

Title: Decoherence as a way to measure extremely soft collisions with Dark Matter

Date: Nov 06, 2015 01:00 PM

URL: <http://pirsa.org/15110083>

Abstract: <p>Quantum superpositions of matter are unusually sensitive to decoherence by tiny momentum transfers, in a way that can be made precise with a new diffusion standard quantum limit. Upcoming matter interferometers will produce unprecedented spatial superpositions of over a million nucleons. What sorts of dark matter scattering events could be seen in these experiments as anomalous decoherence? We show that it is extremely weak but medium range interaction between matter and dark matter that would be most visible, such as scattering through a Yukawa potential. We construct toy models for these interactions, discuss existing constraints, and delineate the expected sensitivity of forthcoming experiments. In particular, the OTIMA interferometer developing at the University of Vienna will directly probe many orders of magnitude of parameter space, and the proposed MAQRO satellite experiment would be vastly more sensitive yet. This is a multidisciplinary talk that will be accessible to a non-specialized audience.</p>

# DECOHERENCE FROM EXTREMELY SOFT COLLISIONS WITH DARK MATTER



**C. Jess Riedel**  
with **Itay Yavin**

PRA **92**, 010101(R) (2015)  
PRD **88**, 116005 (2013)  
arXiv 2015 (forthcoming...)

Slides/video/blog at [jessriedel.com](http://jessriedel.com)

Perimeter Institute

6 November 2015



# Orientation



Particle  
physics

Me

Large superposition  
experiments

(matter interferometers,  
nanomechanical resonators)

Quantum foundations

(decoherence, consistent histories)

# Outline



- Initial motivation: bowling balls and ping-pong balls
- New SQL for detecting diffusion/decoherence

# Outline



- Initial motivation: bowling balls and ping-pong balls
- New SQL for detecting diffusion/decoherence
- Ideal searches for decoherence-based methods:
  - Tiny momentum transfers
  - Soft — not weak — interactions
- Candidate: Dark matter scattering through a medium-range ( $\sim 50$  nm) mediator

# Outline



- Initial motivation: bowling balls and ping-pong balls
- New SQL for detecting diffusion/decoherence
- Ideal searches for decoherence-based methods:
  - Tiny momentum transfers
  - Soft — not weak — interactions
- Candidate: Dark matter scattering through a medium-range ( $\sim 50$  nm) mediator
- Sensitivity results

# Bowling balls and ping-pong balls

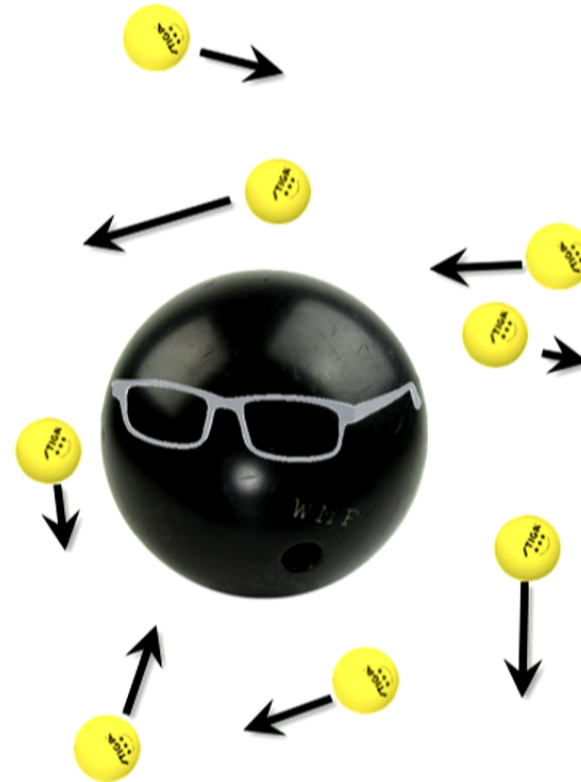


- Suppose everything in the universe—including us—were made of bowling balls



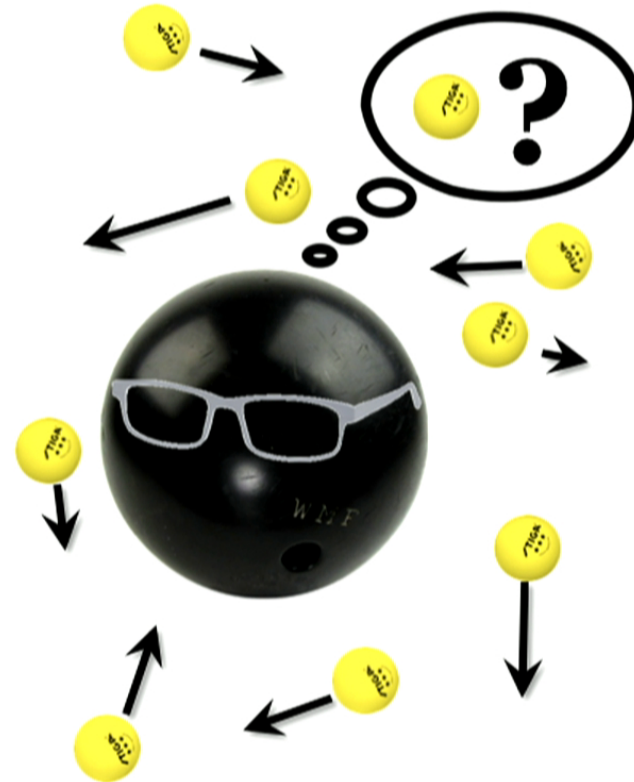
# Bowling balls and ping-pong balls

- Suppose everything in the universe—including us—were made of bowling balls
- Now suppose we were surrounded by a sea of slow-moving ping-pong balls



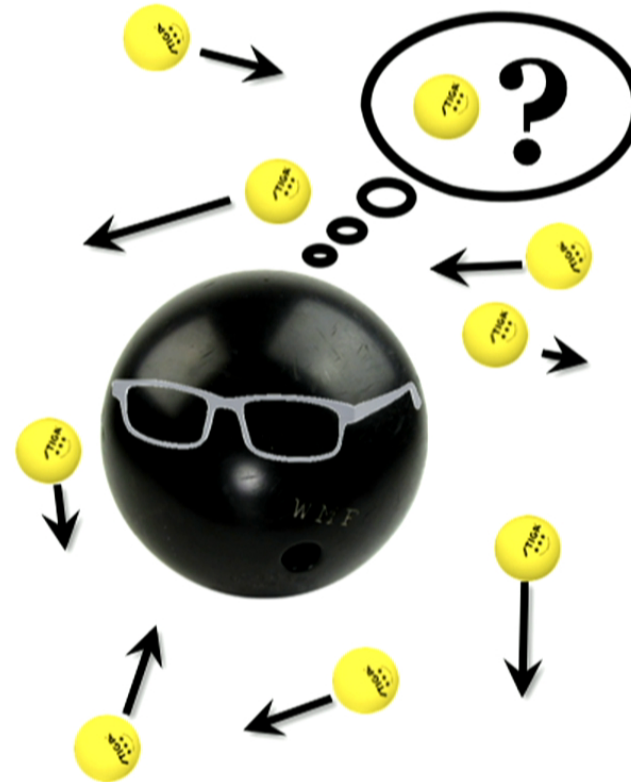
# Bowling balls and ping-pong balls

- Suppose everything in the universe—including us—were made of bowling balls
- Now suppose we were surrounded by a sea of slow-moving ping-pong balls
  - Could we ever tell?



# Bowling balls and ping-pong balls

- Is it possible to influence without being influenced?
- Can there be new degrees of freedom that see and feel us, but which we can't detect? ?
  - Dark matter
  - Supersymmetry
  - New neutrinos
  - Fifth forces
  - ...





# “Force” Standard Quantum Limit



- Suppose we need to measure a weak force  $F$  during a short time period  $T$  acting on a probe  $M$ 
  - For example: gravitational waves, for which the time-averaged force is zero



# “Force” Standard Quantum Limit



- Suppose we need to measure a weak force  $F$  during a short time period  $T$  acting on a probe  $M$ 
  - For example: gravitational waves, for which the time-averaged force is zero

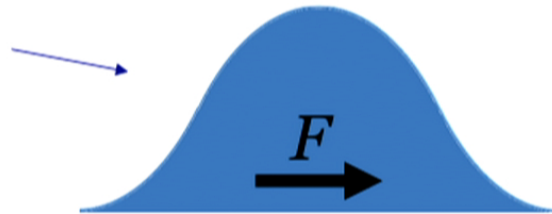


# “Force” Standard Quantum Limit



- Suppose further that we are restricted to position (or position-like) measurements
  - Make initial position measurement
  - Wait time  $T$
  - Make final position measurement

Probe  
wavepacket

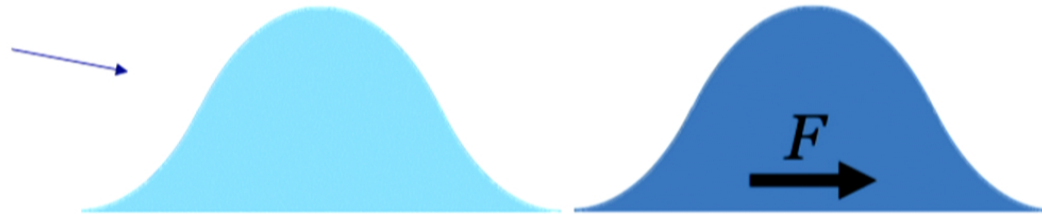


# “Force” Standard Quantum Limit



- Suppose further that we are restricted to position (or position-like) measurements
  - Make initial position measurement
  - Wait time  $T$
  - Make final position measurement

Probe  
wavepacket

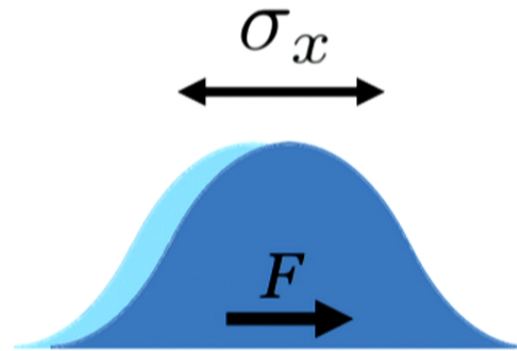


# Force Standard Quantum Limit



- For sufficiently weak forces, the wavepacket is simply not displaced enough to be detectable

$$\Delta x_F = \frac{T^2 F}{2M} \ll \sigma_x$$

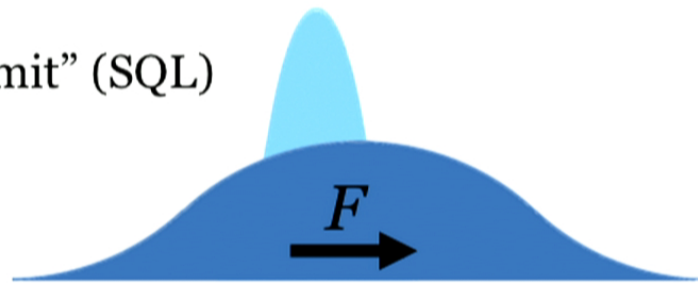


# Force Standard Quantum Limit



- For sufficiently weak forces, the wavepacket is simply not displaced enough to be detectable
- Narrowing the initial wavepacket does not help past a certain point:
  - Smaller initial width causes *faster* spreading during the time interval
- For optimal width, there is a smallest measurable displacement
  - The “standard quantum limit” (SQL)

$$\Delta x_{\text{SQL}} = \sqrt{\frac{\hbar T}{M}}$$



# Force Standard Quantum Limit



- Smallest measurable displacement implies weakest measurable force

$$\Delta x_F = \frac{T^2 F}{2M} \quad \longrightarrow \quad F_{\text{SQL}} = 2\sqrt{\frac{\hbar M}{T^3}}$$
$$\Delta x_{\text{SQL}} = \sqrt{\frac{\hbar T}{M}}$$

# Force Standard Quantum Limit



- Smallest measurable displacement implies weakest measurable force

$$\Delta x_F = \frac{T^2 F}{2M} \quad \longrightarrow \quad F_{\text{SQL}} = 2\sqrt{\frac{\hbar M}{T^3}}$$
$$\Delta x_{\text{SQL}} = \sqrt{\frac{\hbar T}{M}}$$

