Title: Particle-hole symmetry and the nature of the composite fermion

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Abstract:

# Particle-hole symmetry and the nature of the composite fermion

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PI-UIUC workshop, November 2015

DTS, PRX 5, 301027 (2015)

# Plan

- Composite fermions in FQHE
- Problem of particle-hole symmetry
- Composite fermions as Dirac fermions
- Consequences

# Composite fermions

Jain 1989

- CF = electron + even number of flux quanta
  - number of CFs = number of electrons
  - live in a reduced average magnetic field
- provide a unified explanations for a large number of QH plateaux
  - FQH of electrons ~ IQH of CFs
- nu=1/2 state: a Fermi liquid of CFs
  - strong experimental evidence

# HLR field theory

$$\mathcal{L} = i\psi^\dagger(\partial_0 - iA_0 + ia_0)\psi - \frac{1}{2m}|(\partial_i - iA_i + ia_i)\psi|^2 + \frac{1}{2}\frac{1}{4\pi}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda$$

$$b = \nabla \times a = 2 \times 2\pi\psi^\dagger\psi$$ "flux attachment"

mean field: $$B_{\text{eff}} = B - b = B - 4\pi n$$

$$\nu_{\text{CF}}^{-1} = \nu^{-1} - 2$$

Halperin, Lee, Read 1993

# Loose ends of CS field theory

Sometimes I feel a vague uneasiness with the flux-attachment procedure

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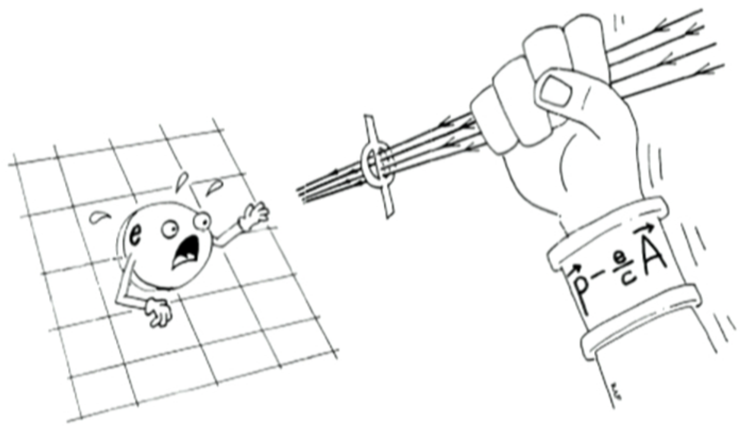


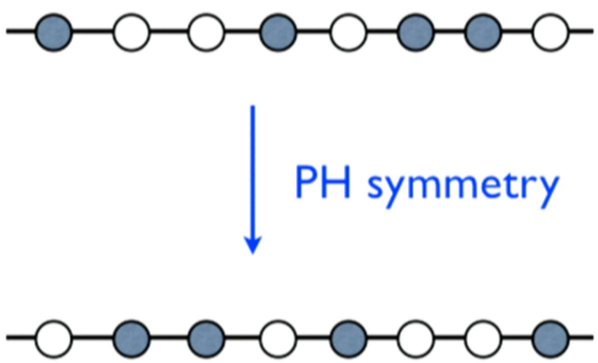

D. Arovas

Flux 4π should do nothing, but does something

# More loose ends

- Most common criticism of the HLR theory: lack of the explicit lowest Landau level projection
- Effective mass of CF finite while electron mass = 0, but in the vanilla HLR theory they are equal
- Can be fixed phenomenologically. Or simply accepted as an input in low-energy effective theory (Landau parameters).
- Perhaps the more fundamental problem is is the lack of particle-hole symmetry

# Particle-hole symmetry




$$\Theta|\text{empty}\rangle = |\text{full}\rangle$$

$$\Theta c_k^\dagger \Theta^{-1} = c_k$$

$$\Theta\, i\, \Theta^{-1} = -i$$

$$\nu \to 1 - \nu$$

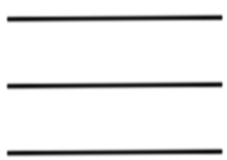
exact symmetry the Hamiltonian on the LLL, when mixing of higher LLs negligible

