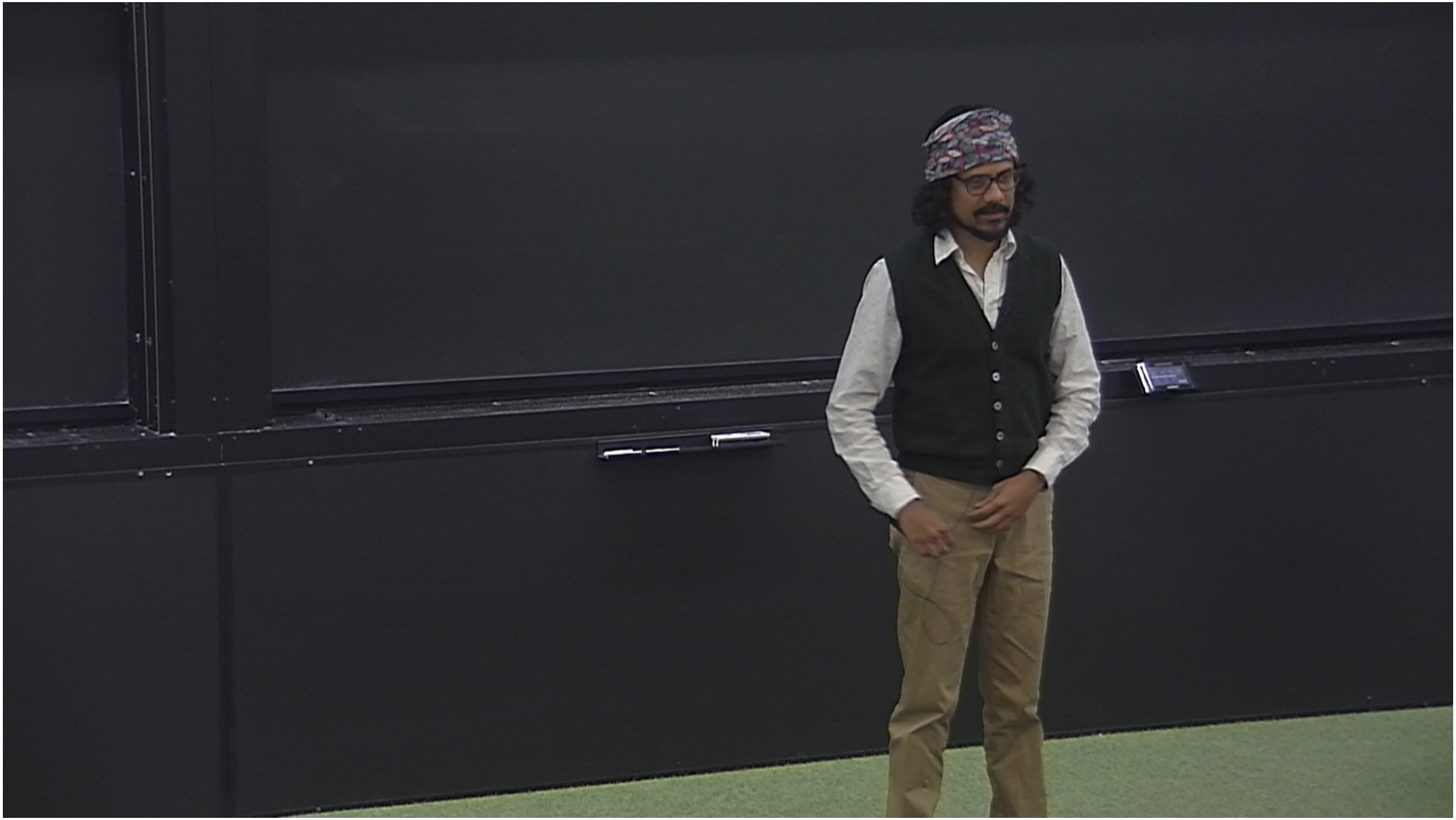
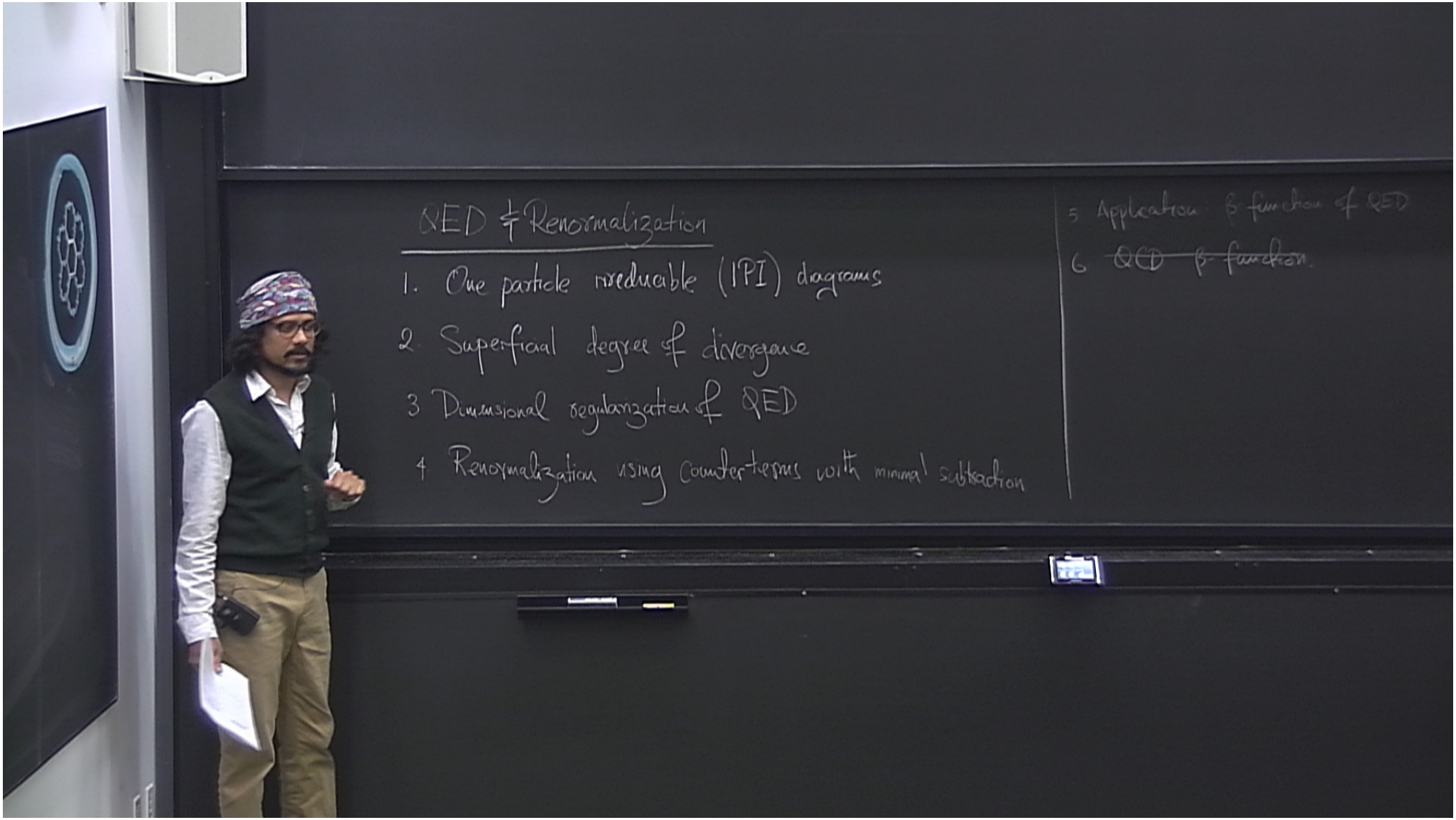
Title: PSI 2015/2016 QED & Renormalization Extra Course - Lecture 1

