Title: Many-body Localization Transition in Rokhsar-Kivelson-type wave functions

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Abstract:

Many-body Localization Transition in Rokhsar-Kivelson-type wave functions

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- What feature in the wave function is responsible for the volume law and area law for entanglement entropy?
- What's the scaling behavior of entanglement entropy around the many-body localization transition?

Outline

- Introduction

  Entanglement entropy in the many-body wave functions
  Quantum thermalization & Many-body localization (MBL)

- Rokhsar-Kivelson (RK)-type wave functions

  Random energy model
  Random sign structure in the wave functions

- Many-body localization (MBL) transition

  Analytical results for upper and lower bounds
  Numerical results for MBL transition
  Multifractality near phase transition

- Discussion and conclusion

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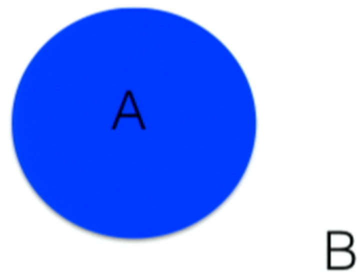
Reduced density matrix



Isolated quantum many-body state

$$\rho_A = e^{-H_E}$$

In general, for the ground state, entanglement entropy satisfies area law

$$H_E \neq H_A$$

Thermal density matrix

Heat Bath

$$\rho_T = e^{-\beta H_A}$$

Thermal entropy satisfies the volume law

$$\rho_A \neq \rho_T$$

When does reduced density matrix take a thermal form?

Quantum thermalization

Isolated quantum
many-body state
=
Heat bath

The reduced density matrix takes a thermal form:

$$\rho_A = \rho_T = e^{-\beta H_A}$$

Entanglement entropy (EE) satisfies volume law

Eigenstate Thermalization Hypothesis (ETH)

Deutsch 1991, Srednicki, 1994, Rigol et al, 2008

Many body localization

- Interacting system with strong static disorder does not quantum thermalize
  Basko, Aleiner, and Altshuler, 2006; Oganesyan and Huse, 2007,…
- Infinite number of local integrals of motion
  Serbyn, Papic and Abanin, 2013; Huse and Oganesyan, 2013, Imbrie, 2014
- Entanglement entropy satisfies the area law
  Bauer and Nayak, 2013,…
- Numerical tools: tensor networks, matrix product states
  Chandran, Carrasquilla, Kim, Abanin, Vidal, 2015; Yu, Pekker and Clark, 2015; Khemani, Pollmann, Sondhi, 2015

Phase transition between ergodic and MBL phase

- Real space RG method
  Vosk, Huse and Altman, 2015
- Exact diagonalization
  Kjall, Bardarson and Pollmann, 2014, Luitz, Alet and Laflorencie, 2014
- Phase transition observed in a many-body wave function

Construct an ensemble of many body wave functions which has a MBL phase transition

Rokhsar-Kivelson (RK) wave function

- Ground state for the quantum dimer model

  Rokhsar, Kivelson, Moessner, Sondhi, Fradkin, Henley

- This wave function can be associated with the partition function of a classical model

$$|\Psi_{RK}\rangle = \frac{1}{\sqrt{Z}} \sum_{\mathcal{C}} e^{-\frac{\beta}{2} E[\mathcal{C}]} |\mathcal{C}\rangle$$

If E(C) is a local function → EE satisfies area law

Stéphan, Furukawa, Misguich and Pasquier, 2009

- Laughlin state, quantum Lifshitz model, Toric code model,…

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Random energy model

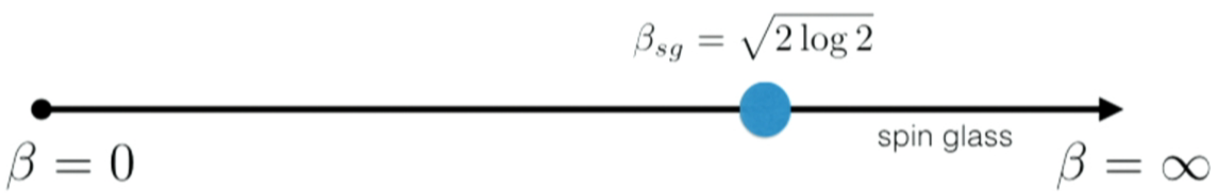
Derrida, 1980

The infinite range coupling limit of the Sherrington-Kirkpatrick spin glass model

