Title: Long-range entangled many-body states

Date: Nov 11, 2015 02:00 PM

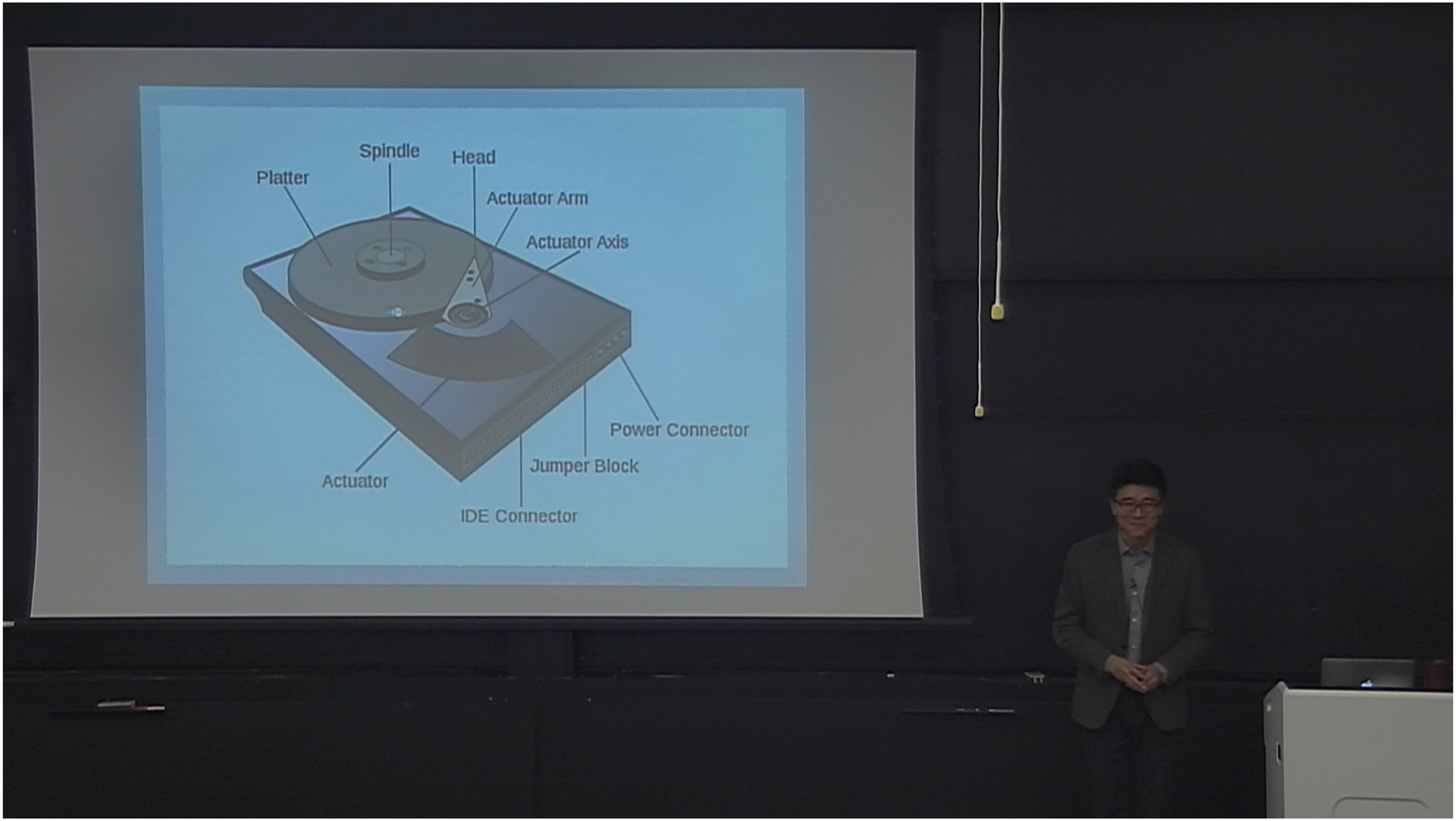
URL: http://pirsa.org/15110080

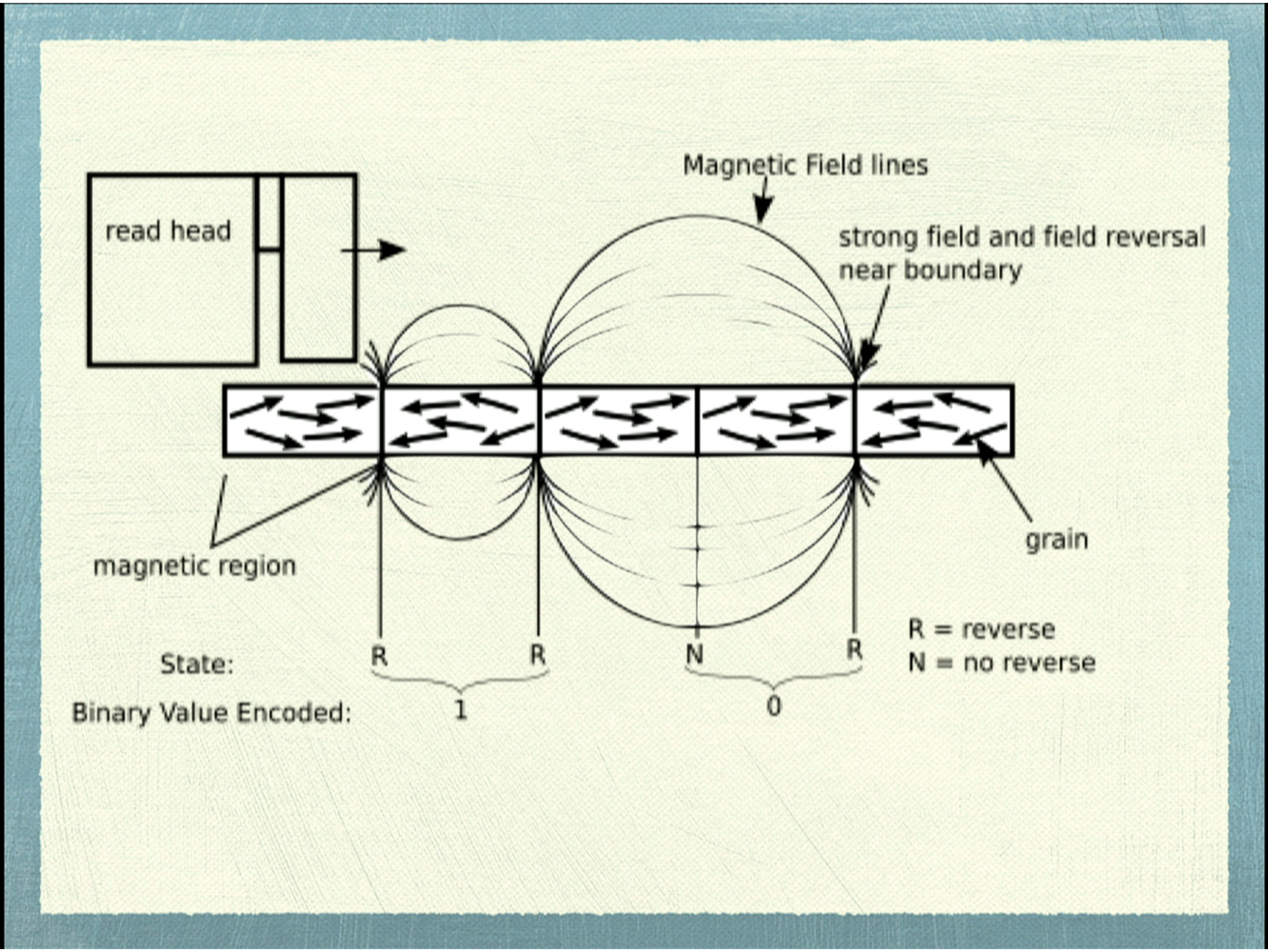
Abstract: <p>A quantum entanglement is a special kind of correlation; it may yield a strong correlation that is not possible in a classical ensemble, or hide the correlation from all local observables. Especially important is the entanglement that arises from local interactions for its implications in many-body physics and futureâ€™s quantum technologies.
I will review a few characteristics of entangled qubits in connection to fault-tolerant quantum information processing, and present a class of long-range entangled many-body states that are ground states of gapped local Hamiltonians on lattices. The class is qualitatively unconventional in many ways, and substantially boosts the richness of many-body entanglement. Implications in mechanisms of localization, renormalization group flow, quantum information storage, and topological order will be discussed.</p>

Long-range Entangled Many-body States

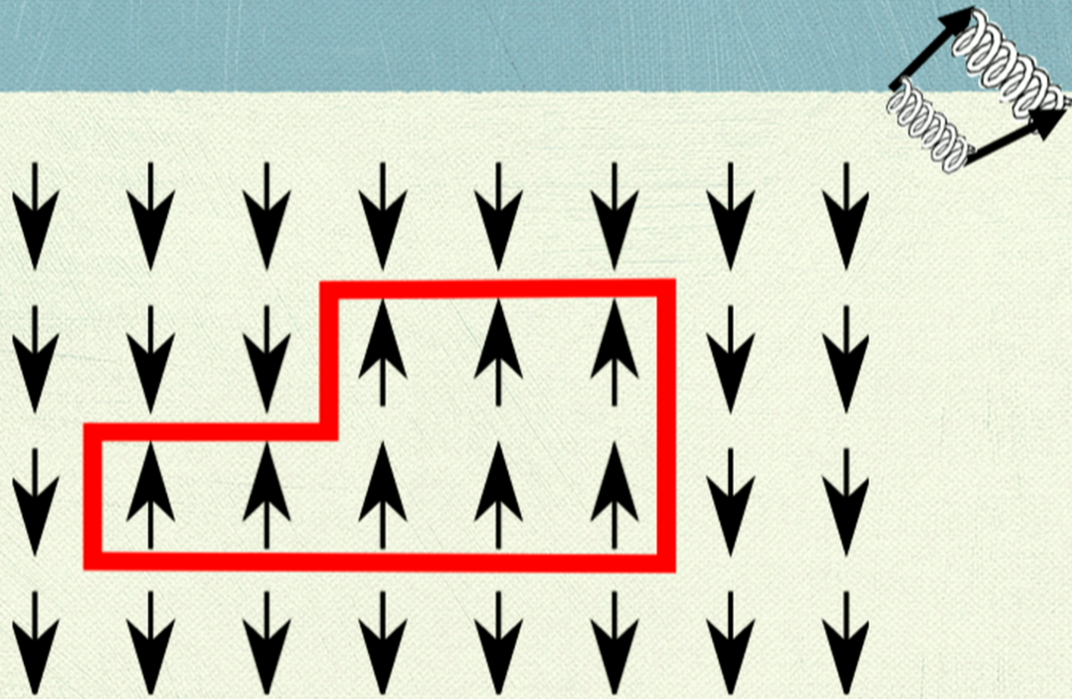
Jeongwan Haah, MIT

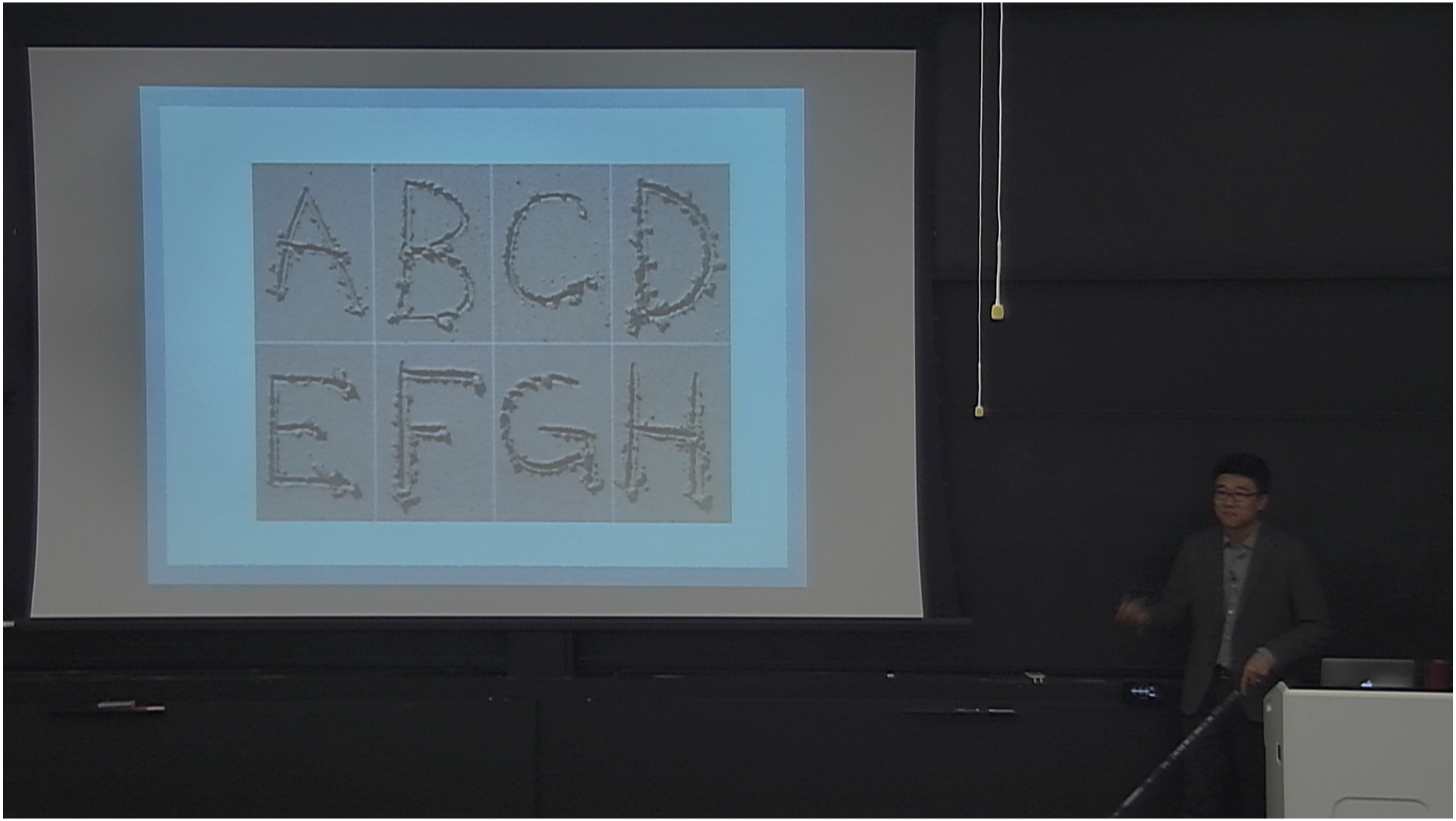
Nov. 11, 2015
Perimeter Institute for Theoretical Physics