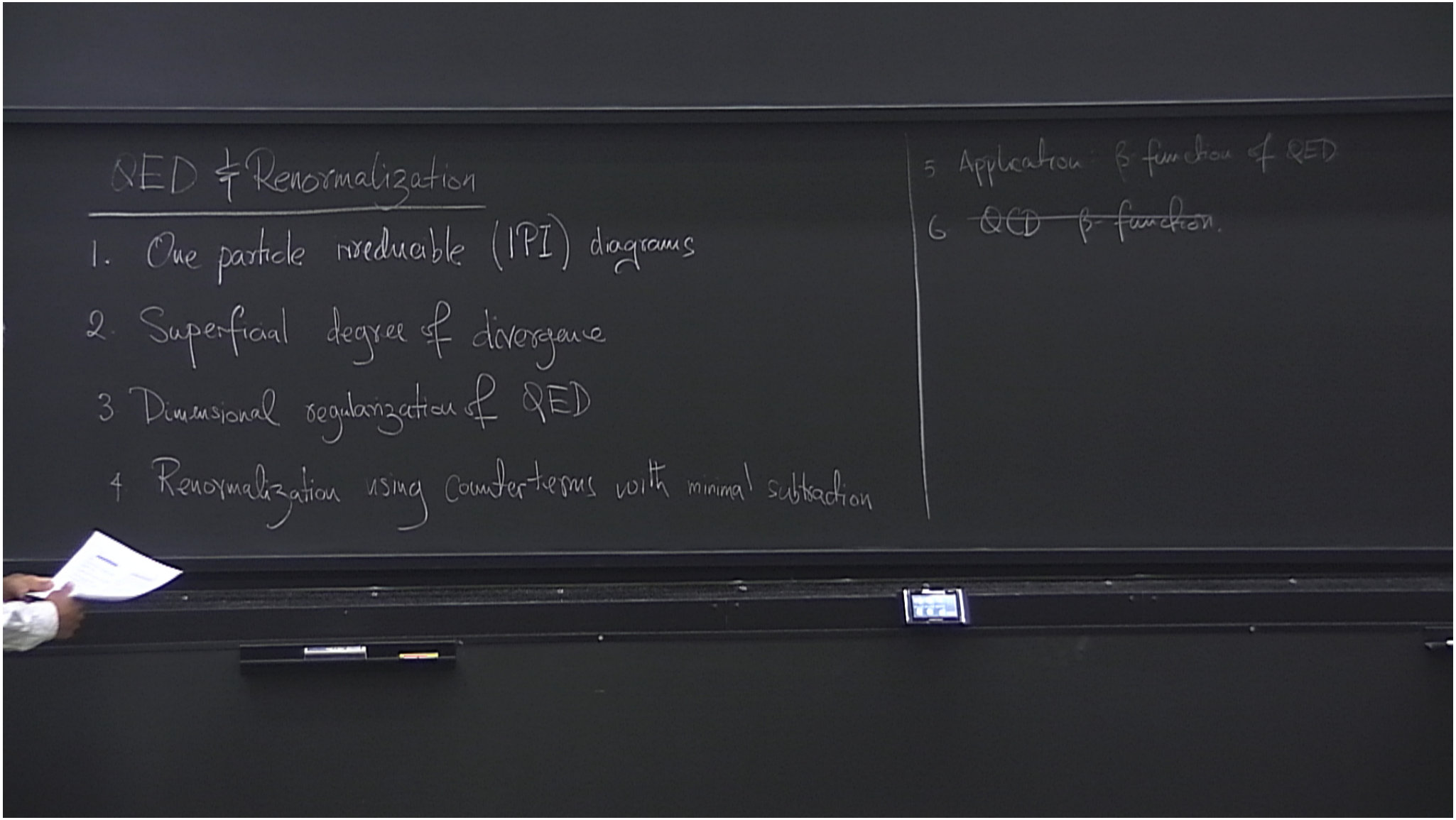
Date: Nov 30, 2015 10:45 AM

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Abstract:




QED & Renormalization
1. One particle irreducible (1PI) diagrams
2. Superficial degree of divergence
3 Dimensional regularization of QED
4 Renormalization using counterterms with minimal subtraction
5 Application: β function of QED
6 QED β-function

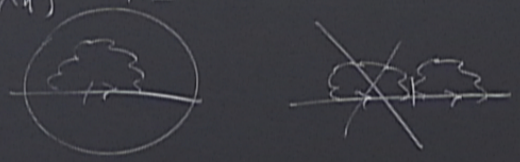

QED & Renormalization
1. One particle irreducible (1PI) diagrams
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3 Dimensional regularization of QED
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5 Application: β-function of QED
6 ~~QCD β-function.~~

