Title: Abelian Topological Phases: Symmetries, Defects, and Entanglement

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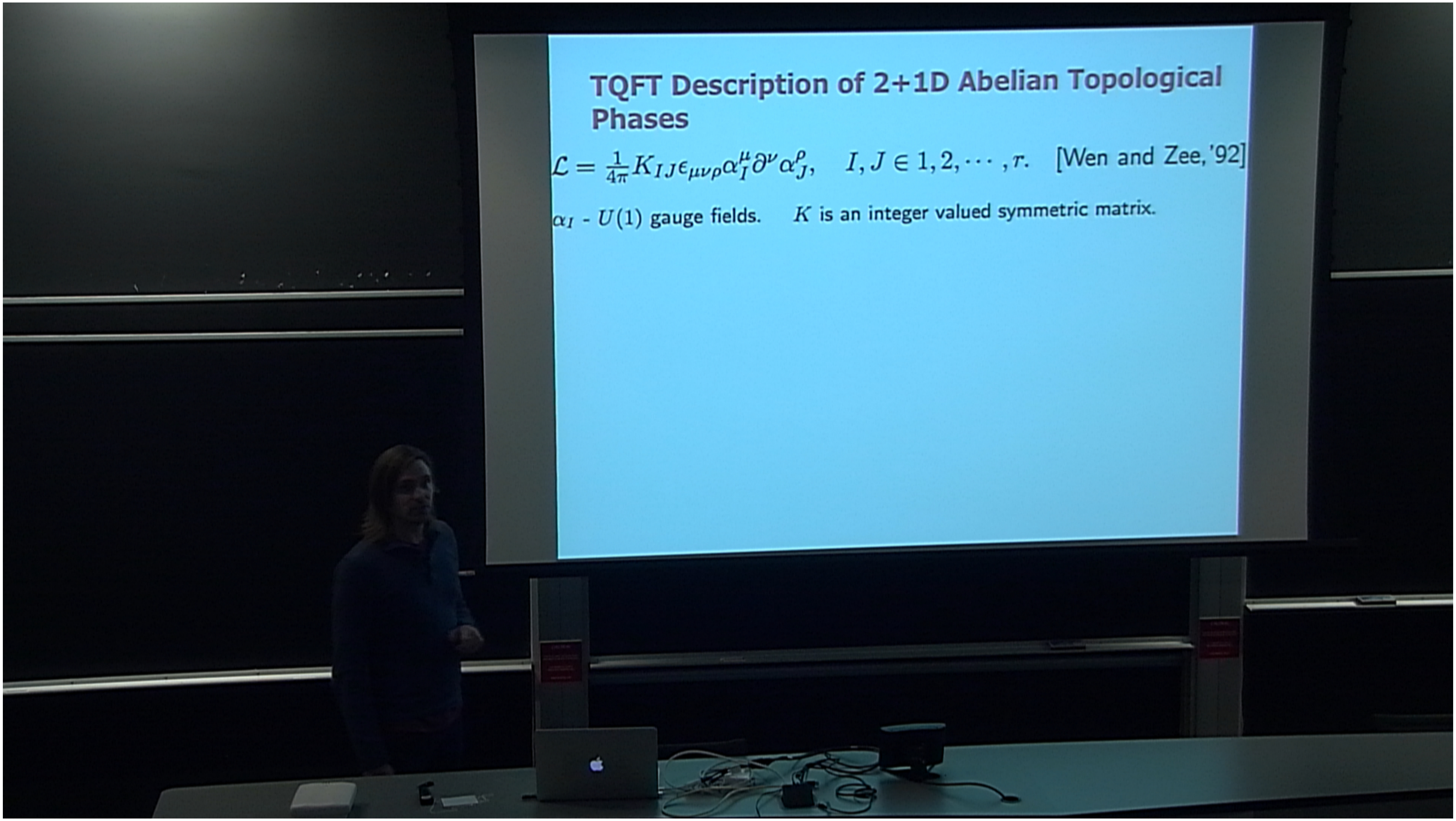
Abstract:


TQFT Description of 2+1D Abelian Topological Phases
$\mathcal{L} = \frac{1}{4\pi} K_{IJ} \epsilon_{\mu\nu\rho} \alpha_I^{\mu} \partial^{\nu} \alpha_J^{\rho}, \quad I, J \in 1, 2, \cdots, r.$ [Wen and Zee, '92]
α_I - $U(1)$ gauge fields. K is an integer valued symmetric matrix.

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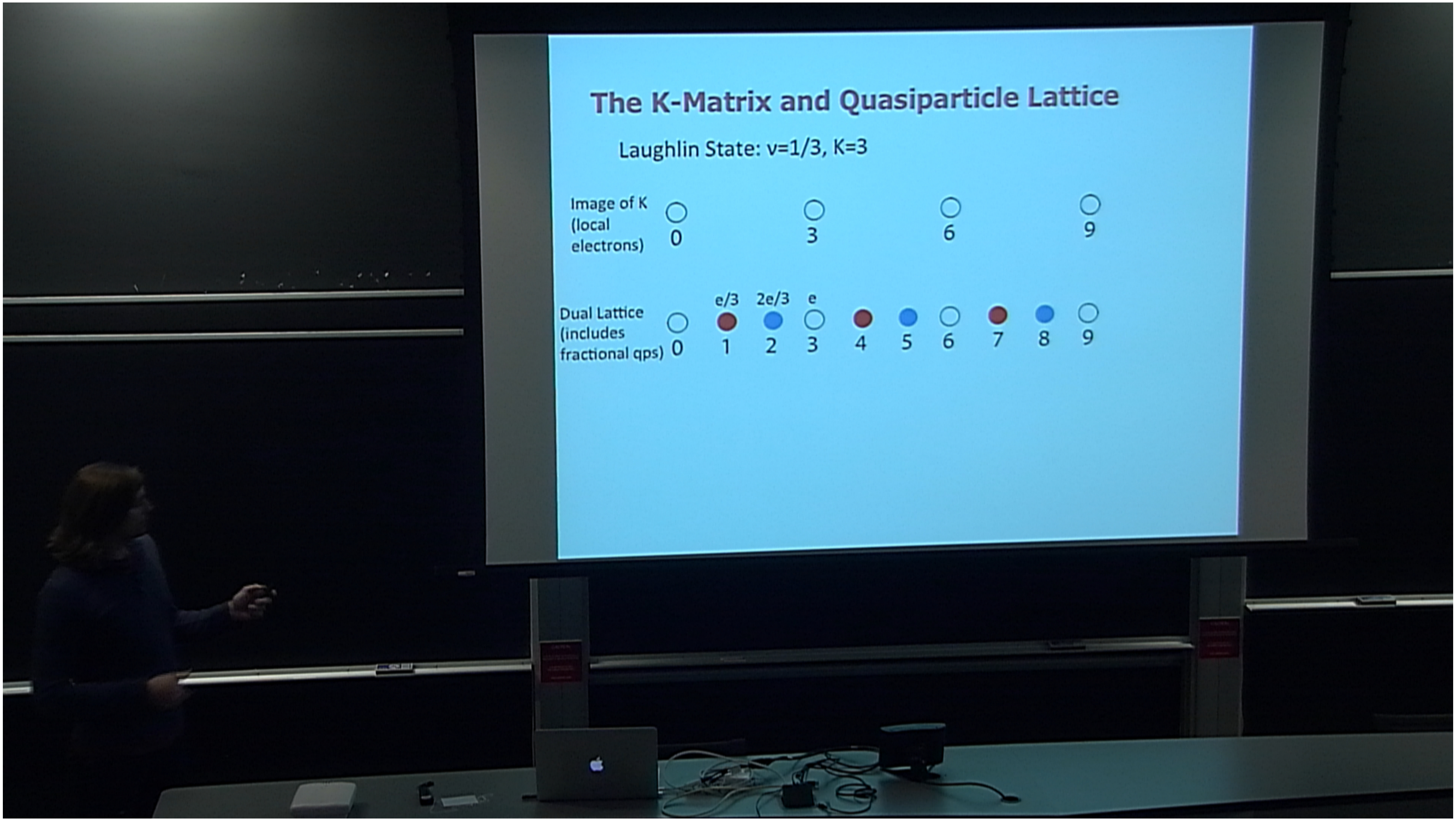
Bulk Topological Data Is Contained in the K-matrix