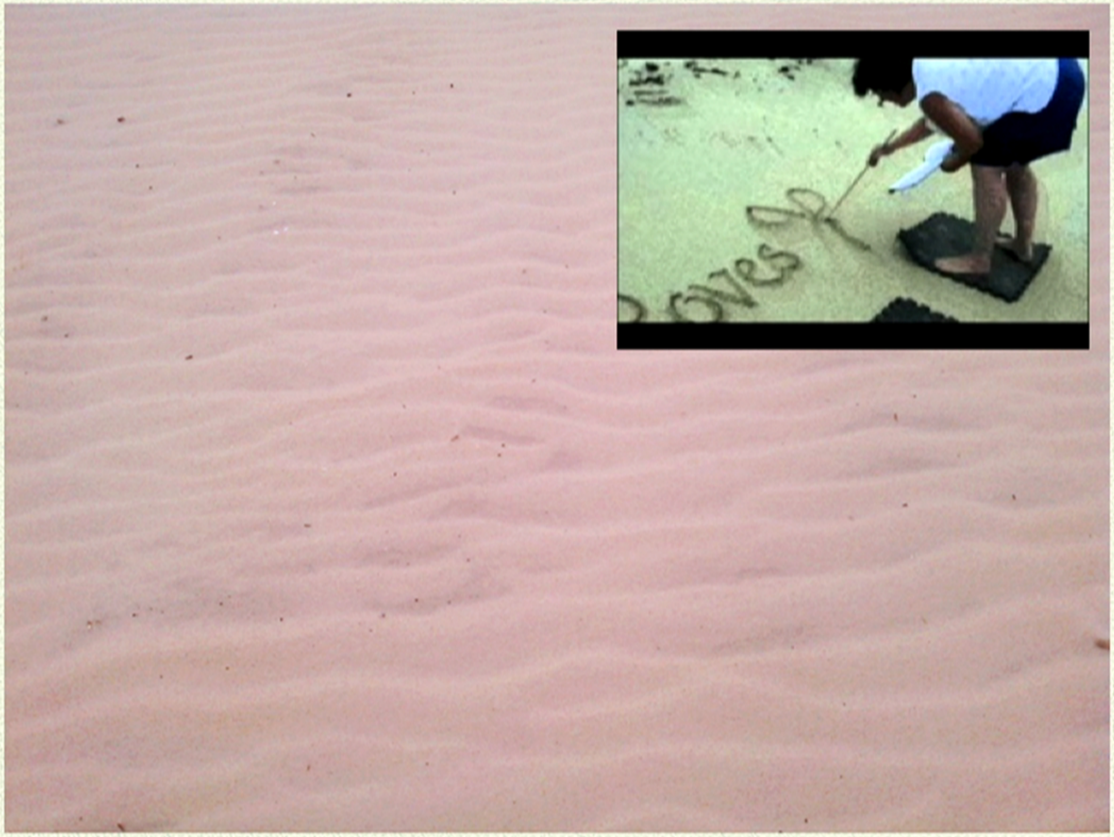

Spindle
Head
Platter
Actuator Arm
Actuator Axis
Power Connector
Jumper Block
Actuator
IDE Connector


Magnetic Field lines
read head
strong field and field reversal
near boundary
magnetic region
grain
R = reverse
N = no reverse
State:
R
R
N
R
Binary Value Encoded:
1
0

Magnet

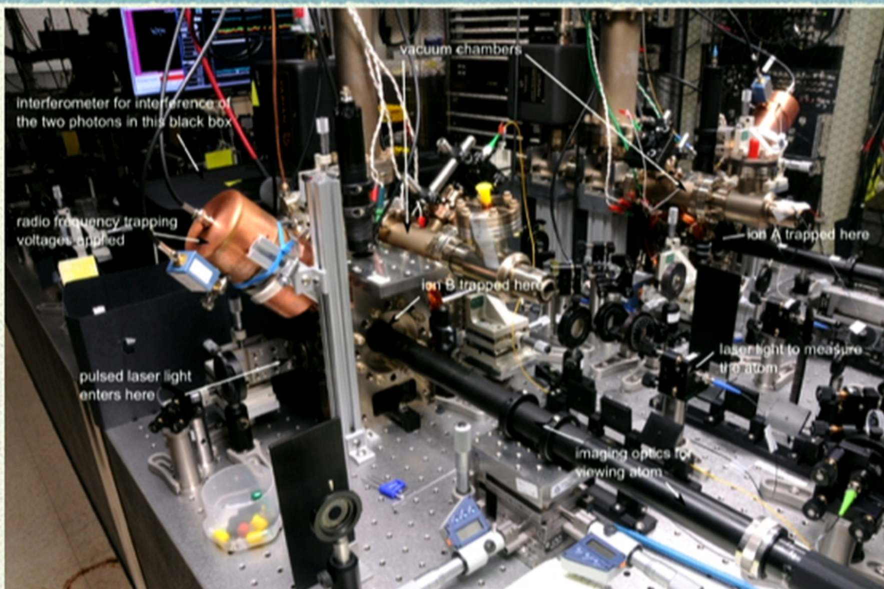


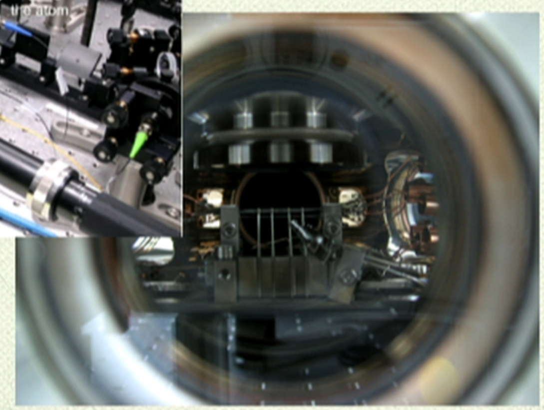

ABCD
EFGH



Writing on sand requires constant monitoring and rewriting.

Experimental Endeavor





Why so complicated?


MOSCOW
INTERNATIONAL
SAND SCULPTURE
COMPETITION
2008

Challenge

- Find a system where **interaction** protects quantum information
- Will reduce the experimental burden.
  Will help building a quantum computer.

Bell pairs

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

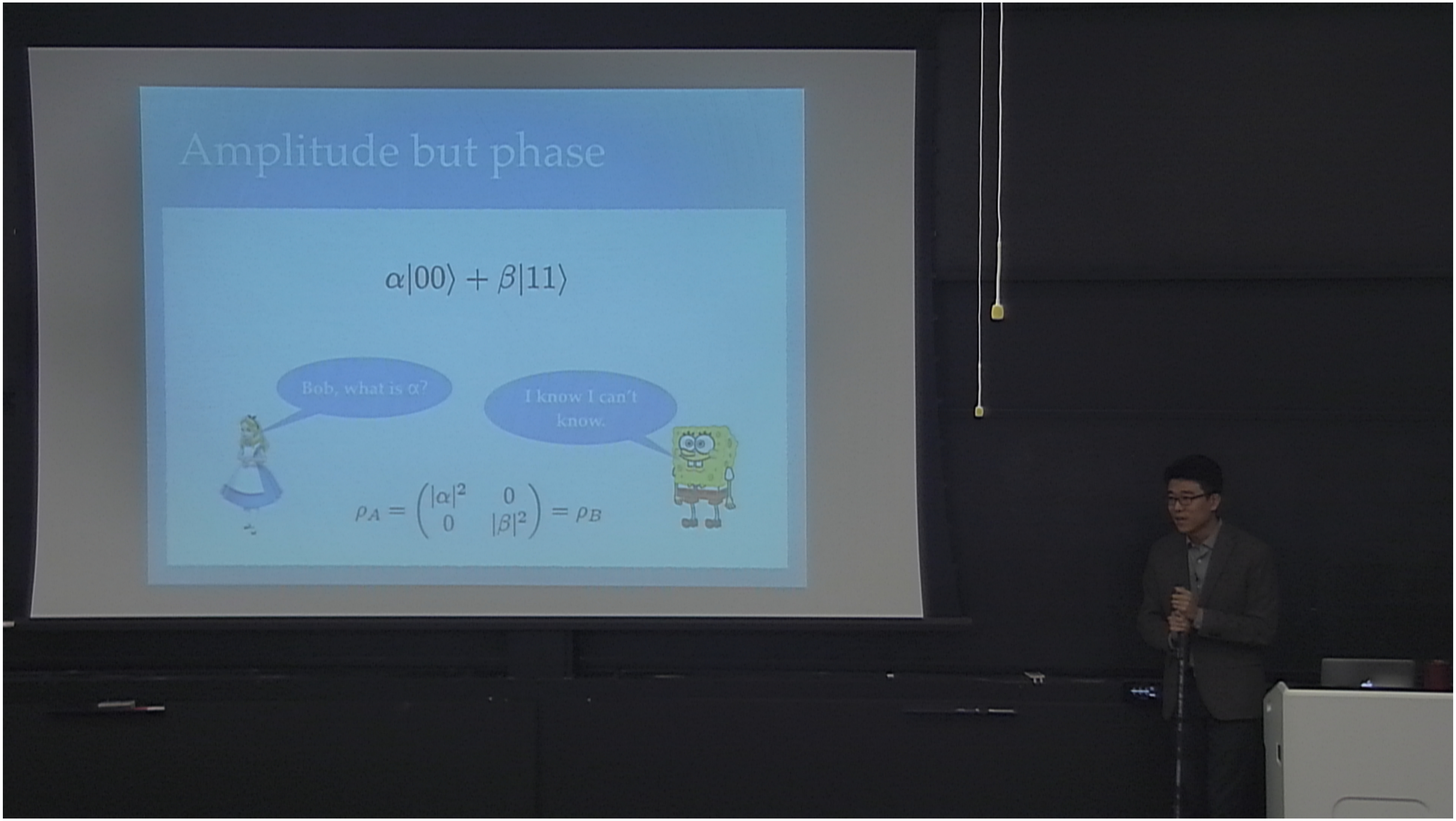
$$\frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

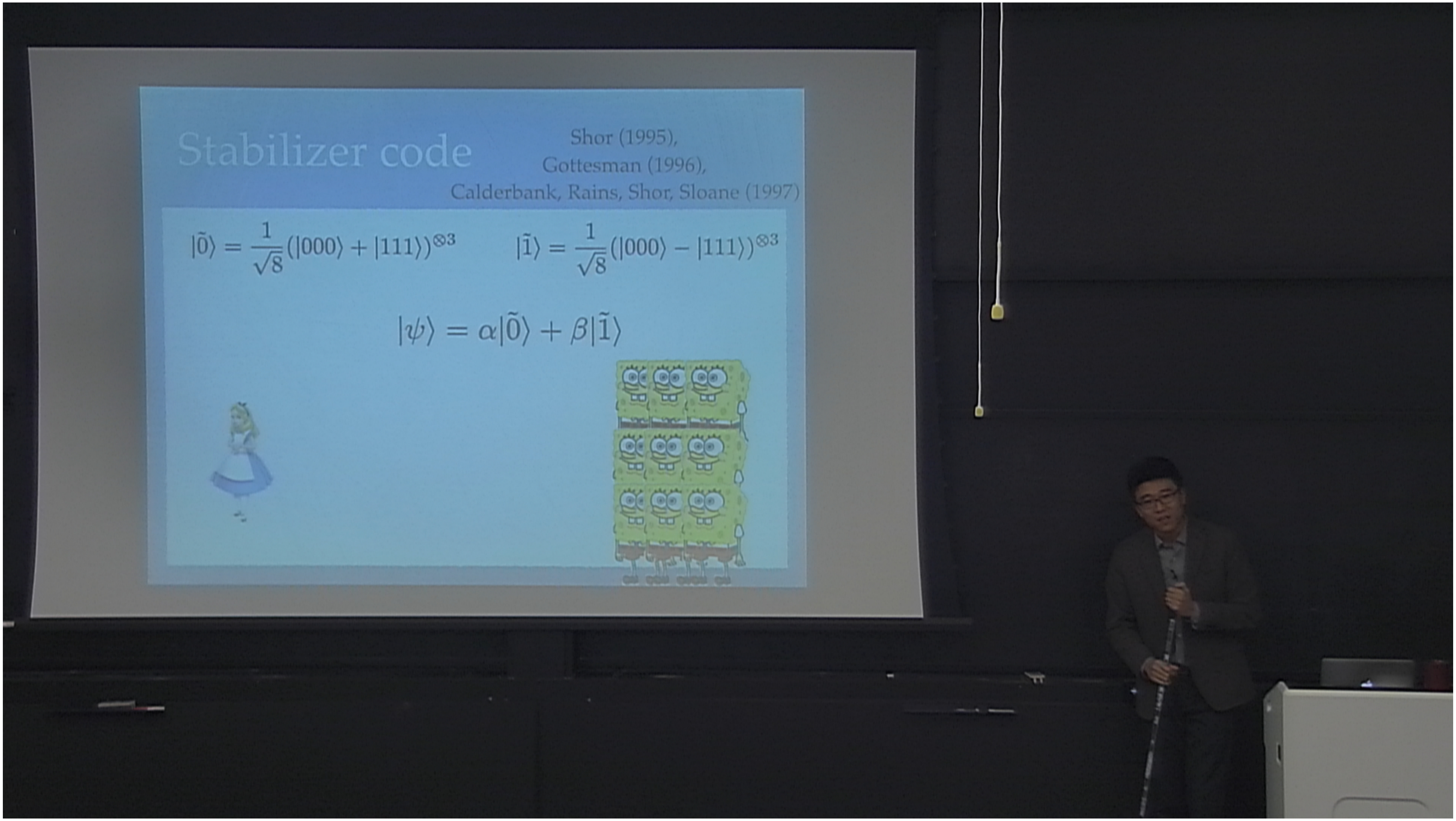
Bell pairs

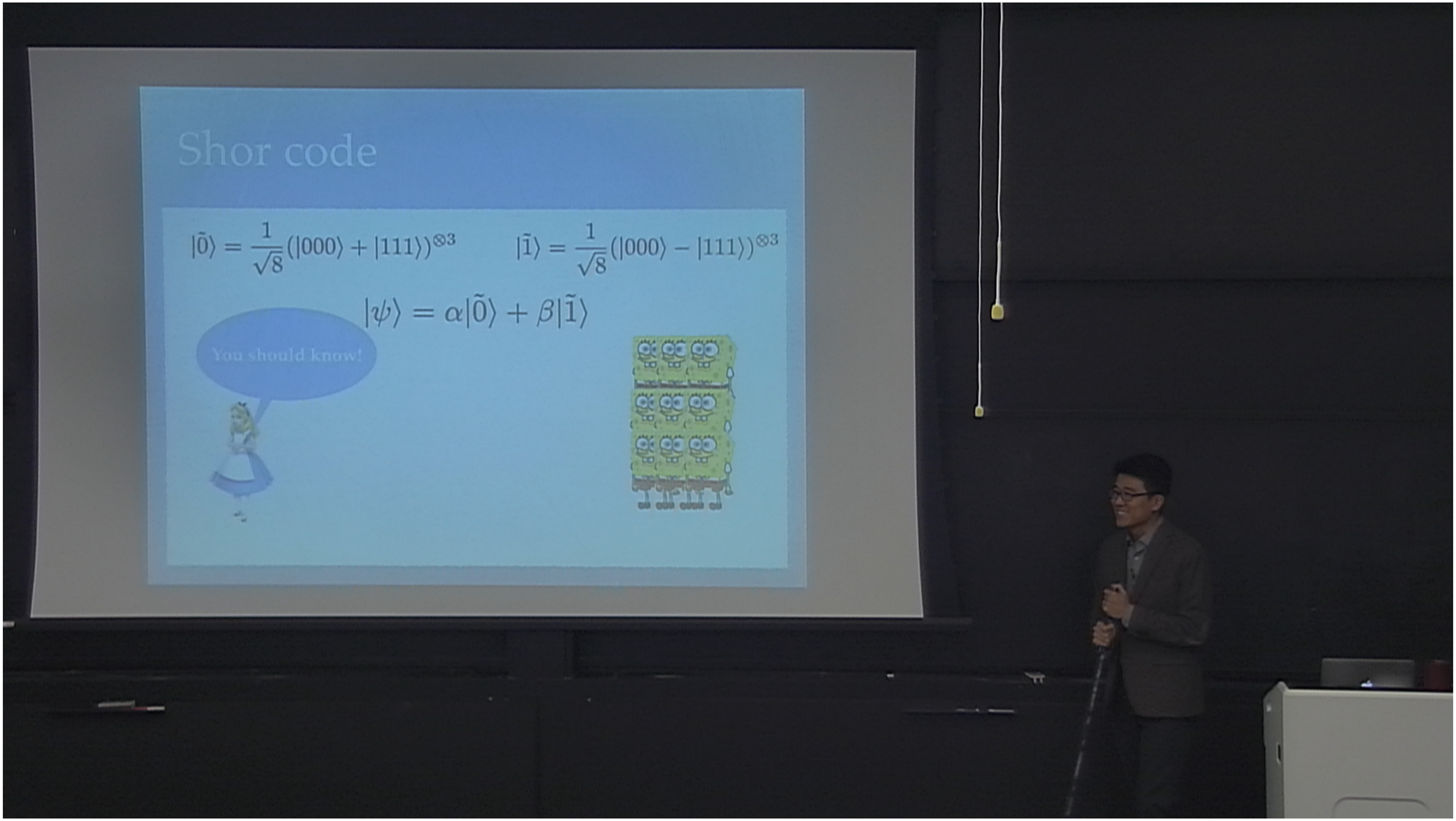
$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$\frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

$$\rho_A = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$


Amplitude but phase
$\alpha|00\rangle + \beta|11\rangle$
Bob, what is α?
I know I can't know.
$\rho_A = \begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix} = \rho_B$


