Title: PSI 2015/2016 Condensed Matter - Oleg Tchernyshyov - Lecture 11

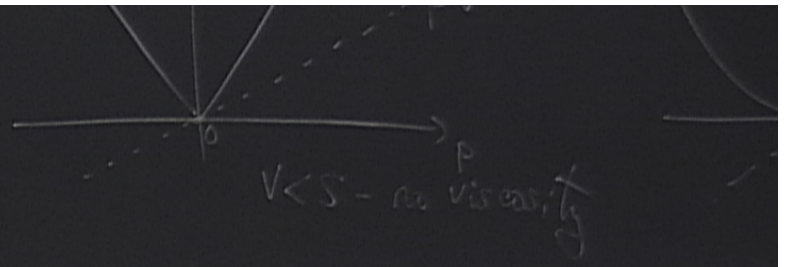
Date: Nov 23, 2015 10:45 AM

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Abstract:

Example: ideal Bose gas, $\epsilon_p = \frac{p^2}{2m}$

Ideal Bose gas is able to create viscous force



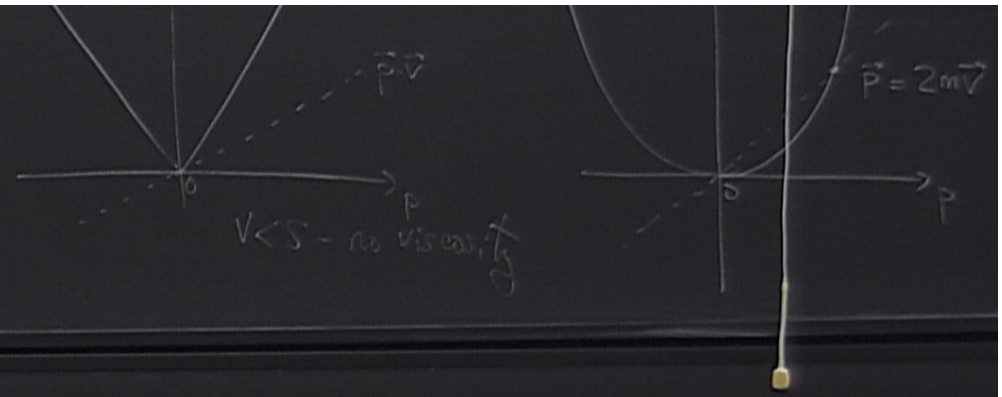
Field theory of an ideal Bose gas. + add interactions later.

Classical complex scalar field $\psi(\vec{r})$ with Lagrangian density $\mathcal{L} = \psi^*$

$\varepsilon_{\vec{p}} = \vec{p}\cdot\vec{v}$ – momentum transfer possible.

Example: ideal Bose gas, $\varepsilon_{\vec{p}} = \frac{\vec{p}^2}{2m}$

Ideal Bose gas is able to create viscous force



Field theory of an ideal Bose gas. + add interactions later.

Classical complex scalar field $\psi(\vec{r})$ with Lagrangian density $\mathcal{L} = i\hbar\psi^*\dot{\psi} - \frac{\hbar^2}{2m}\nabla\psi^*\cdot\nabla\psi$

$-i\hbar\dot{\psi}^*\psi$

Eqns. of motion for ψ^* and ψ:

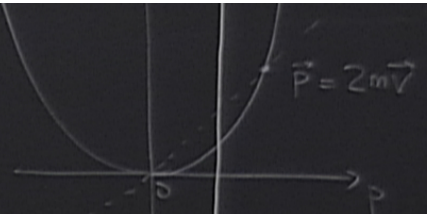
$\varepsilon_{\vec{p}} = \vec{p}\cdot\vec{v}$ – momentum transfer possible.

Example: ideal Bose gas, $\varepsilon_{\vec{p}} = \frac{p^2}{2m}$

Ideal Bose gas is able to create viscous force



$v<s$ – no viscosity



Field theory of an ideal Bose gas. + add interactions later. non-relativistic

Classical complex scalar field $\psi(\vec{r})$ with Lagrangian density $\mathcal{L} = i\hbar\psi^*\dot{\psi} - \frac{\hbar^2}{2m}\nabla\psi^*\cdot\nabla\psi$

$-i\hbar\dot{\psi}^*\psi$

Eqns. of motion for ψ^* and ψ:

$$i\hbar\dot{\psi} + \frac{\hbar^2\nabla^2\psi}{2m} = 0$$

$$-i\hbar\dot{\psi}^* + \frac{\hbar^2\nabla^2\psi^*}{2m} = 0$$

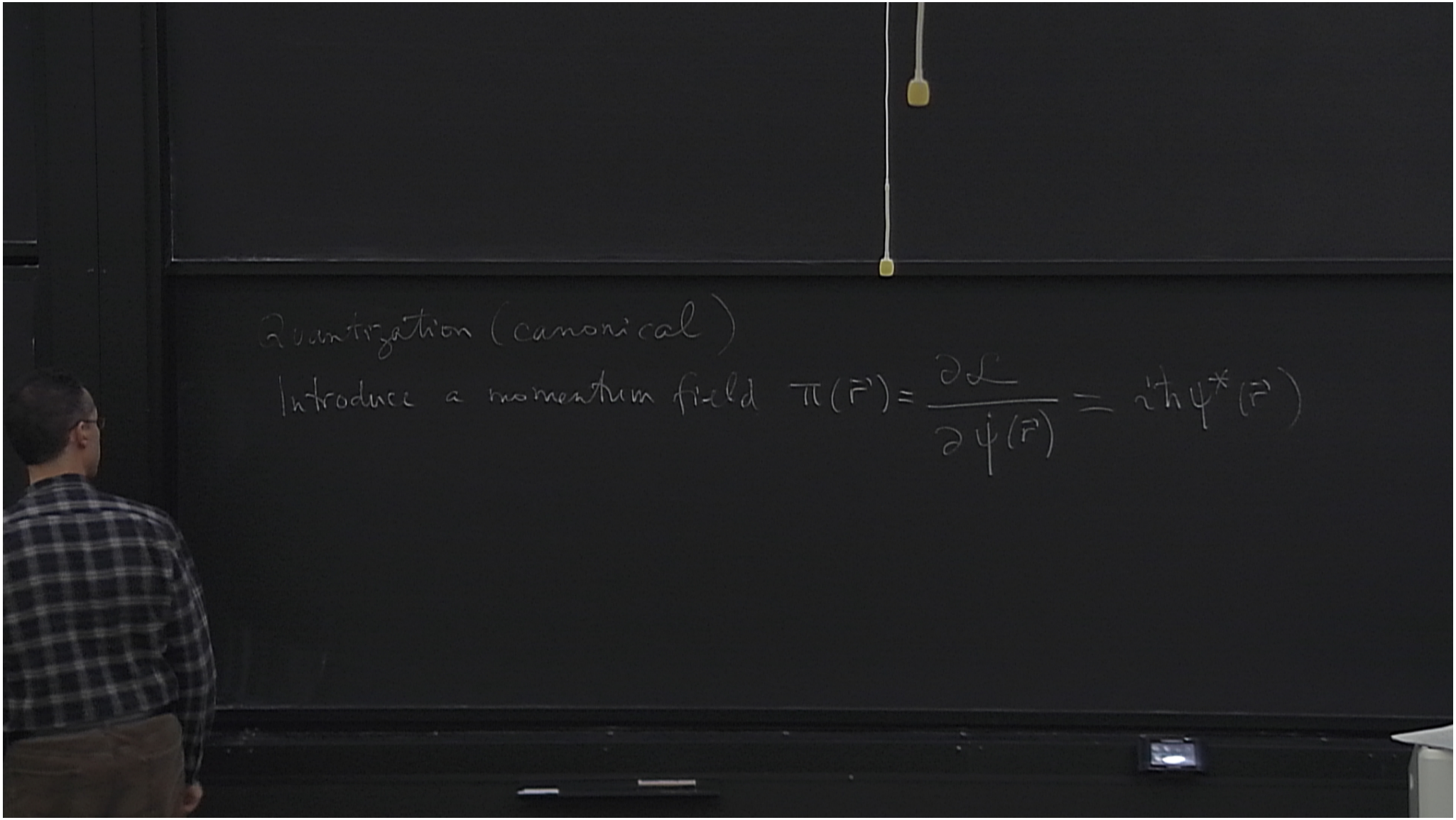
$$\psi(\vec{r},t) = \psi(0,0)e^{-i\omega t + i\vec{k}\cdot\vec{r}} \longrightarrow \hbar\omega - \frac{\hbar^2k^2}{2m} = 0$$

