Title: PSI 2015/2016 CFT Extra PSI Course - Lecture 1

Date: Nov 30, 2015 09:00 AM

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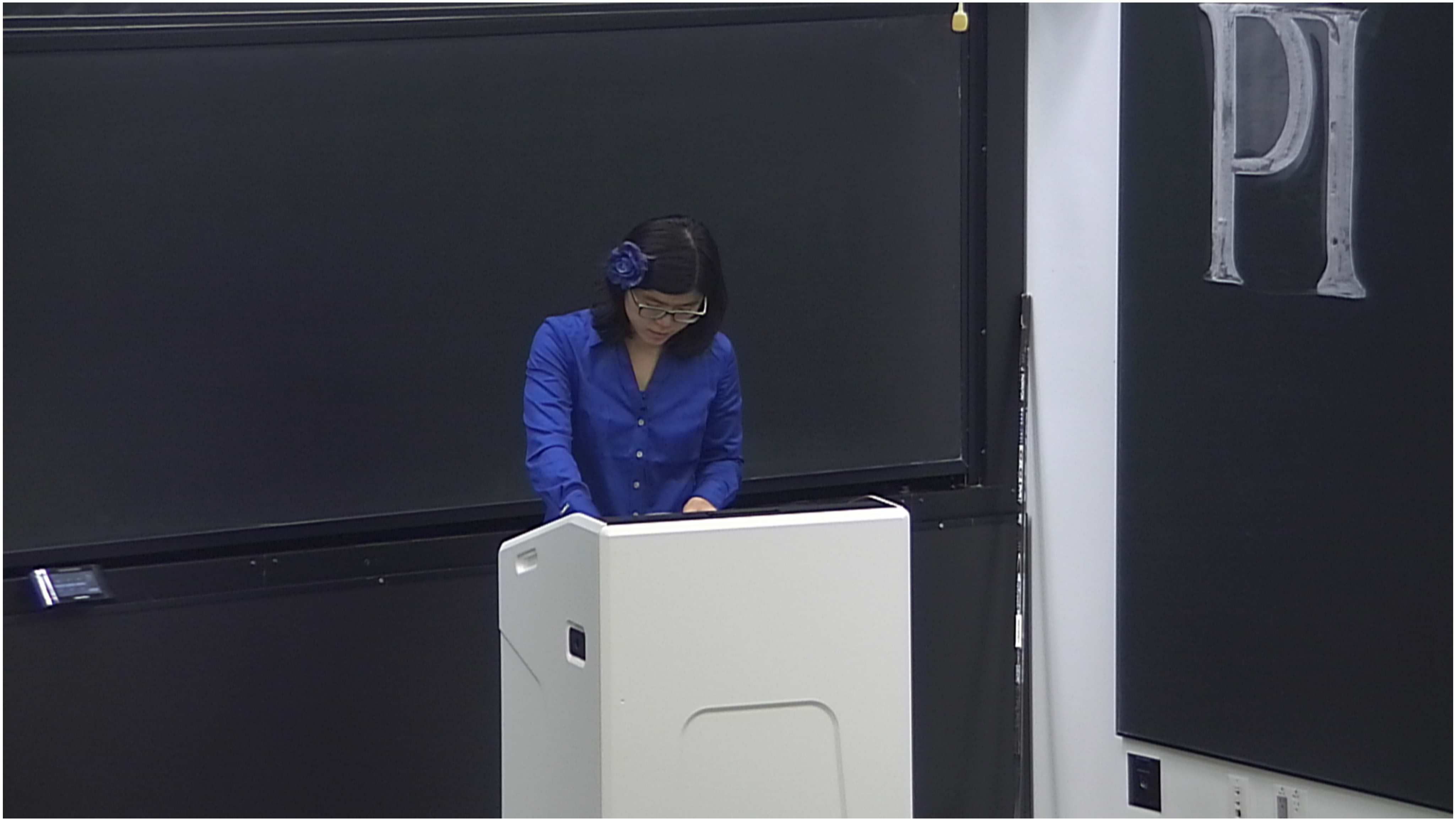
Abstract: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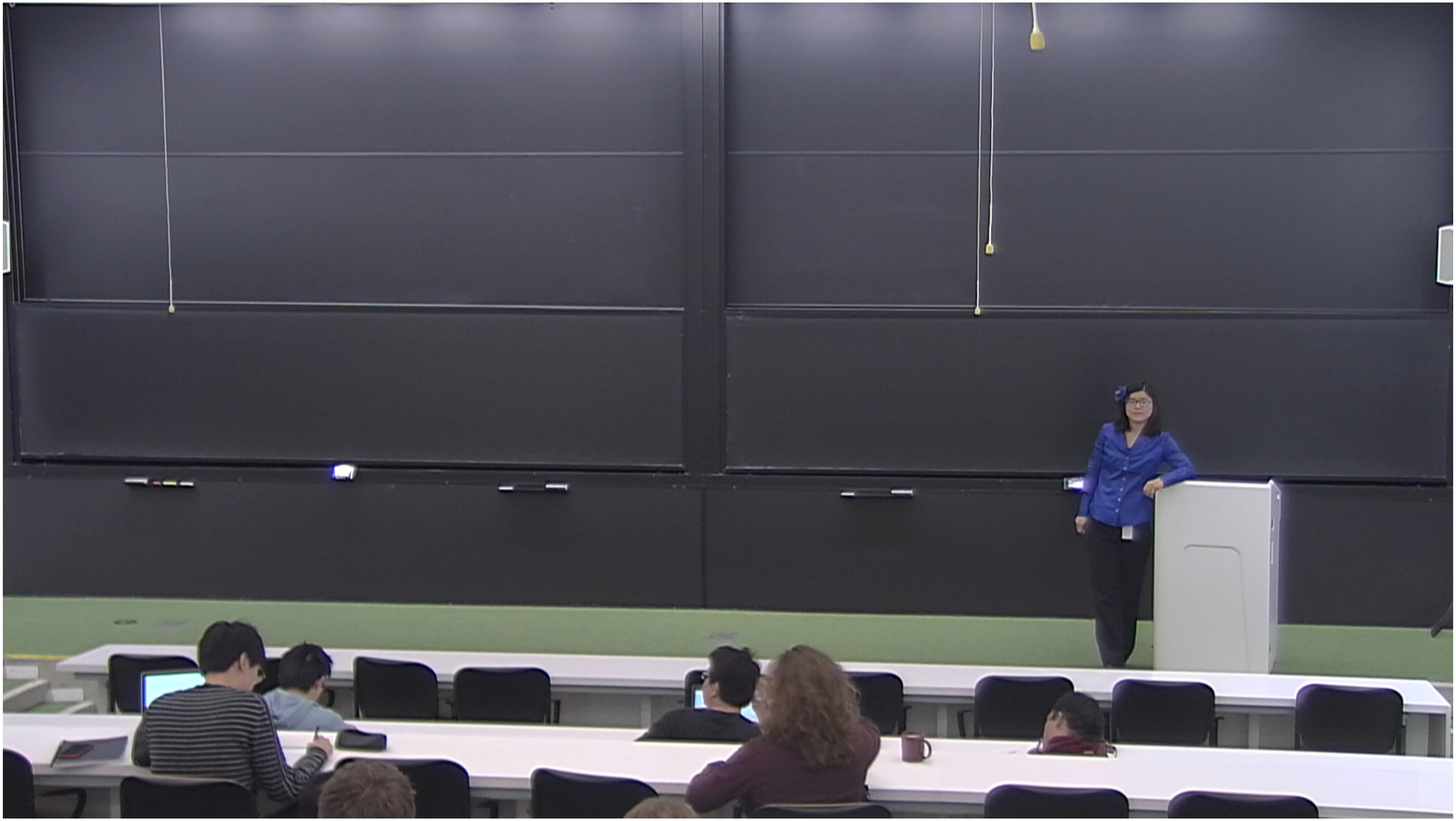
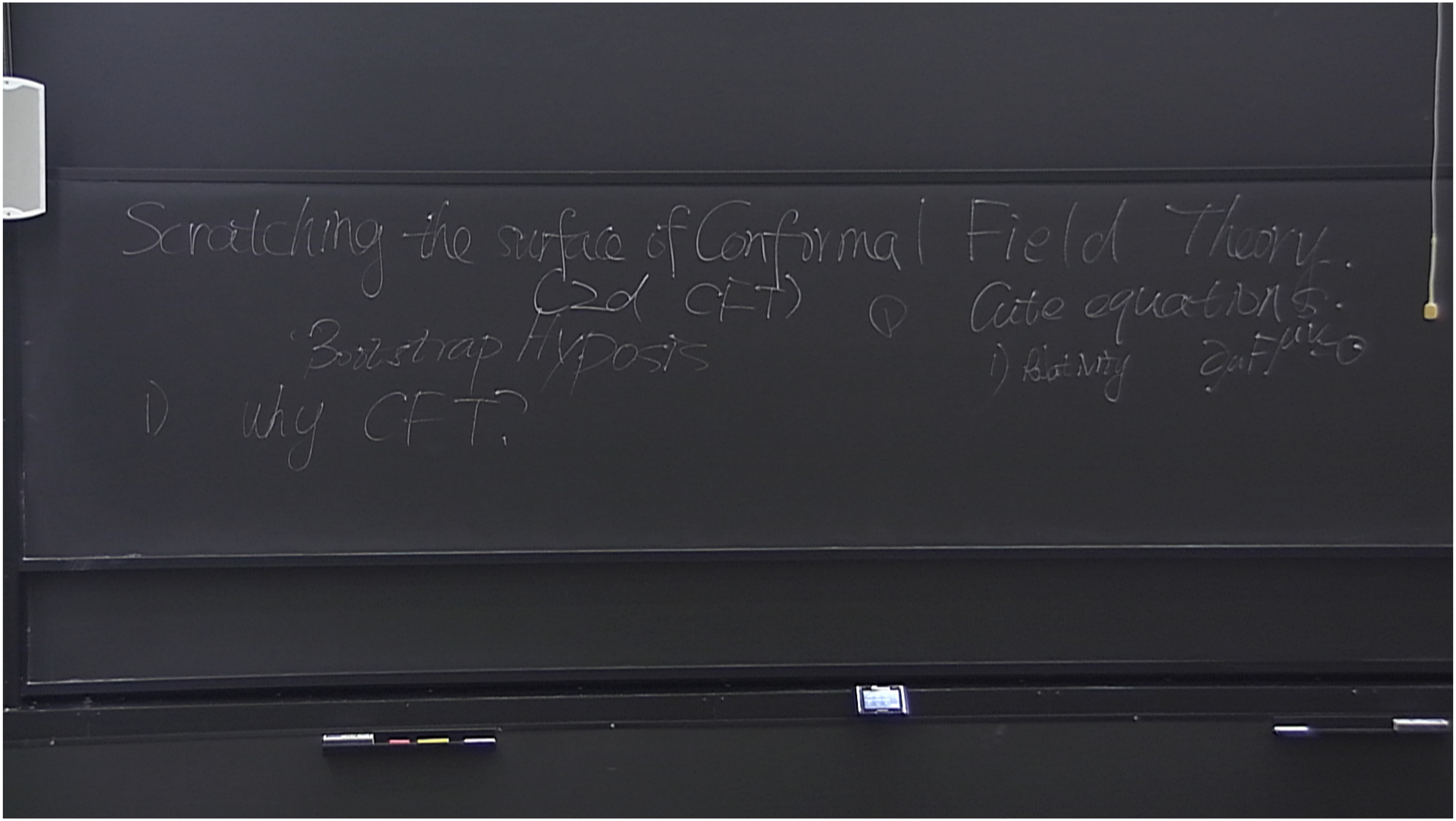




