Title: Composite fermion liquid: from 2DEG to topological insulator surface

Date: Nov 06, 2015 10:30 AM

URL: http://pirsa.org/15110075

Abstract:

Ashvin Vishwanath
(Berkeley)

MM and A. Vishwanath, arXiv:1505.05142

MM, arXiv:1510.05663

Related work:

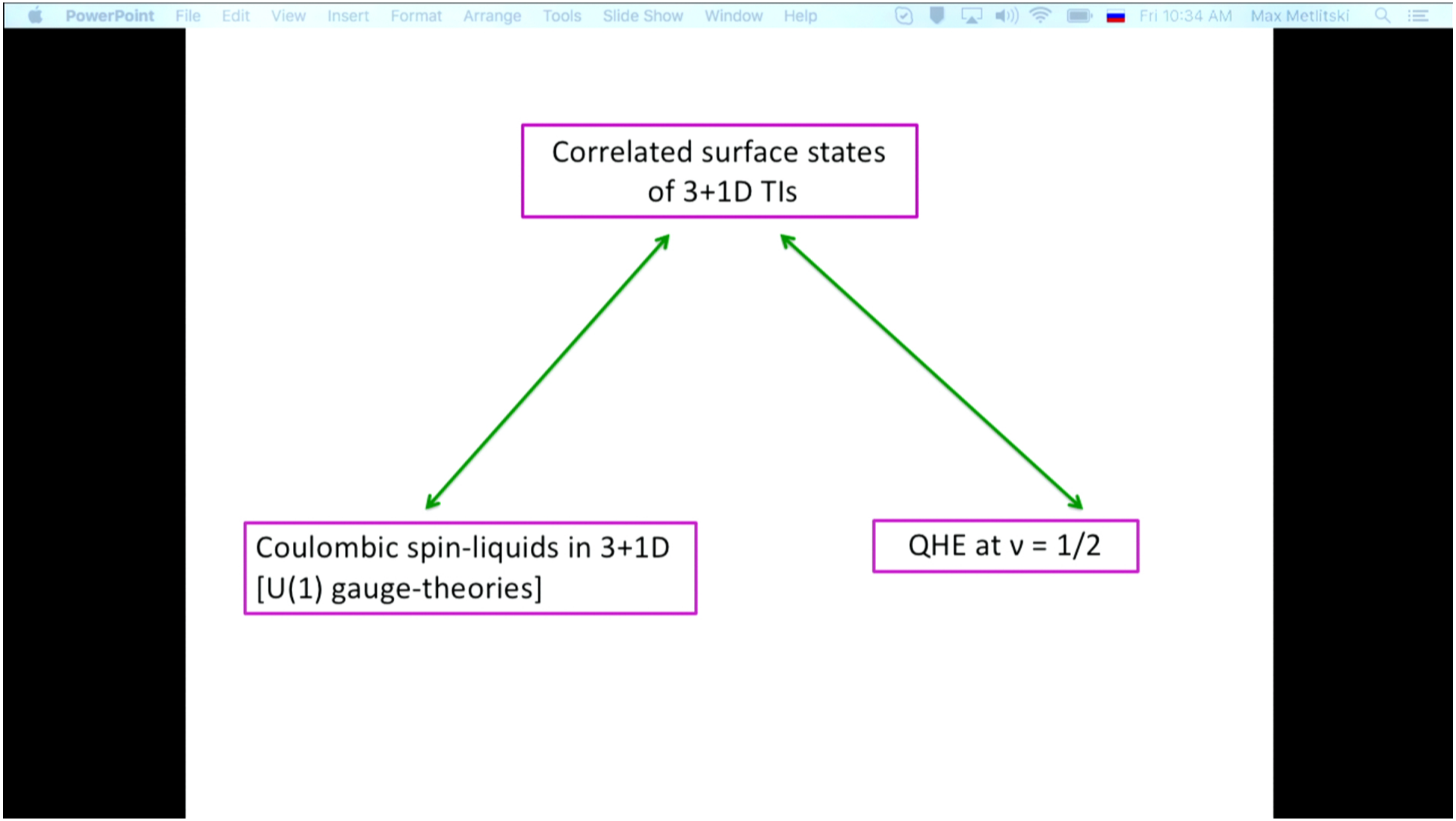
Motivation: D. Son, Phys. Rev. X 5, 031027 (2015)

C. Wang and T. Senthil, arXiv:1505.03520, arXiv:1505.05141

Subsequent work:

G. Murthy, R. Shankar, arXiv:1508.06974

D. F. Mross, J. Alicea, O. I. Motrunich, arXiv:1510.08455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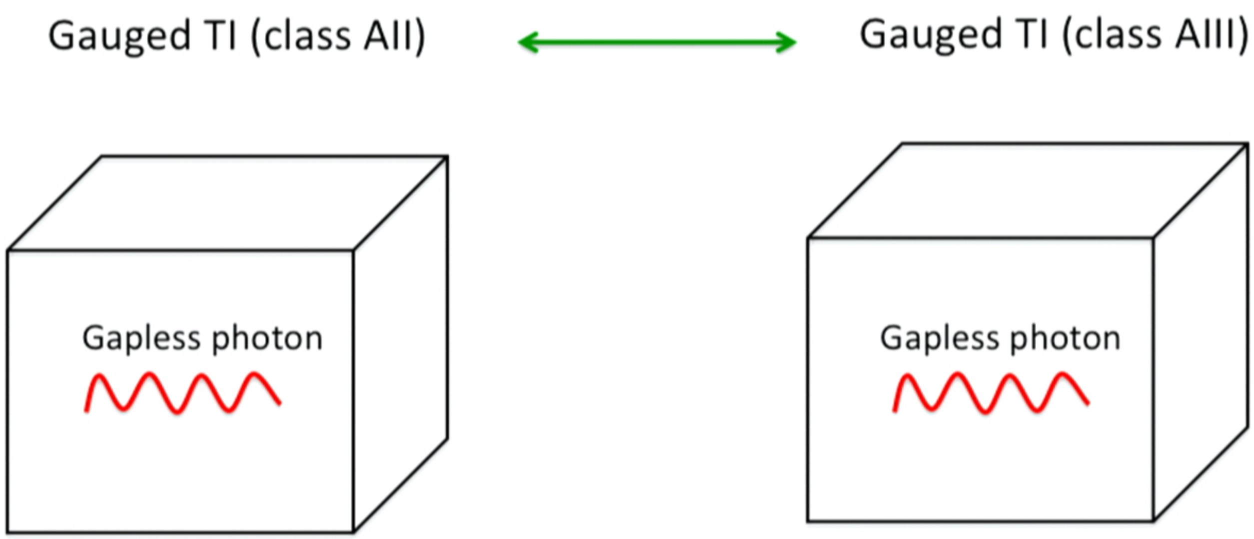
Fri 10:34 AM Max Metlitski
Correlated surface states of 3+1D TIs
Coulombic spin-liquids in 3+1D [U(1) gauge-theories]
QHE at ν = 1/2