$$Z = \sum_{i=1}^{2^N} e^{-\beta E_i} \qquad P(E) = (2N\pi)^{-1/2} e^{-\frac{E^2}{2N}}$$

Saddle point approximation

$$Z = \int dx e^{N\phi(x)} \approx e^{N \max[\phi(x)]}, \quad \phi(x) = \log 2 - \frac{x^2}{2} - \beta x \qquad x = \frac{E}{N}$$



Inverse participation ratio $\quad Y_n(\beta) = \frac{\sum_i^{2^N} e^{-n\beta E_i}}{(\sum_i^{2^N} e^{-\beta E_i})^n} = \frac{Z(n\beta)}{Z(\beta)^n}$

$$\langle Y_2(\beta) \rangle = \begin{cases} 0 & \beta < \beta_{sg} \\ 1 - \frac{\beta_{sg}}{\beta} & \beta \geq \beta_{sg} \end{cases}$$

In the thermodynamic limit

Multifractality in Anderson localization phase transition

$P_n = \int d^d r |\Psi(r)|^{2n}$ single particle wave function

$$\langle P_n \rangle = \begin{cases} L^0 & \text{insulator} \\ L^{-\tau_n} & \text{critical} \\ L^{-d(n-1)} & \text{conductor} \end{cases}$$

τ_n is multifractal spectrum

Thouless, 1974; Evers and Mirlin, 2008

Inverse participation ratio (IPR)

$$|\Psi_{REM+\mathrm{sign}}\rangle = \frac{1}{\sqrt{Z}} \sum_{\mathcal{C}} s_{\mathcal{C}} e^{-\frac{\beta}{2}E[\mathcal{C}]} |\mathcal{C}\rangle$$

The localization properties of the wave function
can be characterized by IPR in configuration space

$$Y_n = \sum_{\mathcal{C}} |\langle \Psi | \mathcal{C} \rangle|^{2n} = \frac{Z(n\beta)}{Z(\beta)^n} \qquad Y_n \sim \mathcal{D}^{-\tau(n)}, \quad \text{with } \mathcal{D} = 2^N$$

$$\tau(n) = \begin{cases} (n-1)(1-\gamma n), & 0 \le \gamma < \frac{1}{n^2} \\ n(1-\sqrt{\gamma})^2, & \frac{1}{n^2} < \gamma < 1 \\ 0, & \gamma > 1 \end{cases} \qquad \gamma = \frac{\beta^2}{2\log 2}$$

Chamon, Mudry and Wen, 1996; Castillo, Chamon, Fradkin, Goldbart and Mudry, 1997

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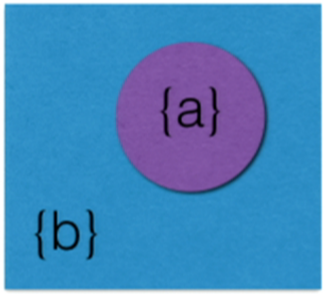
Renyi entanglement entropy