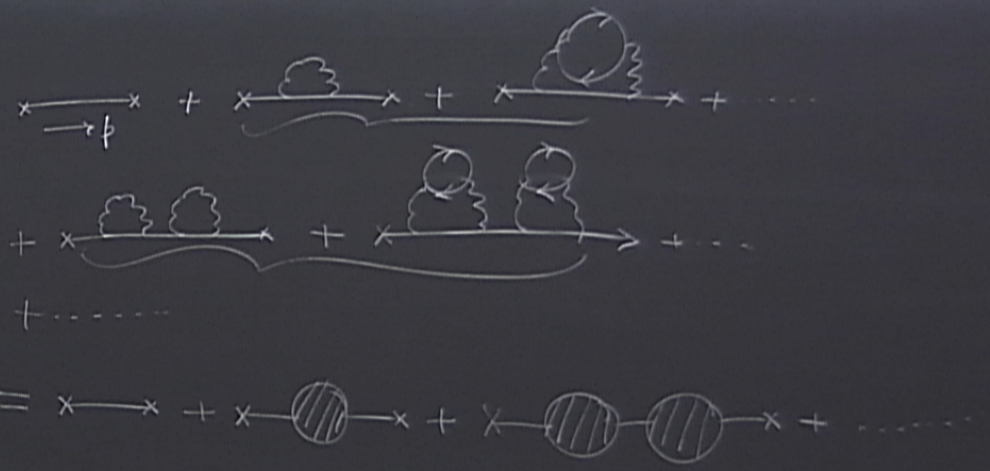
3 Dimensional regularization of QED

4 Renormalization using counterterms with minimal subtraction

1PI or Proper vertices

$G_c^n(x_1, \ldots, x_n)$ 1PI



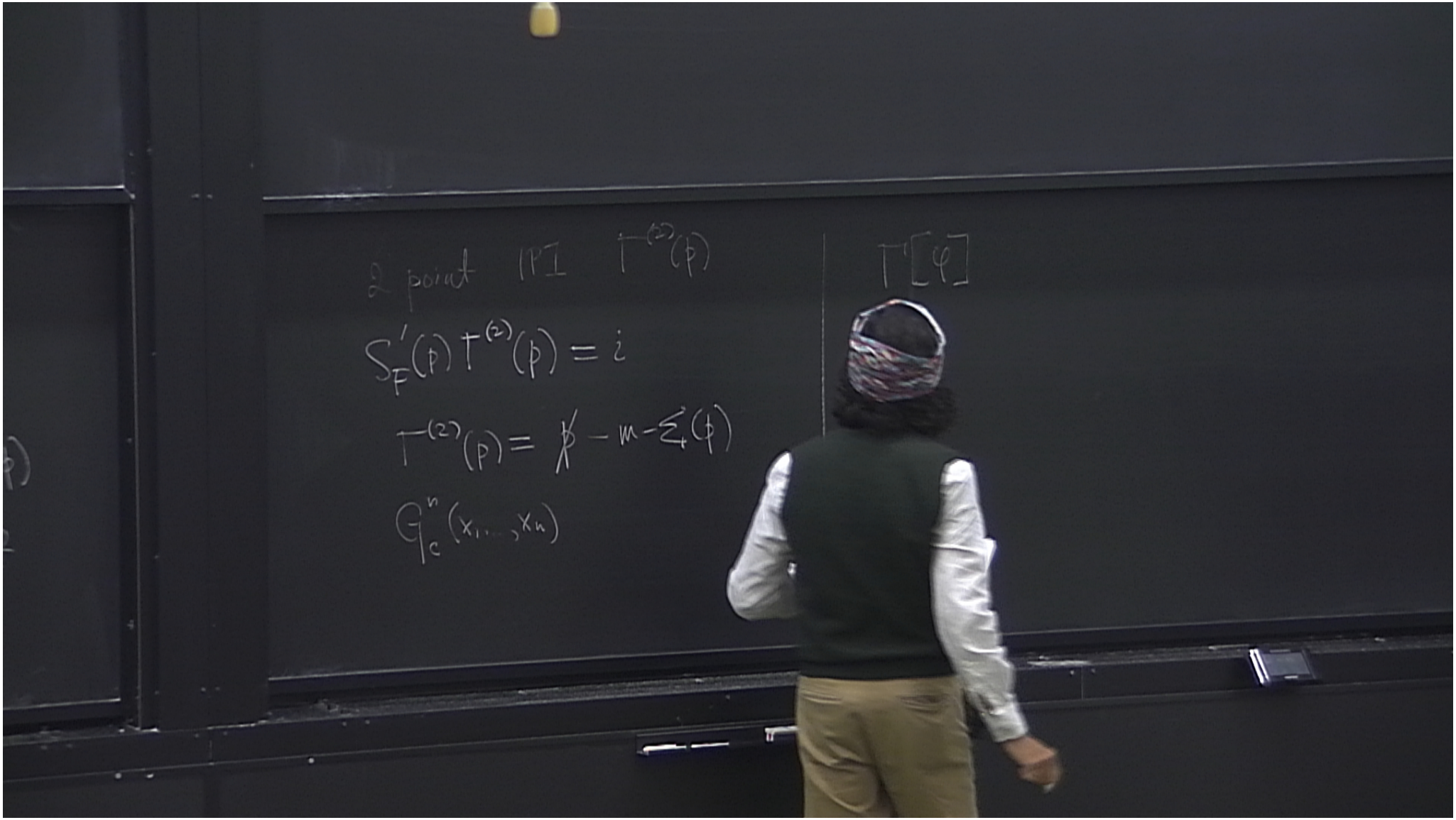


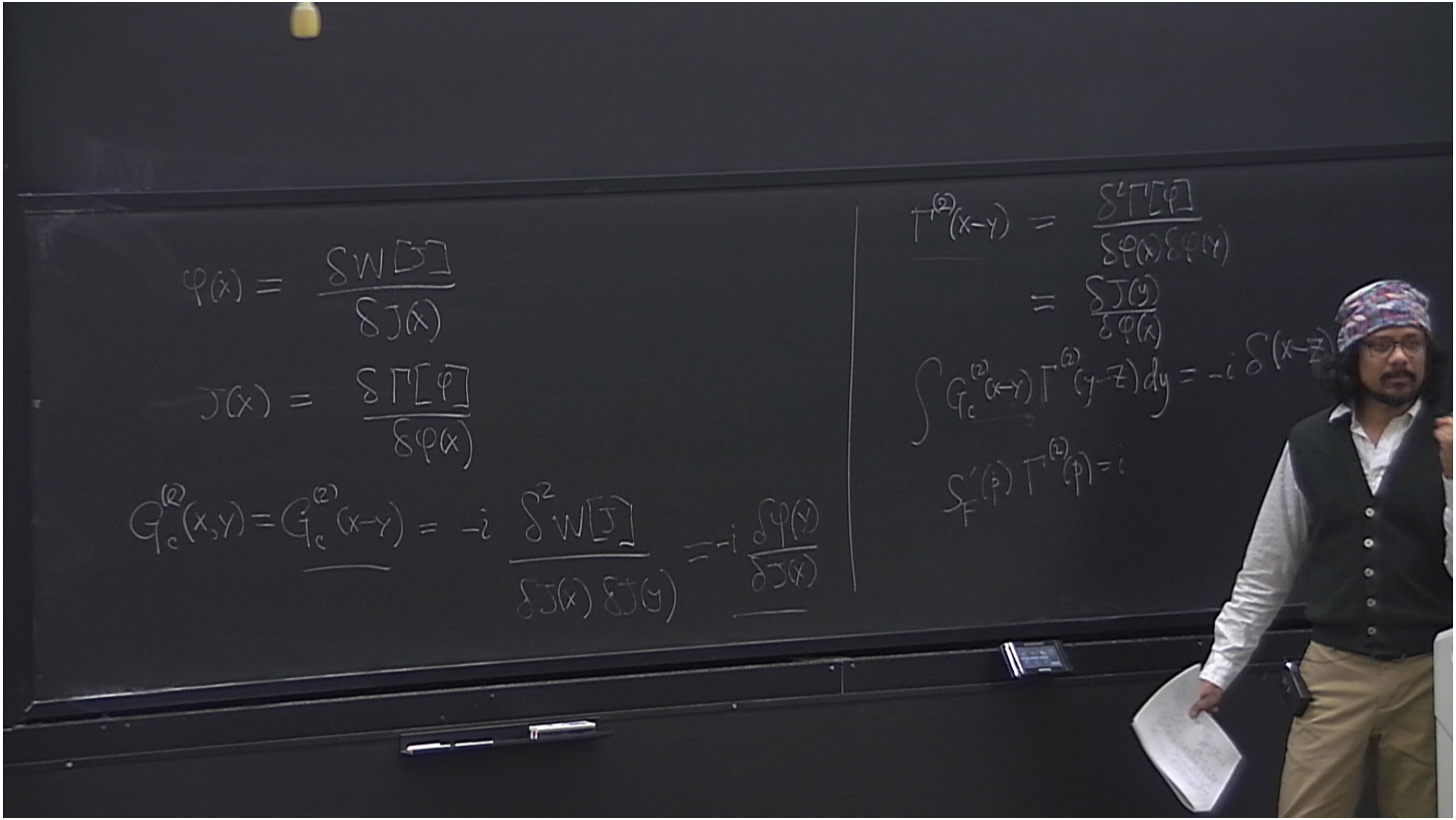
$$\sum_i(p) = S_{F0}^{-1}(p) \times \text{⬭} \times S_{F0}^{-1}(p)$$

