Title: Tensor network renormalization

Date: Nov 05, 2015 10:45 AM

URL: http://pirsa.org/15110068

Abstract:

Motivation:

- Field theory ⟷ space-time symmetries

e.g. translation invariance $x \to x' = x + \epsilon$

at criticality scale invariance (augmented to global/local conformal group?)

$$x \to x' = (1 + \epsilon)x$$

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  e.g. translation invariance $x \to x' = x + \epsilon$

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  $x \to x' = (1+\epsilon)x$

  (augmented to global/local conformal group?)

- On the lattice (genuine physics or UV cut-off)

  translation ? $x \to x' = x + a$ discrete

  $a \equiv$ lattice spacing

Goals:

1) Define a ***global* scale transformation** on the lattice (≈ real space RG transformation)

Hamiltonian (Hilbert space) — Entanglement renormalization / MERA

PRL 2007 (arXiv:cond-mat/0512165)

PRL 2008 (arXiv:quant-ph/0610099)

Lagrangian (Euclidean path integral)

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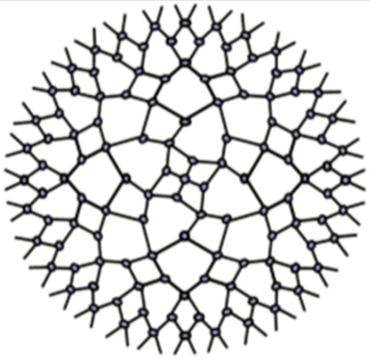
Hamiltonian (Hilbert space)	Entanglement renormalization / MERA	PRL 2007 (arXiv:cond-mat/0512165) PRL 2008 (arXiv:quant-ph/0610099)
Lagrangian (Euclidean path integral)	Tensor network renormalization	Evenbly, Vidal PRL 2015 (arXiv:1412.0732) Evenbly, arXiv:1509.07484

2) Define a ***local*** **scale transformation** on the lattice

Outline:

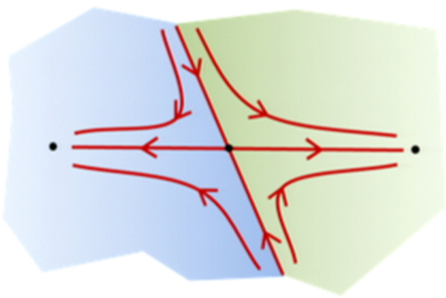


wave-functions / Hamiltonians

- *global* scale transformation (RG transformation)
- *local* scale transformations

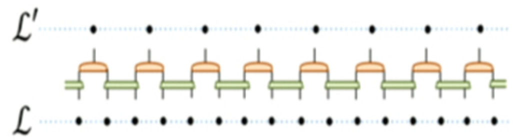
Euclidean path integrals / classical partition functions

- *global* scale transformation (RG transformation)
- *local* scale transformations



How do we define a scale transformation on the lattice?

- many possibilities
- here, isometric map $\mathbb{V}^{\otimes N} \leftrightarrow \mathbb{V}^{\otimes N/2}$



variational

locality

entanglement

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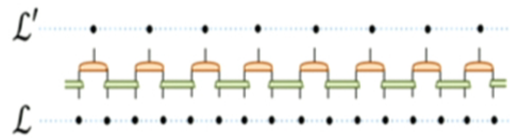

variational

locality

entanglement

- requirement: consistency with the *continuum*

field theory —discretization→ lattice model

scale transformation $x \rightarrow e^{-s}x$ ↓ | **discrete scale transformation** $x \rightarrow 1/2x$ ↓

field theory → lattice model

reproduce expected RG flow,
including *explicit scale invariance* at RG fixed-points

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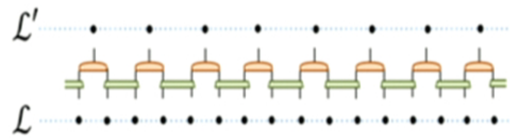

variational

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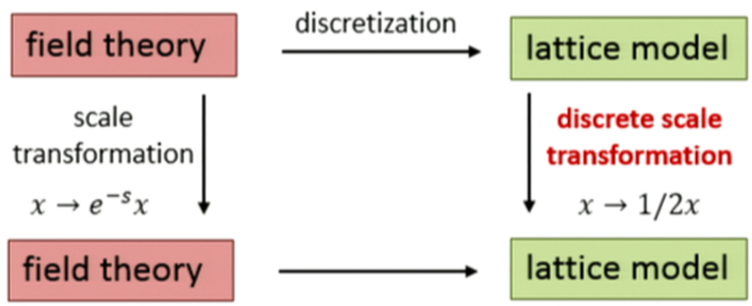

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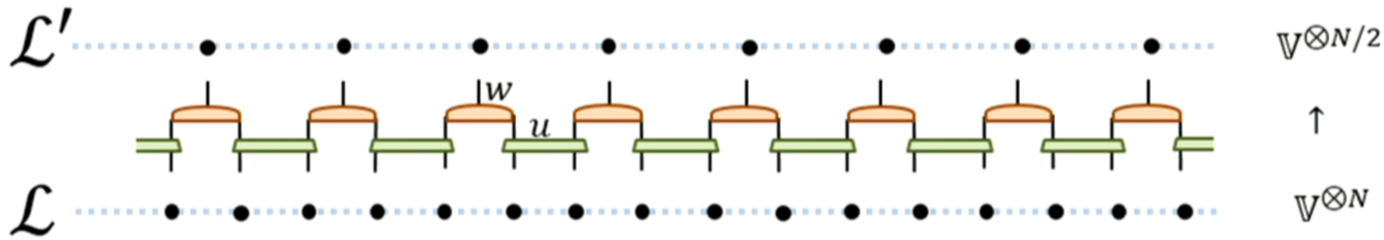
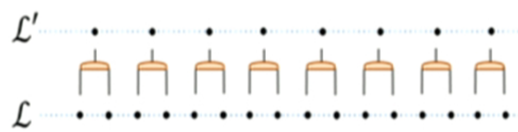
PRL 2007
(arXiv:cond-mat/0512165)