$$S_n(\rho_A) = \frac{1}{1-n} \log \mathrm{Tr} \rho_A^n$$

Two extreme limits

$\beta = 0$ Every configuration contributes equally to the state

$$\begin{aligned}
\langle \mathrm{Tr}\rho_A^2 \rangle &= \sum_{a_1,a_2,b_1,b_2} \langle a_1, b_1|\psi\rangle\langle\psi|a_2,b_1\rangle\langle a_2,b_2|\psi\rangle\langle\psi|a_1,b_2\rangle \\
&= \sum_{a_1,b_1} \frac{1}{2^{2N}} \quad (a_1 = a_2,\ b_1 = b_2) \\
&+ \sum_{a_1,b_1,b_2} \frac{1}{2^{2N}} \quad (a_1 = a_2,\ b_1 \neq b_2) \\
&+ \sum_{a_1,a_2,b_1} \frac{1}{2^{2N}} \quad (a_1 \neq a_2, b_1 = b_2) \\
&+ \sum_{a_1,a_2,b_1,b_2} \frac{1}{2^{2N}} \sum (\text{random sign}) \quad (a_1 \neq a_2,\ b_1 \neq b_2) \\
&= \frac{1}{2^N} + \frac{1}{2^{N_A}} + \frac{1}{2^{N_B}}
\end{aligned}$$



Grover and Fisher, (2013)

$$S_2(\langle\rho_A\rangle) = -\log\langle \mathrm{Tr}\rho_A^2\rangle = N_A \log 2$$

$$\langle S_2(\rho_A)\rangle \geq S_2(\langle\rho_A\rangle) = N_A \log 2$$

volume law at $T = \infty$

$\beta = \infty$ Localized in one configuration $\langle S_n \rangle = 0$

EE is a monotonically function

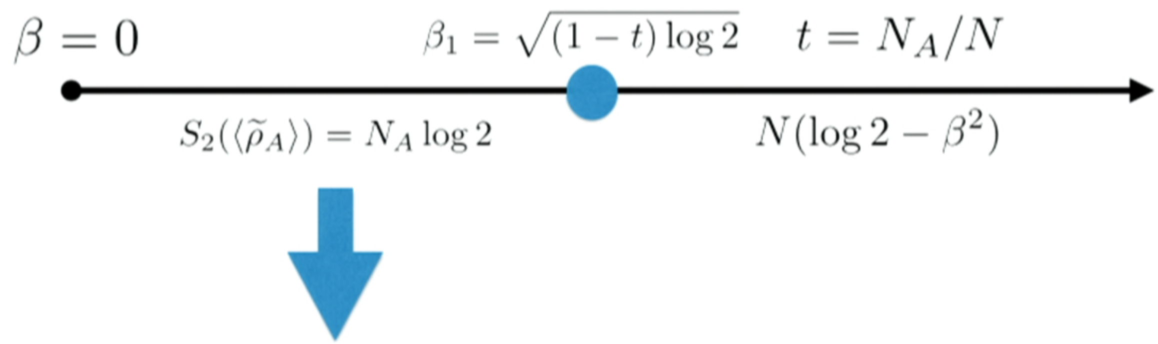
Lower bound

Quenched average $\langle S_n(\rho_A)\rangle = \frac{1}{1-n}\langle \log \mathrm{Tr}\rho_A^n\rangle$

Annealed average $S_n(\langle\tilde{\rho}_A\rangle) = \frac{1}{1-n}\left(\log\langle \mathrm{Tr}\tilde{\rho}_A^n\rangle - n\langle \log Z(\beta)\rangle\right)$

$\rho_A = \tilde{\rho}_A / Z$

$$\langle S_n(\rho_A)\rangle \geq S_n(\langle\tilde{\rho}_A\rangle), \quad \text{when } n > 1$$