Scratching the surface of Conformal Field Theory.

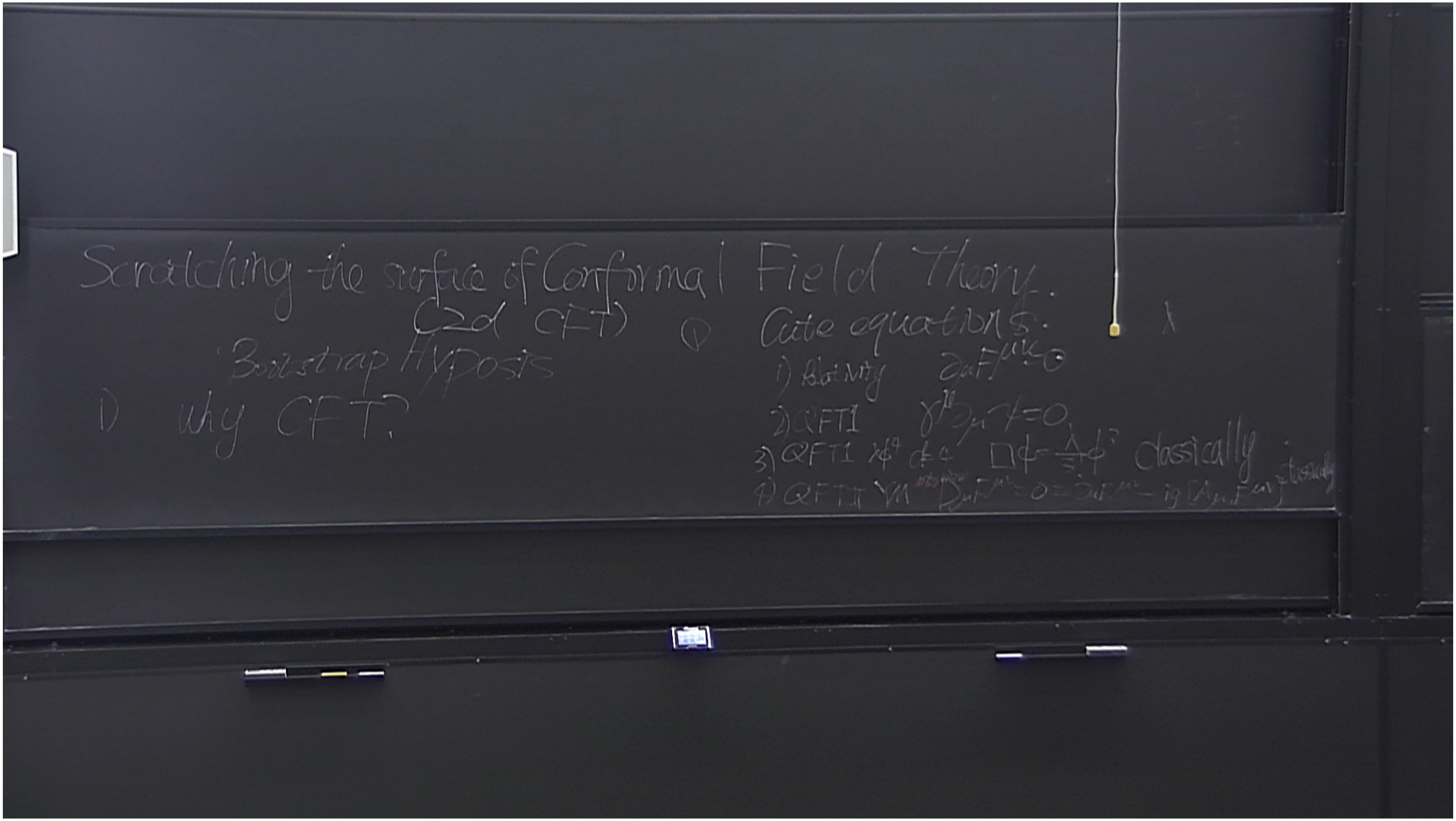
(2d CFT)

'Bootstrap Hyposis

1) why CFT?

♡ Cute equations.

1) Relativity $\partial_\mu F^{\mu\nu} = 0$



Scratching the surface of Conformal Field Theory.

(2d CFT)

Bootstrap Hyposis

1) why CFT?

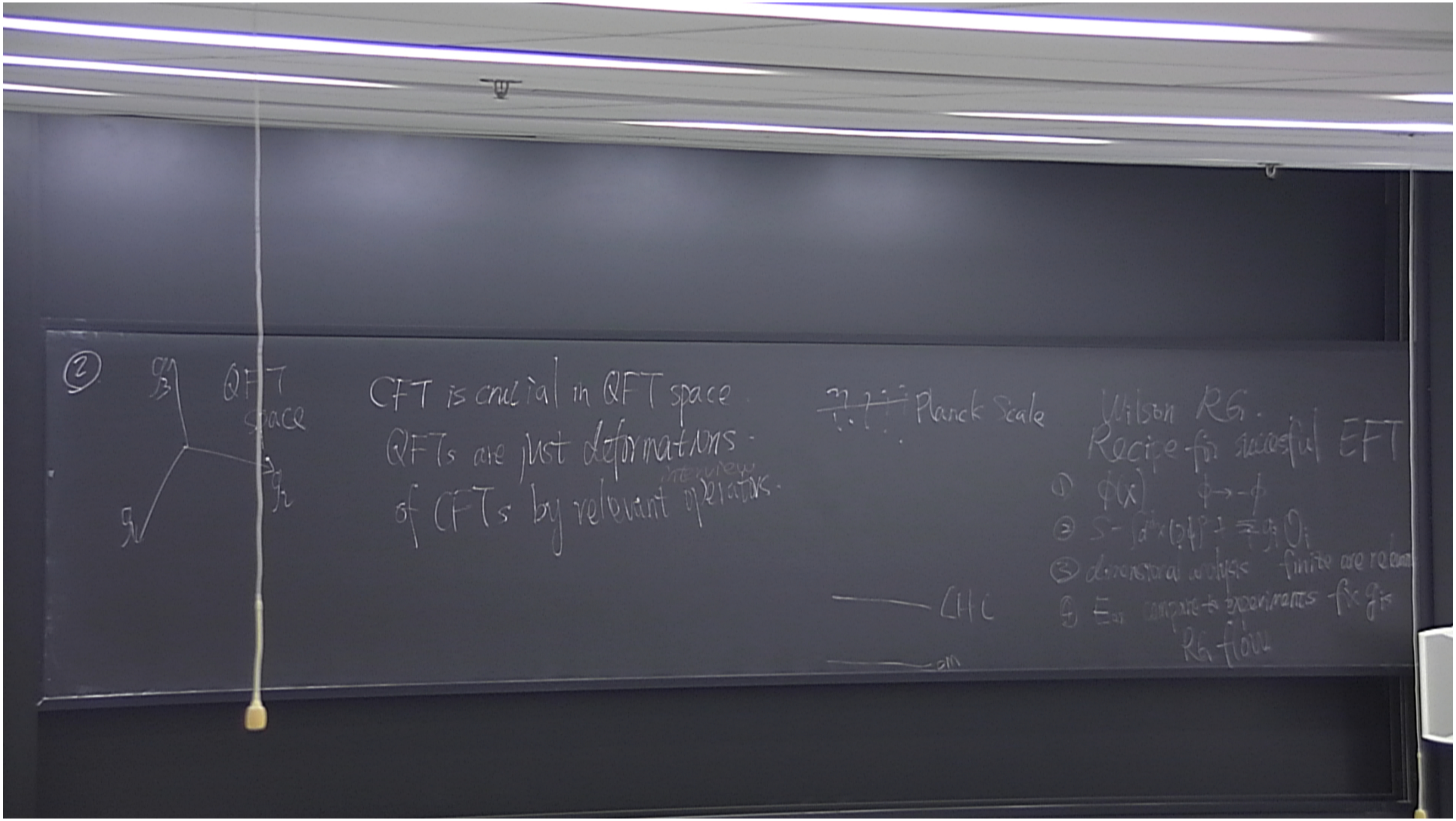
① Cute equations:

1) Relativity $\partial_\mu F^{\mu\nu}=0$

2) QFT I $\gamma^\mu \partial_\mu \psi=0$

3) QFT I $\lambda\phi^4$ $d=4$ $\Box\phi = \frac{\lambda}{3!}\phi^3$ classically

4) QFT II YM interacting $D_\mu F^{\mu\nu}=0=\partial_\mu F^{\mu\nu}-ig[A_\mu,F^{\mu\nu}]$ classically



CFT is crucial in QFT space.