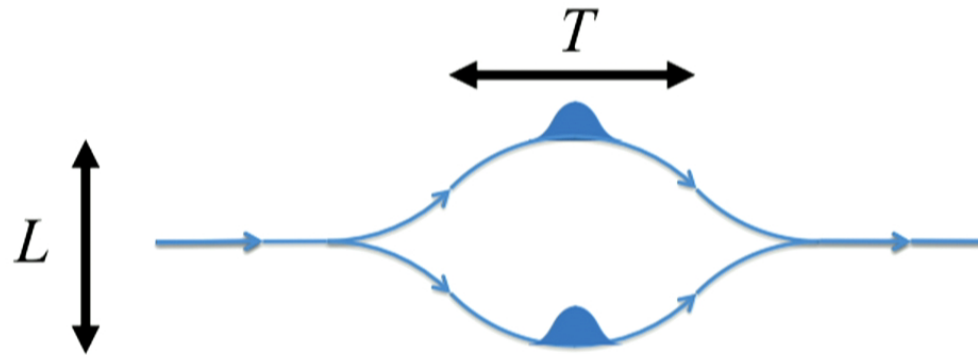
- This is based on a given probe mass  $M$  and a given time interval  $T$
- Crucially, this assumes a particular preparation and measurement



# Beating the force SQL



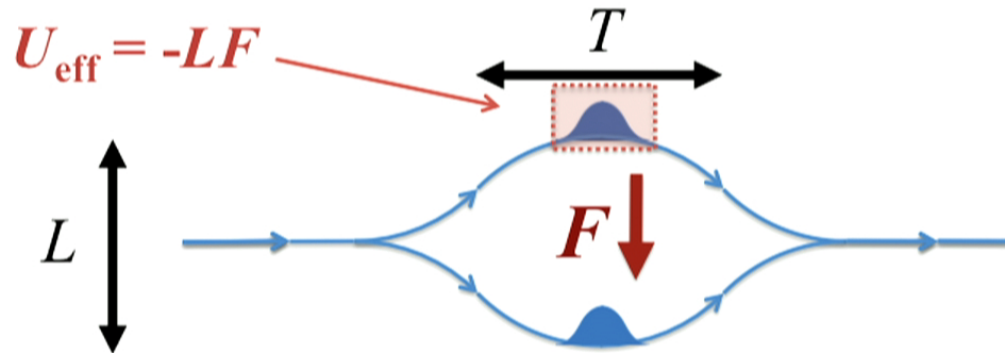
- Alternative: produce superposition of widely separated wavepackets in an interferometer



# Beating the force SQL



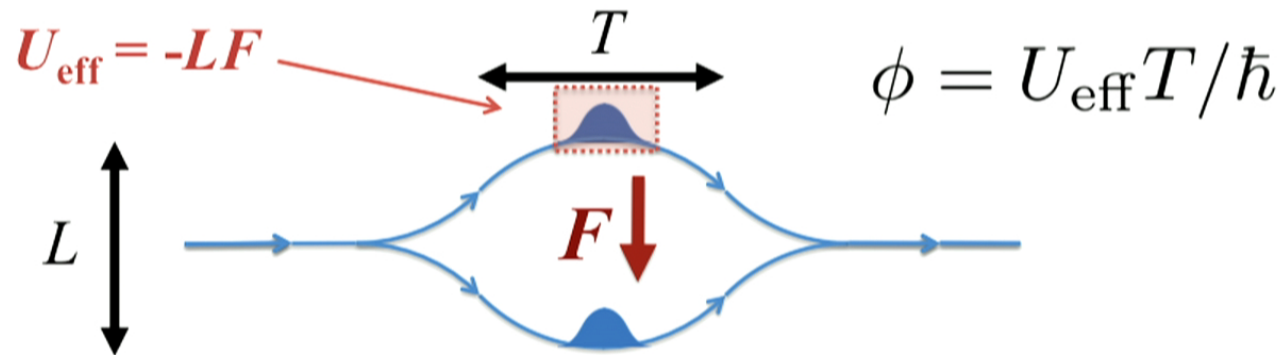
- Alternative: produce superposition of widely separated wavepackets in an interferometer
- The weak force is associated with a potential energy difference  $U_{\text{eff}}$  between arms



# Beating the force SQL



- In principle can measure *arbitrarily* weak forces for sufficiently wide wavepacket separation  $L$



# Force versus diffusion SQL

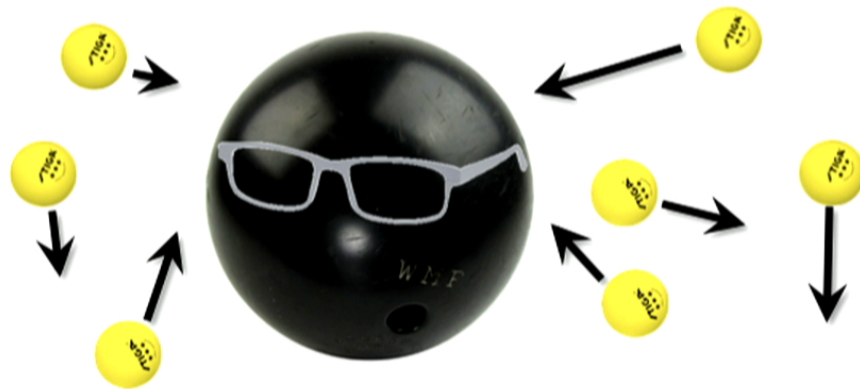


- Gravitational waves are weak *classical* forces

# Force versus diffusion SQL



- Gravitational waves are weak *classical* forces
- A classical influence on a quantum probe can always be modeled as a unitary transformation
- But Brownian baths and other sources of diffusion and decoherence cannot be modeled as unitary
  - Rather, the sources may become entangled with the probe



# Force versus diffusion SQL



- We can prove an analogous limit for detecting collisional decoherence as for detecting weak forces
  - PRA **92**, 010101(R) (2015)

# Force versus diffusion SQL



- We can prove an analogous limit for detecting collisional decoherence as for detecting weak forces
  - PRA **92**, 010101(R) (2015)
- The prototypical source of collisional decoherence is quantum Brownian motion (QBM)

# Force versus diffusion SQL



- We can prove an analogous limit for detecting collisional decoherence as for detecting weak forces
  - PRA **92**, 010101(R) (2015)
- The prototypical source of collisional decoherence is quantum Brownian motion (QBM)
- In the appropriate special case, the idealized probe dynamics are

$$\partial_t \rho = -i[\hat{H}, \rho] - \frac{D}{2}[\hat{x}, [\hat{x}, \rho]]$$

$$\hat{H} = \frac{\hat{p}^2}{2m} - F\hat{x}$$



# Force versus diffusion SQL



- $F$  parameterizes the strength of the force
- $D$  parameterizes the strength of diffusion and decoherence
  - $D$  induces random walk (diffusion) in momentum space
  - For terms sufficiently far off  $\rho$ 's main diagonal that  $H$  can be ignored, we get pure decoherence

$$\partial_t \rho = -i[\hat{H}, \rho] - \frac{D}{2}[\hat{x}, [\hat{x}, \rho]]$$

$$\rho_t(x, x') \approx \rho_0(x, x') e^{-Dt(x-x')^2} \quad \text{for large } x - x'$$

# Force versus diffusion SQL



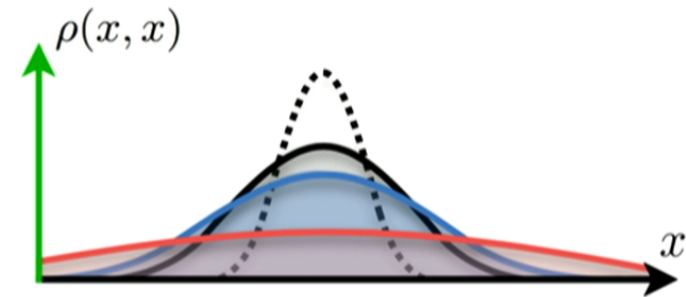
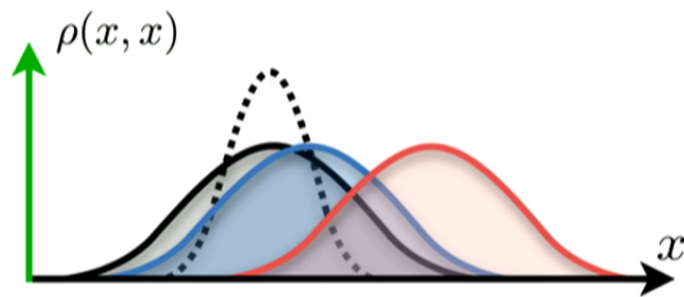
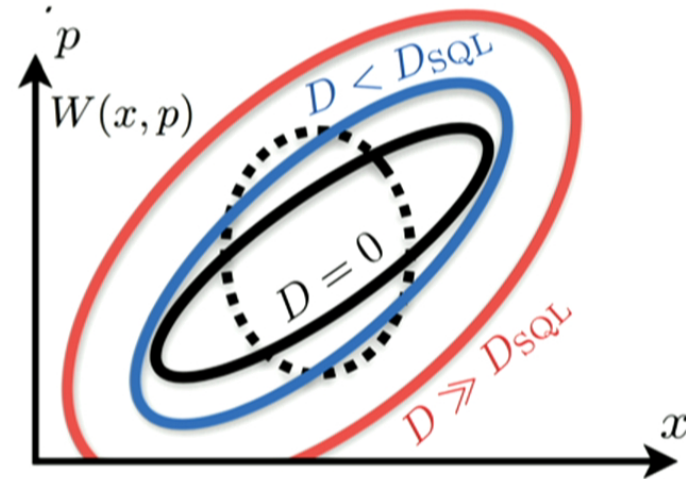
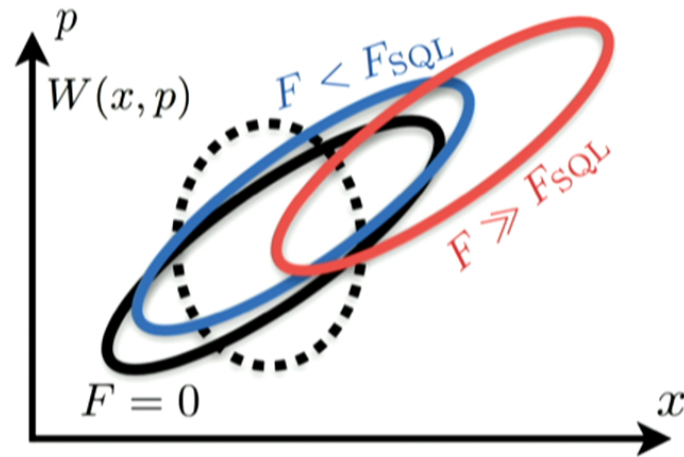
- Along with  $F$ , the coefficient  $D$  is one of a handful of variables which parameterize a class of Markovian dynamics that
  - generalize the Harmonic oscillator to open system and
  - are uniquely preferred by symplectic symmetry

# Force versus diffusion SQL



- Along with  $F$ , the coefficient  $D$  is one of a handful of variables which parameterize a class of Markovian dynamics that
  - generalize the Harmonic oscillator to open system and
  - are uniquely preferred by symplectic symmetry
- “Symplectic covariant” quantum Brownian motion
  - Very satisfying topic if you want to get better intuition for the Wigner function and fully-general linearized evolution in phase space
  - Makes decoherence-diffusion connection transparent
  - See PRA, appendix to arXiv:1507.04083, and citations therein

# SQLs in phase space



# Force versus diffusion SQL



- Just like for the traditional force SQL, diffusion SQL can be beaten by non-classical preparations like cat states and squeezed states

# Force versus diffusion SQL

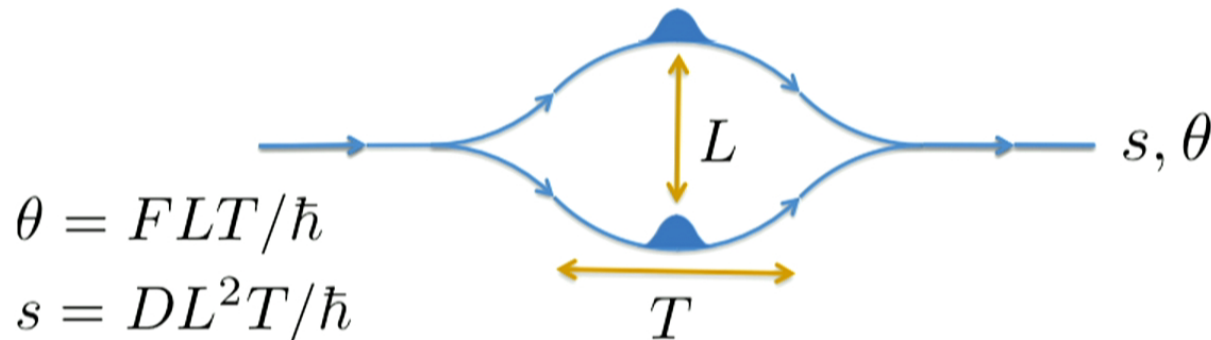
- Just like for the traditional force SQL, diffusion SQL can be beaten by non-classical preparations like cat states and squeezed states

