Title: Technology as Foundation, from the Quantum Informational Viewpoint: The Future (and some Past) of Quantum Theory after the Higgs Boson

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Abstract: <p>The talk first offers a brief assessment of the realist and nonrealist understanding of quantum theory, in relation to the role of probability and statistics there from the perspective of quantum information theory, in part in view of several recent developments in quantum information theory in the work of M. G. Dâ€™Ariano and L. Hardy, among others. It then argues that what defines quantum theory, both quantum mechanics and quantum field theory, most essentially, including as concerns realism or the lack thereof and the probability and statistics, is a new (vs. classical physics or relativity) role of technology in quantum physics. This role was first considered by Bohr in his analysis of the fundamental role of measuring instruments in the constitution of quantum phenomena, which, he argued, is responsible for the difficulties of providing a realist description of quantum objects and their behavior, and, correlatively, for the irreducibly probabilistic or statistical nature of all quantum predictions. In this paper, I mean â€œtechnologyâ€• in a broader sense, akin to what the ancient Greeks called â€œtekhneâ€• (â€œtechniqueâ€•). It refers the means by which we create new mental and material constructions, such as mathematical, scientific, or philosophical theories or works of art and architecture, or machines, and through which we interact with the world. I shall consider three forms of technologyâ€”mathematical, experimental, and digital. The relationships among them were crucial to the discovery of the Higgs boson, and, I argue, are likely to remain equally crucial, indeed unavoidable, in the future of physics, especially quantum physics.</p>

TECHNOLOGY AS FOUNDATION, FROM THE QUANTUM INFORMATIONAL VIEWPOINT: THE FUTURE (AND SOME PAST) OF QUANTUM THEORY AFTER THE HIGGS BOSON

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Related articles (both are on ArXiv):
A. Plotnitsky, "A Matter of Principle: The Principles of Quantum Theory, Dirac's Equation, and Quantum Information," ***Found Phys. DOI 10.1007/s10701-015-9928-z (2015)***

A. Plotnitsky and A. Khrennikov, "Reality Without Realism: On the Ontological and Epistemological Architecture of Quantum Mechanics," ***Found Phys. DOI 10.1007/s10701-015-9942-1(2015)***

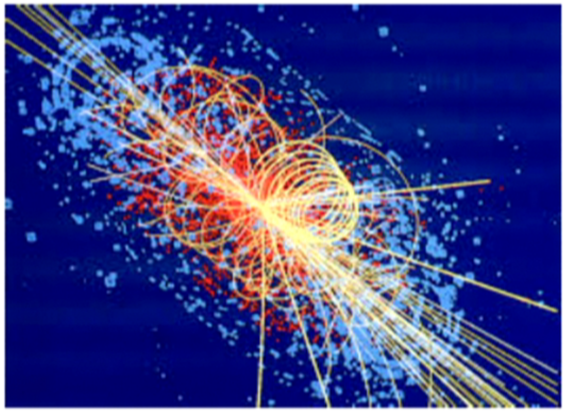
A Philosophy of Physics as A Philosophy of Thought

Philosophical thinking: the invention of new concepts
Examples: Plato's ideas, Descartes' Cogito, Leibniz's Monads, etc.

Mathematical and Physical Thinking: the invention of new mathematical or in physics hybrid concepts and frames of references, and logical relations between frames of reference.

Examples: The concept of motion in classical physics, defined by the concepts of coordinate and velocity or momentum, determinable at any point; Heisenberg's concept of new quantum variables as (unbounded) matrices with complex coefficients, from which one derives (Born's rule) the probability of the electron's transitions between energy levels in an atom.

Art: the invention of new compositions.



What does it really mean and, correlatively, how this is possible?

"Inquiry into nature is a search for the causes of each thing; why each thing comes into existence, why it goes out of existence, why it exists."
Plato, *Phaedo*, 4th Century, BC (96 a 6–10)

"If, therefore, we experience that something happens, then we always presuppose that something else precedes it, which it *follows* in accordance with a rule."
Kant, I. *Critique of Pure Reason*, 1781

"Causality postulates that there are laws by which the occurrence of an entity B of a certain class depends on the occurrence of an entity A of another class, where the word entity means any physical object, phenomenon, situation, or event. A is called the cause, B the effect."
"Antecedence postulates that the cause must be prior to, or at least simultaneous with, the effect."
"Contiguity postulates that cause and effect must be in spatial contact or connected by a chain of intermediate things in contact."
(Born 1949, *Natural Philosophy of Cause and Chance,* 1949, p. 9).

