Title: Localization from superselection rules in translation invariant systems

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Abstract:

Localization from superselection rules in translation-invariant systems

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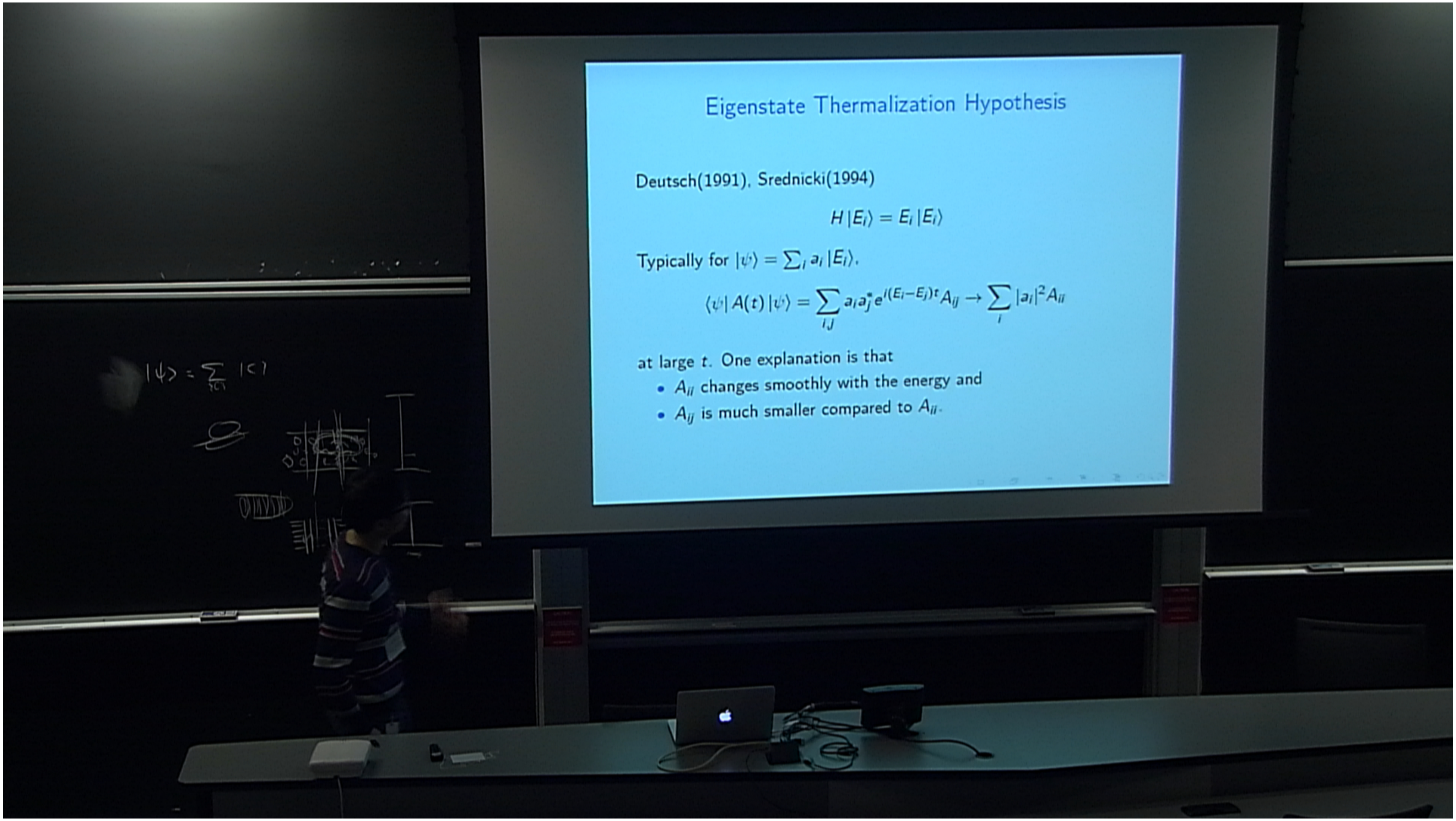
Joint work with Jeongwan Haah(MIT)


Eigenstate Thermalization Hypothesis
Deutsch(1991), Srednicki(1994)
$H|E_i\rangle = E_i|E_i\rangle$
Typically for $|\psi\rangle = \sum_i a_i|E_i\rangle$,
$\langle\psi|A(t)|\psi\rangle = \sum_{i,j} a_i a_j^* e^{i(E_i-E_j)t} A_{ij} \to \sum_i |a_i|^2 A_{ii}$
at large t. One explanation is that
• A_{ii} changes smoothly with the energy and
• A_{ij} is much smaller compared to A_{ii}.