Bulk duality

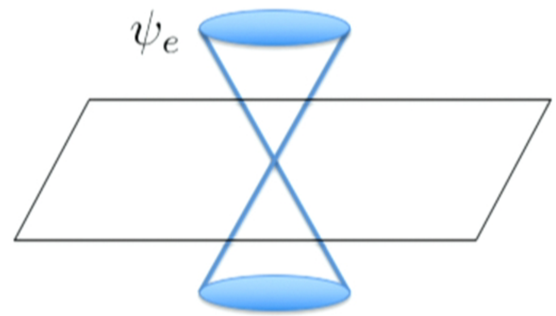
- Electric-magnetic duality of T-invariant Coulomb spin-liquids in 3+1D [U(1) gauge theories].



Surface "duality"

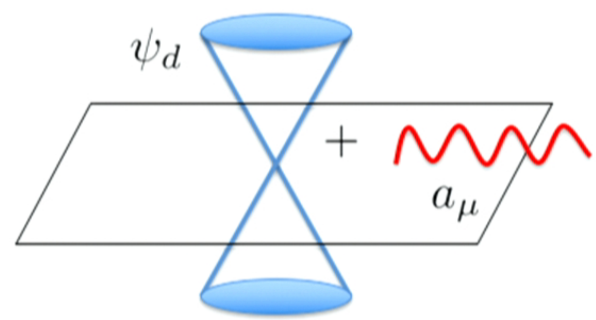
- Particle-vortex duality of single Dirac cone on TI surface

$$L_{free} = \bar{\psi}_e \gamma^\mu (\partial_\mu - i A^{ext}_\mu) \psi_e$$



$$\updownarrow$$

$$L_{QED_3} = \bar{\psi}_d \gamma^\mu (\partial_\mu - i a_\mu) \psi_d + \frac{1}{4g^2} f^2_{\mu\nu} + \frac{i}{4\pi} A^{ext}_\mu \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda$$



Surface dictionary

$$L_{free} = \bar{\psi}_e \gamma^\mu (\partial_\mu - i A^{ext}_\mu) \psi_e$$

$$L_{QED_3} = \bar{\psi}_d \gamma^\mu (\partial_\mu - i a_\mu) \psi_d + \frac{1}{4g^2} f^2_{\mu\nu} + \frac{i}{4\pi} A^{ext}_\mu \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda$$

L_{free}	L_{QED_3}
J^μ_e	$\frac{1}{4\pi} \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda$
ψ_e	Flux 4π instanton of a_μ
Double (flux 4π vortex)	ψ_d

Time-reversal

$$L_{free} = \bar{\psi}_e \gamma^\mu (\partial_\mu - i A_\mu^{ext}) \psi_e$$

$$L_{QED_3} = \bar{\psi}_d \gamma^\mu (\partial_\mu - i a_\mu) \psi_d + \frac{1}{4g^2} f_{\mu\nu}^2 + \frac{i}{4\pi} A_\mu^{ext} \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda$$

L_{free}	L_{QED_3}
$T: \quad \psi_{e,\uparrow} \to \psi_{e,\downarrow}$	$T: \quad \psi_{d,\uparrow} \to \psi^\dagger_{d,\downarrow}$
$\psi_{e,\downarrow} \to -\psi_{e,\uparrow}$	$\psi_{d,\downarrow} \to -\psi^\dagger_{d,\uparrow}$

Particle-hole symmetry

$$L_{free} = \bar{\psi}_e \gamma^\mu (\partial_\mu - i A_\mu^{ext}) \psi_e$$

$$L_{QED_3} = \bar{\psi}_d \gamma^\mu (\partial_\mu - i a_\mu) \psi_d + \frac{1}{4g^2} f_{\mu\nu}^2 + \frac{i}{4\pi} A_\mu^{ext} \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda$$

L_{free}	L_{QED_3}
$PH:$ $\psi_{e,\uparrow} \to \psi^\dagger_{e,\downarrow}$, $\psi_{e,\downarrow} \to -\psi^\dagger_{e,\uparrow}$, $i \to -i$	$PH:$ $\psi_{d,\uparrow} \to -\psi_{d,\downarrow}$, $\psi_{d,\downarrow} \to -\psi_{d,\uparrow}$, $i \to -i$

Particle-vortex duality of bosons

$$L_{XY} = |(\partial_\mu - iA^{ext}_\mu)\phi|^2 + m^2|\phi|^2 + u|\phi|^4$$

$$L_{AH} = |(\partial_\mu - ia_\mu)V|^2 + \tilde{m}^2|V|^2 + \tilde{u}|V|^4 + \frac{i}{2\pi} A^{ext}_\mu \epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda$$

L_{XY}	L_{AH}
J^μ_e	$\frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda$
ψ_e	Flux 2π instanton of a_μ
Double (flux 2π vortex)	V
$T: \quad \phi \to \phi$	$T: \quad V \to V^\dagger$

Surface "duality"

$$L_{free} = \bar{\psi}_e \gamma^\mu (\partial_\mu - i A^{ext}_\mu) \psi_e$$

$$\updownarrow$$

$$L_{QED_3} = \bar{\psi}_d \gamma^\mu (\partial_\mu - i a_\mu) \psi_d + \frac{1}{4g^2} f^2_{\mu\nu} + \frac{i}{4\pi} A^{ext}_\mu \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda$$

Same anomaly under $U(1)_{ext}$ and T symmetries - yes!