Stabilizer code
Shor (1995),
Gottesman (1996),
Calderbank, Rains, Shor, Sloane (1997)
$|\tilde{0}\rangle = \frac{1}{\sqrt{8}}(|000\rangle + |111\rangle)^{\otimes 3}$
$|\tilde{1}\rangle = \frac{1}{\sqrt{8}}(|000\rangle - |111\rangle)^{\otimes 3}$
$|\psi\rangle = \alpha|\tilde{0}\rangle + \beta|\tilde{1}\rangle$


Shor code
$|\tilde{0}\rangle = \frac{1}{\sqrt{8}}(|000\rangle + |111\rangle)^{\otimes 3}$
$|\tilde{1}\rangle = \frac{1}{\sqrt{8}}(|000\rangle - |111\rangle)^{\otimes 3}$
$|\psi\rangle = \alpha|\tilde{0}\rangle + \beta|\tilde{1}\rangle$
You should know!

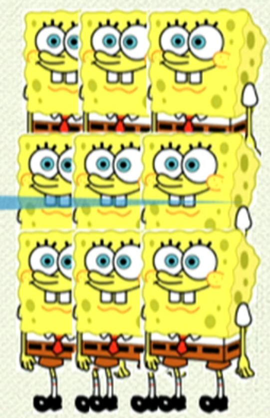
Shor code

$$|\tilde{0}\rangle = \frac{1}{\sqrt{8}}(|000\rangle + |111\rangle)^{\otimes 3} \qquad |\tilde{1}\rangle = \frac{1}{\sqrt{8}}(|000\rangle - |111\rangle)^{\otimes 3}$$

$$|\psi\rangle = \alpha|\tilde{0}\rangle + \beta|\tilde{1}\rangle$$

You should know!

We three know $|\alpha|^2$.



Shor code

$$|\tilde{0}\rangle = \frac{1}{\sqrt{8}}(|000\rangle + |111\rangle)^{\otimes 3} \qquad |\tilde{1}\rangle = \frac{1}{\sqrt{8}}(|000\rangle - |111\rangle)^{\otimes 3}$$

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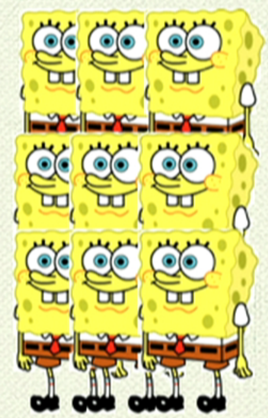
Shor code

$$|\tilde{0}\rangle = \frac{1}{\sqrt{8}}(|000\rangle + |111\rangle)^{\otimes 3} \qquad |\tilde{1}\rangle = \frac{1}{\sqrt{8}}(|000\rangle - |111\rangle)^{\otimes 3}$$

$$|\psi\rangle = \alpha|\tilde{0}\rangle + \beta|\tilde{1}\rangle$$

The global amplitude is accessible in multiple ways, though it is completely hidden from local observers.



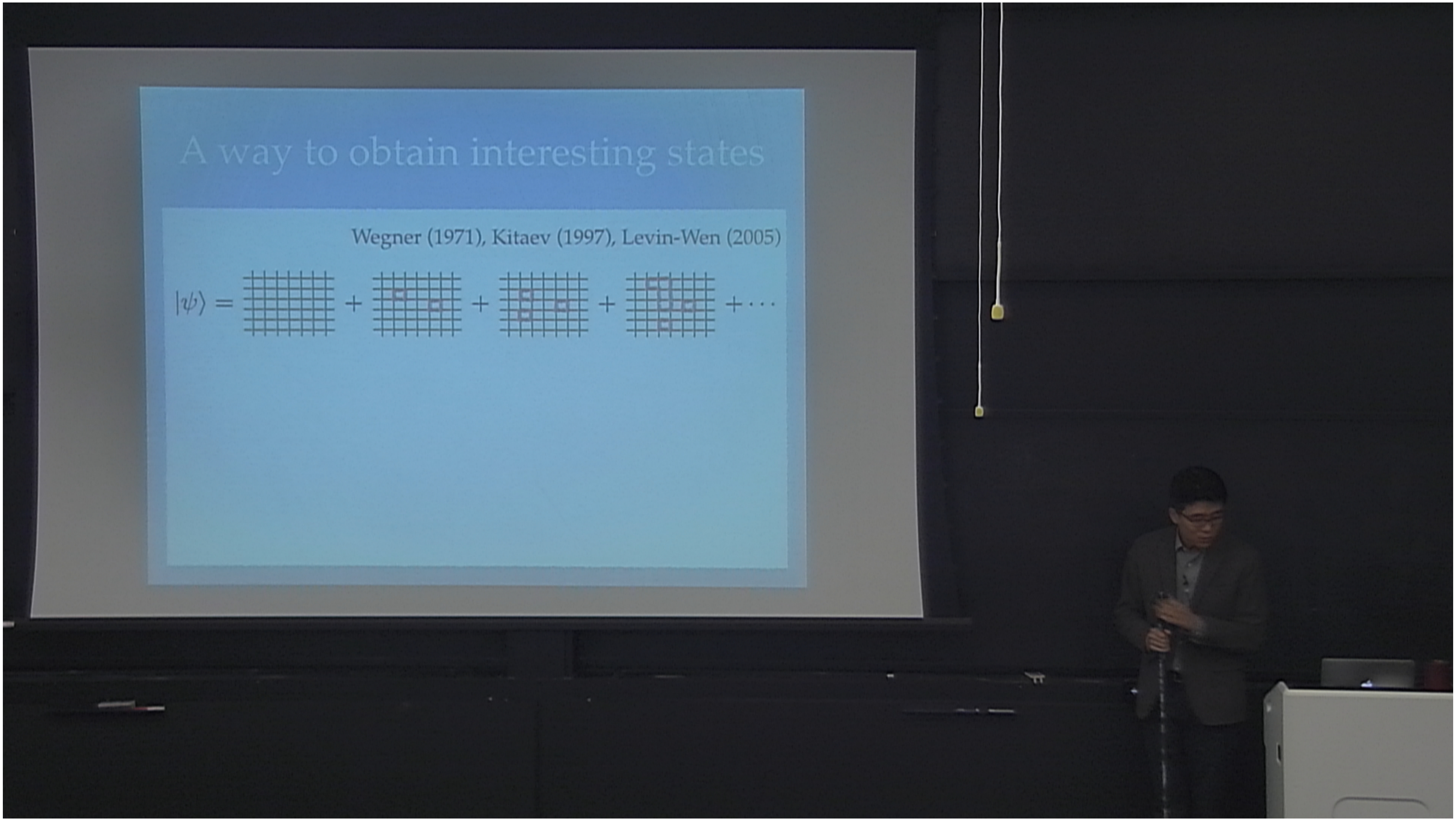
Protecting Information by Entanglement

- Entanglement can **hide** some degree of freedom from local observers.

Protecting Information by Entanglement

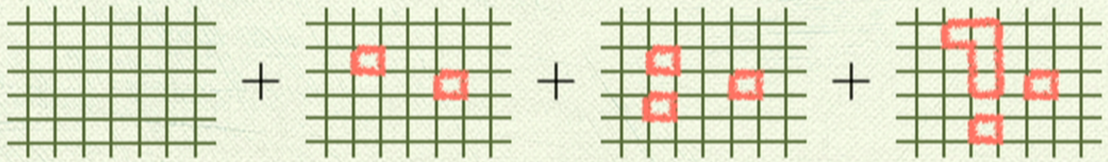
- Entanglement can **hide** some degree of freedom from local observers.
- Inevitably, the non-local info. is accessible in **multiple** ways
- Underlying idea of fault-tolerant quantum computing


Protecting Information by Entanglement
Entanglement can hide some degree of freedom from local observers.
Inevitably, the non-local info. is accessible in multiple ways
Underlying idea of fault-tolerant quantum computing
More interesting if this happens in ground states.


A way to obtain interesting states
Wegner (1971), Kitaev (1997), Levin-Wen (2005)
|ψ⟩ = + + + + ···

A way to obtain interesting states

Wegner (1971), Kitaev (1997), Levin-Wen (2005)

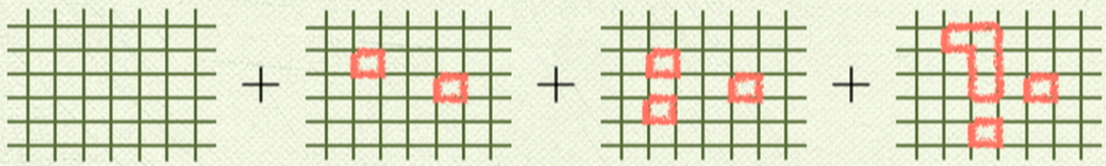
$|\psi\rangle =$  $+ \cdots$

- Qubits on edges
- Superposition of all deformations by small loops

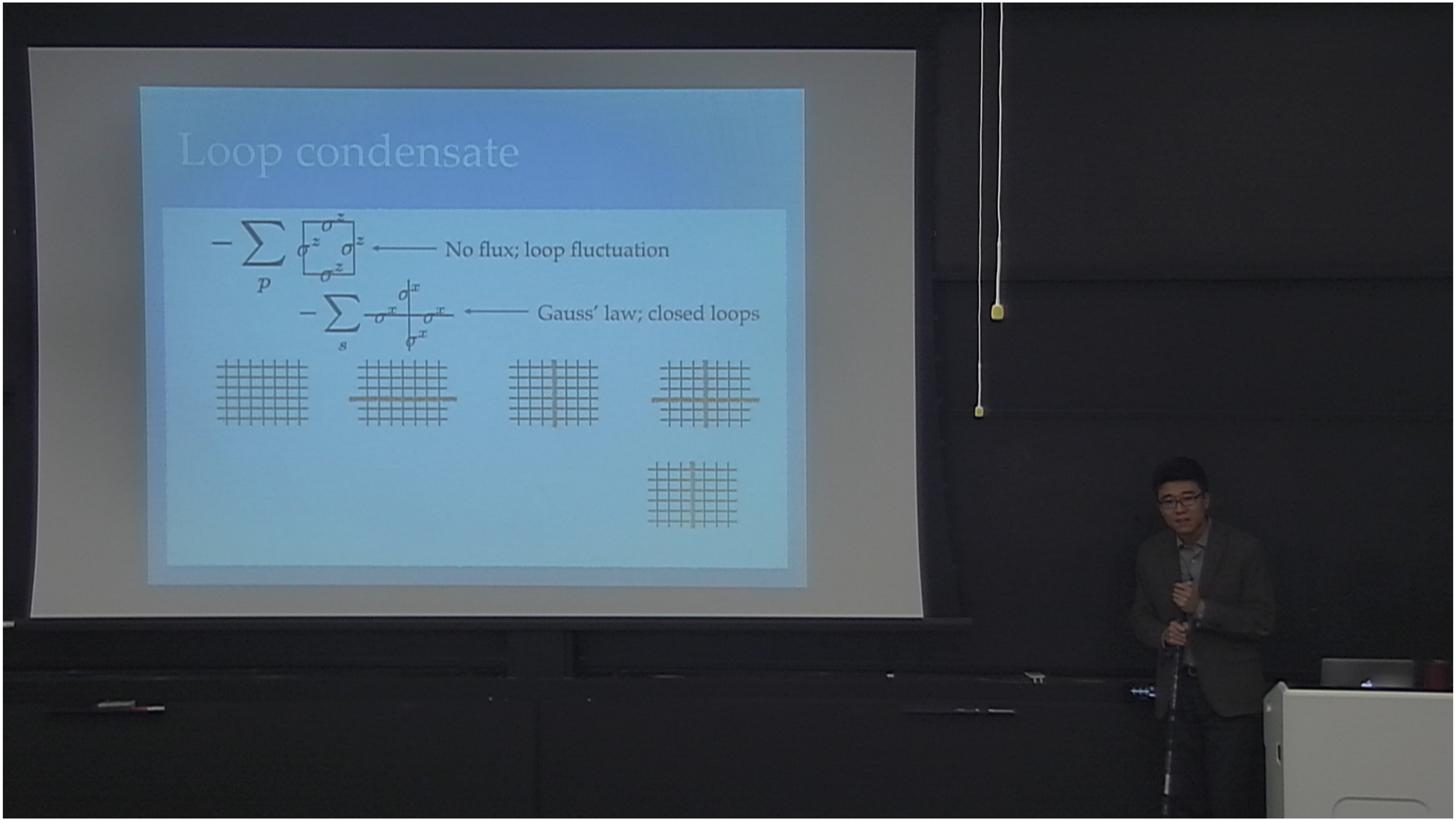
Loop condensate

$$-\sum_p \prod_{\square} \sigma^z \longleftarrow \text{No flux; loop creation/annihilation}$$

$$-\sum_s \prod_{+} \sigma^x \longleftarrow \text{Gauss' law; closed loops}$$

$|\psi\rangle =$  $+ \cdots$

Loop fluctuation cannot change the topology.