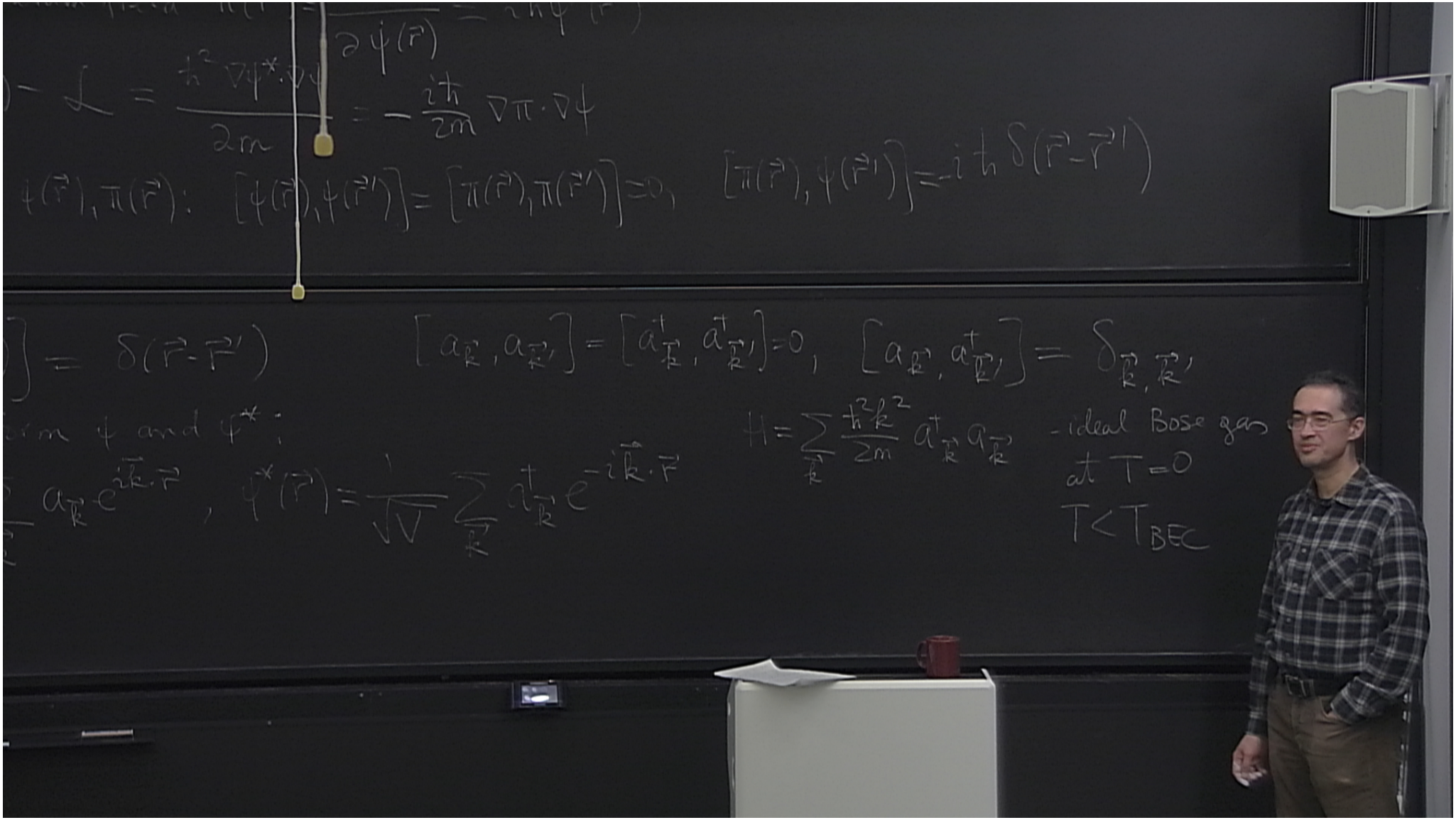
$\varepsilon = \hbar\omega$

$\vec{p} = \hbar\vec{k}$

$\varepsilon = \frac{p^2}{2m}$

$\frac{\hbar^2}{2mc^2}\dot{\psi}^*\dot{\psi}$


Quantization (canonical)
Introduce a momentum field $\pi(\vec{r}) = \frac{\partial \mathcal{L}}{\partial \dot{\psi}(\vec{r})} = i\hbar\psi^*(\vec{r})$


$\mathcal{L} = \frac{\hbar^2 \nabla\psi^* \cdot \nabla\psi}{2m}$
$= -\frac{i\hbar}{2m} \nabla\pi \cdot \nabla\psi$
$\frac{\partial\psi(\vec{r})}{\partial t} = \frac{i}{\hbar}[H, \psi(\vec{r})]$
$[\psi(\vec{r}), \pi(\vec{r})]$: $[\psi(\vec{r}), \psi(\vec{r}')] = [\pi(\vec{r}), \pi(\vec{r}')] = 0$, $[\pi(\vec{r}), \psi(\vec{r}')] = -i\hbar\delta(\vec{r}-\vec{r}')$
$= \delta(\vec{r}-\vec{r}')$
$[a_{\vec{k}}, a_{\vec{k}'}] = [a^\dagger_{\vec{k}}, a^\dagger_{\vec{k}'}] = 0$, $[a_{\vec{k}}, a^\dagger_{\vec{k}'}] = \delta_{\vec{k},\vec{k}'}$
$H = \sum_{\vec{k}} \frac{\hbar^2 k^2}{2m} a^\dagger_{\vec{k}} a_{\vec{k}}$
- ideal Bose gas
at $T = 0$
$T < T_{BEC}$
$\psi^*(\vec{r}) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} a^\dagger_{\vec{k}} e^{-i\vec{k}\cdot\vec{r}}$

