Title: What the Reeh-Schielder theorem tells us about relativistic causality, or, Can experimenters in a lab on Earth create a Taj Mahal on the back of the moon?

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Abstract: <p>The Reeh-Schlieder theorem says, roughly, that, in any reasonable quantum field theory, for any bounded region of spacetime R, any state can be approximated arbitrarily closely by operating on the vacuum state (or any state of bounded energy) with operators formed by smearing polynomials in the field operators with functions having support in R. This strikes many as counterintuitive, and Reinhard Werner has glossed the theorem as saying that â€œBy acting on the vacuum with suitable operations in a terrestrial laboratory, an experimenter can create the Taj Mahal on (or even behind) the Moon!â€• This talk has two parts. First, I hope to convince listeners that the theorem is not counterintuitive, and that it follows immediately from facts that are already familiar fare to anyone who has digested the opening chapters of any standard introductory textbook of QFT. In the second, I will discuss what we can learn from the theorem about how relativistic causality is implemented in quantum field theories.</p>

What the Reeh-Schlieder Theorem Tells us About Relativistic Causality or, Can we create the Taj Mahal on the back of the moon?

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Quantum Foundations Seminar
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The Reeh-Schlieder theorem

- Under mild assumptions, for any quantum field theory in Minkowski spacetime, for any open spacetime region R, any state can be approximated arbitrarily closely by acting on the vacuum with an operator associated with R.

This strikes some as counterintuitive

Rainer Verch: This result by Reeh and Schlieder appears entirely counterintuitive since it says that every state of the theory can be approximated to arbitrary precision by acting with operators (operations) localized in any arbitrarily given spacetime region on the vacuum. To state it in a rather more drastic and provocative way (which I learned from Reinhard Werner): By acting on the vacuum with suitable operations in a terrestrial laboratory, an experimenter can create the Taj Mahal on (or even behind) the Moon!

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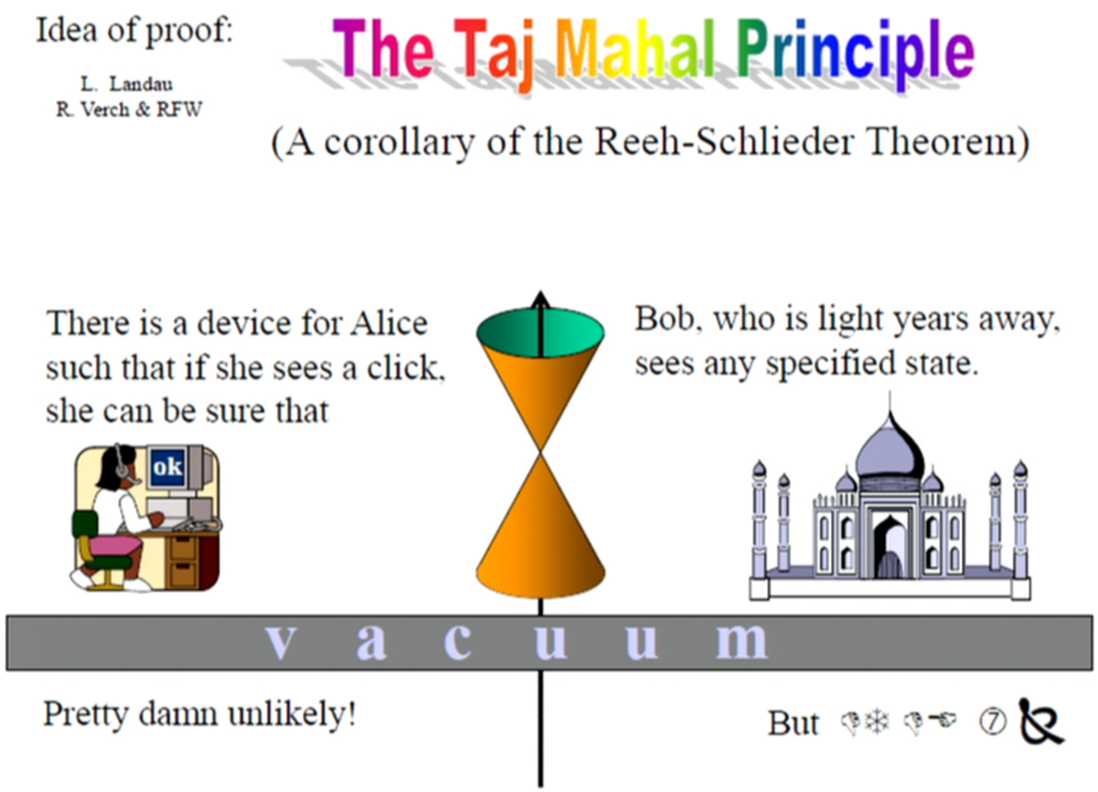
The “Taj Mahal Principle”

- From Reinhard Werner:



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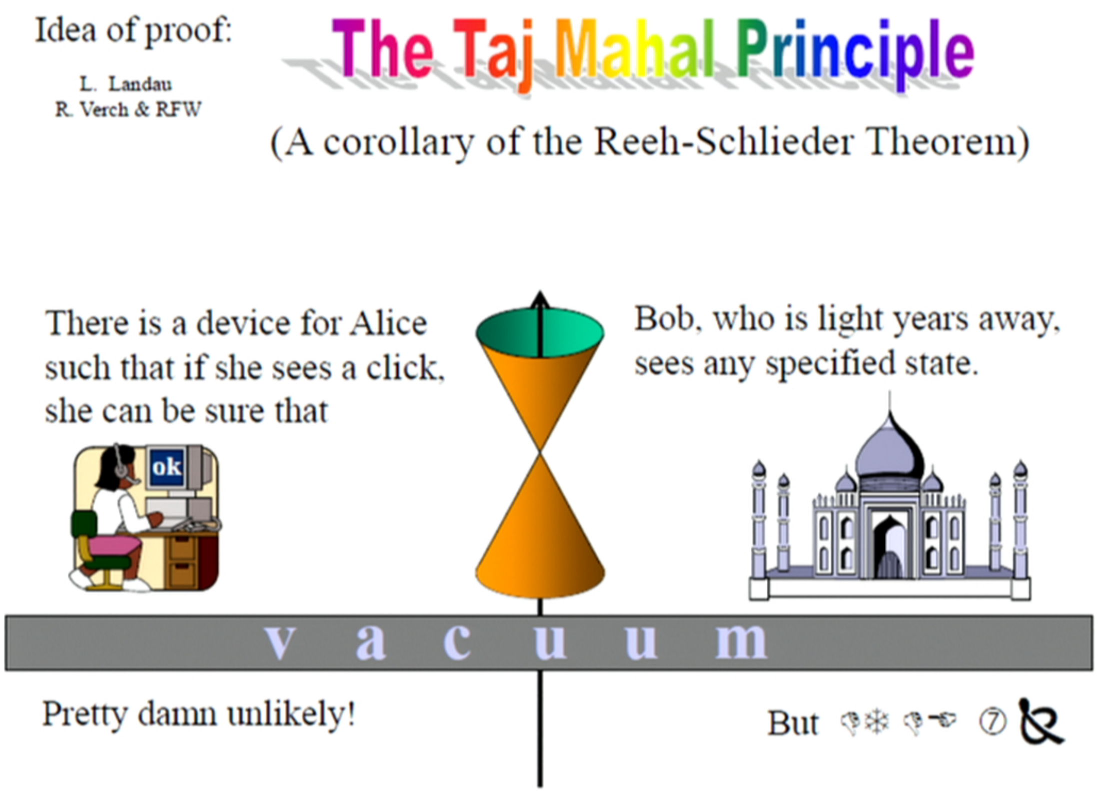

Plan of talk

- The Reeh-Schlieder theorem is not counter-intuitive.
- Nevertheless, thinking about the theorem is helpful in thinking about how relativistic causality is implemented in the theory.

The set-up

- Assume we have a quantum field theory, with field operators $\hat{\phi}_\alpha(x)$. Observables are self-adjoint operators formed from polynomials in these fields, smeared with test functions.

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- For any spacetime region R, let $\mathcal{P}(R)$ be the algebra of operators formed by smearing polynomials in the fields with test functions with support in R.

Assumptions

- **Translation covariance.** Assume we have a unitary group of operators implementing spacetime translations:

$$\hat{\phi}_\alpha(x + a) = \hat{U}^\dagger(a)\, \hat{\phi}_\alpha(x)\, \hat{U}(a).$$

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- There are self-adjoint operators $\hat{P}_\mu, \mu = 0, \ldots 4$, that are infinitesimal generators of these spacetime translations,

$$\hat{U}(a) = e^{-i\hat{P}\cdot a}.$$

  These correspond to energy-momentum.

Assumptions, cont'd

- **Spectrum Condition.** For any vector a in the positive light-cone, the spectrum of the operator $\hat{P} \cdot a$ consists of positive reals.

Microcausality

- Not needed for R-S theorem, but will be used in a corollary:

  Microcausality. If $\hat{A}$ and $\hat{B}$ are self-adjoint operators associated with spacelike separated regions, then they commute.

The theorem

- For any open region R, let $\mathcal{H}(R)$ be the norm closure of $\mathcal{P}(R)|0\rangle$.

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- For any open region R, let $\mathcal{H}(R)$ be the norm closure of $\mathcal{P}(R)|0\rangle$.
- *Theorem* (Reeh & Schlieder 1961). Under the stated assumptions, for any open region R, $\mathcal{H}(R) = \mathcal{H}$.

How to prove it: one step at a time

- Pick one field $\hat{\phi}_\alpha$, and consider vectors of the of the form

$$|\Phi\rangle = \int d^4x \, f(x) \, \hat{\phi}^\dagger_\alpha(x) \, |0\rangle.$$

- Let $\mathcal{H}_\alpha$ be the subspace spanned by vectors like that.
- For any open region R, let $\mathcal{H}_\alpha(R)$ be the subspace spanned by vectors like that, for f with support in R.

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Proving Lemma 1.

- Let $|x\rangle = \hat{\phi}^\dagger_\alpha(x)|0\rangle$, and define wavefunctions:

$$\phi_\alpha(x) = \langle x|\Phi\rangle = \langle 0|\hat{\phi}_\alpha(x)|\Phi\rangle.$$

- Any $|\Phi\rangle \in \mathcal{H}_\alpha$ can be uniquely characterized by its wavefunction.

- **Unsurprising fact 1.** If $\phi_\alpha(x)$ vanishes everywhere in some open set R, it vanishes everywhere.

Proving Lemma 1.

- **Unsurprising fact 1.** If $\phi_\alpha(x)$ vanishes everywhere in some open set R, it vanishes everywhere.

- To show: For any open region R, $\mathcal{H}_\alpha(R) = \mathcal{H}_\alpha$.

- If $\mathcal{H}_\alpha(R) \neq \mathcal{H}_\alpha$, there must be some $|\Psi\rangle \in \mathcal{H}$ in the orthogonal complement of $\mathcal{H}_\alpha(R)$
- If

$$|\Psi\rangle = \int d^4x\, f(x)\, \hat{\phi}^\dagger_\alpha(x)\, |0\rangle,$$

then

$$\begin{aligned} \langle\Psi|\Phi\rangle &= \int d^4x\, f(x)^* \langle 0|\hat{\phi}_\alpha(x)|\Phi\rangle \\ &= \int d^4x\, f(x)^*\, \phi_\alpha(x). \end{aligned}$$

Two-particle sector

- For any fields $\hat{\phi}_\alpha, \hat{\phi}_\beta$, consider vectors of the form

$$|\Phi\rangle = \int d^4x_1 \int d^4x_2\, f(x_1, x_2)\, \hat{\phi}^\dagger_\alpha(x_1)\, \hat{\phi}^\dagger_\beta(x_2)\, |0\rangle.$$

Et cetera!

