Title: Supersymmetric flavors on curved space and a precision test of AdS/CFT

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Abstract: <p>Quantum field theory on curved space has long been studied for its interesting phenomenology, and more recently also as a means to obtain non-perturbative results in supersymmetric theories. In this talk I will describe the holographic dual for N=4 SYM coupled to massive N=2 flavors on spaces of constant curvature. With that in hand, I will discuss a topology-changing phase transition on S^4 and confront holographic computations with exact field theory results obtained using supersymmetric localization.</p>

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Two kinds of reasons:

(i) phenomenological:

- Minkowski space only an approximation on short enough length scales
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(ii) formal:

- compact Euclidean spaces are nice – no IR issues, path integrals
- in combination with supersymmetry: localization

$\longrightarrow$ exact non-perturbative QFT results

Why curved space in AdS/CFT?

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- perturbation theory challenging beyond free theory
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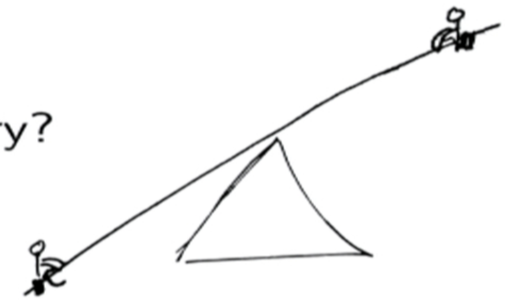
(ii) AdS $\leftarrow$ CFT:

- AdS/CFT not formally proven
  precise range of validity? complete dictionary?
- even testing is hard: conservation of evil



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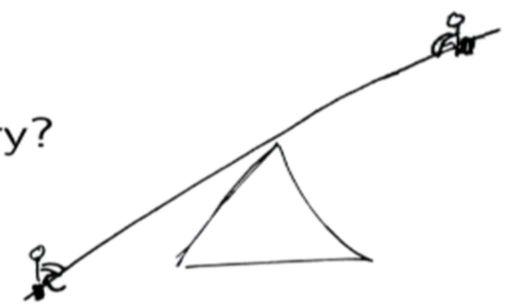

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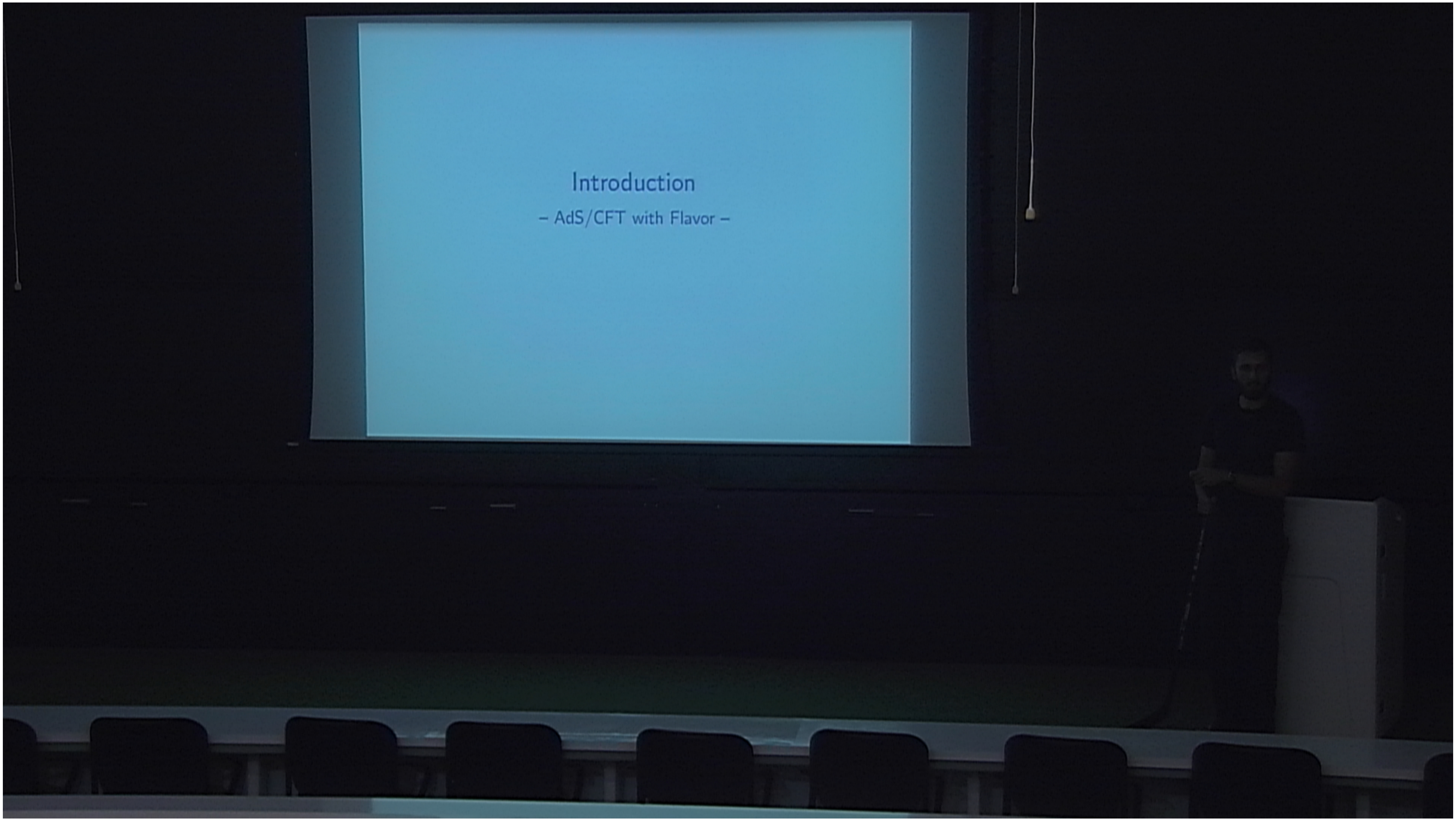
Litmus test for supergravity duals of $\mathcal{N}=2^\star$:

- Pilch-Warner flow ruled out for $\mathcal{N}=2^\star$ on S^4 [Buchel '13]
- extra scalar mass terms for susy on S^4 need larger truncation, dual constructed numerically in [Bobev,Elvang,Freedman,Pufu '13]


Introduction
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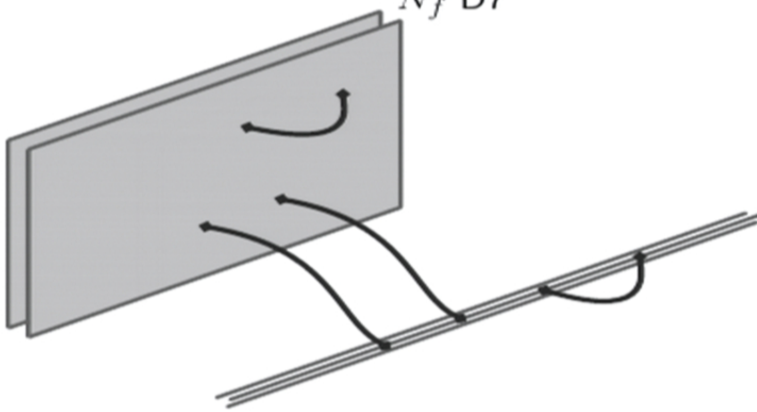
$\rightarrow$ add "quarks" [Karch,Katz '02]