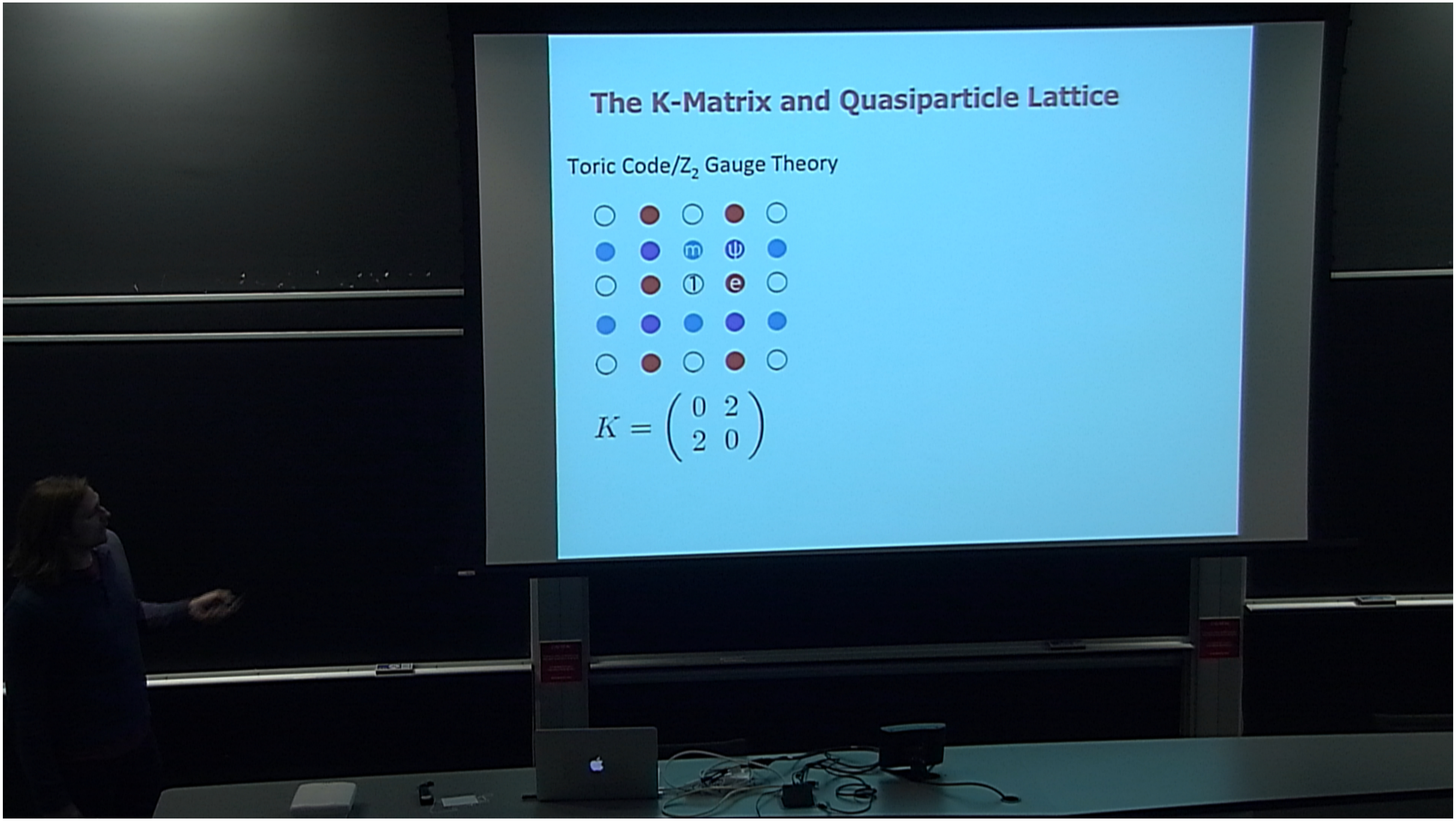
GSD on Torus = Det(K)

Chiral Central Charge = Signature of K

The K-Matrix and Quasiparticle Lattice

Laughlin State: ν=1/3, K=3


The K-Matrix and Quasiparticle Lattice
Laughlin State: ν=1/3, K=3
Image of K (local electrons)
0
3
6
9
Dual Lattice (includes fractional qps)
e/3
2e/3
e
0
1
2
3
4
5
6
7
8
9


The K-Matrix and Quasiparticle Lattice
Toric Code/Z2 Gauge Theory
m
ψ
1
e
K = (0 2 ; 2 0)

Anyonic Symmetries

Laughlin State: ν=1/3, K=3.

Point Group Type:
$WKW^T=K$

W is a unimodular matrix with integer entries. Acts as point group operations on lattice that take **local** particles to **local** particles. Preserves S and T.

Anyonic Symmetries

$WKW^T = K$

- Set of all W's satisfying this are Aut(K).
- Inner(K) are all automorphisms that take a qp to the same qp modulo local particles
- (Point-group type) Anyonic Symmetries are given by Outer(K)=Aut(K)/Inner(K)

Anyonic Symmetries of Bosonic ADE Quantum Hall States

Given a simply-laced Lie Algebra A_n, D_n, E_n its Cartan matrix is symmetric and integer valued. If we use the Cartan matrix as the K-matrix for a Chern Simons theory we get an Abelian topological phase with local bosons whose edge theory is the corresponding ADE Wess-Zumino-Witten CFT at level 1.

$A_n = su(n)_1$ have n anyon sectors with statistics like the $\nu=1/n$ Laughlin state but bosonic versions

$D_n = so(2n)_1$ have 4 anyon sectors. When n is even the 4 quasiparticles are like $1,e,m,\psi$ of the toric code. When n is odd the are like $1,s,s^*, ss^*$ of the double semion theory. Statistics depend on n.