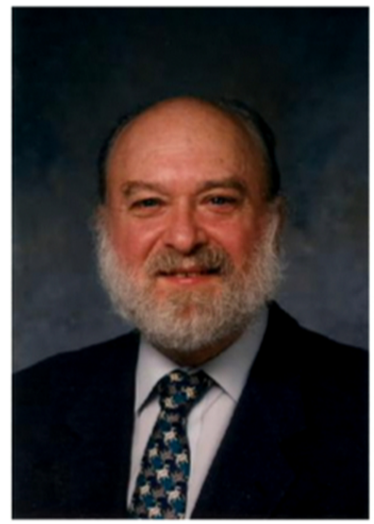
Entanglement renormalization



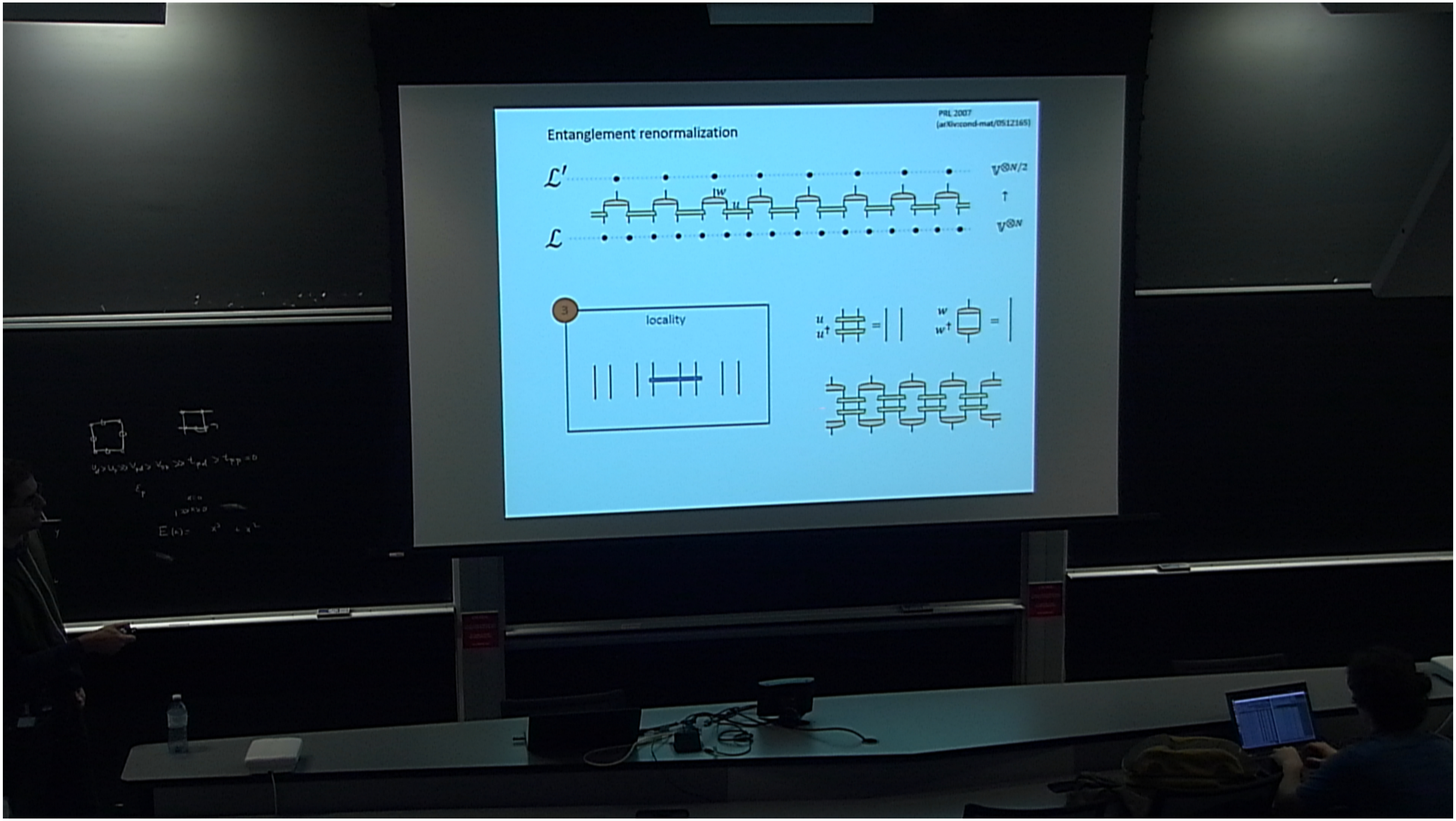
1

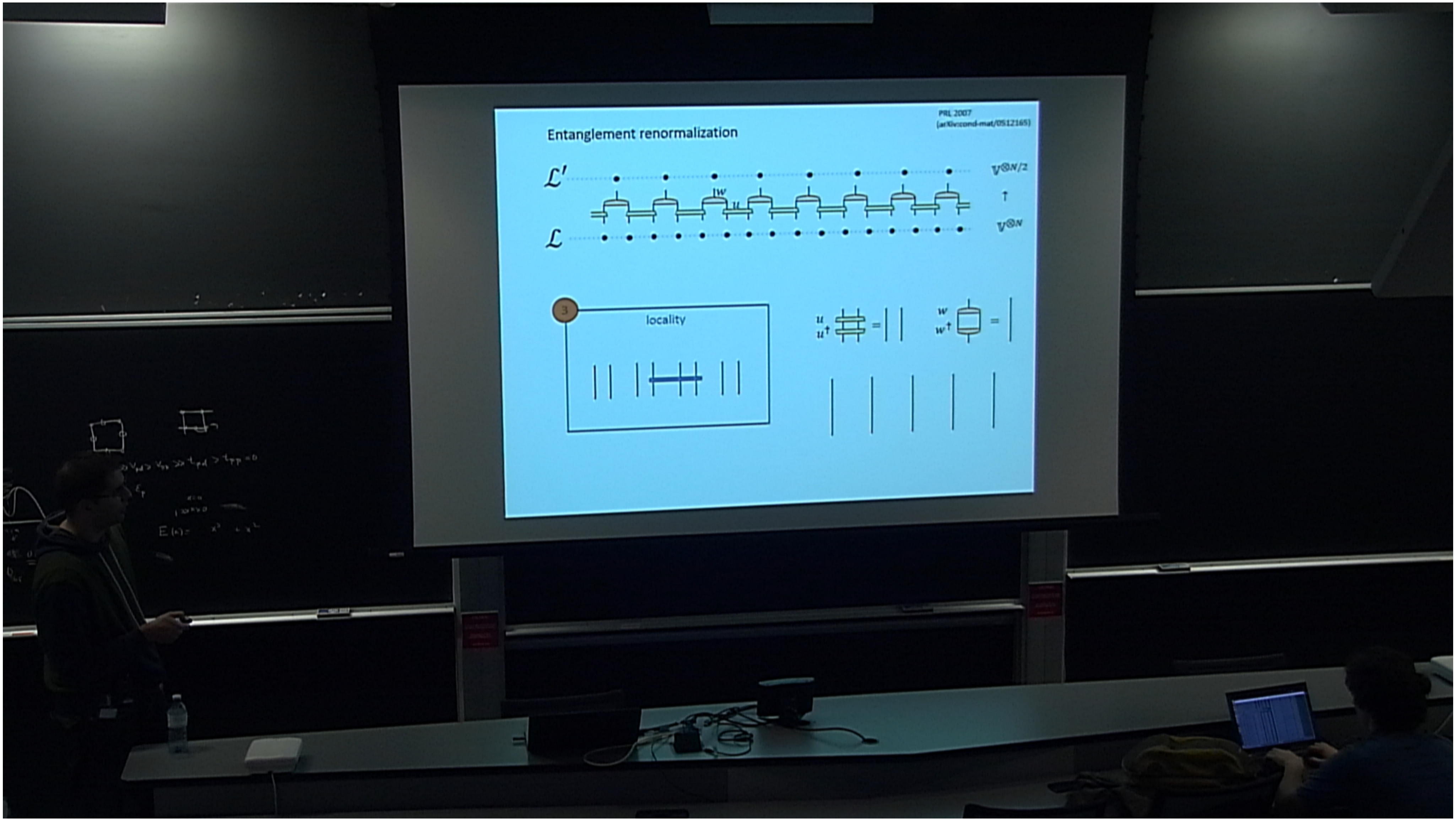
spin blocking
(as Kadanoff 1966)





Leo Kadanoff
1937 - 2015


PRL 2007
(arXiv:cond-mat/0512165)
Entanglement renormalization
$\mathcal{L}'$
$\mathbb{V}^{\otimes N/2}$
w
u
↑
$\mathcal{L}$
$\mathbb{V}^{\otimes N}$
3
locality
u
$u^\dagger$
=
w
$w^\dagger$
=