$\mathcal{N}=4$ SYM + flavor

$\cong$

D3 and D7 in IIB

Spicing it up. . . moderately

N_f (# D7) $\ll N_c$ (# D3): quenched/probe approximation

- D3 create $\mathrm{AdS}_5 \times \mathrm{S}^5$ background
- D7 embeddings $\sim$ extremal $S_{\mathrm{DBI}} = N_f T_7 \int d^8 y \sqrt{\det(\gamma + F)}$

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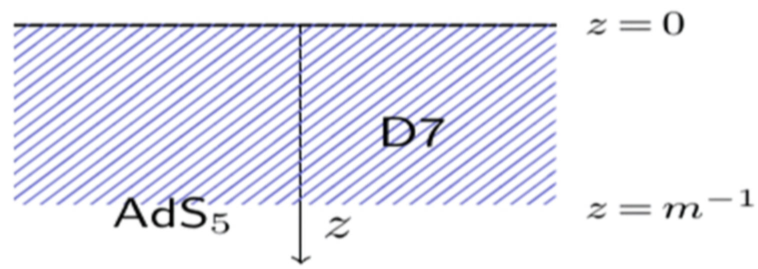

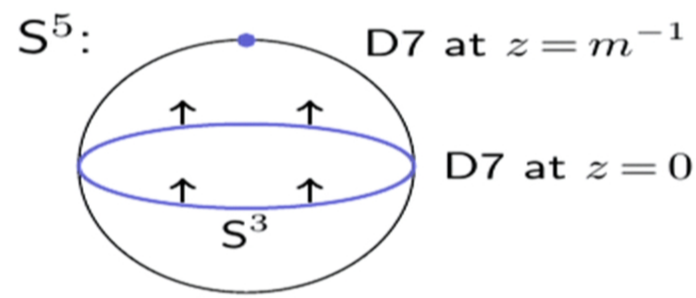

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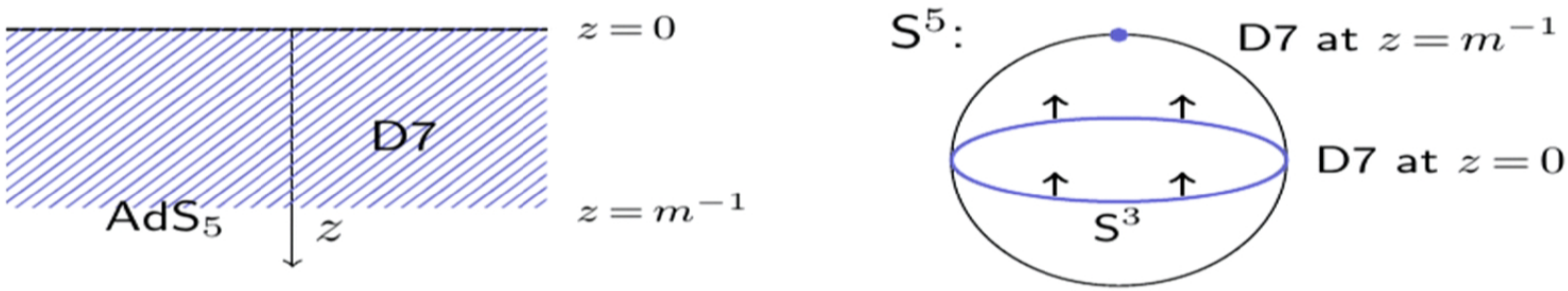

UV fixed point only in quenched limit.

AdS/CFT with curved boundaries

In simple cases: just choose different coordinates on AdS_5, different representative of boundary conformal structure

$$\frac{dz^2 + (1 - \frac{z^2}{4}) g_{S^4}}{z^2}$$

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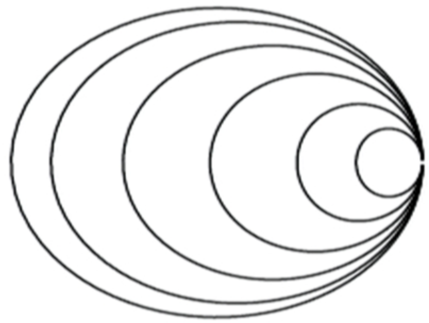

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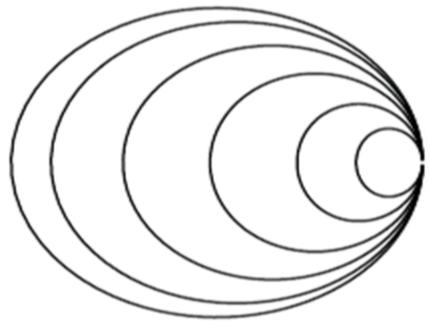

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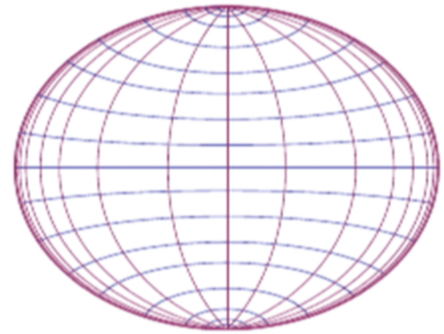

$$\frac{dz^2 + (1 + \frac{z^2}{4}) g_{AdS_4}}{z^2}$$

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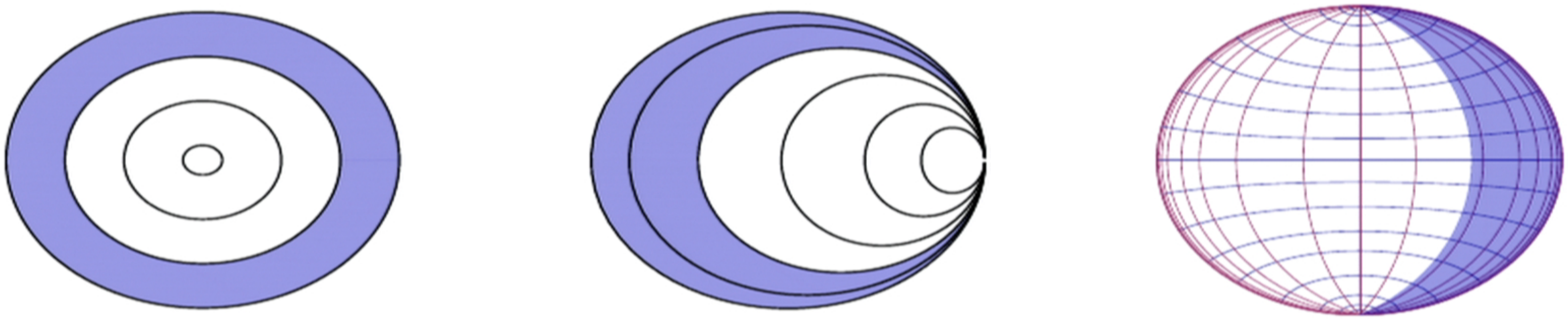


$$\frac{dz^2 + (1 - \frac{z^2}{4}) g_{S^4}}{z^2} \qquad \frac{dz^2 + \eta}{z^2} \qquad \frac{dz^2 + (1 + \frac{z^2}{4}) g_{AdS_4}}{z^2}$$

Boundary geometry: S^4, $\mathbb{R}^{1,3}$ and two copies of AdS_4

A distinction without much of a difference for conformal theories.
Not with massive flavors: geometrically different D7 embeddings.