Khan, Teo, Hughes, PRB (2014)

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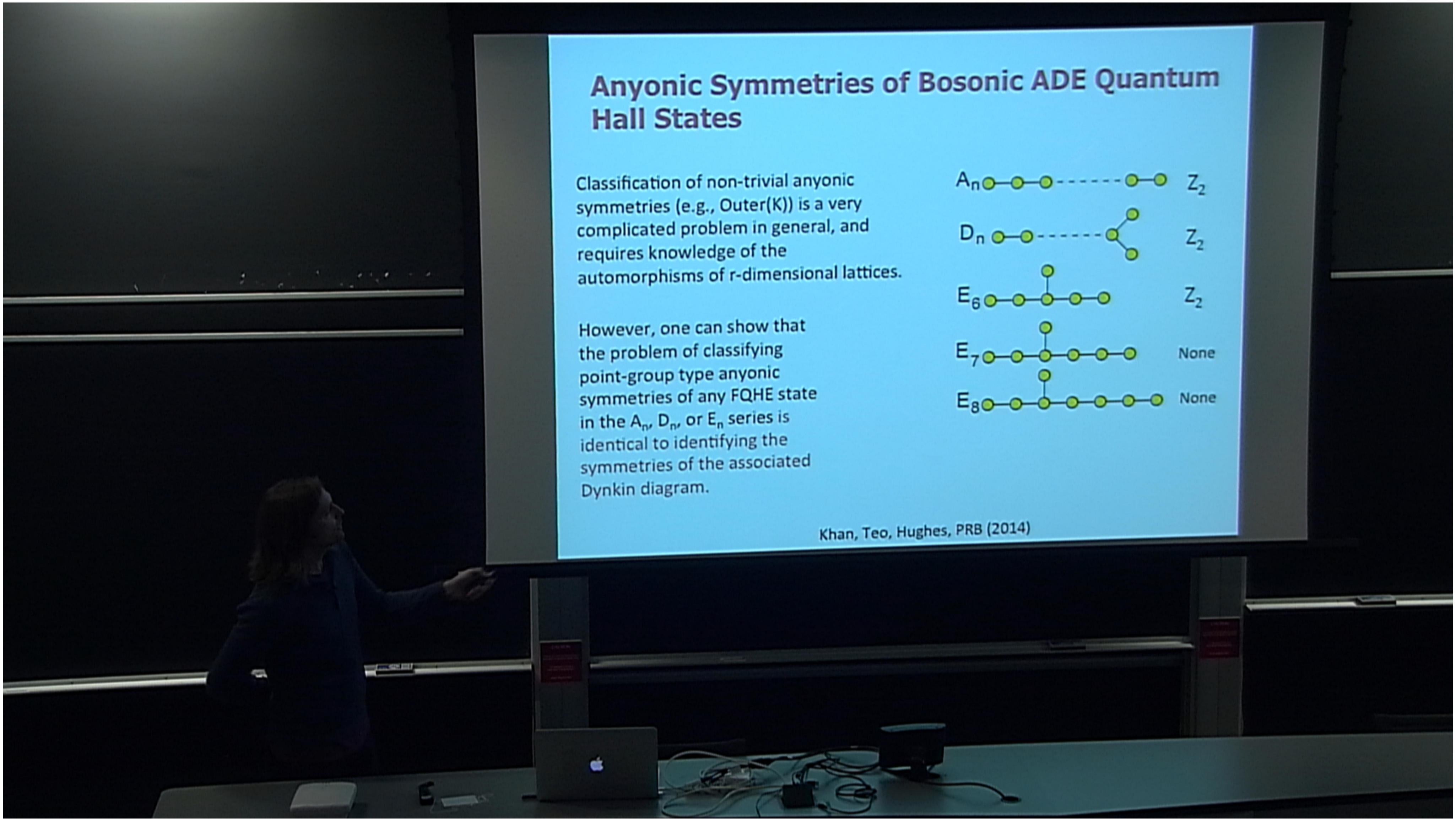
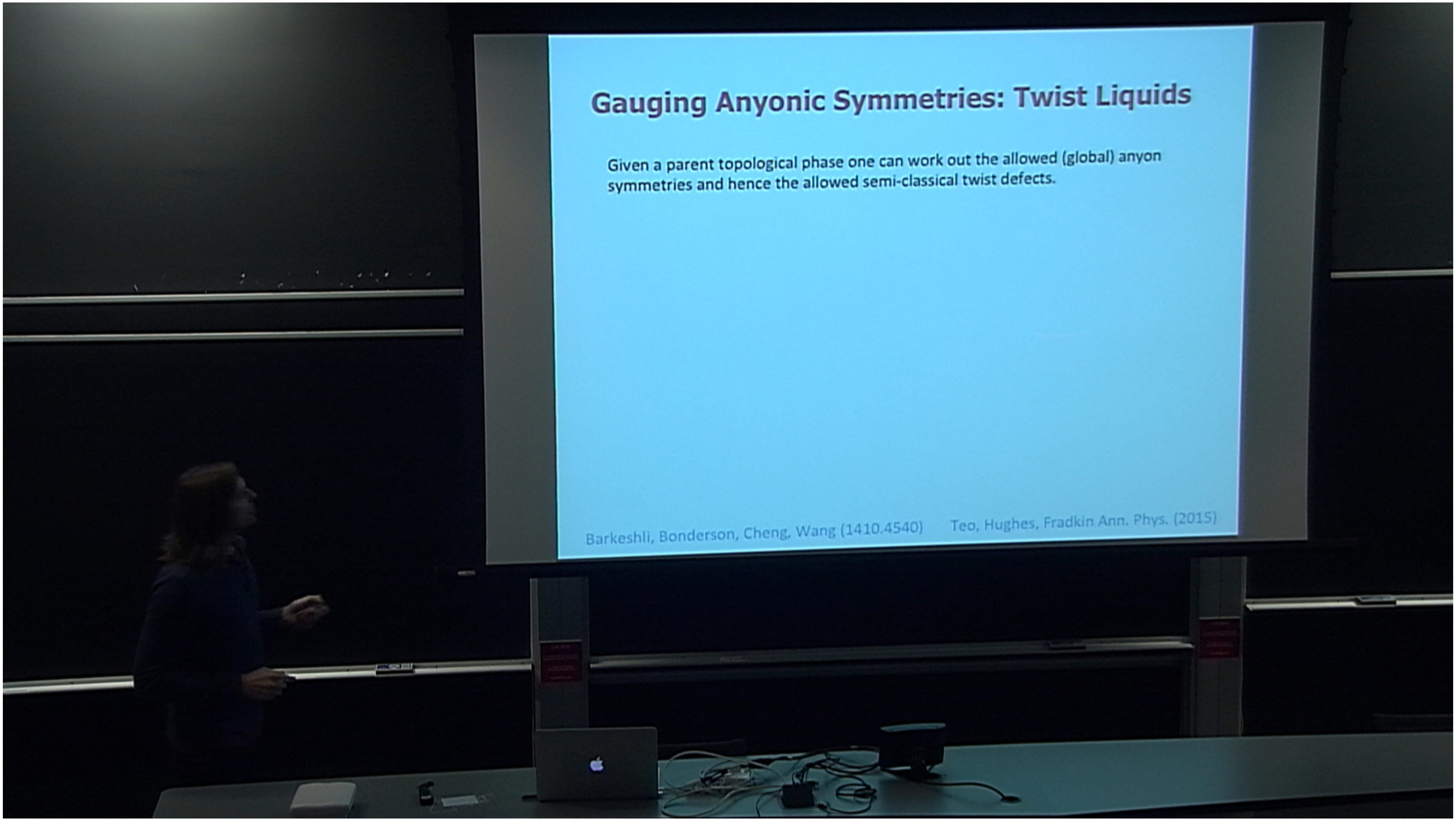