QFTs are just deformations of CFTs by relevant operators.
interview

Wilson RG.

Recipe for successful EFT

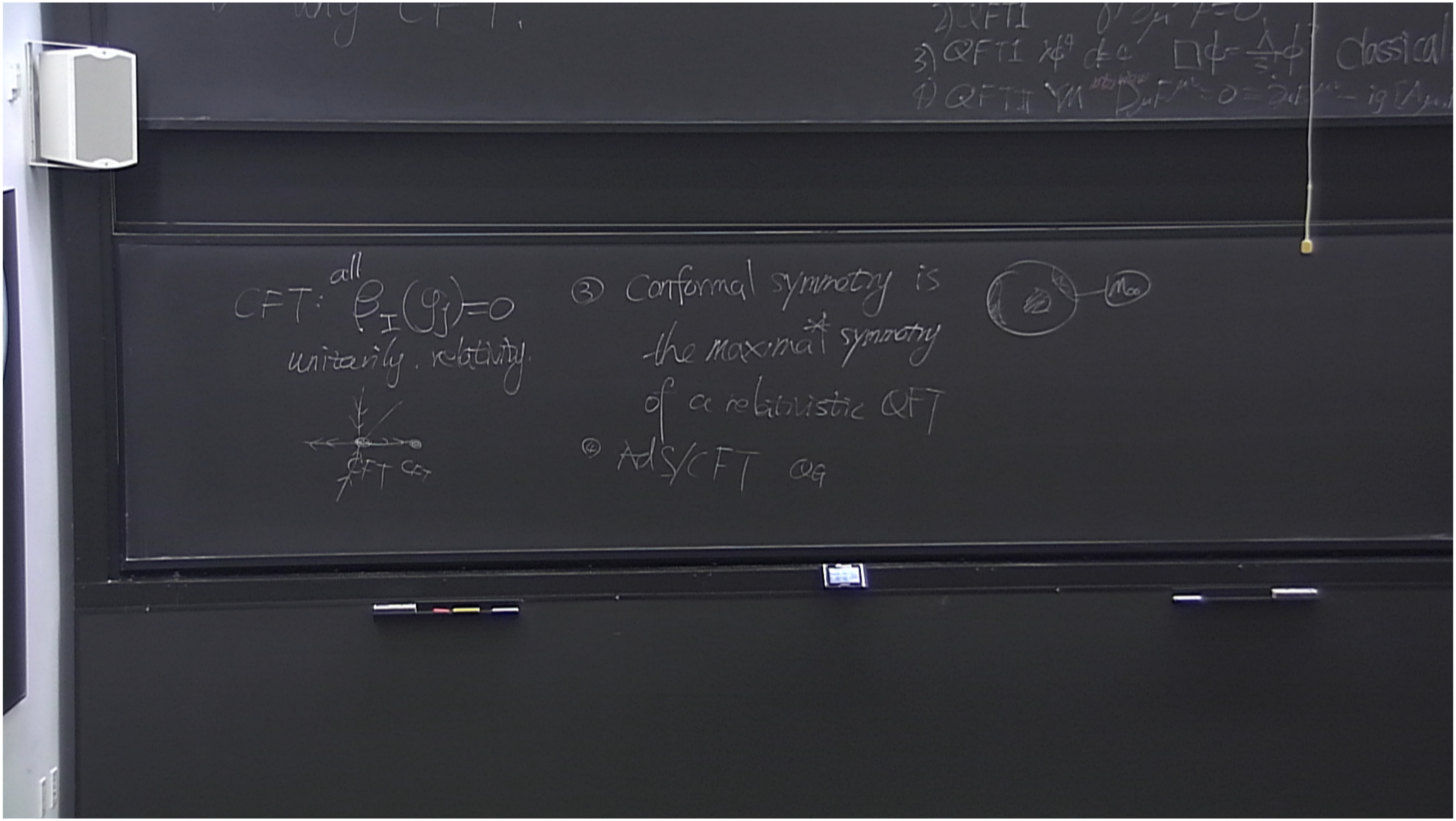
① $\phi(x)$  $\phi \to -\phi$

② $S = \int d^dx (\partial\phi)^2 + \sum_i g_i O_i$

③ dimensional analysis  finite are relev

④ $E_{ex}$ compare to experiments fix $g_i$s

RG flow



any CFT.

2) QFT I $= 0,$

3) QFT I $\lambda\phi^4$ $d=4$ $\Box\phi = \frac{\lambda}{3!}\phi^3$ classical

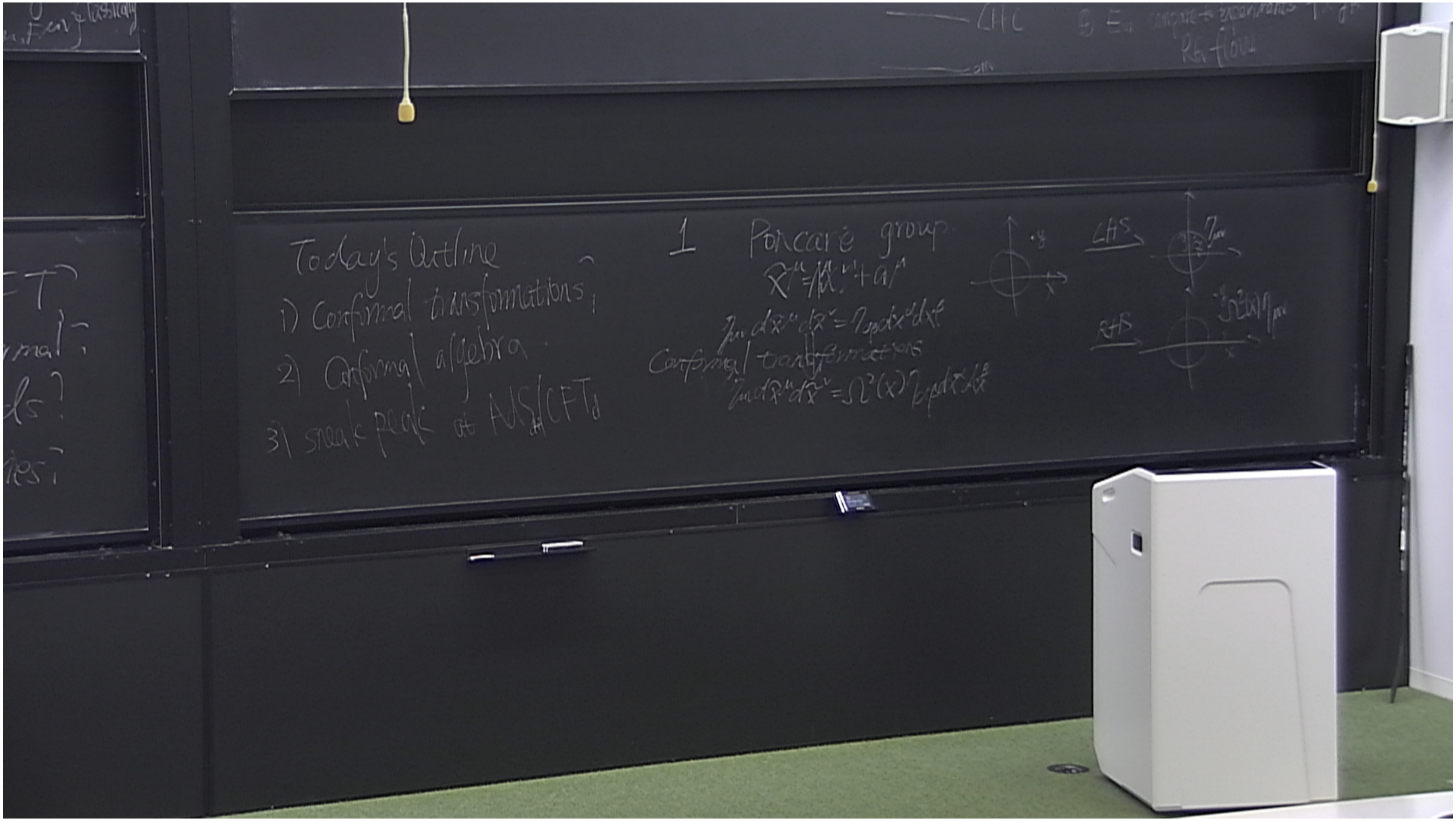
4) QFT II YM interaction $D_\mu F^{\mu\nu} = 0 = \partial_\mu F^{\mu\nu} - ig[A_\mu,$

CFT: all $\beta_I(g_j) = 0$

unitarily. relativity.

③ Conformal symmetry is the maximal* symmetry of a relativistic QFT

④ AdS/CFT QG



Today's Outline

1) Conformal transformations;