"the discovery of the quantum of action … implies a shortcoming of the causality ideal itself. … "
N. Bohr "The Causality Problem in Atomic Physics," 1938)

"[I]t is most important to realize that the recourse to probability laws under such circumstances is essentially different in aim from the familiar application of statistical considerations as practical means of accounting for the properties of mechanical systems of great structural complexity. In fact, in quantum physics we are presented not with intricacies of this kind, but with the inability of the classical frame of concepts to comprise the peculiar feature of indivisibility, or 'individuality,' characterizing the elementary processes" N. Bohr, "Discussion with Einstein," 1949)

“Constructive theories build up a picture of the more complex phenomena out of the materials of a relatively simple formal scheme from which they start out,” which implies that this simpler formal scheme describes, at least ideally and in principle, the ultimate underlying reality responsible for this phenomena.

Example: the kinetic theory of gases, which “seeks to reduce mechanical, thermal, and diffusional processes to movements of molecules—i.e., to build them up out of the hypothesis of molecular motion,” described by the laws of classical mechanics.

“Principle theories employ the analytic, not the synthetic, method. The elements which form their basis and starting point are not hypothetically constructed but empirically discovered ones, general characteristics of natural processes, principles that give rise to *mathematically formulated criteria* which the separate processes or the theoretical representations of them have to satisfy.”

Example: thermodynamics, (parallel to the kinetic theory of gases as a constructive theory), which “seeks by analytical means to deduce necessary conditions, which separate events have to satisfy, from the universally experienced fact that perpetual motion is impossible.”

A. Einstein, “What is Relativity?” 1920

Definition: “Principles are empirically [established or assumed] general characteristics of natural processes, … that give rise to *mathematically formulated criteria* which the separate processes or the theoretical representations of them have to satisfy.”
The requirement that principles give rise to “mathematically formulated criteria” is crucial.

In both Heisenberg's initial approach to quantum mechanics and Bohr's initial interpretation of the theory the key principles were:

(1) the principle of quantum discreteness or the QD principle (initially defined by what Bohr called "the quantum postulate," "symbolized," by Planck's quantum of action, *h*, according to which all observed quantum phenomena are individual and discrete in relation to each other, which is not the same as the (Democretian) atomic discreteness of quantum objects themselves;
(2) the principle of the probabilistic or statistical nature of quantum predictions, even, in contradistinction to classical statistical mechanics, in the case of elemental, unsubdivisible, quantum processes and events—the quantum probability, QP, principle, and
(3) the correspondence principle, which, as initially used by Bohr required that the predictions of quantum theory must coincide with those of classical mechanics at the classical limit (even though the processes themselves were still quantum at this limit), but which was given by Heisenberg a new, more rigorous and precise form, "the mathematical correspondence principle," requiring that the equations of quantum mechanics convert into those of classical mechanics at the classical limit.

Bohr's interpretation added a new principle:

(4) the complementarity principle, which stems from the concept of complementarity, introduced by Bohr a bit later (following Heisenberg's discovery of the uncertainty relations), and which requires:

***(a)* a mutual exclusivity of certain phenomena, entities, or conceptions; and yet**

***(b)* the possibility of applying each one of them separately at any given point, and**

***(c)* the necessity of using all of them at different moments for a comprehensive account of the totality of phenomena that one must consider in quantum physics.**

The REALITY WITHOUT REALISM (RWR) principle:
"In quantum mechanics we are not dealing with an arbitrary renunciation of a more detailed analysis of atomic phenomena, but with a recognition that such an analysis is *in principle* excluded [beyond a certain point]."
--Niels Bohr, "Discussion with Einstein on Epistemological Problems in Atomic Physics" (1949)

The RWR Principle:

Incompatible with classical causality:

"if a classical state [defined by the ideally definite position and the definite momentum of an object at any moment of time] an object does not exist at any moment, it can hardly change causally"

Schrödinger 1935, "The Present Day Situation in Quantum Mechanics" [The Cat-Paradox Paper], p.154.