Girvin 1984

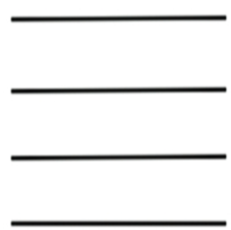
# PH symmetry in the CF theory

PH conjugate pairs of FQH states

$$\nu = \frac{n}{2n+1} \qquad \nu = \frac{n+1}{2n+1}$$



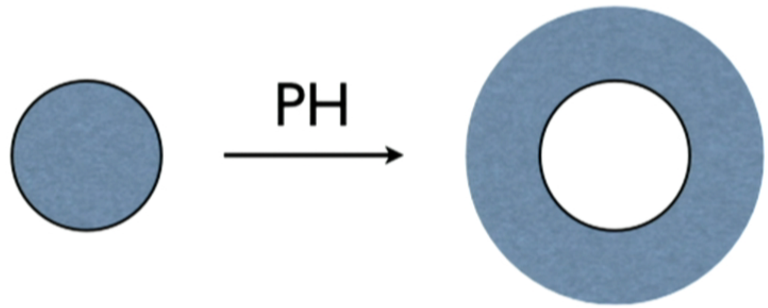
ν=3/7



ν=4/7

CF picture does not respect PH symmetry

# PH symmetry of CF Fermi liquid?



# Anti-Pfaffian state

- The Pfaffian state = p+ip BCS paired state of CFs
- What about the anti-Pfaffian state (particle-hole conjugate of Pfaffian)?
- Can it be p-ip BCS paired state of CFs?
  - No: the shift would be wrong (1 instead of -1)
- can the anti-Pfaffian be

# PH symmetry within HLR theory

- at $\nu=1/2$: PH symmetry requires $\sigma_{xy}=1/2$
- HLR + mean field: $\rho_{xy}=2$ $\sigma_{xy}<1/2$ Barkeshli Fisher Mulligan 2015
- Kivelson et al (1997): $\sigma_{xy}=1/2$ requires CFs to have nonzero Hall conductivity at zero field

$$\sigma_{xy}^{\mathrm{CF}} = -\frac{1}{2}\frac{e^2}{h}$$

while within HLR: $\sigma_{xy}^{\mathrm{CF}} = 0$

Kivelson, Lee, Krotov, Gan PRB 55 (1997)

# A new view on the CFs

- CF is a Dirac fermion: Berry phase π around Fermi surface
- CF transforms to CF (not hole) under PH conjugation
- CFs interact through an emergent U(1) gauge field, whose action does not have the Chern-Simons term
- The number of CFs is half the magnetic flux, in general not equal to the number of electrons

DTS, PRX 5, 301027 (2015)
Wang, Senthil
Metlitski, Vishwanath; Metlitski
Geraedts et al., 1508.04140
Mross, Alicea, Motrunich, 1510.08455
...