2) Conformal algebra

3) sneak peak at $AdS_{d+1}/CFT_d$

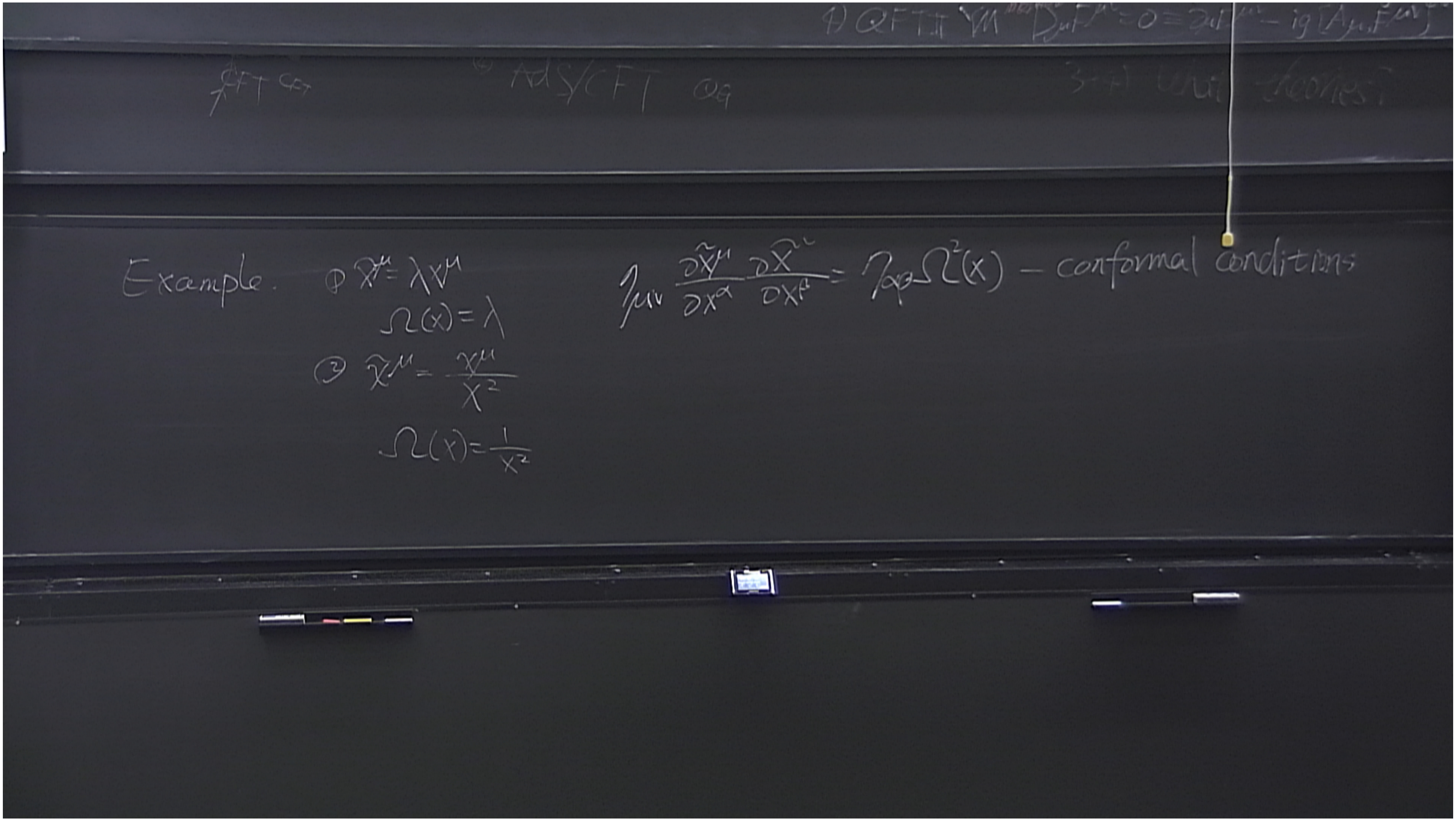
1 Poincaré group

$$\tilde{x}^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu$$

$$\eta_{\mu\nu} d\tilde{x}^\mu d\tilde{x}^\nu = \eta_{\alpha\beta} dx^\alpha dx^\beta$$

Conformal transformations

$$\eta_{\mu\nu} d\tilde{x}^\mu d\tilde{x}^\nu = \Omega^2(x)\eta_{\alpha\beta} dx^\alpha dx^\beta$$



④ QFT II YM $D_\mu F^{\mu\nu} = 0 \equiv \partial_\mu F^{\mu\nu} - ig[A_\mu, F^{\mu\nu}]$

CFT CFT ④ AdS/CFT QG 3+1) What theories?

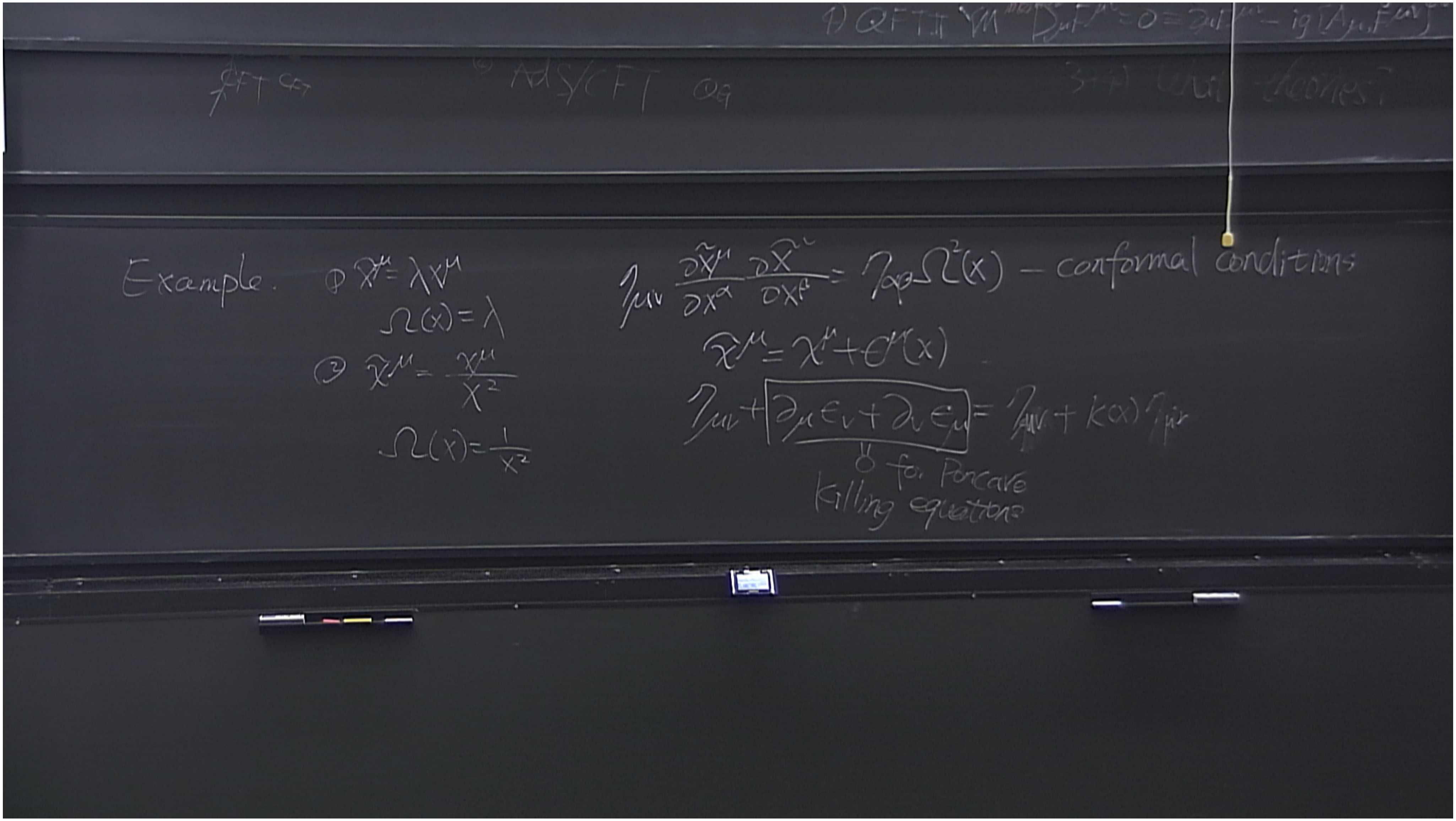
Example. ① $\tilde{x}^\mu = \lambda x^\mu$

$\Omega(x) = \lambda$

② $\tilde{x}^\mu = \frac{x^\mu}{x^2}$

$\Omega(x) = \frac{1}{x^2}$

$\eta_{\mu\nu} \frac{\partial \tilde{x}^\mu}{\partial x^\alpha} \frac{\partial \tilde{x}^\nu}{\partial x^\beta} = \eta_{\alpha\beta} \Omega^2(x)$ – conformal conditions



1) QFT II YM $D_\mu F^{\mu\nu}=0=\partial_\mu F^{\mu\nu}-ig[A_\mu,F^{\mu\nu}]$

CFT

AdS/CFT QG

3+1) What theories?