- Repeat for all sequences $\alpha_1, ...\alpha_n$: $\mathcal{H}_{\alpha_1,...\alpha_n}(R)$ is dense in $\mathcal{H}_{\alpha_1,...\alpha_n}$.
- Anything that is in the orthogonal complement of $\mathcal{H}(R)$ must be in the orthogonal complement of $\mathcal{H}_{\alpha_1,...\alpha_n}(R)$, and hence, in the orthogonal complement of $\mathcal{H}_{\alpha_1,...\alpha_n}$.

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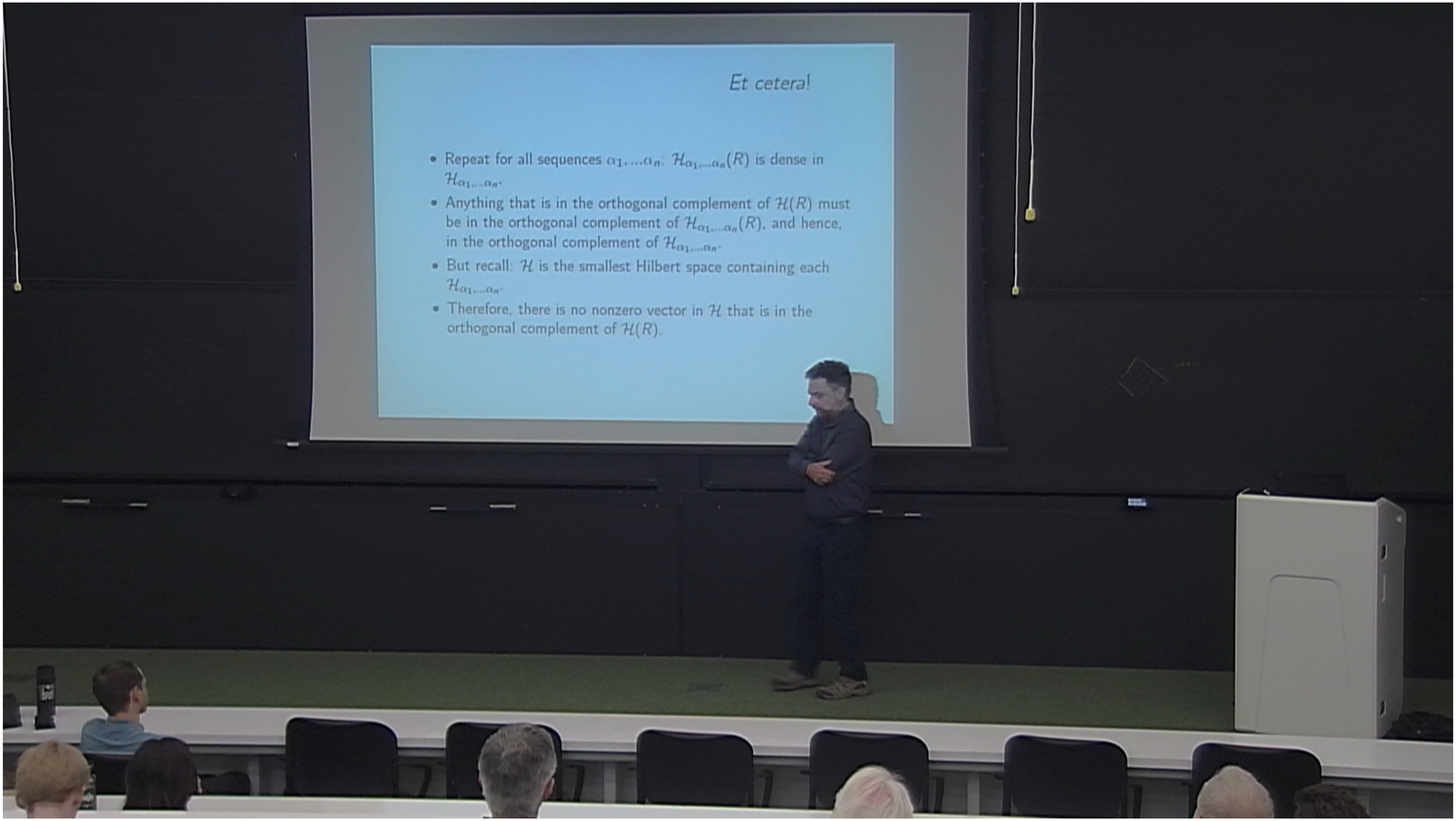
- But recall: $\mathcal{H}$ is the smallest Hilbert space containing each $\mathcal{H}_{\alpha_1,...\alpha_n}$.
- Therefore, there is no nonzero vector in $\mathcal{H}$ that is in the orthogonal complement of $\mathcal{H}(R)$.