Entanglement renormalization
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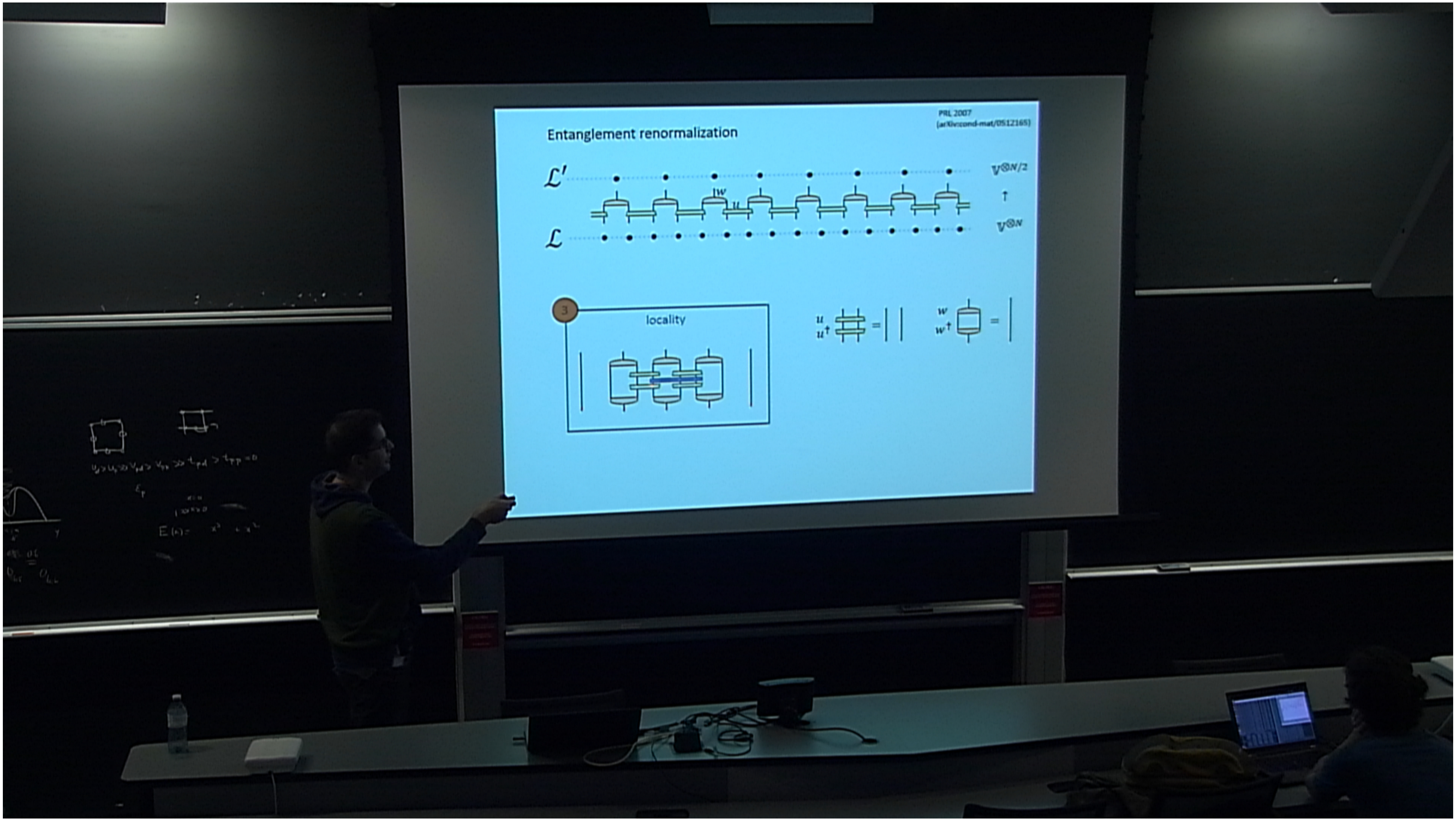
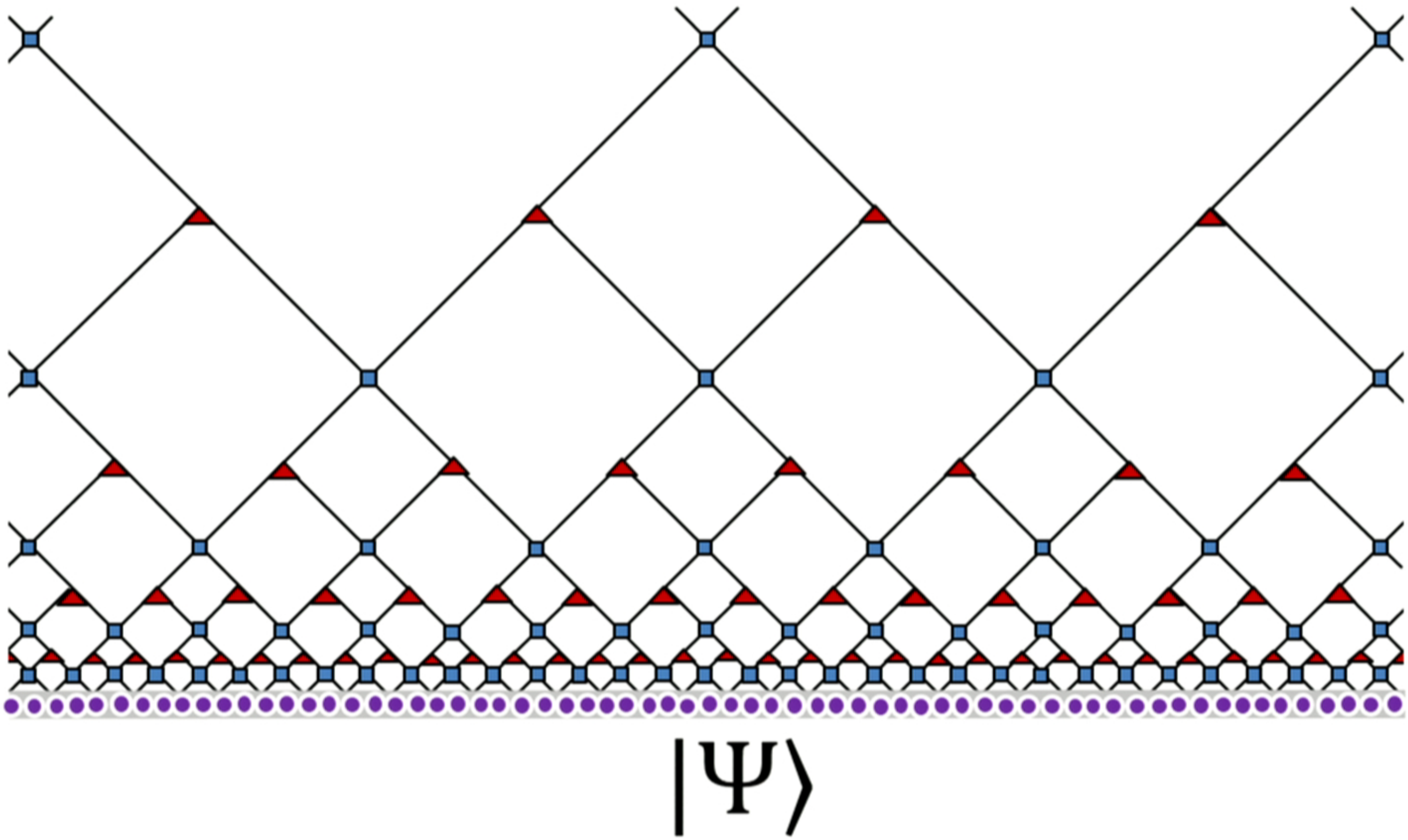

PRL 2007
(arXiv:cond-mat/0512165)
Entanglement renormalization
locality


PRL 2007
(arXiv:cond-mat/0512165)
Entanglement renormalization
failure to remove
some short-range
entanglement !
entanglement
removal of all
short-range
entanglement !