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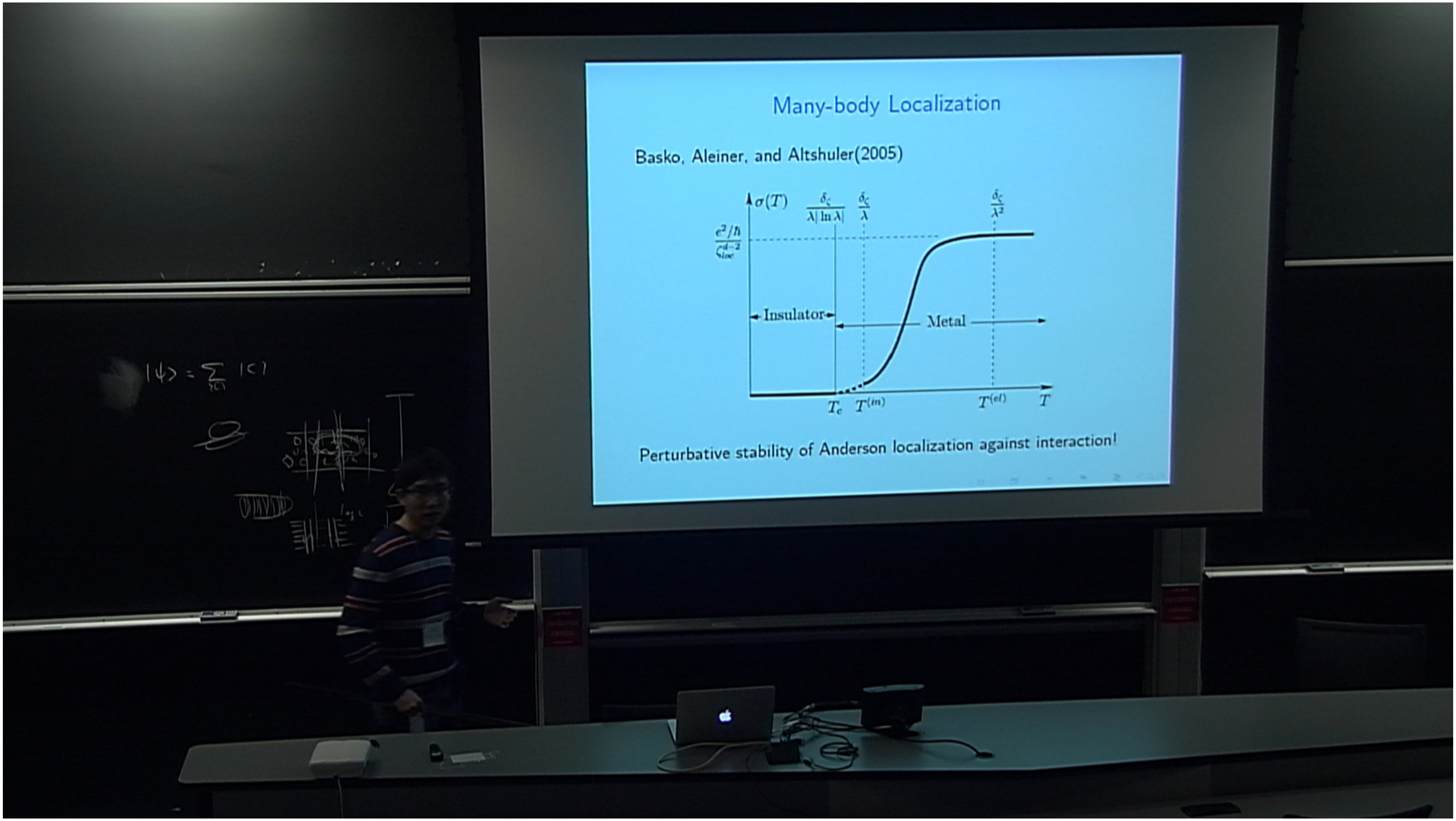

Many-body Localization
Basko, Aleiner, and Altshuler(2005)
$\sigma(T)$
$\frac{\delta_\zeta}{\lambda|\ln\lambda|}$
$\frac{\delta_\zeta}{\lambda}$
$\frac{\delta_\zeta}{\lambda^2}$
$\frac{e^2/\hbar}{\zeta_{loc}^{d-2}}$
Insulator
Metal
T_c
$T^{(in)}$
$T^{(el)}$
T
Perturbative stability of Anderson localization against interaction!
$|\psi\rangle = \sum_{\{C\}} |C\rangle$

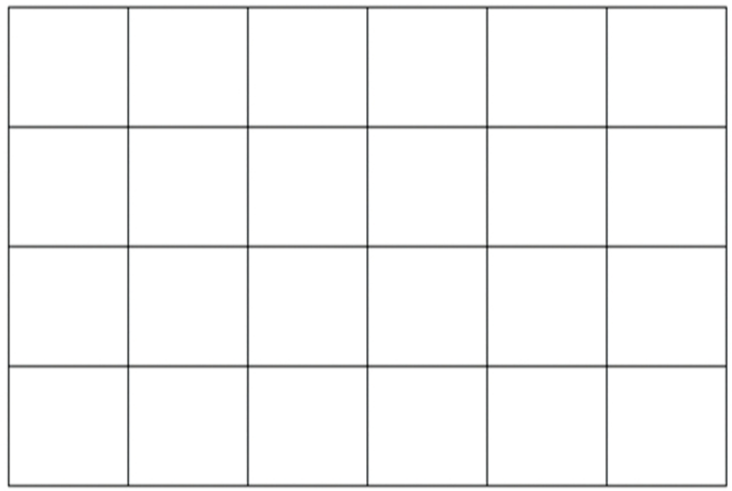
Localization from superselection rules

We propose a different mechanism for localization, which is based on an **emergent superselection rule.**

- A specific model is studied under an arbitrary but weak, local perturbation.
- Almost all states with $O(1)$ energy are localized.
- Translation invariant or not, perturbative stability guaranteed against **any** locally interacting perturbation.
- Very different from Mott insulator.
- Unfortunately, the case for the finite energy density remains open.