Example. ① $\tilde{x}^\mu=\lambda x^\mu$

$\Omega(x)=\lambda$

② $\tilde{x}^\mu=\frac{x^\mu}{x^2}$

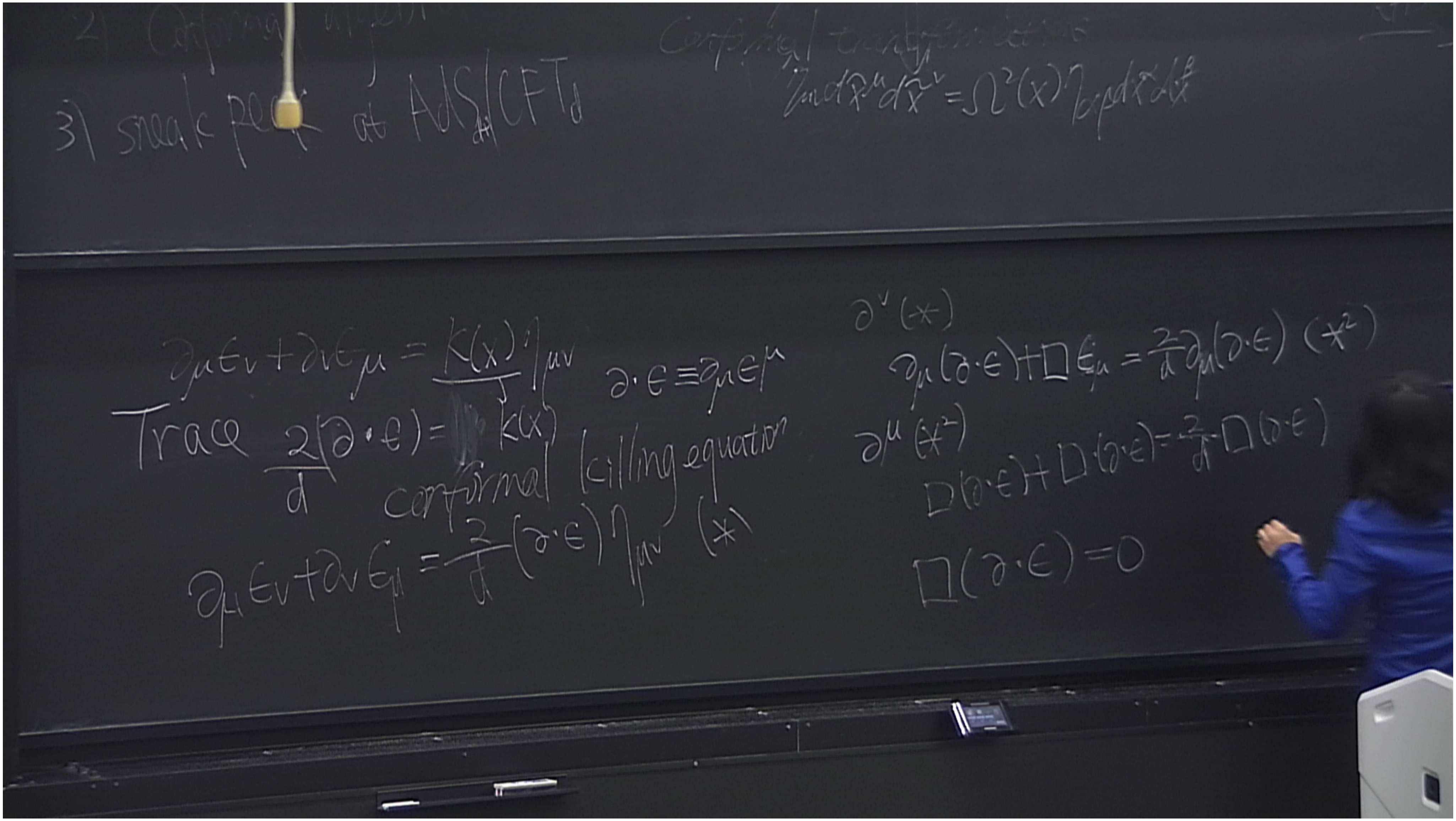
$\Omega(x)=\frac{1}{x^2}$

$$\eta_{\mu\nu}\frac{\partial\tilde{x}^\mu}{\partial x^\alpha}\frac{\partial\tilde{x}^\nu}{\partial x^\beta}=\eta_{\alpha\beta}\Omega^2(x)$$ – conformal conditions

$$\tilde{x}^\mu=x^\mu+\epsilon^\mu(x)$$

$$\eta_{\mu\nu}+\underbrace{\boxed{\partial_\mu\epsilon_\nu+\partial_\nu\epsilon_\mu}}_{0 \text{ for Poincare}}=\eta_{\mu\nu}+k(x)\eta_{\mu\nu}$$

Killing equations



2) Conformal algebra

3) sneak peek at $AdS_{d+1}/CFT_d$

Conformal transformations

$$\eta_{\mu\nu}\, d\tilde{x}^\mu d\tilde{x}^\nu = \Omega^2(x)\,\eta_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = \frac{K(x)}{d}\eta_{\mu\nu} \qquad \partial\cdot\epsilon \equiv \partial_\mu \epsilon^\mu$$

Trace $\frac{2(\partial\cdot\epsilon)}{d} = k(x)$

conformal killing equation

$$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = \frac{2}{d}(\partial\cdot\epsilon)\,\eta_{\mu\nu} \quad (*)$$

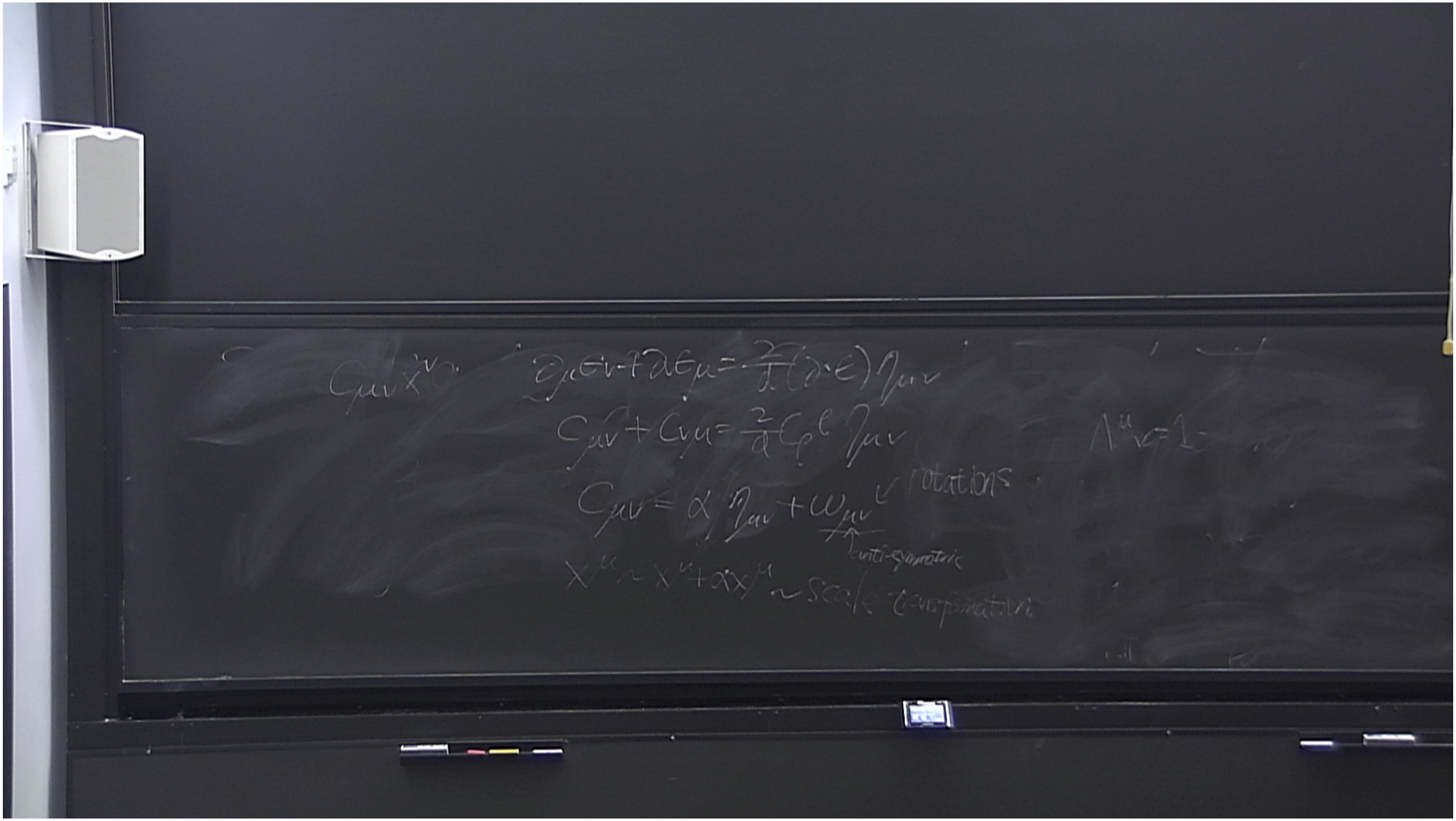
$\partial^\nu (*)$

$$\partial_\mu(\partial\cdot\epsilon) + \Box\,\epsilon_\mu = \frac{2}{d}\partial_\mu(\partial\cdot\epsilon) \quad (*^2)$$

$\partial^\mu (*^2)$

$$\Box(\partial\cdot\epsilon) + \Box\cdot(\partial\cdot\epsilon) = \frac{2}{d}\Box(\partial\cdot\epsilon)$$

$$\Box(\partial\cdot\epsilon) = 0$$



$C_{\mu\nu} x^{\nu}$

$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = \frac{2}{d}(\partial\cdot\epsilon)\eta_{\mu\nu}$

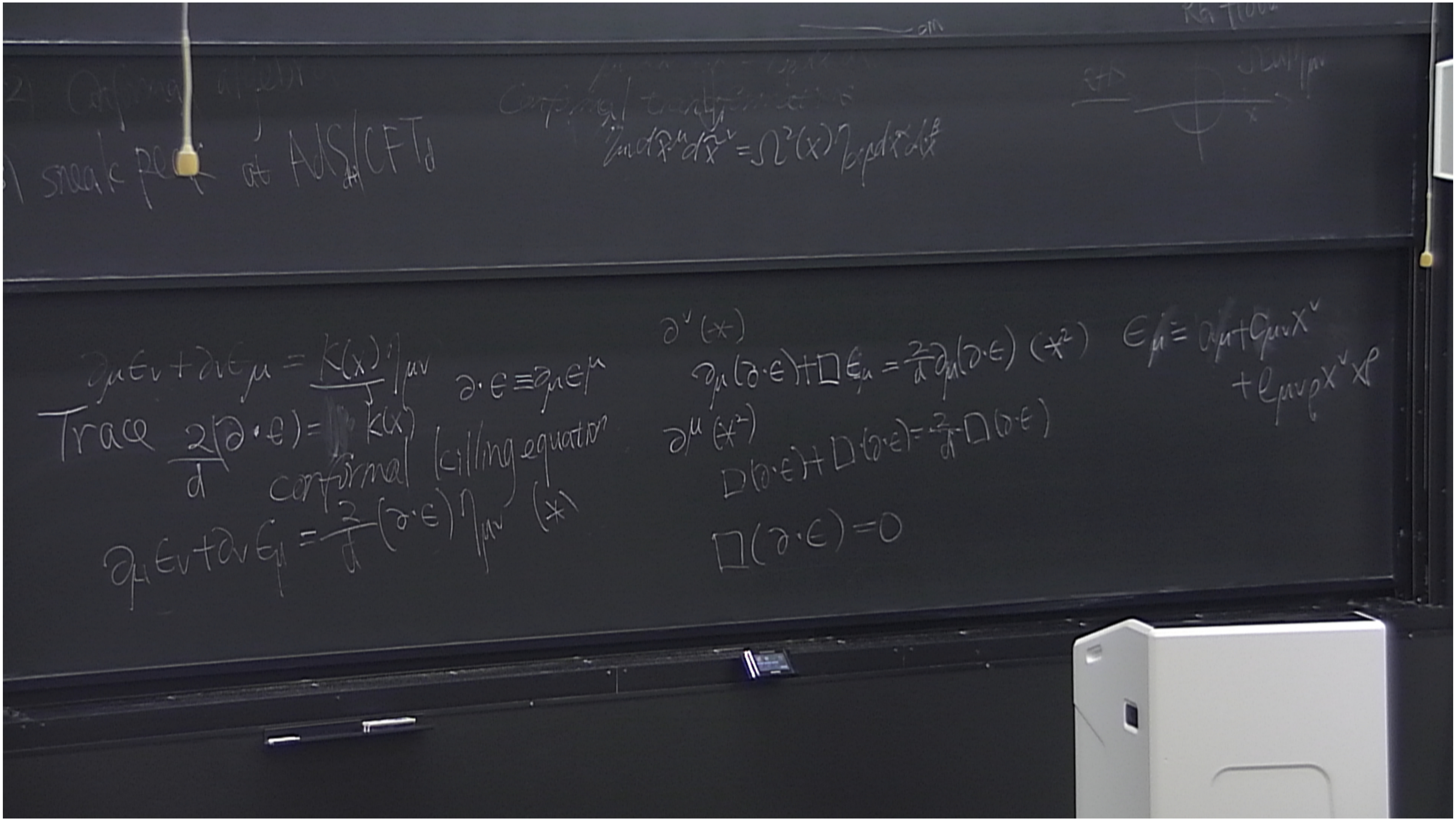
$C_{\mu\nu} + C_{\nu\mu} = \frac{2}{d} C_\rho^{\ \rho}\,\eta_{\mu\nu}$