Introduce a momentum field $\pi(\vec{r}) = \frac{\partial \mathcal{L}}{\partial \dot{\psi}(\vec{r})} = i\hbar\psi^*(\vec{r})$

$\mathcal{H} = \pi(\vec{r})\dot{\psi}(\vec{r}) - \mathcal{L} = \frac{\hbar^2 \nabla\psi^* \cdot \nabla\psi}{2m} = -\frac{i\hbar}{2m}\nabla\pi\cdot\nabla\psi$

Operator fields $\psi(\vec{r}), \pi(\vec{r})$: $[\psi(\vec{r}), \psi(\vec{r}')] = [\pi(\vec{r}), \pi(\vec{r}')] = 0$, $[\pi(\vec{r}), \psi(\vec{r}')] = -i\hbar\delta(\vec{r}$

At $T < T_{BEC}$, $n_0 = a_0^\dagger a_0 = N_0 \gg 1$. Treat $a_0^\dagger$ and a_0 as numbers

$a_0 a_0^\dagger = N_0 + 1$

$a_0 = \sqrt{N_0}\, e^{i\theta}$, $a_0^\dagger = \sqrt{N_0}\, e^{-i\theta}$

$\psi(\vec{r}) = \underbrace{\sqrt{\frac{N_0}{V}} e^{i\theta}}_{\psi_0} + \frac{1}{\sqrt{V}}\sum_{\vec{k}\neq 0} a_{\vec{k}} e^{i\vec{k}\cdot\vec{r}}$, $\psi^*(\vec{r}) = \underbrace{\sqrt{\frac{N_0}{V}} e^{-i\theta}}_{\psi_0^*} + \frac{1}{\sqrt{V}}\sum_{\vec{k}} a_{\vec{k}}^\dagger e^{-i\vec{k}\cdot\vec{r}}$

Condensate amplitude, $|\psi_0|^2 = N_0 = \#$ of bosons in $\vec{k} = 0$

Introduce a momentum field $\pi(\vec{r}) = \frac{\partial \mathcal{L}}{\partial \dot{\psi}(\vec{r})} = i\hbar\psi^*(\vec{r})$

$\mathcal{H} = \pi(\vec{r})\dot{\psi}(\vec{r}) - \mathcal{L} = \frac{\hbar^2 \nabla\psi^* \cdot \nabla\psi}{2m} = -\frac{i\hbar}{2m}\nabla\pi \cdot \nabla\psi$

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At $T < T_{BEC}$, $n_0 = a_0^\dagger a_0 = N_0 \gg 1$ Treat $a_0^\dagger$ and a_0 as numbers

$a_0 a_0^\dagger = N_0 + 1$

$a_0 = \sqrt{N_0}\, e^{i\theta}$, $a_0^\dagger = \sqrt{N_0}\, e^{-i\theta}$

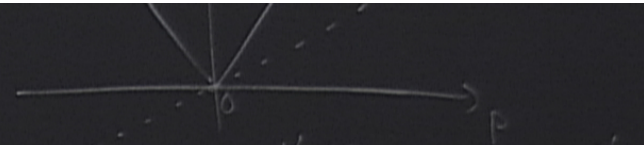
$\psi(\vec{r}) = \underbrace{\sqrt{\frac{N_0}{V}} e^{i\theta}}_{\psi_0} + \frac{1}{\sqrt{V}} \sum_{\vec{k}\neq 0} a_{\vec{k}} e^{i\vec{k}\cdot\vec{r}}$, $\psi^*(\vec{r}) = \underbrace{\sqrt{\frac{N_0}{V}} e^{-i\theta}}_{\psi_0^*} + \frac{1}{\sqrt{V}} \sum_{\vec{k}} a_{\vec{k}}^\dagger e^{-i\vec{k}\cdot\vec{r}} = \psi_0^* + \delta\psi^*(\vec{r})$