Loop condensate
$-\sum_p \sigma^z\sigma^z\sigma^z\sigma^z$ ← No flux; loop fluctuation
$-\sum_s \sigma^x\sigma^x\sigma^x\sigma^x$ ← Gauss' law; closed loops

Entanglement is an Invariant

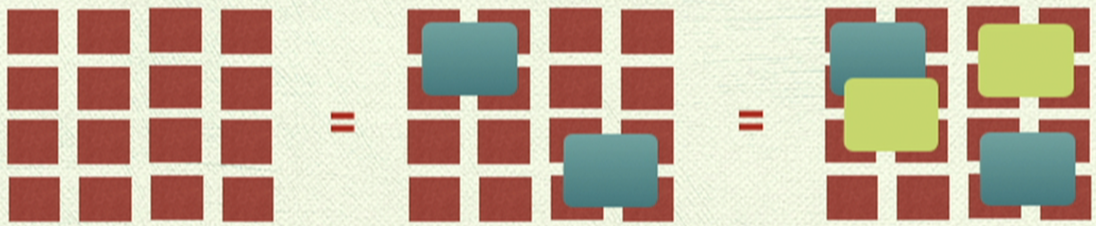
- Entanglement = invariant under local unitaries.

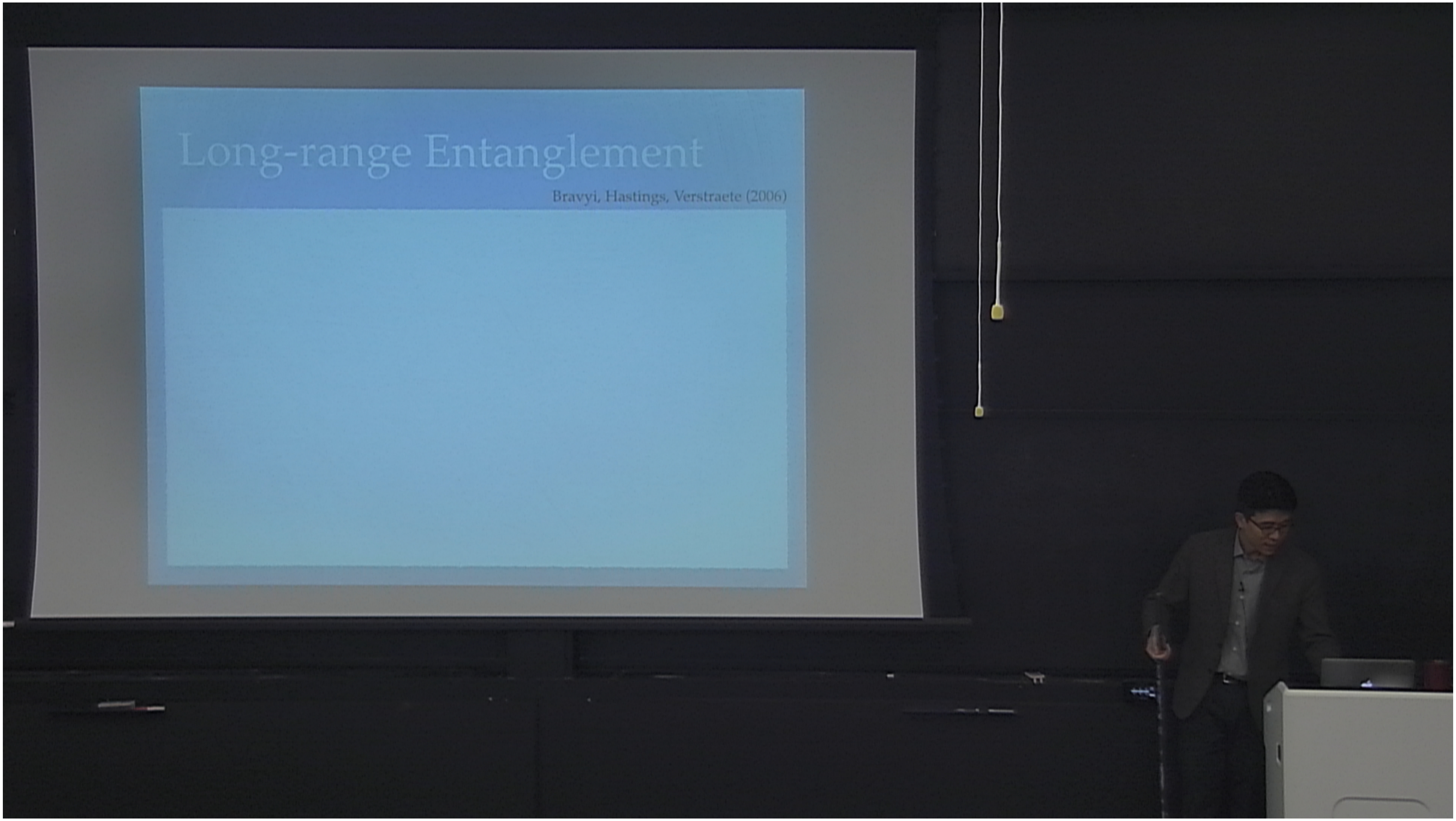
$$|\psi\rangle = \sum_r \sqrt{\lambda_r} |\phi_r^A\rangle |\phi_r^B\rangle$$

Many-body Entanglement

Many-body Entanglement

- Local entanglement is washed away by local unitaries.

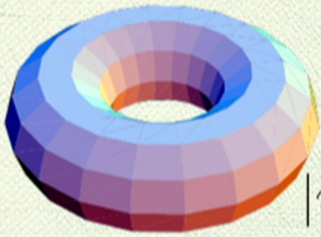



Long-range Entanglement
Bravyi, Hastings, Verstraete (2006)

Long-range Entanglement

Bravyi, Hastings, Verstraete (2006)

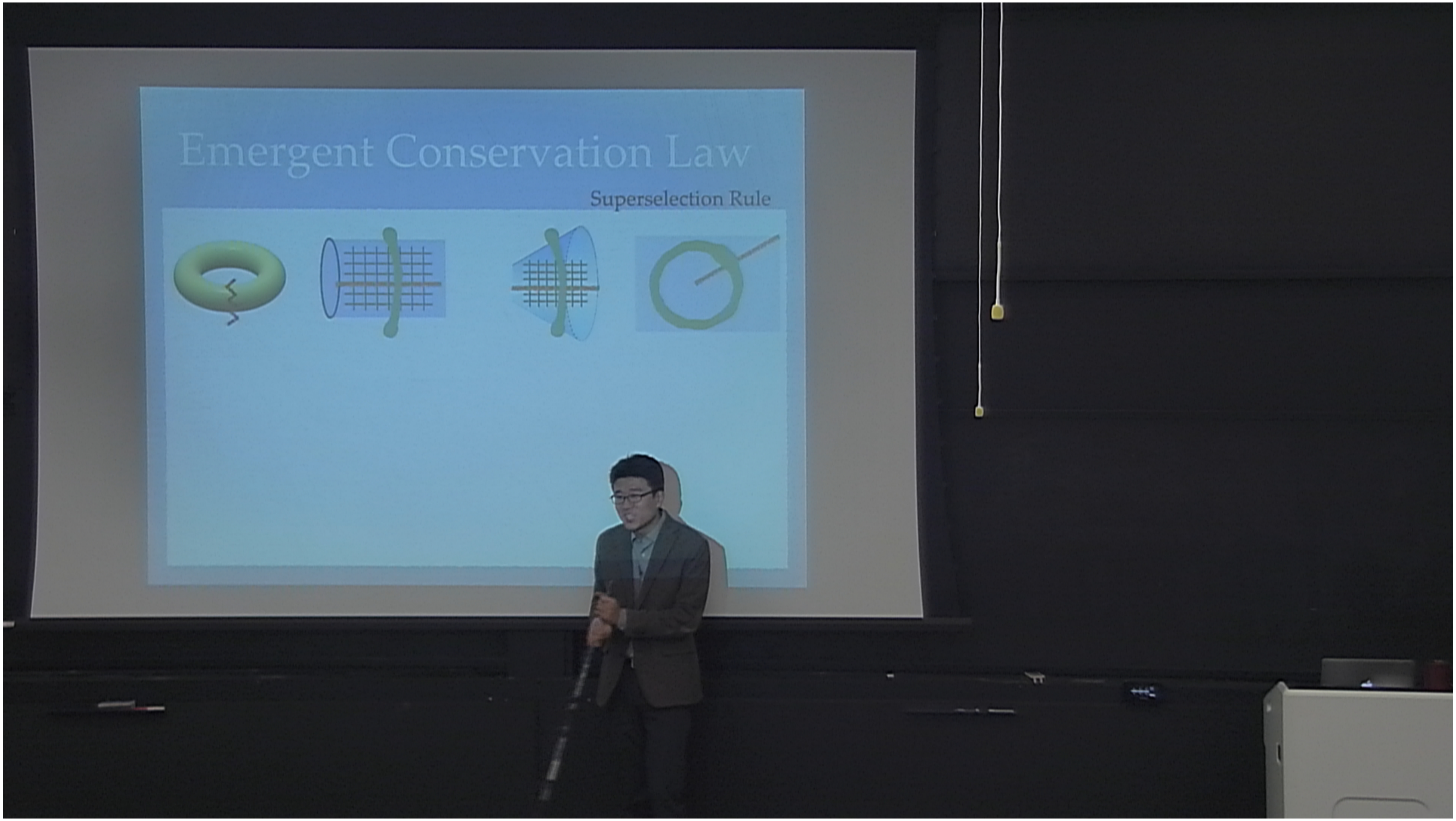
[Wolfram MathWorld]