Same at the IR fixed point, i.e. at $\omega \ll g^2 \ll \Lambda_{UV}$?

T-Pfaffian$_+$

$$L_{QED_3} = \bar{\psi}_d \gamma^\mu (\partial_\mu - i a_\mu)\psi_d + \frac{1}{4g^2} f_{\mu\nu}^2 + \frac{i}{4\pi} A^{ext} \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda$$

Charge 2 Higgs: $\langle \psi_d^T C \psi_d \rangle \neq 0$

Symmetric gapped topologically ordered surface of TI: T-Pfaffian$_+$

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Charge 2 Higgs: $\langle \psi_d^T C \psi_d \rangle \neq 0$

Symmetric gapped topologically ordered surface of TI: T-Pfaffian$_+$

$$\mathrm{T} - \mathrm{Pfaffian}_+ \subset \mathrm{Ising} \times U(1)_{-8}$$

$k \rightarrow$	0	1	2	3	4	5	6	7
I	1		$-i$		1		$-i$	
σ		1		-1		-1		1
ψ	-1		i		-1		i	
T^2	1	η		$-\eta$	-1	$-\eta$		η

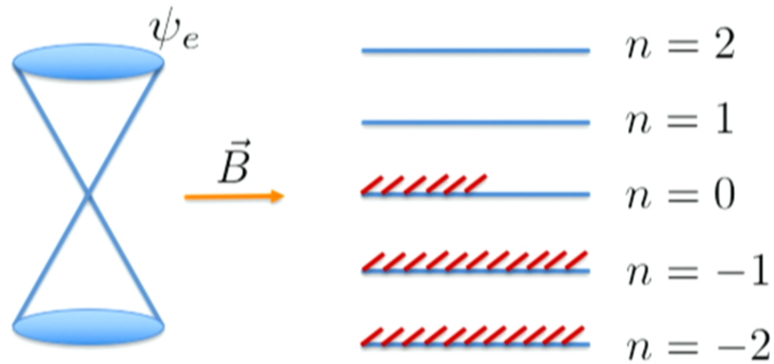
$Q_{ext} = q/4$

$\eta = \pm 1$

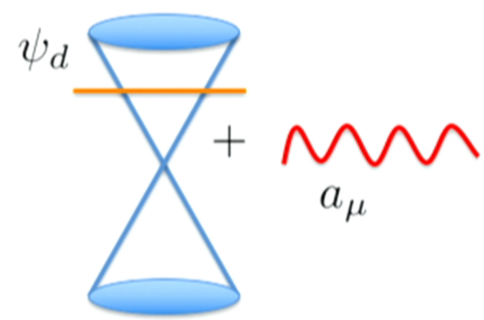
P. Bonderson, C. Nayak, X.L. Qi (2013), X. Chen, L. Fidkowski, A. Vishwanath (2013)

Duality in B-field

$$L_{free} = \bar{\psi}_e \gamma^\mu (\partial_\mu - i A^{ext}_\mu) \psi_e$$



$$L_{QED_3} = \bar{\psi}_d \gamma^\mu (\partial_\mu - i a_\mu) \psi_d + \frac{1}{4g^2} f^2_{\mu\nu} + \frac{i}{4\pi} A^{ext}_\mu \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda$$



Duality in B-field

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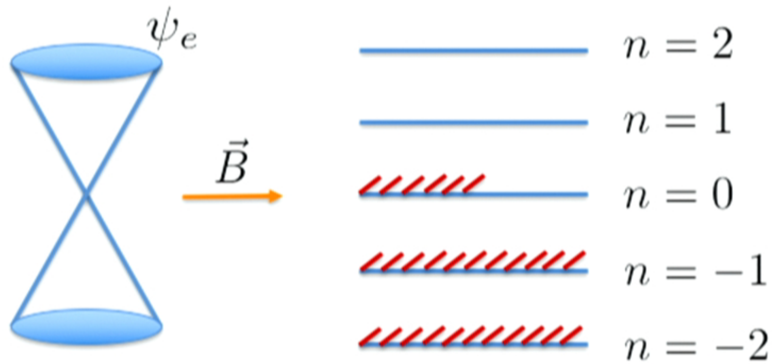
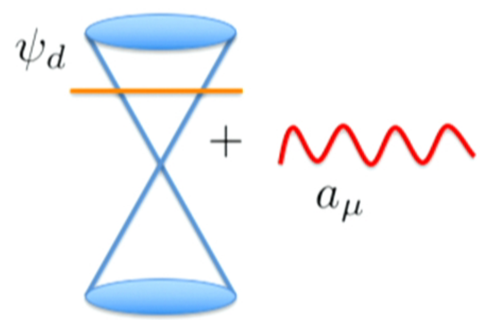


$$L_{QED_3} = \bar{\psi}_d \gamma^\mu (\partial_\mu - i a_\mu) \psi_d + \frac{1}{4g^2} f_{\mu\nu}^2 + \frac{i}{4\pi} A_\mu^{ext} \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda$$

$$PH: \quad \psi_{e,\uparrow} \to \psi_{e,\downarrow}^\dagger \qquad \psi_{d,\uparrow} \to -\psi_{d,\downarrow}$$

$$\psi_{e,\downarrow} \to -\psi_{e,\uparrow}^\dagger \qquad \psi_{d,\downarrow} \to -\psi_{d,\uparrow}$$



Spin-liquid 1

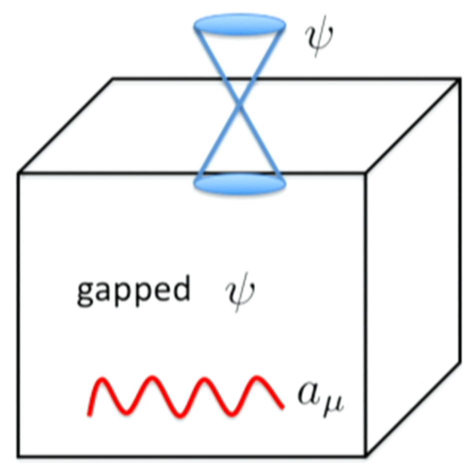
- Parton construction: $\vec{S} = \psi^\dagger \vec{\Gamma} \psi$ ψ - fermion