- Cats in “wavepacket basis”:

$$\begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \rightarrow \begin{pmatrix} \rho_{11} & e^{-s+i\theta} \rho_{12} \\ e^{-s-i\theta} \rho_{21} & \rho_{22} \end{pmatrix}$$

Phase (force)

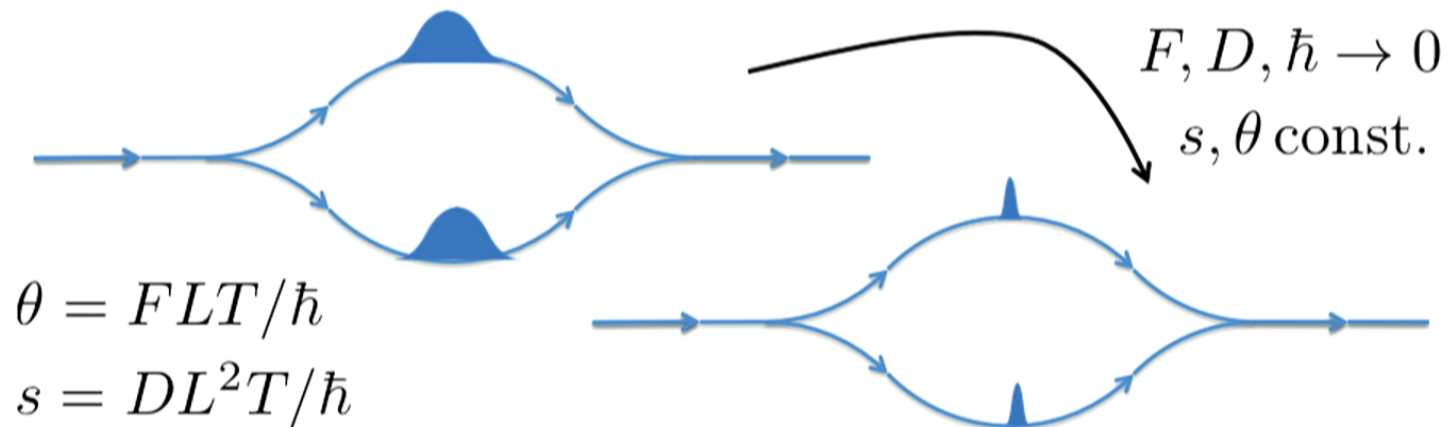
Decoherence (diffusion)



# Classical undetectability



- These can detect *classically undetectable* phenomena
  - In  $\hbar \rightarrow 0$  limit, wavepackets become points in phase space
  - Can simultaneously take  $D$  and  $F \rightarrow 0$ , while phase shift and decoherence remain finite.



# Classical undetectability

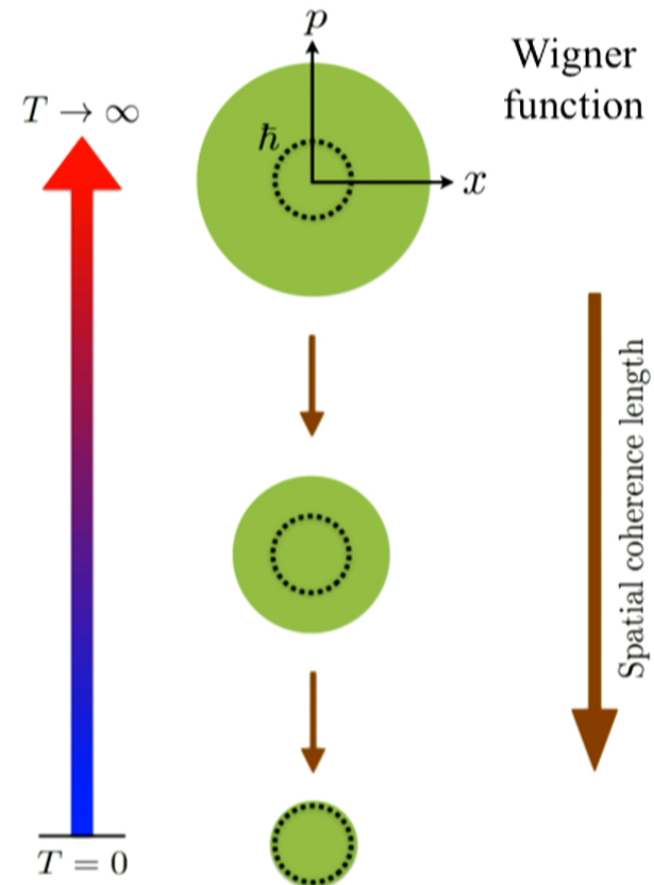


- These can detect *classically undetectable* phenomena
  - In  $\hbar \rightarrow 0$  limit, wavepackets become points in phase space
  - Can simultaneously take  $D$  and  $F \rightarrow 0$ , while phase shift and decoherence remain finite.
- **Large** ( $\sim$ mesoscopic) superpositions are uniquely sensitive detectors



# Classical-to-quantum sensitivity

- More generally:  
Increased sensitivity to  
small momentum  
transfers comes as  
soon as you start  
cooling down



# Classical-to-quantum sensitivity



- Once ground state is reached – the SQL – further sensitivity requires non-classical states

$T = 0$

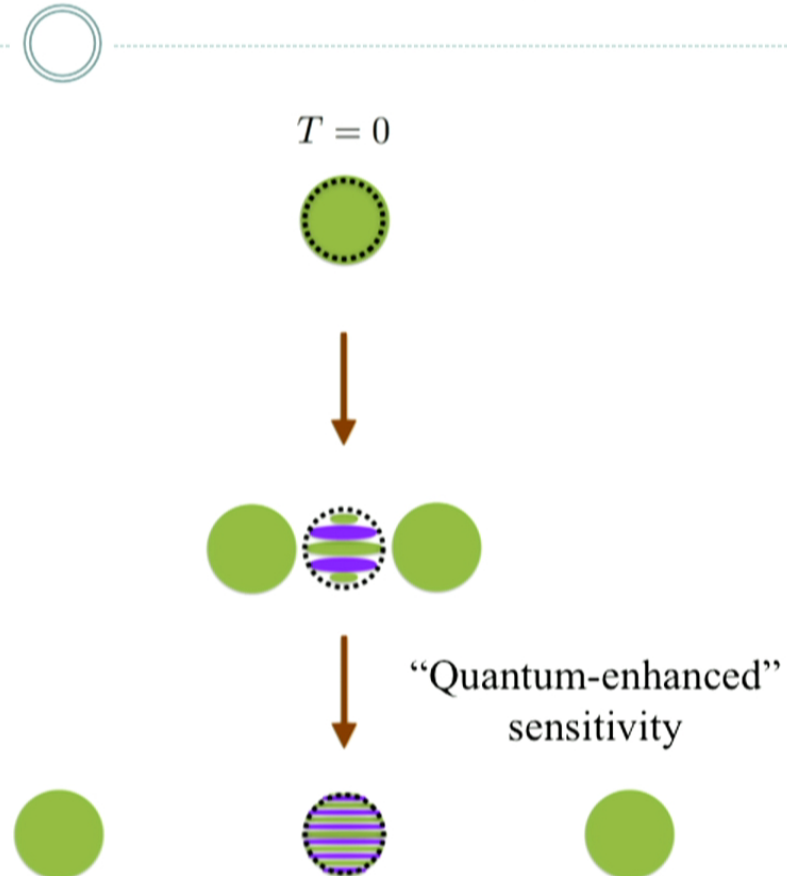


← SQL  
sensitivity

# Classical-to-quantum sensitivity

- Once ground state is reached – the SQL – further sensitivity requires non-classical states

- Cat states

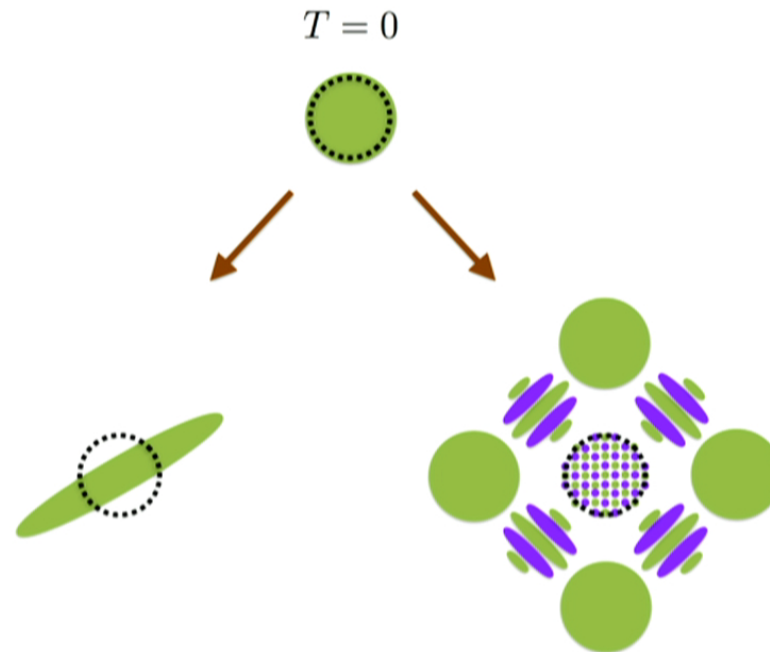


# Classical-to-quantum sensitivity



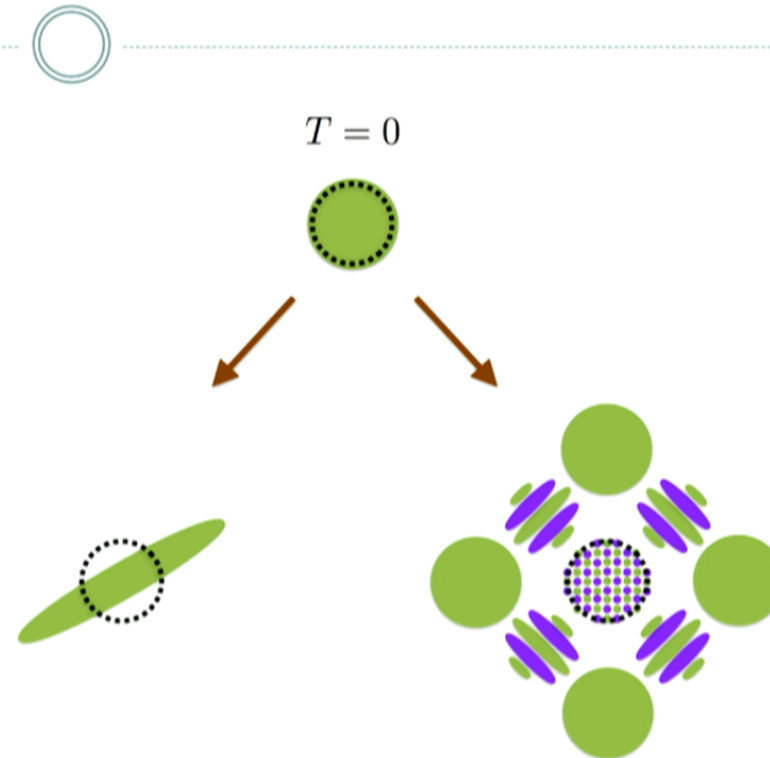
- Once ground state is reached – the SQL – further sensitivity requires non-classical states

- Cat states
- Squeezed states
- More exotic...