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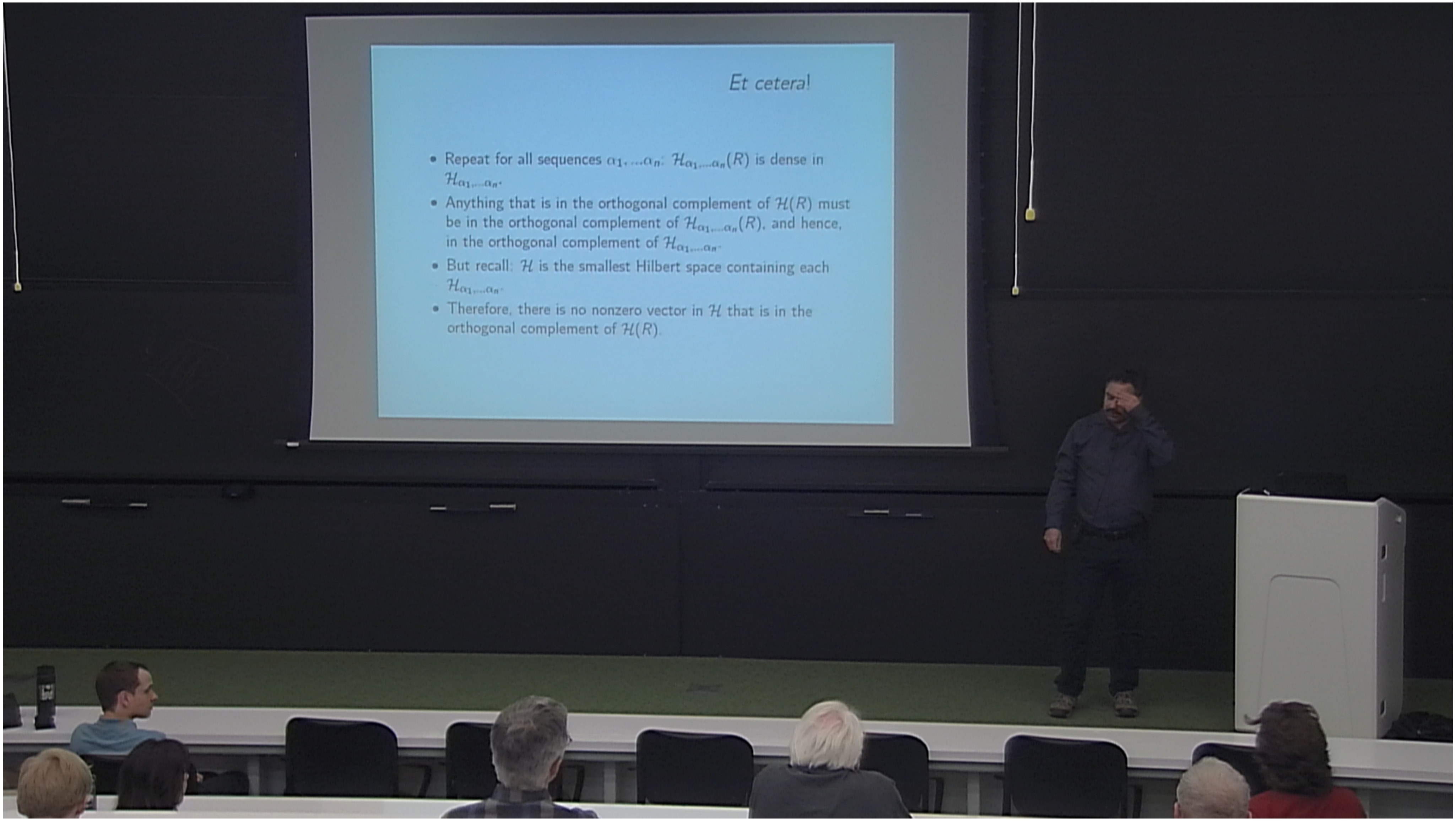
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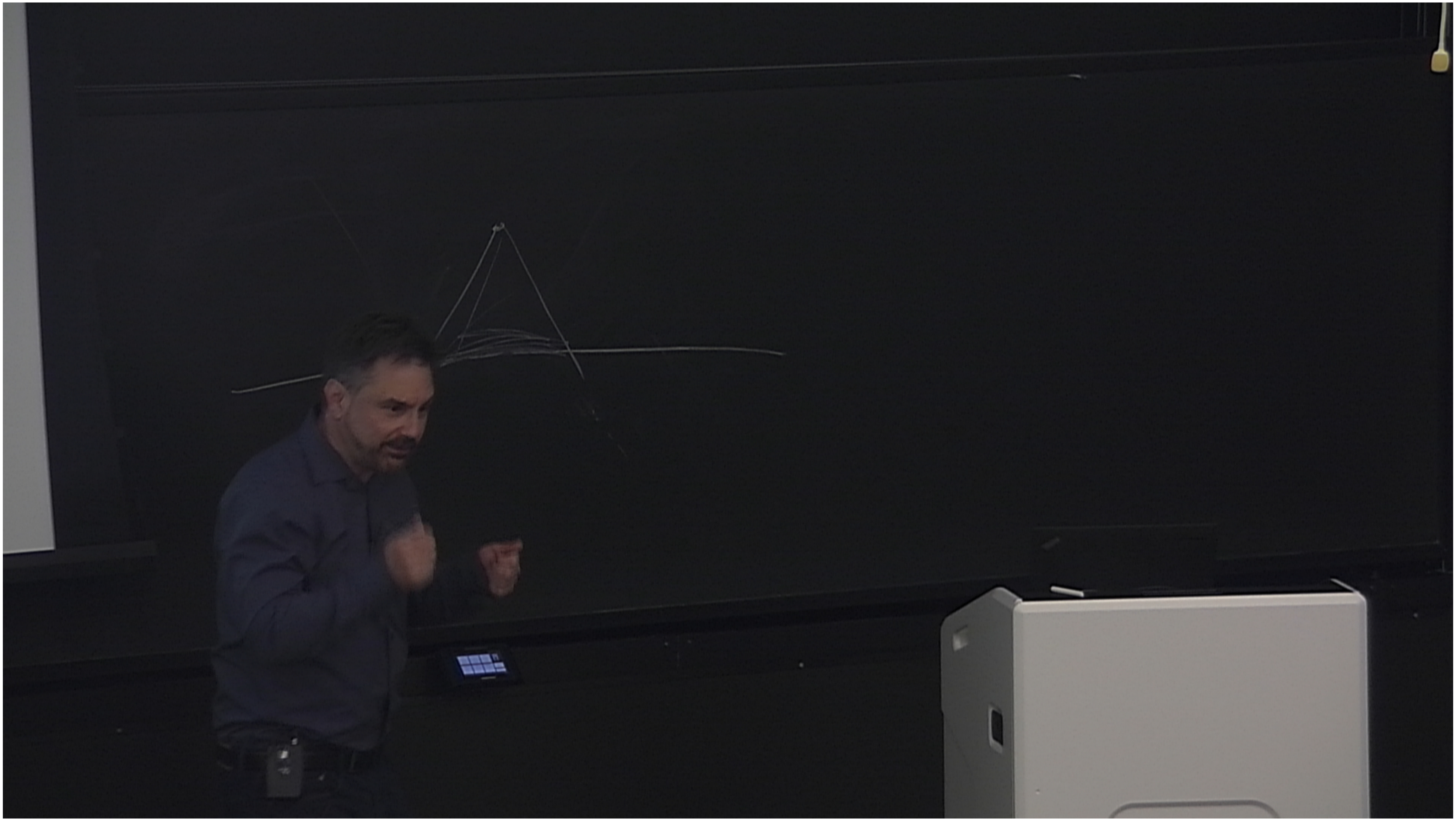
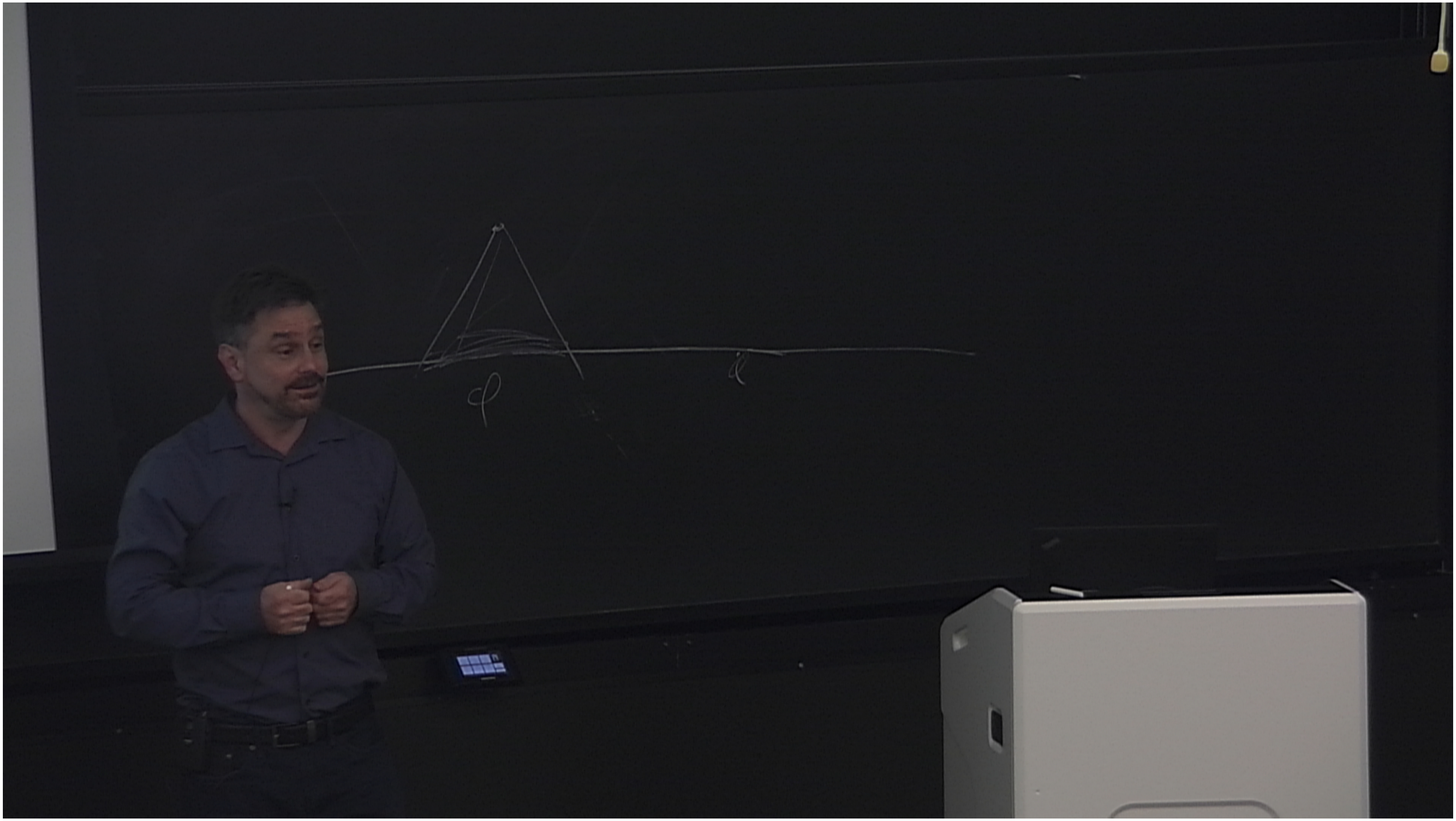
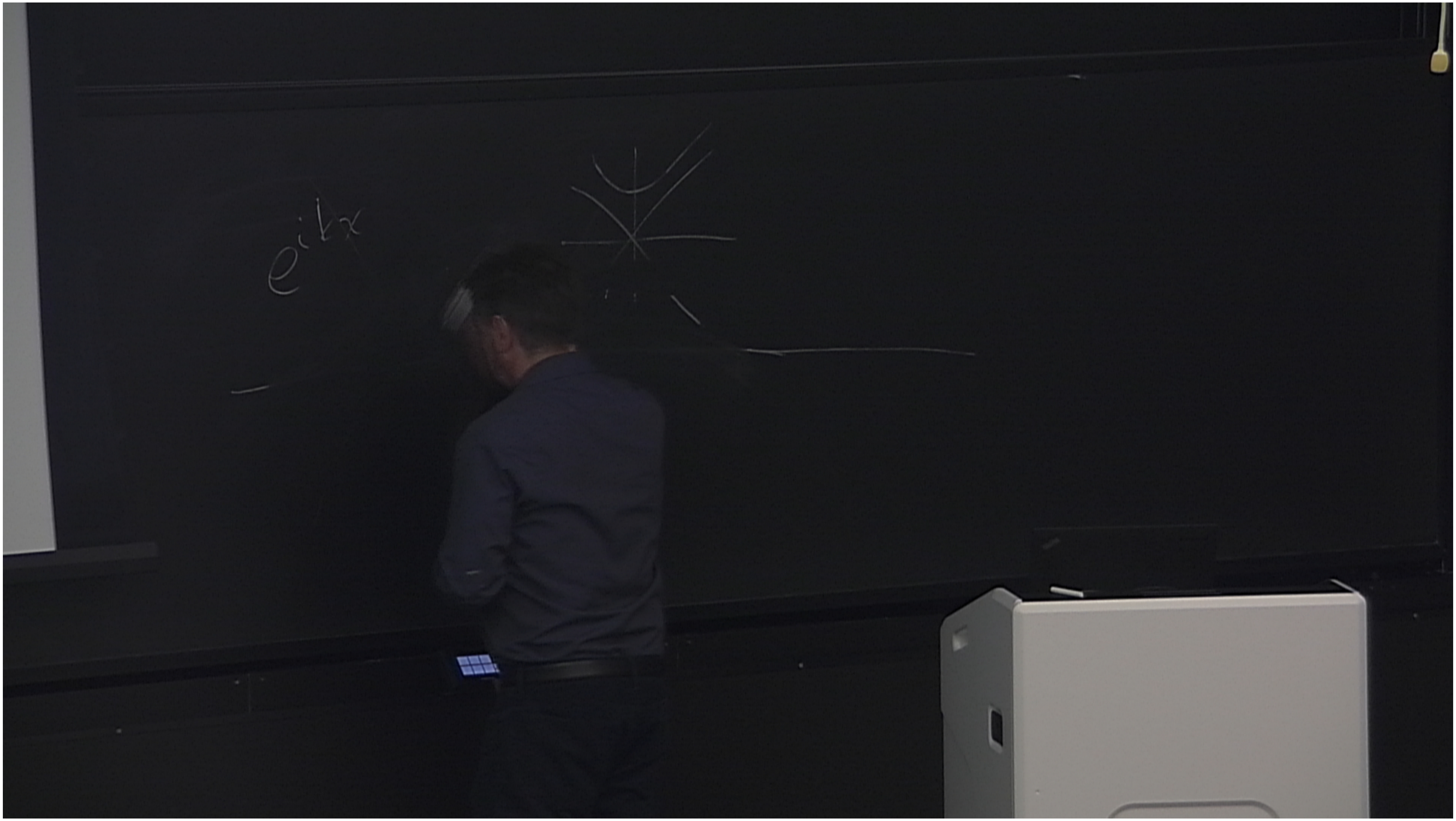
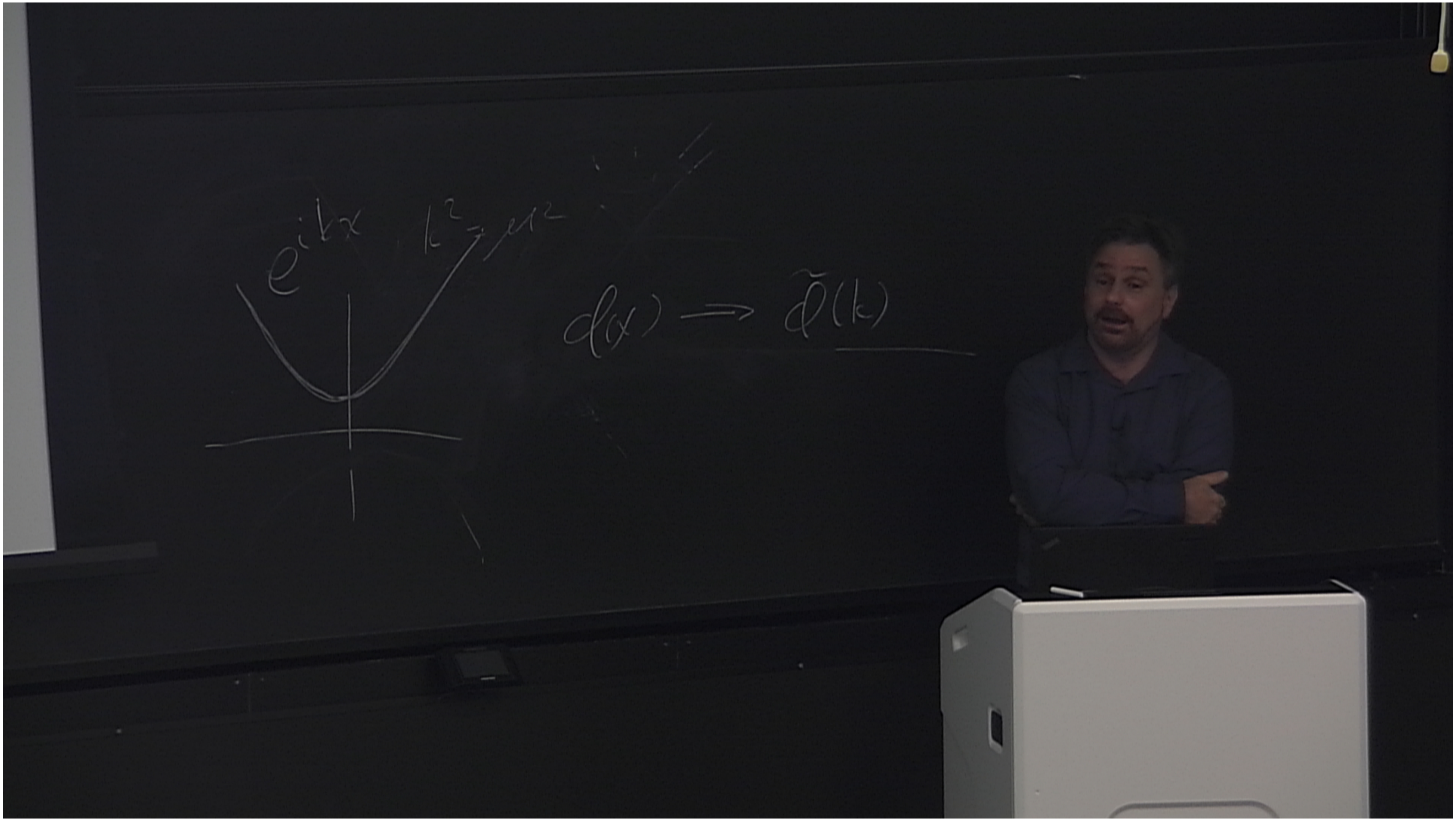
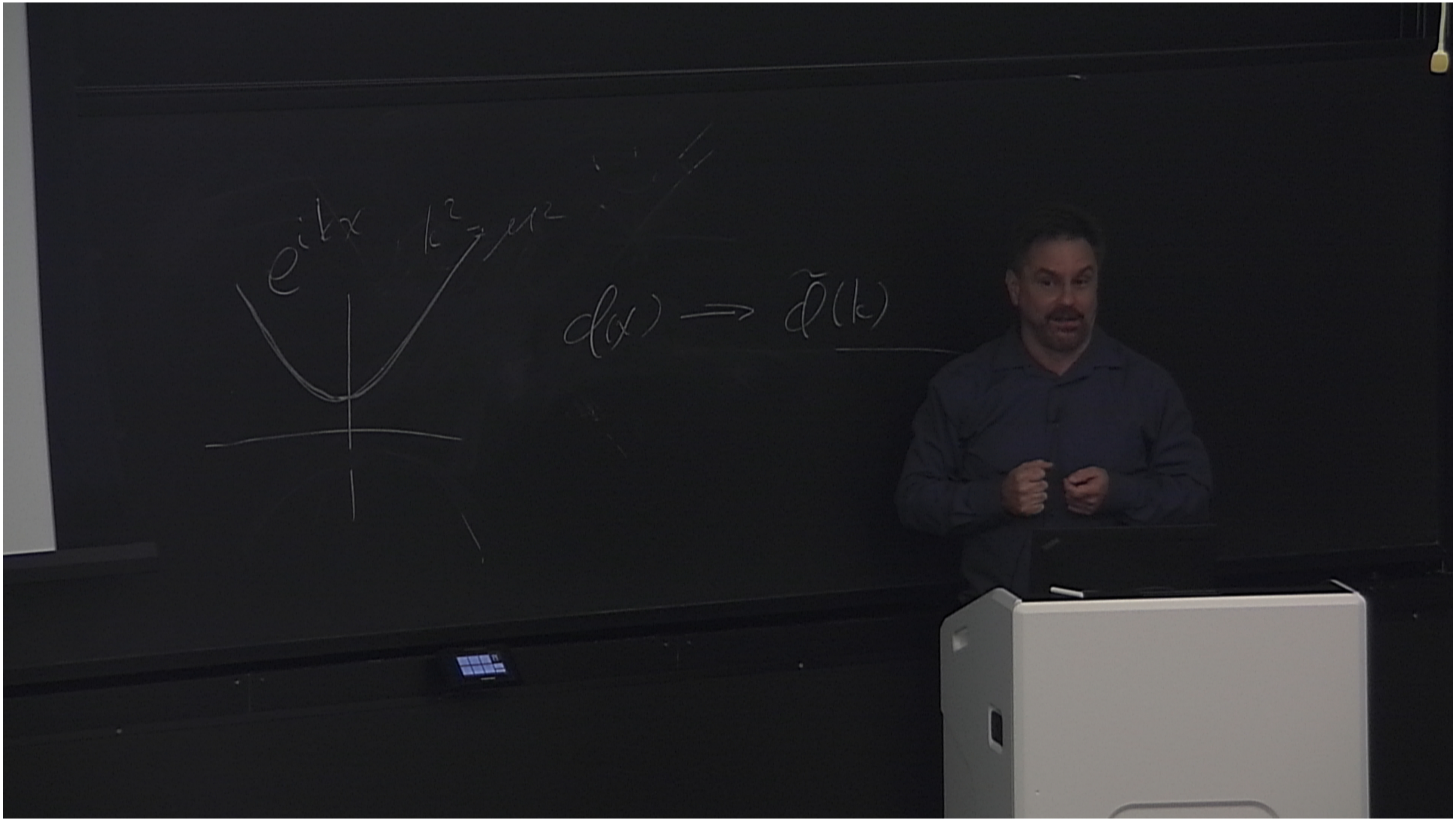
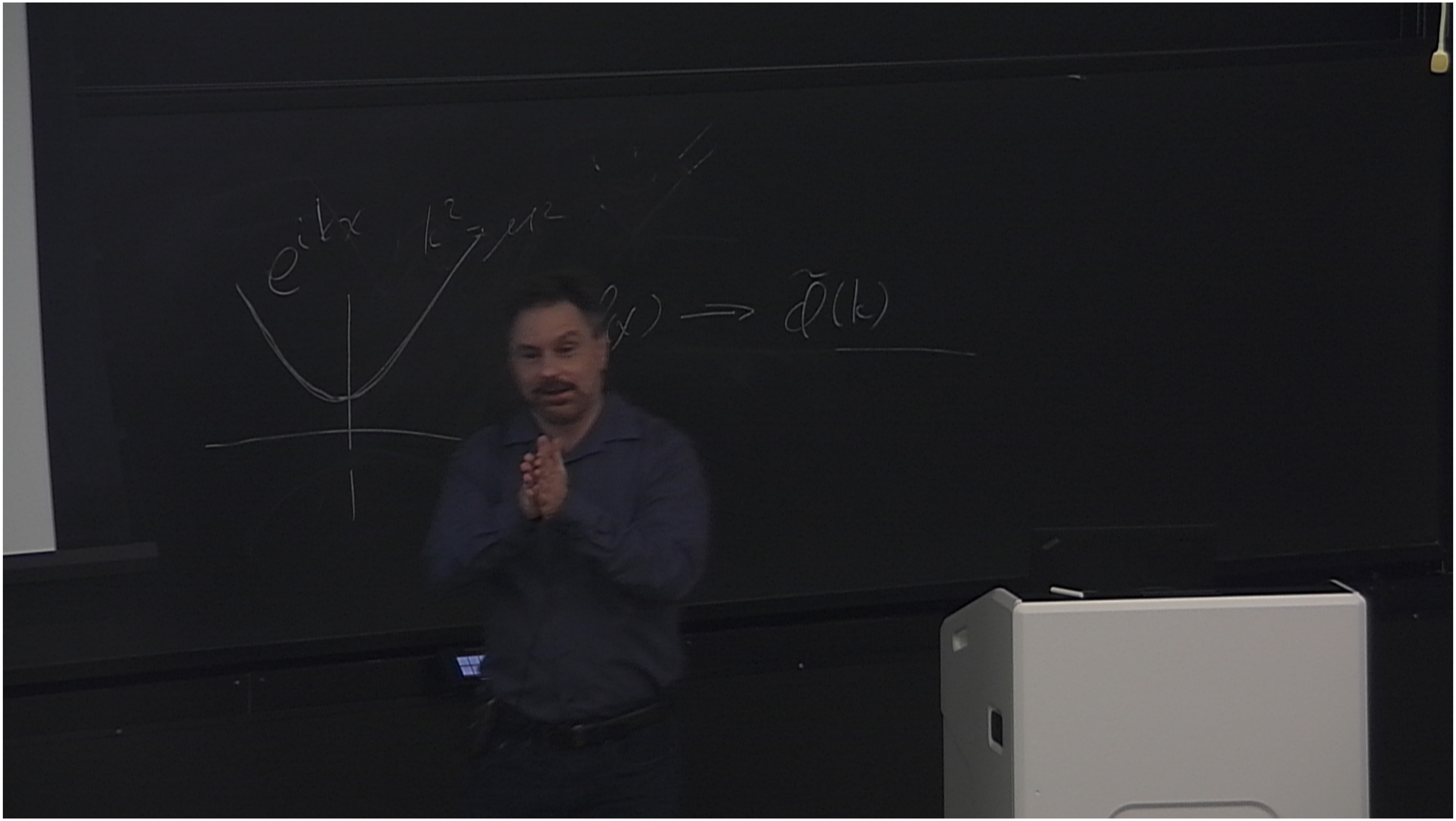