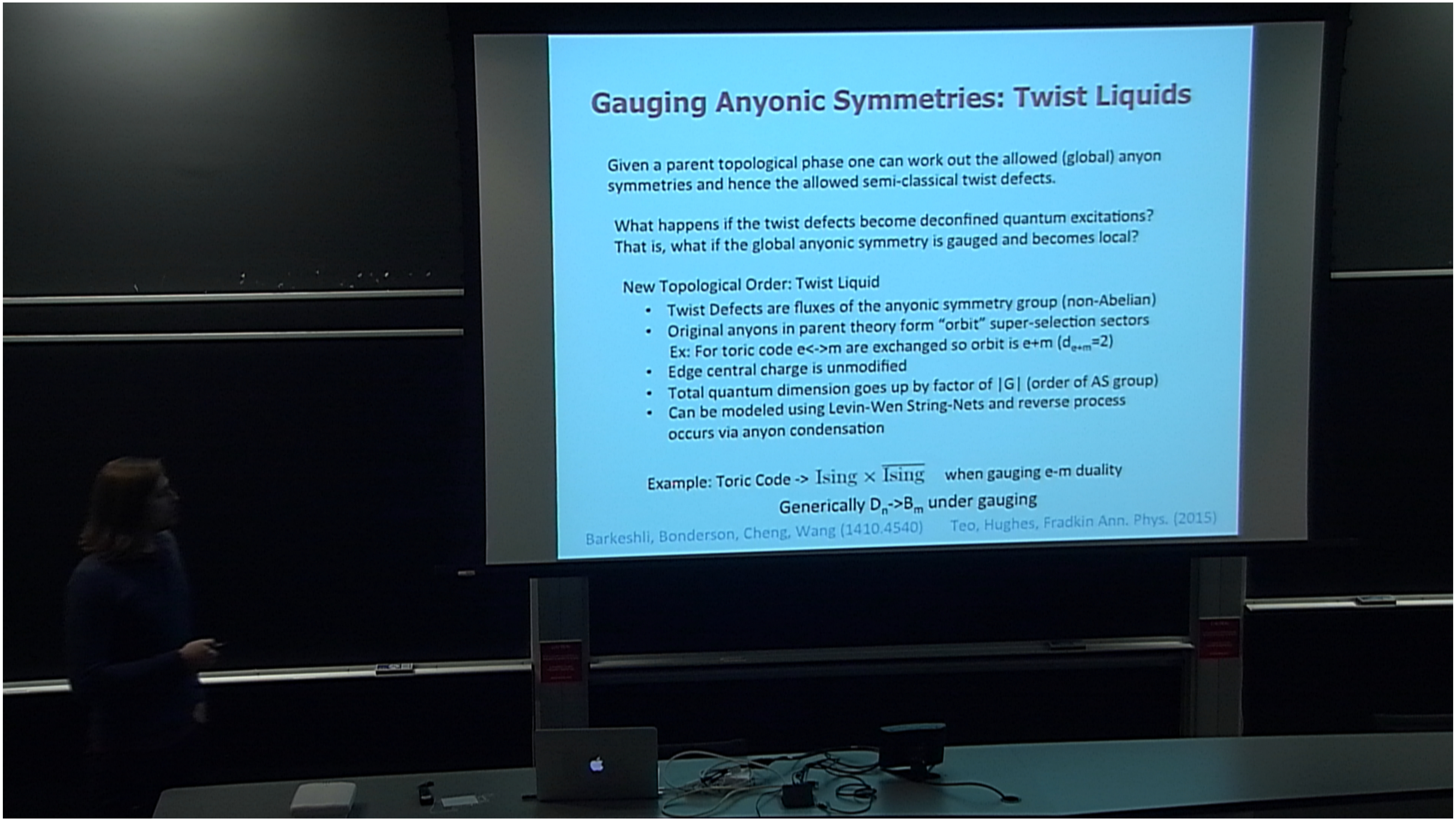
$E_6 \sim su(2)_1$ has 2 anyon sectors
$E_7 \sim su(3)_1$ has 3 anyon sectors
E_8 is a bosonic integer quantum Hall state with no anyons