# Classical-to-quantum sensitivity

- Once ground state is reached – the SQL – further sensitivity requires non-classical states
  - Cat states
  - Squeezed states
  - More exotic...
- Key feature: spatial coherence
  - Manifests as fine structure in Wigner function



# Force versus diffusion SQL



- Sensitivity of cat states is proportional to  $L^2$

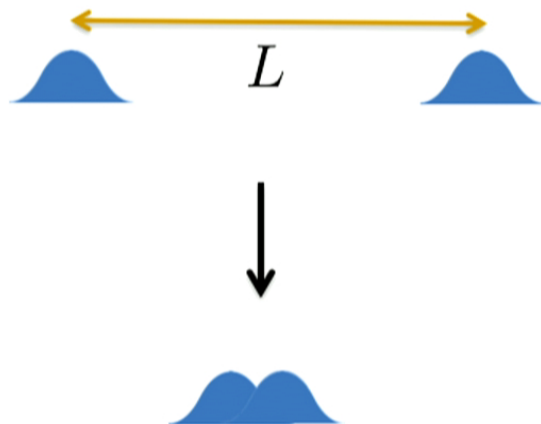


$$D_{\text{sense}} = \frac{\hbar^2}{L^2 T}$$


# Force versus diffusion SQL



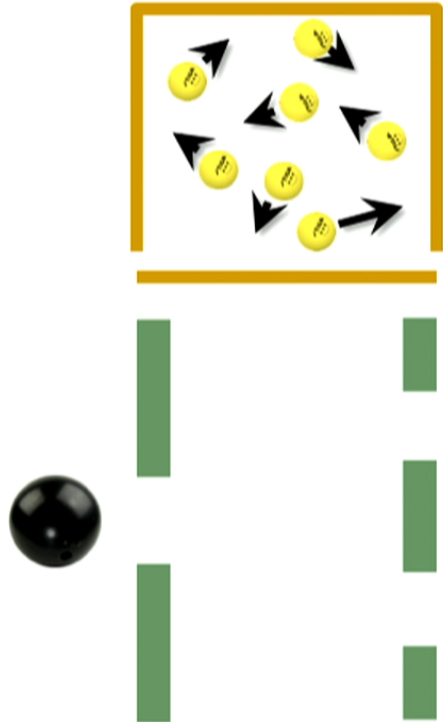
- Sensitivity of cat states is proportional to  $L^2$
- Reduces to SQL sensitivity as  $L$  goes to zero



$$D_{\text{sense}} = \frac{\hbar^2}{L^2 T}$$


$$D_{\text{SQL}}$$

# Bowling-ball interferometry





# Solution in search of a problem



- Quantum superpositions can detect momentum transfer that can't be detected any other way
- 1: What sorts of new particles and forces fit the bill?
  - *High* flux particles scattering *elastically* and *often* but transferring *tiny* amounts of momentum
- 2: What's best superposition probe (target)? Some considerations:
  - Momentum sensitivity determined by superposition separation (coherence length), not recoil
  - Heavier targets reduce recoil but increase scattering cross-section
  - Important: “coherent elastic scattering”

## Aside: Coherent elastic scattering

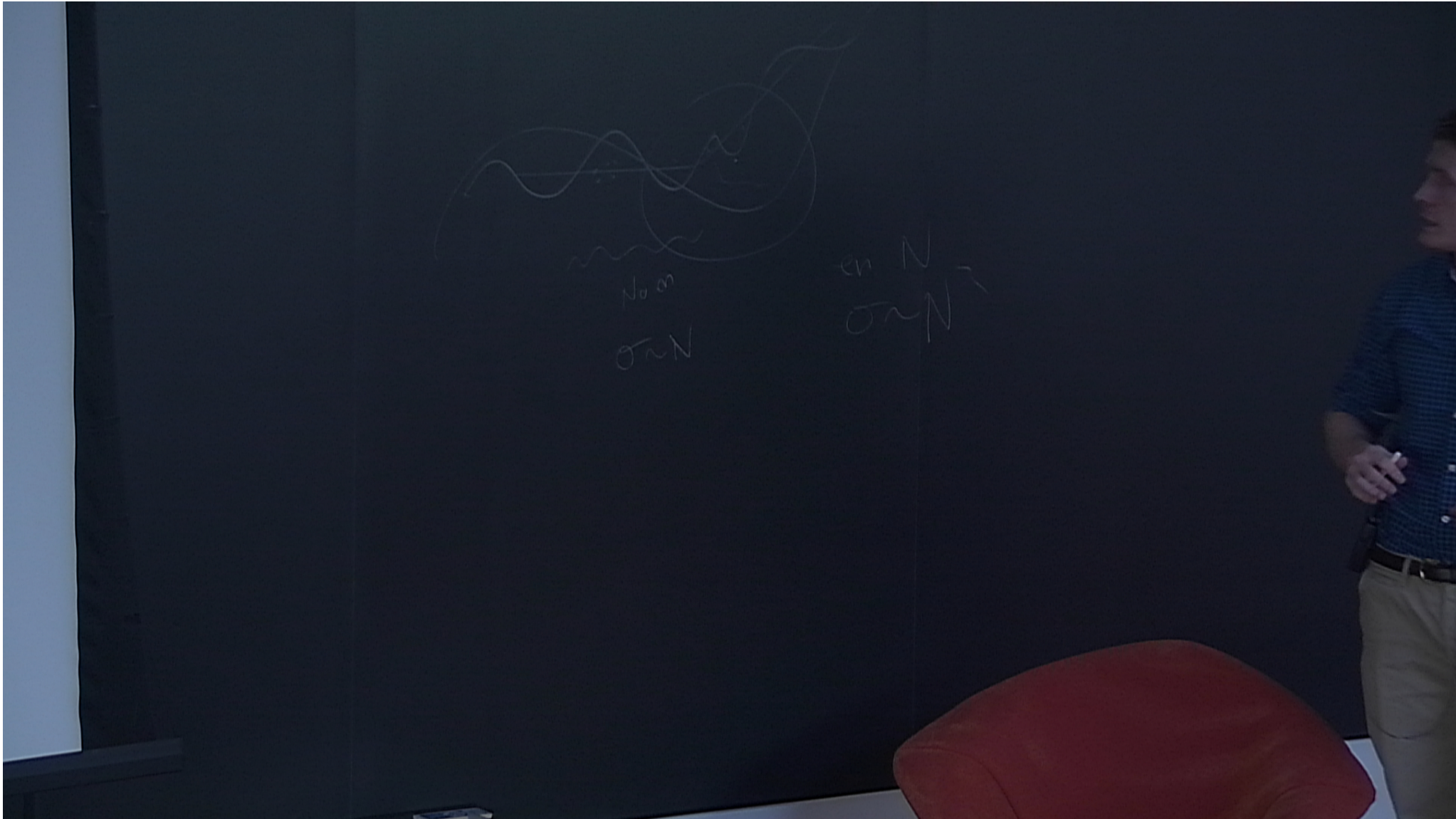


- Very general property of soft elastic scattering from targets composed of multiple ( $N$ ) charges
  - Not dependent on detail of particles or mediators

## Aside: Coherent elastic scattering



- Very general property of soft elastic scattering from targets composed of multiple ( $N$ ) charges
  - Not dependent on detail of particles or mediators
- When wavelength of incident particle is larger than target size,  $\lambda \gg R$ , one gets  $\sigma \sim N^2$

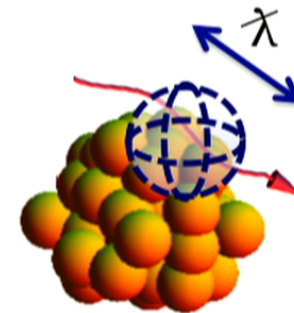




## Aside: Coherent elastic scattering



- Very general property of soft elastic scattering from targets composed of multiple ( $N$ ) charges
  - Not dependent on detail of particles or mediators
- When wavelength of incident particle is larger than target size,  $\lambda \gg R$ , one gets  $\sigma \sim N^2$
- When  $\text{\AA} \ll \lambda \ll R$ , there are complicated interference effects (constructive and destructive)
- Rule of thumb: enhancement is proportional to number of charges in “coherent scattering volume”
  - Scale set by momentum transfer)



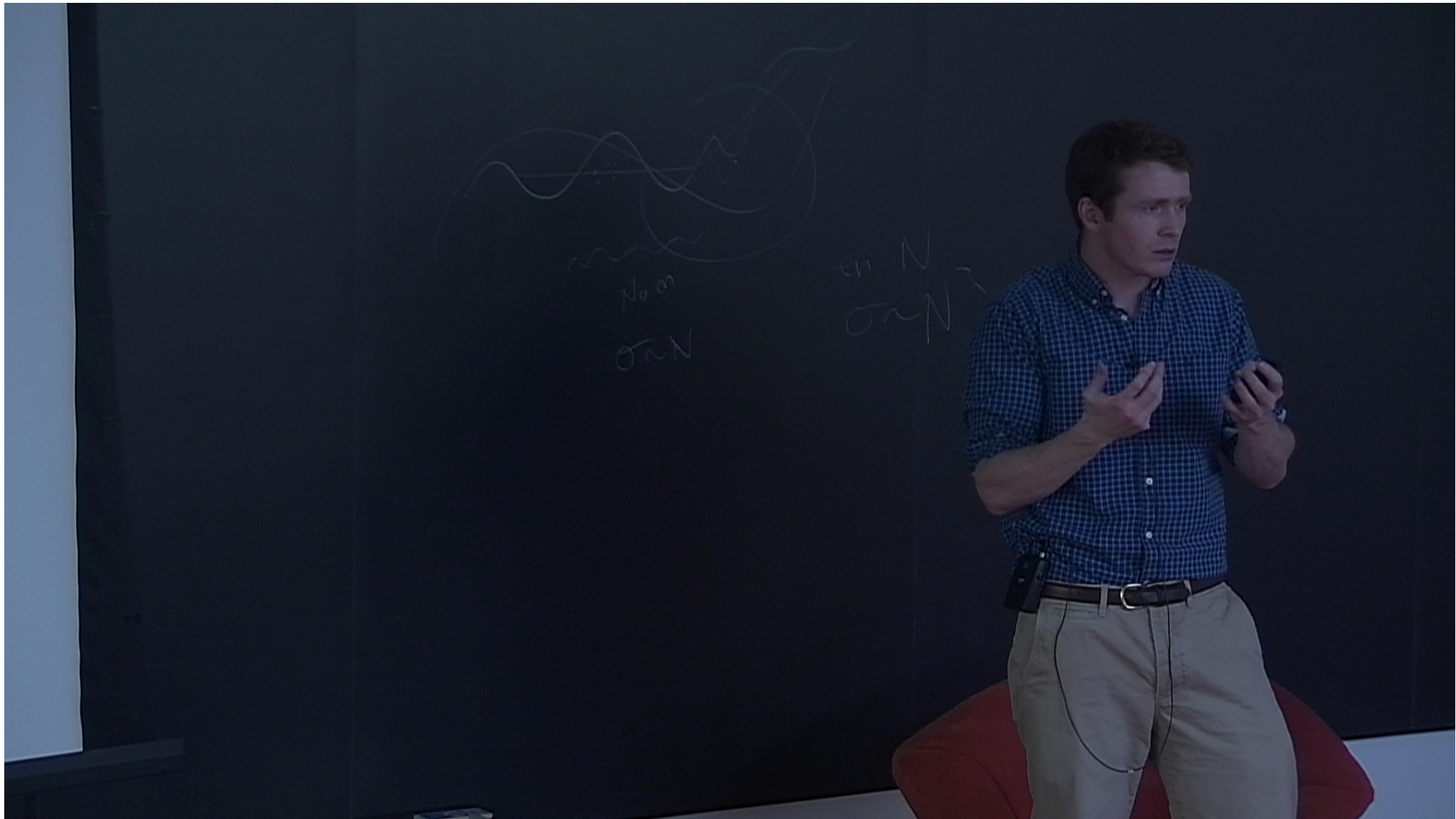
# What type of superposition targets?