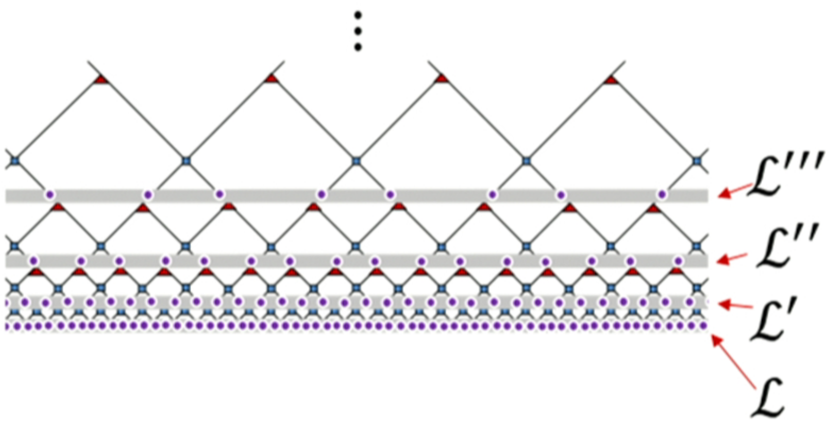
PRL 2008
(arXiv:quant-ph/0610099)

multi-scale entanglement renormalization ansatz (MERA)

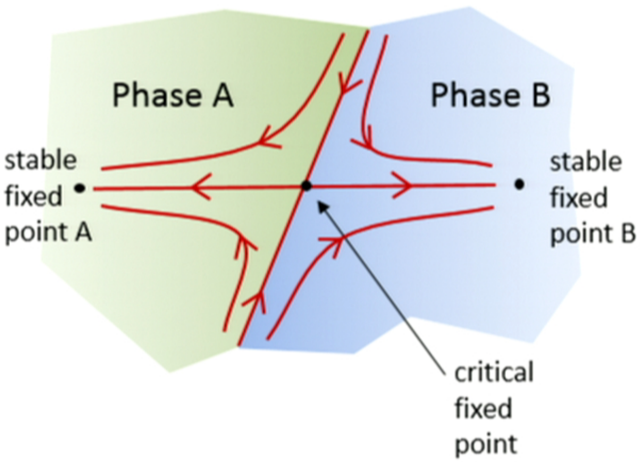


MERA defines an RG flow in the space of wave-functions

PRL 2008
(arXiv:quant-ph/0610099)



$$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to |\Psi'''\rangle \cdots$$



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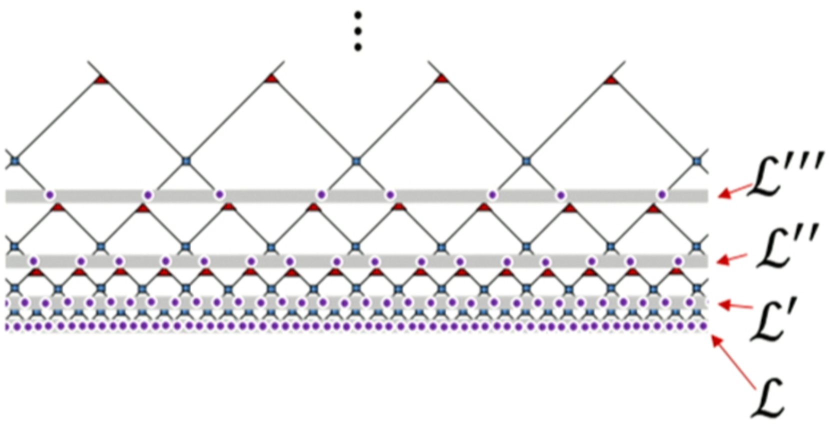

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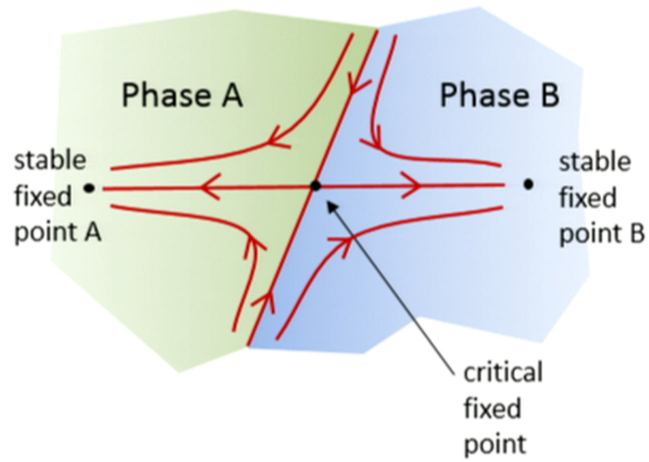
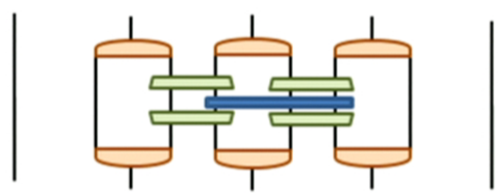


... and in the space of Hamiltonians



$$H \rightarrow H' \rightarrow H'' \rightarrow H''' \cdots$$

Claim: MERA defines a *proper* scale transformation on the lattice

In particular, at criticality, we recover *explicit* scale invariance

input

1D quantum Hamiltonian on the lattice

- **at a critical point**

→

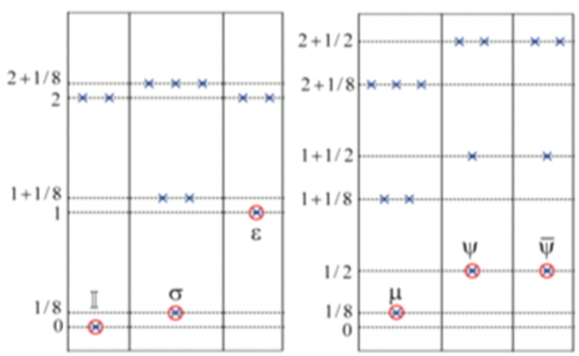
output

Numerical extraction of conformal data of underlying CFT:

- **central charge** c
- **scaling dimensions** $\Delta_\alpha \equiv h_\alpha + \bar{h}_\alpha$ **and conformal spins** $s_\alpha \equiv h_\alpha - \bar{h}_\alpha$
- **OPE coefficients** $C_{\alpha\beta\gamma}$

e.g. critical Ising model (approx. an hour on your laptop)