$|\psi_0\rangle$

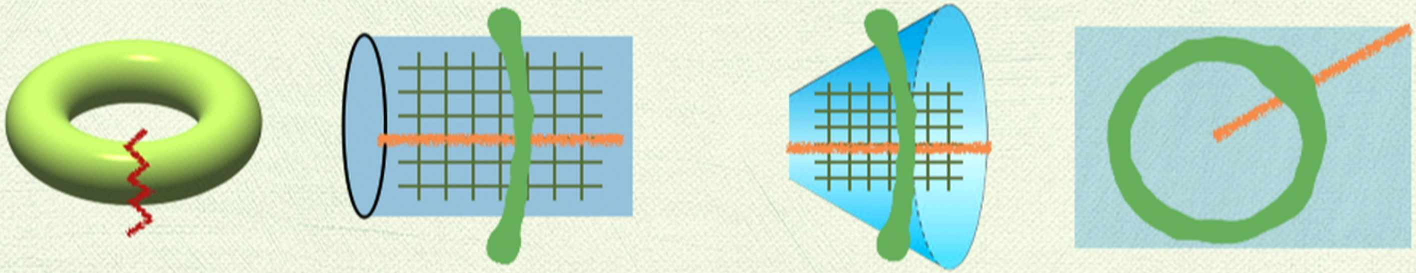
If $=$

$|01001\cdots011\rangle$


Emergent Conservation Law
Superselection Rule

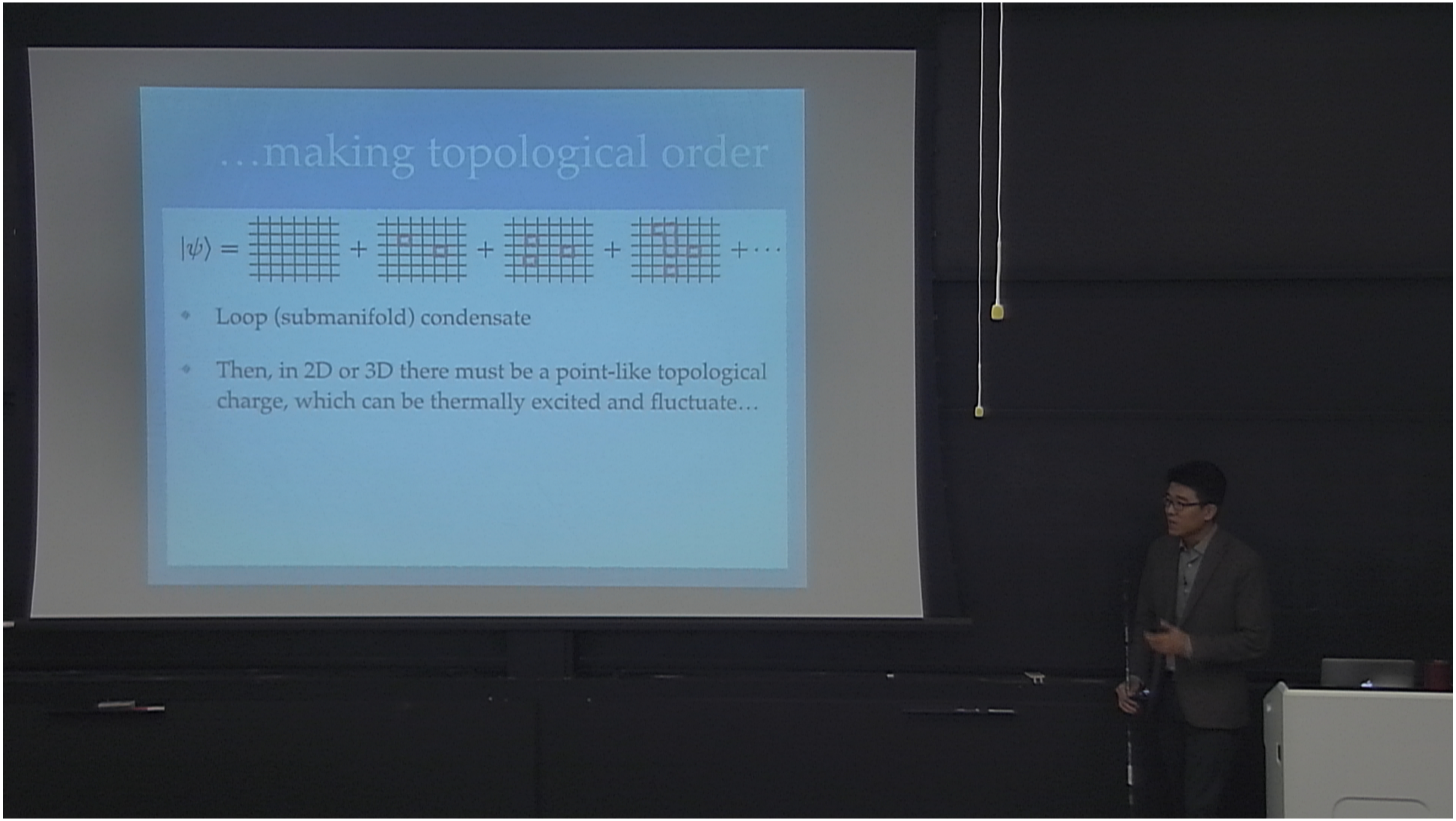
Emergent Conservation Law

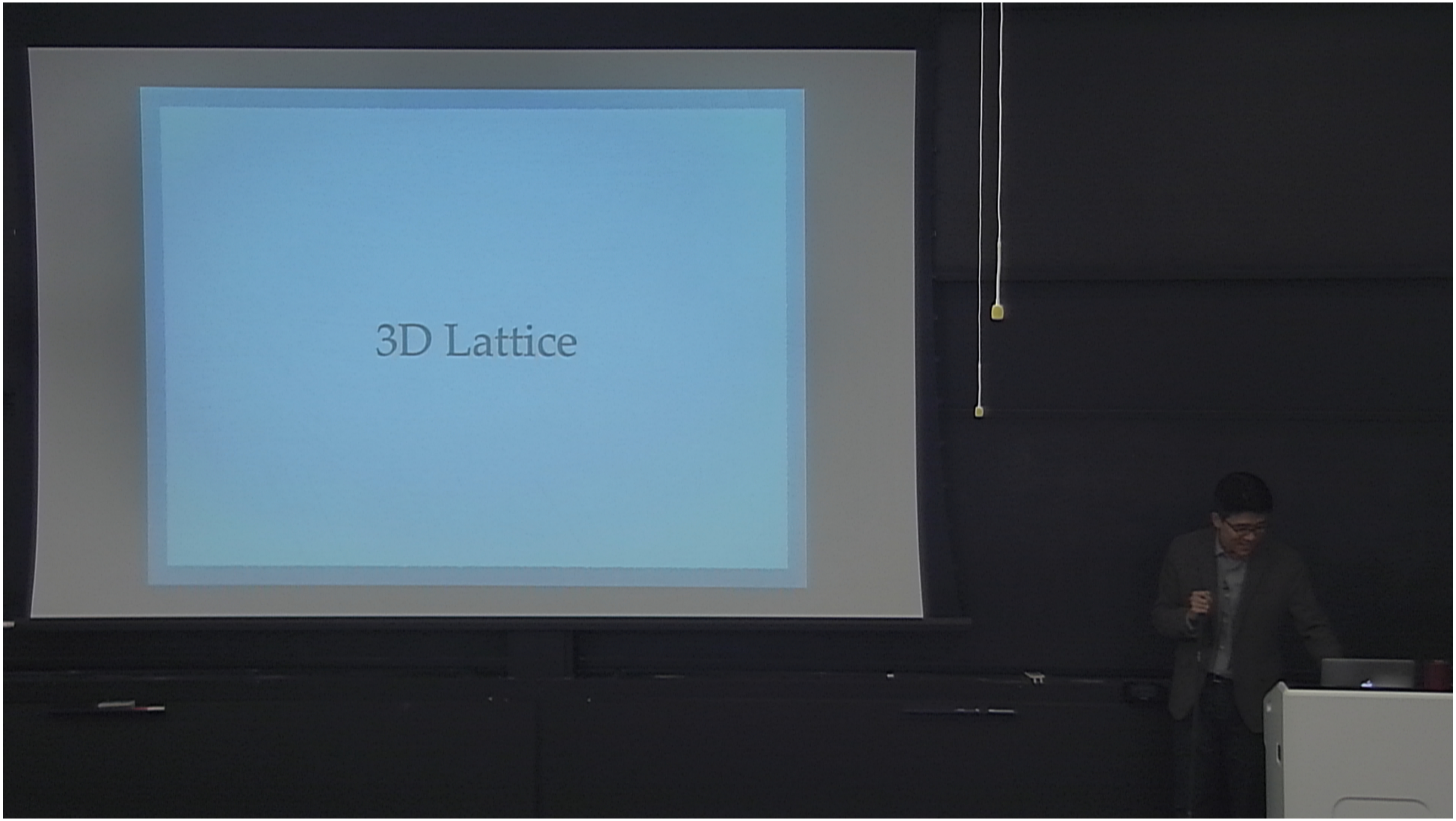
Superselection Rule



- The **presence of open end**
  can be detected by a loop operator **far away**.

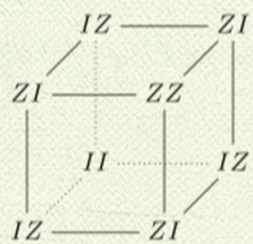
- Therefore,
  Conservation Law
  it cannot be changed by **any** operation near the open end.

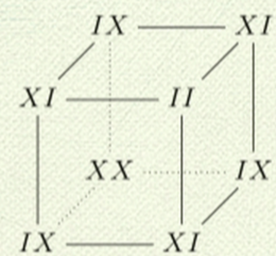

…making topological order
|ψ⟩ = + + + + ···
Loop (submanifold) condensate
Then, in 2D or 3D there must be a point-like topological charge, which can be thermally excited and fluctuate…


3D Lattice

My Interaction Design

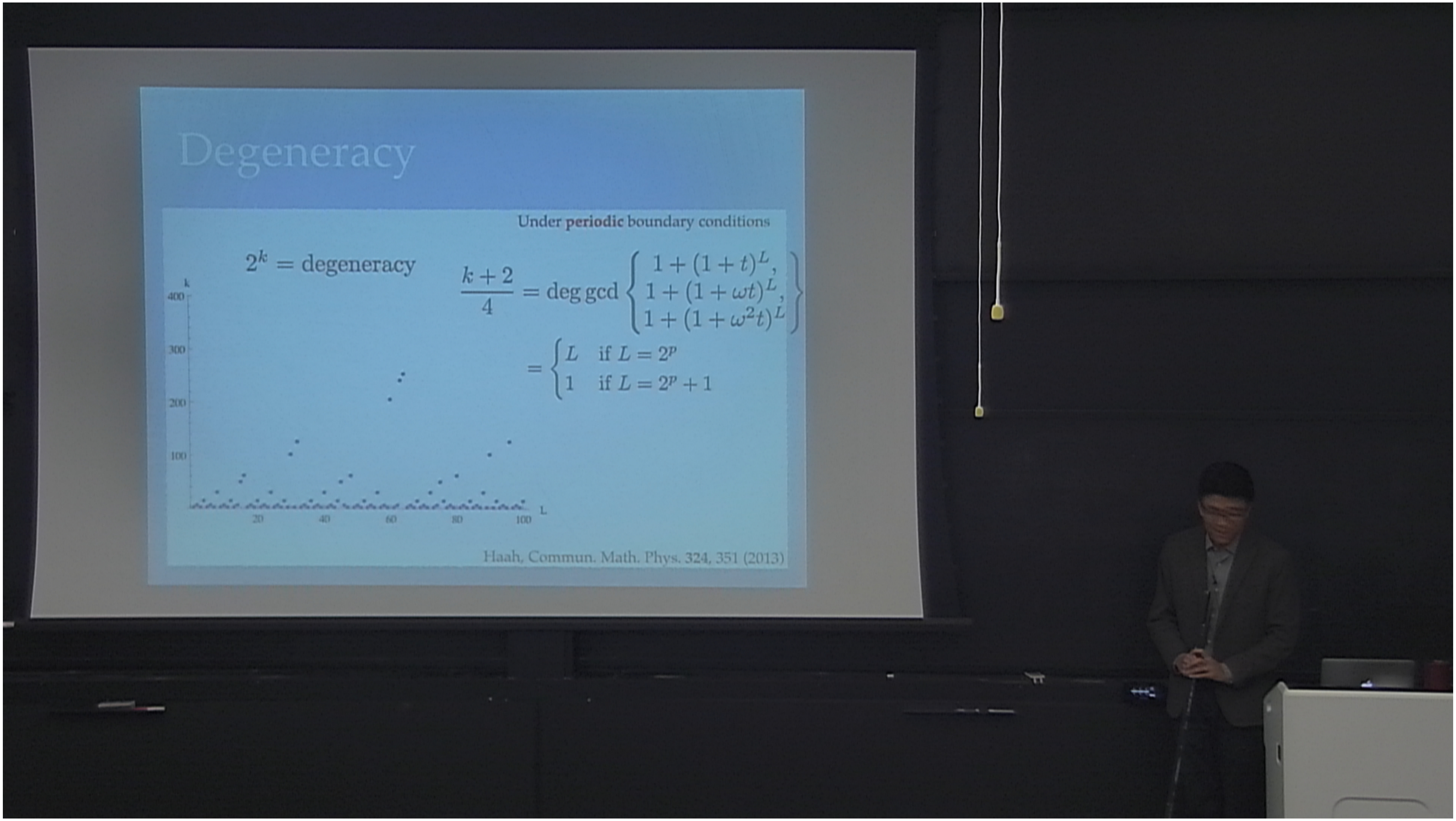
- Simple cubic lattice, with 2 qubits per site

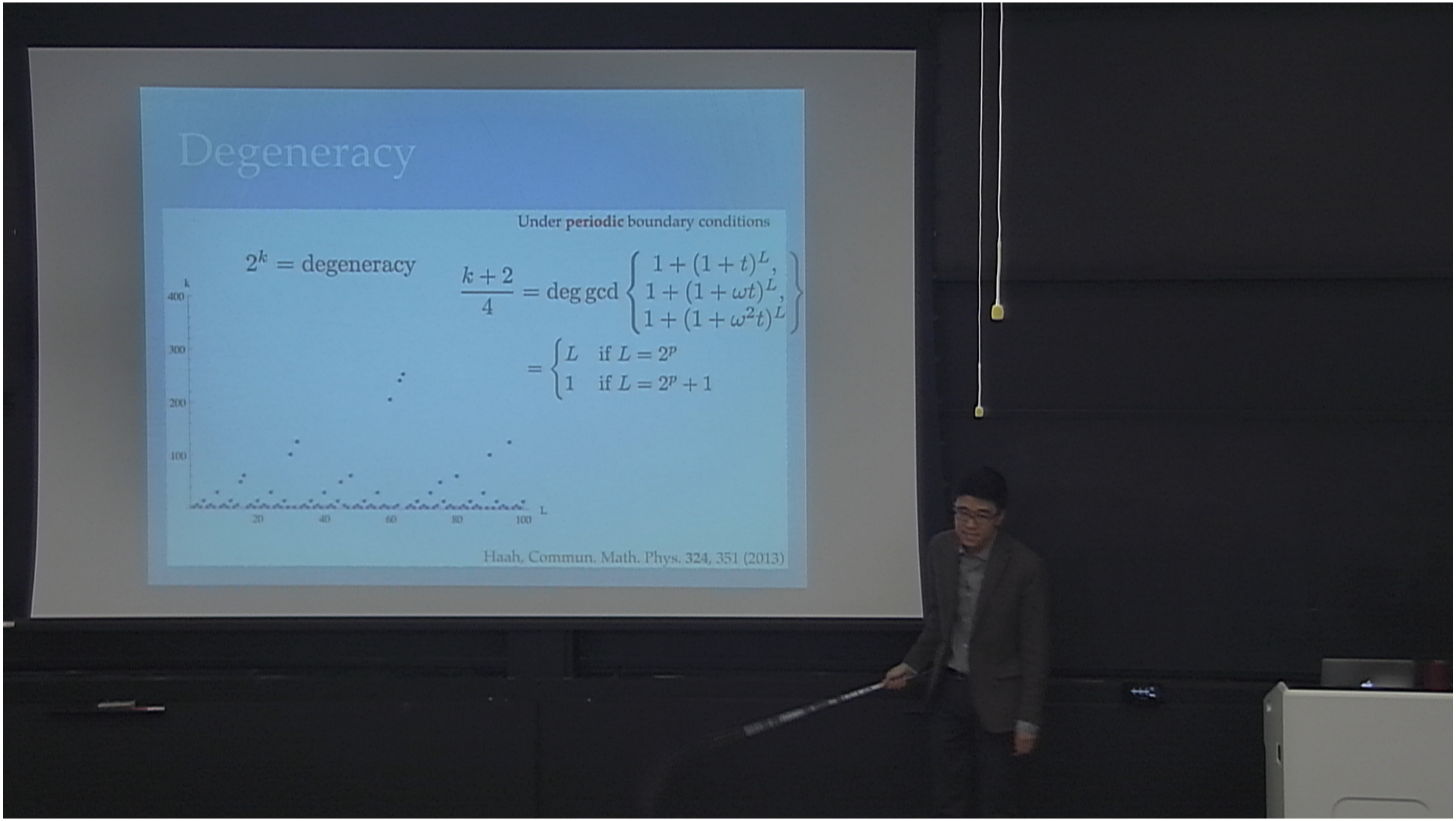
$-\sum_{c}$ 

$-\sum_{c}$ 

$$H = -J \sum_{i\in\Lambda} \Big(\sigma^x_{i,1}\sigma^x_{i,2}\sigma^x_{i+\hat{x},1}\sigma^x_{i+\hat{y},1}\sigma^x_{i+\hat{z},1}\sigma^x_{i+\hat{y}+\hat{z},2}\sigma^x_{i+\hat{z}+\hat{x},2}\sigma^x_{i+\hat{x}+\hat{y},2} + \sigma^z_{i,1}\sigma^z_{i,2}\sigma^z_{i-\hat{x},2}\sigma^z_{i-\hat{y},2}\sigma^z_{i-\hat{z},2}\sigma^z_{i-\hat{y}-\hat{z},1}\sigma^z_{i-\hat{z}-\hat{x},1}\sigma^z_{i-\hat{x}-\hat{y},1} \Big)$$

Haah, Phys. Rev. A **83** 042330 (2011)


Degeneracy
Under periodic boundary conditions
$2^k =$ degeneracy
$\frac{k+2}{4} = \deg \gcd \begin{Bmatrix} 1+(1+t)^L, \\ 1+(1+\omega t)^L, \\ 1+(1+\omega^2 t)^L \end{Bmatrix}$
$= \begin{cases} L & \text{if } L = 2^p \\ 1 & \text{if } L = 2^p + 1 \end{cases}$
k
400
300
200
100
L
20
40
60
80
100
Haah, Commun. Math. Phys. 324, 351 (2013)


Degeneracy
Under periodic boundary conditions
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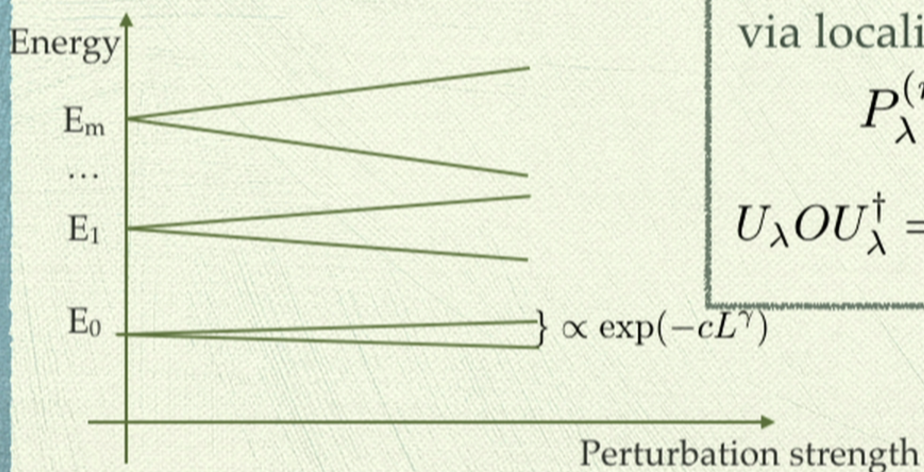
$= \begin{cases} L & \text{if } L = 2^p \\ 1 & \text{if } L = 2^p + 1 \end{cases}$
k
400
300
200
100
20
40
60
80
100
L
Haah, Commun. Math. Phys. 324, 351 (2013)

Stability under Perturbations

- Bravyi, Michalakis, Hastings, J. Math.Phys. (2010)
- Hastings, Wen, PRB (2005)
- Kitaev, Ann. Phys. (2003)
- Wen, Niu, PRB (1990)

Local indistinguishability implies:

(Any local operator that acts on the ground-state subspace is trivial.)