Compatible with quantum causality.

The RWR principle could be inferred from the complementarity principle, because the latter prevents us from ascertaining the complete composition of the "whole from parts," to the degree, this concept applies, because the complementary parts never add to a whole in the way they do in classical physics or relativity. This became especially apparent in view of the EPR-type experiments

Why RWR principle?:

"It is not surprising that our language should be incapable of describing processes occurring within atoms, for, as has been remarked, it was invented to describe the experiences of daily life, and these consist only of processes involving exceedingly large numbers of atoms. Furthermore, it is very difficult to modify our language so that it will be able to describe these atomic processes, for words can only describe things of which we can form mental pictures, and this ability, too, is a result of daily experience. Fortunately, mathematics is not subject to this limitation, and it has been possible to invent a mathematical scheme—the quantum theory—which seems entirely adequate for the treatment of atomic processes."
Werner Heisenberg, *The Physical Principles of the Quantum Theory, 1930*

One can get an idea of the quantum smallness in relation to observation from the following perspective. We may, it is sometimes observed, be lucky to have the solar system, visible to our naked eye, as a model on which, suitably idealized, we were able to build classical physics. If, on the other hand, in order to observe the Moon we would have to shoot Jupiter into it and study the debris, the situation would be quite different. We would see a picture not entirely unlike photographs from which we infer the existence of the Higgs boson.

"In this theory [matrix mechanics] the attempt is made to transcribe every use of mechanical concepts in a way suited to the nature of the quantum theory, and such that in every stage of the computation only directly observable quantities enter. In contrast to ordinary mechanics, the new mechanics does not deal with a space-time description of the motion of atomic particles. It operates with manifolds of quantities which replace the harmonic oscillating components of the motion and symbolize the possibilities of transitions between stationary states [No longer orbits!]. . . . These quantities satisfy certain relations which take the place of the mechanical equations of motion and the quantization rules [of the old quant.um theory]."
N. Bohr, "Atomic Theory and Mechanics" (1925)

A hydrogen atom = data/information ---------------→ spectra = data/information

1) Mathematical scheme = Matrix (non)mechanics
2) Physical Interpretation/information processing (between technological devices)
3) Philosophical interpretation (ontology, epistemology, etc.)

"It should be distinctly understood, however, that this [the deduction of the fundamental equation of quantum mechanics] cannot be a deduction in the mathematical sense of the word, since the equations to be obtained form themselves the *postulates* of the theory. Although made highly plausible by the follows considerations [given in the Appendix], their ultimate justification lies in the agreement of their predictions with the experiment" W. Heisenberg, *The Physical Principles of Quantum Theory*, 1930, p. 108; emphasis added.

Heisenberg's new variables, the main elements of his deduction is a hybrid—physical-mathematical—object.

"[I]n quantum theory it has not been possible to associate the electron with a point in space, *considered as a function of time*, by means of observable quantities. However, even in quantum theory it is possible to ascribe to an electron the emission of radiation. In order to characterize this radiation we first need the frequencies which appear as functions of two variables. In quantum theory these functions are in the form [originally introduced by Bohr]:

$$\nu(n, n-\alpha) = 1/h\ \{W(n) - W(n-\alpha)\} \quad (1)$$

and in classical theory in the form

$$\nu(n,\alpha) = \alpha\nu(n) = \alpha/h(dW/dn)\text{"}$$ (Heisenberg 1925, p. 263).

This difference, reflecting the quantum discreteness (QD) principle, leads to a difference between classical and quantum theories as concerns the combination relations for frequencies, which correspond to the Rydberg-Ritz combination rules.

However, "in order to complete the description of radiation [in accordance, by the mathematical correspondence principle, with the Fourier representation of classical kinematics] it is necessary to have not only frequencies but also the amplitudes" [p. 263]. The crucial point is that, in Heisenberg's theory and in quantum mechanics since then, these "amplitudes" are no longer amplitudes of any physical motions, which makes the name "amplitude" itself an artificial, *symbolic* term. These amplitudes are linked to the probabilities of transitions between stationary states: they are what we now call probability amplitudes. The corresponding probabilities are derived, from Heisenberg's matrices, by a form of Born's rule for this limited case (Born's rule is more general).