Outline

D3/D7 for susy flavors on curved space

- D7-brane embeddings from κ-symmetry
- AdS_4 embeddings

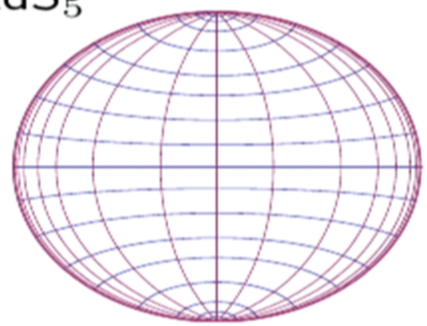
A precision test for AdS/CFT with flavor

- topology-changing phase transition on S^4
- AdS/CFT vs. supersymmetric localization

D3/D7 for susy flavors on curved space

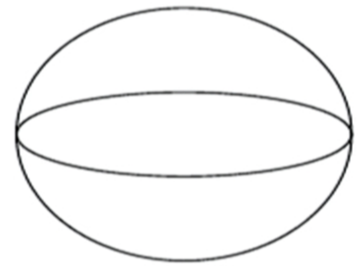
Background and DBI action

AdS$_5$



$\times$

S^5



$$g_{\mathrm{AdS}_5} = d\rho^2 + \cosh^2\!\rho \, g_{\mathrm{AdS}_4}$$

$$g_{\mathrm{S}^5} = d\theta^2 + \sin^2\theta \, g_{\mathrm{S}^3} + \cos^2\theta \, d\psi^2$$

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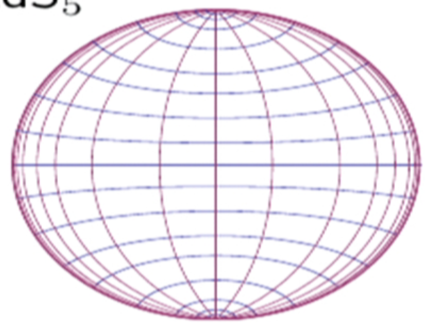

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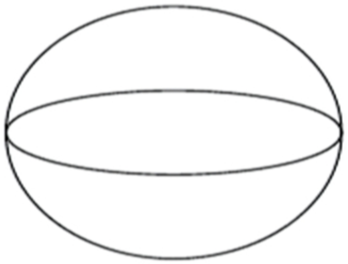

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Background and DBI action

DBI action for D7-branes with gauge field in that background:

$$S_{\mathrm{D7}} = -T_7 \int d^8\xi \sqrt{-\det\left(g+F\right)} + 2T_7 \int C_4 \wedge F \wedge F$$

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$$\begin{aligned}
0 = {} & \frac{1}{4}\theta' \sin\theta \tanh\rho \left(32f^2 - 4\cos(2\theta) + \cos(4\theta) + 3\right) \left(2f'^2 - \left(\theta'^2+1\right)\cos(2\theta) + \theta'^2 + 1\right) \\
& - \cos\theta \left(2\sin^2\theta \left(2f^2\left(\theta'^2+1\right) + f'^4\right) + f'^2\left(\theta'^2+5\right)\sin^4\theta + 3\left(\theta'^2+1\right)\sin^6\theta\right) \\
& + 4f^2 f'^2 \cos\theta \left(\theta'^2 - 1\right) + 4ff'\sin\theta\left(ff'\theta'' - ff''\theta' + f'^2\theta'\right) \\
& + 4f\sin^3\theta\left(f\theta'' + f'\left(\theta'^3+\theta'\right)\right) + f'\sin^5\theta(f'\theta'' - f''\theta') + \theta''\sin^7\theta
\end{aligned}$$

$$\begin{aligned}
0 = {} & 8f\cosh\rho\left(f'^2 + \left(\theta'^2+1\right)\sin^2\theta\right)\sqrt{(4f^2+\sin^4\theta)\left(f'^2 + (\theta'^2+1)\sin^2\theta\right)} \\
& + \sin^3\theta\cosh\rho\left(f'\cos\theta\left(2f'^2\theta' + \left(\theta'^3+\theta'\right)\sin^2\theta\right) - \sin^6\theta\left(f'\theta'\theta'' - f''\theta'^2 - f''\right)\right) \\
& + 2f^2\left(2\sin\theta\cosh\rho\left(\sin\theta\left(-f'\theta'\theta'' + f''\theta'^2 + f''\right) - f'\left(\theta'^3+\theta'\right)\cos\theta\right)\right) \\
& - 2f\cosh\rho\left(f'^2 + \left(\theta'^2+1\right)\sin^2\theta\right)\left(2\theta'^2\sin^2\theta - \cos(2\theta) + 1\right) \\
& + 4\left(4f^2+\sin^4\theta\right)f'\sinh\rho\left(f'^2 + \left(\theta'^2+1\right)\sin^2\theta\right)
\end{aligned}$$

Supersymmetry to the rescue

IIB sugra background: δfermions=0 $\rightarrow$ BPS/Killing spinor eq.

Adding probe D-branes:

- no effect on background or Killing spinor eq. @LO
- superspace embedding $\rightarrow$ too many fermions
- fermionic κ gauge symmetry for $\#\text{bosons} = \#\text{fermions}$

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Supersymmetries compatible with κ-symmetry: [Bergshoeff, Townsend]

$$\Gamma_\kappa \epsilon = \epsilon$$

↖ background Killing spinor

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$\nearrow$ projector, encodes D7 embedding

$\nwarrow$ background Killing spinor

Looks simple enough: feed in $\mathrm{AdS}_5 \times \mathrm{S}^5$ Killing spinors and a a generic embedding, demand that there be solutions.

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$$J_{(p)}^{(n)} = (-1)^n \, (\sigma_3)^{n+(p-3)/2} \, i\sigma_2 \otimes \Gamma_{(0)}$$

$$\Gamma_{(0)} = \frac{1}{(p+1)!\sqrt{-\det g}} \, \varepsilon^{i_1 \dots i_{p+1}} \gamma_{i_1 \dots i_{p+1}} \; , \qquad \gamma_m = e^a_\mu \Gamma_a \partial_m X^\mu$$

Not exactly pretty, but only 1^{st}-order in derivatives of X^μ, A_μ.
$\mathrm{AdS}_5 \times \mathrm{S}^5$ Killing spinors:

$$\epsilon = e^{\frac{\theta}{2} i \Gamma^{\underline{\psi}} \Gamma_{\vec{\chi}}} \, e^{\frac{\psi}{2} i \Gamma_{\vec{\chi}} \Gamma^{\underline{\theta}}} \, e^{\frac{1}{2}\chi_1 \Gamma^{\underline{\theta \chi_1}}} \, e^{\frac{1}{2}\chi_2 \Gamma^{\underline{\chi_1 \chi_2}}} \, e^{\frac{1}{2}\chi_3 \Gamma^{\underline{\chi_2 \chi_3}}}$$
$$\times \, e^{\frac{\rho}{2} i \Gamma_{\underline{\rho}} \Gamma_{\mathrm{AdS}}} \left[e^{\frac{r}{2} i \Gamma_{\underline{r}} \Gamma_{\mathrm{AdS}}} + i e^{r/2} x^\mu \Gamma_{\underline{x_\mu}} \Gamma_{\mathrm{AdS}} P_{r-} \right] P_{\mathrm{L}} \epsilon_0$$

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regular matrix ($\mathcal{R}$), constant projector (P_{L}), constant spinor (ϵ_0)

$\Rightarrow M := \mathcal{R}^{-1}\Gamma_\kappa \mathcal{R} P_{\mathrm{L}}$ has to be *constant* on some subspace to preserve part of the supersymmetries

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Solving the κ-symmetry constraint

Can be done systematically, separating internal space and AdS.

Internal space: 2 projectors $\sim$ 1/4 BPS ($\mathcal{N}=4$ conf. $\rightarrow$ $\mathcal{N}=2$)
ω in $A = f\omega$ fixed.