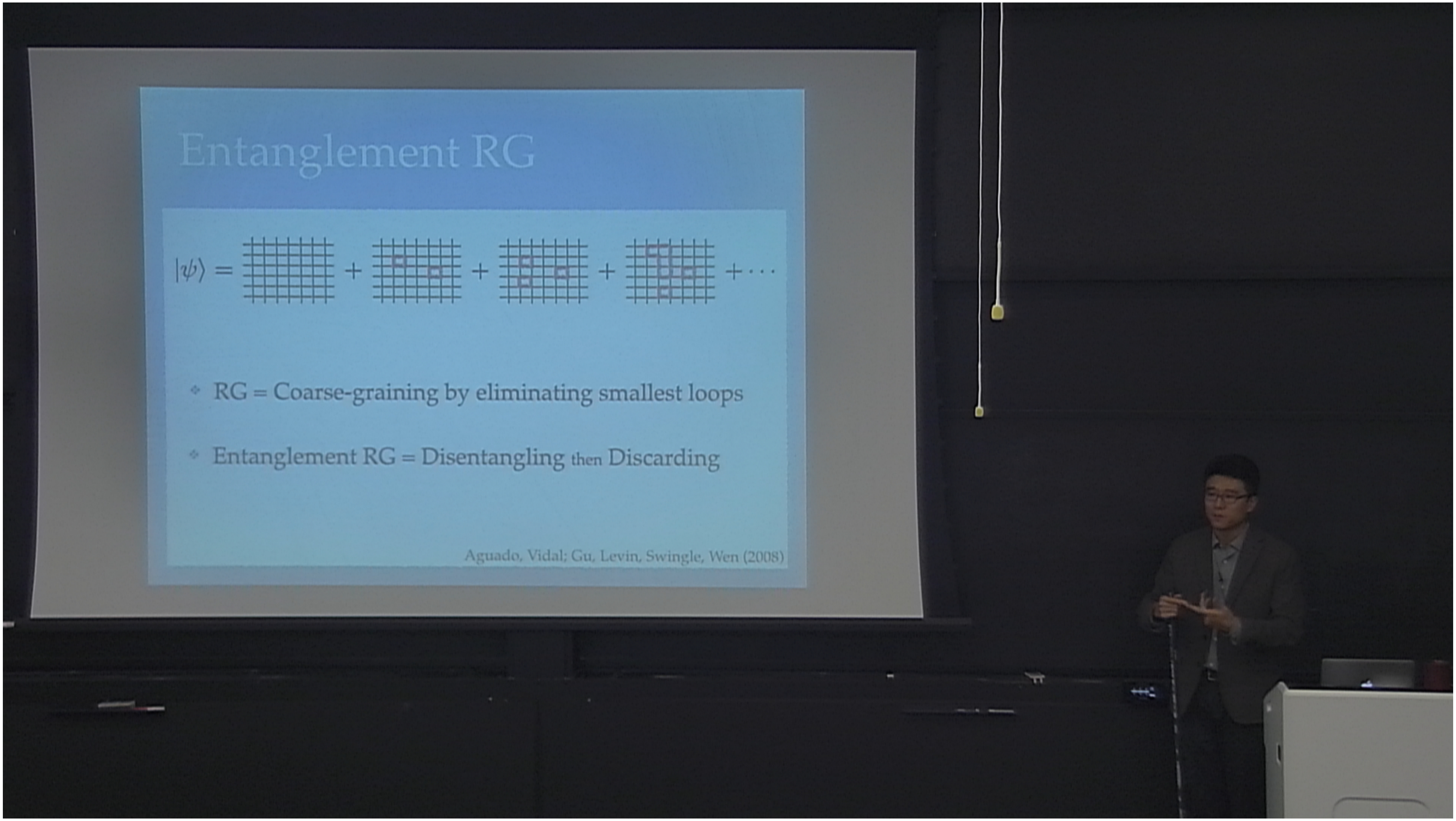
Each subspace is connected
via locality-preserving unitary.

$$P_\lambda^{(n)} = U_\lambda P_0^{(n)} U_\lambda^\dagger$$

$$U_\lambda O U_\lambda^\dagger = \sum_r O'_r, \quad \|O'_r\| \sim e^{-cr^\gamma}$$

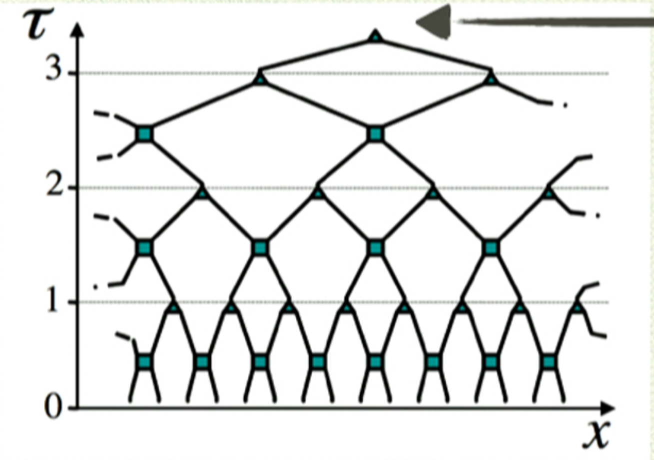
Entanglement RG

Aguado, Vidal; Gu, Levin, Swingle, Wen (2008)


Entanglement RG
$|\psi\rangle = $ + + + + ⋯
RG = Coarse-graining by eliminating smallest loops
Entanglement RG = Disentangling then Discarding
Aguado, Vidal; Gu, Levin, Swingle, Wen (2008)

MERA

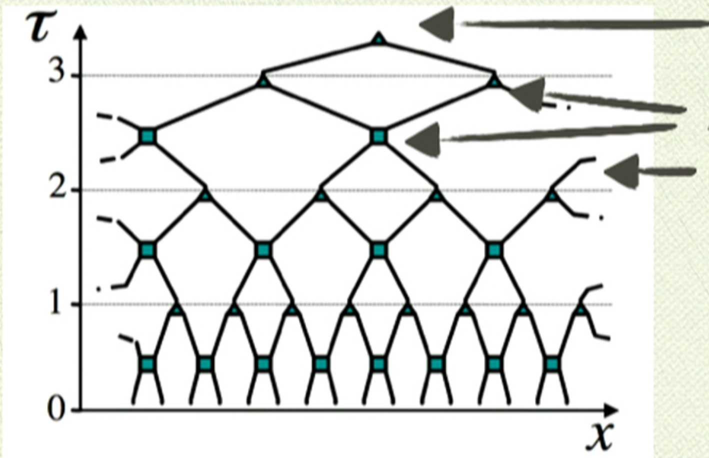
Multi-scale Entanglement Renormalization Ansatz (Vidal 2006)



Start with a qubit

MERA

Multi-scale Entanglement Renormalization Ansatz (Vidal 2006)