- We can superpose...
  - photons over many km, but they have no inertia
  - neutrons over macroscopic distances, but their inertia is tiny









# Basic dark matter



- All evidence is essentially gravitational
- Many, many competing ideas
- Candidate explanations must satisfy a wide range of experiments and observations stretching back decades
  - Many indirect, model-dependent restrictions
- Relatively few model-independent results

# The dark matter halo



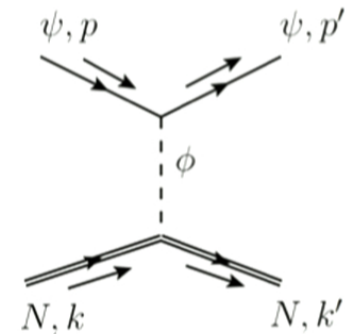
- But we have a **generic** local prediction: roughly spherical, virialized halo of dark matter enveloping the Milky Way
  - Isotropic in galactic rest frame
  - Maxwellian velocity distribution
  - Local density  $\sim 0.4 \text{ GeV/cm}^3$
  - Typical velocity  $\sim 230 \text{ km/s}$
- Assumed for limits set by underground detectors
- Based only on local, present-day observation
  - (no cosmology necessary)



# Toward a bespoke theory of dark matter...



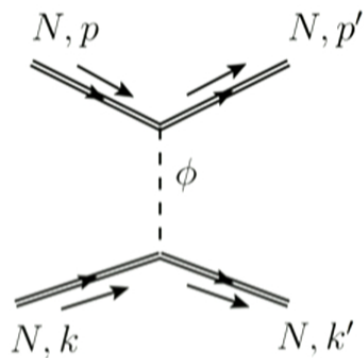
- Want scattering dominated by small momentum transfer  $\sim \hbar/(50 \text{ nm})$
- Otherwise, should be rarely interacting
- Itay Yavin: forward scattering off atoms through new massive scalar mediator (“heavy photon”)



# Interactions

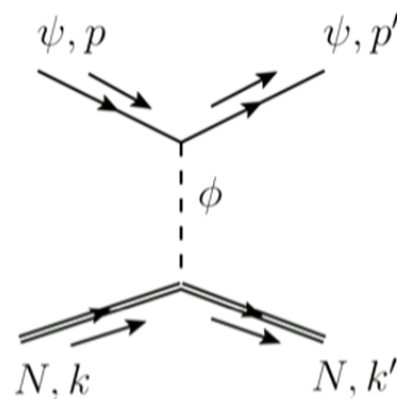


- DM-nucleon interaction →
- Nucleon-nucleon Yukawa potential ↓



⇒

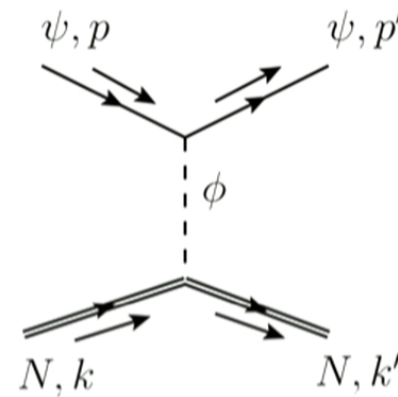
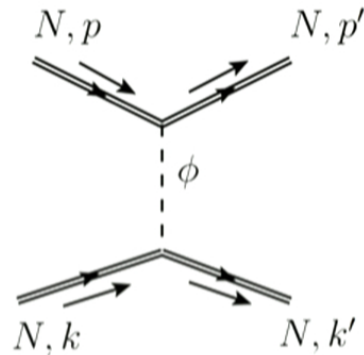
$$V_M(r) = -\frac{g_M^2}{4\pi} \frac{1}{r} e^{-mr} \quad (5)$$



# Interactions



- DM-nucleon interaction →
- Nucleon-nucleon Yukawa potential ↓



$$\Rightarrow V_M(r) = -\frac{g_M^2}{4\pi} \frac{1}{r} e^{-mr} \quad (5)$$

- Strongest constraints on  $\alpha_M$  and  $m_{\text{med}}$  come mostly from Casimir-force and neutron scattering experiments
  - Nucleon-Nucleon rather than DM-Nucleon

# Particle dark matter



- No value of  $\alpha_{\text{DM}}$ ...
  - avoids DM self-interaction constraints (e.g., bullet cluster) and
  - still detectable

# Particle dark matter



- No value of  $\alpha_{\text{DM}}$ ...
  - avoids DM self-interaction constraints (e.g., bullet cluster) and
  - still detectable
- Take  $\alpha_{\text{DM}} \sim 1$
- So must be no more than an 10% component of DM
  - We imagine a dark sector that's complicated, just like regular matter (bound states, multiple forces, etc.)
  - Would be just one part of a vast dark sea
- Can construct arbitrarily complicated models to boost or suppress signal (e.g. large dark nuclei – Robert Lasenby)
  - Sensitivity plots are just a guide

# Particle dark matter



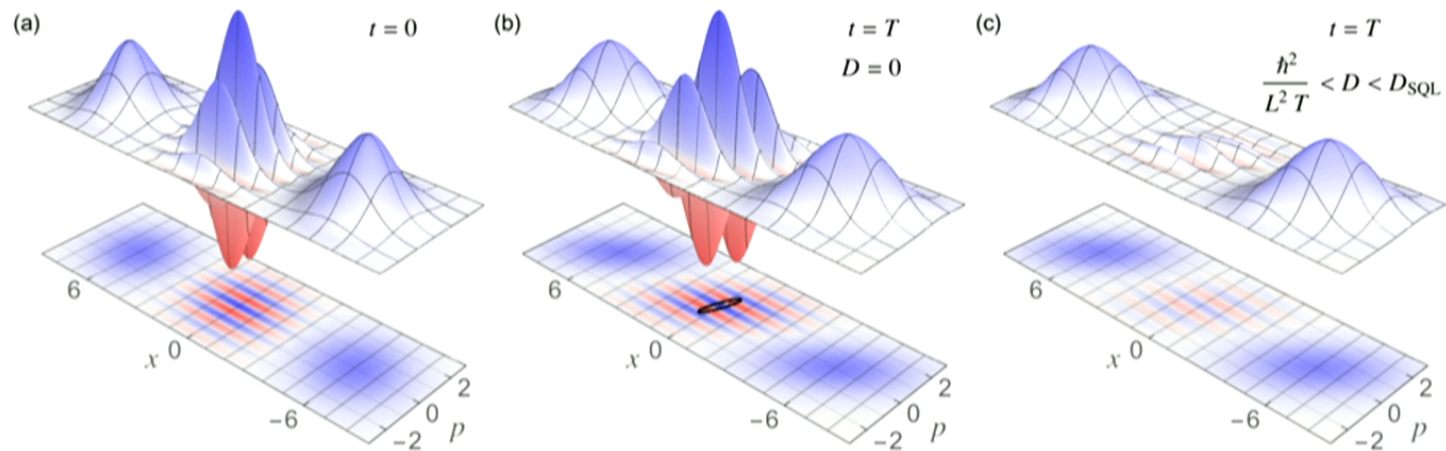
- Model then has three free parameters:
  - Mediator-Nucleon coupling:  $\alpha_M = g_M^2 / 4\pi$
  - Mediator mass:  $m_{\text{med}}$  (with corresponding length scale  $\lambda_{\text{Yukawa}}$ )
  - Dark matter mass  $m_{\text{DM}}$



# What's the signal?



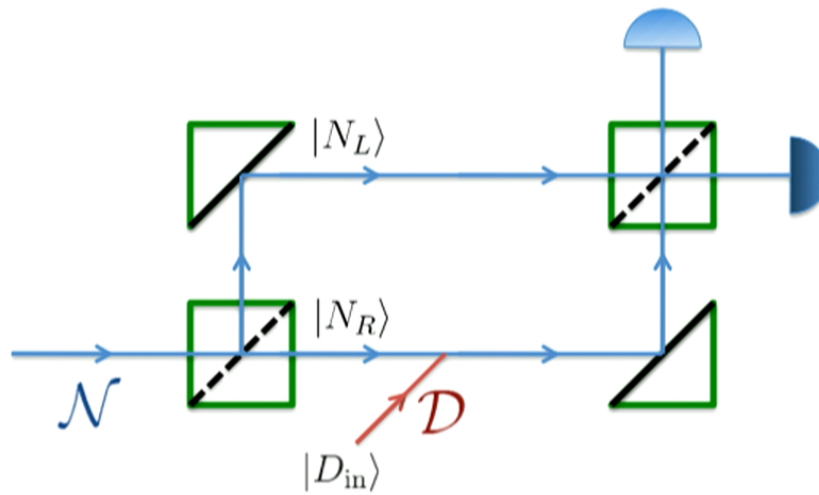
- For widely separated wavepackets in interferometers, DM would act as a source of decoherence



# Detection through decoherence



- Initial state:  $[|\mathcal{N}_L\rangle + |\mathcal{N}_R\rangle] |D_{\text{in}}\rangle$
- Final state:
- Measurement:



# Anomalous decoherence



- There are many possible sources of decoherence and noise
- Major challenge of nanomechanics is identifying and defeating one level of noise after another

# Anomalous decoherence



- There are many possible sources of decoherence and noise
- Major challenge of nanomechanics is identifying and defeating one level of noise after another
- Anomalous decoherence does not imply dark matter
  - However, the inverse statement is true: a cold (or quantum) resonator implies *all* sources of noise above some threshold have been eliminated
  - This can establish robust dark matter exclusion limits
- But if we think anomalous decoherence might be due to dark matter, how could we be sure?

# Establishing convincing evidence



- Try varying experimental parameters, e.g.
  - Size and shape of the resonator
  - Applied driving/cooling
  - Elemental composition of resonator
  - Isotopic composition of elements
  - (Analogous parameters exist for interferometers)

# Establishing convincing evidence



- Try varying experimental parameters, e.g.
  - Size and shape of the resonator
  - Applied driving/cooling
  - Elemental composition of resonator
  - Isotopic composition of elements
  - (Analogous parameters exist for interferometers)
- General sources of decoherence will not have same dependence on these parameters

# Establishing convincing evidence



- Try influencing expected dark matter flux
  - Shield experiment from dark matter (concrete, lead, underground)
  - Strength of dark matter wind will vary by order unity over day, and several percentage points over the year, due to Earth's motion

# Establishing convincing evidence



- Try influencing expected dark matter flux
  - Shield experiment from dark matter (concrete, lead, underground)
  - Strength of dark matter wind will vary by order unity over day, and several percentage points over the year, due to Earth's motion
- In general, the *orientation* of the resonator will give order-unity change to  $D$ 
  - Resonators are naturally *directional* detectors!



# Existing and proposed interferometers

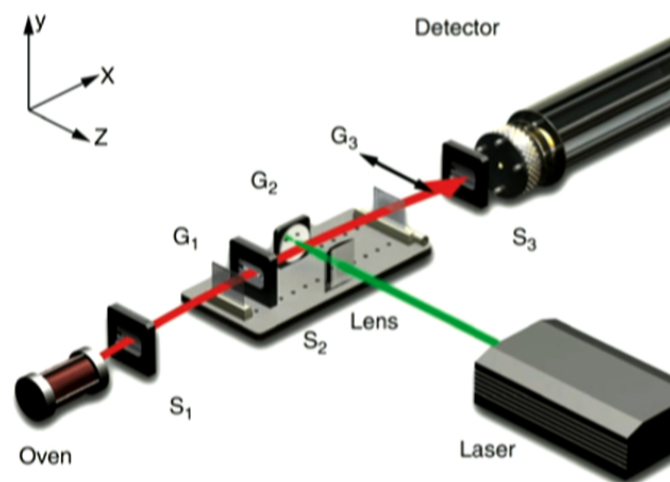
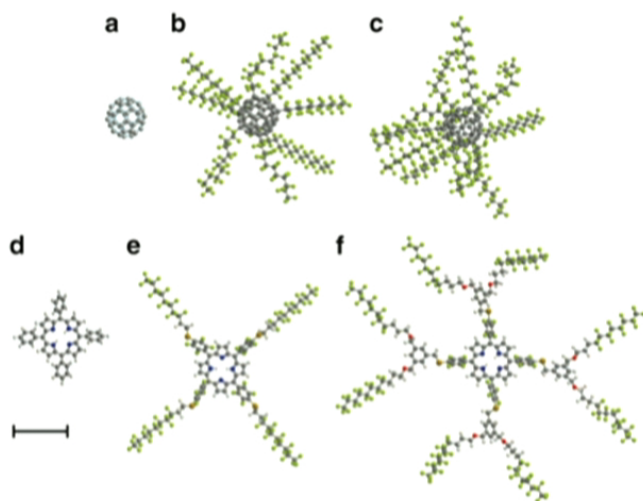


- Look at 4 benchmark interferometers to see the sort of sensitivity that's possible for our proof-of-concept model of DM
  - #1: KDTL – exists; operating at design specification
  - #2: OTIMA – exists; currently scaling up
  - #3: Bateman et al. – proposal; reasonable
  - #4: MAQRO – proposal; satellite (speculative)