The mathematical structure thus emerging is in effect that of vectors and (in general, noncommuting) Hermitian operators in Hilbert spaces over complex numbers, which spaces are in this case, infinite-dimensional, given that we deal with continuous variable. This structure may be seen as a mathematical expression of the QP principle. Heisenberg explains the situation in these, more rigorous, terms in his 1930 book. In his original paper, he argues as follows:

> The amplitudes may be treated as complex vectors, each determined by six independent components, and they determine both the polarization and the phase. As the amplitudes are also functions of the two variables n and α, the corresponding part of the radiation is given by the following expressions:
>
> **Quantum-theoretical:**
>
> $$\mathrm{Re}\{A(n, n-\alpha)e^{i\omega(n, n-\alpha)t}\}$$
>
> **Classical:**
>
> $$\mathrm{Re}\{A_\alpha(n)e^{i\omega(n)\alpha t}\}$$ (p. 263)

The problem—a difficult and, "at first sight," even insurmountable problem—is now apparent: "[T]he phase contained in A would seem to be devoid of physical significance in quantum theory, since in this theory frequencies are in general not commensurable with their harmonics" [25, pp. 263-264]. Heisenberg now proceeds to inventing a new theory around this problem, in effect, by making it into a solution, by, as it were, saying: "However, we shall see presently that also in quantum theory the phase had a definitive significance which is *analogous* to its significance in quantum theory" [25, p. 264; emphasis added). "Analogous" could only mean here that, rather than being analogous physically, the way the phase functions mathematically is analogous to the way the classical phase functions mathematically in classical theory, or analogous in accordance with the *mathematical* form of the correspondence principle, insofar as quantum-mechanical equations are formally the same as those of classical physics.

In this way, Heisenberg gave the correspondence principle a mathematical expression, indeed changed it into the mathematical correspondence principle. The variables to which these equations apply cannot, however, be the same, because, if they are, they will not give us correct predictions for low quantum numbers. As Heisenberg explains, if one considers "a given quantity $x(t)$ [a coordinate as a function of time] in classical theory, this can be regarded as represented by a set of quantities of the form

$$A_\alpha(n) e^{i\omega(n)\alpha t},$$

which, depending upon whether the motion is periodic or not, can be combined into a sum or integral which represents $x(t)$:

$$x(n, t) = \sum_{-\infty}^{+\infty} A_\alpha(n)\, e^{i\omega(n)\alpha t}$$

or

$$x(n, t) = \int_{-\infty}^{+\infty} A_\alpha(n)\, e^{i\omega(n)\alpha t}\, d\alpha \text{ ''} \quad \text{[p. 264].}$$

He notes that "a similar combination of the corresponding quantum-theoretical quantities seems to be impossible in a unique manner and therefore not meaningful, in view of the equal weight of the variables n and $n - \alpha$" (p. 264). "However," he says, "one might readily regard the ensemble of quantities $A\,(n, n - \alpha)e^{i\omega(n,\, n - \alpha)t}$ [an infinite square matrix] as a representation of the quantity $x(t)$" (p. 264).

The arrangement of the data into square tables is a brilliant and, as I said, in retrospect, but, again, only in retrospect, natural way to connect the relationships (transitions) between two stationary states. However, it does not by itself establish an *algebra* of these arrangements, for which one needs to find the rigorous rules for adding and multiplying these elements—rules without which Heisenberg cannot use his new variables in the equations of the new mechanics. To produce a *quantum-theoretical interpretation* (which, again, abandons motion and other concepts of classical physics at the quantum level) of the classical equation of motion that he considered, as applied to these new variables, Heisenberg needs to be able to construct the powers of such quantities, beginning with $x(t)^2$, which is actually all that he needs for his equation. The answer in classical theory is obvious and, for the reasons just explained, obviously unworkable in quantum theory. Now, "in quantum theory," Heisenberg proposes, "it seems that the simplest and most natural assumption would be to replace classical [Fourier] equations … by

$$B(n, n-\beta)e^{i\omega(n,\, n-\beta)t} = \sum_{+\infty}^{+\infty} A(n, n-\alpha)A(n-\alpha, n-\beta)e^{i\omega(n,\, n-\beta)t}$$

or

$$= \int_{-\infty}^{+\infty} A(n, n-\alpha)A(n-\alpha, n-\beta)e^{i\omega(n,\, n-\beta)t}\, d\alpha$$

" (p. 265).