$$= \cdots \text{⬭} \cdots$$

$$S_F'(p) = S_F^0(p) + S_F^0(p)\sum_i(p)S_F^0(p) + S_F^0(p)\sum_i(p)S_F^0(p)\sum_i(p)S_F^0(p) + \cdots$$

$$= S_F^0(p)\left[1 - \sum_i(p)S_F^0(p)\right]^{-1}$$

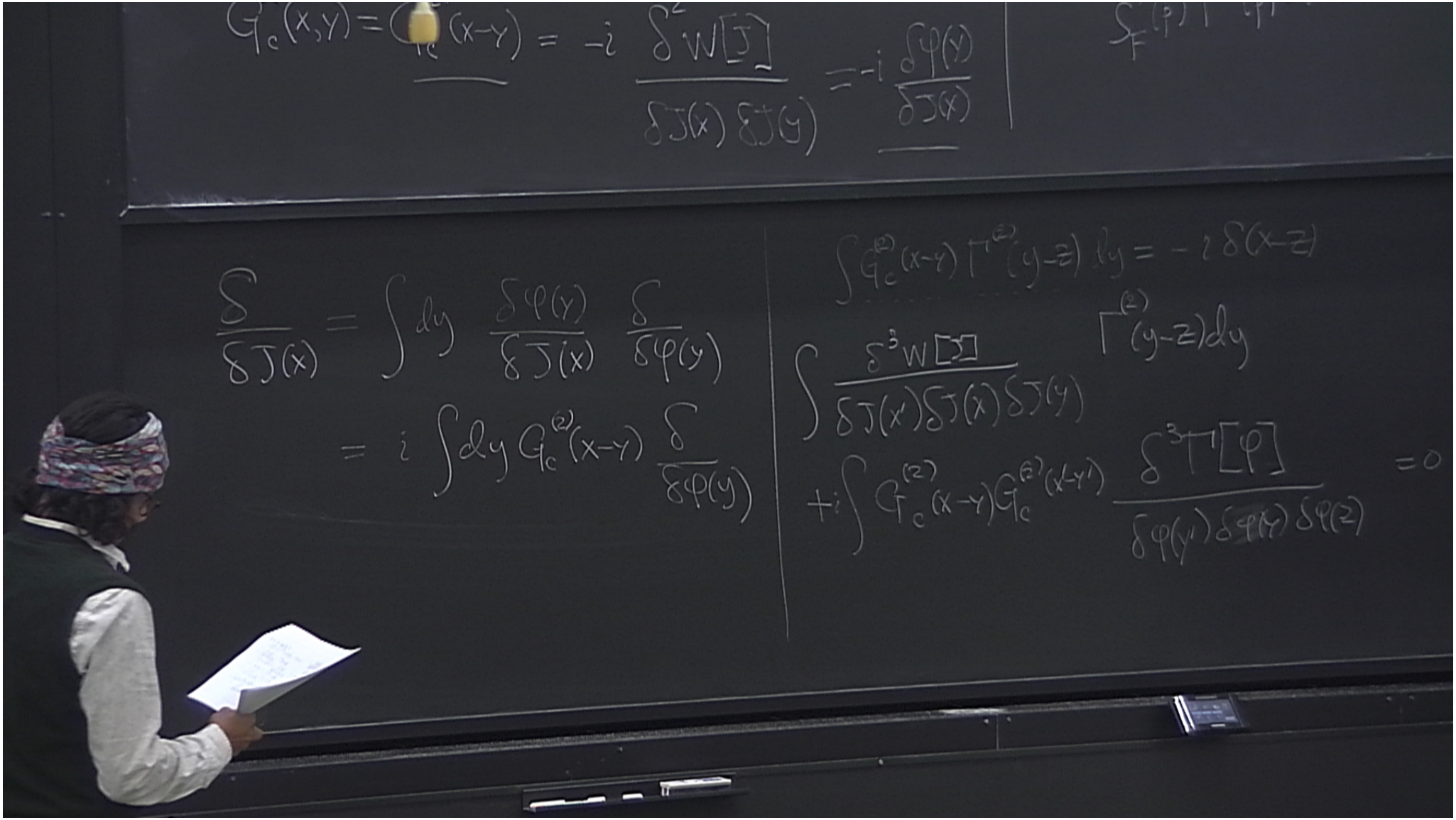

2 point 1PI $\Gamma^{(2)}(p)$
$S_F'(p)\,\Gamma^{(2)}(p) = i$
$\Gamma^{(2)}(p) = \not{p} - m - \Sigma_*(p)$
$G_c^n(x_1, \ldots, x_n)$
$\Gamma[\varphi]$


$\varphi(x) = \frac{\delta W[J]}{\delta J(x)}$
$J(x) = \frac{\delta \Gamma[\varphi]}{\delta \varphi(x)}$
$G_c^{(2)}(x,y) = G_c^{(2)}(x-y) = -i \frac{\delta^2 W[J]}{\delta J(x)\delta J(y)} = -i \frac{\delta \varphi(y)}{\delta J(x)}$
$\Gamma^{(2)}(x-y) = \frac{\delta^2 \Gamma[\varphi]}{\delta \varphi(x)\delta \varphi(y)}$
$= \frac{\delta J(y)}{\delta \varphi(x)}$
$\int G_c^{(2)}(x-y)\,\Gamma^{(2)}(y-z)\,dy = -i\,\delta(x-z)$
$S_F'(p)\,\Gamma'^{(2)}(p) = i$