$C_{\mu\nu} = \alpha\,\eta_{\mu\nu} + \omega_{\mu\nu}$ ✓ rotations

↑ anti-symmetric

$x^\mu \sim x^\mu + \alpha x^\mu$ ~ scale transformation

$\Lambda^\mu{}_\nu = 1 -$



2) Conformal algebra

3) sneak peek at $AdS_{d+1}/CFT_d$

Conformal transformations

$$\eta_{\mu\nu}\, d\tilde{x}^\mu d\tilde{x}^\nu = \Omega^2(x)\,\eta_{\rho\sigma}\, dx^\rho dx^\sigma$$

$$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = \frac{K(x)}{d}\eta_{\mu\nu} \qquad \partial\cdot\epsilon \equiv \partial_\mu \epsilon^\mu$$

Trace $\frac{2}{d}(\partial\cdot\epsilon) = K(x)$

conformal killing equation

$$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = \frac{2}{d}(\partial\cdot\epsilon)\eta_{\mu\nu} \quad (*)$$

$\partial^\nu (*)$

$$\partial_\mu(\partial\cdot\epsilon) + \Box\epsilon_\mu = \frac{2}{d}\partial_\mu(\partial\cdot\epsilon) \quad (*^2)$$

$\partial^\mu (*^2)$

$$\Box(\partial\cdot\epsilon) + \Box(\partial\cdot\epsilon) = \frac{2}{d}\Box(\partial\cdot\epsilon)$$

$$\Box(\partial\cdot\epsilon) = 0$$

$$\epsilon_\mu = a_\mu + b_{\mu\nu}x^\nu + c_{\mu\nu\rho}x^\nu x^\rho$$

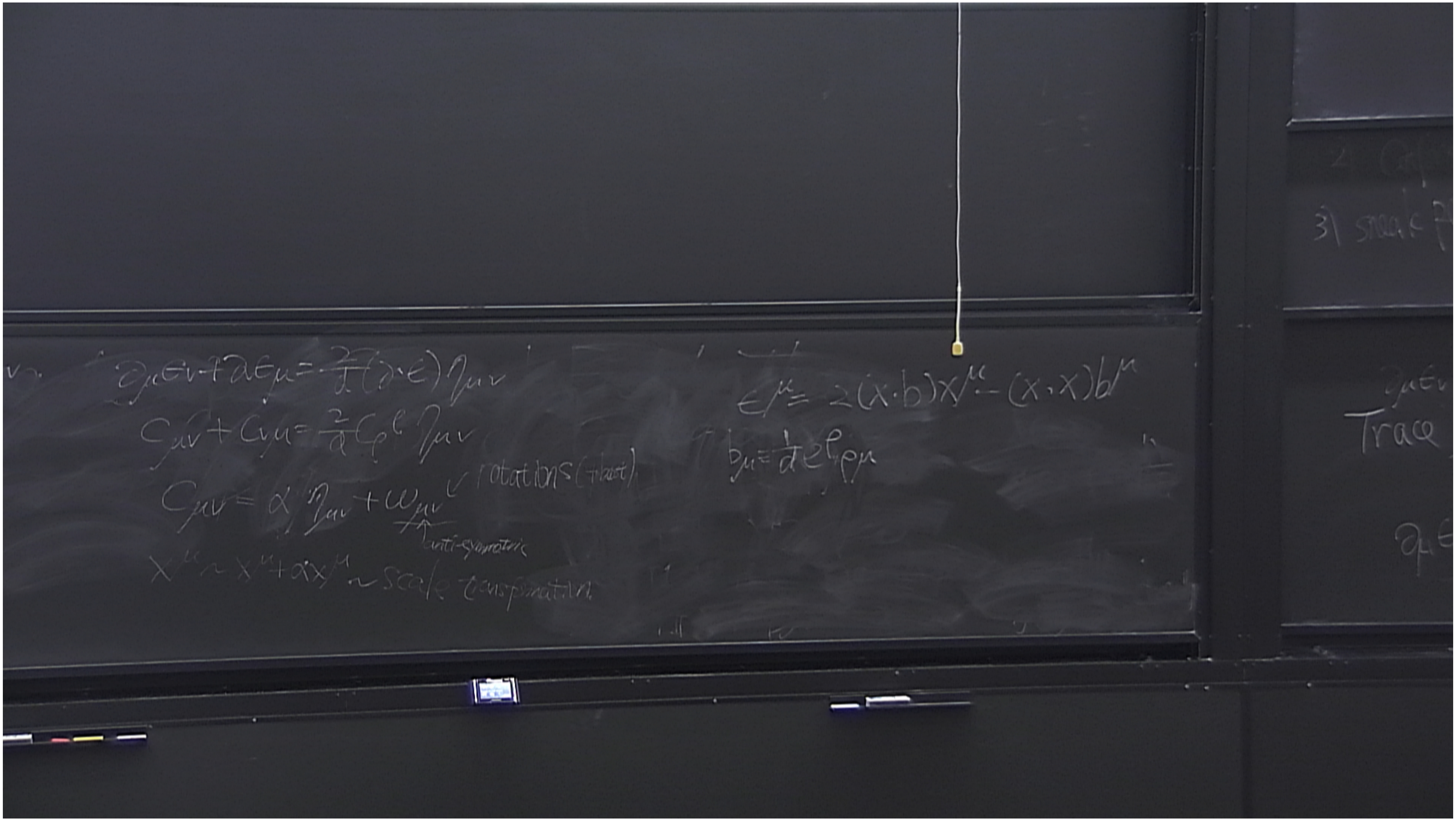
$c_{\mu\nu}x^{\nu}$

$$\partial_\mu\epsilon_\nu+\partial\epsilon_\mu=\frac{2}{d}(\partial\cdot\epsilon)\eta_{\mu\nu}$$

$$c_{\mu\nu}+c_{\nu\mu}=\frac{2}{d}c_\rho{}^{\rho}\eta_{\mu\nu}$$

$$c_{\mu\nu}=\alpha\,\eta_{\mu\nu}+\underbrace{\omega_{\mu\nu}}_{\text{anti-symmetric}}$$ ✓ rotations (+boost)

$x^\mu \sim x^\mu+\alpha x^\mu$ ~ scale transformation.



$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = \frac{2}{d}(\partial \cdot \epsilon)\eta_{\mu\nu}$

$c_{\mu\nu} + c_{\nu\mu} = \frac{2}{d} c_\rho{}^\rho \eta_{\mu\nu}$

$c_{\mu\nu} = \alpha\, \eta_{\mu\nu} + \underset{\uparrow \text{anti-symmetric}}{\omega_{\mu\nu}}$ ✓ rotations (+boost)

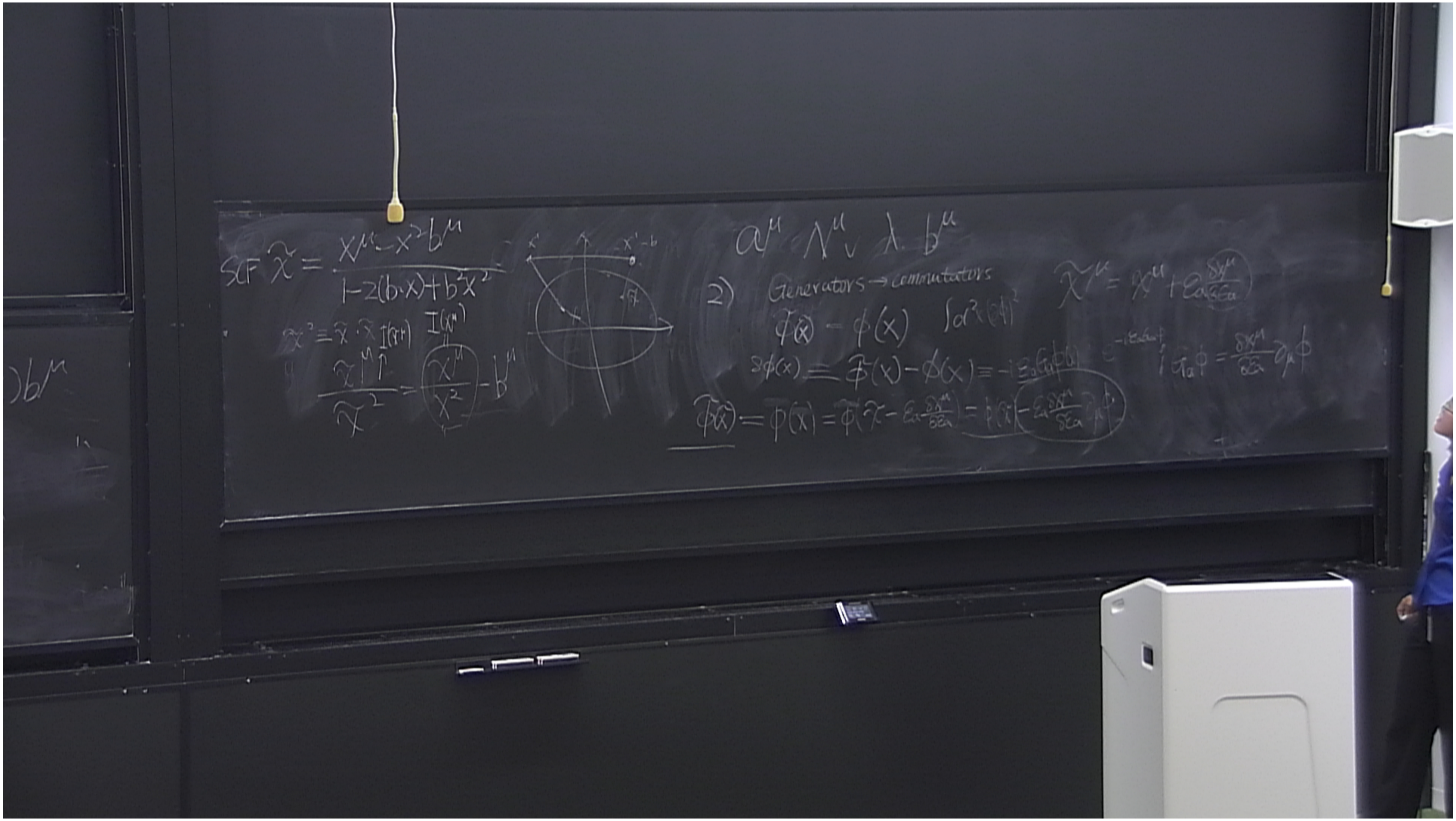
$x^\mu \sim x^\mu + \alpha x^\mu \sim$ scale transformation.