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From AdS part w/o additional projectors:

$$\begin{aligned} \cot\theta + \csc\theta\, f' \cosh\rho + 2f \csc^3\theta \sinh\rho = 0 \\ \theta' + \csc\theta\, f' \sinh\rho + 2f \csc^3\theta \cosh\rho = 0 \end{aligned}$$

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Still non-linear, but 1^{st} order and much prettier than the EOM before. Bonus: they can be decoupled!

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These equations can be solved *analytically*:

$$\cos\theta = 2\cos\frac{4\pi + \cos^{-1}\tau}{3}\ , \quad \tau = \frac{6(m\rho - c) + 3m\sinh(2\rho)}{4\cosh^3\rho}$$

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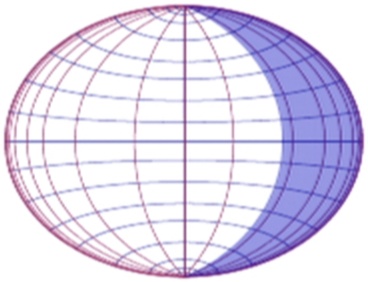
This simple embedding indeed solves the complicated DBI EOM ✓
$m \sim$ flavor mass $M = m\sqrt{\lambda}/2\pi$, $c \sim$ chiral condensate $\langle\overline{\psi}\psi\rangle$

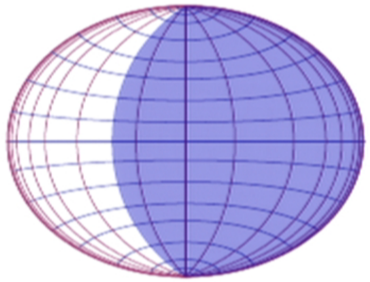
$\theta \sim$ superpotential mass term, $A = f\omega \sim$ extra scalar mass term, with correct relative coefficient [Kruczenski et al. '03;Festuccia,Seiberg '11]

AdS_4 embeddings

Fixing c from IR regularity yields 3 types of embeddings:



short
$\forall m$



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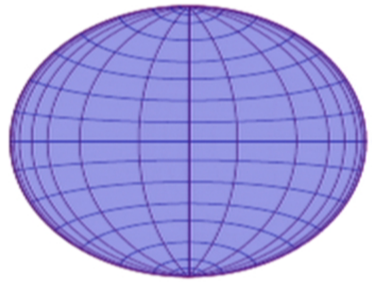
$0 \leq m < m_{\text{crit}}$



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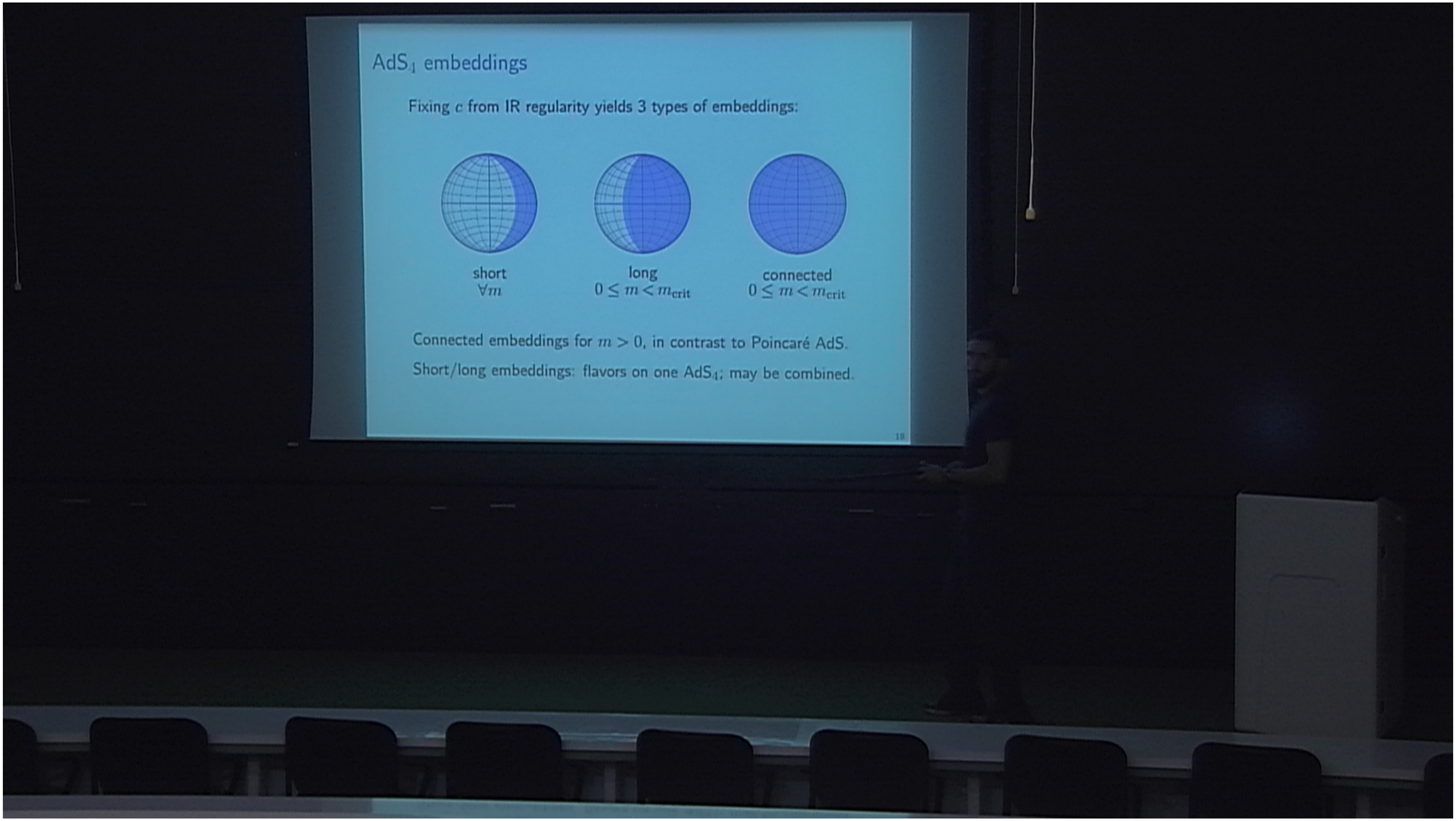
Connected embeddings for $m > 0$, in contrast to Poincaré AdS.

Short/long embeddings: flavors on one AdS_4; may be combined.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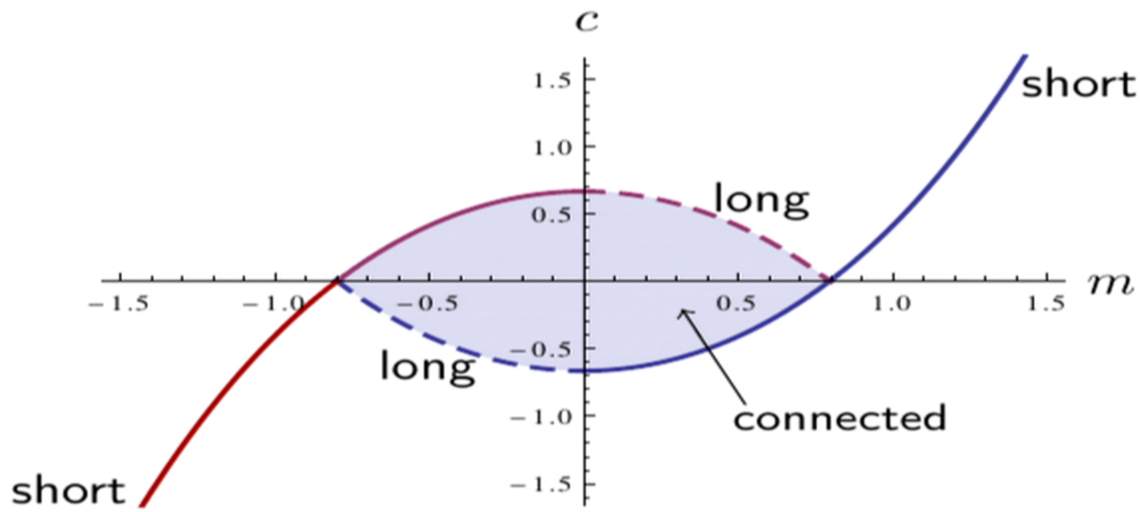

Wrapping up:

D3/D7 configurations to describe $\mathcal{N}=4$ SYM coupled to quenched massive $\mathcal{N}=2$ flavors on curved space.