Pfeifer, Evenbly, Vidal 08



$(\Delta_{\mathbb{I}} = 0)$

$\Delta_\sigma \approx 0.124997$

$\Delta_\varepsilon \approx 0.99993$

$\Delta_\mu \approx 0.125002$

$\Delta_\psi \approx 0.500001$

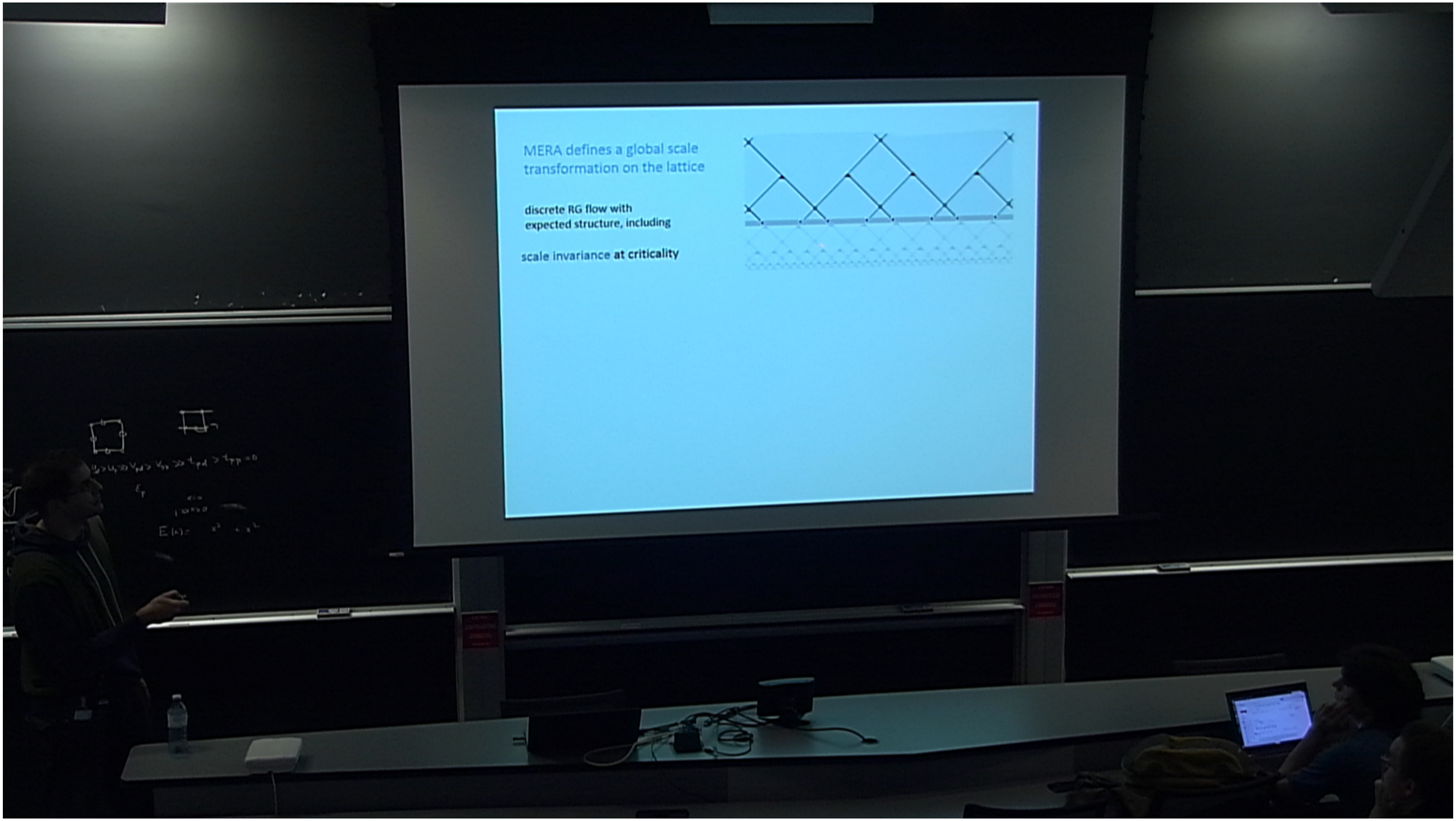
$\Delta_{\bar{\psi}} \approx 0.500001$

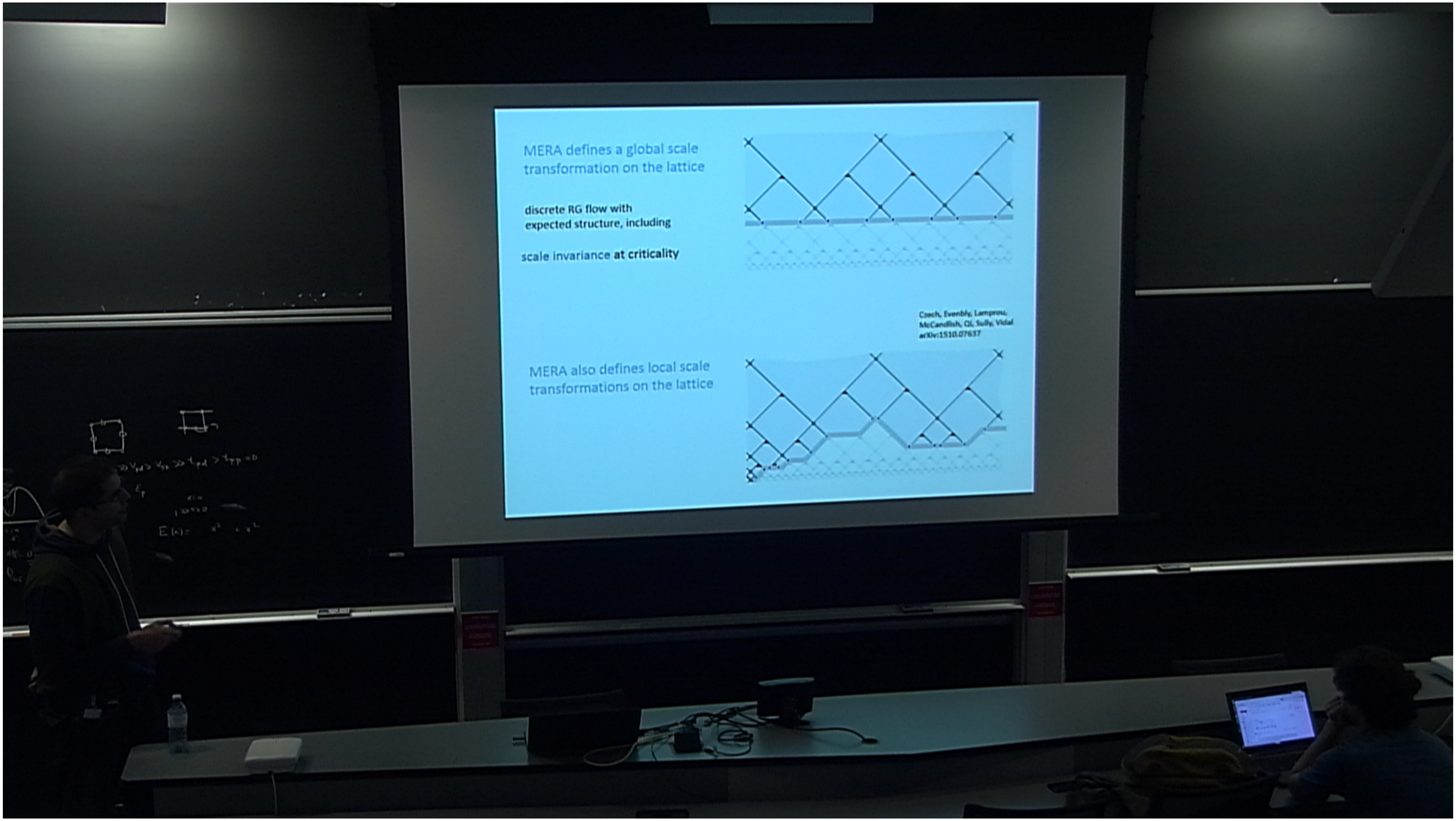
$C_{\epsilon\sigma\sigma} = \frac{1}{2}$ $C_{\epsilon\mu\mu} = -\frac{1}{2}$

$C_{\epsilon\psi\bar{\psi}} = i$ $C_{\epsilon\bar{\psi}\psi} = -i$

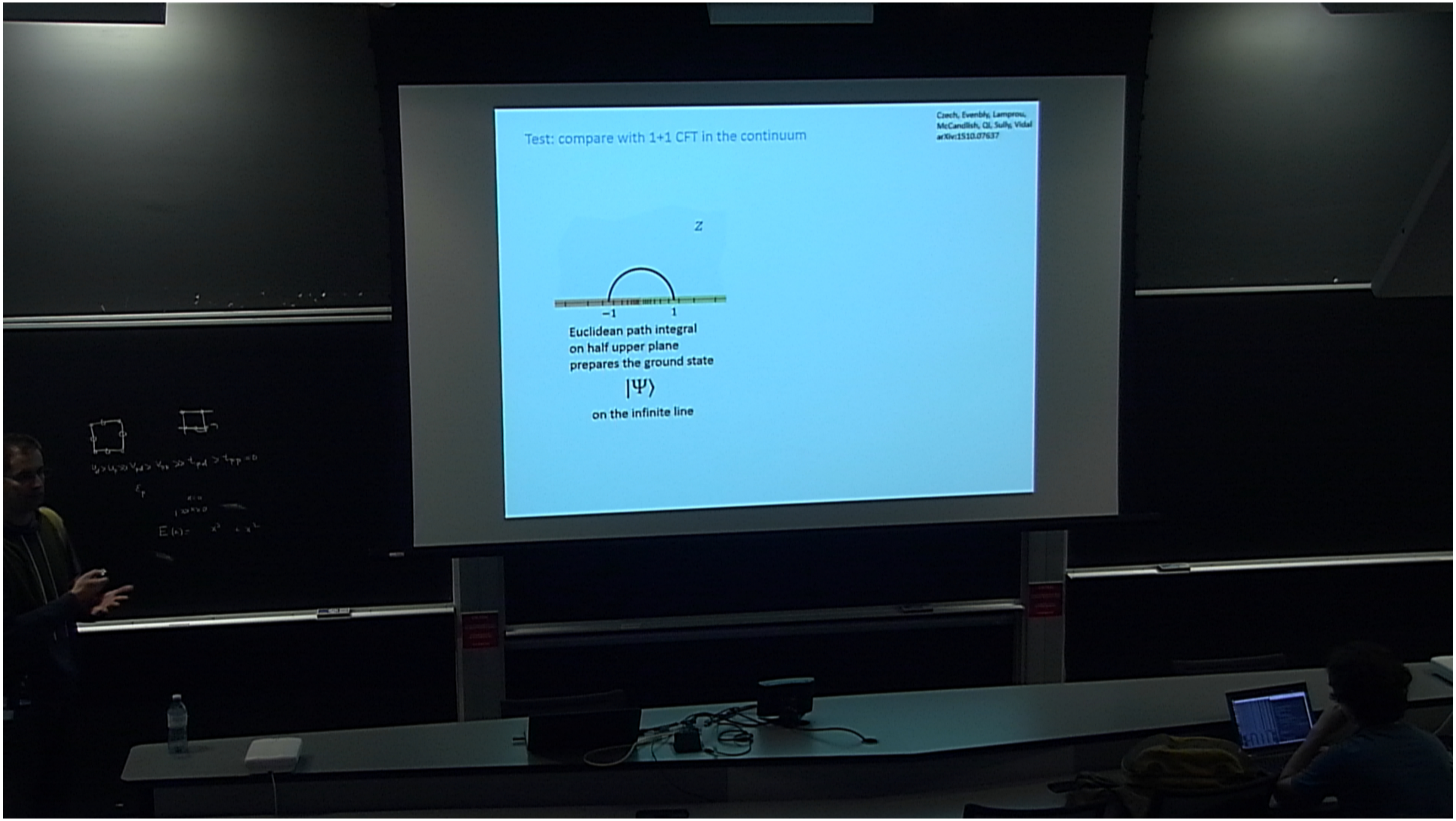
$C_{\psi\mu\sigma} = \frac{e^{-\frac{i\pi}{4}}}{\sqrt{2}}$ $C_{\bar{\psi}\mu\sigma} = \frac{e^{\frac{i\pi}{4}}}{\sqrt{2}}$