$$\varphi(x) = \frac{\delta W[J]}{\delta J(x)}$$

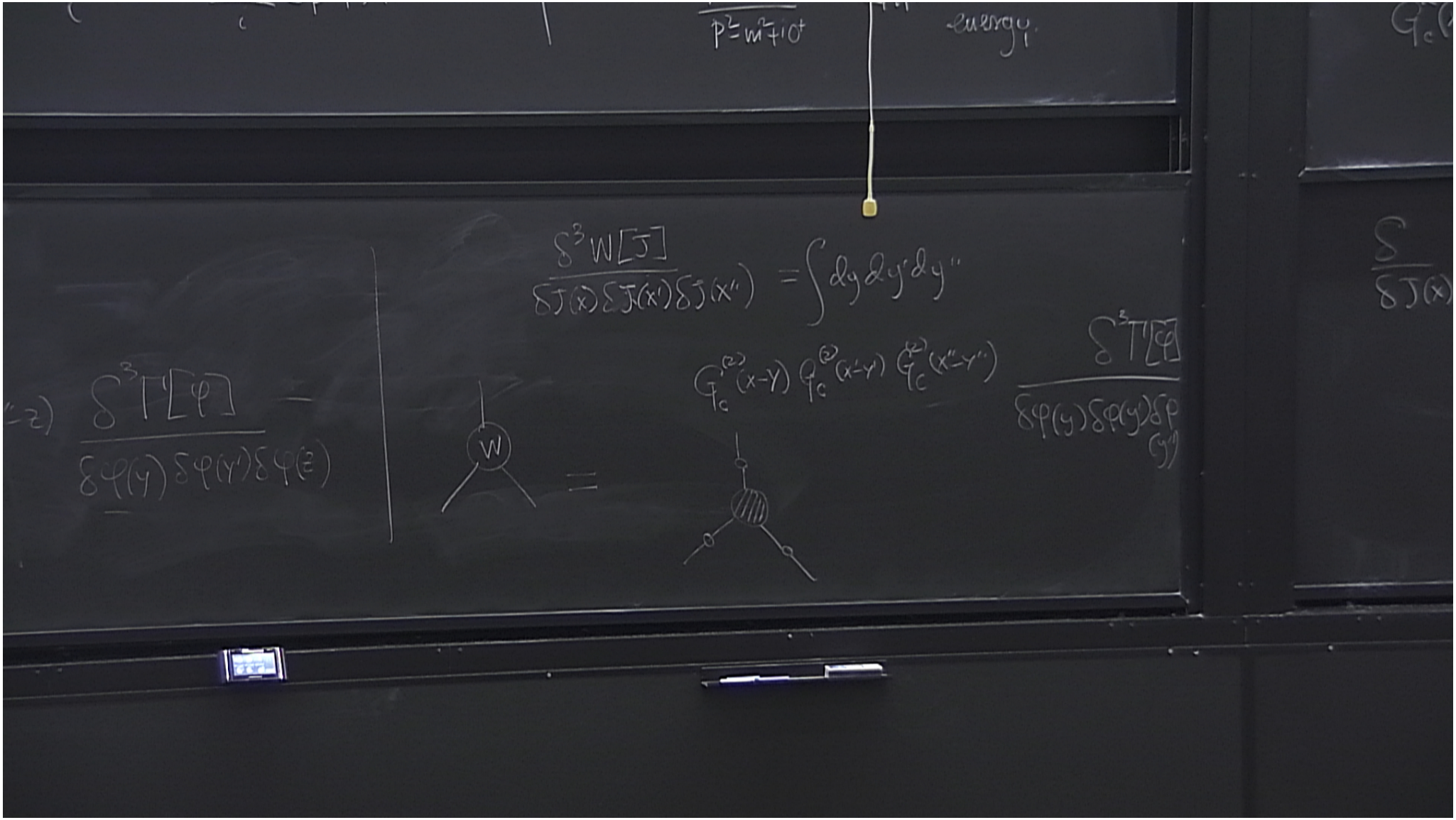
$$J(x) = \frac{\delta \Gamma[\varphi]}{\delta \varphi(x)}$$

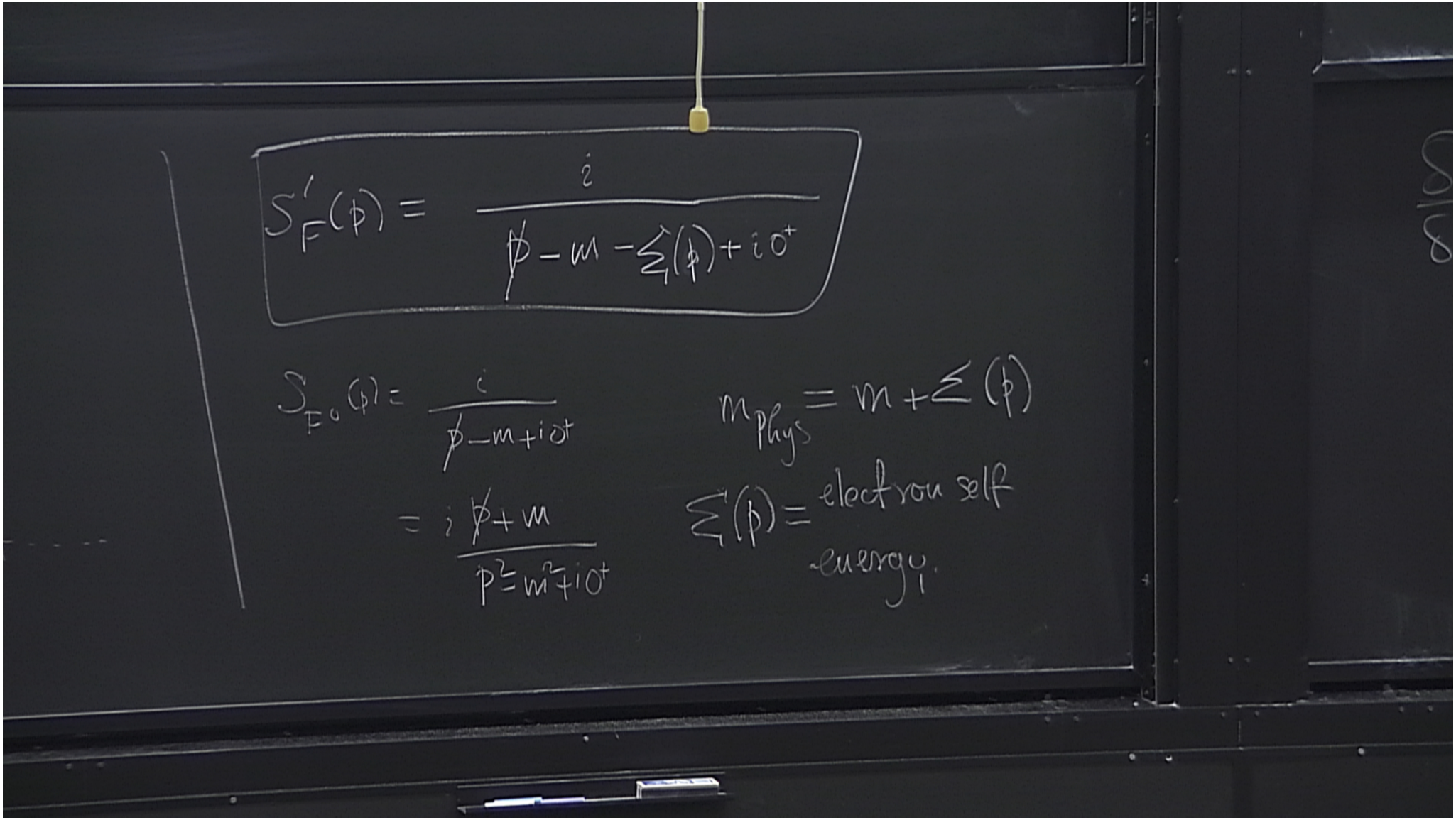
$$G_c^{(2)}(x,y) = G_c^{(2)}(x-y) = -i \frac{\delta^2 W[J]}{\delta J(x)\, \delta J(y)} = -i \frac{\delta \varphi(x)}{\delta J(y)}$$