$\epsilon^\mu = 2(x \cdot b)x^\mu - (x \cdot x)b^\mu$

$b_\mu = \frac{1}{d} e_{\rho\mu}^{\rho}$

3) scale

Trace



$)b^\mu$

SCF $\tilde{x} = \frac{x^\mu - x^2 b^\mu}{1-2(b\cdot x)+b^2x^2}$

$\tilde{x}^2 \equiv \tilde{x}\cdot\tilde{x}$ $\quad I(\tilde{x}^\mu) \quad I(x^\mu)$

$\frac{\tilde{x}^\mu}{\tilde{x}^2} = \boxed{\frac{x^\mu}{x^2}} - b^\mu$

$a^\mu \quad \Lambda^\mu{}_\nu \quad \lambda. \quad b^\mu$

2) Generators → commutators

$\tilde{\Phi}(\tilde{x}) - \phi(x) \qquad \int d^2x\,(\partial\phi)^2$

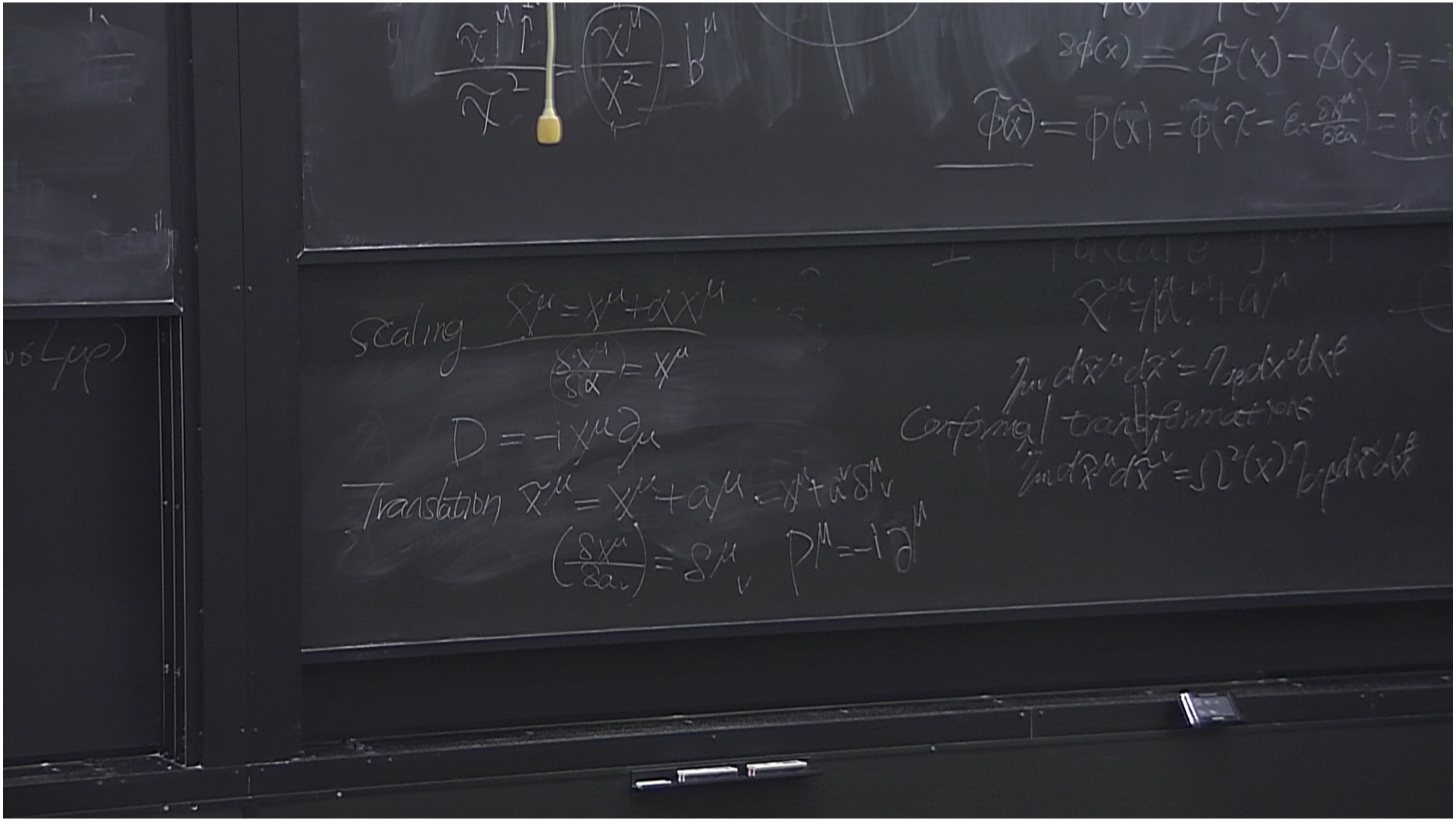
$\tilde{x}^\mu = x^\mu + \epsilon_a \frac{\delta x^\mu}{\delta \epsilon_a}$

$e^{-i\epsilon_a G_a}$

$iG_a\phi = \frac{\delta x^\mu}{\delta \epsilon_a}\partial_\mu \phi$

$\delta\phi(x) \equiv \tilde{\Phi}(x) - \phi(x) \equiv -i\epsilon_a G_a \phi(x)$

$\tilde{\Phi}(\tilde{x}) = \phi(x) = \phi\left(\tilde{x} - \epsilon_a \frac{\delta x^\mu}{\delta \epsilon_a}\right) = \phi(\tilde{x}) - \epsilon_a \frac{\delta x^\mu}{\delta \epsilon_a}\partial_\mu \phi$



$$\frac{\tilde{x}^\mu}{\tilde{x}^2} = \frac{x^\mu}{x^2} - b^\mu$$

$$\delta\phi(x) \equiv \tilde{\Phi}(x) - \phi(x) \equiv -$$

$$\tilde{\Phi}(\tilde{x}) = \phi(x) = \phi\left(\tilde{x} - \epsilon_a \frac{\delta x^\mu}{\delta \epsilon_a}\right) = \phi(\tilde{x}$$

Scaling $\tilde{x}^\mu = x^\mu + \alpha x^\mu$

$$\frac{\delta x^\mu}{\delta \alpha} = x^\mu$$

$$D = -i x^\mu \partial_\mu$$

Translation $\tilde{x}^\mu = x^\mu + a^\mu = x^\mu + a^\nu \delta^\mu{}_\nu$

$$\left(\frac{\delta x^\mu}{\delta a_\nu}\right) = \delta^\mu{}_\nu \quad P^\mu = -i\partial^\mu$$