Gauge-symmetry: $u(1): \ \psi(x) \to e^{i\alpha(x)}\psi(x)$

Emergent gauge field: a_μ

$T:$ $\psi_\uparrow \to \psi_\downarrow$, $\psi_\downarrow \to -\psi_\uparrow$ $T^2 = -1$

Place ψ into TI bandstructure:



Spin-liquid 2

- Parton construction: $\vec{S} = \tilde{\psi}^\dagger \vec{\tilde{\Gamma}} \tilde{\psi}$ $\qquad \tilde{\psi}$ - fermion

Gauge-symmetry: $\tilde{u}(1): \ \tilde{\psi}(x) \to e^{i\alpha(x)} \tilde{\psi}(x)$

Emergent gauge field: $\tilde{a}_\mu$

$$T: \quad \tilde{\psi}_\uparrow \to \tilde{\psi}_\downarrow^\dagger \qquad \tilde{\psi}_\downarrow \to -\tilde{\psi}_\uparrow^\dagger$$

$\tilde{u}(1)$ charge inverted by T

Non-interacting symmetry class AIII (T and S^z preserving SCs)

$$T: \quad \psi_\uparrow \to \psi_\downarrow \qquad \psi_\downarrow \to -\psi_\uparrow$$

$u(1)$ charge preserved by T

Ordinary TIs: class AII

Spin-liquid 2

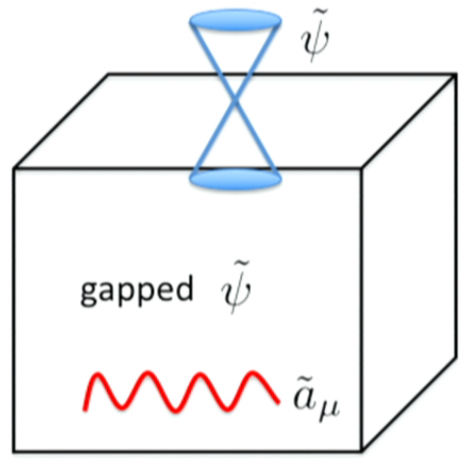
- Parton construction: $\vec{S} = \tilde{\psi}^{\dagger}\vec{\tilde{\Gamma}}\tilde{\psi}$ $\qquad$ $\tilde{\psi}$ - fermion

Gauge-symmetry: $\tilde{u}(1):\ \tilde{\psi}(x) \to e^{i\alpha(x)}\tilde{\psi}(x)$

Emergent gauge field: $\tilde{a}_{\mu}$

$$T: \quad \tilde{\psi}_{\uparrow} \to \tilde{\psi}_{\downarrow}^{\dagger}$$
$$\tilde{\psi}_{\downarrow} \to -\tilde{\psi}_{\uparrow}^{\dagger}$$

Place $\tilde{\psi}$ into $\nu = 1$
class AIII band-structure



Bulk duality

Spin-liquid 1 (gauged TI)

$$T: \quad \psi_\uparrow \to \psi_\downarrow$$
$$\psi_\downarrow \to -\psi_\uparrow$$

Spin-liquid 2 (gauged AIII)

$$T: \quad \tilde{\psi}_\uparrow \to \tilde{\psi}_\downarrow^\dagger$$
$$\tilde{\psi}_\downarrow \to -\tilde{\psi}_\uparrow^\dagger$$

Same T-invariant 3+1D phase!

$\psi \longleftrightarrow$ Double monopole of $\tilde{a}_\mu$

Bulk duality

Spin-liquid 1 (gauged TI)

$$T: \quad \psi_\uparrow \to \psi_\downarrow$$
$$\psi_\downarrow \to -\psi_\uparrow$$

Spin-liquid 2 (gauged AIII)

$$T: \quad \tilde{\psi}_\uparrow \to \tilde{\psi}_\downarrow^\dagger$$
$$\tilde{\psi}_\downarrow \to -\tilde{\psi}_\uparrow^\dagger$$

Same T-invariant 3+1D phase!

$\psi \longleftrightarrow$ Double monopole of $\tilde{a}_\mu$

Spin-liquid 2

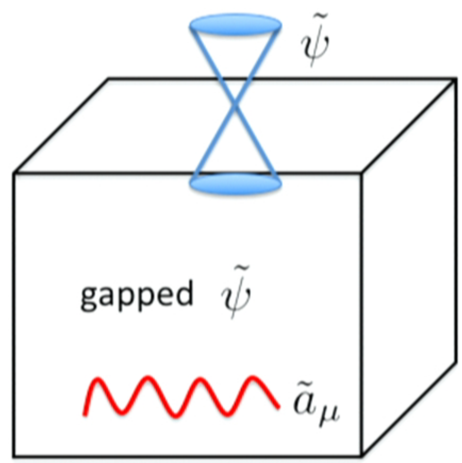
- Parton construction: $\vec{S} = \tilde{\psi}^\dagger \vec{\tilde{\Gamma}} \tilde{\psi}$ $\qquad$ $\tilde{\psi}$ - fermion

Gauge-symmetry: $\tilde{u}(1): \ \tilde{\psi}(x) \to e^{i\alpha(x)}\tilde{\psi}(x)$

Emergent gauge field: $\tilde{a}_\mu$

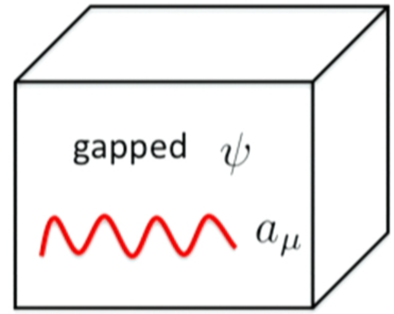
$$T: \quad \begin{aligned} \tilde{\psi}_\uparrow &\to \tilde{\psi}_\downarrow^\dagger \\ \tilde{\psi}_\downarrow &\to -\tilde{\psi}_\uparrow^\dagger \end{aligned}$$