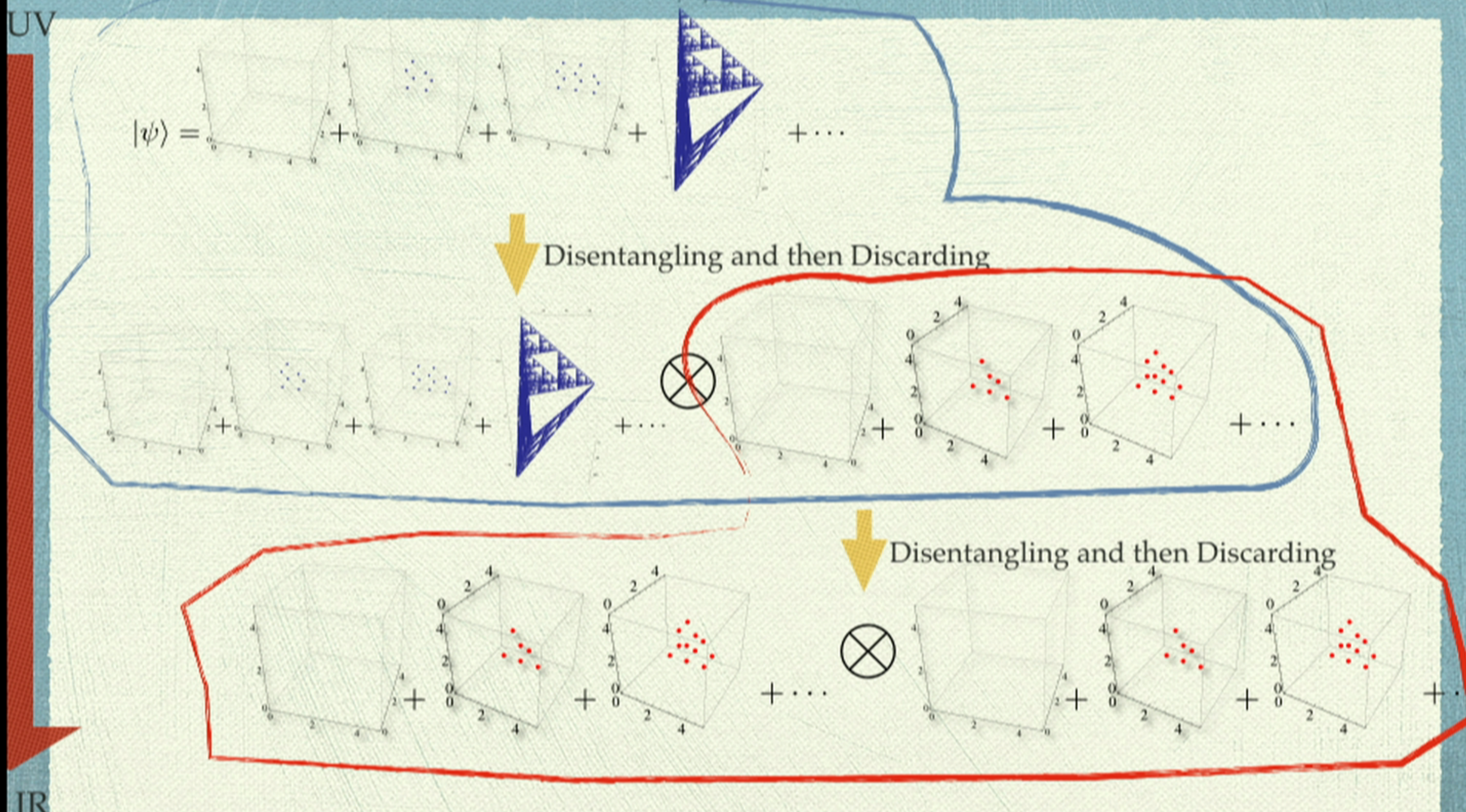
Start with a qubit

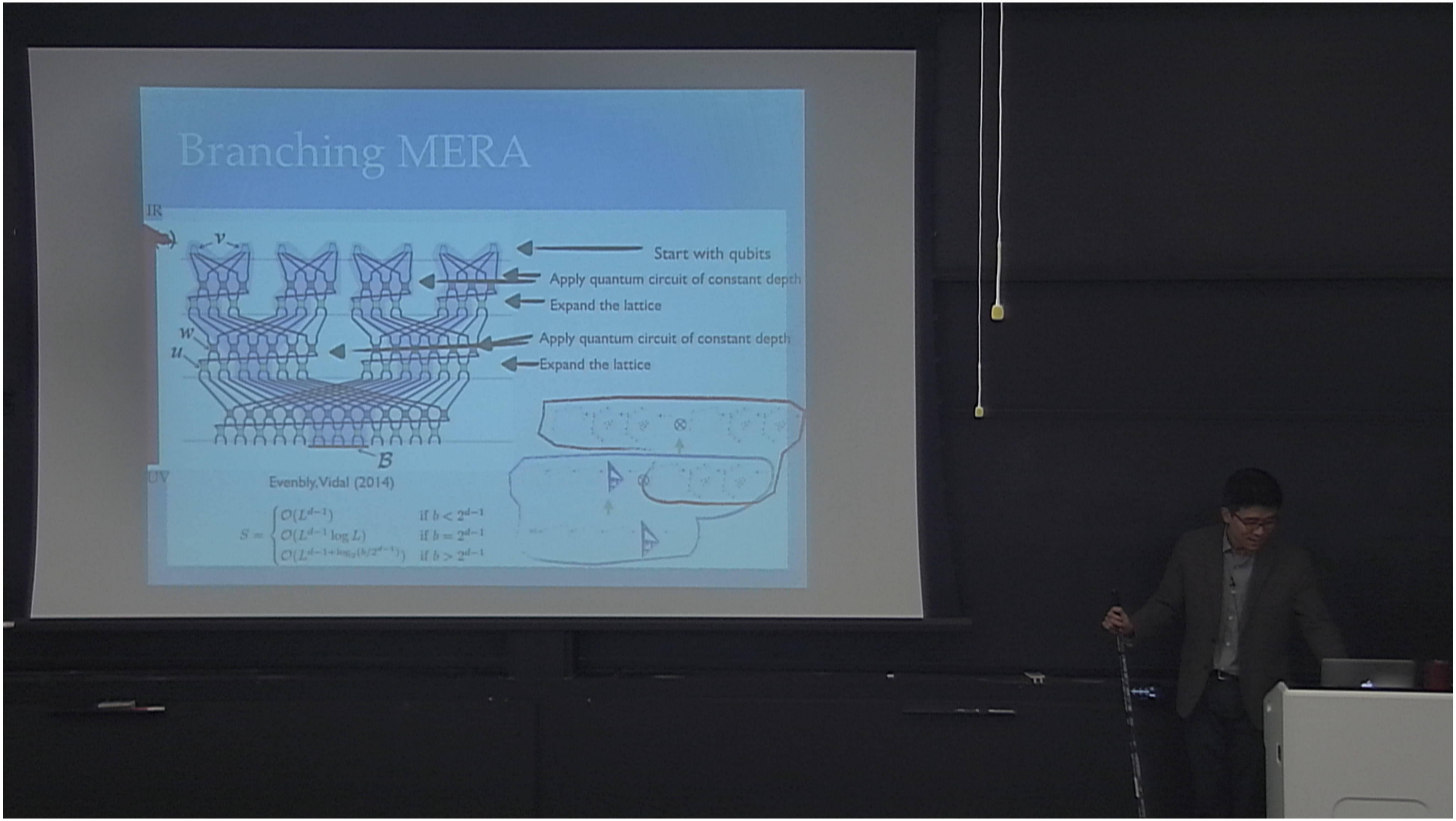
Apply quantum circuit of constant depth

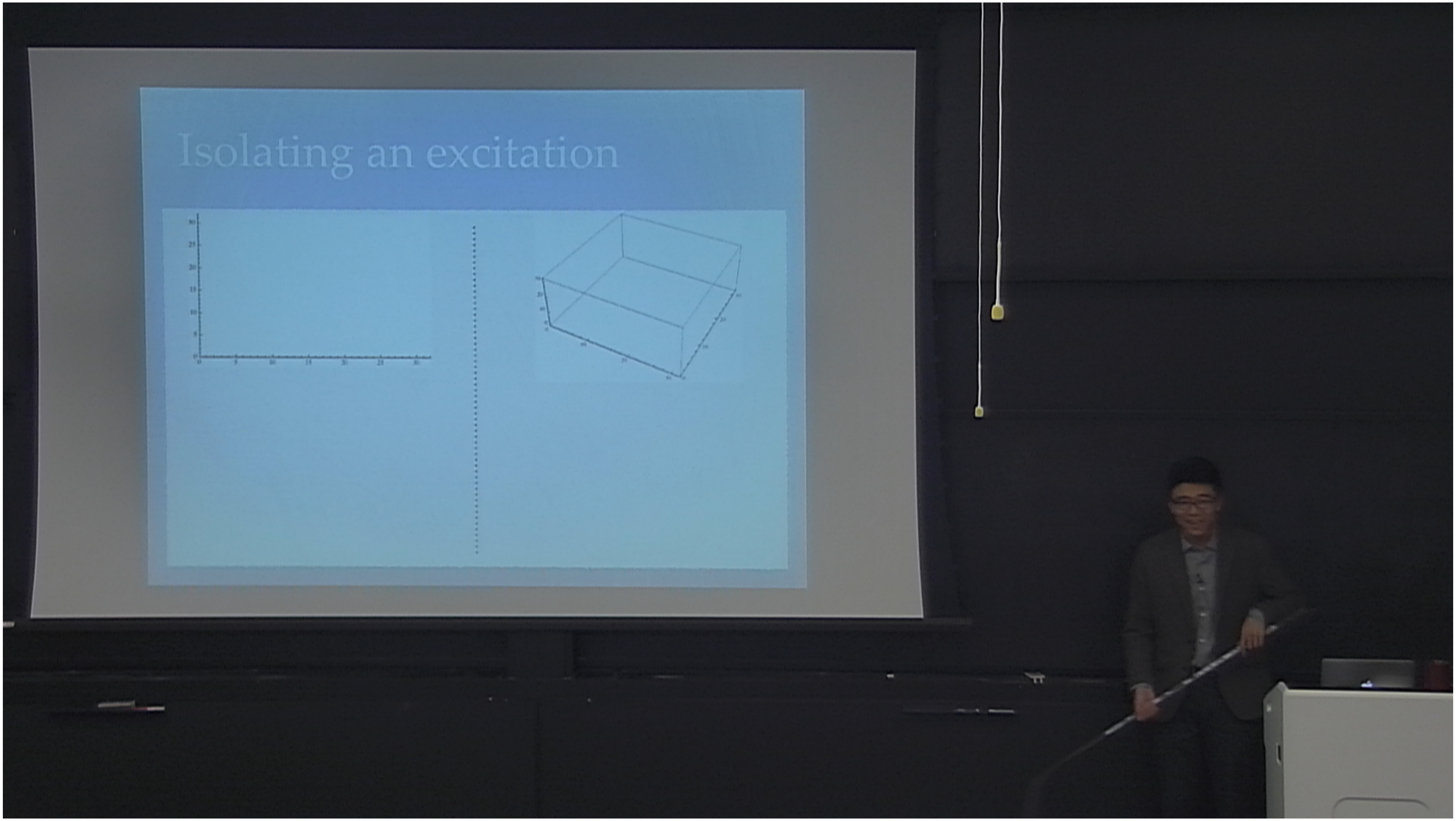
Expand the lattice

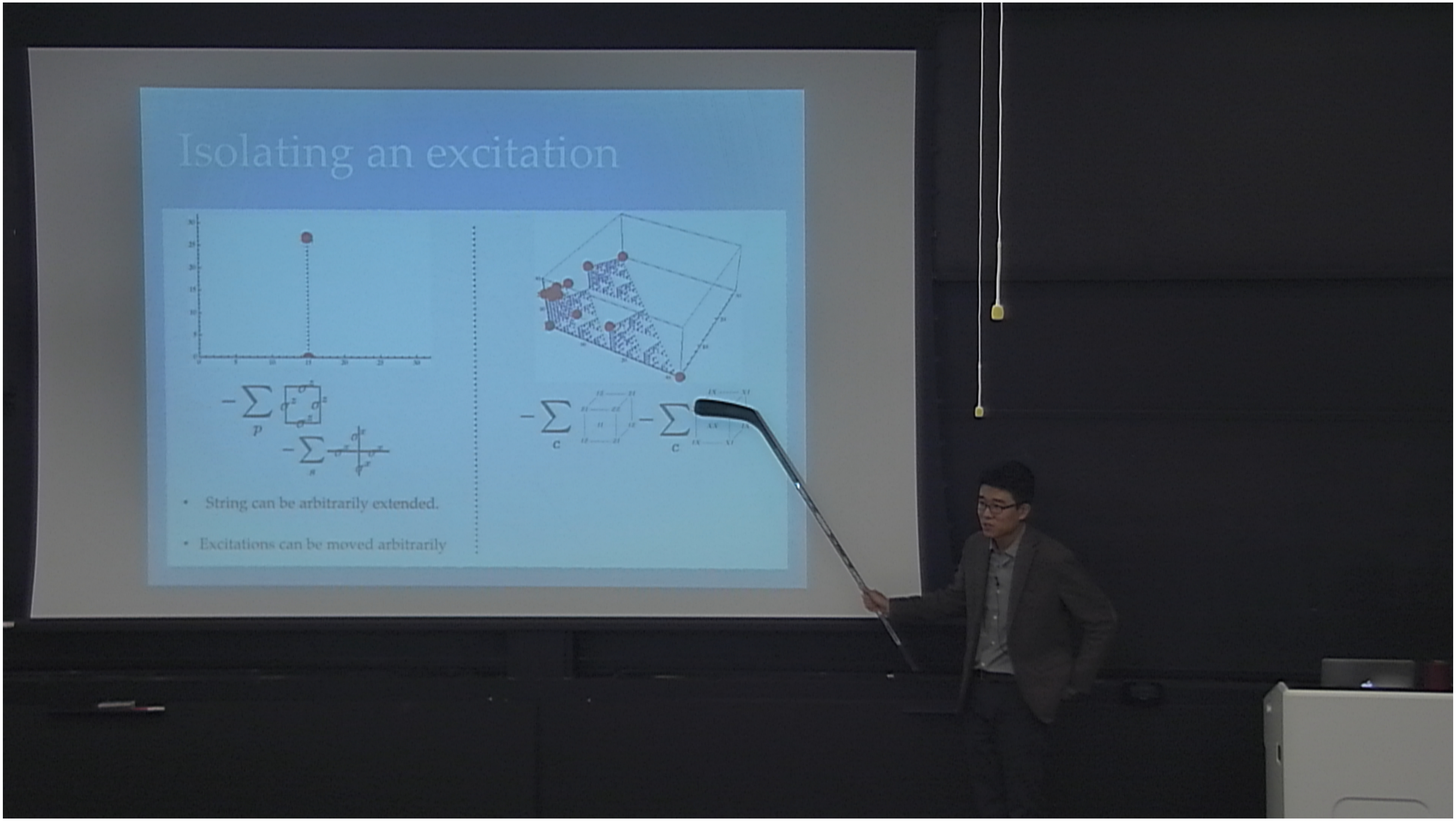
Entanglement RG

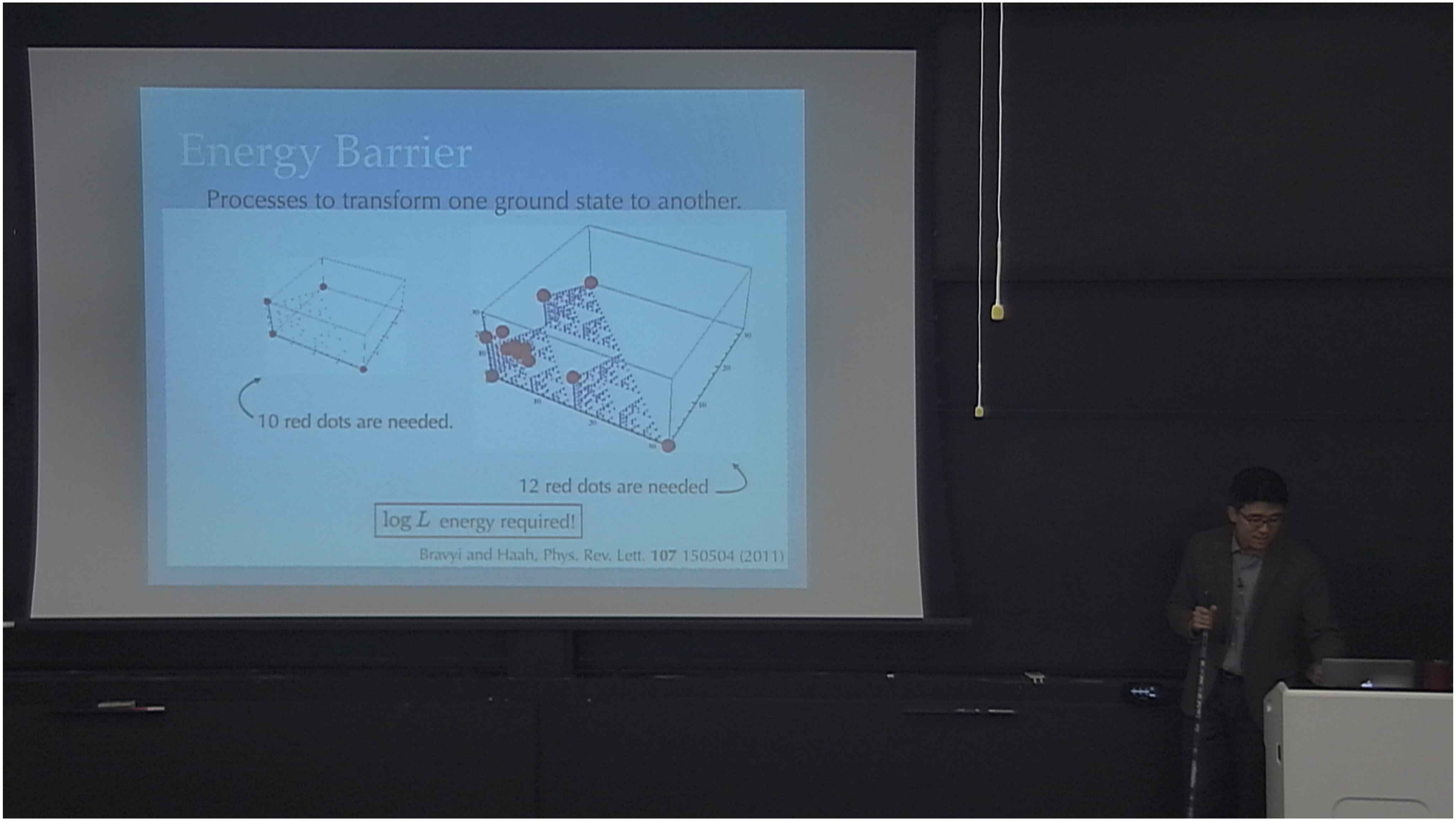
Haah, Phys. Rev. B **89**, 075119 (2014)

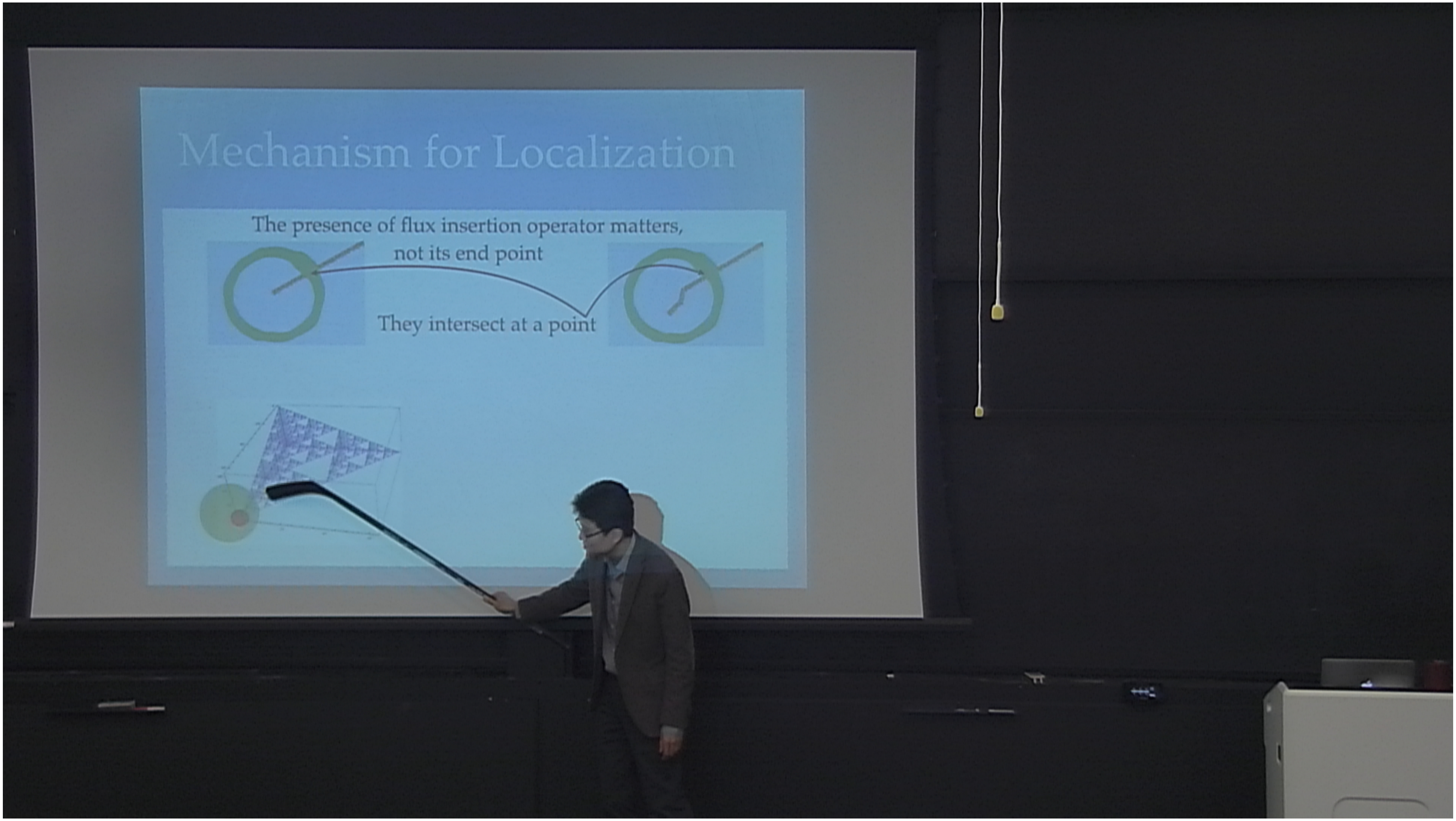



Branching MERA
IR
v
w
u
UV
Start with qubits
Apply quantum circuit of constant depth
Expand the lattice
Apply quantum circuit of constant depth
Expand the lattice
Evenbly, Vidal (2014)
$S = \begin{cases} O(L^{d-1}) & \text{if } b < 2^{d-1} \\ O(L^{d-1}\log L) & \text{if } b = 2^{d-1} \\ O(L^{d-1+\log_2(b/2^{d-1})}) & \text{if } b > 2^{d-1} \end{cases}$


