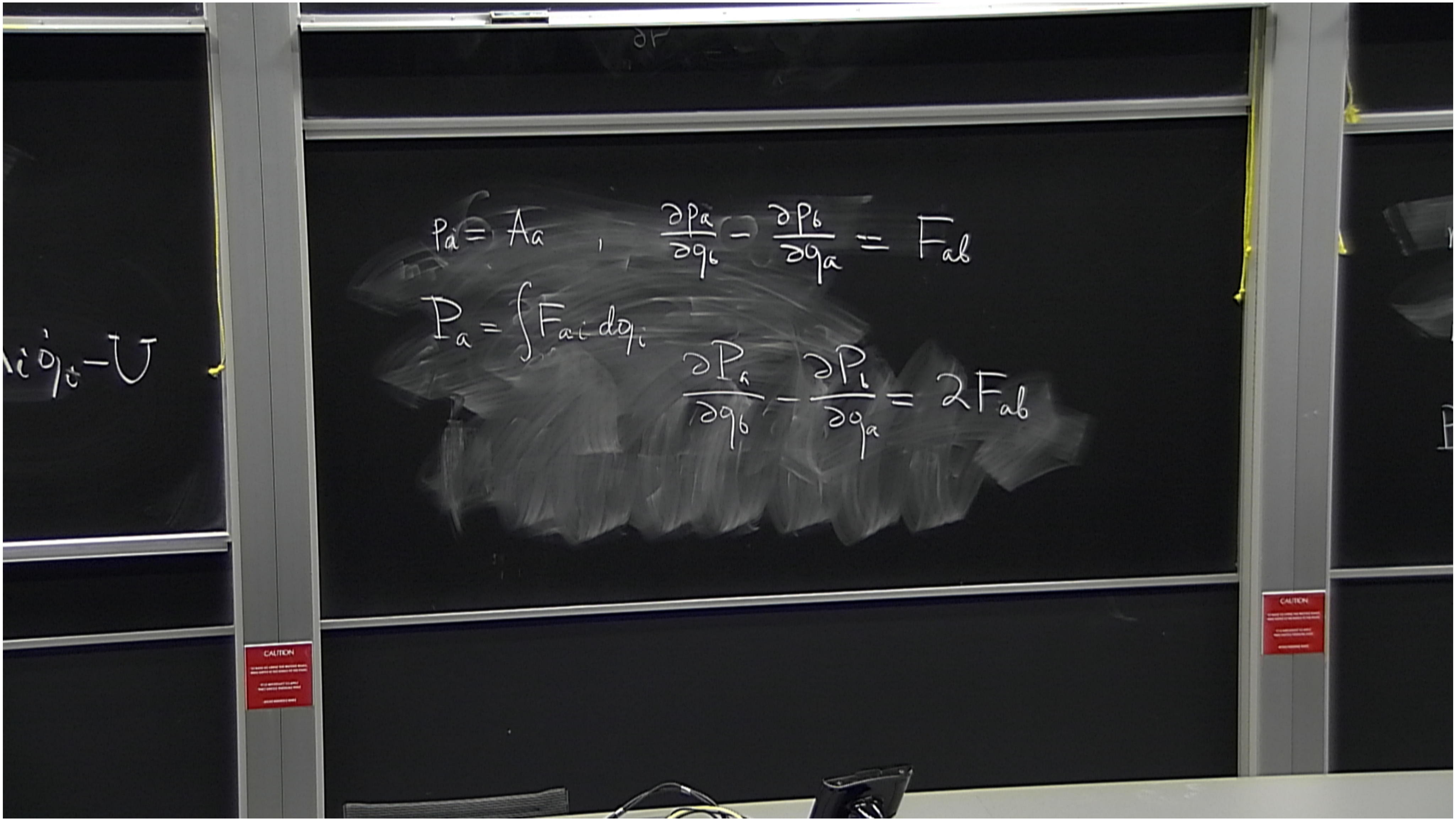

$m\ddot{q}_a = -\frac{\partial U}{\partial q_a} + F_{ai}\dot{q}_i$
$m\ddot{q}_a - F_{ai}\dot{q}_i = -\frac{\partial U}{\partial q_a}$
$P_a = m\ddot{q}_a - \int F_{ai}\dot{q}_i\,dt =$
CAUTION


$m\ddot{q}_a = -\frac{\partial U}{\partial q_a} + F_{ai}\dot{q}_i$
$m\ddot{q}_a - F_{ai}\dot{q}_i = -\frac{\partial U}{\partial q_a} = 0$
$P_a = m\dot{q}_a - \int F_{ai}\dot{q}_i\,dt = \text{Const}$
$P_a = -\int F_{ai}\,dq_i$


∮ F_ai dq_i = 0
∂F_ai/∂q_j − ∂F_aj/∂q_i = 0
∂F_ia/∂q_j + ∂F_ja/∂q_i + ∂F_ij/∂q_a − ∂F_ij/∂q_a = 0


$\oint F_{ai}\,dq_i = 0$
$\frac{\partial F_{ai}}{\partial q_j} - \frac{\partial F_{aj}}{\partial q_i} = 0$
$\frac{\partial F_{ai}}{\partial q_j} + \frac{\partial F_{ja}}{\partial q_i} + \frac{\partial F_{ij}}{\partial q_a} - \frac{\partial F_{ij}}{\partial q_a} = 0$


$p_a = A_a \quad , \quad \frac{\partial p_a}{\partial q_b} - \frac{\partial p_b}{\partial q_a} = F_{ab}$


p_a = A_a , ∂p_a/∂q_b − ∂p_b/∂q_a = F_ab
P_a = ∫F_ai dq_i
∂P_a/∂q_b − ∂P_b/∂q_a = 2F_ab
CAUTION