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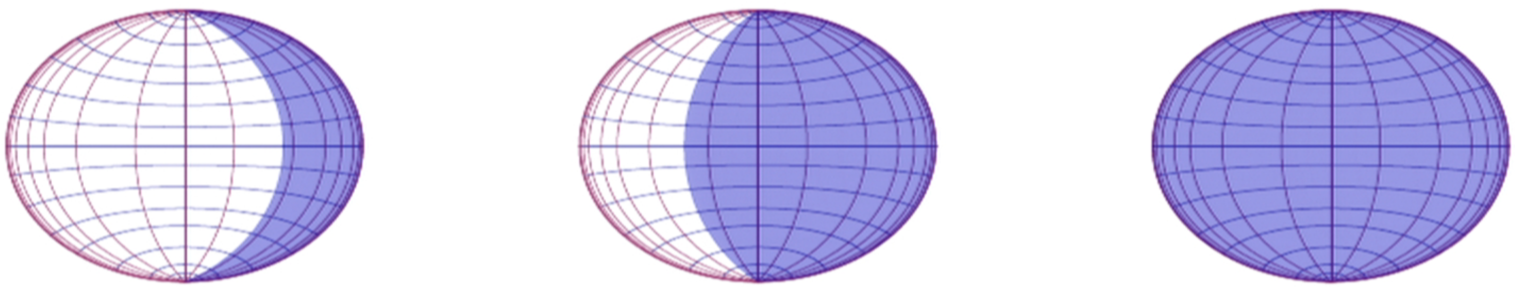

short $\forall m$ | long $0 \leq m < m_{\text{crit}}$ | connected $0 \leq m < m_{\text{crit}}$

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A precision test for AdS/CFT with flavor

Program:

- study phase structure of flavored $\mathcal{N}=4$ SYM on S^4 as function of mass holographically, compute free energy
- rewind & repeat, using supersymmetric localization
- confront flavored AdS/CFT with field theory results

D7-brane embeddings $S^4 \hookrightarrow AdS_5$

Background from AdS_4 slicing via analytic continuation

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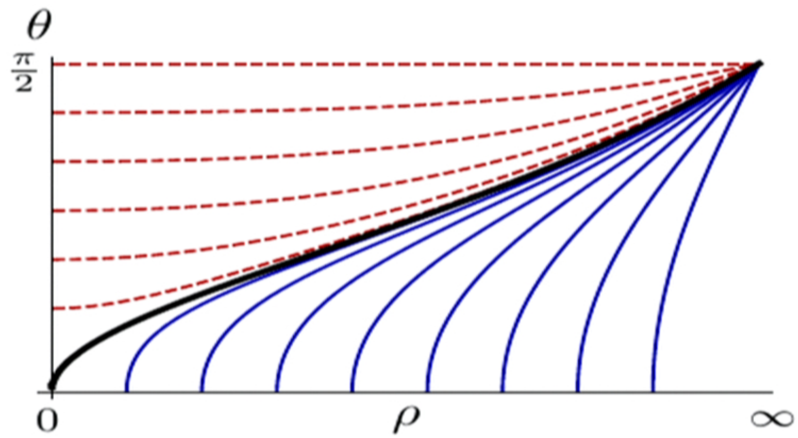
Final solutions still related by $\rho \to \rho - \frac{i\pi}{2}$

$$\theta = \theta_{AdS_4}\big|_{\rho\to\rho-\frac{i\pi}{2}} \qquad f = \frac{i}{2}\sin^3\theta\left(\sinh\rho\,\cot\theta - \theta'\cosh\rho\right)$$

Imaginary scalar mass term – non-unitary on dS_4 [Pestun '07]

D7-brane embeddings $S^4 \hookrightarrow AdS_5$



- phase transition at $m = 1$ from degenerating S^3 to S^4
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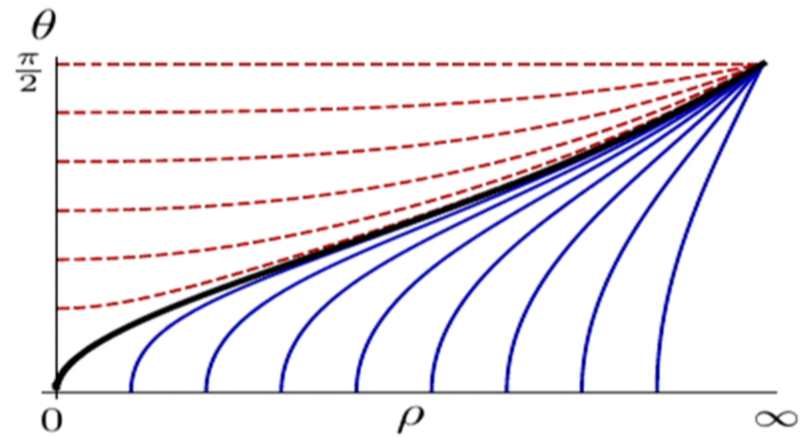

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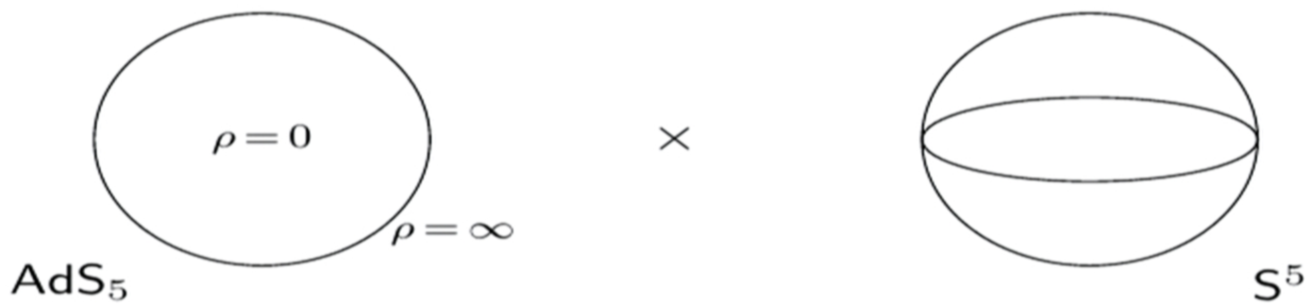