# First indication of Dirac nature of CFs

$$\nu = \frac{n}{2n+1} \longrightarrow \nu_{\mathrm{CF}} = n$$

$$\nu = \frac{n+1}{2n+1} \longrightarrow \nu_{\mathrm{CF}} = n+1$$

# First indication of Dirac nature of CFs

$$\nu = \frac{n}{2n+1} \rightarrow \nu_{\mathrm{CF}} = n + \frac{1}{2} \ ?$$

$$\nu = \frac{n+1}{2n+1} \rightarrow \nu_{\mathrm{CF}} = n + \frac{1}{2} \ ?$$

CFs form an IQH state at half-integer filling factor: must be a Dirac fermion

# IQHE in graphene

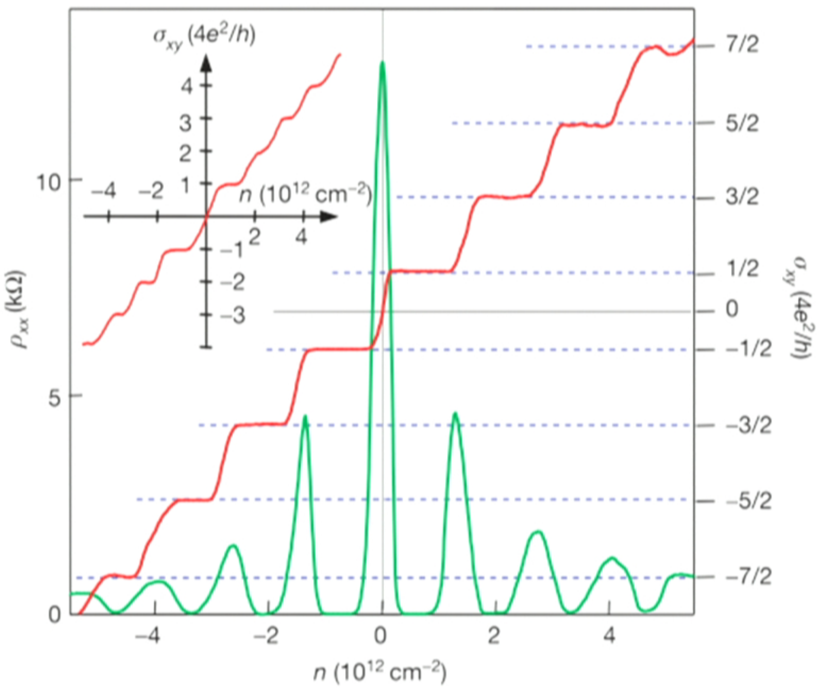


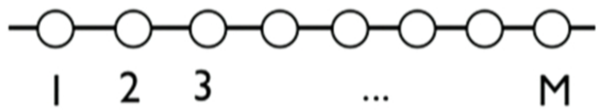

**Figure 4** | **QHE for massless Dirac fermions.** Hall conductivity $\sigma_{xy}$ and longitudinal resistivity $\rho_{xx}$ of graphene as a function of their concentration at $B = 14$ T and $T = 4$ K. $\sigma_{xy} \equiv (4e^2/h)\nu$ is calculated from the measured

$$\sigma_{xy} = \left(n + \frac{1}{2}\right) \frac{e^2}{2\pi\hbar}$$

# Origin of anomalous Hall conductivity of the CFs

- PH symmetry requires the CFs to have nonzero anomalous Hall conductivity equal $-1/2\ e^2/h$ (Kivelson et al 1997)
- Haldane and others: fractional part of anomalous Hall conductivity = Berry phase around Fermi surface
- Thus the Berry phase of the CFs have to be $\pi$

# PH conjugation




$$\Theta|\text{empty}\rangle = |\text{full}\rangle = c_1^\dagger c_2^\dagger \cdots c_M^\dagger |\text{empty}\rangle \qquad \Theta c_k^\dagger \Theta^{-1} = c_k$$

$$\Theta^2|\text{empty}\rangle = (-1)^{M(M-1)/2}|\text{empty}\rangle$$

$$\Theta^2|\text{any}\rangle = (-1)^{M(M-1)/2}|\text{any}\rangle \qquad |\text{any}\rangle = c_{n_1}^\dagger c_{n_2}^\dagger \ldots c_{n_{N_e}}^\dagger |\text{empty}\rangle$$

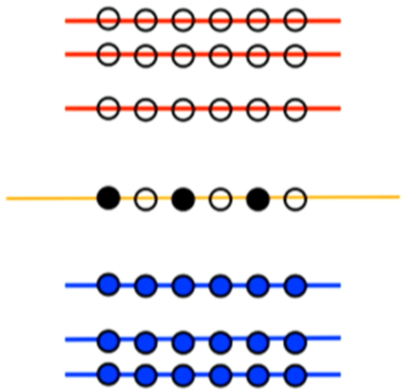
$$M = 2N_{\text{CF}} \qquad \Theta^2 = (-1)^{N_{\text{CF}}}$$