# Interferometric benchmarks



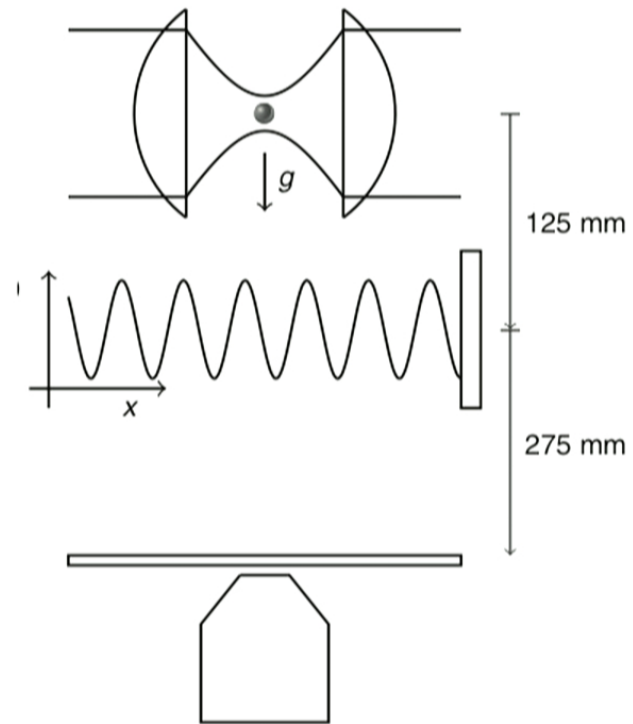
- #1: Kapitza-Dirac-Talbot-Lau (KDTL) interferometer
  - Reached  $>10^3$  amu (later  $>10^4$ )
  - Gerlich et al. *Natural Communications* **2**, 263 (2011)



# Interferometric benchmarks



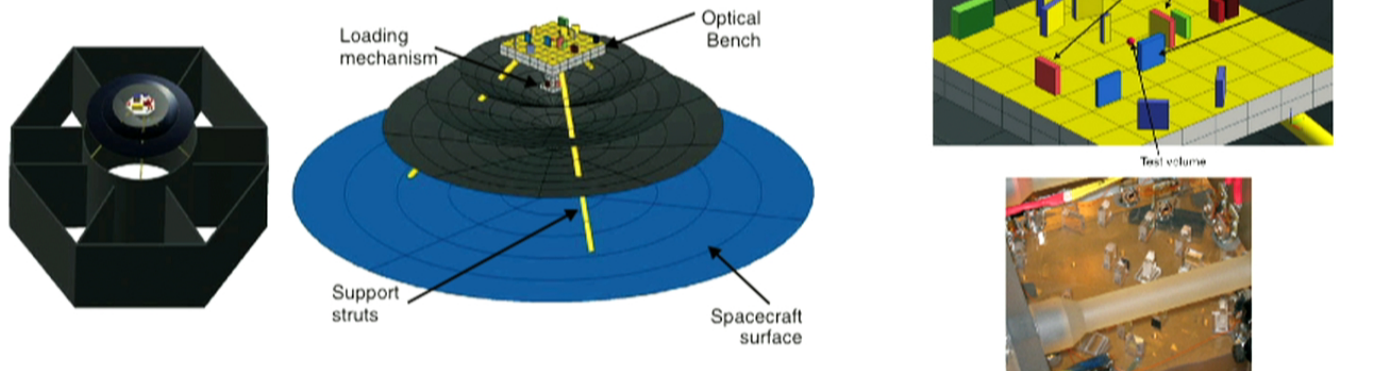
- #3: Bateman et al. “point source” proposal
  - Optical hybrid: 5 nm,  $10^6$  amu nanoparticle suspended in trap, cooled, then dropped through laser grating
  - J. Bateman et al. *Nature Communications* **5**, 4788 (2014)



# Interferometric benchmarks



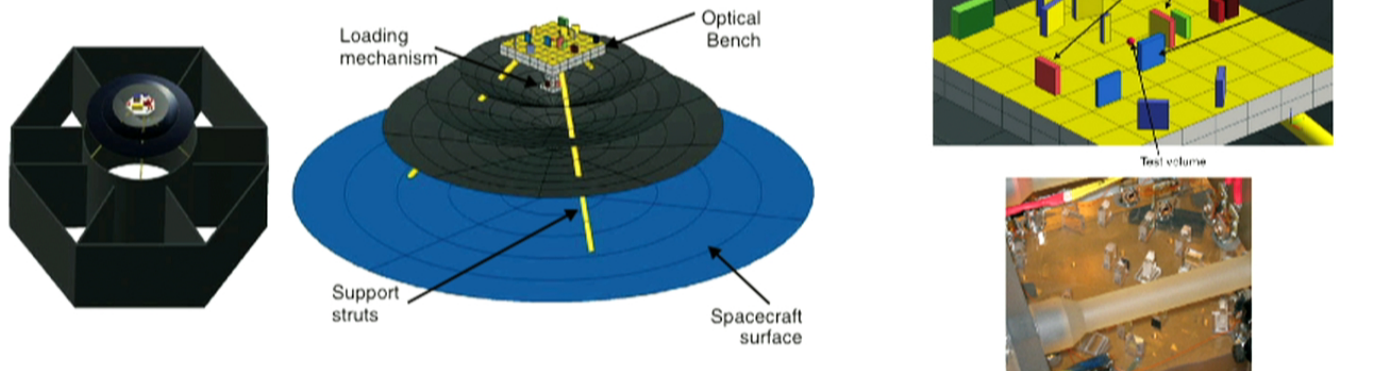
- #4: Optically trapped 120 nm silica nanoparticle on board the MAQRO satellite proposal
  - Nanoparticle suspended, cooled to COM motional ground state, then brought into superposition
  - Tech under development. See M4 mission proposal made to European Space Agency (arXiv:1503.02640)



# Interferometric benchmarks



- #4: Optically trapped 120 nm silica nanoparticle on board the MAQRO satellite proposal
  - Nanoparticle suspended, cooled to COM motional ground state, then brought into superposition
  - Tech under development. See M4 mission proposal made to European Space Agency (arXiv:1503.02640)

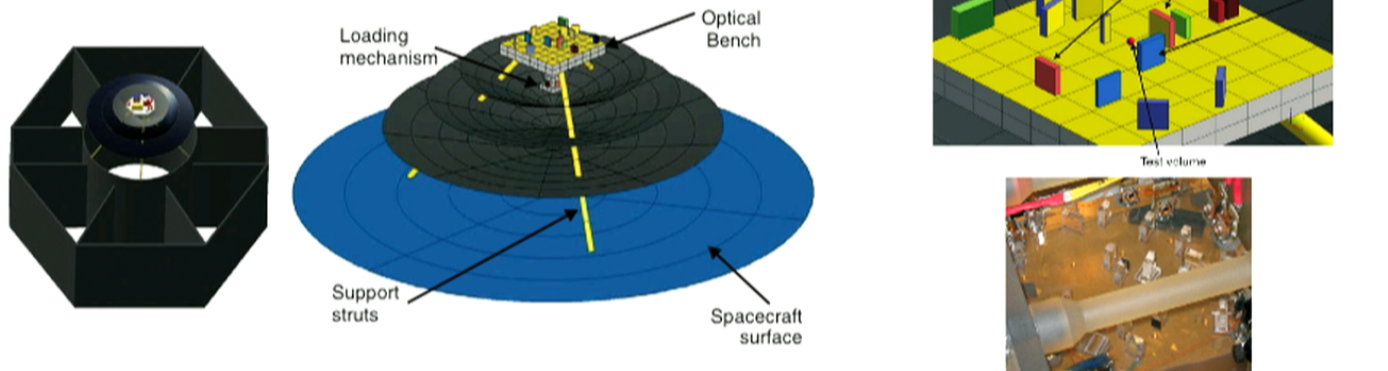




# Interferometric benchmarks



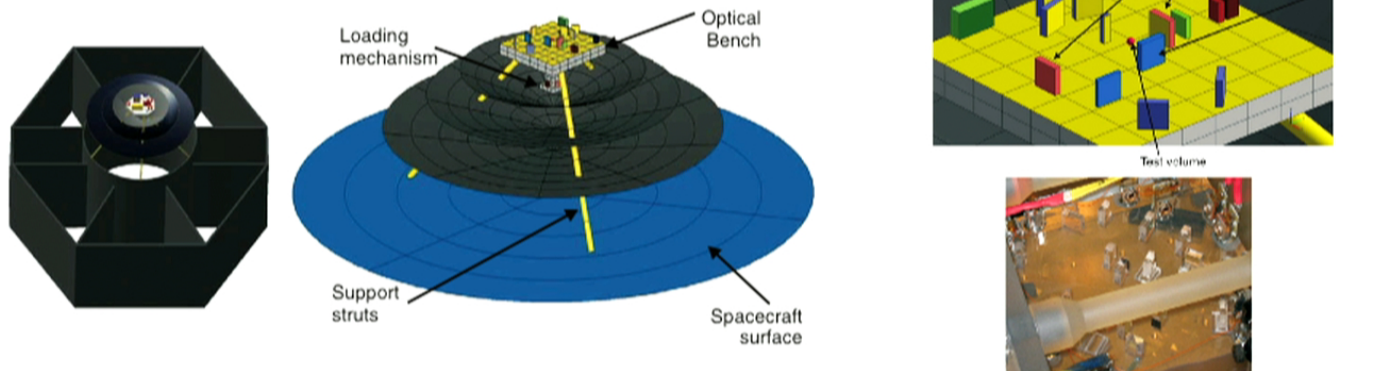
- #4: Optically trapped 120 nm silica nanoparticle on board the MAQRO satellite proposal
  - Nanoparticle suspended, cooled to COM motional ground state, then brought into superposition
  - Tech under development. See M4 mission proposal made to European Space Agency (arXiv:1503.02640)



# Interferometric benchmarks



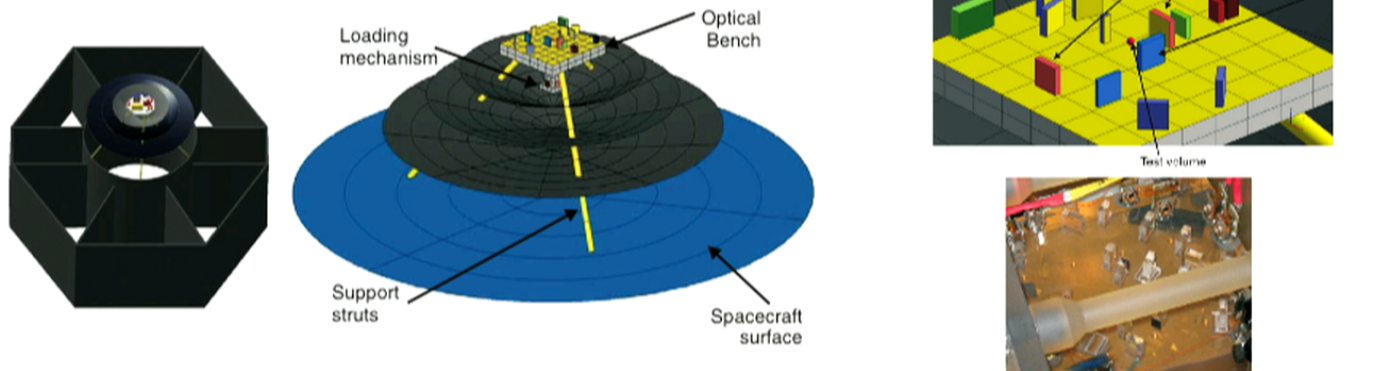
- #4: Optically trapped 120 nm silica nanoparticle on board the MAQRO satellite proposal
  - Nanoparticle suspended, cooled to COM motional ground state, then brought into superposition
  - Tech under development. See M4 mission proposal made to European Space Agency (arXiv:1503.02640)



# Interferometric benchmarks



- #4: Optically trapped 120 nm silica nanoparticle on board the MAQRO satellite proposal
  - Nanoparticle suspended, cooled to COM motional ground state, then brought into superposition
  - Tech under development. See M4 mission proposal made to European Space Agency (arXiv:1503.02640)