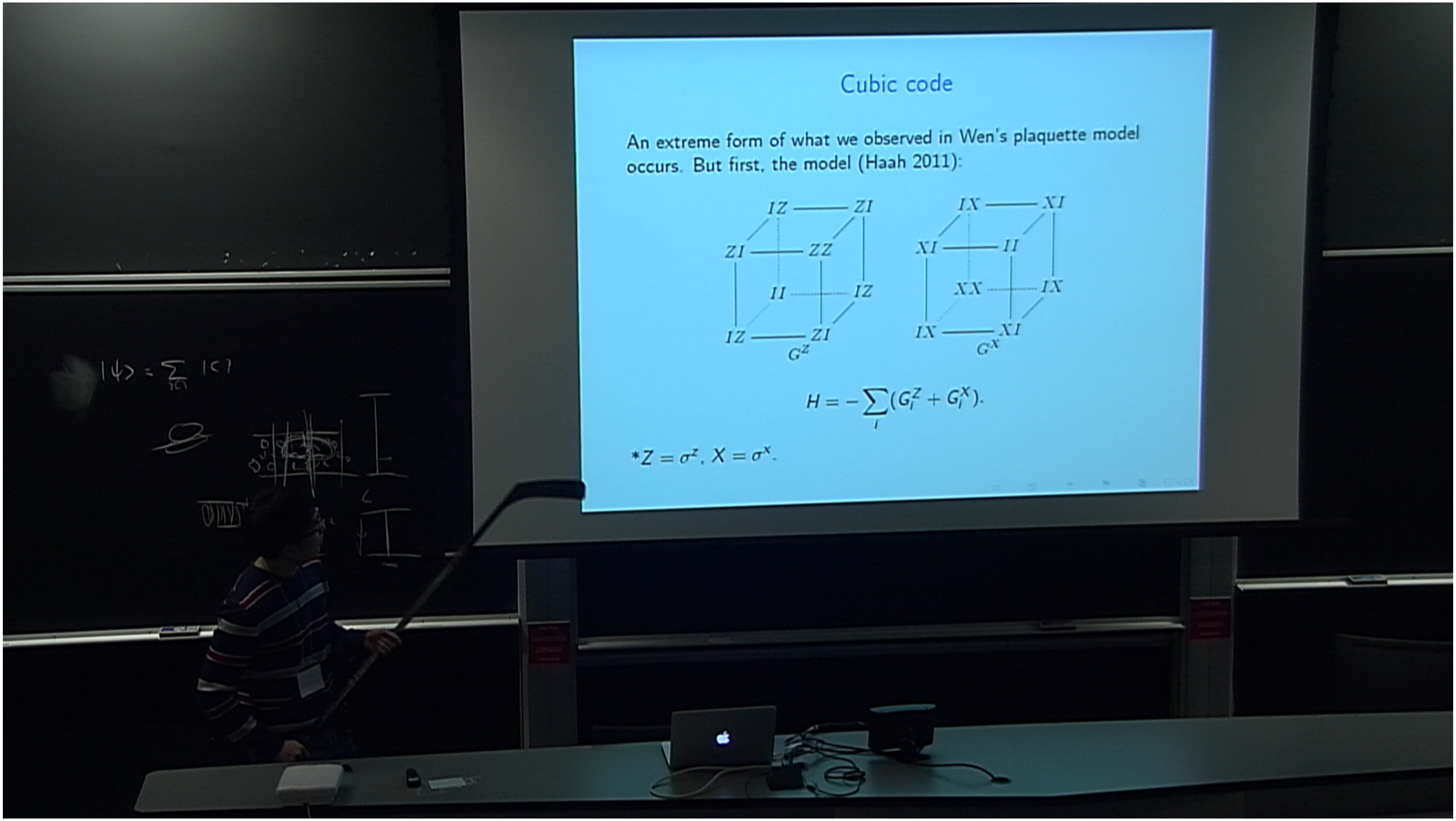
Position-dependent superselection rule : Wen's plaquette model

Wen (2003)



$$H = -\sum_{(i,j)} \sigma^{x}_{(i,j)} \sigma^{z}_{(i+1,j)} \sigma^{x}_{(i+1,j+1)} \sigma^{z}_{(i,j+1)}$$

This is formally equivalent to the Toric code.[Kitaev (1996)]


Cubic code
An extreme form of what we observed in Wen's plaquette model occurs. But first, the model (Haah 2011):
IZ ZI ZI ZZ II IZ IZ ZI
G^Z
IX XI XI II XX IX IX XI
G^X
$H = -\sum_i (G_i^Z + G_i^X).$
$*Z = \sigma^z, X = \sigma^x.$

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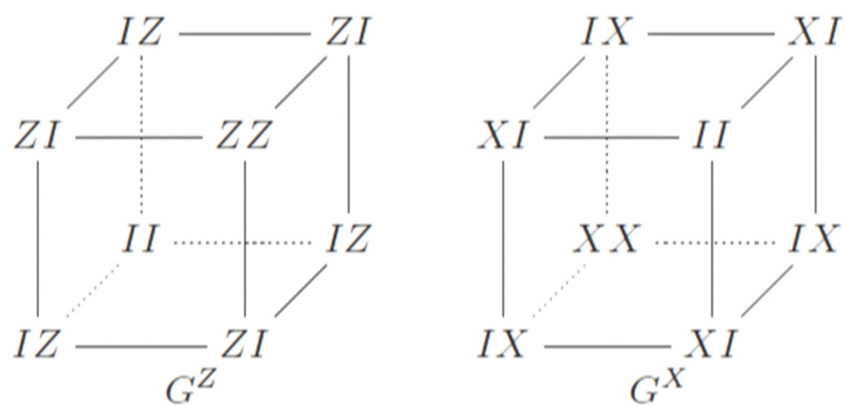