Heisenberg's overall scheme essentially amounts to the Hilbert-space formalism (with Heisenberg's matrices as operators), introduced by J. von Neumann shortly thereafter, thus giving firm and rigorous mathematical foundations to Heisenberg's scheme, by then developed more properly by Heisenberg himself, M. Bohr and P. Jordan, and, differently (in terms of *q*-numbers), by Dirac. Dirac, who followed Heisenberg's principle way of thinking in his work on both quantum mechanics and quantum electrodynamics, was also the first to fully realize that noncommutativity was the most essential mathematical feature of Heisenberg's scheme. Remarkably, Heisenberg himself, as well as Pauli, not only did not think it to be essential but also thought that ultimately the theory should be freed from it, and Pauli initially thought that the theory should not be probabilistic either, and changed his mind on both counts only after Schrödinger's equation was introduced.

"It will interest mathematical circles that the mathematical *instruments* created by the higher algebra play an essential part in the rational formulation of the new quantum mechanics. Thus, the general proofs of the conservation theorems in Heisenberg's theory carried out by Born and Jordan are based on the use of the theory of matrices, which go back to Cayley and were developed especially by Hermite. It is to be hoped that a new era of mutual stimulation of mechanics and mathematics has commenced. To the physicists it will at first seem deplorable that in atomic problems we have apparently met with such a limitation of our usual means of visualization. This regret will, however, have to give way to thankfulness that mathematics in this field, too, presents us with the tools to prepare the way for further progress."

--Niels Bohr, "Atomic Theory and Mechanics" (1925)

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"*Physics to mathematics correspondence principle*. For any physical theory, there exists a small number of simple hybrid statement that enable us to translate from the physical description to the corresponding mathematical calculation such that the mathematical calculation (in appropriate notation) looks the same as the physical description (in appropriate notation).

Such a principle might be useful in obtaining new physical theories (such as a theory of quantum gravity). Related ideas to this have been considered by category theorists [B. Coecke, E. O. Paquette, Categories for the practising physicist, arXiv:0905.3010 (2009).]. A category of physical processes can be defined corresponding to the physical description. A category corresponding to the mathematical calculation can also be given. The mapping from the first category to the second is given by a functor (this takes us from one category to another)."

L. Hardy, "A formalism-local framework for general probabilistic theories including quantum theory," 2010

* * *

An Interlude: Brief Reflections of QBism/Cubism

Pablo Picasso, Le Vieux Marc (oil on canvas), 1912.

QBism puts the scientist back into science
N. David Mermin, Nature, 26 March 2014

Jean Metzinger, ***La Femme au Cheval, A Woman on a Horse 1911, purchased by Niels Bohr in 1932.***

"The artist has broken down the picture plane into facets, presenting multiple aspects of the subject simultaneously. This concept first pronounced by Metzinger in 1910—since considered a founding principle of Cubism—would soon find its way, via complementarity, into the foundations of the Copenhagen interpretation of quantum mechanics; the fact that a complete description of one and the same subject may require diverse points of view which defy a unique description."

--Wikipedia, La Femme au Cheval

Georges Braque, *Violin and Candlestick*, 1910

1. **One could experience a cubist painting aesthetically, compositionally, and in this sense as abstracted from reality, the reality of the material or human world.**

2. **If, however, one wants to *relate* such a painting to either reality, one could only do so insofar as one, treating the painting similarly to quantum phenomena, reasonably predict, bet on, such a reality, which is thus constructible from this painting.**

Heisenberg's revolutionary thinking not only introduced a new mathematical model in physics but also established a new way of doing theoretical physics. Indeed, this way of thinking also redefined experimental physics, *or at least one type of understanding of what experimental physics is in quantum regimes.*

(Some might think otherwise concerning what they are doing, for example, discovering and representing the actual nature of quantum objects and processes. Weather they actually do this is another matter.