Isolating an excitation

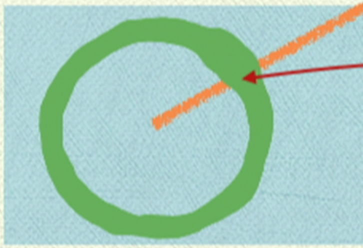

Isolating an excitation
String can be arbitrarily extended.
Excitations can be moved arbitrarily

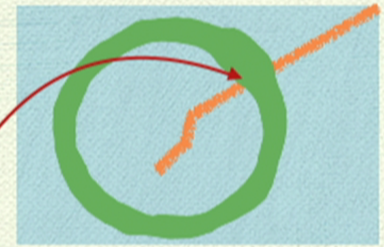

Energy Barrier
Processes to transform one ground state to another.
10 red dots are needed.
12 red dots are needed
log L energy required!
Bravyi and Haah, Phys. Rev. Lett. 107 150504 (2011)


Mechanism for Localization
The presence of flux insertion operator matters,
not its end point
They intersect at a point

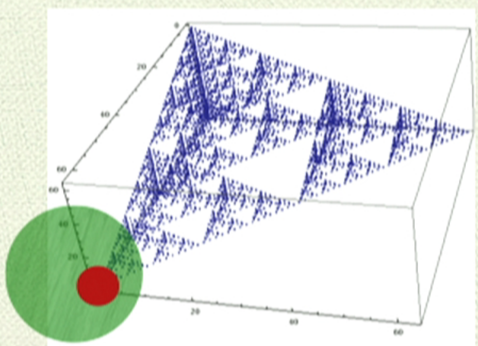
Mechanism for Localization

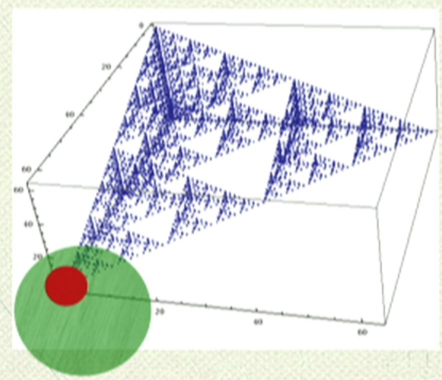
The presence of flux insertion operator matters,
not its end point

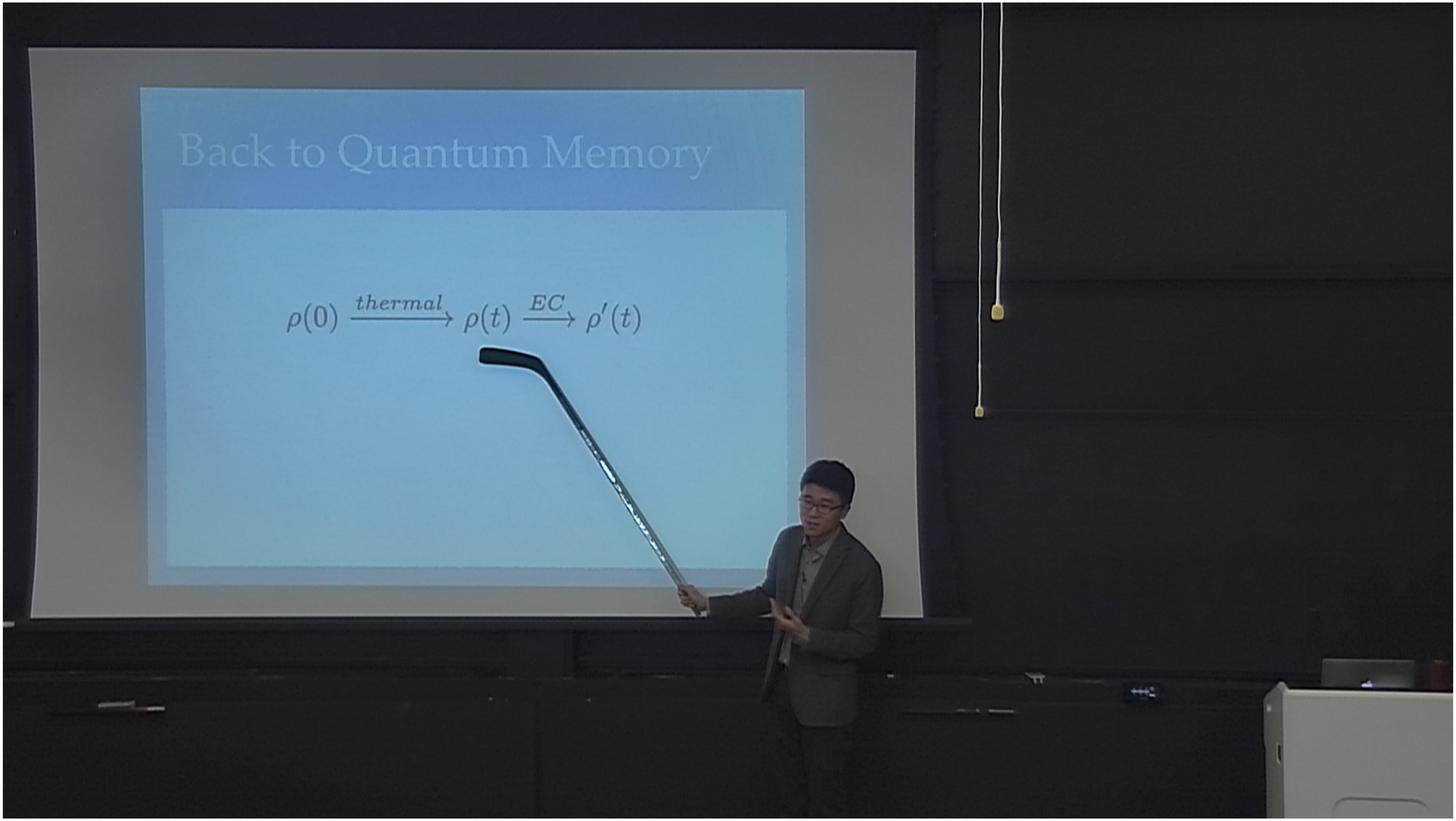


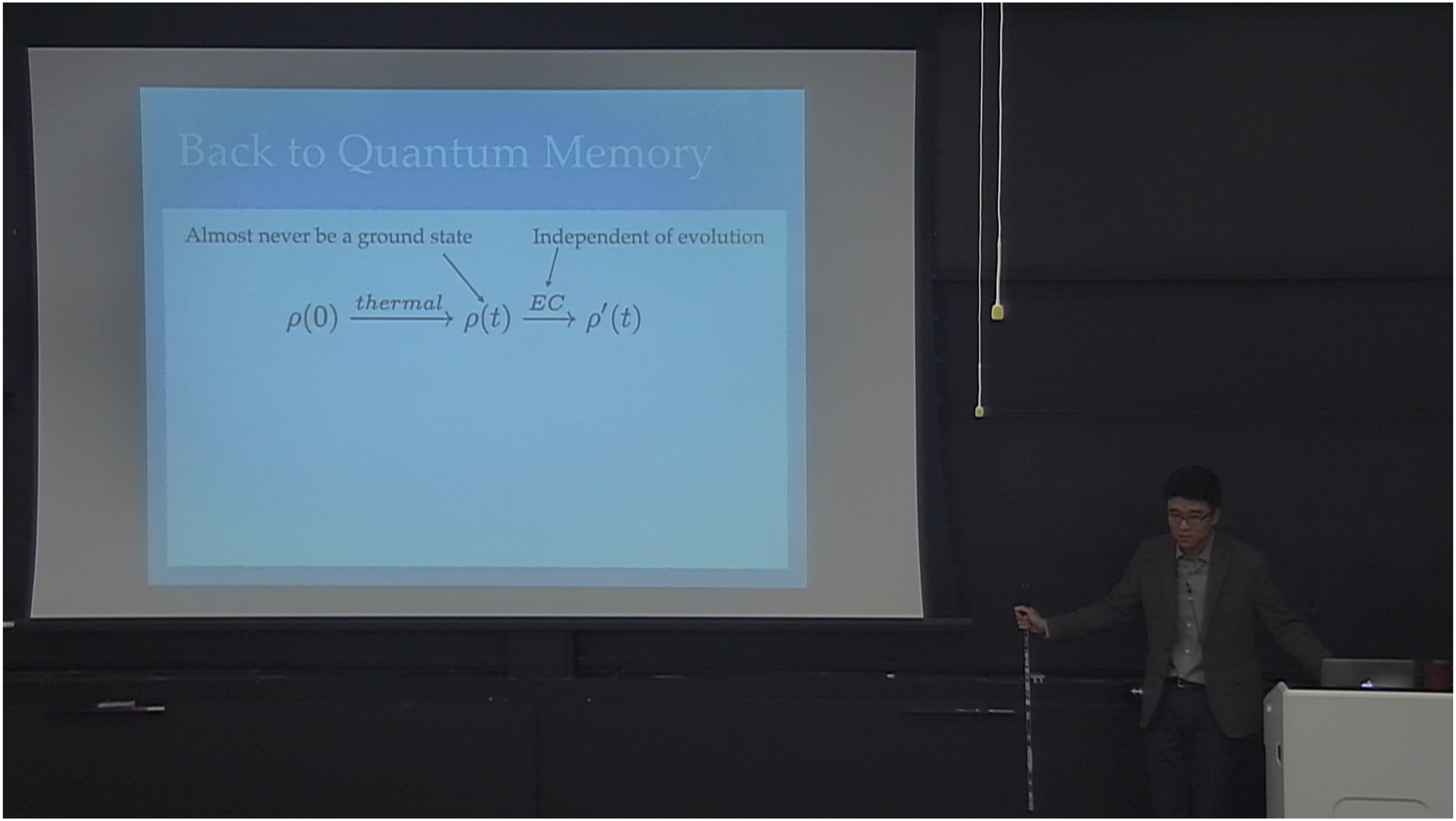


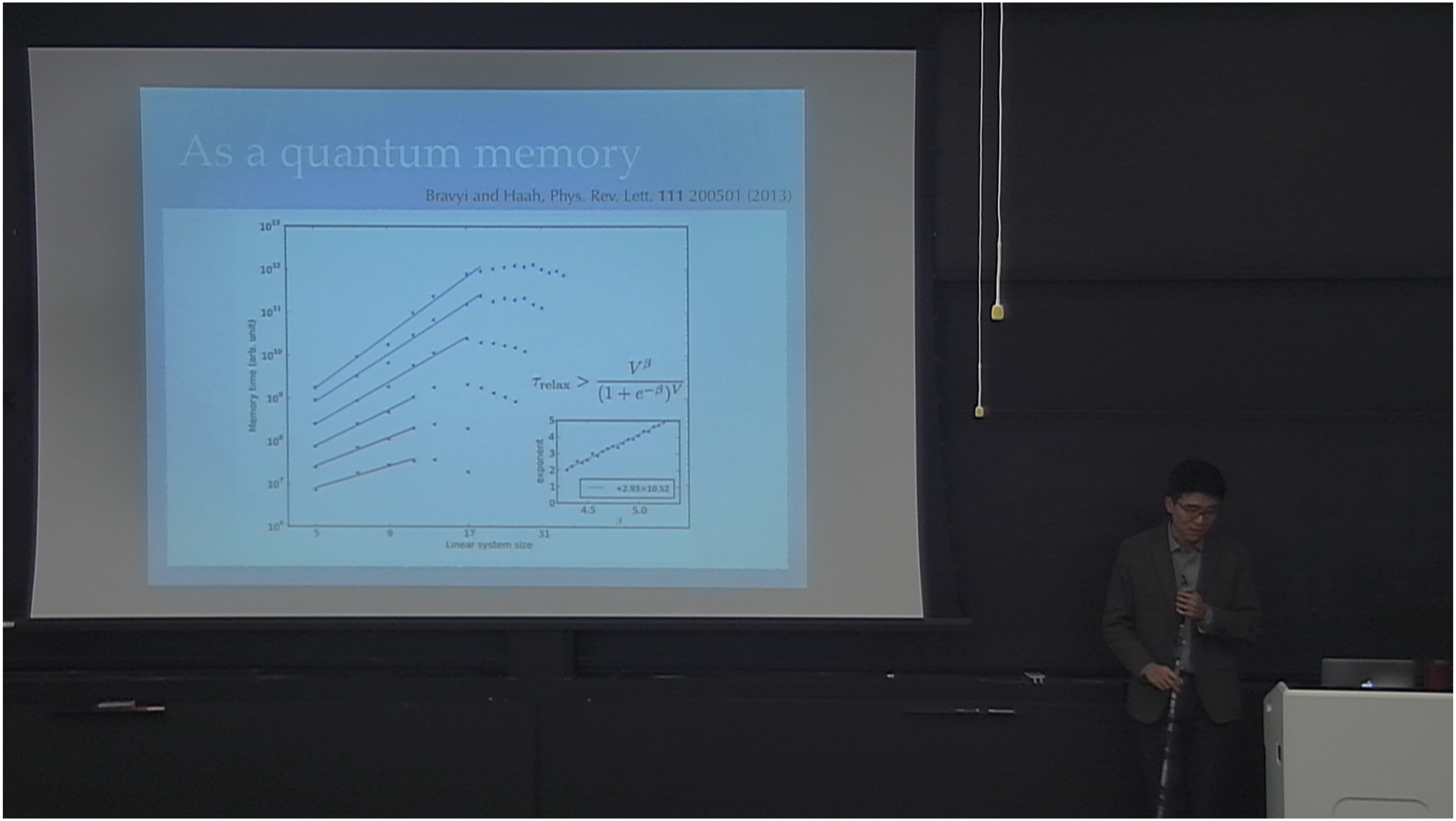
They intersect at a point





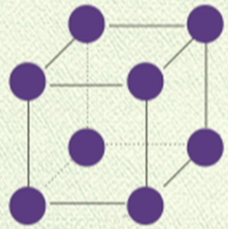

Back to Quantum Memory
$\rho(0) \xrightarrow{thermal} \rho(t) \xrightarrow{EC} \rho'(t)$


Back to Quantum Memory
Almost never be a ground state
Independent of evolution
$\rho(0) \xrightarrow{thermal} \rho(t) \xrightarrow{EC} \rho'(t)$


As a quantum memory
Bravyi and Haah, Phys. Rev. Lett. 111 200501 (2013)
Memory time (arb. unit)
Linear system size
$\tau_{\text{relax}} > \frac{V^\beta}{(1+e^{-\beta})^V}$
exponent

Other models

Chamon (2005),
Haah (2011),
Kim (2011),
Yoshida (2013),
Vijay, Haah, Fu (2015)

$$-\sum_c$$ 

Why cube?
Nothing special, just local interaction

- All beyond TQFT
- Immobile point-like excitation
- Composite excitations:
  - Still immobile
  - Trapped along a "submanifold."

[Model Factory]

$$\begin{cases} \sigma^\dagger \lambda \sigma = 0 \\ \ker \sigma^\dagger \lambda = \mathrm{im} \sigma \end{cases}$$

over polynomial rings with coefficients in finite prime field.

Long-range Entangled Many-body States

$$\alpha|00\rangle + \beta|11\rangle$$