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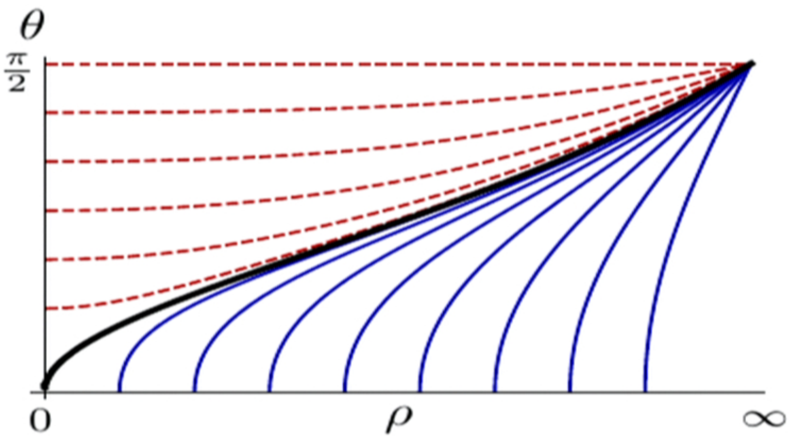

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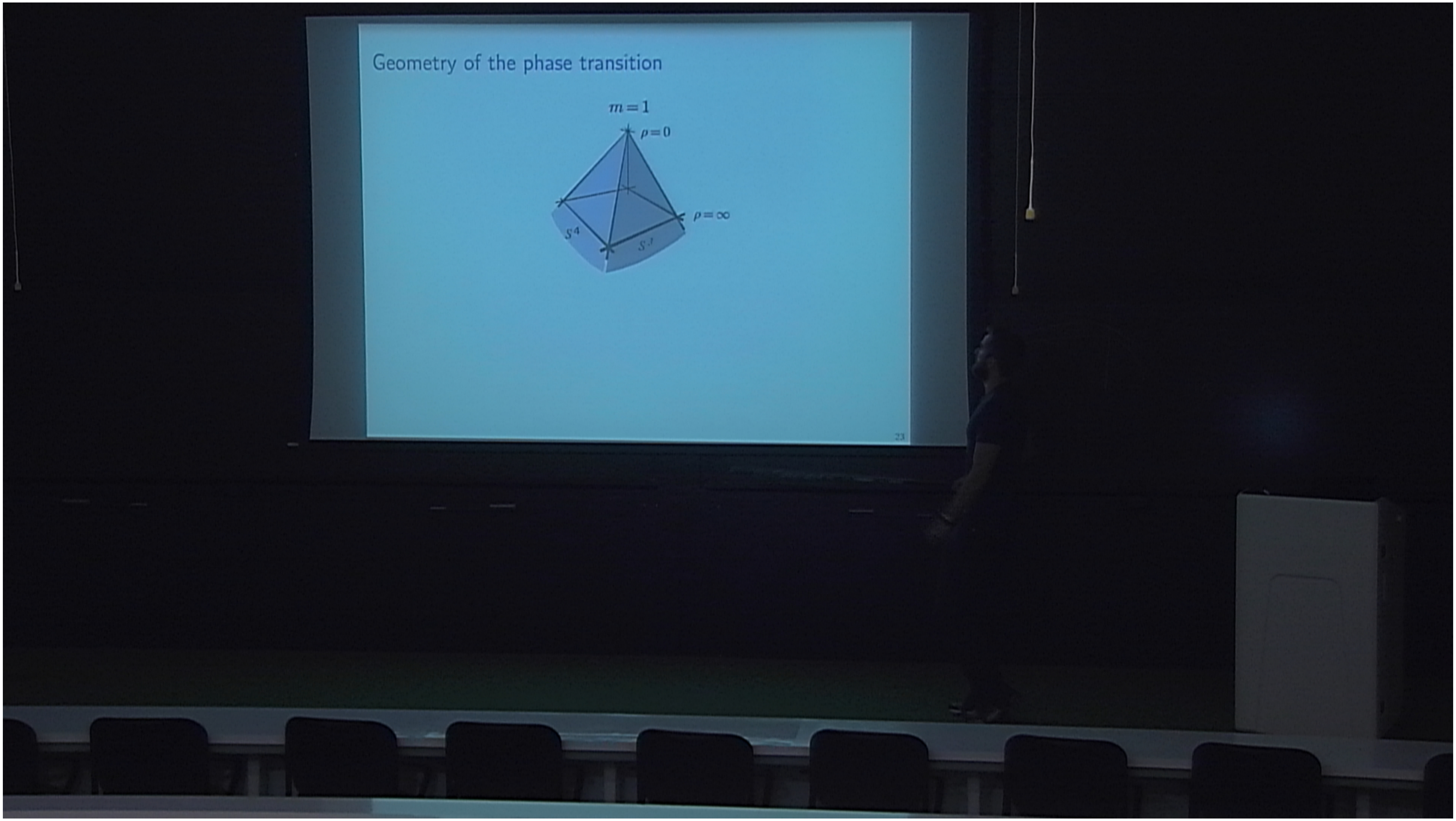
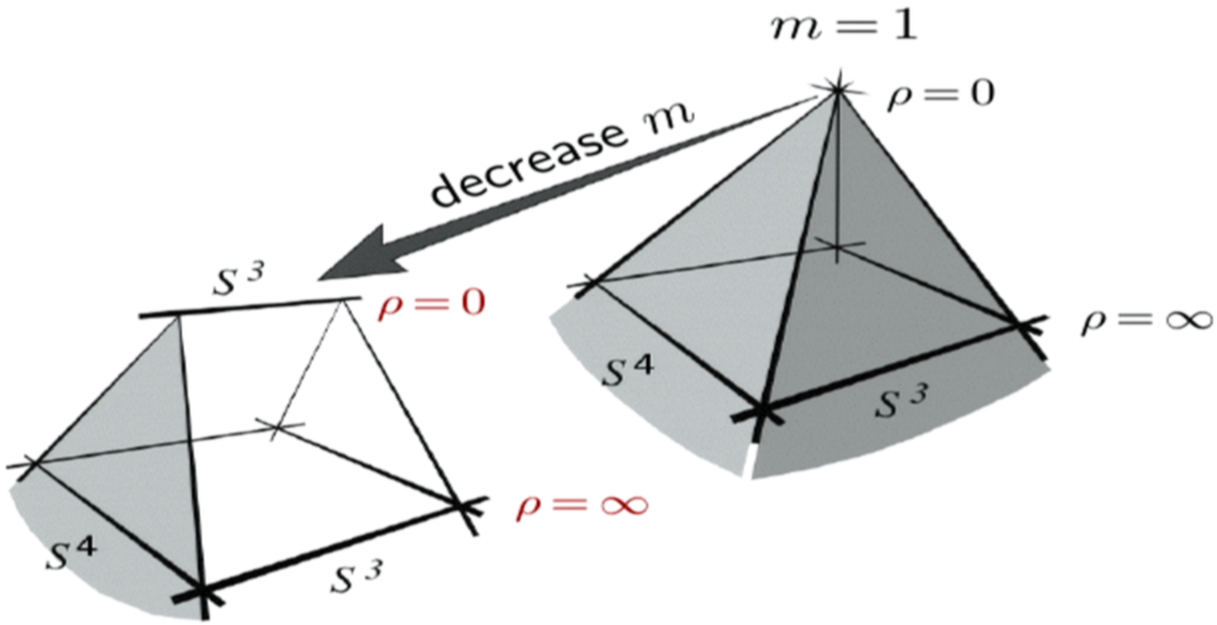
$m = 1$:


Geometry of the phase transition
$m=1$
$\rho=0$
$\rho=\infty$
S^4
23

Geometry of the phase transition



S^4 in AdS_5 degenerates
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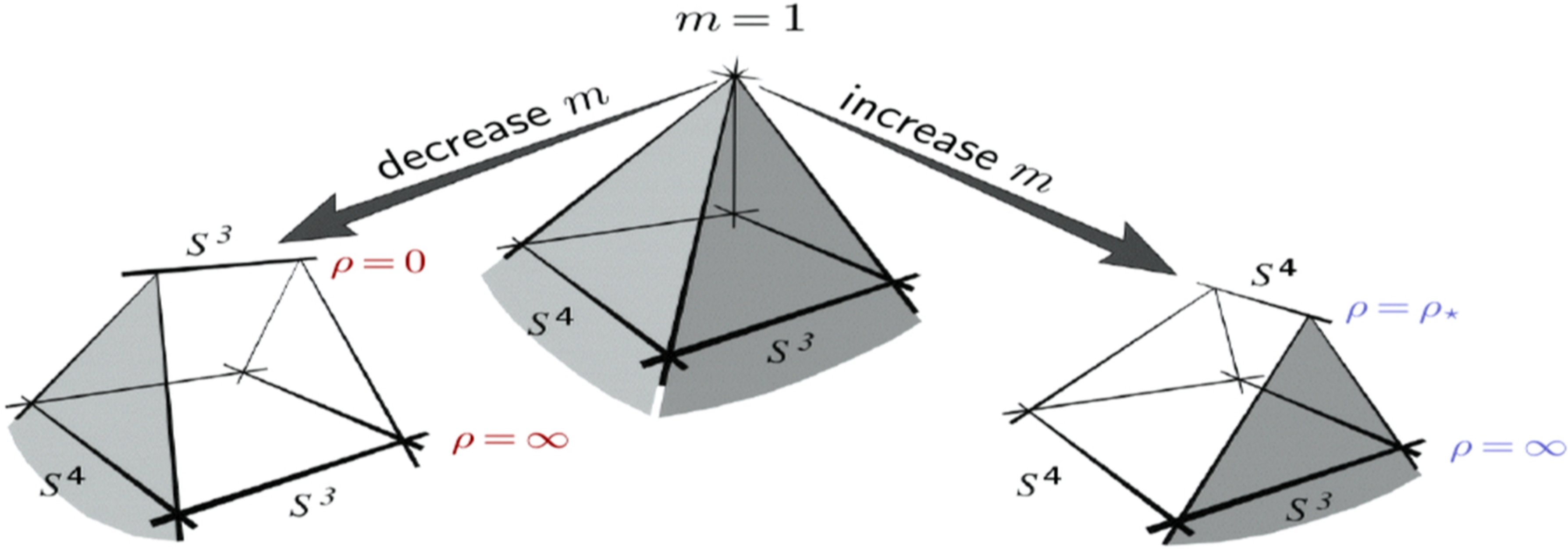
Geometry of the phase transition



S^4 in AdS_5 degenerates at $\rho = 0$

S^3 in S^5 degenerates at $\rho = \rho_\star$

Free energy and critical exponent

Two one-point functions from holographically renormalized on-shell action ($\mu \equiv \sqrt{\lambda}/2\pi$):

$$\mu\langle\mathcal{O}_\theta\rangle = -\frac{1}{\sqrt{g_{S^4}}}\frac{\delta S_{\mathrm{D7,ren}}}{\delta\theta^{(0)}} \qquad \mu\langle\mathcal{O}_f\rangle = -\frac{1}{\sqrt{g_{S^4}}}\frac{\delta S_{\mathrm{D7,ren}}}{\delta f^{(0)}}$$

Varying *within* susy configurations: $\delta\theta^{(0)} = i\delta f^{(0)}$.

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$\rightarrow$ flavor contribution to free energy $F^{(1)}$:

$$\langle\mathcal{O}_s\rangle := \langle\mathcal{O}_\theta\rangle + i\langle\mathcal{O}_f\rangle = \frac{1}{V_{\mathrm{S}^4}}\frac{dF^{(1)}}{dM}$$

Free energy and critical exponent

Finite counterterms $\sim M^4, M^2R^2$ introduce scheme dependence

$$V_{S^4}\langle\mathcal{O}_s\rangle = \frac{2}{3}\mu N_f N\left[3c + \frac{2+12\alpha_1}{3}m^3 - \frac{7+4\beta}{2}m\right]$$

$\alpha_1 = -\frac{5}{12}$ to preserve susy, term linear in m scheme dependent

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The interesting part is c, determined from IR regularity:

$$c_{m>1} = \frac{m^2+2}{3}\sqrt{m^2-1} + m\log\left(m - \sqrt{m^2-1}\right)$$
$$c_{m\leq 1} = 0$$

This $V_{S^4}\langle\mathcal{O}_s\rangle$ is what we will compare to the QFT calculation.

Free energy and critical exponent

Condensate $\langle\mathcal{O}_s\rangle$ non-analytic at $m = 1$. For $m = 1 + \epsilon$:

$$\langle\mathcal{O}_s\rangle = \frac{\mu N_f N_c}{V_{S^4}}\left[\frac{1}{3} - (1+\epsilon)\log\frac{\mu^2}{4} - \epsilon - 2\epsilon^2 + \frac{16\sqrt{2}}{15}\epsilon^{5/2} + \dots\right]$$

$\rightarrow$ first non-analytic term $\propto \epsilon^{5/2}$

Compare to non-susy embeddings: [Karch,O'Bannon,Yaffe '09]

$$\langle\mathcal{O}_\theta\rangle = \text{analytic} + \#\epsilon^\alpha + \dots \qquad \alpha = \frac{4+\sqrt{2}}{4-\sqrt{2}}$$

Imaginary gauge field changes scaling analysis – susy's different.

Supersymmetric localization

Assume there is a symmetry generator Q with $Q^2 \approx 0$:

$$\mathcal{Z} = \int D\phi e^{-S[\phi]}$$

∞-dim configuration space

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With $\mathcal{Z}$ independent of t, might as well do $t \to \infty$:

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Reduced integral from generic configurations to extrema of QV.

$\mathcal{N}=4$ SYM with quenched $\mathcal{N}=2$ flavors

$\mathcal{N}=4$ SYM on S^4: [Pestun '07]

- pick supercharge Q, $QV = (Q\Psi, \overline{Q\Psi})$ positive semidefinite

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$\mathcal{N}=4$ SYM on S^4: [Pestun '07]

- pick supercharge Q, $QV = (Q\Psi, \overline{Q\Psi})$ positive semidefinite
- extrema of QV parametrized by <u>one constant</u> $\mathfrak{u}(N)$ matrix

$\rightarrow$ Gaussian matrix model

$$\mathcal{Z} = \int da^{N-1} \prod_{i<j} a_{[ij]}^2 \, e^{S_0} \,, \qquad S_0 = -\frac{8\pi^2}{\lambda} N \sum_i a_i^2$$

At large N: Wigner semicircle distribution ρ_{w}

$\mathcal{N}=4$ SYM with quenched $\mathcal{N}=2$ flavors

Adding fundamental hypers: only new contribution are 1-loop fluctuations

$$\mathcal{Z} = \int d^{N-1}a \, \frac{\prod_{i<j} a_{[ij]}^2}{\prod_i \sqrt{H_+^{N_f}(a_i) H_-^{N_f}(a_i)}} \, e^{S_0}$$

$H_\pm(x) = H(x \pm M)$, $H(x) = G(1 - ix)G(1 + ix)$

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Leading-order flavor correction to $F = -\log \mathcal{Z}$ at large N:

$$F^{(1)} = -\delta \hat{S} = -\delta_\rho \hat{S}^{(0)}\Big|_{\rho_{\mathrm{w}}} \delta\rho \; - \; \hat{S}^{(1)}\big|_{\rho_{\mathrm{w}}}$$

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ρ_{w} extremizes $\hat{S}^{(0)}$