The practice of experimental physics no longer consists, as in classical physics, in tracking the independent behavior of the systems considered. Instead it consists in *unavoidably* creating configurations, by now almost unbelievable in their complexity (think of the Linear Hadron Collider, where the Higgs boson was discovered), of experimental technology that reflect the fact that what happens is unavoidably defined by what experiments we perform, how we interact quantum objects, rather than only by their independent behavior. These configurations embody the effects of the interactions between quantum objects and measuring instruments, through which effect quantum objects are defined and, when possible, distinguished from one another, while always remaining, qua quantum objects, beyond the rich of quantum theory and even thought itself. My emphasis on "unavoidably" reflects the fact that, while the behavior of classical physical objects is sometimes affected by experimental technology, in general, as Bohr stressed, we can observe classical physical objects without appreciably affecting their behavior, or else we can compensate for this interference so that we can still describe classical objects independently. This does not appear possible in quantum experiments.

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The practice of theoretical physics no longer consists in offering an idealized mathematical description of quantum objects and their behavior. Instead it consists in developing mathematical machinery that is able to predict, in general (again, in accordance with what obtains in experiments) probabilistically, the effects in question, manifested the outcomes of quantum events and of correlations between some of these events.

The situation takes a more radical form in quantum field theory (QFT) and the experimental physics in the corresponding (high-energy) quantum regimes than in quantum mechanics (QM) and the corresponding (low-energy) experimental regimes. While, at least in this type of interpretation, retaining the nonrealist and noncausal epistemology of quantum phenomena in low-energy (QM) regimes, the QFT situation is characterized by, correlatively:

(1) more complex configurations of observed phenomena, defined by the effects of the interaction between quantum objects and measuring instruments and, hence, of configurations of such instruments, in part due to the loss of the identity of a quantum object even within a single experiment, first, exemplified the phenomena of annihilation and creation of electrons in QED regimes;

(2) a more complex structure of theoretical predictions and, hence, of the relationships between a possible mathematical formalism and the measuring instruments involved, and hence quantum objects and ultimately quantum fields, responsible for the effects observed in these instruments;

(3) a more complex than in QM nature of the mathematical formalism of QFT and the mathematical procedures involved, responding to the situation described in (1) and (2).

The discovery of the Higgs boson may be the most recent spectacular example of this situation. In this case, this technology, staggering to begin with, was also supplemented by, arguably, from now on, in turn irreducible, role of computer technology, which thus becomes, and has been from a while a fundamental part of mathematical modeling in physics and beyond. However, many other examples are found throughout the history of high-energy physics, beginning with the discovery of antimatter, a consequence of Dirac's equation, the inaugural event of this history, which reflects and indeed establishes that much more complex mathematical models than those of quantum mechanics are required in quantum field theory.

Dirac's equation:

$$(\beta mc^2 + \sum_{k=1}^{3} \alpha_k p_k c)\psi(x,t) = i\hbar \frac{\partial\psi(x,t)}{\partial t}$$

The new mathematical elements here are the 4×4 matrices α_k and β and the four-component wave function ψ. The Dirac matrices are all Hermitian,

$$\alpha_i^2 = \beta^2 = I_4$$

(I_4 is the identity matrix), and the mutually anticommute:

$$\alpha_i\beta + \beta\alpha_i = 0$$
$$\alpha_i\alpha_j + \alpha_j\alpha_i = 0$$

The above single symbolic equation unfolds into four coupled linear first-order partial differential equations for the four quantities that make up the wave function. The matrices form a basis of the corresponding Clifford algebra. One can think of Clifford algebras as quantizations of Grassmann's exterior algebras, in the same way that the Weyl algebra is a quantization of symmetric algebra. Here, *p* is the momentum operator in Schrödinger's sense, but in a more complicated Hilbert space than in the standard quantum mechanics. The wave function ψ *(t, x)* takes value in a Hilbert space $X = C^4$ (Dirac's spinors are elements of *X*). For each *t*, *y (t, x)* is an element of

$$H = L^2 (R^3; X) = L^2 (R^3) \otimes X = L^2 (R^3) \otimes C^4.$$

This mathematical architecture allows one to predict the probabilities of quantum-electro-dynamical (high-energy) events, which, as explained below, have a greater complexity than quantum-mechanical (low-energy) events.