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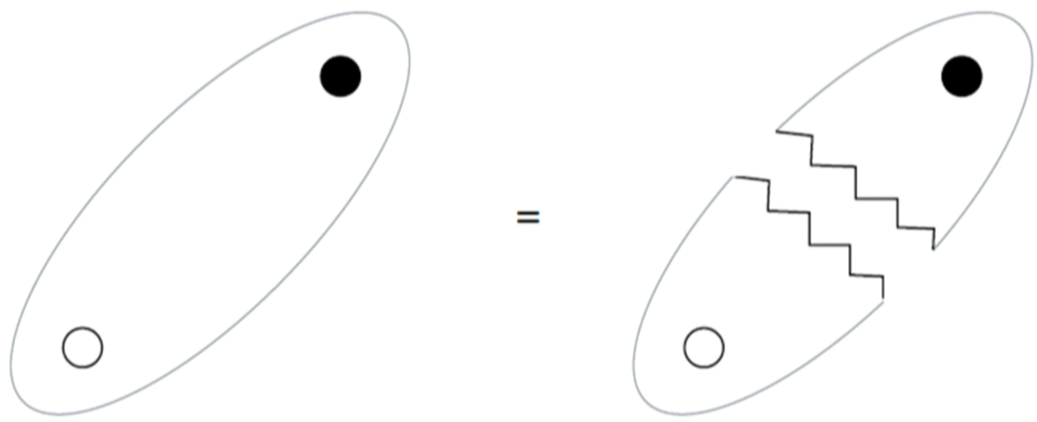
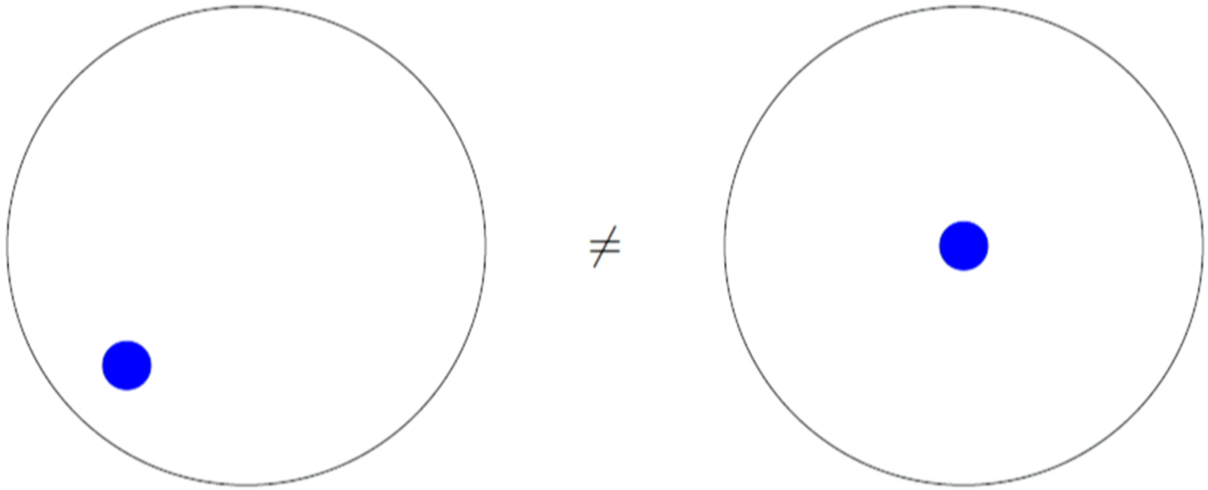
No-strings rule

Haah (2011)

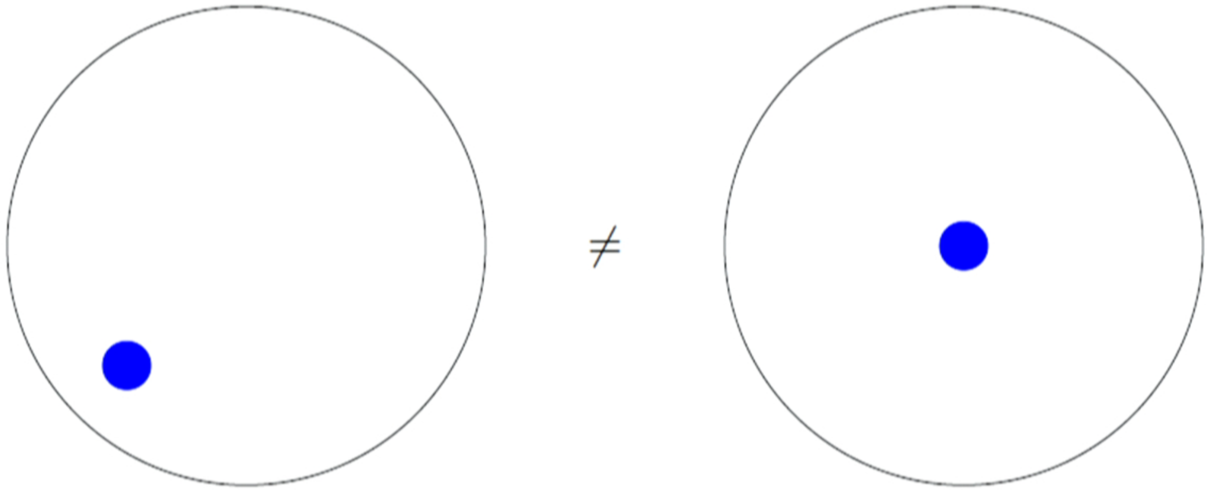


Superselection rules in cubic code



- An isolated defect cannot be moved by applying an operator inside the ball.
- There are infinitely many topological charges.

Superselection rules in cubic code



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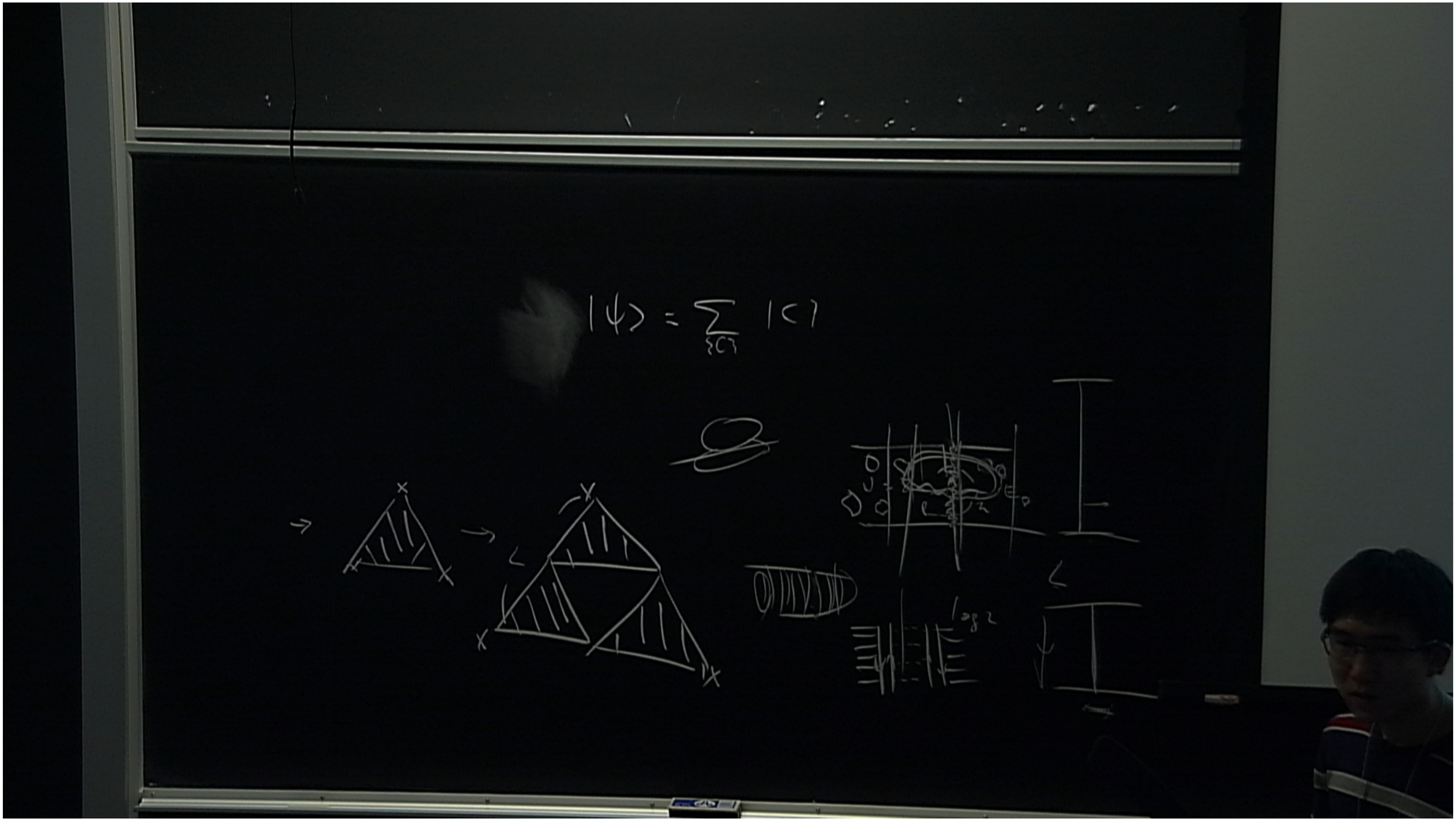
|ψ> = Σ_{C} |C>
log 2