Non-simply laced have integer, but not symmetric Cartan matrices. These theories are intrinsically non-Abelian even at level 1.

Khan, Teo, Hughes, PRB (2014)


Anyonic Symmetries of Bosonic ADE Quantum Hall States
Classification of non-trivial anyonic symmetries (e.g., Outer(K)) is a very complicated problem in general, and requires knowledge of the automorphisms of r-dimensional lattices.
However, one can show that the problem of classifying point-group type anyonic symmetries of any FQHE state in the A_n, D_n, or E_n series is identical to identifying the symmetries of the associated Dynkin diagram.
A_n Z_2
D_n Z_2
E_6 Z_2
E_7 None
E_8 None
Khan, Teo, Hughes, PRB (2014)


Gauging Anyonic Symmetries: Twist Liquids
Given a parent topological phase one can work out the allowed (global) anyon symmetries and hence the allowed semi-classical twist defects.
Barkeshli, Bonderson, Cheng, Wang (1410.4540)
Teo, Hughes, Fradkin Ann. Phys. (2015)


Gauging Anyonic Symmetries: Twist Liquids
Given a parent topological phase one can work out the allowed (global) anyon symmetries and hence the allowed semi-classical twist defects.
What happens if the twist defects become deconfined quantum excitations? That is, what if the global anyonic symmetry is gauged and becomes local?
New Topological Order: Twist Liquid
• Twist Defects are fluxes of the anyonic symmetry group (non-Abelian)
• Original anyons in parent theory form "orbit" super-selection sectors
Ex: For toric code e<->m are exchanged so orbit is e+m (d_{e+m}=2)
• Edge central charge is unmodified
• Total quantum dimension goes up by factor of |G| (order of AS group)
• Can be modeled using Levin-Wen String-Nets and reverse process occurs via anyon condensation
Example: Toric Code -> $\mathrm{Ising} \times \overline{\mathrm{Ising}}$ when gauging e-m duality
Generically D_n->B_m under gauging
Barkeshli, Bonderson, Cheng, Wang (1410.4540)
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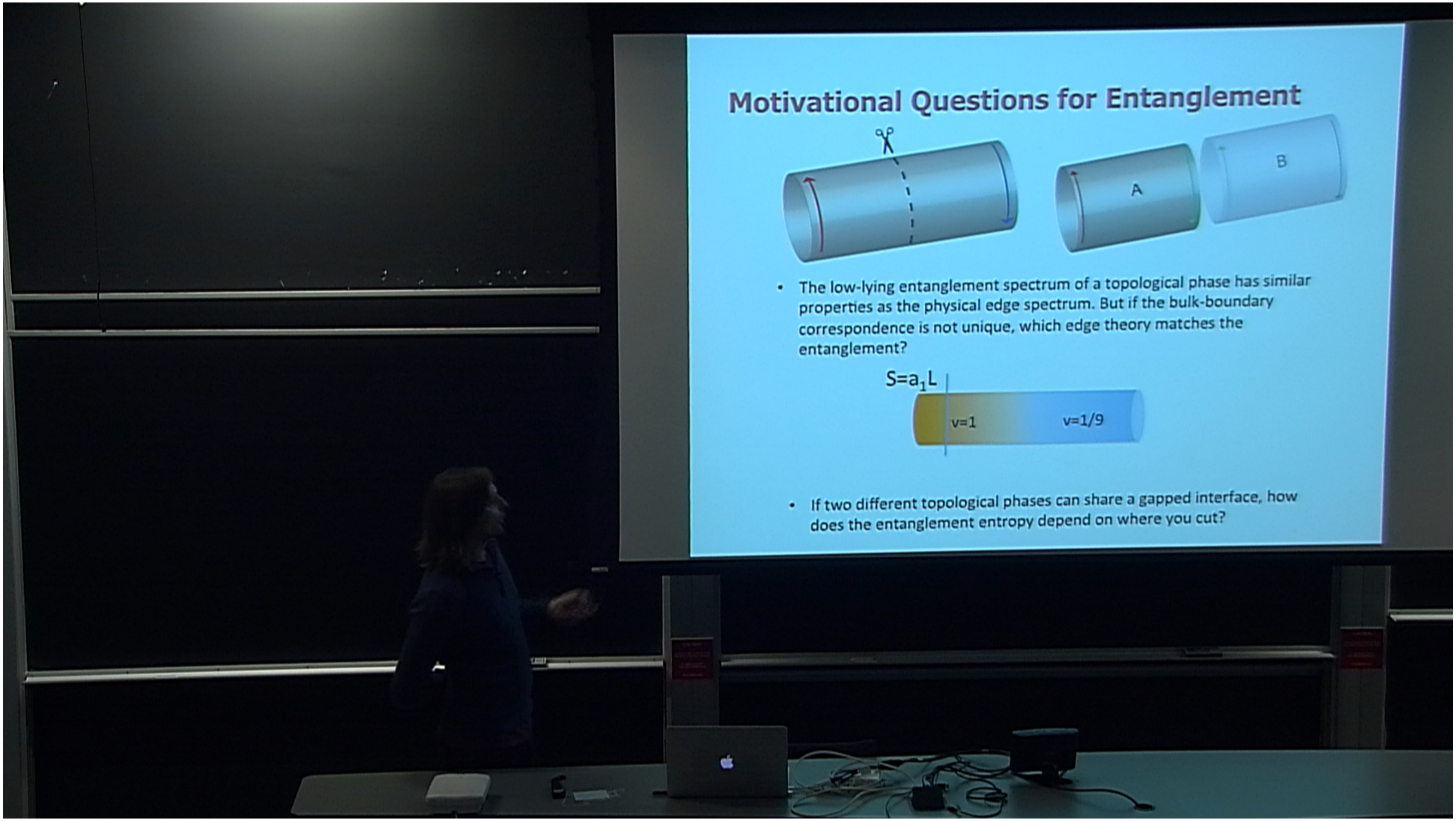
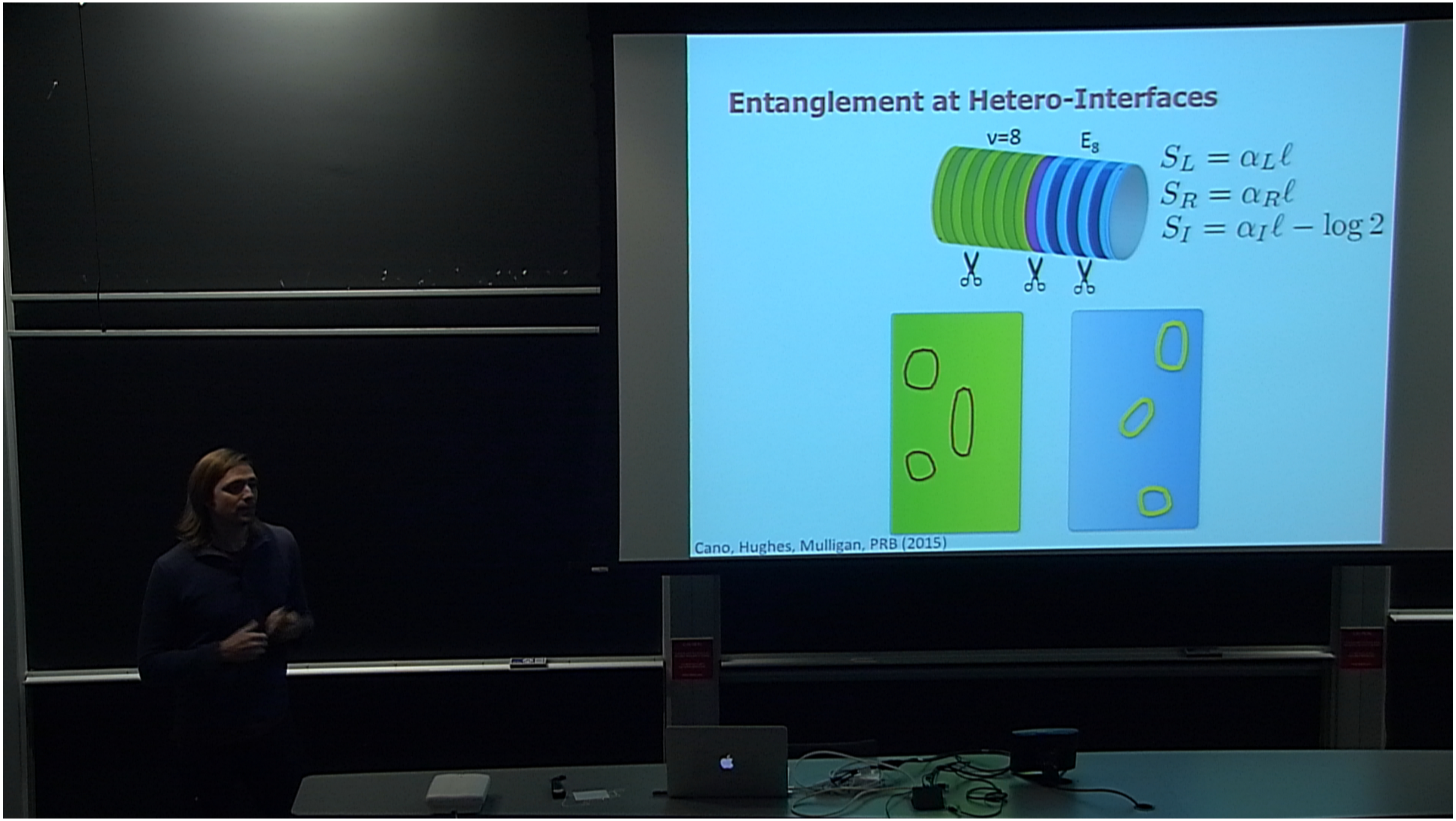
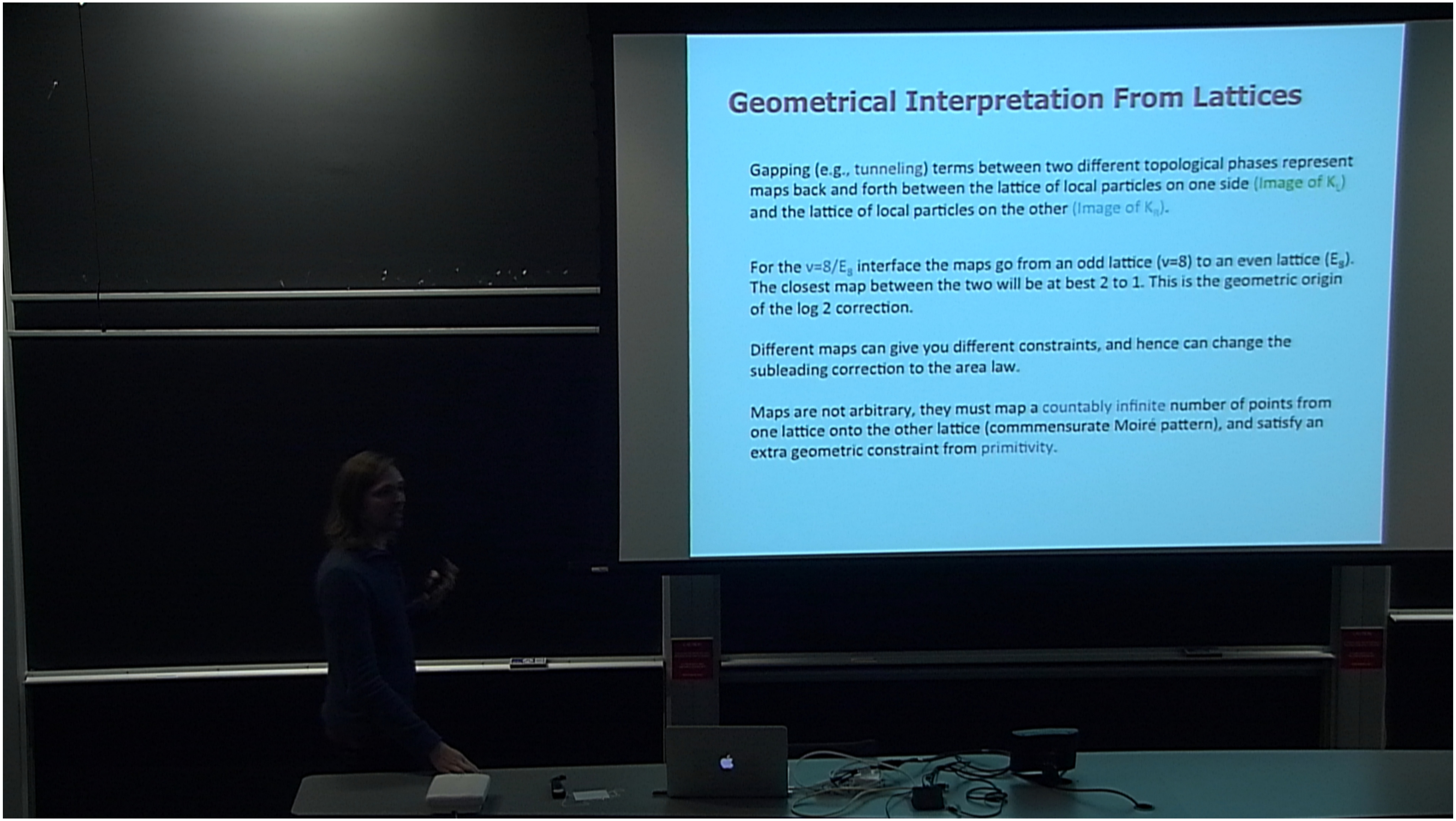
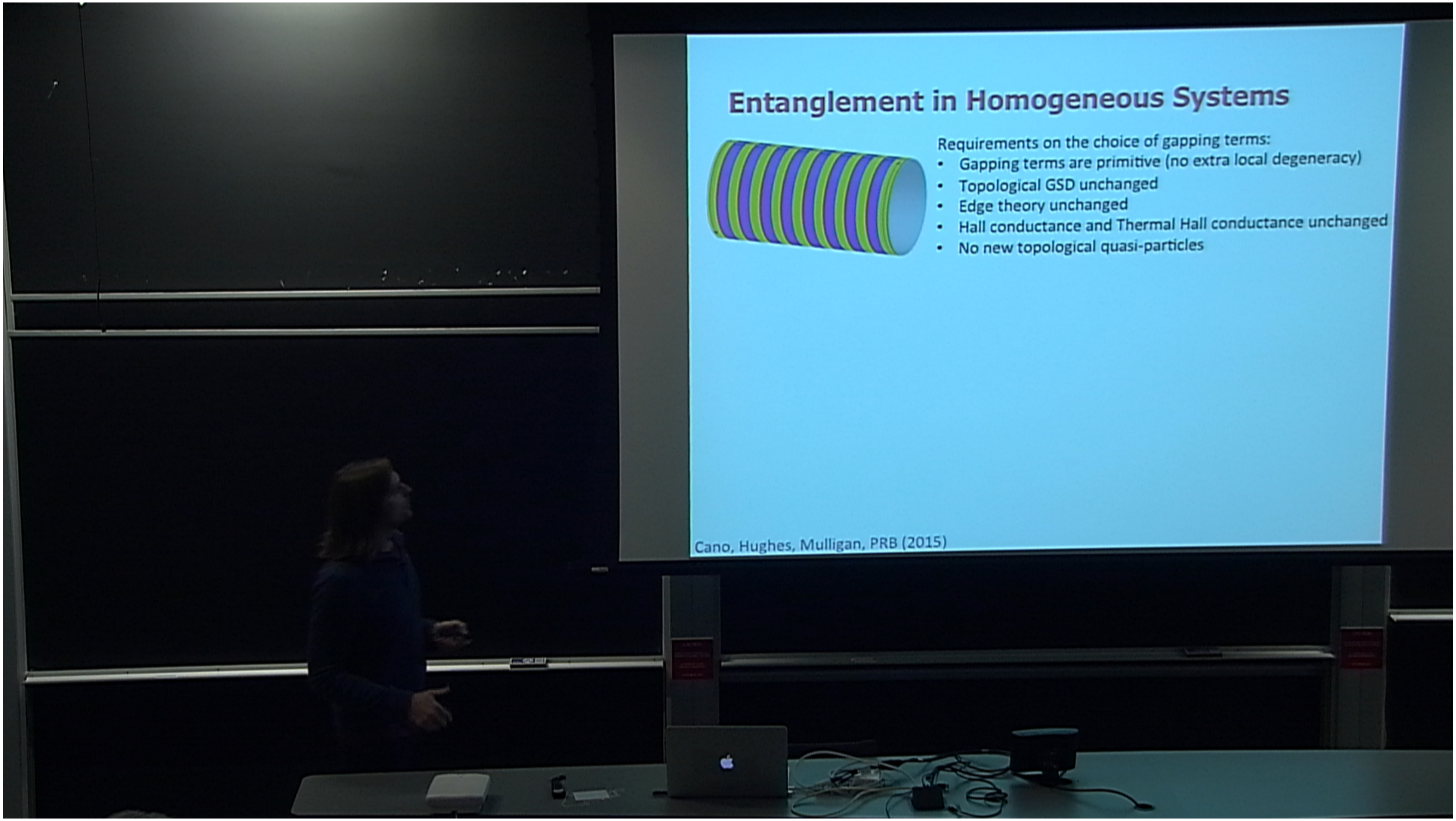
New Developments in Abelian Topological Phases
1. Non-uniqueness of bulk-boundary correspondence [Cano, Cheng, Mulligan et. al., PRB]