Finding new matrix-type variables or, more generally, Hilbert-space operators became the defining mathematical element of quantum theory. The current theories of weak forces, electroweak unifications, and strong forces (quantum chromodynamics) were all discovered by finding such variables. This is correlative to establishing the transformation group, a Lie group, of the theory and finding representations of this group in the corresponding Hilbert spaces. n modern elementary-particle theory, irreducible representations of such groups correspond to elementary particles, the idea that was one of Wigner's major contributions to quantum physics. This was, for example, how M. Gell-Mann discovered quarks, because at the time there were no particles corresponding to the irreducible representations (initially there were three of those, corresponding to three quarks) of the symmetry group of the theory, the so-called SO (3). It is the group of all rotations around the origin in the three-dimensional space, R^3, rotations represented by all three by three orthogonal matrices with determinant 1. (This group is noncommutative.) The electroweak group that Gell-Mann helped to find as well is SU (2), the group of two by two matrices with the determinant 1. Quarks are part of both theories.

The genealogy of this group-theoretical thinking extends from Dirac's four by four matrices and, earlier, Pauli's two by two spin matrices.

$$H = L^2 (R^3; X) = L^2 (R^3) \otimes X = L^2 (R^3) \otimes C^4$$

Other forms of quantum field theory give this type of architecture an even greater complexity, keeping in mind that, as quantum mechanics, Dirac's equation or, more generally, quantum field theory only provides probabilities for the outcomes of quantum events, registered in measuring instruments.

[Dirac's discovery of antimatter is] perhaps the biggest change of all the big changes in physics of our century … because it changed our whole picture of matter … It was one of the most spectacular consequences of Dirac's discovery that the old concept of the elementary particle collapsed completely"

--Werner Heisenberg, "What is an Elementary Particle?" (1973)

D'Ariano et al and Hardy—the general informational probability principles of probabilistic to quantum information principle:

D'Ariano et al: Purification postulate:

"The main message of our work is simple: within a standard class of theories of information processing, quantum theory is uniquely identified by a single postulate: *purification*. The purification postulate, introduced in Ref. [10], expresses a distinctive feature of quantum theory, namely that the ignorance about a part is always compatible with the maximal knowledge of the whole. The key role of this feature was noticed already in 1935 by Schrödinger in his discussion about entanglement (Schrödinger 1935b), of which he famously wrote "I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought." In a sense, our work can be viewed as the concrete realization of Schrödinger's claim: the fact that every physical state can be viewed as the marginal of some pure state of a compound system is indeed the key to single out quantum theory within a standard set of possible theories. It is worth stressing, however, that the purification principle assumed in this paper includes a requirement that was not explicitly mentioned in Schrödinger's discussion: if two pure states of a composite system AB have the same marginal on system A, then they are connected by some reversible transformation on system B. In other words, we assume that all purifications of a given mixed state are equivalent under local reversible operations. "

There are instructive specific parallels (not identical features!) between D'Ariano and coauthors' and Hardy's approaches, on the one hand, and Heisenberg's, on the other. The QP principle present in both cases, given that D'Ariano et al (rightly) see quantum mechanics an "operational-probabilistic theory" of a special type, defined by the purification postulate [9, p. 3].

"The operational-probabilistic framework combines the operational language of circuits with the toolbox of probability theory: on the one hand experiments are described by circuits resulting from the connection of physical devices, on the other hand each device in the circuit can have classical outcomes and the theory provides the probability distribution of outcomes when the devices are connected to form closed circuits (that is, circuits that start with a preparation and end with a measurement)."

(D'Ariano et al)

This is close to Heisenberg's initial thinking, keeping in mind the difference defined by the concept of "circuit," not found in Heisenberg and closer to Bohr's view of the role of measuring apparatus, on which I shall comment presently. Heisenberg found his formalism by using the mathematical correspondence principle, not exactly the first principle, because it depended on the equations of classical mechanics at the classical limit where h could be neglected. However, Heisenberg needed new variable because the classical variables (as functions of real variables) do not give Bohr's frequencies rules for spectra, or the probabilities or statistics he needed. Heisenberg discovered that these rules are satisfied by, in general, noncommuting matrix variables with complex coefficients, related to amplitudes, from which one derives, in essence by means of Born's rule for this case, the probabilities (or probability distributions) or, again, statistics for transitions between stationary states (no longer assumed to be orbits) defining spectra, which are observed in measuring devices.