$\mathcal{N}=4$ SYM with quenched $\mathcal{N}=2$ flavors

Leading-order correction to F:

$$F^{(1)} = \frac{N_f N}{2} \int_{-\mu}^{\mu} dx \rho_{\rm w}(x) \log\left[H_+(x) H_-(x)\right]$$

Wigner semicircle: $\rho_{\rm w} = \dfrac{2}{\pi\mu^2}\sqrt{\mu^2 - x^2}$, $\mu = \sqrt{\lambda}/2\pi$

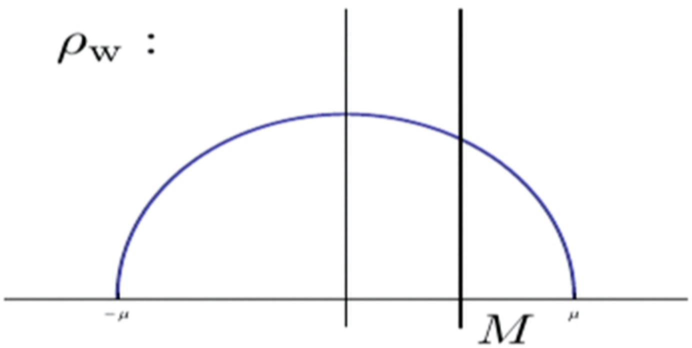
$\mathcal{N}=4$ SYM with quenched $\mathcal{N}=2$ flavors

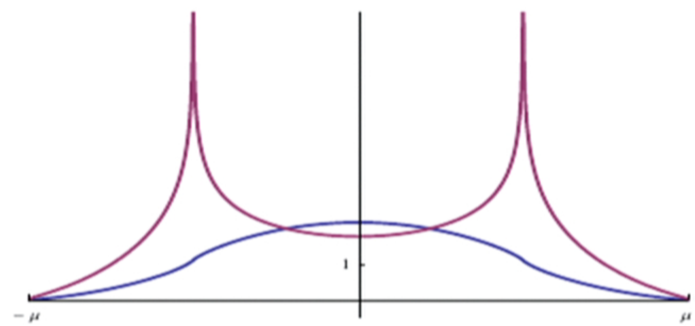
Leading-order correction to F' with $M, \lambda \gg 1$:

$$\frac{dF^{(1)}}{dM} = \frac{N_f N}{2} \int_{-\mu}^{\mu} dx \rho_{\mathrm{w}}(x) \left[4M - x_+ \log x_+^2 - x_- \log x_-^2\right]$$

Wigner semicircle: $\rho_{\mathrm{w}} = \dfrac{2}{\pi\mu^2}\sqrt{\mu^2 - x^2}$, $\mu = \sqrt{\lambda}/2\pi$

Phase transition: $M > \mu$ vs. $M < \mu$





AdS vs. CFT face-off:

Evaluating the matrix-model integral:

$$M > \mu : \quad F'^{(1)} = \frac{N_f N}{3\mu^2}\Big[2\sqrt{M^2-\mu^2}(M^2+2\mu^2) - 2M^3 + 3M\mu^2\Big(1 - 2\log\frac{M+\sqrt{M^2-\mu^2}}{2}\Big)\Big]$$

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Phase transition at same mass as in holographic analysis ✓

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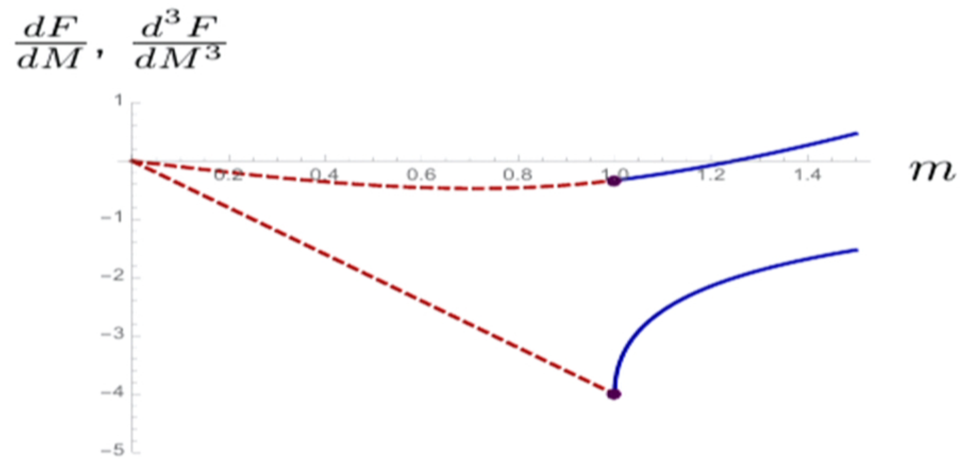
universal non-analytic terms in $F'^{(1)}$ match as functions ✓

scheme-dependent terms can be matched across transition ✓

AdS vs. CFT face-off:

A single continuous quantum phase transition in holographic and QFT analyses – at the same mass and with matching free energies.



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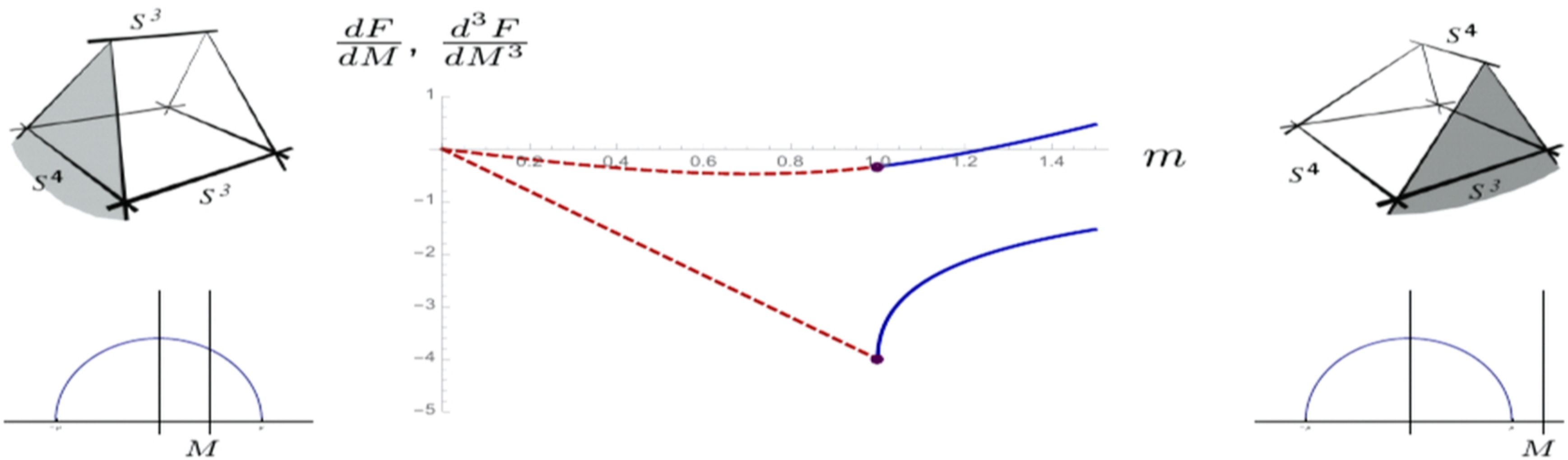

Conclusion

Holographic description for $\mathcal{N}=4$ SYM coupled to quenched massive $\mathcal{N}=2$ flavors on AdS_4, S^4, dS_4.

In spite of (understandable) skepticism towards probe brane constructions and D3/D7: AdS/CFT seems to work fine.

Interesting phenomenology – topology-changing quantum phase transition on S^4, rich phase diagram on AdS_4