New Developments in Abelian Topological Phases
1. Non-uniqueness of bulk-boundary correspondence [Cano, Cheng, Mulligan et. al., PRB]
Bulk properties determined by coarse information of the lattice: genus
Boundary properties determined by a lattice within a genus
Bulk -> Many Boundary Theories [Stable Equivalence]
2. Gapable interfaces between very different topological phases
[Levin PRX 2013, Plamadeala,Mulligan, Nayak PRB 2013, Cano et al.]
There are conditions under which even very different topological phases can share a gapped interface
Examples: (v=8)/E8, (v=1)/(v=1/9)
v=1
v=1/9

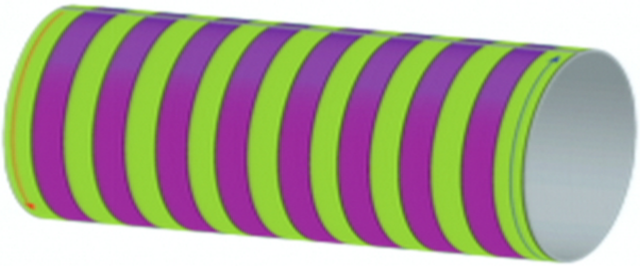

Motivational Questions for Entanglement
A
B
The low-lying entanglement spectrum of a topological phase has similar properties as the physical edge spectrum. But if the bulk-boundary correspondence is not unique, which edge theory matches the entanglement?
S=a1L
v=1
v=1/9
If two different topological phases can share a gapped interface, how does the entanglement entropy depend on where you cut?


Entanglement at Hetero-Interfaces
ν=8
E_8
$S_L = \alpha_L \ell$
$S_R = \alpha_R \ell$
$S_I = \alpha_I \ell - \log 2$
Cano, Hughes, Mulligan, PRB (2015)


Geometrical Interpretation From Lattices
Gapping (e.g., tunneling) terms between two different topological phases represent maps back and forth between the lattice of local particles on one side (Image of K_L) and the lattice of local particles on the other (Image of K_R).
For the v=8/E8 interface the maps go from an odd lattice (v=8) to an even lattice (E8). The closest map between the two will be at best 2 to 1. This is the geometric origin of the log 2 correction.
Different maps can give you different constraints, and hence can change the subleading correction to the area law.
Maps are not arbitrary, they must map a countably infinite number of points from one lattice onto the other lattice (commmensurate Moiré pattern), and satisfy an extra geometric constraint from primitivity.


Entanglement in Homogeneous Systems
Requirements on the choice of gapping terms:
• Gapping terms are primitive (no extra local degeneracy)
• Topological GSD unchanged
• Edge theory unchanged
• Hall conductance and Thermal Hall conductance unchanged
• No new topological quasi-particles
Cano, Hughes, Mulligan, PRB (2015)

Entanglement in Homogeneous Systems



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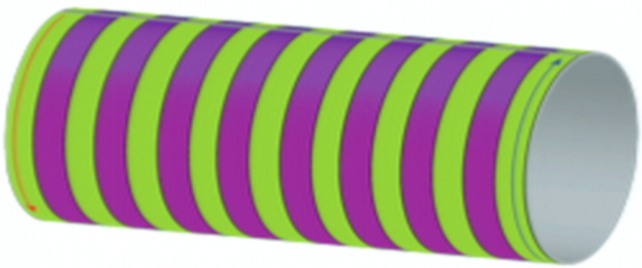

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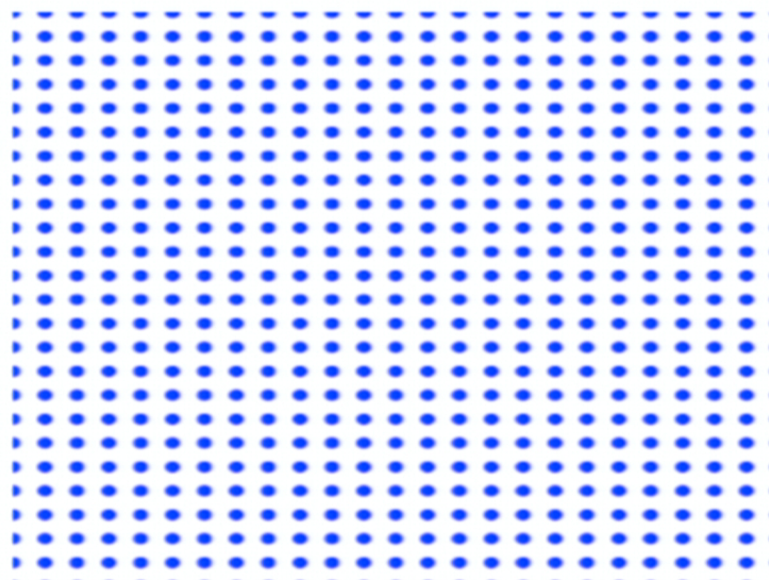
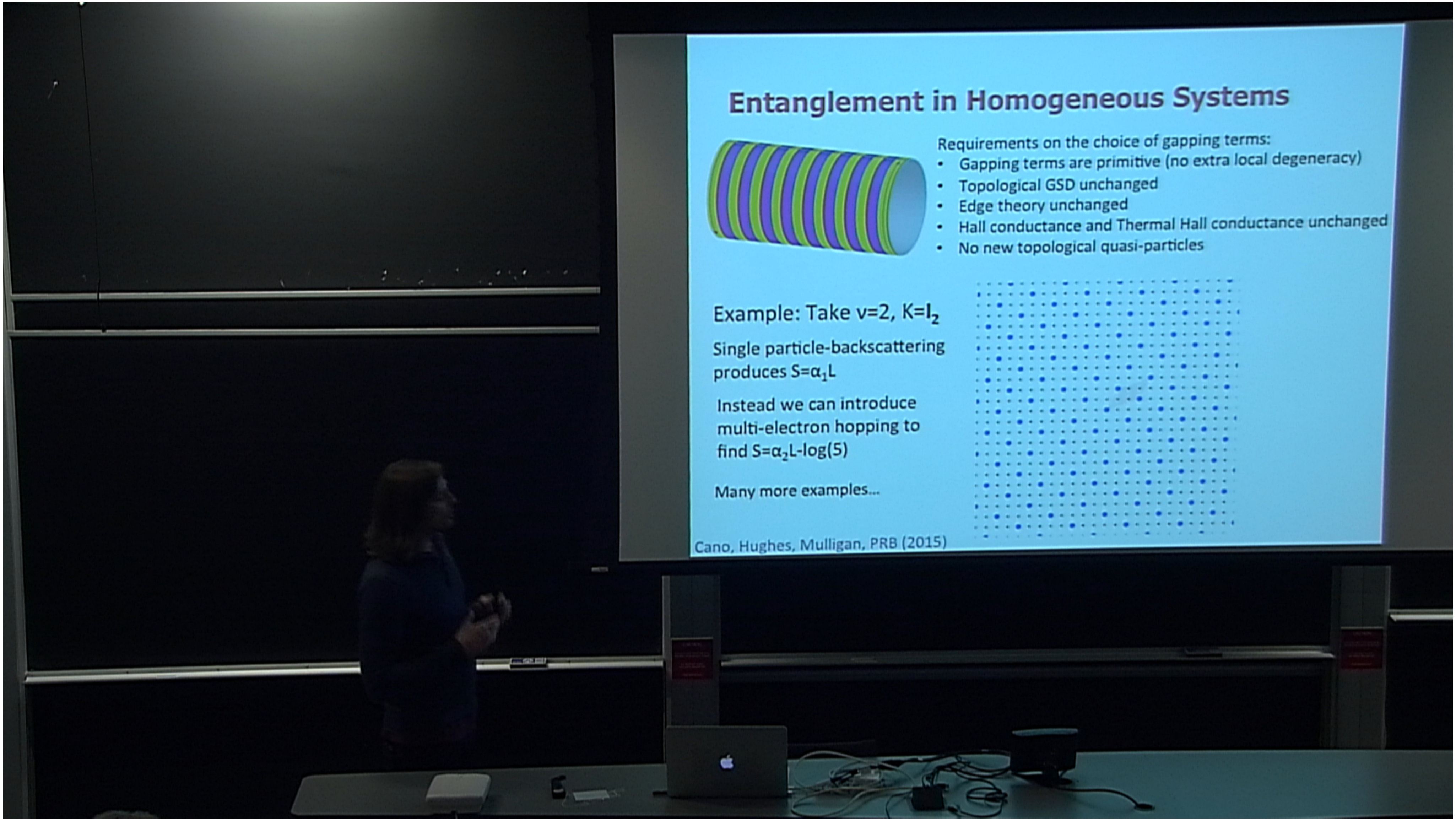
Example: Take $\nu=2$, $K=\mathbf{I}_2$