$(\pm 6 \times 10^{-4})$


MERA defines a global scale transformation on the lattice
discrete RG flow with expected structure, including
scale invariance at criticality



MERA defines a global scale transformation on the lattice
discrete RG flow with expected structure, including
scale invariance at criticality
Czech, Evenbly, Lamprou, McCandlish, Qi, Sully, Vidal arXiv:1510.07637
MERA also defines local scale transformations on the lattice


Test: compare with 1+1 CFT in the continuum
Czech, Evenbly, Lamprou, McCandlish, Qi, Sully, Vidal arXiv:1510.07637
z
−1
1
Euclidean path integral
on half upper plane
prepares the ground state
|Ψ⟩
on the infinite line


Czech, Evenbly, Lamprou,
McCandlish, Qi, Sully, Vidal
arXiv:1510.07637
Test: compare with 1+1 CFT in the continuum
conformal transformation
$z \to w = \frac{\beta}{\pi}\log(z)$
z
W
$i\beta$
β
-1
1
0
Euclidean path integral
on half upper plane
prepares the ground state
$|\Psi\rangle$
on the infinite line
Euclidean path integral
on infinite strip
prepares thermal state
ρ_β
on the infinite line

Test: compare with 1+1 CFT in the continuum

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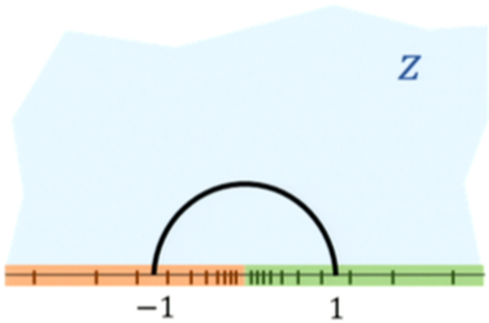
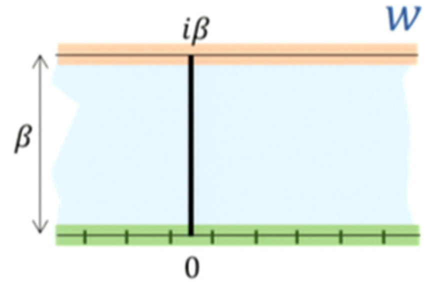
conformal transformation