$= \psi_0 + \delta\psi(\vec{r})$

Condensate amplitude, $|\psi_0|^2 = N_0 =$ # of bosons with $\vec{p} = 0$

Example: ideal Bose gas, $\epsilon_p = \frac{p^2}{2m}$

Ideal Bose gas is able to create viscous force

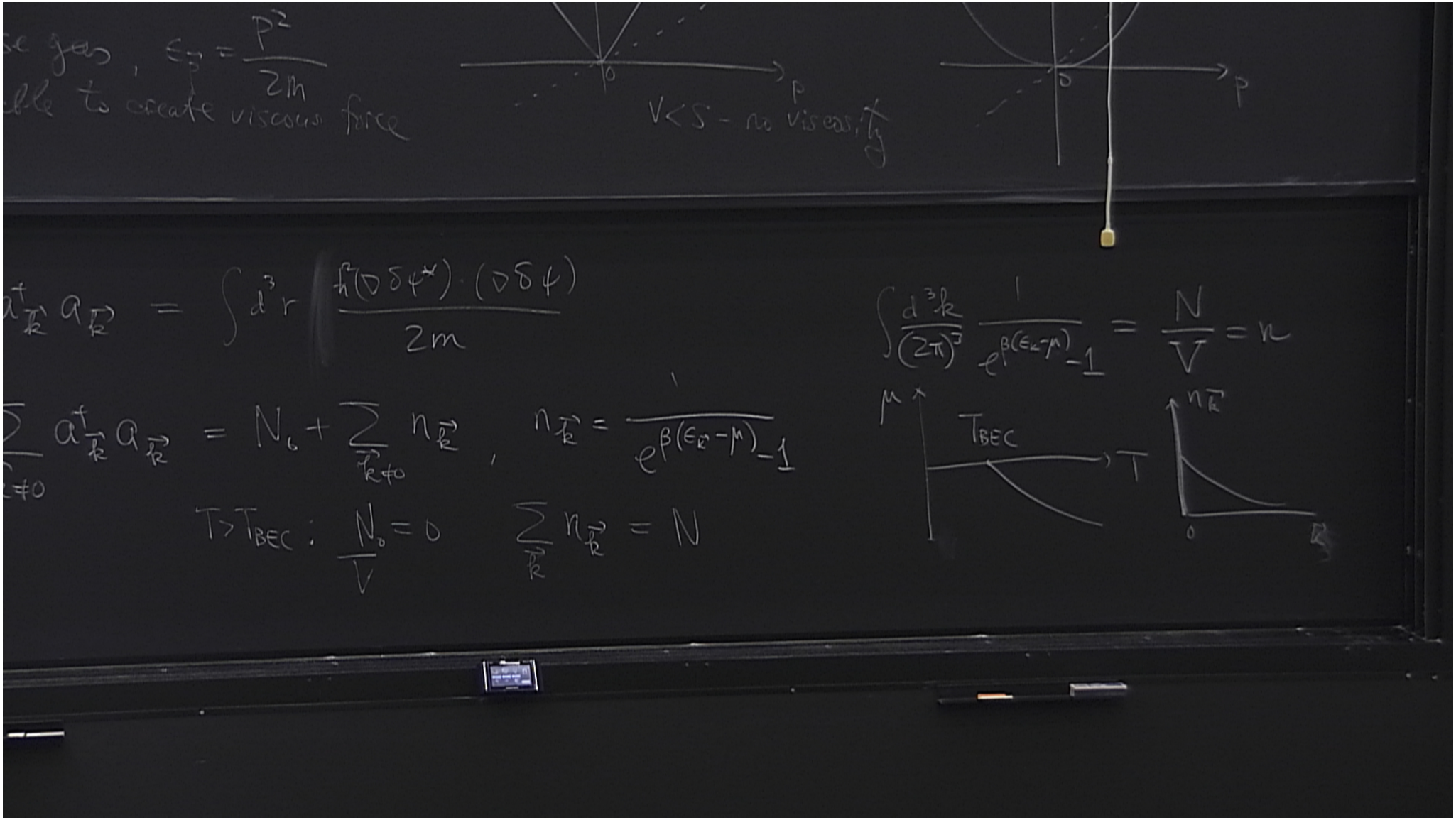


$$H - \mu N = \sum_{\vec{k}\neq 0} \frac{\hbar^2 k^2}{2m} a^\dagger_{\vec{k}} a_{\vec{k}} = \int d^3 r \, \frac{\hbar(\nabla\delta\psi^*)\cdot(\nabla\delta\psi)}{2m}$$

$$\int \frac{d^3k}{(2\pi)}$$

$$N = |\psi_0|^2 V + \sum_{\vec{k}\neq 0} a^\dagger_{\vec{k}} a_{\vec{k}} = N_0 + \sum_{\vec{k}\neq 0} n_{\vec{k}}, \quad n_{\vec{k}} = \frac{1}{e^{\beta(\epsilon_{\vec{k}} - \mu)} - 1}$$

$T > T_{BEC}$: $N_0 = 0$ $\quad \sum_{\vec{k}} n_{\vec{k}} = N$


$\epsilon_p = \frac{p^2}{2m}$
to create viscous force
p
V<S - no viscosity
p
$\int d^3r \frac{\hbar(\nabla\delta\varphi^*)\cdot(\nabla\delta\varphi)}{2m}$
$\int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\beta(\epsilon_k-\mu)}-1} = \frac{N}{V} = n$
$\sum_{\vec{k}\neq 0} a^\dagger_{\vec{k}} a_{\vec{k}} = N_0 + \sum_{\vec{k}\neq 0} n_{\vec{k}}$, $n_{\vec{k}} = \frac{1}{e^{\beta(\epsilon_k-\mu)}-1}$
$T > T_{BEC}$: $\frac{N_0}{V} = 0$ $\sum_{\vec{k}} n_{\vec{k}} = N$
μ
T_{BEC}
T
$n_{\vec{k}}$
0

Introduce a momentum field $\pi(\vec{r}) = \frac{\partial \mathcal{L}}{\partial \dot{\psi}(\vec{r})} = i\hbar\psi^*(\vec{r})$

$\mathcal{H} = \pi(\vec{r})\dot{\psi}(\vec{r}) - \mathcal{L} = \frac{\hbar^2 \nabla\psi^* \cdot \nabla\psi}{2m} = -\frac{i\hbar}{2m}\nabla\pi \cdot \nabla\psi$

Operator fields $\psi(\vec{r}), \pi(\vec{r})$: $[\psi(\vec{r}), \psi(\vec{r}')] = [\pi(\vec{r}), \pi(\vec{r}')] = 0$, $[\pi(\vec{r}), \psi(\vec{r}')] = -i\hbar\delta(\vec{r}-\vec{r}')$

Interacting Bose gas.

$\mathcal{L} = i\hbar\psi^*\dot{\psi} - \frac{\hbar^2 \nabla\psi^* \cdot \nabla\psi}{2m} - \frac{1}{2}g\psi^*\psi^*\psi\psi + \mu\psi^*\psi$ for interacting Bose gas.

$\mathcal{H} = \pi(\vec{r})\dot{\psi}(\vec{r}) - \mathcal{L} = \frac{\hbar^2 \nabla\psi^* \cdot \nabla\psi}{2m} = -\frac{i\hbar}{2m}\nabla\pi \cdot \nabla\psi$

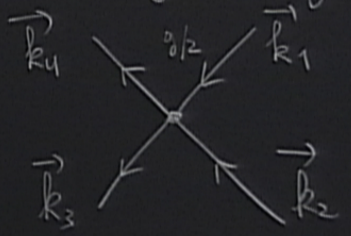
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Interacting Bose gas.

$\mathcal{L} = i\hbar\psi^*\dot{\psi} - \frac{\hbar^2 \nabla\psi^* \cdot \nabla\psi}{2m} - \frac{1}{2}g\psi^*\psi^*\psi\psi + \mu\psi^*\psi$, $\mu \neq 0$ for interacting Bose gas.

$H - \mu N = \sum_{\vec{k}} \left(\frac{\hbar^2 k^2}{2m} - \mu\right) a^\dagger_{\vec{k}} a_{\vec{k}} + \frac{1}{V}\sum_{\vec{k}\vec{k}'} \frac{g}{2} a^\dagger_{\vec{k}_4} a^\dagger_{\vec{k}_3} a_{\vec{k}_2} a_{\vec{k}_1}$

$g > 0$ repulsive interaction



$\mathcal{H} = \pi(\vec{r})\dot{\psi}(\vec{r}) - \mathcal{L} = \frac{\hbar^2 \nabla\psi^* \cdot \nabla\psi}{2m} = -\frac{i\hbar}{2m}\nabla\pi\cdot\nabla\psi$