Place $\tilde{\psi}$ into $\nu = 1$
class AIII band-structure



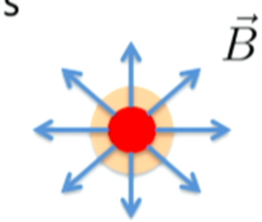
Dyon excitations: spin-liquid 1 (AII)

$$L_{bulk} = \frac{1}{4e^2} f_{\mu\nu}^2 + \frac{i\theta}{32\pi^2}\epsilon^{\mu\nu\lambda\sigma} f_{\mu\nu} f_{\lambda\sigma}$$



Maxwell theory with $\theta = \pi$

Bulk excitations: electric charges and magnetic monopoles

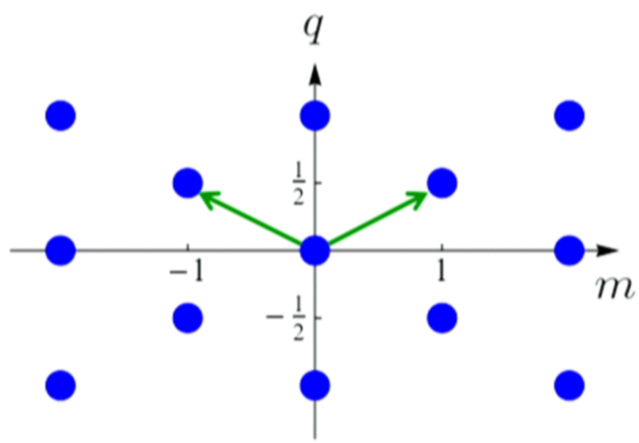


Witten effect: $q = n + \frac{\theta m}{2\pi}$

$$\theta = \pi : \quad q = n + \frac{m}{2}$$

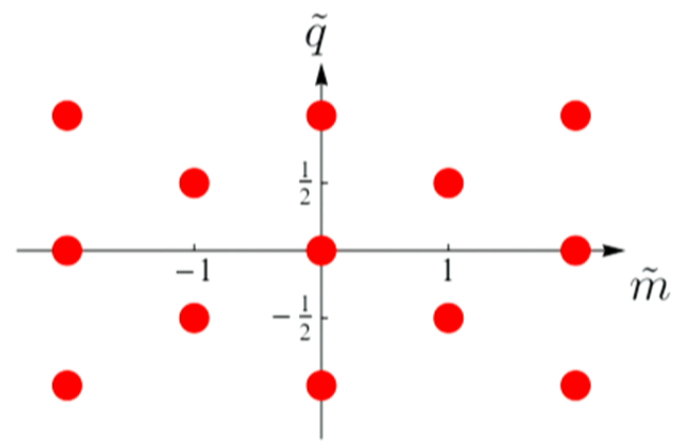
Dyon excitations

AII



$$T: \quad q \to q$$
$$m \to -m$$

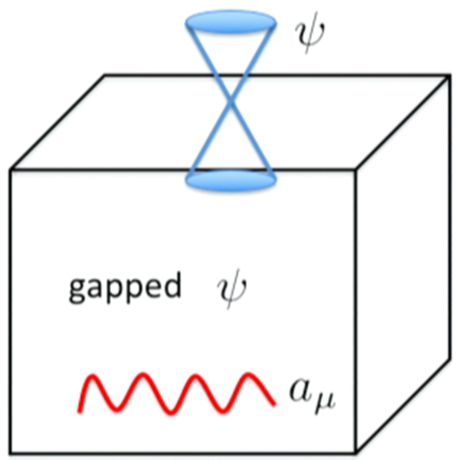
AIII



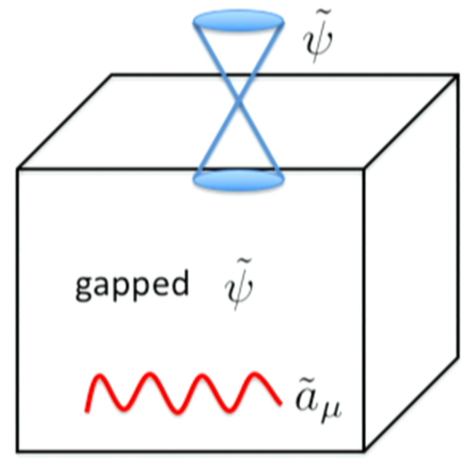
$$T: \quad q \to -q$$
$$m \to m$$

Surface ``duality”

Spin-liquid 1 (gauged TI)



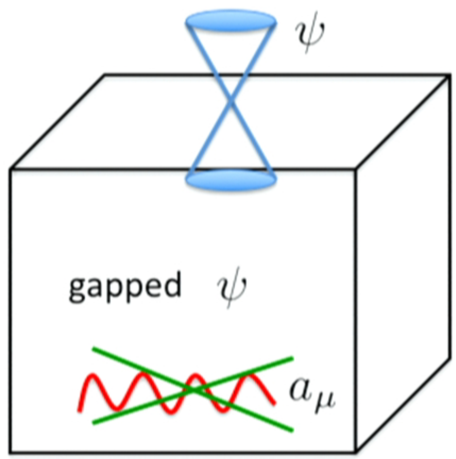
Spin-liquid 2 (gauged AIII)



Introduce electron ψ_e : put into a trivial insulator

Higgs transition

Spin-liquid 1 (gauged TI)

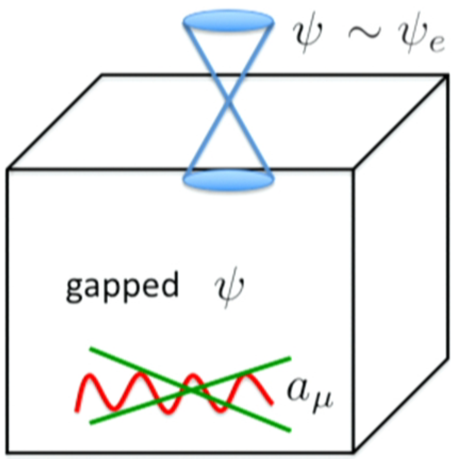


Condense $\psi_e^\dagger \psi$

Higgs effect

Higgs transition

Spin-liquid 1 (gauged TI)