$$z \to w = \frac{\beta}{\pi}\log(z)$$



→



Euclidean path integral on half upper plane prepares the ground state

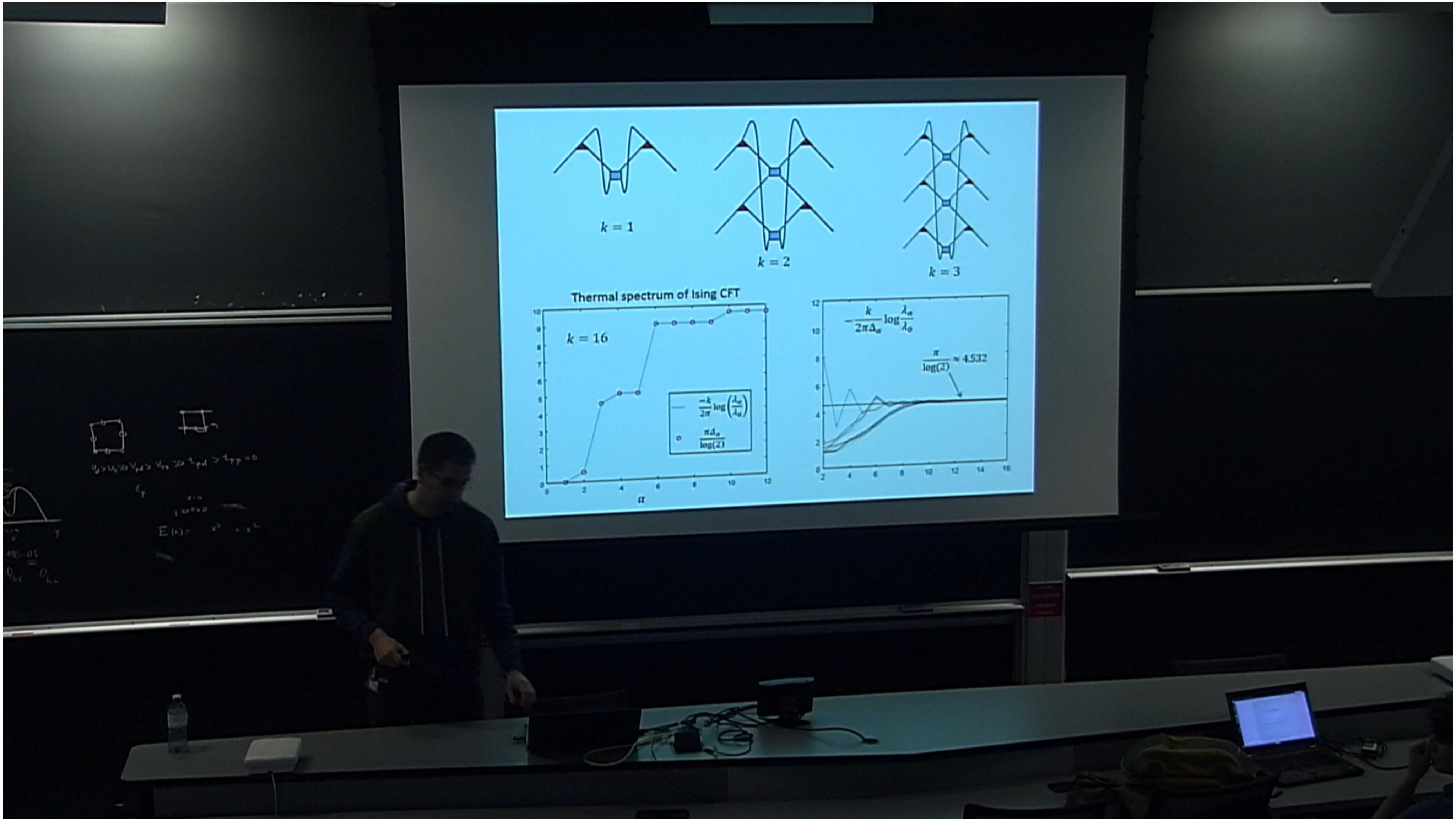
$$|\Psi\rangle$$

on the infinite line

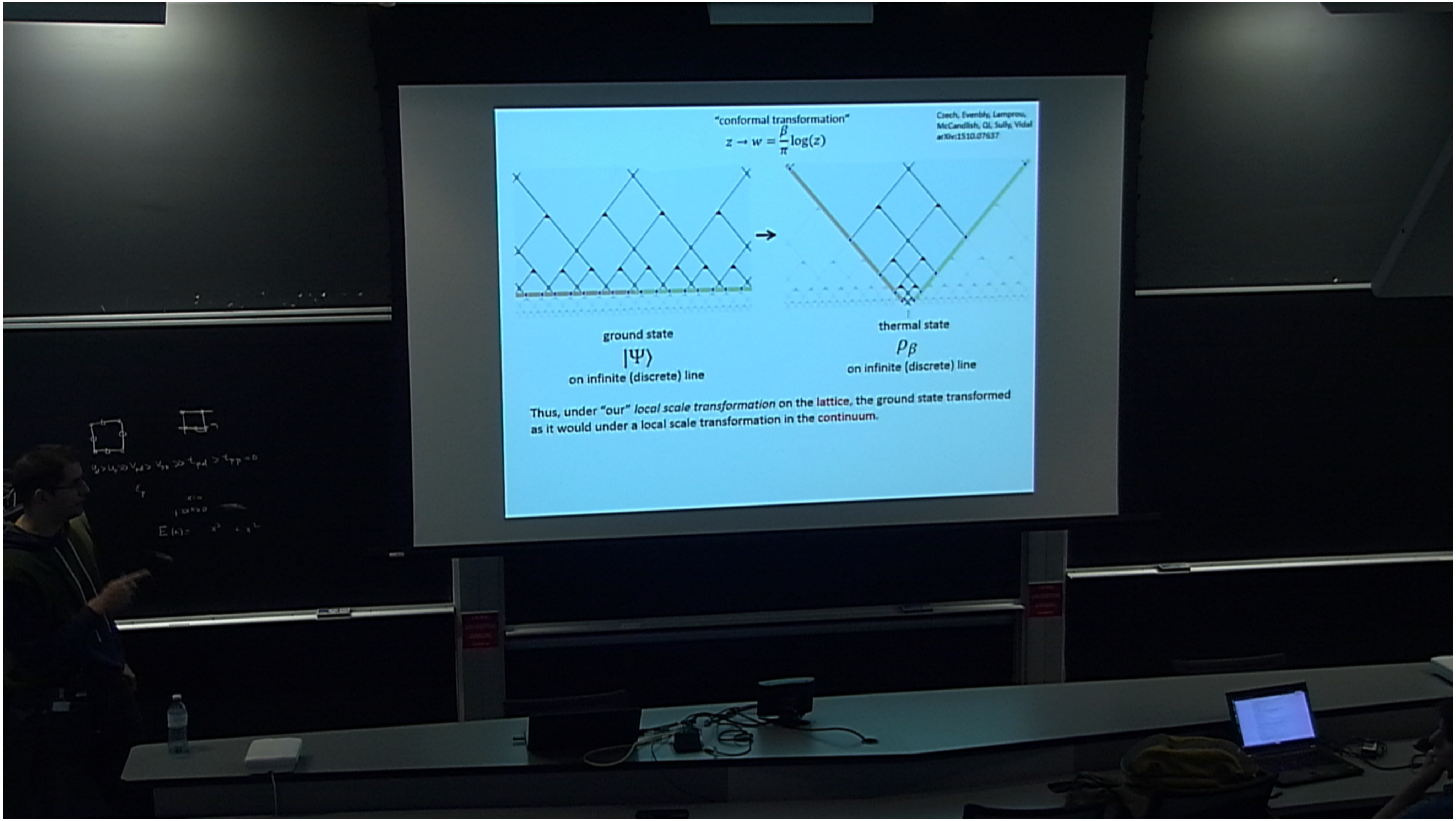
Euclidean path integral on infinite strip prepares thermal state

$$\rho_\beta$$

on the infinite line



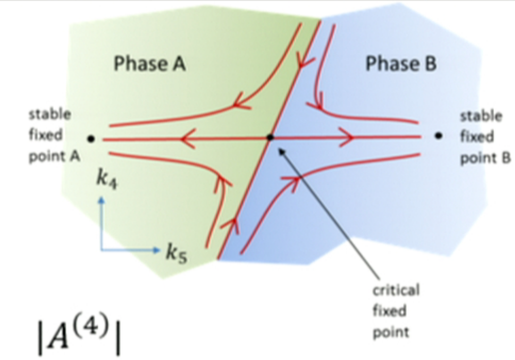
k = 1
k = 2
k = 3
Thermal spectrum of Ising CFT
k = 16
π/log(2) ≈ 4.532
α



"conformal transformation"
$z \to w = \frac{\beta}{\pi} \log(z)$
Czech, Evenbly, Lamprou, McCandlish, Qi, Sully, Vidal arXiv:1510.07637
ground state
$|\Psi\rangle$
on infinite (discrete) line
thermal state
ρ_β
on infinite (discrete) line
Thus, under "our" *local scale transformation* on the lattice, the ground state transformed as it would under a local scale transformation in the continuum.

Tensor Network Renormalization (TNR)
defines a proper RG flow in the space of tensors