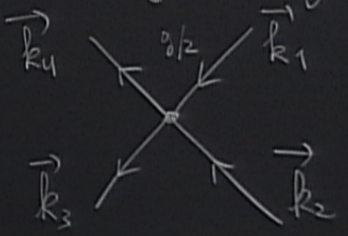
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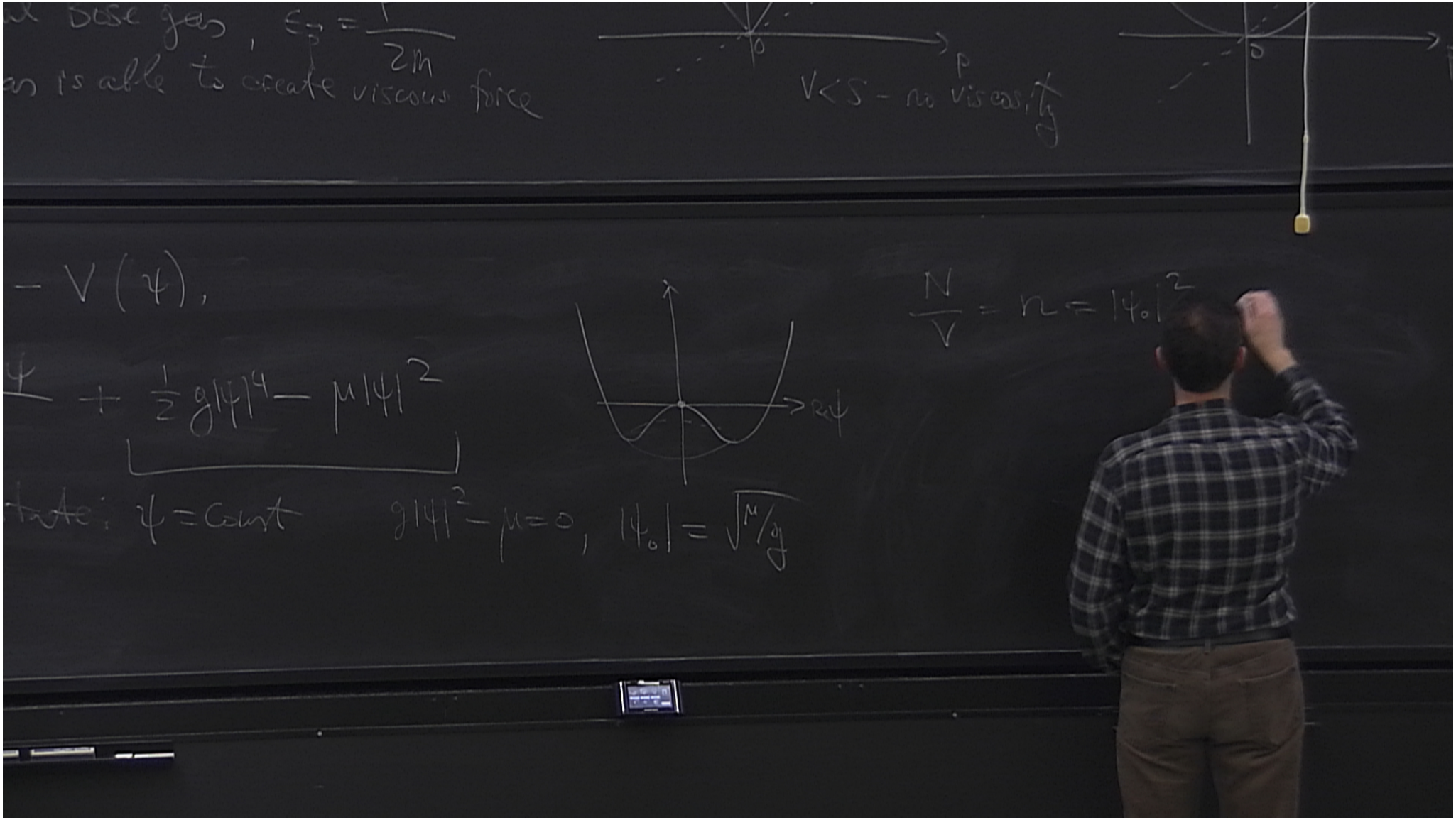
Interacting Bose gas. $\quad \frac{1}{2}gn^2 \quad \frac{1}{2}\int d^3r \int d^3r'\, n(\vec{r})\, g(\vec{r}-\vec{r}')\, n(\vec{r}')$