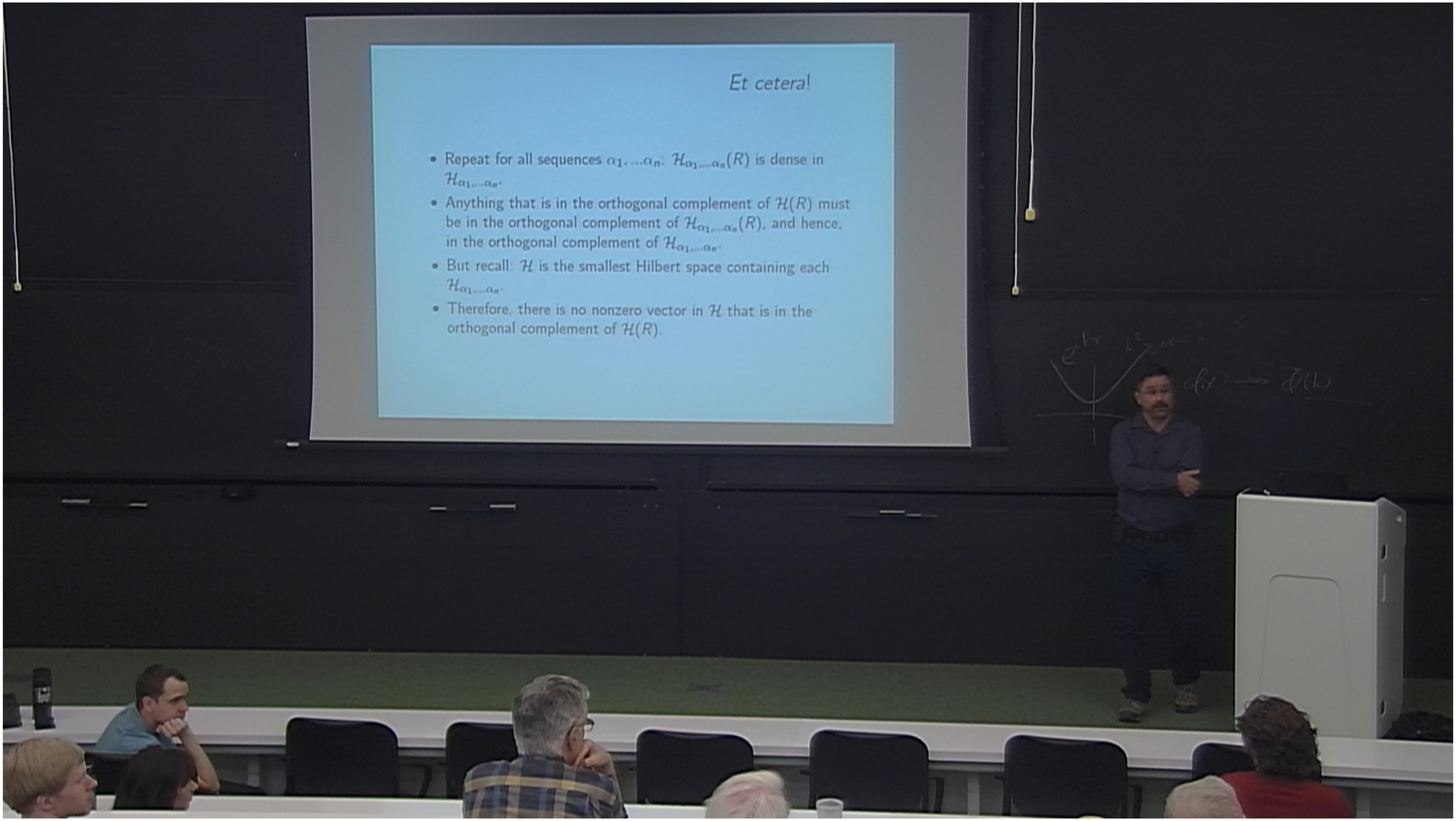



eikx




e^{ikx}
$k^2 = \mu^2$
$\phi(x) \longrightarrow \tilde{\phi}(k)$

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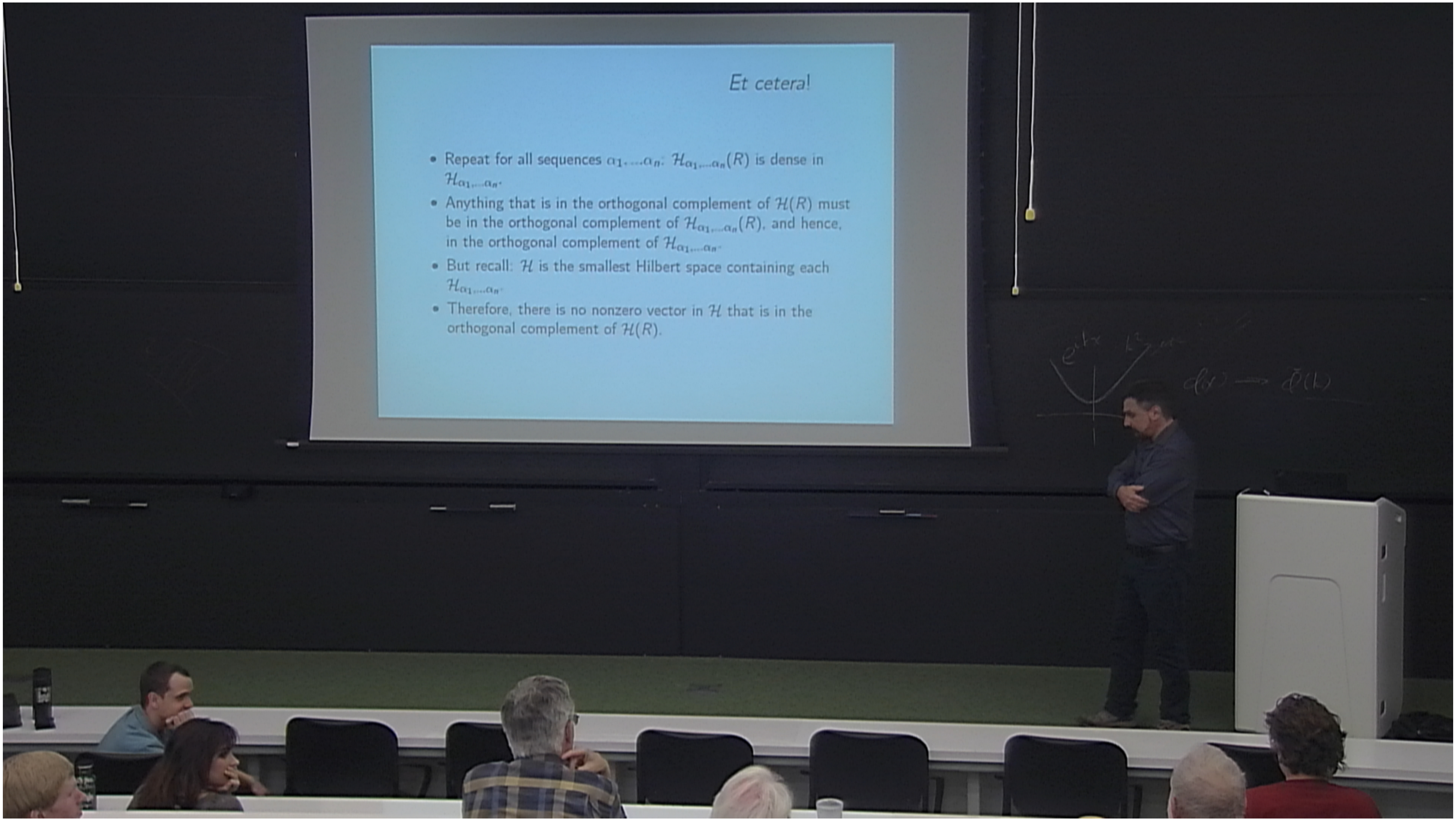
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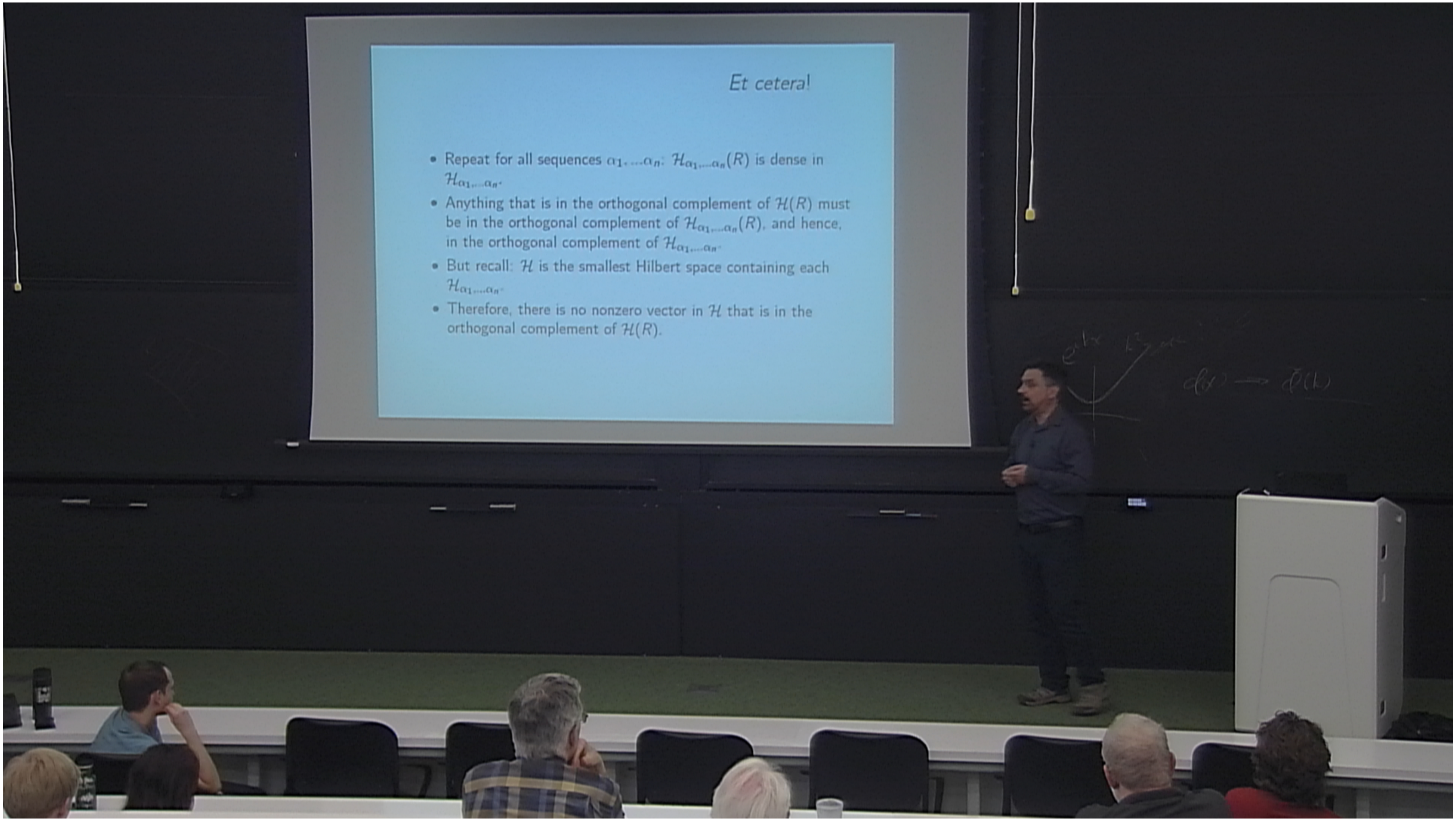
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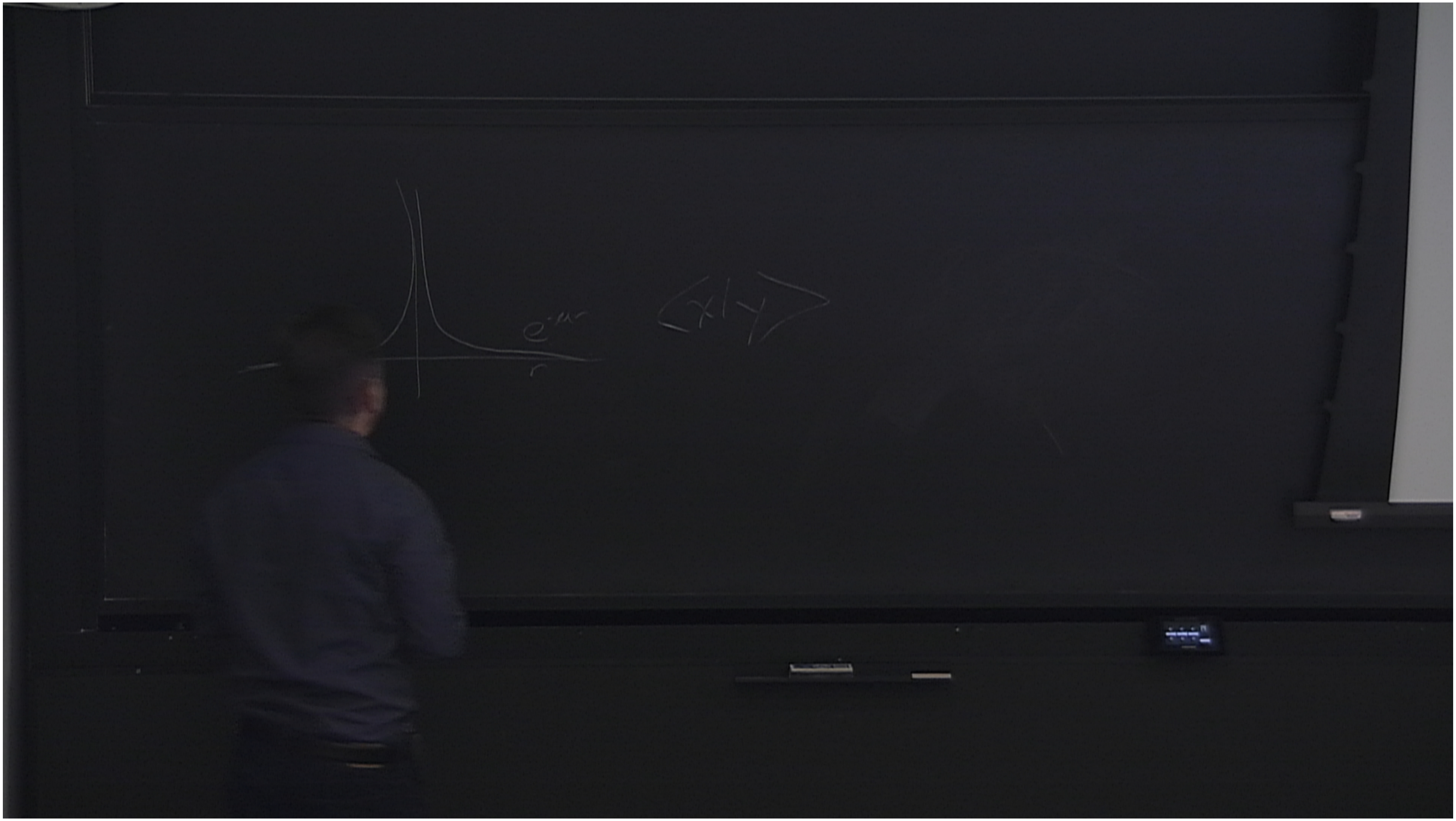
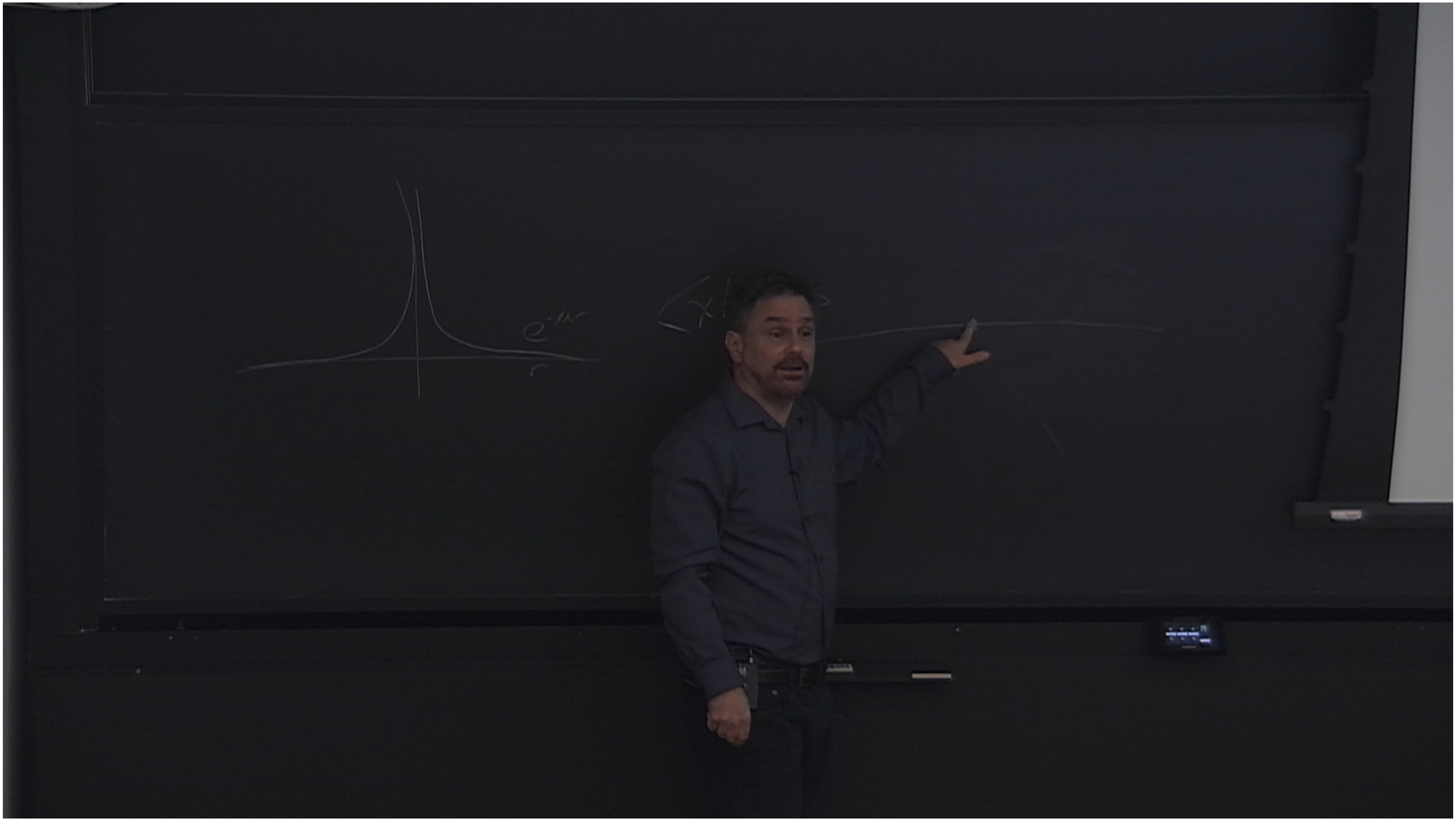
An easy corollary