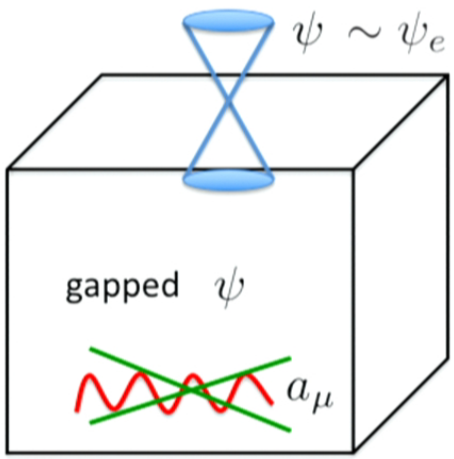


Condense $\psi_e^\dagger \psi$

Higgs effect

Dual picture

Spin-liquid 1 (gauged TI)

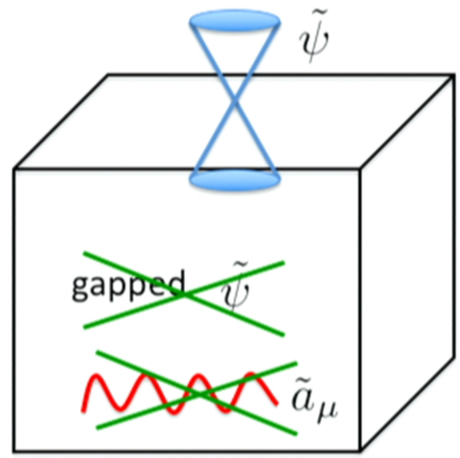


Condense $\psi_e^\dagger \psi$

Higgs

Result: ordinary TI

Spin-liquid 2 (gauged AIII)

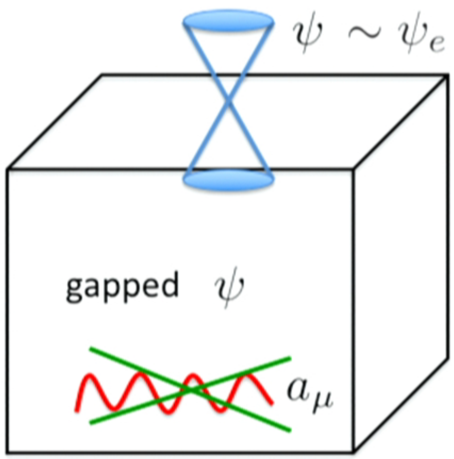


Condense $\psi_e^\dagger \times$ monopole^2

Bulk confinement

Dual picture

Spin-liquid 1 (gauged TI)

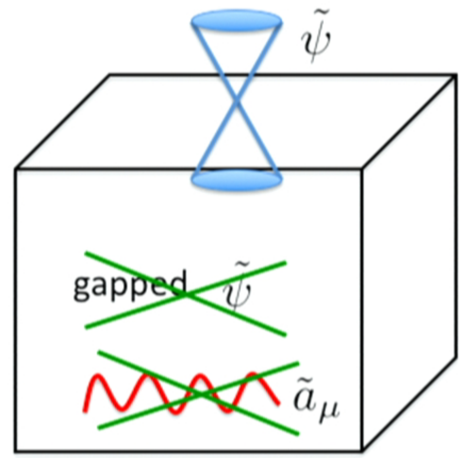


Condense $\psi_e^\dagger \psi$

Higgs

Result: ordinary TI

Spin-liquid 2 (gauged AIII)



Condense $\psi_e^\dagger \times$ monopole^2

Bulk confinement

More explicit derivation

$$L_{free} = \bar{\psi}\gamma^{\mu}(\partial_{\mu} - iA^{ext}_{\mu})\psi$$

$$L_{QED_3} = \bar{\psi}_d\gamma^{\mu}(\partial_{\mu} - ia_{\mu})\psi_d + \frac{1}{4g^2}f^2_{\mu\nu} + \frac{i}{4\pi}A^{ext}_{\mu}\epsilon^{\mu\nu\lambda}\partial_{\nu}a_{\lambda}$$

Exact quasi-1d ``lattice” mapping:
D. Mross, J. Alicea, O. I. Motrunich, arXiv:1510.08455

Caveat i): dual theory strongly coupled: $g^2 \sim \Lambda_{UV}$

More explicit derivation

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Exact quasi-1d ``lattice" mapping:

D. Mross, J. Alicea, O. I. Motrunich, arXiv:1510.08455

Caveat i): dual theory strongly coupled: $g^2 \sim \Lambda_{UV}$

Caveat ii): $T^2 =$ translation

Fixing the ambiguity

Spin-liquid 1 (gauged AII)

$$\begin{aligned} S_1(e) &= \frac{1}{4e^2}\int d^4x\sqrt{g}f_{\mu\nu}f^{\mu\nu} \\ &+ \pi i N[a] \end{aligned}$$

Spin-liquid 2 (gauged ν=1 AIII)

$$\begin{aligned} S_2(\tilde{e}) &= \frac{1}{4\tilde{e}^2}\int d^4x\sqrt{g}\tilde{f}_{\mu\nu}\tilde{f}^{\mu\nu} \\ &+ 2\pi i\eta[\tilde{a}] \end{aligned}$$

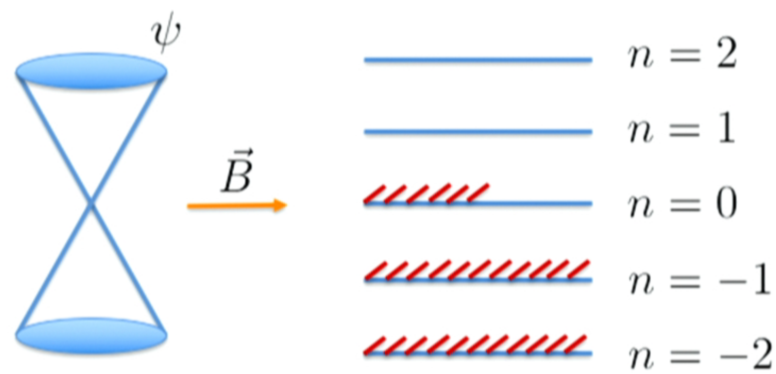
$$Z_1(e) = Z_2\left(\frac{4\pi}{e}\right)$$