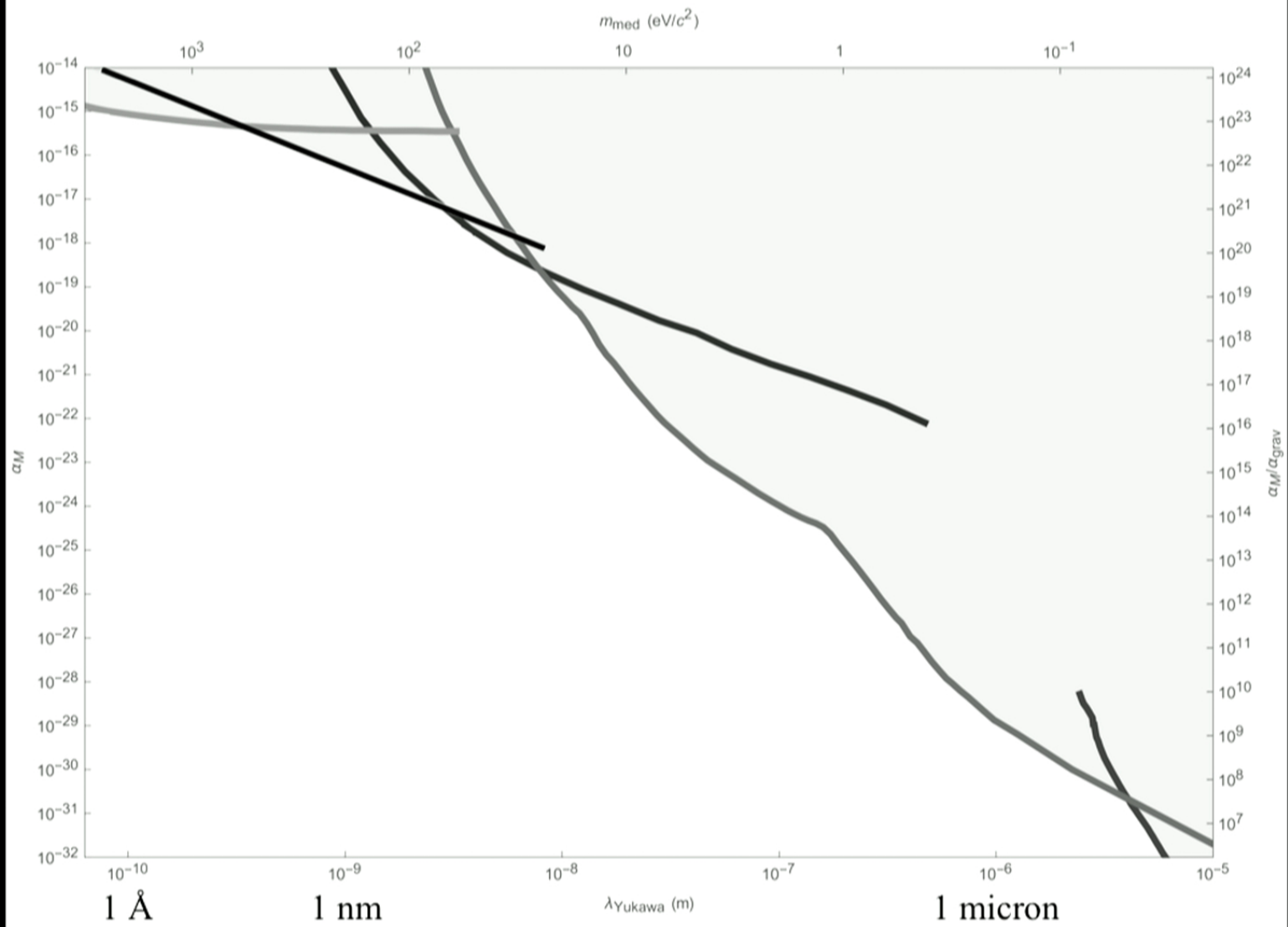


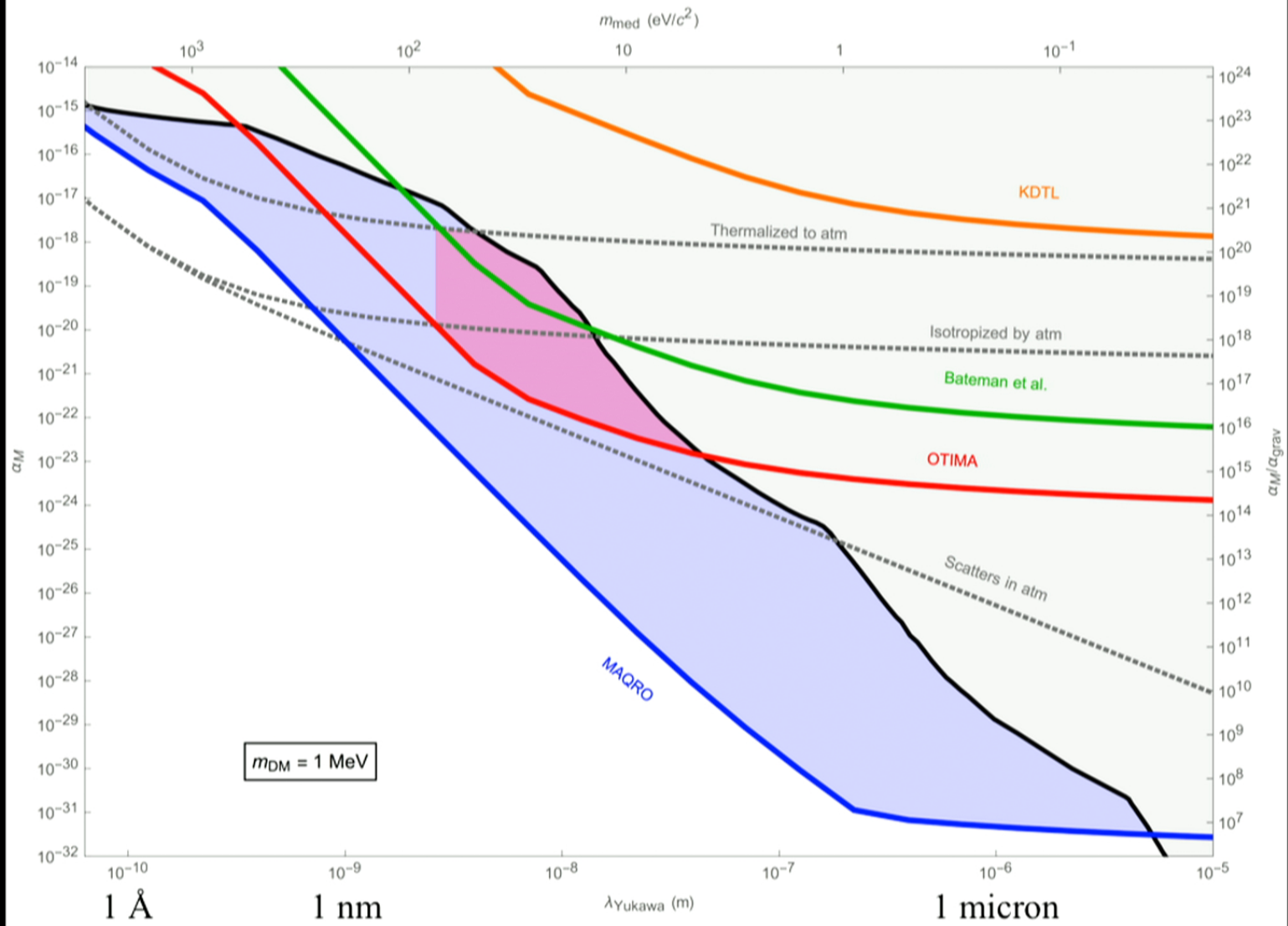
# Conservative assumptions

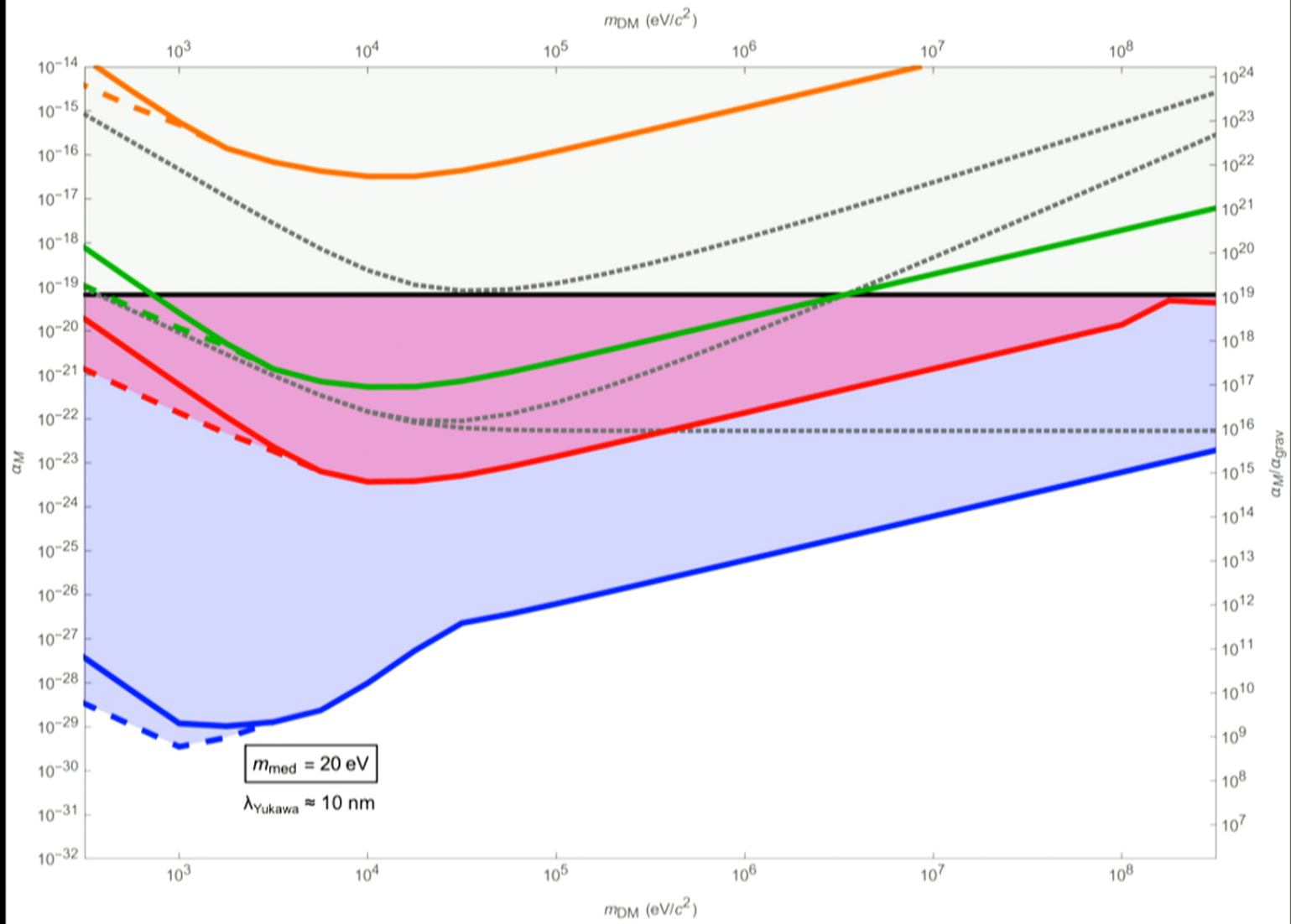


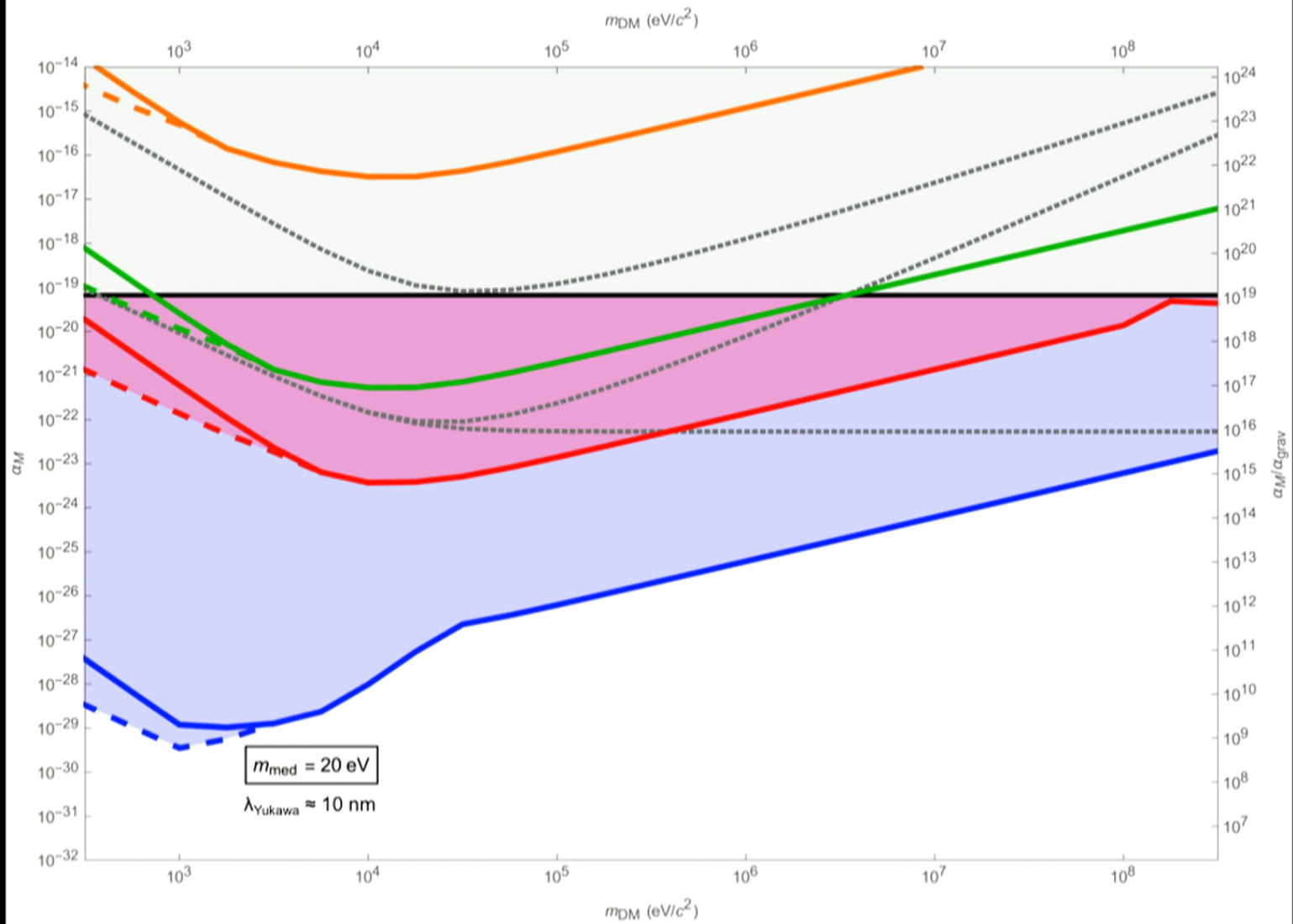
- Assume: dark matter entering the Earth is absorbed and disappears,
- Rather than...
  - reflected (isotropically) or