- Assume Microcausality.
- The vacuum is not an eigenstate of any observable belonging to a region R with non-empty complement.


<x|y>



A mistaken intuition

- One might be tempted to think of the state

$$|x\rangle = \hat{\phi}^{\dagger}_{\alpha}(x)\,|0\rangle$$

  as one such that, on any hypersurface containing x, there is a particle located at x, and is just like the vacuum at all points spacelike from x.
- Not so: field quanta cannot be localized!
- Following Knight (1961), say that a state is *strictly localized* in a region R if expectation values for all observables pertaining to the complement of R are the same as their vacuum expectation values.
- *Theorem* (Knight 1961). No state of finite particle number is strictly localized in any region whose complement contains an open set.

Strictly localized states

- Well, then, are there *any* states strictly localized in some region that isn't the whole of spacetime?
- Yes: take self-adjoint $\hat{A}$ belonging to some bounded region R; then (assuming Microcausality)

$$e^{i\hat{A}}|0\rangle$$

is strictly localized in the light-cones of R.

- *Example*: particle creation by a classical source field $j(y)$ localized in R.
- Single-particle wave function:

$$\phi(x) = \int d^4y \, \langle x|y\rangle j(y).$$

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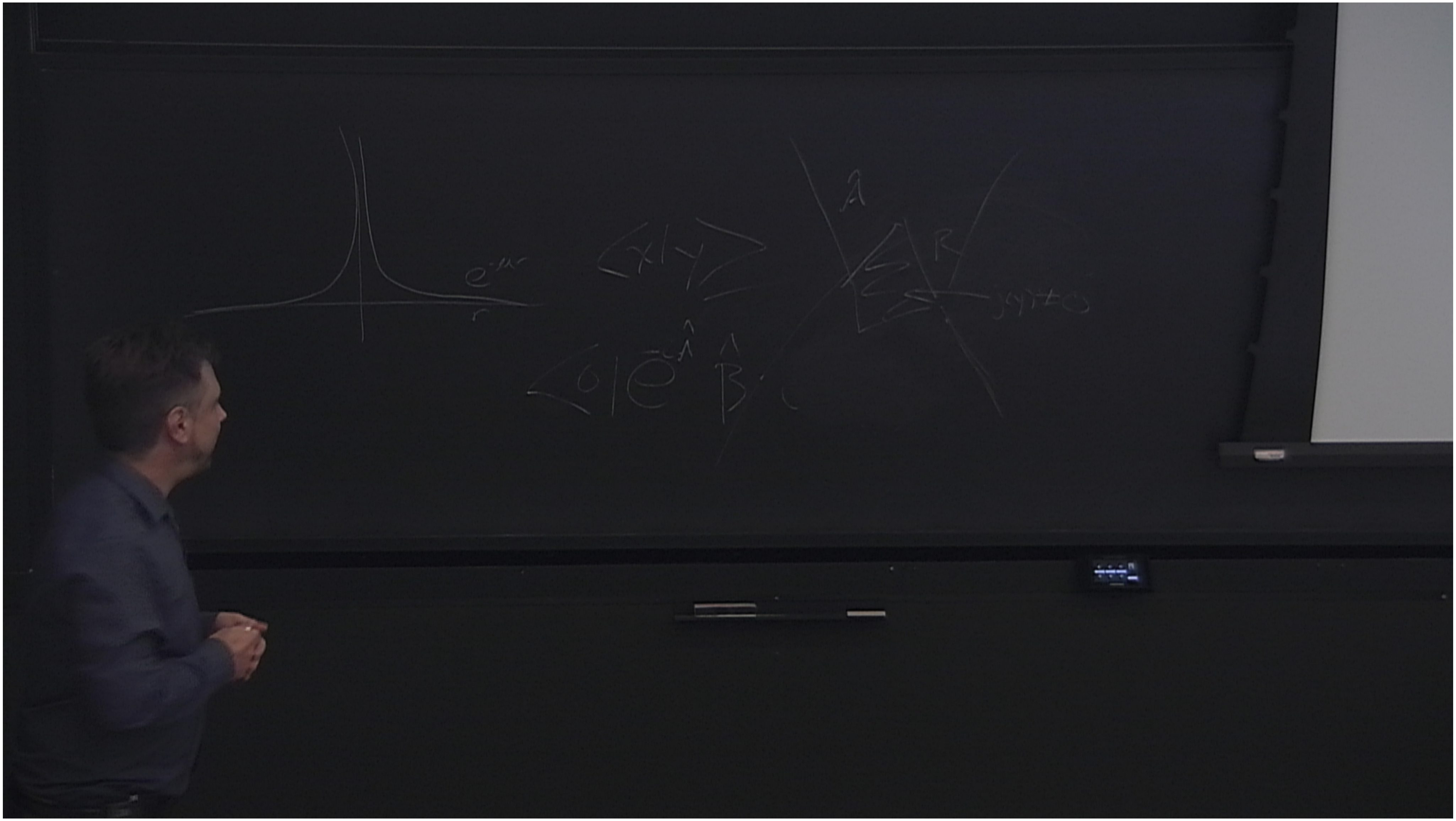
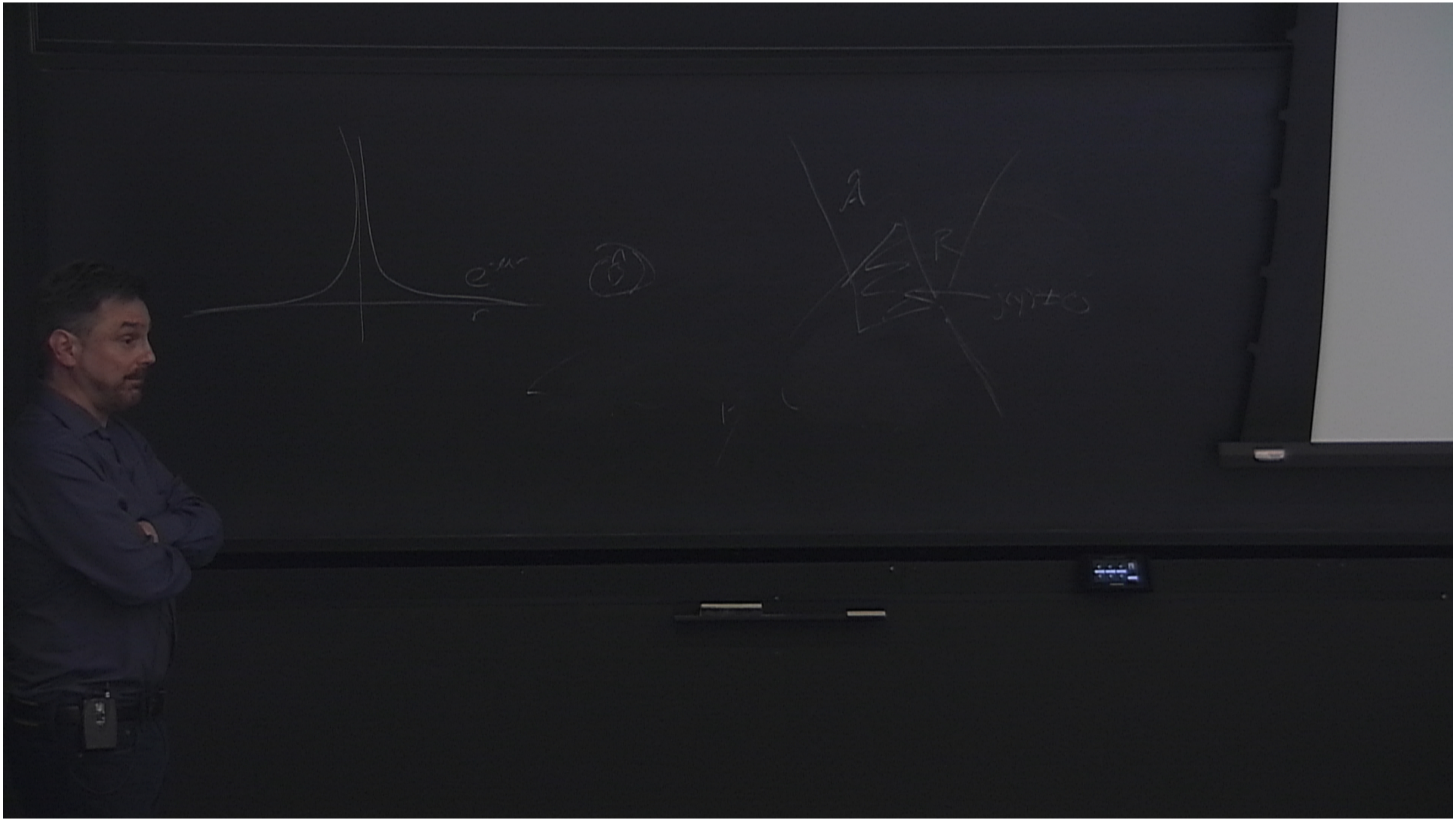
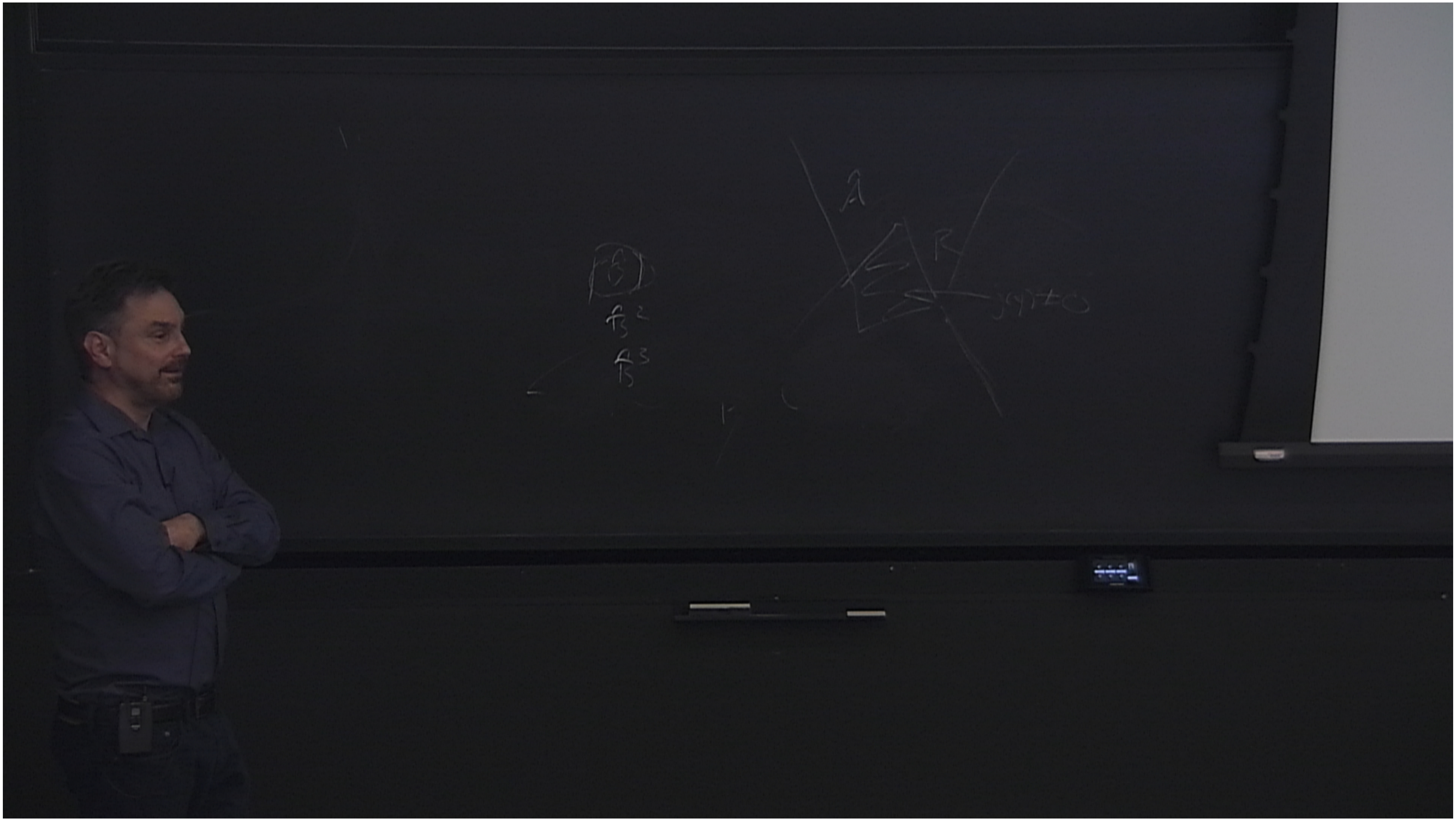
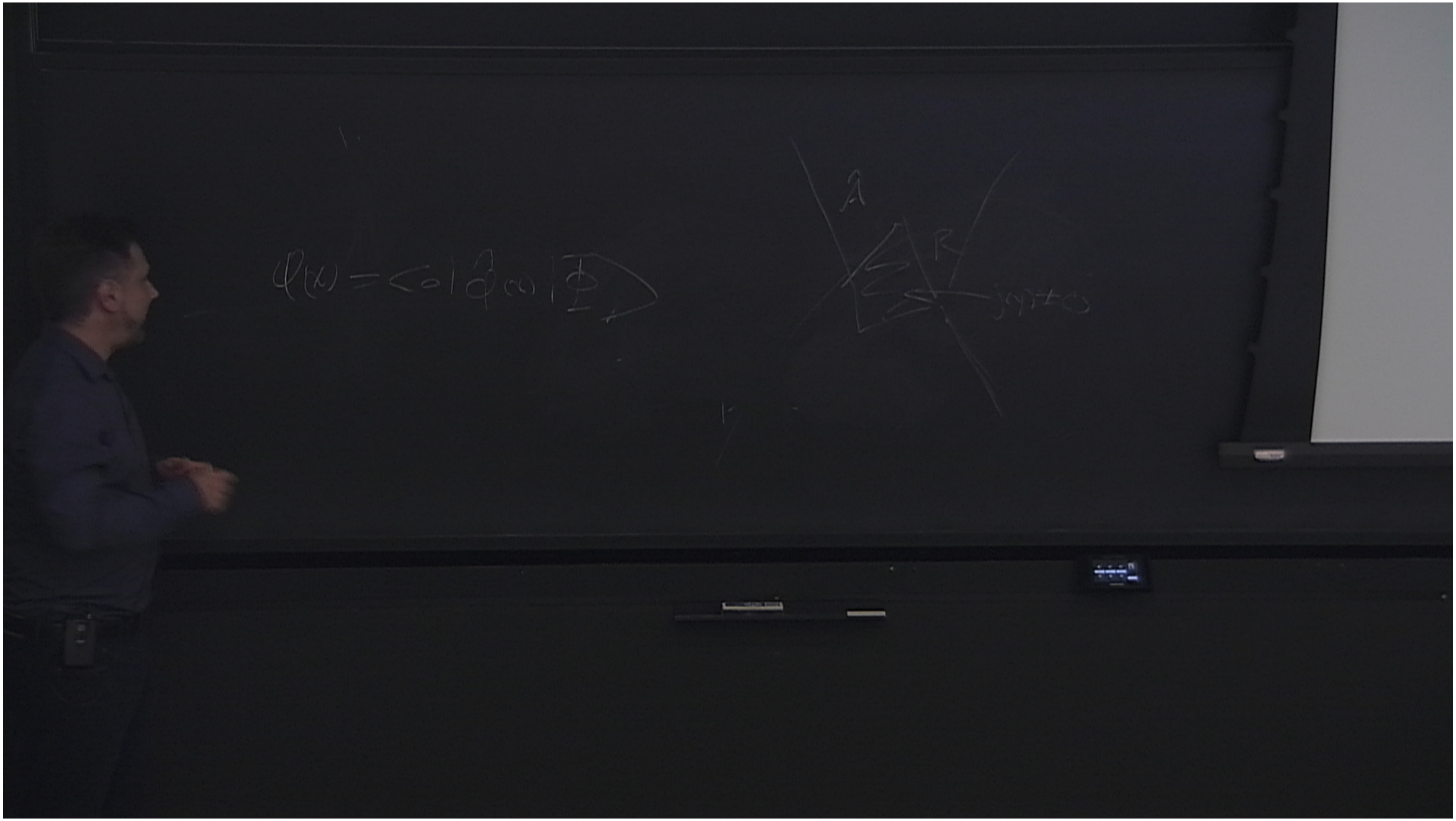
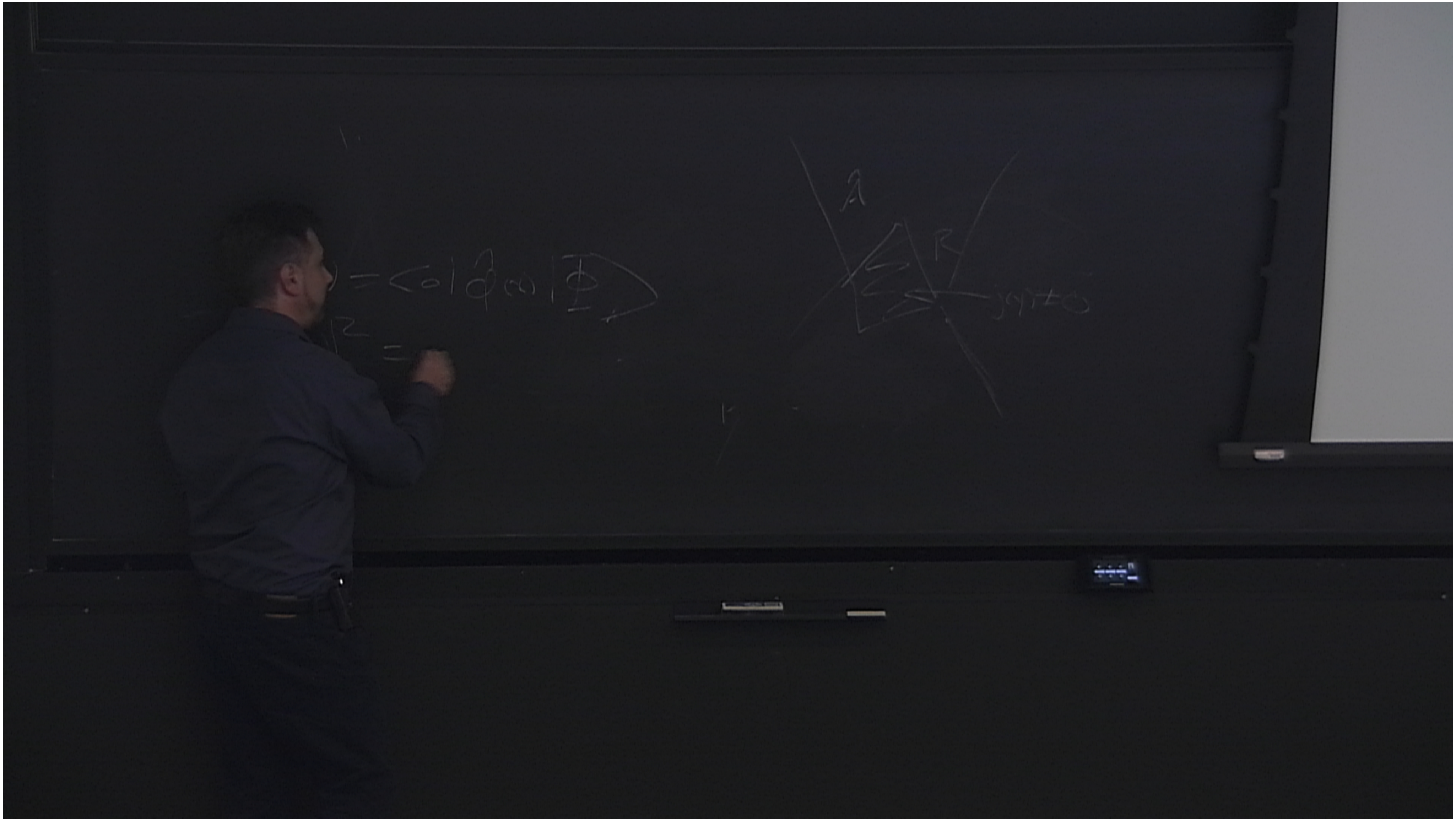
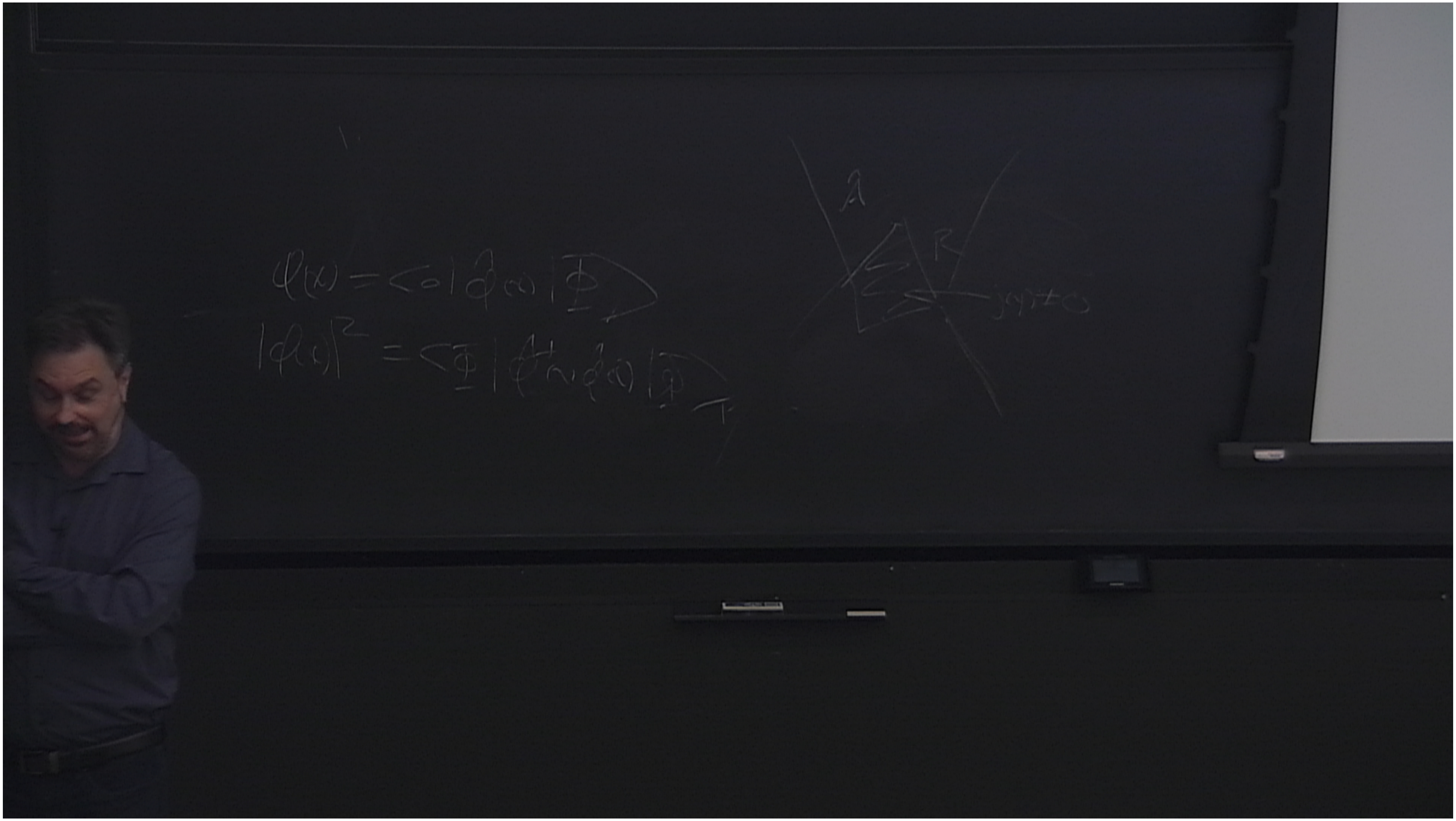