For consistent $\Theta^2$ assignment, number of CFs should not be the number of electrons

Geraedts, Zaletel, Mong, Metlitski, Vishwanath, Motrunich; Levin, Son

# Dirac composite fermions

- The notion of a Dirac composite fermion was proposed by Mross, Essin, Alicea (2014) for the interacting surface of TI
- In the context of QHE, it may seem strange: the original electron is nonrelativistic
- But in the lowest Landau level limit there is no difference between nonrelativistic or relativistic electrons



$$\nu_{\mathrm{rel}} = \nu_{\mathrm{NR}} - \frac{1}{2}$$

# Flux attachment?

- Since number of CFs $\neq$ number of electrons, flux attachment is not the right picture
- What is the alternative?

# Jain sequences

In relativistic convention $\nu_{\rm rel} = \nu_{\rm NR} - \frac{1}{2}$

Jain's sequences $\nu_{\rm NR} = \frac{n}{2n+1}$ and $\nu_{\rm NR} = \frac{n+1}{2n+1}$

correspond to $\nu_{\rm rel} = \mp \frac{1}{2(2n+1)}$

We want $\nu_{\rm CF} = n + \frac{1}{2}$

$$2\nu_{\rm rel} = \frac{1}{2\nu_{\rm CF}}$$

# Particle-vortex duality

- If electron-electron interaction is short-ranged, the CF theory is QED3: non-Fermi liquid
- For Coulomb interactions: marginal Fermi liquid
- Also useful for understanding strongly interacting surface of TI (Metlitski's talk)

# EM response

- Electron conductivity tensor σ in terms of the CF conductivity tensor $\tilde{\sigma}$

$$\sigma_{xx} = \frac{1}{4}\frac{\tilde{\sigma}_{xx}}{\tilde{\sigma}_{xx}^2 + \tilde{\sigma}_{xy}^2} ,$$

$$\sigma_{xy} = -\frac{1}{4}\frac{\tilde{\sigma}_{xy}}{\tilde{\sigma}_{xx}^2 + \tilde{\sigma}_{xy}^2} + \left[\tfrac{1}{2}\right]_{\mathrm{NR}} .$$

(all in units of $e^2/h$)

# Galilean invariance and Kohn's theorem

$$\mathcal{L} = i\psi^\dagger(D_0 + v_{\rm F}\,\boldsymbol{\sigma}\cdot\mathbf{D})\psi + \frac{i}{2}\mathbf{v}\cdot\psi^\dagger\overset{\leftrightarrow}{\mathbf{D}}\psi - \frac{1}{4}(\boldsymbol{\nabla}\times\mathbf{v})\psi^\dagger\sigma_3\psi + \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda - \frac{b}{8\pi}(\boldsymbol{\nabla}\times\mathbf{v})$$

where $D_\mu\psi = (\partial_\mu + 2ia_\mu)\psi$.

relaxation time approximation

$$\sigma_{xx} = \frac{1}{4}\frac{\tilde{\sigma}_{xx}}{\tilde{\sigma}_{xx}^2 + \tilde{\sigma}_{xy}^2} + \frac{m_*}{2B}i\omega\,,$$
$$\sigma_{xy} = -\frac{1}{4}\frac{\tilde{\sigma}_{xy}}{\tilde{\sigma}_{xx}^2 + \tilde{\sigma}_{xy}^2} + \left[\tfrac{1}{2}\right]_{\rm NR}\,.$$

$$\sigma_{xx} = \frac{m_*}{2B\tau}\,,$$
$$\sigma_{xy} = \frac{b}{B} + \frac{1}{2} = \nu.$$