$$A \to A' \to A'' \to \cdots \to A^{fp}$$



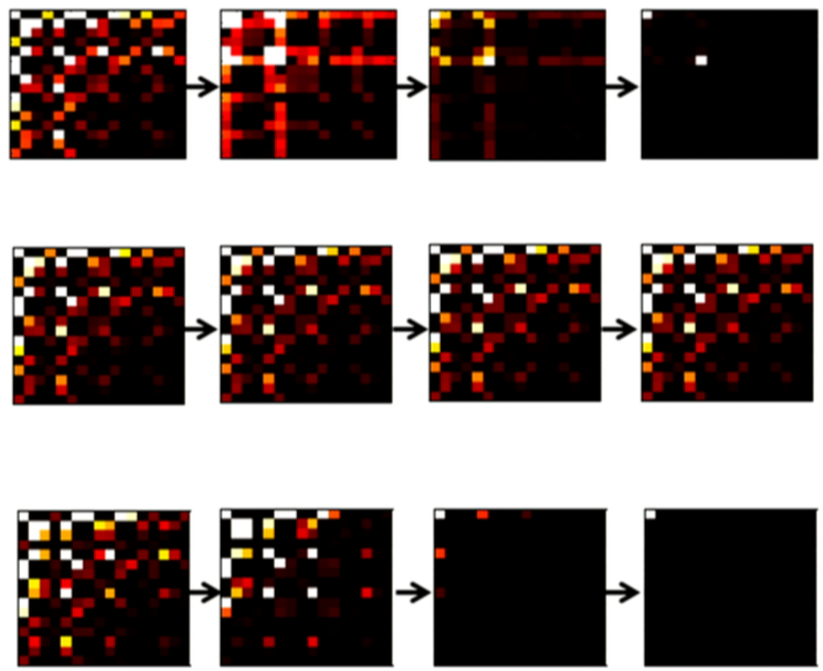
Example: 2D classical Ising model

$|A^{(1)}|$ $|A^{(2)}|$ $|A^{(3)}|$ $|A^{(4)}|$

below critical
$T = 0.9\, T_c$

critical
$T = T_c$

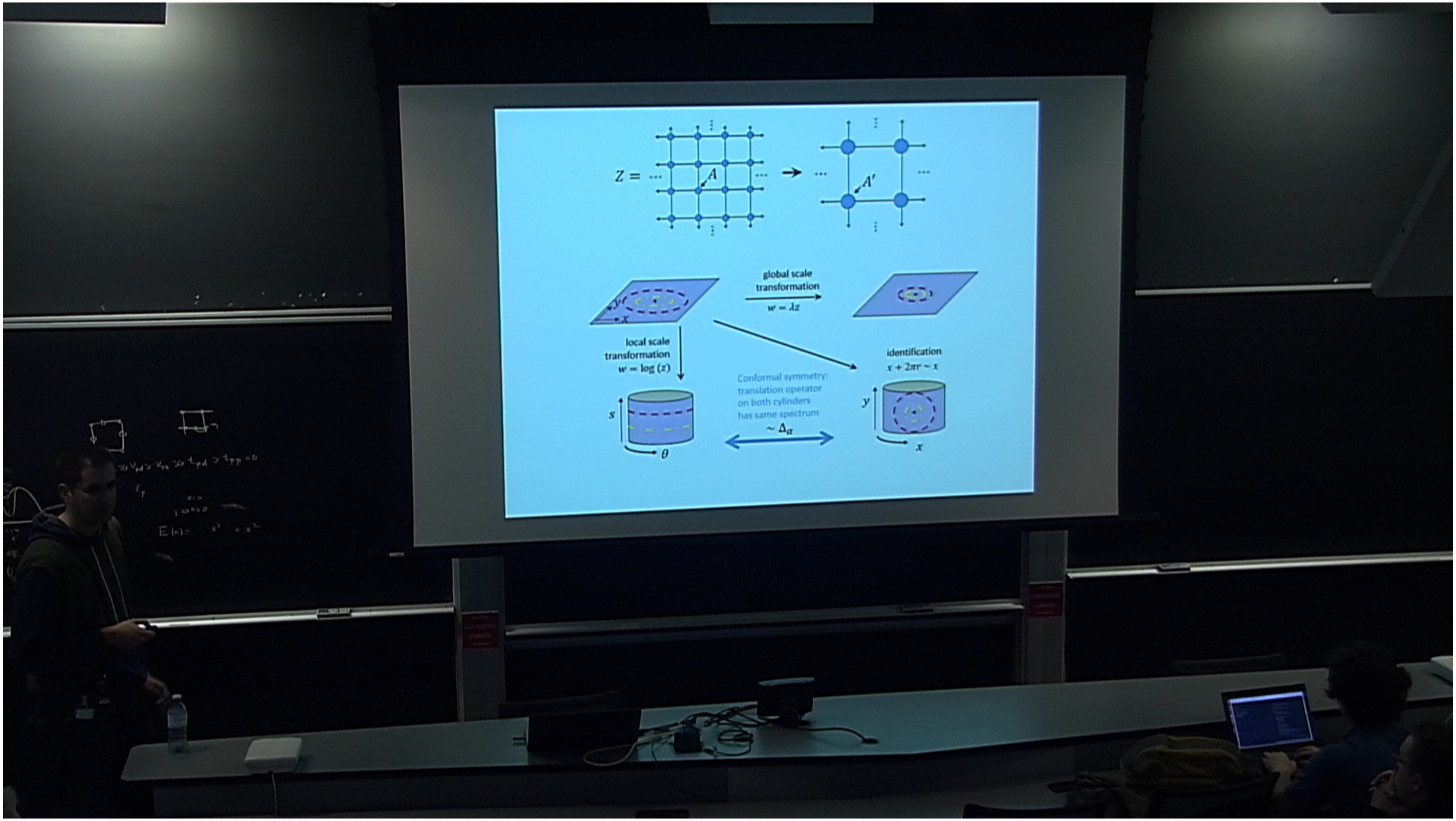
above critical
$T = 1.1\, T_c$



ordered (Z2) fixed point
also TEFR of Gu, Wen (2009)

critical fixed point

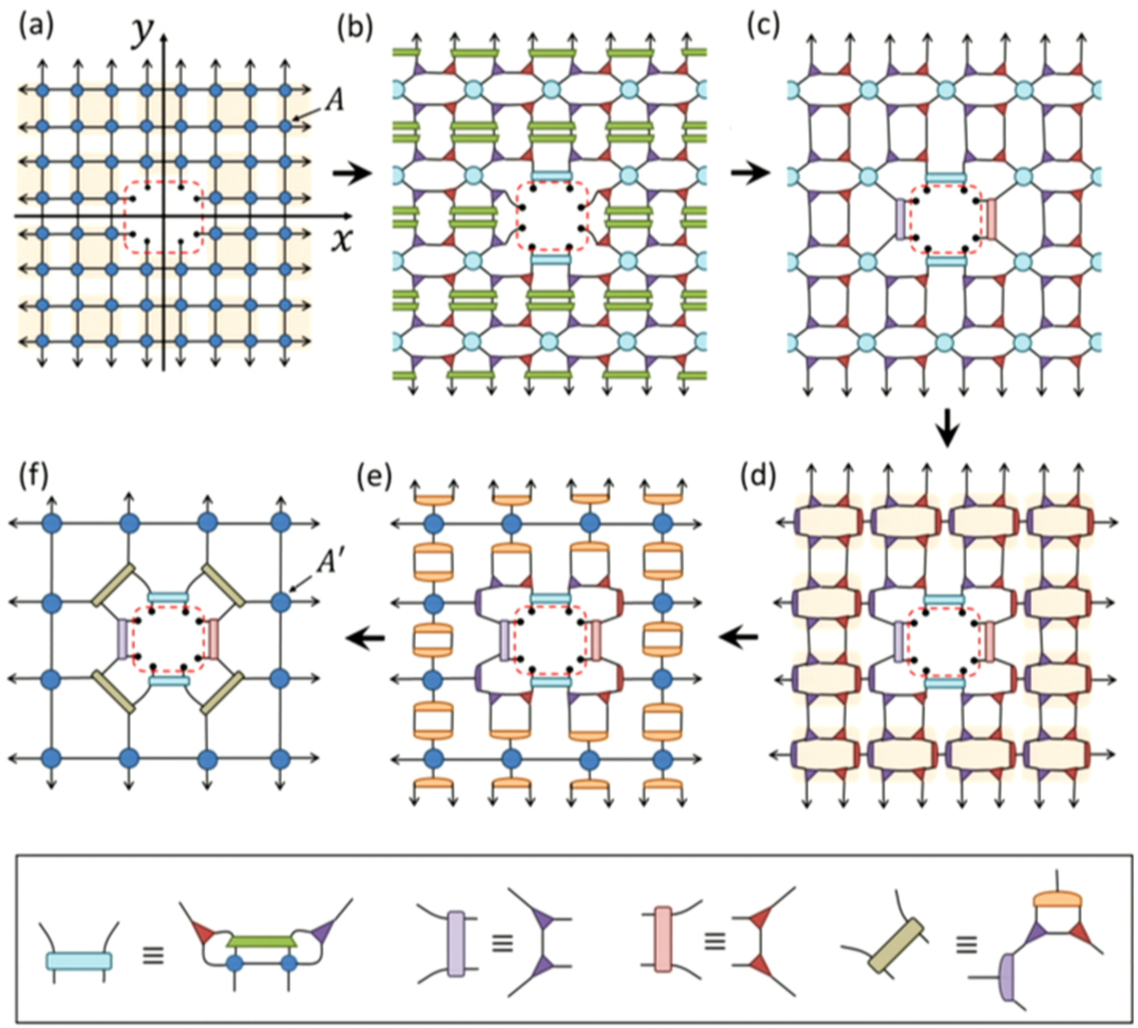
disordered (trivial) fixed point
also TEFR of Gu, Wen (2009)



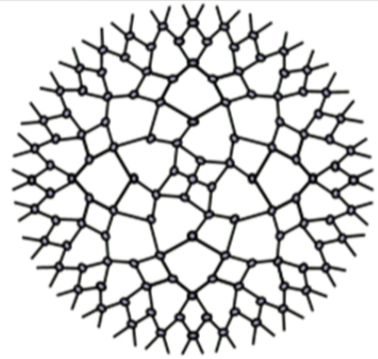
Z =
A
A'
global scale
transformation
w = λz
local scale
transformation
w = log (z)
identification
x + 2πr ~ x
Conformal symmetry:
translation operator
on both cylinders
has same spectrum
~ Δ_α
s
θ
y
x

Plane to cylinder

Evenbly, Vidal
arXiv:1510.00689



Summary:



wave-functions / Hamiltonians

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Euclidean path integrals / classical partition functions

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on the lattice !