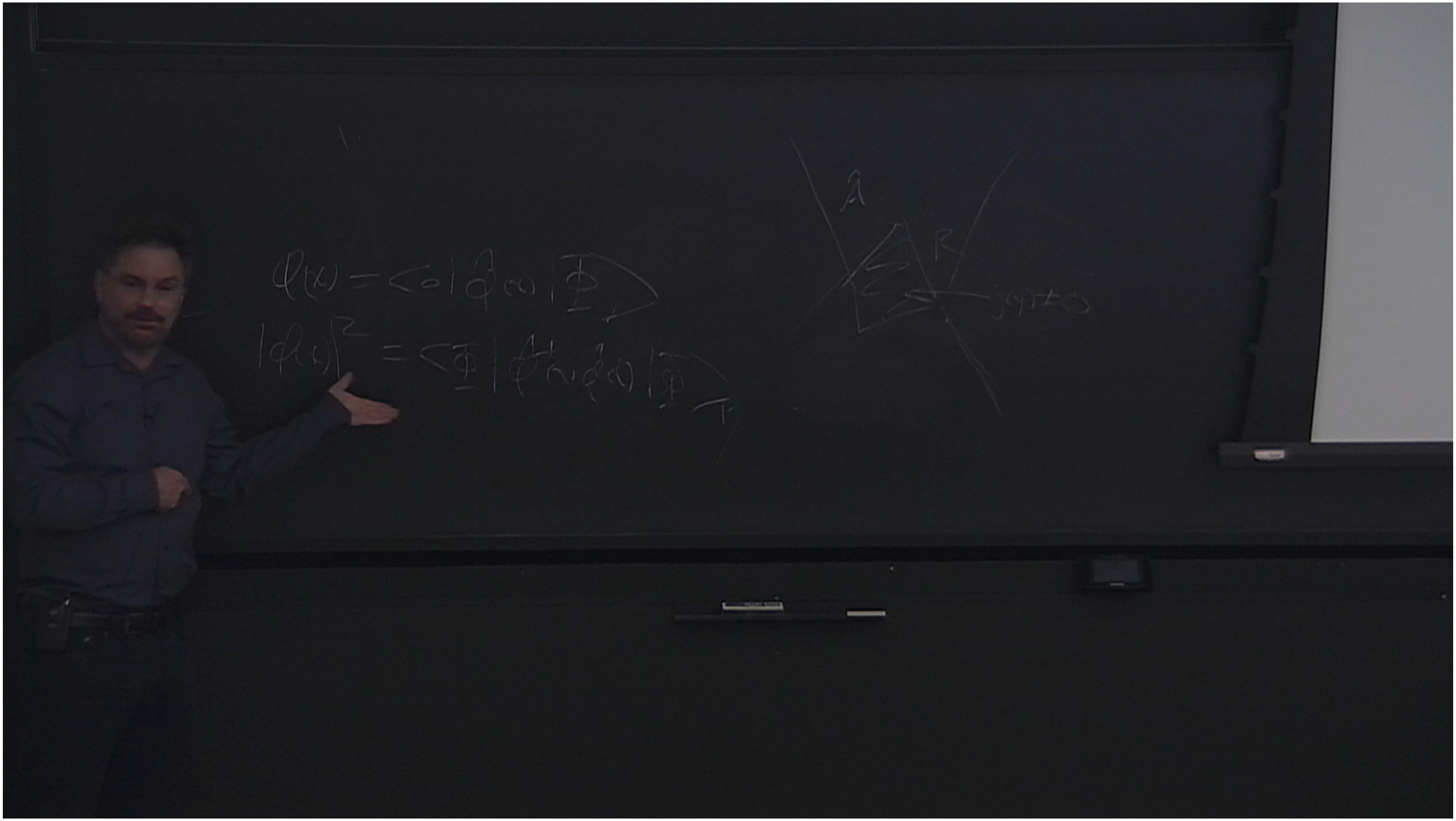





φ(x) = <0|φ̂(x)|Φ>
R
j(y) ≠ 0







Can we create the Taj Mahal?

- How to model our interventions?
- Assume: for any operation we can perform, there is a set $\{\hat{K}_i\}$ of operators, such that

$$\sum_i \hat{K}_i^\dagger \hat{K}_i = I.$$

- If the initial state-vector is $|\Psi\rangle$, the result of our operation will be $\hat{K}_i\,|\Psi\rangle$, for some i, and the probabilities for *which* one it will be are given by

$$p_i = \frac{\|\hat{K}_i|\Psi\rangle\|^2}{\||\Psi\rangle\|^2}.$$

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- What we have is a correlation: Bob's results are correlated with Alice's.
- This relation is symmetric: nothing distinguishes one as cause, the other, effect.
- Conceptually, we're on familiar territory: correlated results of experiments in an entangled state.