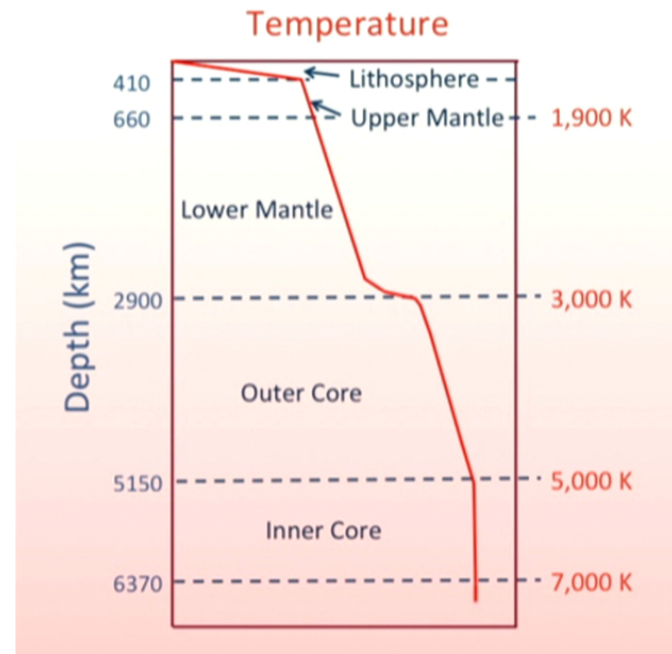




# Optimism



- Lets relax conservative assumptions
  - Allow for greenhouse effect, reflection from ground, etc.
- Several different parameters (effective  $T_{\text{Earth}}$ , reflectivity, etc.)
  - Just fill in everything that's possible
  - Most optimistic

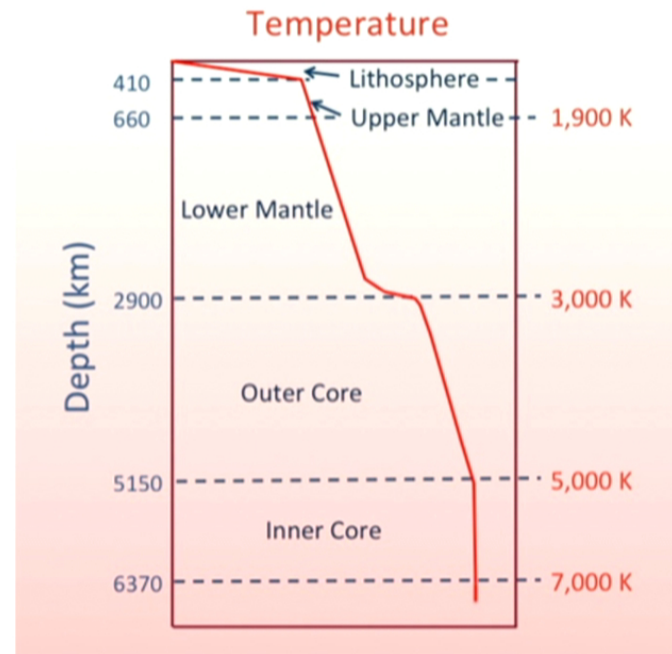


$$T_{\text{DM}} \sim 2300 \text{ K} \left( \frac{m_{\text{DM}}}{1 \text{ MeV}} \right)$$

# Optimism



- Lets relax conservative assumptions
  - Allow for greenhouse effect, reflection from ground, etc.
- Several different parameters (effective  $T_{\text{Earth}}$ , reflectivity, etc.)
  - Just fill in everything that's possible
  - Most optimistic



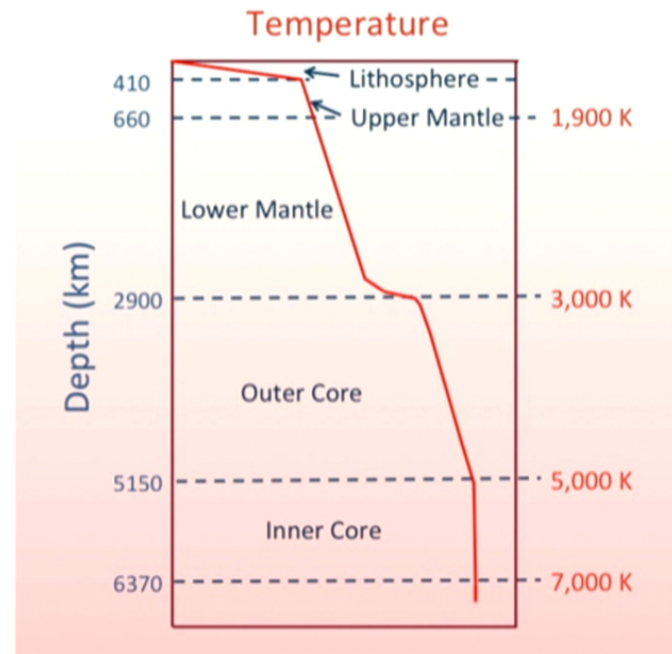
$$T_{\text{DM}} \sim 2300 \text{ K} \left( \frac{m_{\text{DM}}}{1 \text{ MeV}} \right)$$



# Optimism



- Lets relax conservative assumptions
  - Allow for greenhouse effect, reflection from ground, etc.
- Several different parameters (effective  $T_{\text{Earth}}$ , reflectivity, etc.)
  - Just fill in everything that's possible
  - Most optimistic



$$T_{\text{DM}} \sim 2300 \text{ K} \left( \frac{m_{\text{DM}}}{1 \text{ MeV}} \right)$$



# Take home



- Colder targets are more sensitive to small kicks

# Take home



- Colder targets are more sensitive to small kicks
- Large superpositions have *quantum-enhanced* (“ $T < 0$ ”) sensitivity
  - Supremacy precise in sense of SQL, just like squeezed light in LIGO
- Interferometers especially sensitive to medium-range forces
  - Most competitive when range  $\sim$  size of superposed object  $\sim 10$  nm

# Superposition motivations



- There are several reasons to build large quantum superpositions
  - Push the limits of quantum mechanics
  - Search for a “Sure/Shor separator”
  - Improve techniques of quantum control
  - Sensitive measurement of short-range classical forces

# Superposition motivations

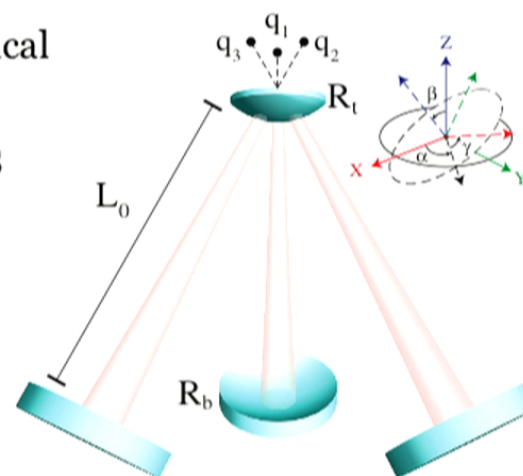


- There are several reasons to build large quantum superpositions
  - Push the limits of quantum mechanics
  - Search for a “Sure/Shor separator”
  - Improve techniques of quantum control
  - Sensitive measurement of short-range classical forces

# Superposition motivations



- There are several reasons to build large quantum superpositions
  - Push the limits of quantum mechanics
  - Search for a “Sure/Shor separator”
  - Improve techniques of quantum control
  - Sensitive measurement of short-range classical forces
- This talk gives another one: searches for
  - tiny momentum transfers
  - diffusive forces
- Gives new motivations for large-separation (i.e., *unclamped*) nanomechanical resonators

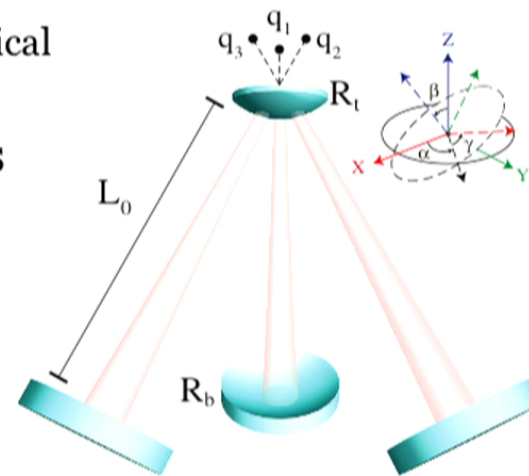


Guccione et al., *PRL* **111**,  
183001 (2013)

# Superposition motivations



- There are several reasons to build large quantum superpositions
  - Push the limits of quantum mechanics
  - Search for a “Sure/Shor separator”
  - Improve techniques of quantum control
  - Sensitive measurement of short-range classical forces
- This talk gives another one: searches for
  - tiny momentum transfers
  - diffusive forces
- Gives new motivations for large-separation (i.e., *unclamped*) nanomechanical resonators

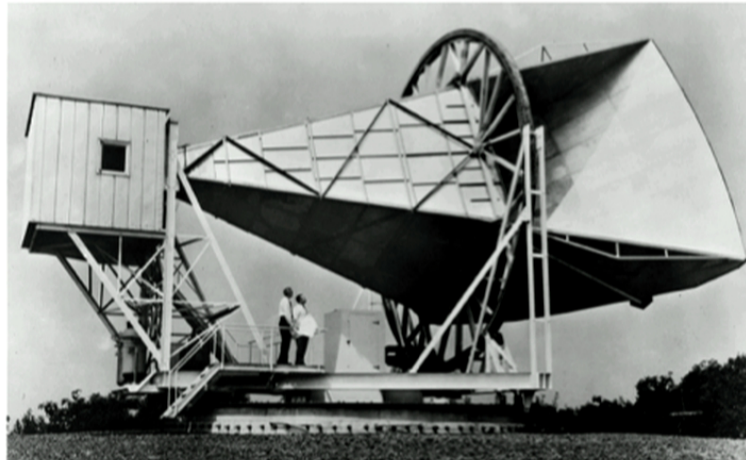


Guccione et al., *PRL* **111**,  
183001 (2013)

# The End

Write [jessriedel@gmail.com](mailto:jessriedel@gmail.com) to get  
reminder email when this is  
posted to arXiv

PRA **92**, 010101(R) (2015)  
PRD **88**, 116005 (2013)  
Slides/video/blog: [jessriedel.com](http://jessriedel.com)





$$\langle D_{out}^L | D_{out}^R \rangle$$

$$\Pi \langle D_{out}^L | D_{out}^R \rangle$$

$$V_{15} = 0.4$$

$$0.4 \text{ GeV/cm}^3$$