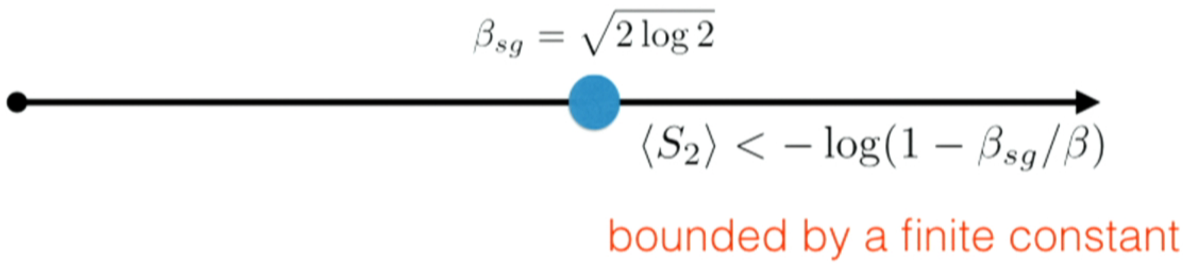
$$\langle S_2\rangle = N_A \log 2$$

$$\langle S_n\rangle \geq \langle S_2\rangle, \quad \text{if } n < 2$$

Similar method can be used to calculate Renyi entropy when n>2

Upper bound

$$\mathrm{Tr}\rho_A^2 > \frac{Z(2\beta)}{Z(\beta)^2} = Y_2(\beta) \quad \Rightarrow \quad \langle S_2 \rangle < \tau_2(\beta) N \log 2$$



Upper bound

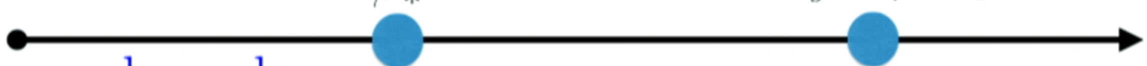
$\mathrm{Tr}\rho_A^2 > \frac{Z(2\beta)}{Z(\beta)^2} = Y_2(\beta)$ ⇒ $\langle S_2 \rangle < \tau_2(\beta) N \log 2$