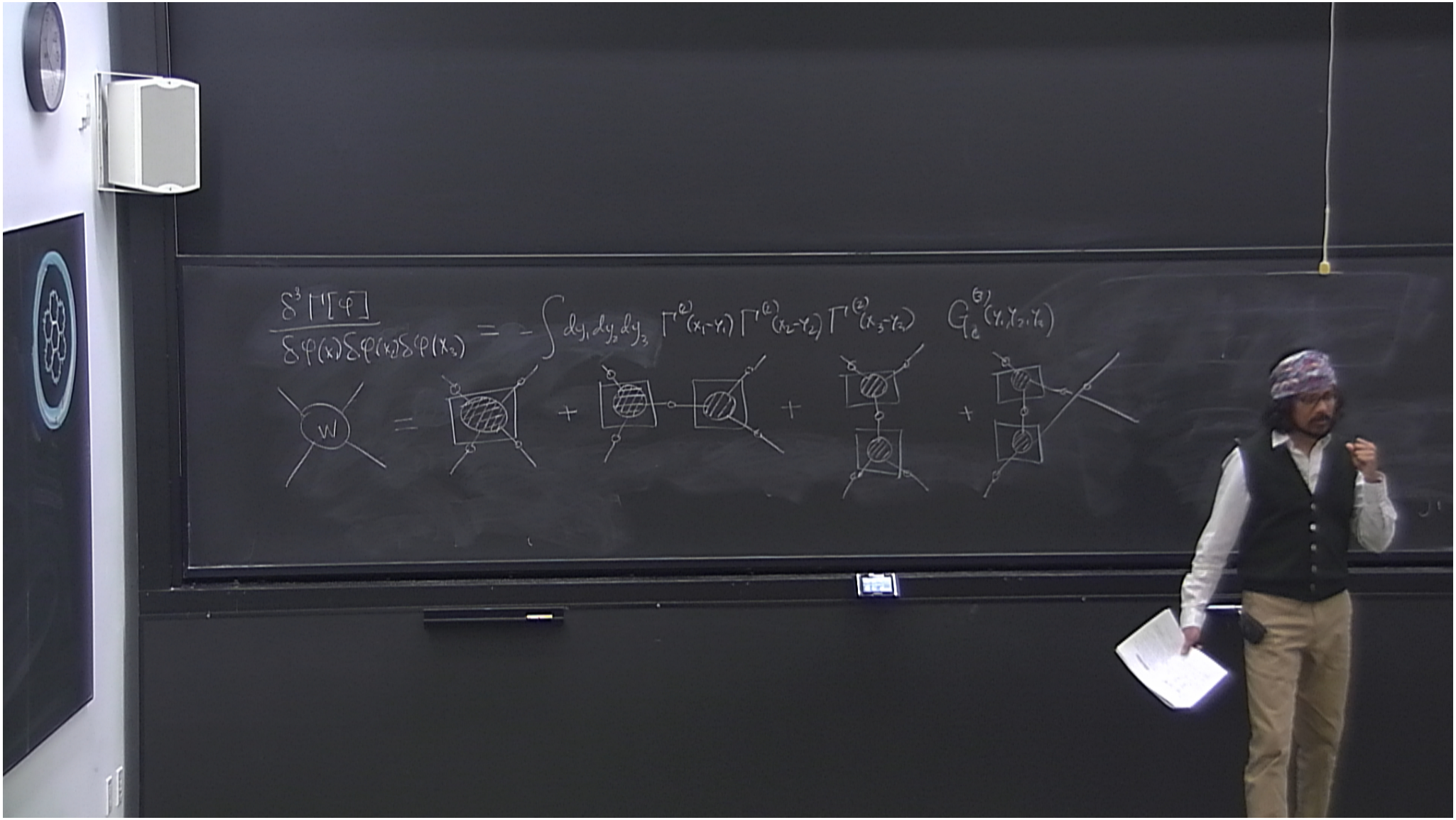

$G_c(x,y) = G_c(x-y) = -i \frac{\delta^2 W[J]}{\delta J(x)\delta J(y)} = -i \frac{\delta \varphi(y)}{\delta J(x)}$
$\frac{\delta}{\delta J(x)} = \int dy \frac{\delta \varphi(y)}{\delta J(x)} \frac{\delta}{\delta \varphi(y)}$
$= i \int dy\, G_c^{(2)}(x-y) \frac{\delta}{\delta \varphi(y)}$
$\int G_c^{(2)}(x-y)\Gamma^{(2)}(y-z)\,dy = -i\delta(x-z)$
$\int \frac{\delta^3 W[J]}{\delta J(x')\delta J(x)\delta J(y)} \Gamma^{(2)}(y-z)dy$
$+i \int G_c^{(2)}(x-y) G_c^{(2)}(x-y') \frac{\delta^3 \Gamma[\varphi]}{\delta\varphi(y')\delta\varphi(y)\delta\varphi(z)} = 0$

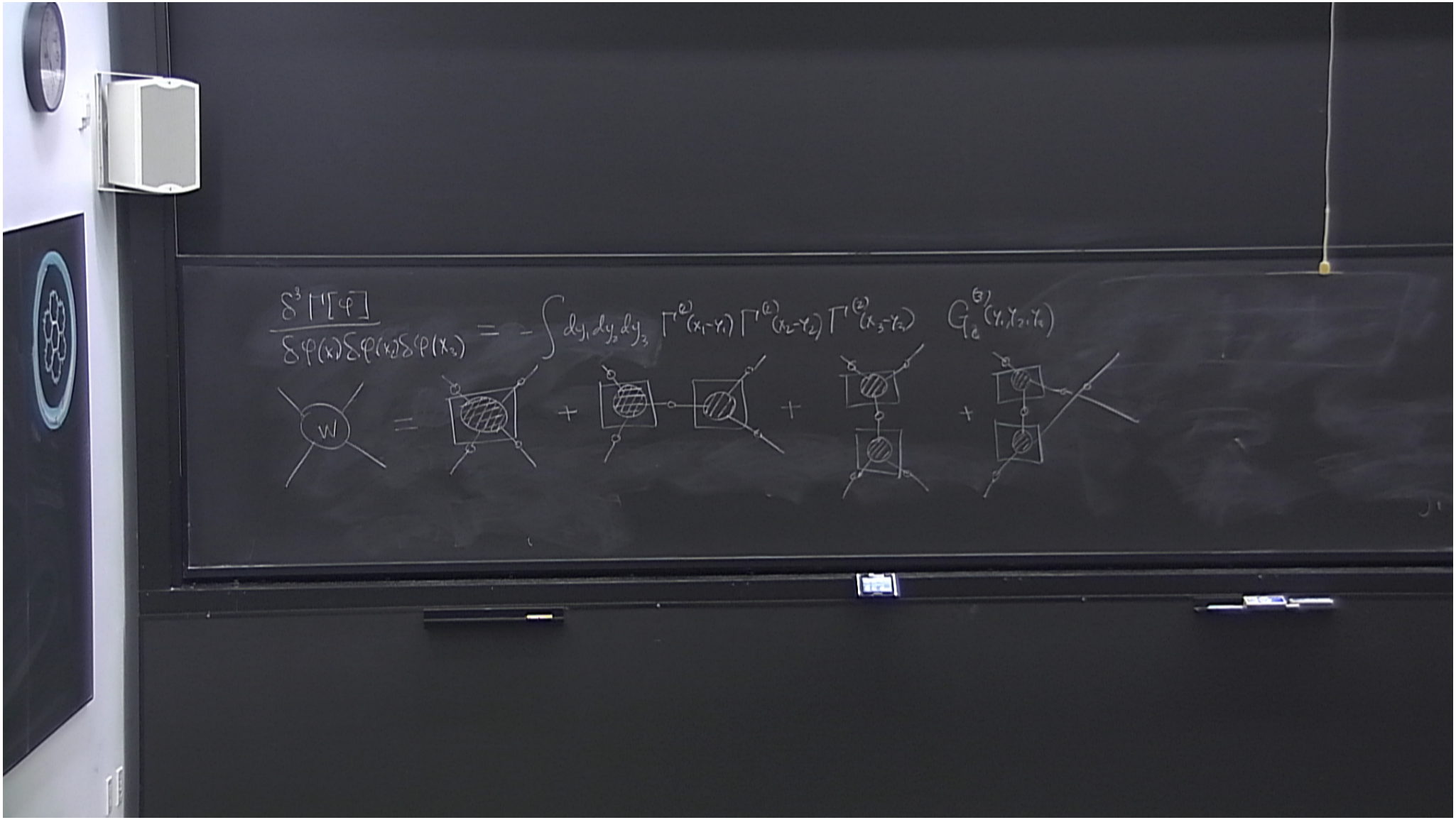
$$-i \int \frac{\delta^3 W[J]}{\delta J(x') \delta J(x) \delta J(y)} \delta(y - y'') \, dy$$