Checked on $\mathbb{RP}^4, \mathbb{CP}^2$ - rules out possible eTmT, FFF factors

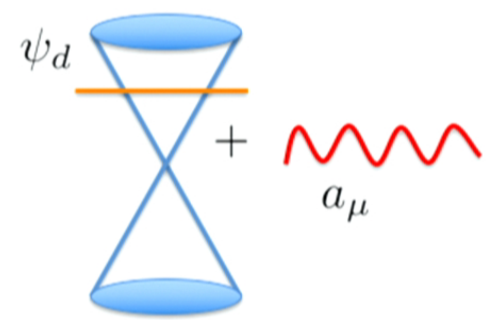
MM, arXiv:1510.05663

QH fluid at $\nu = 1/2$

$$L_{free} = \bar{\psi}\gamma^{\mu}(\partial_{\mu} - iA^{ext}_{\mu})\psi$$



$$L_{QED_3} = \bar{\psi}_d\gamma^{\mu}(\partial_{\mu} - ia_{\mu})\psi_d + \frac{1}{4g^2}f^2_{\mu\nu} + \frac{i}{4\pi}A^{ext}_{\mu}\epsilon^{\mu\nu\lambda}\partial_{\nu}a_{\lambda}$$



DMRG of QH fluid at ν = 1/2

L_y up to $24 l_B$

- LLL, ν = 1/2, $V(r) = \frac{1}{r} e^{-r^2/2\lambda^2}, \quad \lambda = 6 l_B$

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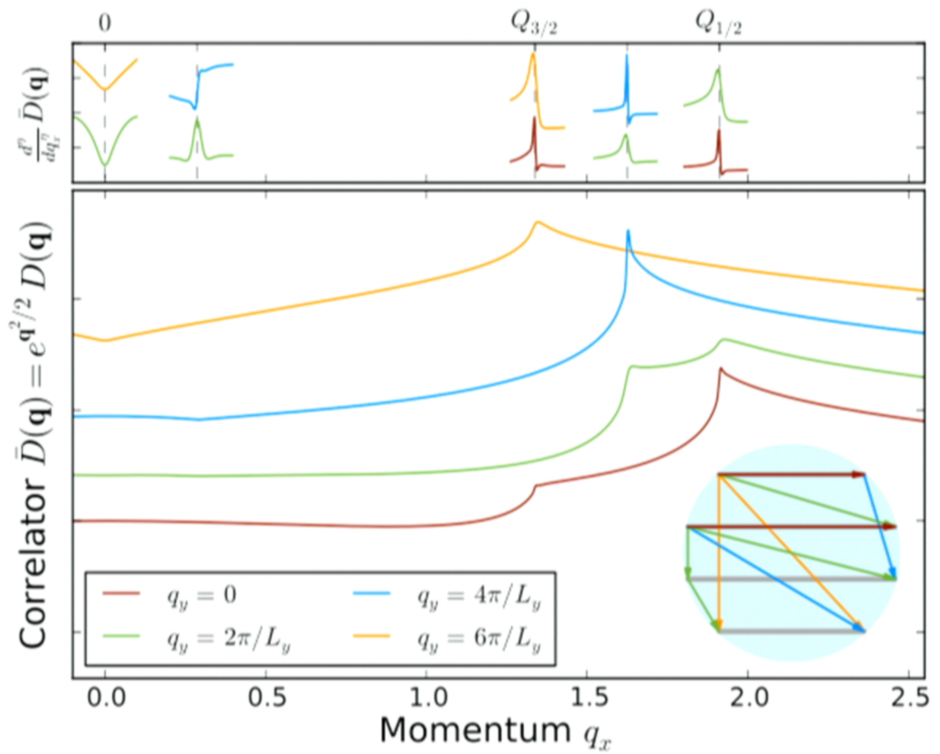
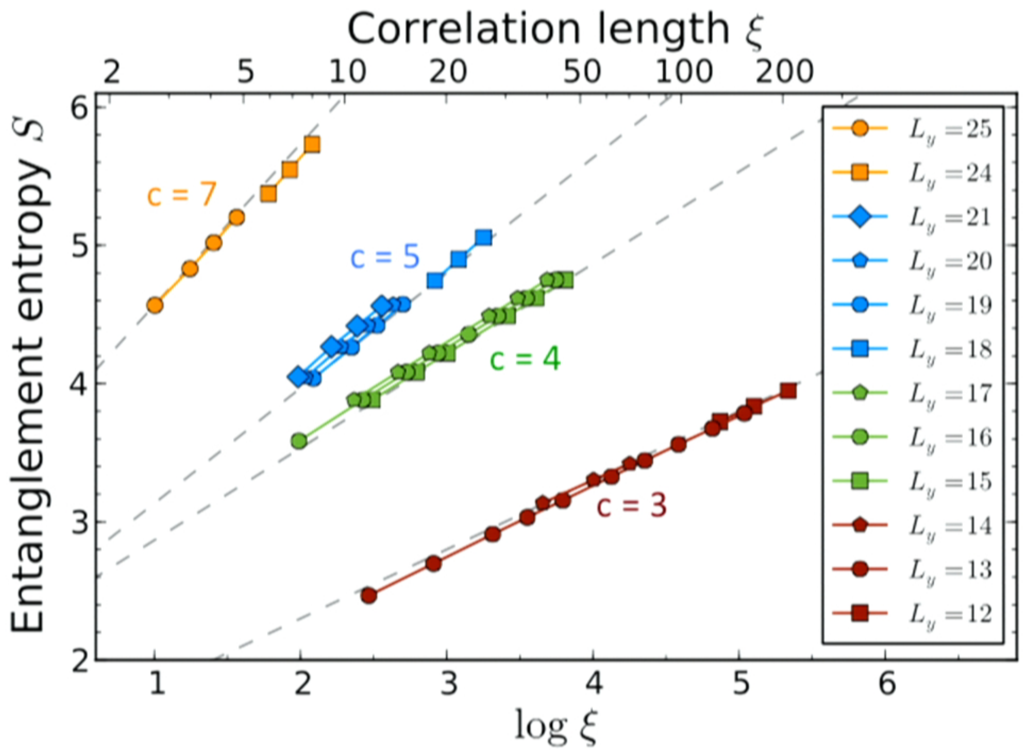
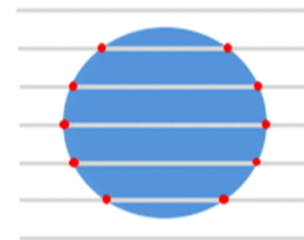
Density-density correlator, $L_y = 13\ l_B$