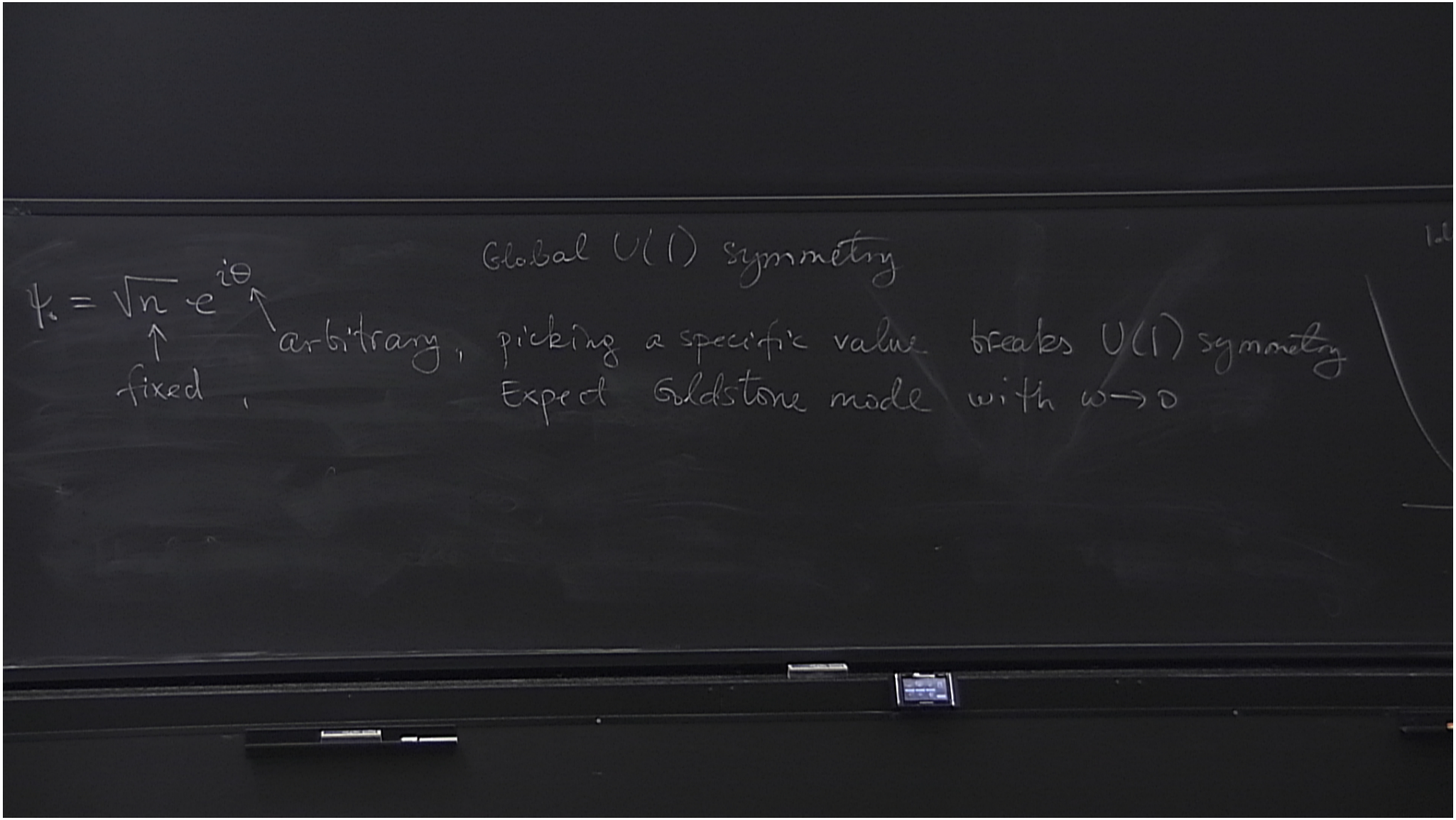
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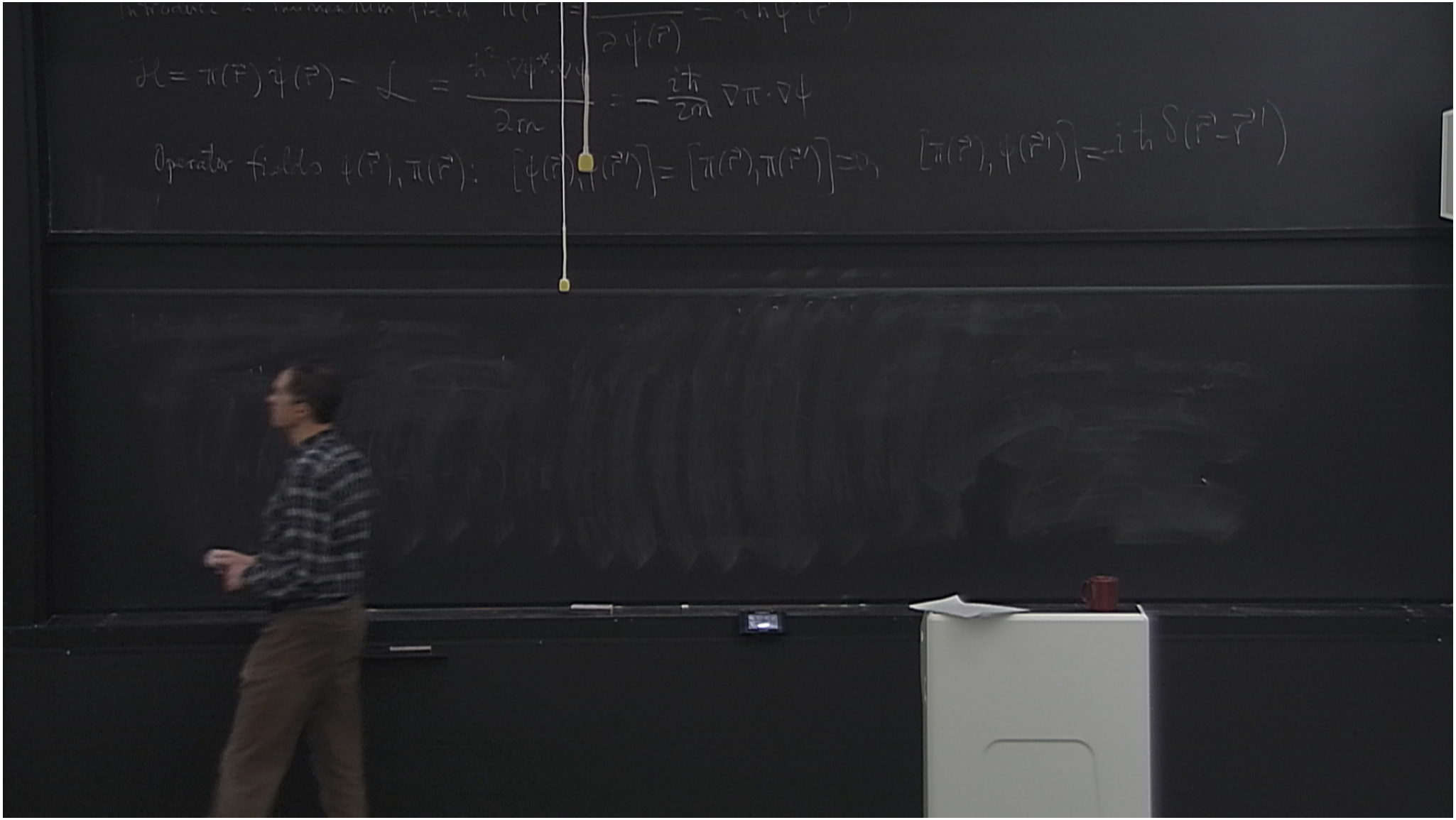
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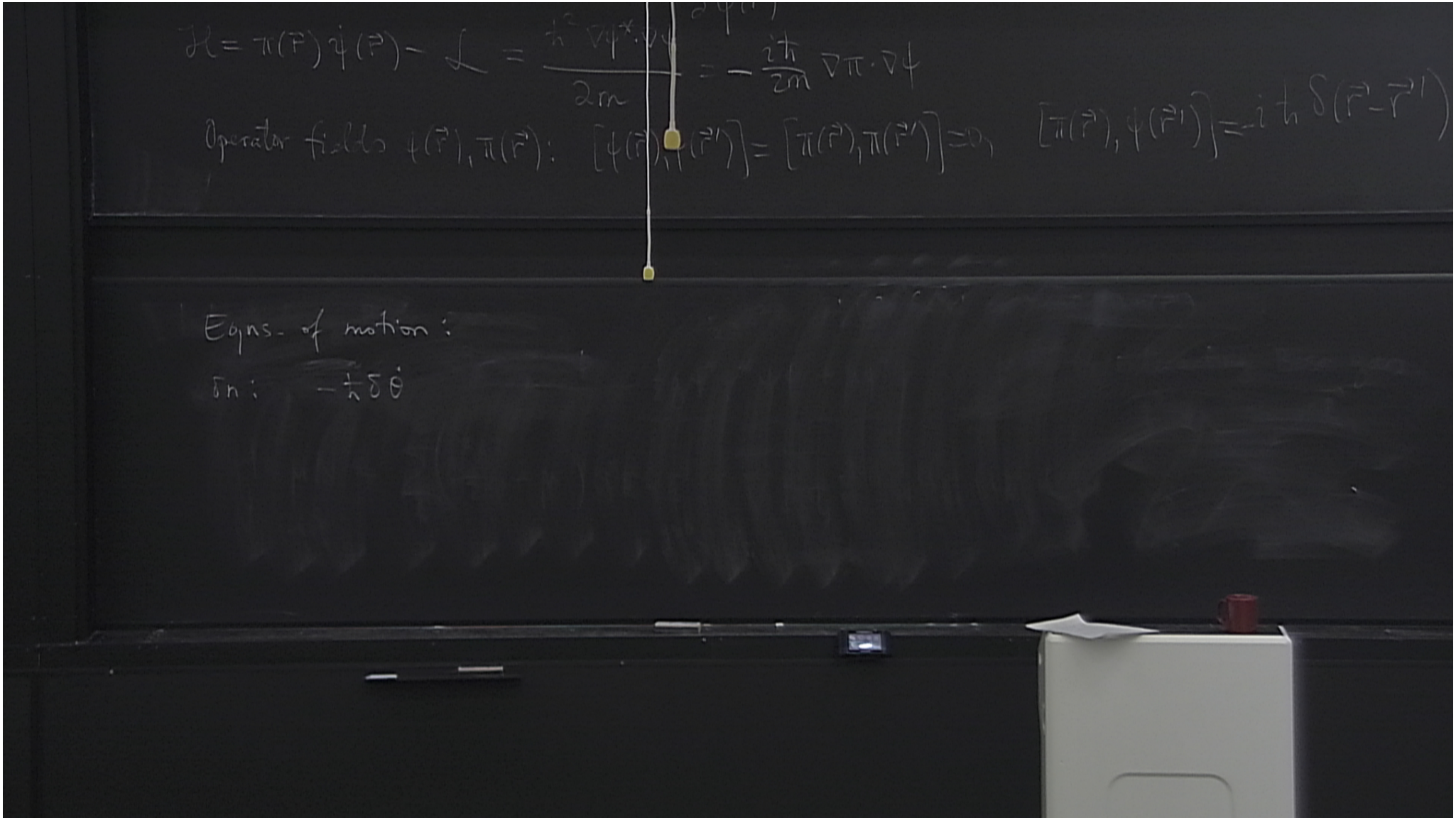
$g > 0$ repulsive interaction