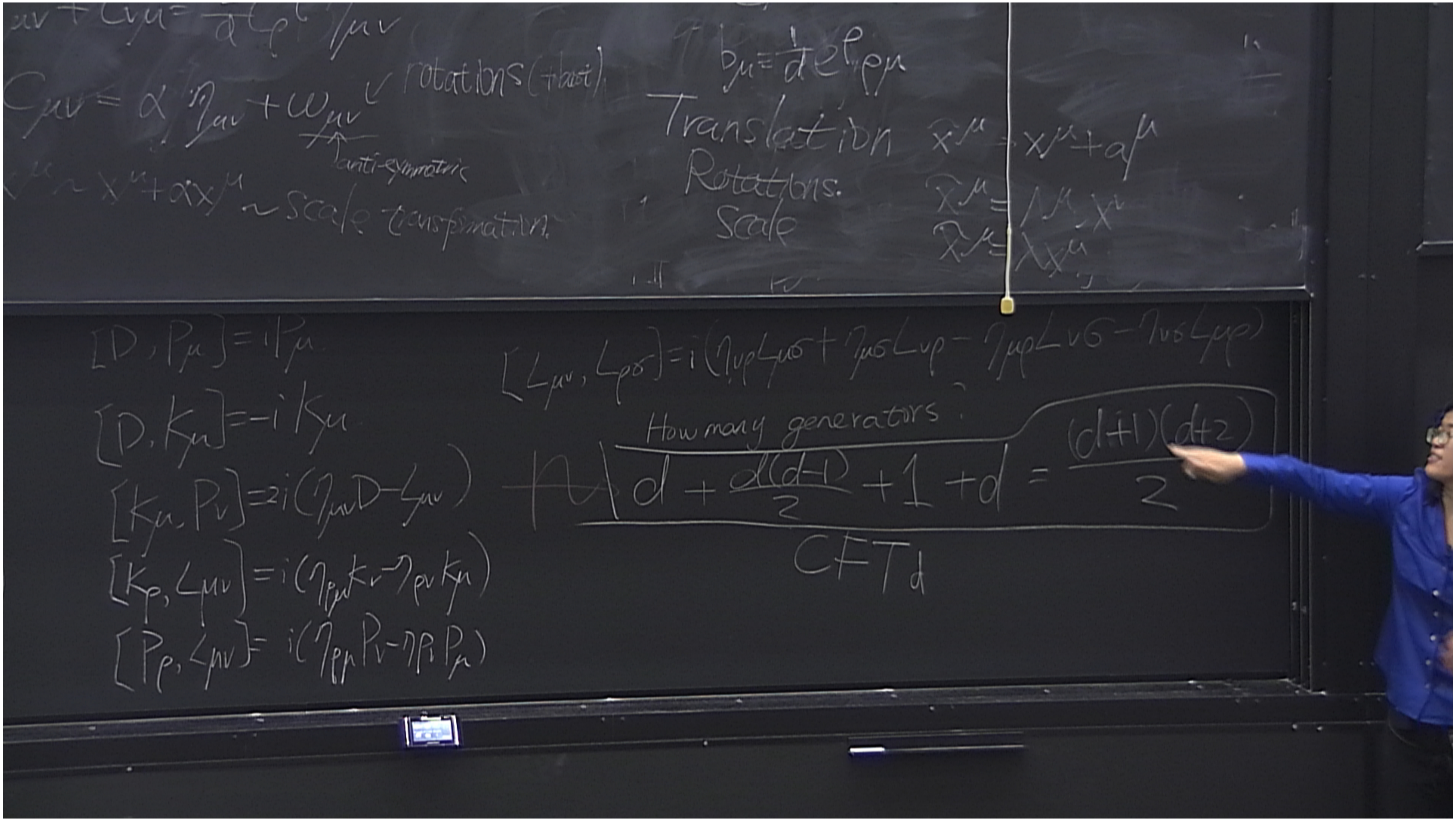
Poincaré group

$$\tilde{x}^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu$$

$$\eta_{\mu\nu} d\tilde{x}^\mu d\tilde{x}^\nu = \eta_{\alpha\beta} dx^\alpha dx^\beta$$

Conformal transformations

$$\eta_{\mu\nu} d\tilde{x}^\mu d\tilde{x}^\nu = \Omega^2(x) \eta_{\alpha\beta} dx^\alpha dx^\beta$$



$\ldots_{\mu\nu} + c_{\nu\mu} = \frac{\;}{d} c_\rho{}^\rho \eta_{\mu\nu}$

$c_{\mu\nu} = \alpha\, \eta_{\mu\nu} + \underset{\uparrow \text{anti-symmetric}}{\omega_{\mu\nu}}$ ✓ rotations (+boost)

$x^\mu \sim x^\mu + \alpha x^\mu \sim$ scale transformation.

$b_\mu = \frac{1}{d} e^{\rho}{}_{\rho\mu}$

Translation $\tilde{x}^\mu = x^\mu + a^\mu$

Rotations. $\tilde{x}^\mu = \Lambda^\mu{}_\nu x^\nu$

Scale $\tilde{x}^\mu = \lambda x^\mu$

$[D, P_\mu] = iP_\mu$

$[D, K_\mu] = -iK_\mu$

$[K_\mu, P_\nu] = 2i(\eta_{\mu\nu} D - L_{\mu\nu})$

$[K_\rho, L_{\mu\nu}] = i(\eta_{\rho\mu} K_\nu - \eta_{\rho\nu} K_\mu)$

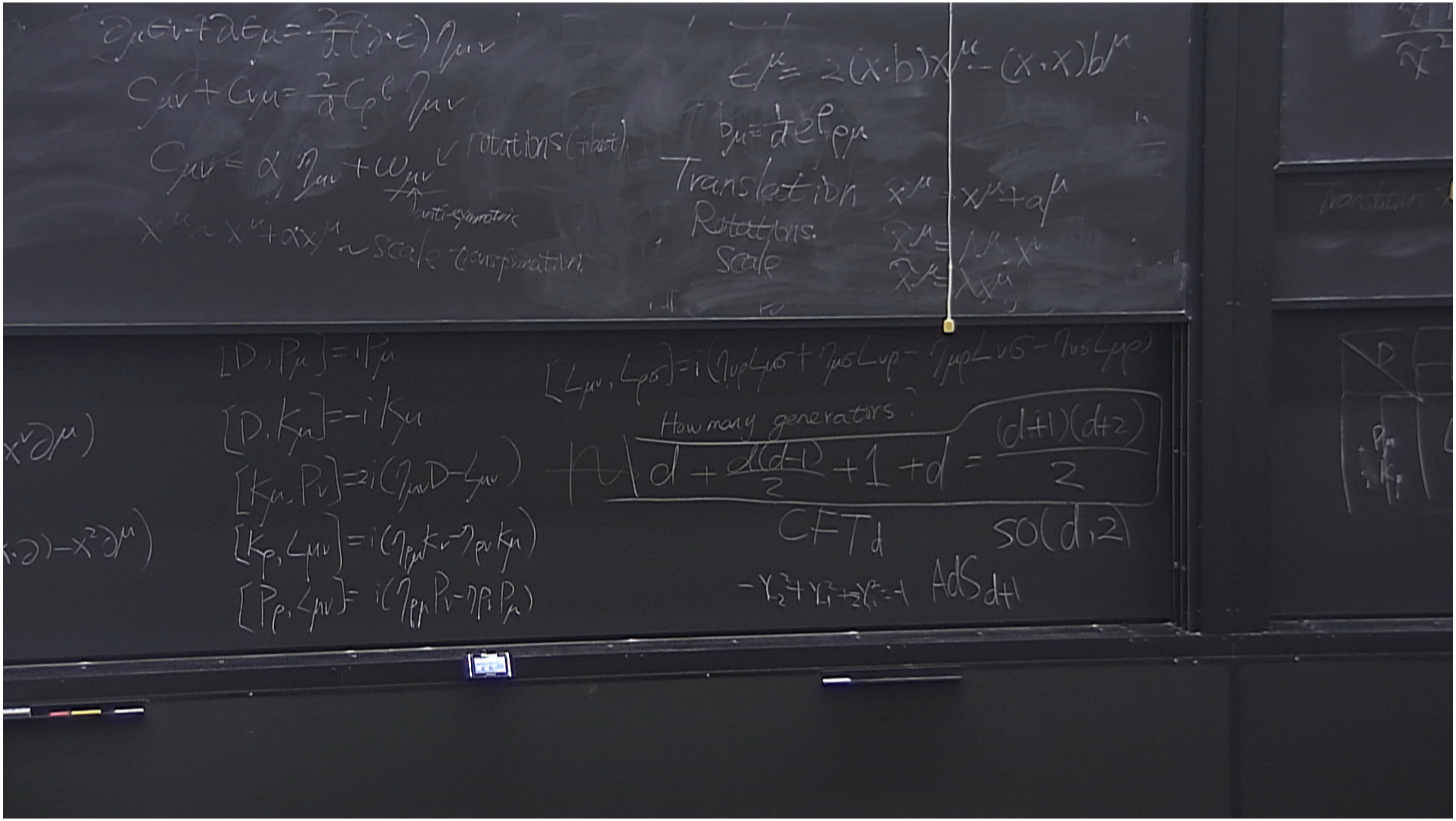
$[P_\rho, L_{\mu\nu}] = i(\eta_{\rho\mu} P_\nu - \eta_{\rho\nu} P_\mu)$

$[L_{\mu\nu}, L_{\rho\sigma}] = i(\eta_{\nu\rho} L_{\mu\sigma} + \eta_{\mu\sigma} L_{\nu\rho} - \eta_{\mu\rho} L_{\nu\sigma} - \eta_{\nu\sigma} L_{\mu\rho})$

How many generators?

$$d + \frac{d(d-1)}{2} + 1 + d = \frac{(d+1)(d+2)}{2}$$

$\mathrm{CFT}_d$



$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = \frac{2}{d}(\partial\cdot\epsilon)\eta_{\mu\nu}$

$C_{\mu\nu} + C_{\nu\mu} = \frac{2}{d} C_\rho{}^\rho \eta_{\mu\nu}$

$C_{\mu\nu} = \alpha\,\eta_{\mu\nu} + \underset{\uparrow}{\omega_{\mu\nu}}$ ✓ rotations (+boost)

anti-symmetric

$x'^\mu \sim x^\mu + \alpha x^\mu \sim$ scale transformation

$\epsilon^\mu = 2(x\cdot b)x^\mu - (x\cdot x)b^\mu$

$b_\mu = \frac{1}{d}\partial^\rho C_{\rho\mu}$

Translation $\tilde{x}^\mu = x^\mu + a^\mu$

Rotations. $\tilde{x}^\mu = \Lambda^\mu{}_\nu x^\nu$

Scale $\tilde{x}^\mu = \lambda x^\mu$

$x^\nu\partial^\mu)$

$\cdot\partial) - x^2\partial^\mu)$

$[D, P_\mu] = iP_\mu$

$[D, K_\mu] = -iK_\mu$

$[K_\mu, P_\nu] = 2i(\eta_{\mu\nu}D - L_{\mu\nu})$

$[K_\rho, L_{\mu\nu}] = i(\eta_{\rho\mu}K_\nu - \eta_{\rho\nu}K_\mu)$

$[P_\rho, L_{\mu\nu}] = i(\eta_{\rho\mu}P_\nu - \eta_{\rho\nu}P_\mu)$

$[L_{\mu\nu}, L_{\rho\sigma}] = i(\eta_{\nu\rho}L_{\mu\sigma} + \eta_{\mu\sigma}L_{\nu\rho} - \eta_{\mu\rho}L_{\nu\sigma} - \eta_{\nu\sigma}L_{\mu\rho})$

How many generators?

$$\boxed{d + \frac{d(d-1)}{2} + 1 + d = \frac{(d+1)(d+2)}{2}}$$

$CFT_d$ $\quad so(d,2)$

$-Y_{-2}^2 + Y_{-1}^2 + \sum Y_i^2 = -1$ $\quad AdS_{d+1}$

Translation

$D$

$P_\mu$

$K_\mu$