Perturbation theory

Let's start with an excited state of the original Hamiltonian, and perturb it by $V = \sum_n v_n$. How does the state change?

$$\delta |\psi_i\rangle = \sum_{j \neq i} \frac{\langle \psi_i| V |\psi_j\rangle}{E_i - E_j} |\psi_j\rangle .$$

At low order of the perturbation theory, $|E_i - E_j| > O(1)$ because you end up creating more and more defects. $|E_i - E_j| \approx 0$ only at high order, at which point the contribution becomes negligible.

Dynamical properties

There are states $|\psi\rangle$ of a perturbed system which have sparse defect configuration such that

$$|\langle\psi|\, e^{i(H+V)t}\, |\psi\rangle\,| \geq 1 - tL^{\alpha}e^{-cL^{\eta}}.$$

* System size is L^3.
* Defects are effectively frozen forever.

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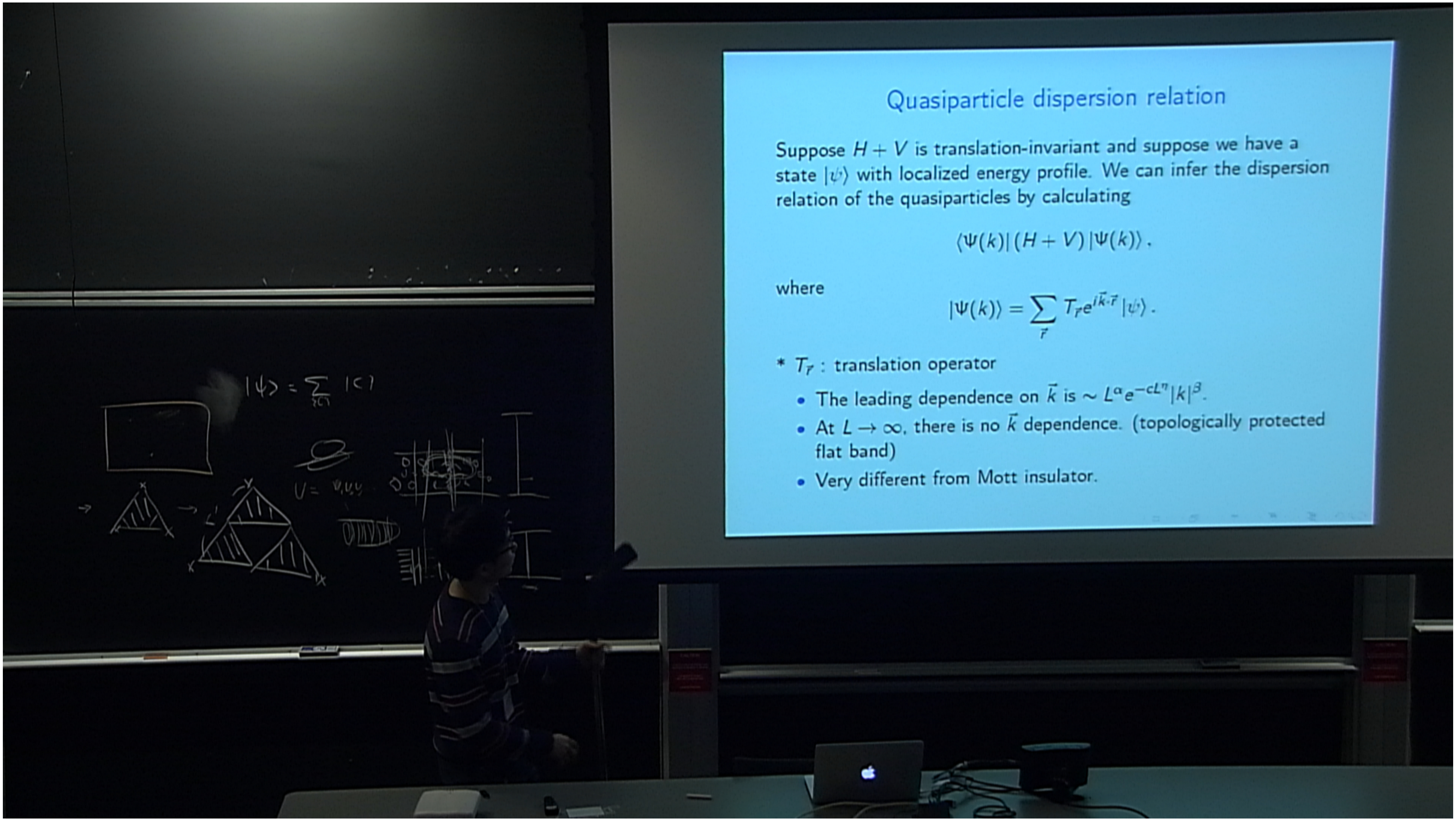
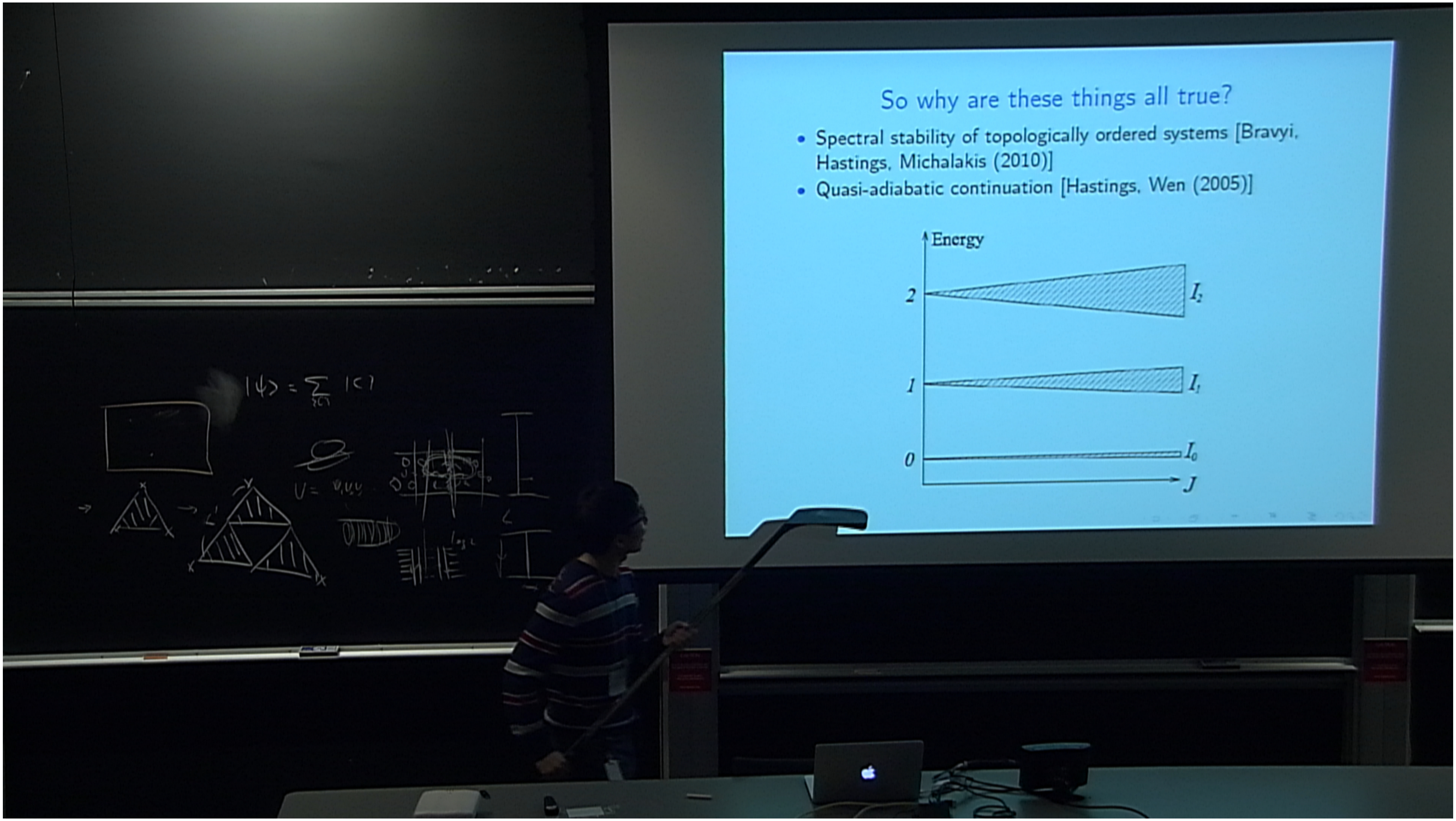
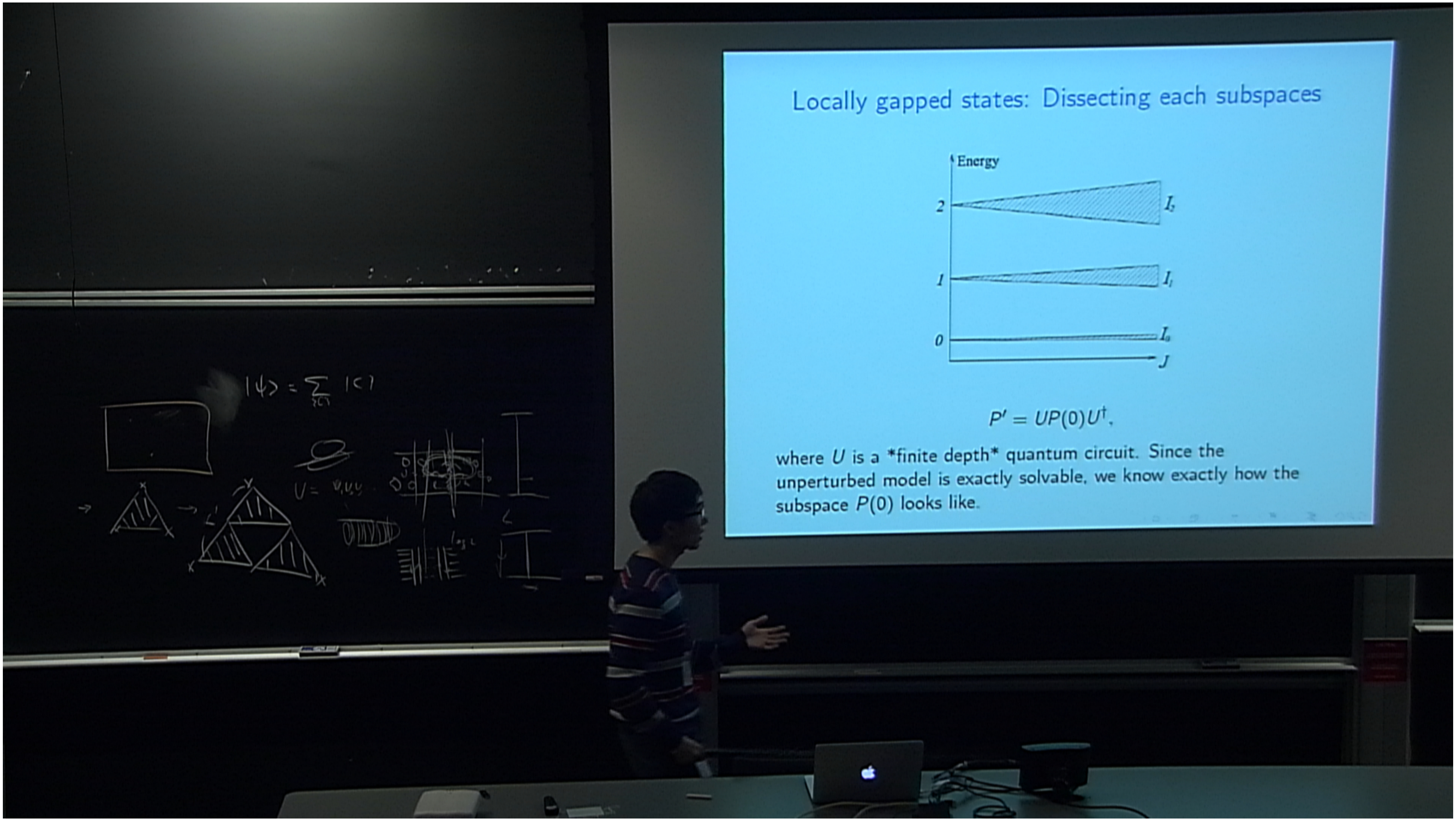
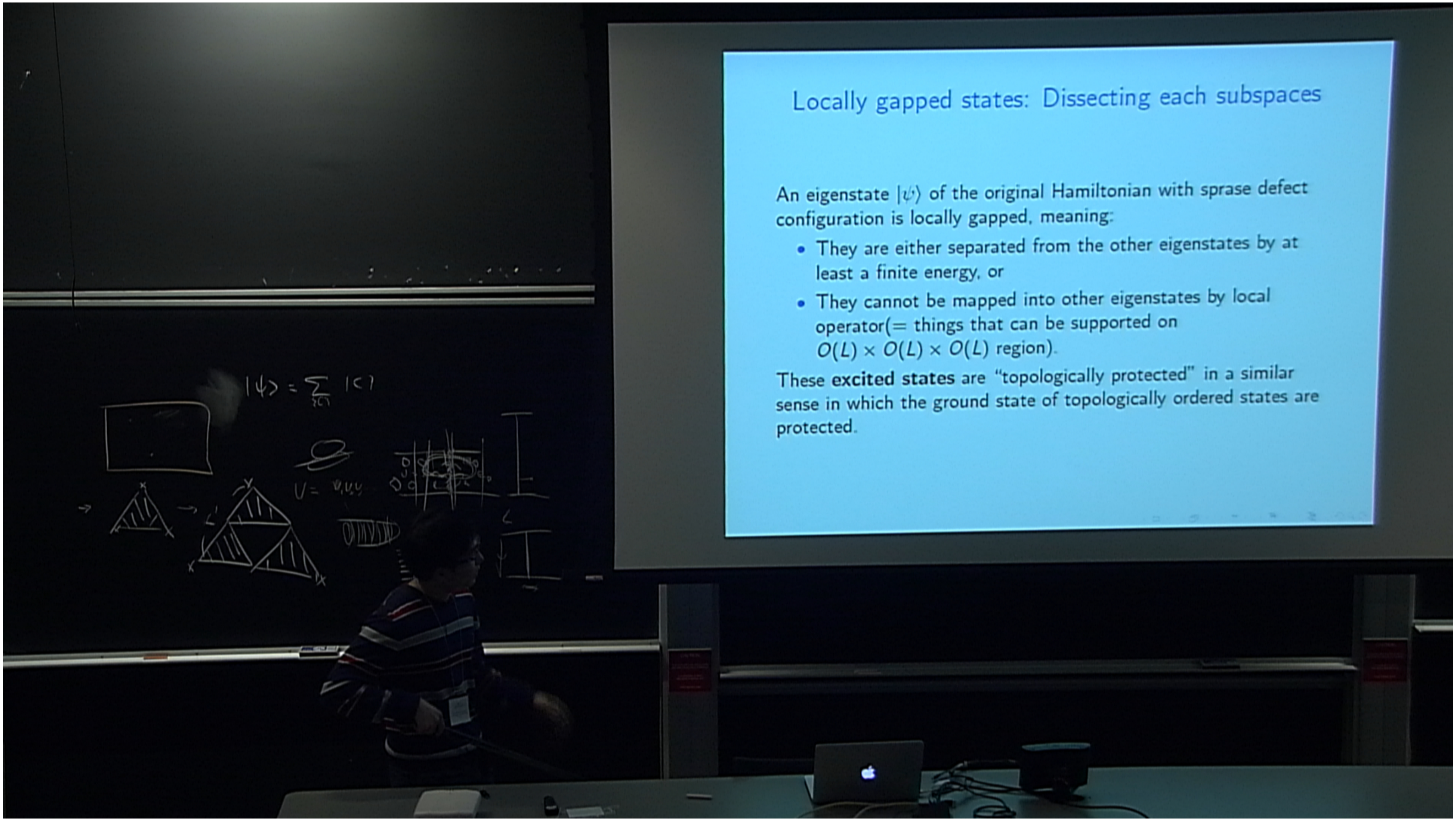
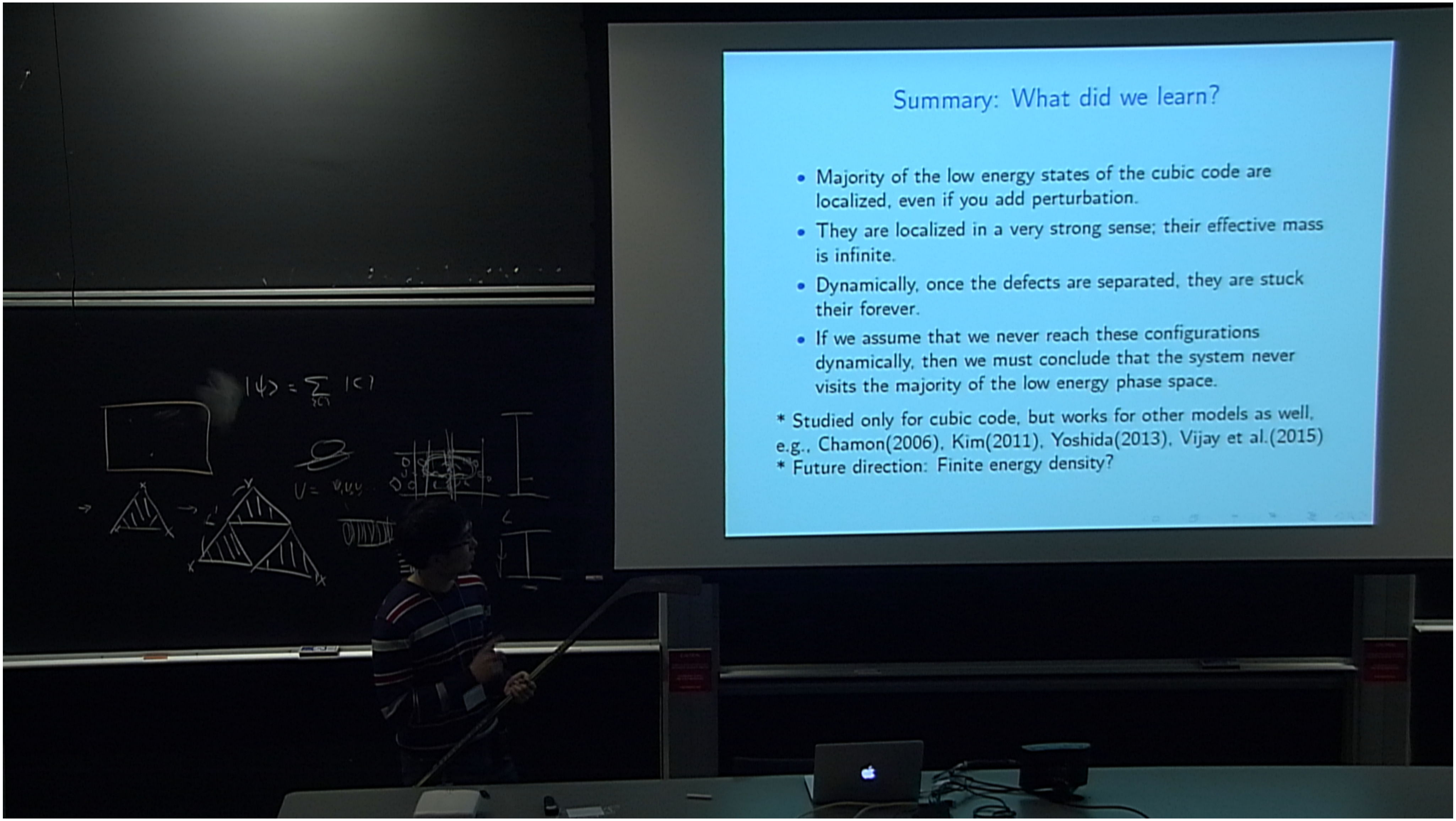