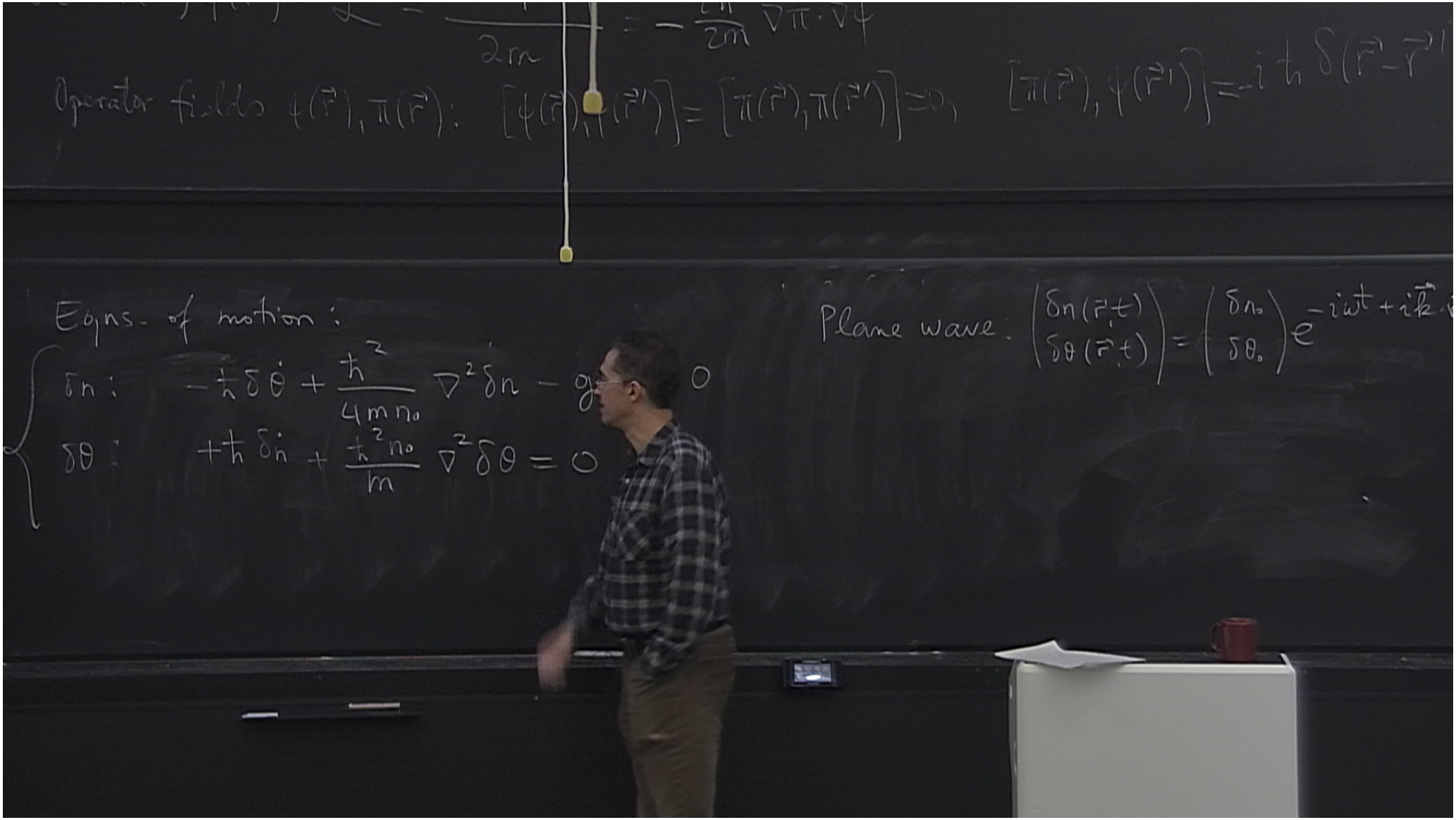


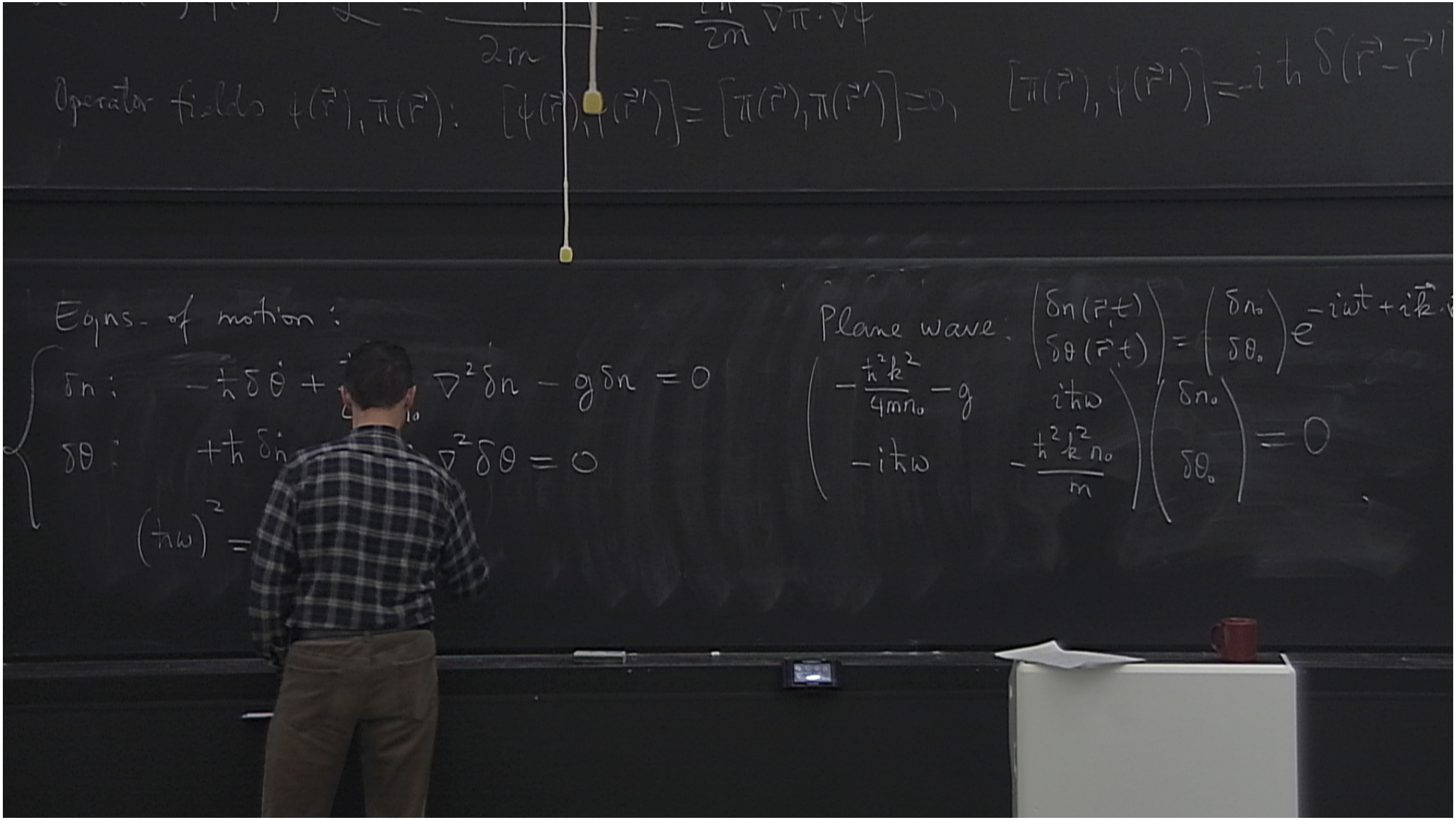

Bose gas, $\epsilon_p = \frac{p^2}{2m}$
is able to create viscous force
V<S - no viscosity
$-V(\psi)$,
$+\frac{1}{2}g|\psi|^4 - \mu|\psi|^2$
$\frac{N}{V} = n = |\psi_0|^2$
$g|\psi|^2 - \mu = 0$, $|\psi_0| = \sqrt{\mu/g}$


Global U(1) symmetry
$\psi_0 = \sqrt{n}\, e^{i\theta}$
fixed, arbitrary, picking a specific value breaks U(1) symmetry
Expect Goldstone mode with $\omega \to 0$


$\mathcal{H} = \pi(\vec{r})\dot{\psi}(\vec{r}) - \mathcal{L} = \frac{\hbar^2}{2m}\nabla\psi^*\cdot\nabla\psi = -\frac{i\hbar}{2m}\nabla\pi\cdot\nabla\psi$
Operator fields $\psi(\vec{r}), \pi(\vec{r})$: $[\psi(\vec{r}),\psi(\vec{r}')] = [\pi(\vec{r}),\pi(\vec{r}')] = 0$, $[\pi(\vec{r}),\psi(\vec{r}')] = -i\hbar\,\delta(\vec{r}-\vec{r}')$


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Eqns. of motion:
δn: $-\frac{1}{\hbar}\delta\dot{\theta}$


Operator fields $\psi(\vec{r}), \pi(\vec{r})$: $[\psi(\vec{r}), \pi(\vec{r}')] = [\pi(\vec{r}), \pi(\vec{r}')] = 0$, $[\pi(\vec{r}), \psi(\vec{r}')] = -i\hbar\delta(\vec{r}-\vec{r}')$
Eqns. of motion:
$\delta n: \quad -\frac{1}{\hbar}\dot{\delta\theta} + \frac{\hbar^2}{4mn_0}\nabla^2\delta n - g\frac{}{} = 0$
$\delta\theta: \quad +\hbar\dot{\delta n} + \frac{\hbar^2 n_0}{m}\nabla^2\delta\theta = 0$
Plane wave: $\begin{pmatrix}\delta n(\vec{r},t)\\ \delta\theta(\vec{r},t)\end{pmatrix} = \begin{pmatrix}\delta n_0\\ \delta\theta_0\end{pmatrix} e^{-i\omega t + i\vec{k}\cdot}$