$\beta_{sg} = \sqrt{2\log 2}$

$\langle S_2 \rangle < -\log(1 - \beta_{sg}/\beta)$

bounded by a finite constant

β_* $\beta_{sg} = \sqrt{2\log 2}$

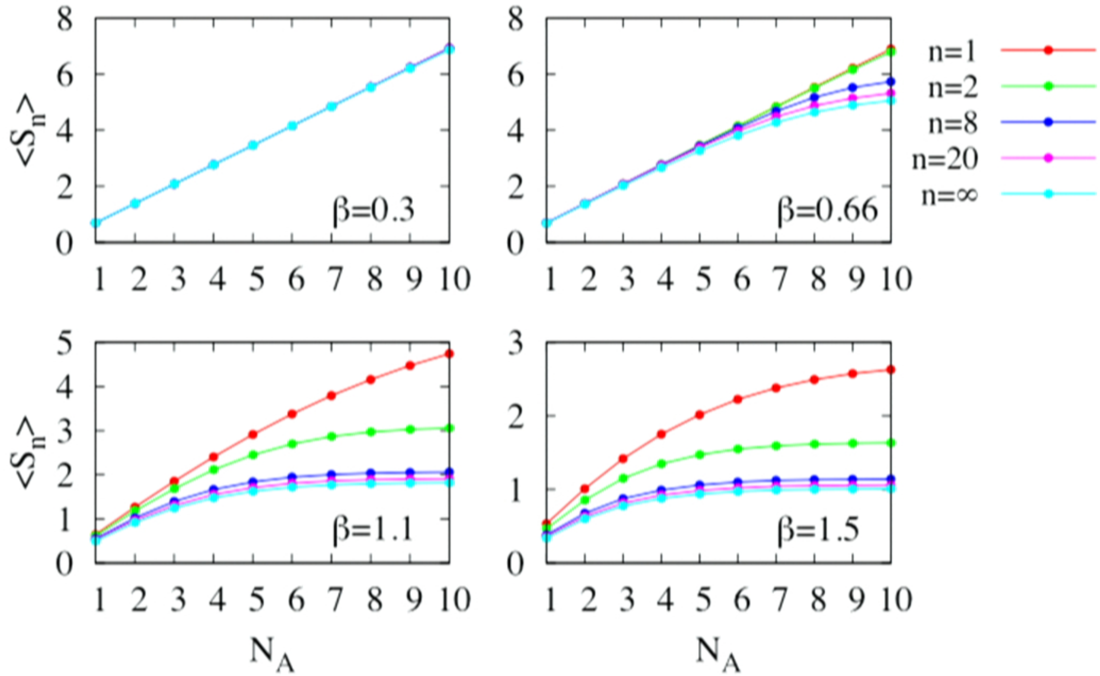


volume law
$T = \infty$

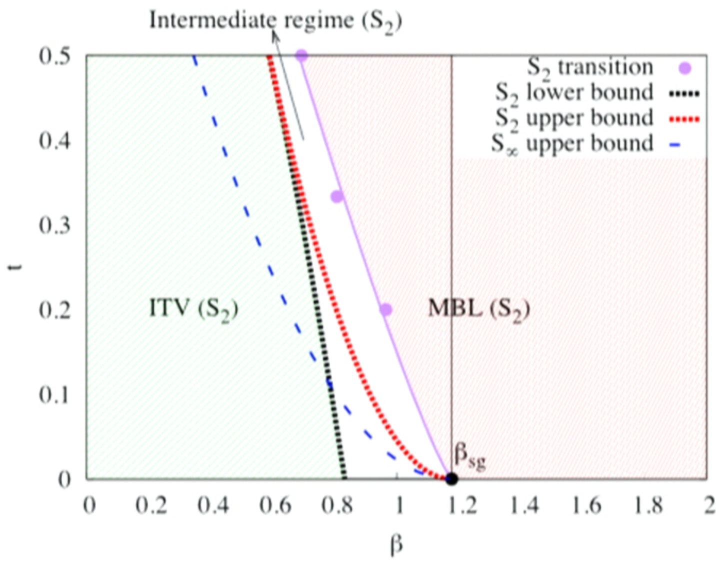
MBL transition?

finite constant

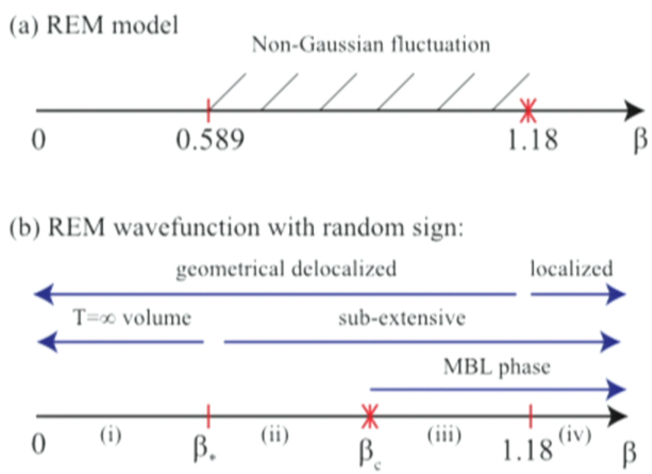
Numerical results $N = 30$



A summary of our knowledge for phase diagram



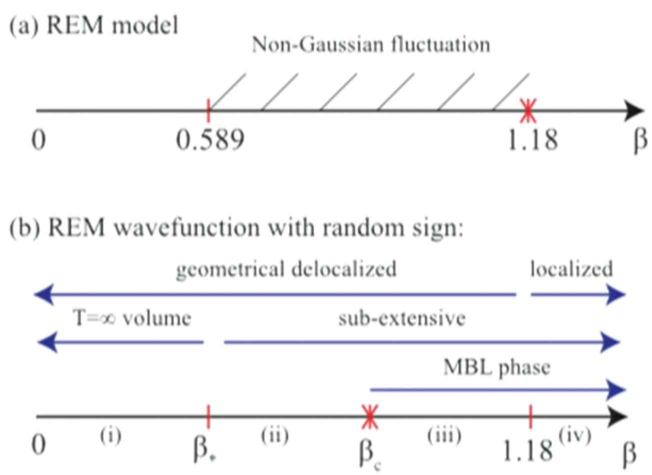
Conclusion:



Future direction

1. RK state(local) with sign structure
2. Multifractality in a real system
3. Translational invariant system
4. Sign structure in experiments

Conclusion:



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