$$\vec{v} = \frac{\vec{E} \times \hat{z}}{B}$$

# Consequences

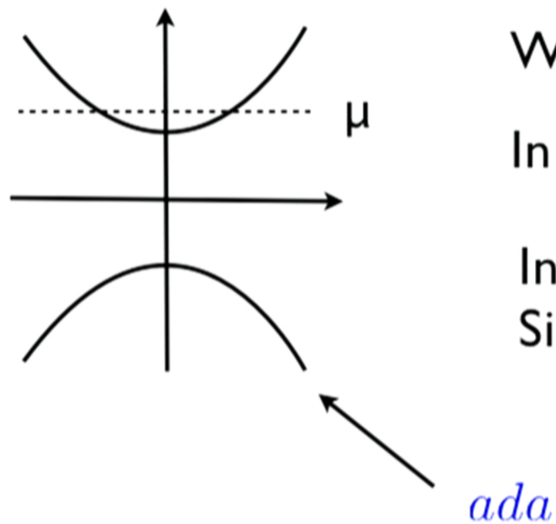
- Exact particle hole symmetry in linear response
  - at $\nu = \frac{1}{2}$ , $\sigma_{xy} = \frac{1}{2}$ exactly (HLR: $\rho_{xy}$=2)
- New particle-hole symmetric gapped nonabelian state at $\nu$=1/2 "PH-Pfaffian"

$$\langle \epsilon^{\alpha\beta} \psi_\alpha \psi_\beta \rangle \neq 0$$

Quantum Hall version of the T-Pfaffian in TI

Pfaffian and anti-Pfaffian states: pairing of Dirac CFs with angular momentum 2 and -2

# HLR theory as the NR limit




When CP is broken, CF has mass

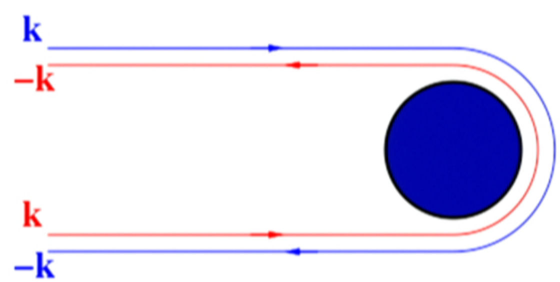
In the NR limit: NR action for CF

Integrating out Dirac sea: Chern-Simons interaction between CF

Old HLR theory is reproduced
Particle-hole symmetry broken by the CF Dirac mass

# Numerical evidence of Dirac CFs

- Suppression of back-scattering on particle-hole symmetric defect




- Observed

Geraedts, Zaletel, Mong, Metlitski, Vishwanath, Motrunich

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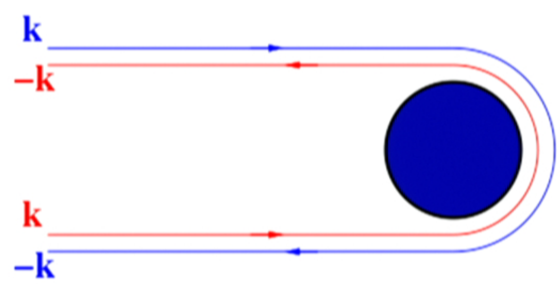


- Observed

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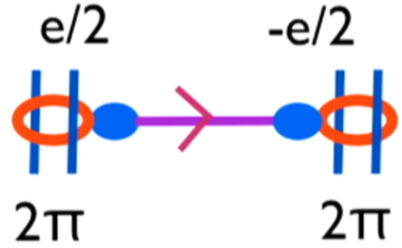
# Wang-Senthil picture of the CF




Wang, Sentil 1507.08290

# Conclusion and open questions

- PH symmetry: a challenge for old CF picture
- Proposal: Dirac CF with gauge, non-CS interaction
- particle-vortex duality instead of flux attachment
- Open questions:
  - a better derivation of the effective theory (but see Mross, Alicea, Motrunich 1010....)
  - experimental measurement of the Berry phase?
  - other experimental signatures?