Single particle-backscattering produces $S=\alpha_1 L$



Cano, Hughes, Mulligan, PRB (2015)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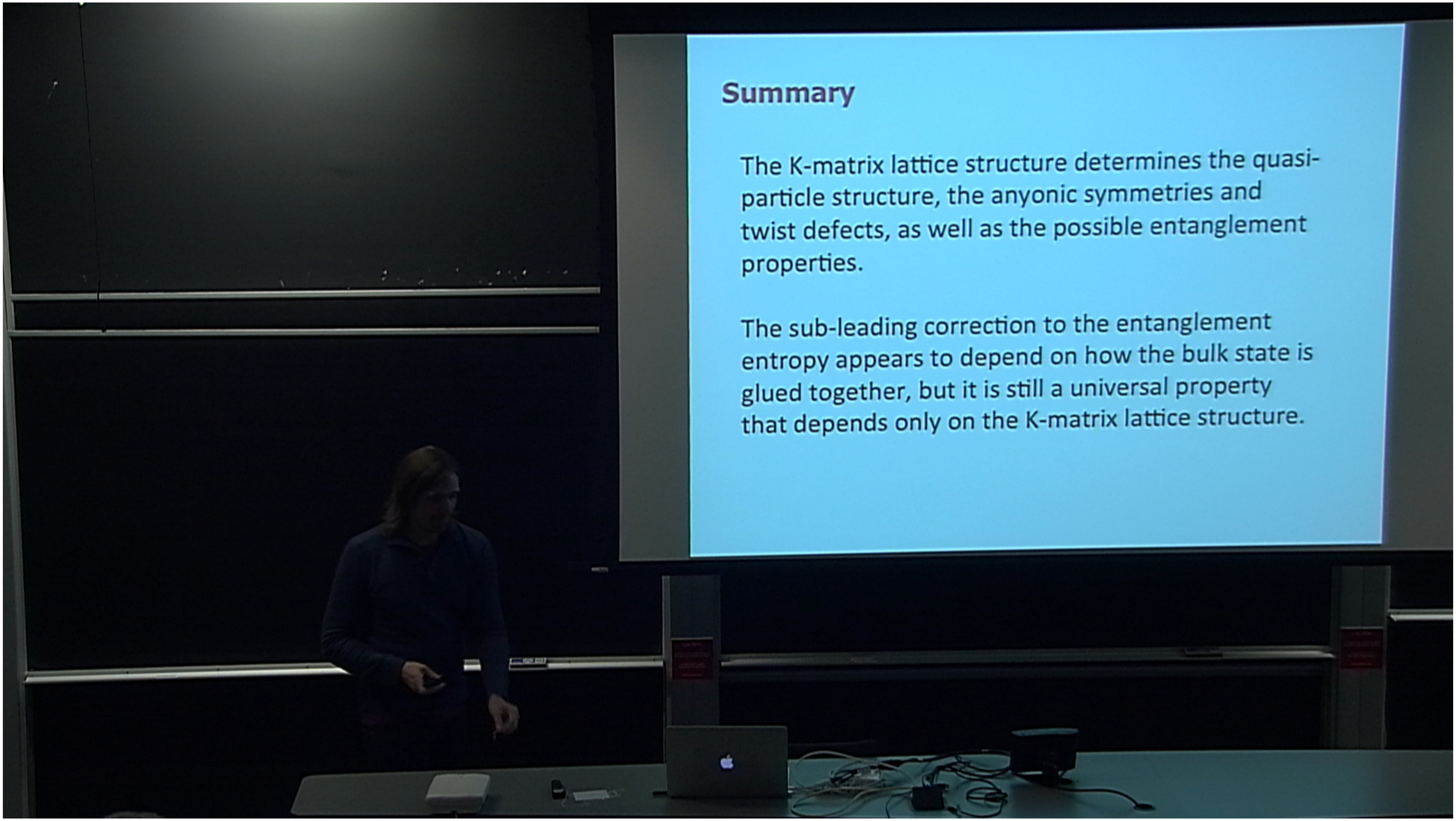
Example: Take ν=2, K=I2
Single particle-backscattering produces S=α1L
Instead we can introduce multi-electron hopping to find S=α2L-log(5)
Many more examples...
Cano, Hughes, Mulligan, PRB (2015)


Summary
The K-matrix lattice structure determines the quasi-particle structure, the anyonic symmetries and twist defects, as well as the possible entanglement properties.
The sub-leading correction to the entanglement entropy appears to depend on how the bulk state is glued together, but it is still a universal property that depends only on the K-matrix lattice structure.