Quasiparticle dispersion relation
Suppose $H + V$ is translation-invariant and suppose we have a state $|\psi\rangle$ with localized energy profile. We can infer the dispersion relation of the quasiparticles by calculating
$\langle \Psi(k) | (H + V) | \Psi(k) \rangle .$
where
$|\Psi(k)\rangle = \sum_{\vec{r}} T_{\vec{r}} e^{i\vec{k}\cdot\vec{r}} |\psi\rangle .$
* $T_{\vec{r}}$: translation operator
The leading dependence on $\vec{k}$ is $\sim L^{\alpha} e^{-cL^{\eta}} |k|^{\beta}$.
At $L \to \infty$, there is no $\vec{k}$ dependence. (topologically protected flat band)
Very different from Mott insulator.


So why are these things all true?
Spectral stability of topologically ordered systems [Bravyi, Hastings, Michalakis (2010)]
Quasi-adiabatic continuation [Hastings, Wen (2005)]
Energy
2
1
0
I_2
I_1
I_0
J


Locally gapped states: Dissecting each subspaces
Energy
2
1
0
I_2
I_1
I_0
J
$P' = UP(0)U^\dagger$,
where U is a *finite depth* quantum circuit. Since the unperturbed model is exactly solvable, we know exactly how the subspace $P(0)$ looks like.


Locally gapped states: Dissecting each subspaces
An eigenstate $|\psi\rangle$ of the original Hamiltonian with sprase defect configuration is locally gapped, meaning:
• They are either separated from the other eigenstates by at least a finite energy, or
• They cannot be mapped into other eigenstates by local operator(= things that can be supported on $O(L) \times O(L) \times O(L)$ region).
These **excited states** are "topologically protected" in a similar sense in which the ground state of topologically ordered states are protected.


Summary: What did we learn?
Majority of the low energy states of the cubic code are localized, even if you add perturbation.
They are localized in a very strong sense; their effective mass is infinite.
Dynamically, once the defects are separated, they are stuck their forever.
If we assume that we never reach these configurations dynamically, then we must conclude that the system never visits the majority of the low energy phase space.
* Studied only for cubic code, but works for other models as well, e.g., Chamon(2006), Kim(2011), Yoshida(2013), Vijay et al.(2015)
* Future direction: Finite energy density?