Operator fields $\hat{\psi}(\vec{r}), \hat{\pi}(\vec{r})$: $[\hat{\psi}(\vec{r}), \hat{\psi}(\vec{r}')] = [\hat{\pi}(\vec{r}), \hat{\pi}(\vec{r}')] = 0$, $[\hat{\pi}(\vec{r}), \hat{\psi}(\vec{r}')] = -i\hbar\delta(\vec{r}-\vec{r}')$
Eqns. of motion:
$\delta n: \quad -\frac{1}{\hbar}\delta\dot{\theta} + \nabla^2\delta n - g\delta n = 0$
$\delta\theta: \quad +\hbar\delta\dot{n} \quad \nabla^2\delta\theta = 0$
$(\hbar\omega)^2 =$
Plane wave: $\begin{pmatrix}\delta n(\vec{r},t)\\ \delta\theta(\vec{r},t)\end{pmatrix} = \begin{pmatrix}\delta n_0\\ \delta\theta_0\end{pmatrix} e^{-i\omega t + i\vec{k}\cdot}$
$\begin{pmatrix} -\frac{\hbar^2k^2}{4mn_0} - g & i\hbar\omega \\ -i\hbar\omega & -\frac{\hbar^2k^2 n_0}{m}\end{pmatrix}\begin{pmatrix}\delta n_0\\ \delta\theta_0\end{pmatrix} = 0$

full time deriv.

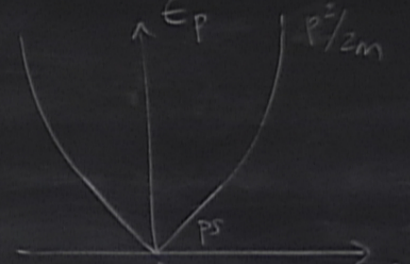
Expand in fluctuations from equilibrium, $n = n_0 + \delta n(\vec{r})$, $\theta = \theta_0 + \delta\theta(\vec{r})$

$$\mathcal{L} = -\hbar n_0 \dot{\delta\theta} - \hbar \delta n \dot{\delta\theta} - \frac{\hbar^2 n_0}{2m}\left[\frac{(\nabla\delta n)^2}{4n_0^2} + (\nabla\delta\theta)^2\right] - \frac{1}{2}g n_0^2 + \mu n_0 - \frac{1}{2}g(\delta n)^2 + \text{terms high}$$

$$\hbar\omega = \sqrt{\frac{\hbar^2 k^2}{2m}\left(\frac{\hbar^2 k^2}{2m} + 2gn_0\right)}$$

$k \to \infty$: $\hbar\omega \sim \frac{\hbar^2 k^2}{2m}$

$k \to 0$: $\hbar\omega \sim \hbar k s$, $s = \sqrt{gn_0/m}$



$v < s$: no dissipation