$$\rho_e(\vec{q}) \sim \rho_d(\vec{q})$$

Central charge





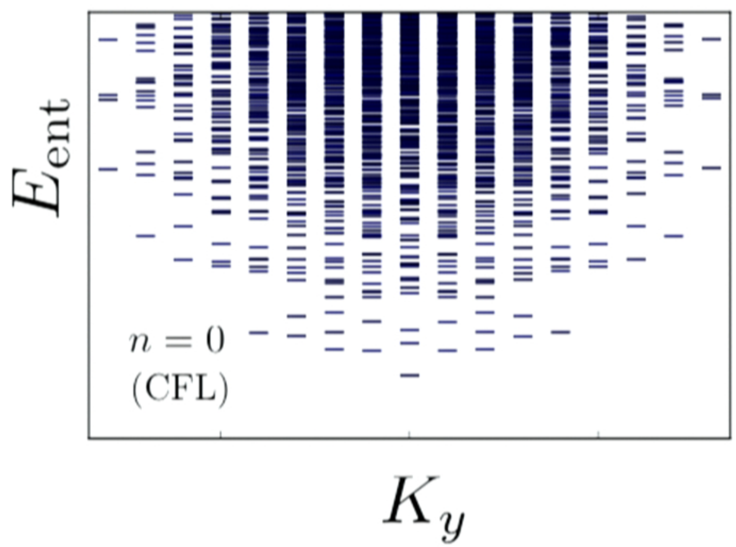
$$S = \frac{c}{6} \log \xi$$

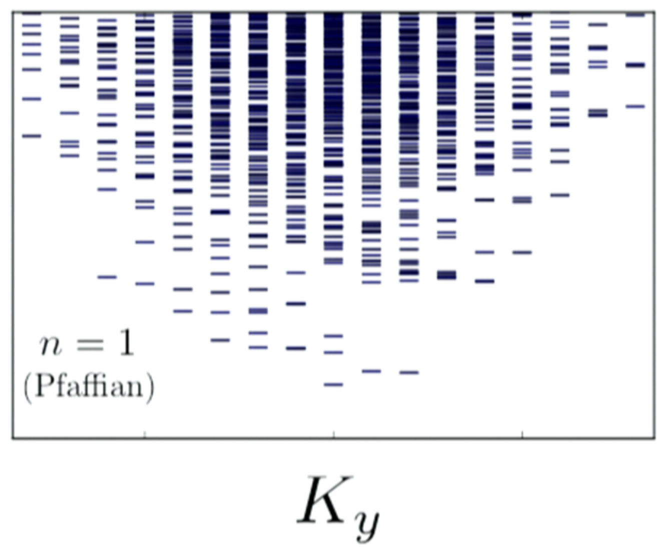
c = cuts – 1, overall charge mode killed by a_μ in 1d limit

Particle-hole symmetry

$$\langle \Psi | PH | \Psi \rangle = (1 - \epsilon)^{N_\Phi}$$

$\epsilon < 6 \times 10^{-5}$ $\qquad\qquad$ $\epsilon \approx 0.022$





Is the composite fermion a Dirac fermion?

$$PH^2 = (-1)^{N_\Phi/2}, \quad N_\Phi \text{ - even}$$

N_Φ fluxes of A_μ^{ext}

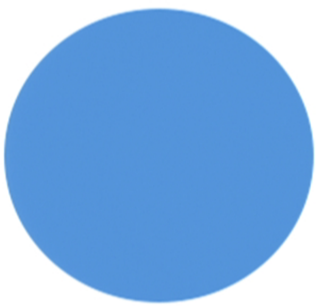
D. Son, M. Levin (unpublished)
S. Geraedts et al

- To add one CF, add an electron and two flux-quanta

$$PH^2 = (-1)^{N_{CF}}$$

- Consistent with Son's theory: $PH: \ \psi_{d,\uparrow} \to -\psi_{d,\downarrow}$

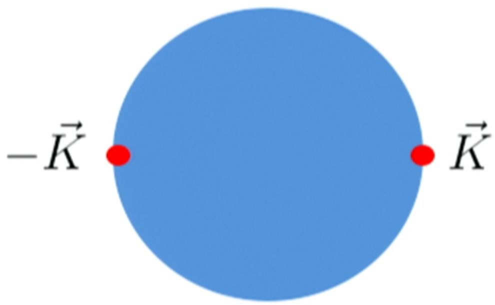
$$\psi_{d,\downarrow} \to -\psi_{d,\uparrow}$$



- Non-degenarate Fermi-surface
- Kramers doublet
- Must have a Berry's phase of π, i.e. Dirac!

$2K_F$ extinction

- TI surface:
  No $2K_F$ backscattering of T-symmetric disorder



$$T O(\vec{x}) T^\dagger = O(\vec{x})$$

$$O(x) \cancel{\sim} \psi^\dagger(\vec{K})\psi(-\vec{K}) e^{2i\vec{K}\vec{x}}$$

- QH at $\nu = 1/2$

$$PH\, O(x) PH^\dagger = O(x)$$

$$O(x) \cancel{\sim} \psi_d^\dagger(\vec{K})\psi_d(-\vec{K}) e^{2i\vec{K}\vec{x}}$$

- PH even operator:

$\rho_e(x)$ ✗

$\rho_e(x)\nabla^2\rho_e(x)$ ✔