$$= -i \int dy dy' dz \; G_c^{(2)}(x - y) G_c^{(2)}(x' - y') G_c^{(2)}(x'' - z) \frac{\delta^3 \Gamma[\varphi]}{\delta\varphi(y) \delta\varphi(y') \delta\varphi(z)}$$


$p^2 = m^2 + i0^+$
energy.
$\frac{\delta^3 W[J]}{\delta J(x)\delta J(x')\delta J(x'')} = \int dy\, dy'\, dy''$
$G_c^{(2)}(x-y)\, G_c^{(2)}(x'-y')\, G_c^{(2)}(x''-y'')$
$\frac{\delta^3 \Gamma[\varphi]}{\delta\varphi(y)\delta\varphi(y')\delta\varphi(y'')}$
$\frac{\delta^3 \Gamma[\varphi]}{\delta\varphi(y)\delta\varphi(y')\delta\varphi(z)}$
W
=


$S'_F(p) = \frac{i}{\not{p} - m - \Sigma(p) + i0^+}$
$S_{F0}(p) = \frac{i}{\not{p} - m + i0^+}$
$= i \frac{\not{p} + m}{p^2 - m^2 + i0^+}$
$m_{phys} = m + \Sigma(p)$
$\Sigma(p) =$ electron self energy.


$\frac{\delta^3 \Gamma[\varphi]}{\delta\varphi(x_1)\delta\varphi(x_2)\delta\varphi(x_3)} = -\int dy_1 dy_2 dy_3 \; \Gamma^{(2)}(x_1-y_1)\,\Gamma^{(2)}(x_2-y_2)\,\Gamma^{(2)}(x_3-y_3)\; G_c^{(3)}(y_1,y_2,y_3)$
W
=
+
+
+


$\frac{\delta^3 \Gamma[\varphi]}{\delta\varphi(x_1)\delta\varphi(x_2)\delta\varphi(x_3)} = -\int dy_1 dy_2 dy_3 \; \Gamma^{(2)}(x_1-y_1)\,\Gamma^{(2)}(x_2-y_2)\,\Gamma^{(2)}(x_3-y_3)\; G_0^{(3)}(y_1,y_2,y_3)$
W
=
+
+
+

Degree of divergence:

d D ~ Superficial degree of divergence

$$D = dL - 2P_i - E_i$$

# of independent momenta = L = # of internal lines − n + 1

$n \rightarrow$ # of vertices

$L = P_i + E_i - n + 1$

$2n = E_e + 2E_i$

$n = P_e + 2P_i$

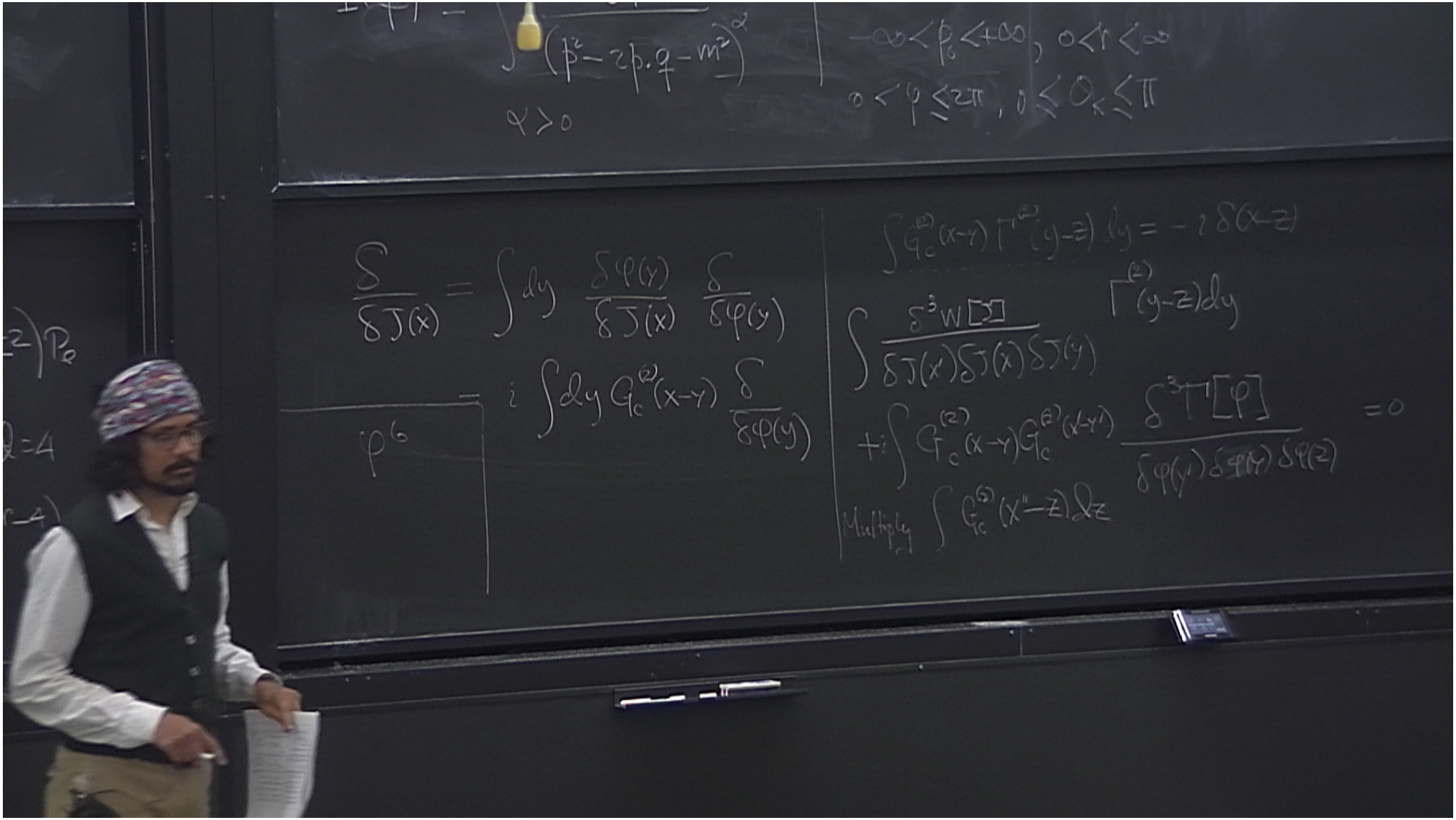
$$D = d + n\left(\frac{d}{2} - 2\right) - \left(\frac{d-1}{2}\right)E_e - \left(\frac{d-2}{2}\right)P_e$$