By contrast, D'Ariano et al arrive at mathematical architecture of (finite-dimensional) quantum in a more first-principle-like way, in particular, independently of classical physics. (The latter, to begin with, does not have discrete variables, such as spin, which are purely quantum, with which the finite-dimensional quantum theory could be associated.) This is accomplished by using the rules governing the structure of operational devices. These rules are more empirical, but not completely, because they are given a mathematical representation or expression, as they must be, in accordance with the authors' and the present view. They are hybrid.

This is not unsimilar to Heisenberg's arrangement of quantities he used into the square tables, matrices, which we now take for granted, but which was in itself an mathematical invention giving the structure, architecture to the manifolds of physical quantities, ultimately linked to probabilities of transitions between stationary stated. Here, again, we deal with the architecture, circuits, of measuring devices. As they say: "The rules summarized in this section define the operational language of circuits, which has been discussed in detail in a series of inspiring works by Coecke:"

"The underlying mathematical foundation of this high-level diagrammatic formalism relies on so-called *monoidal categories*, a product of a fairly recent development in mathematics. Its logical underpinning is *linear logic*, an even more recent product of research in logic and computer science. These monoidal categories do not only provide a natural foundation for physical theories, but also for proof theory, logic, programming languages, biology, cooking … *So the challenge is to discover the necessary additional pieces of structure that allow us to predict genuine quantum phenomena.* These additional pieces of structure represent the capabilities nature has provided us with to manipulate entities subject to the laws of quantum theory. [Coecke, 2009]

This may indeed be a more natural way to give the fundamental structures and principles of quantum theory a proper mathematical expression, and thus also connect the mathematical and experimental technology of quantum mechanics. Hybrid Heisenberg arrangement is a hybrid.

Hardy:

"Circuits have:

• A setting, s(H), given by specifying the setting on each operation.

• An outcome set, o(H), given by specifying the outcome set at each op- eration (equals o(A) × o(B) × o(C) × o(D) × oE in this case). We say the fragment "happened" if the outcome is in the outcome set.

• A wiring, w(E), given by specifying which input/output pairs are wired together.

L. Hardy, "Reconstructing Quantum Theory," 2013.

"We need three background assumptions in setting up the circuit framework. The first is the following.

"Assumption 1 The probability, Prob(A), for any circuit, A (this has no open inputs or outputs), is well conditioned. It is therefore determined by the operations and the wiring of the circuit alone and is independent of settings and outcomes elsewhere."

L. Hardy, "A formalism-local framework for general probabilistic theories including quantum theory," 2010

This is a physical postulate, essentially that of spatial and temporal locality (causality). (Arkady Plotnitsky)

"We now introduce our second assumption:

Assumption 2: Operations are fully decomposable.

In words we will say that any operation is equivalent to a linear combination of operations each of which consists of an effect for each input and a preparation for each output.

We allow the possibility that the entries in $^{d_4 e_5 \ldots f_6}A_{a_1 b_2 \ldots c_3}$ are negative (and this will, indeed, be the case in quantum theory). Hence, in general, this cannot be thought of as physical mixing.

Assumption 2 introduces a subtly different attitude than the usual one concerning how we think about what an operation is. Usually we think of operations as effecting a transformation on systems as they pass through. Here we think of an operation as corresponding to a bunch of separate effects and preparations. We need not think of systems as things that preserve their identity as they pass through - we do not use the same labels for wires coming out as going in. This is certainly a more natural attitude when there can be different numbers of input and output systems and when they can be of different types. Both classical and quantum transformations satisfy this assumption. In spite of the different attitude just mentioned, we can implement arbitrary transformations, such as unitary transformation in quantum theory, by taking an appropriate sum over such effect and preparation operations.

Physics to mathematics correspondence principle. For any physical theory, there exists a small number of simple hybrid statement that enable us to translate from the physical description to the corresponding mathematical calculation such that the mathematical calculation (in appropriate notation) looks the same as the physical description (in appropriate notation).

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