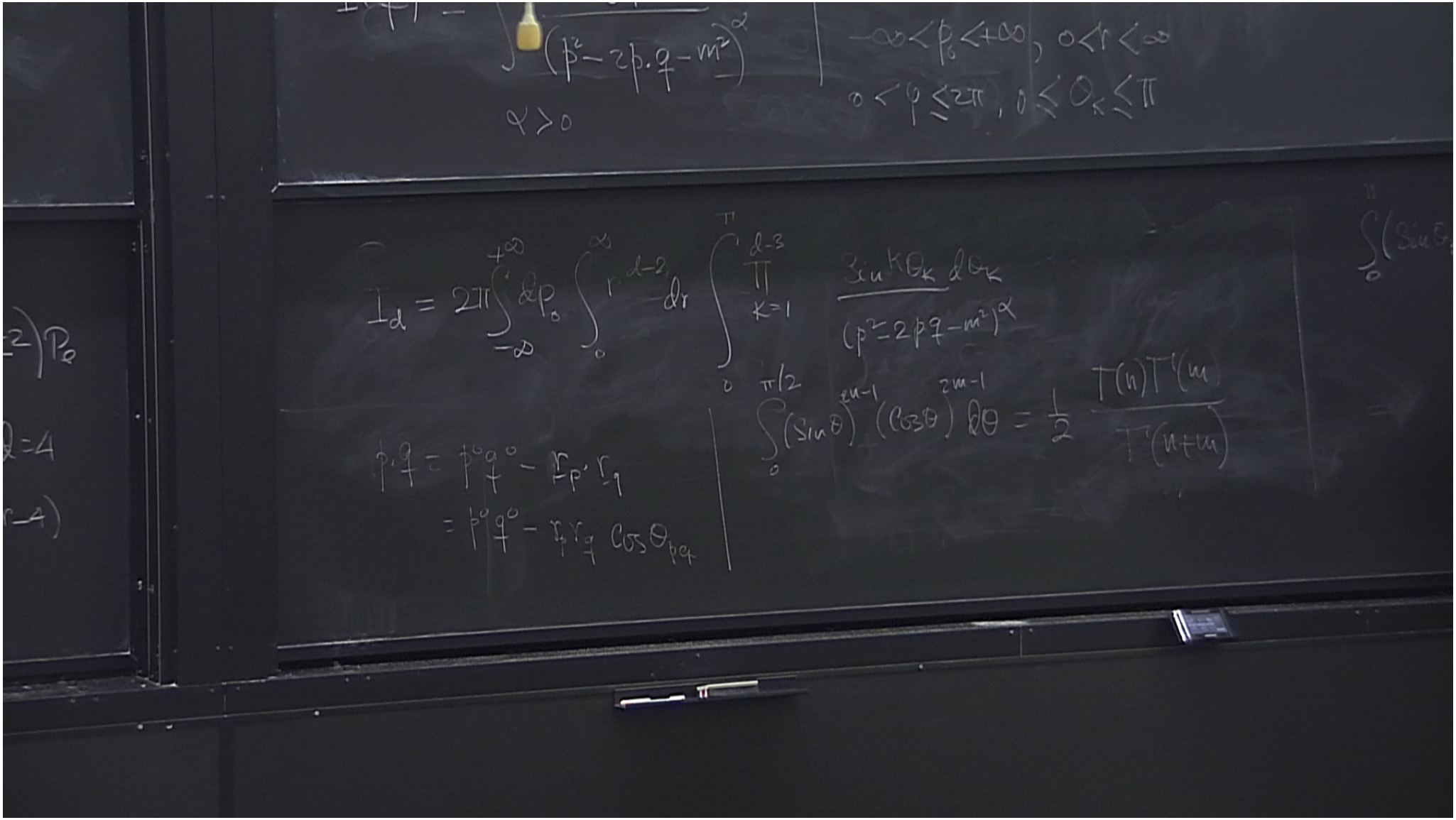
when $d = 4$

$$D = 4 - \frac{3E_e}{2} - P_e$$

φ^r theory in $d = 4$

$$D = 4 - E_\varphi + n(r-4)$$


$\frac{\delta}{\delta J(x)} = \int dy \frac{\delta \varphi(y)}{\delta J(x)} \frac{\delta}{\delta \varphi(y)}$
$= i \int dy\, G_c^{(2)}(x-y) \frac{\delta}{\delta \varphi(y)}$
$\int \frac{\delta^3 W[J]}{\delta J(x) \delta J(x) \delta J(y)} \Gamma^{(3)}(y-z) dy$
$+ i \int G_c^{(2)}(x-y) G_c^{(2)}(x-y') \frac{\delta^3 \Gamma[\varphi]}{\delta \varphi(y') \delta \varphi(y) \delta \varphi(z)} = 0$
$-\infty < \beta_0 < +\infty, \; 0 < r < \infty$
$0 < \varphi \leq 2\pi, \; 0 \leq \theta_k \leq \pi$
$q > 0$


$\int \frac{}{(p^2 - 2p\cdot q - m^2)^\alpha}$
$q > 0$
$-\infty < p_0 < +\infty$, $0 < r < \infty$
$0 < \varphi \leq 2\pi$, $0 \leq \theta_k \leq \pi$
$I_d = 2\pi \int_{-\infty}^{+\infty} dp_0 \int_0^\infty r^{d-2} dr \int_0^\pi \prod_{k=1}^{d-3} \frac{\sin^k \theta_k \, d\theta_k}{(p^2 - 2pq - m^2)^\alpha}$
$p\cdot q = p^0 q^0 - \underline{r}_p \cdot \underline{r}_q$
$= p^0 q^0 - r_p r_q \cos\theta_{pq}$
$\int_0^{\pi/2} (\sin\theta)^{2n-1} (\cos\theta)^{2m-1} d\theta = \frac{1}{2} \frac{\Gamma(n)\Gamma(m)}{\Gamma(n+m)}$

$\alpha \rightarrow$ positive integer

$\Gamma(\alpha-2)$ blows up $\alpha \leq 2$

$d = 4 - \varepsilon$

$$\Gamma(-n+\varepsilon) = \frac{(-1)^n}{n!}\left[\frac{1}{\varepsilon} + \psi_1(n+1) + \mathcal{O}(\varepsilon)\right]$$

$(p^2 - 2p\cdot q - m^2)$

$q > 0$

$0 < \varphi \leq 2\pi, \quad 0 \leq \theta_k \leq \pi$

$$\int \frac{d^d\ell}{(2\pi)^d} \frac{\ell^2}{(\ell^2-\Delta)^n} = \frac{(-1)^{n-1}}{(4\pi)^{d/2}} i \frac{d}{2} \frac{\Gamma\left(n-\frac{d}{2}-1\right)}{\Gamma(n)} \times \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1}$$

$$\int \frac{d^d\ell}{(2\pi)^d} \frac{\ell^\mu \ell^\nu}{(\ell^2-\Delta)^n} = \frac{(-1)^{n-1}}{(4\pi)^{d/2}} \frac{i\eta^{\mu\nu}}{2} \frac{\Gamma\left(n-\frac{d}{2}-1\right)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1}$$