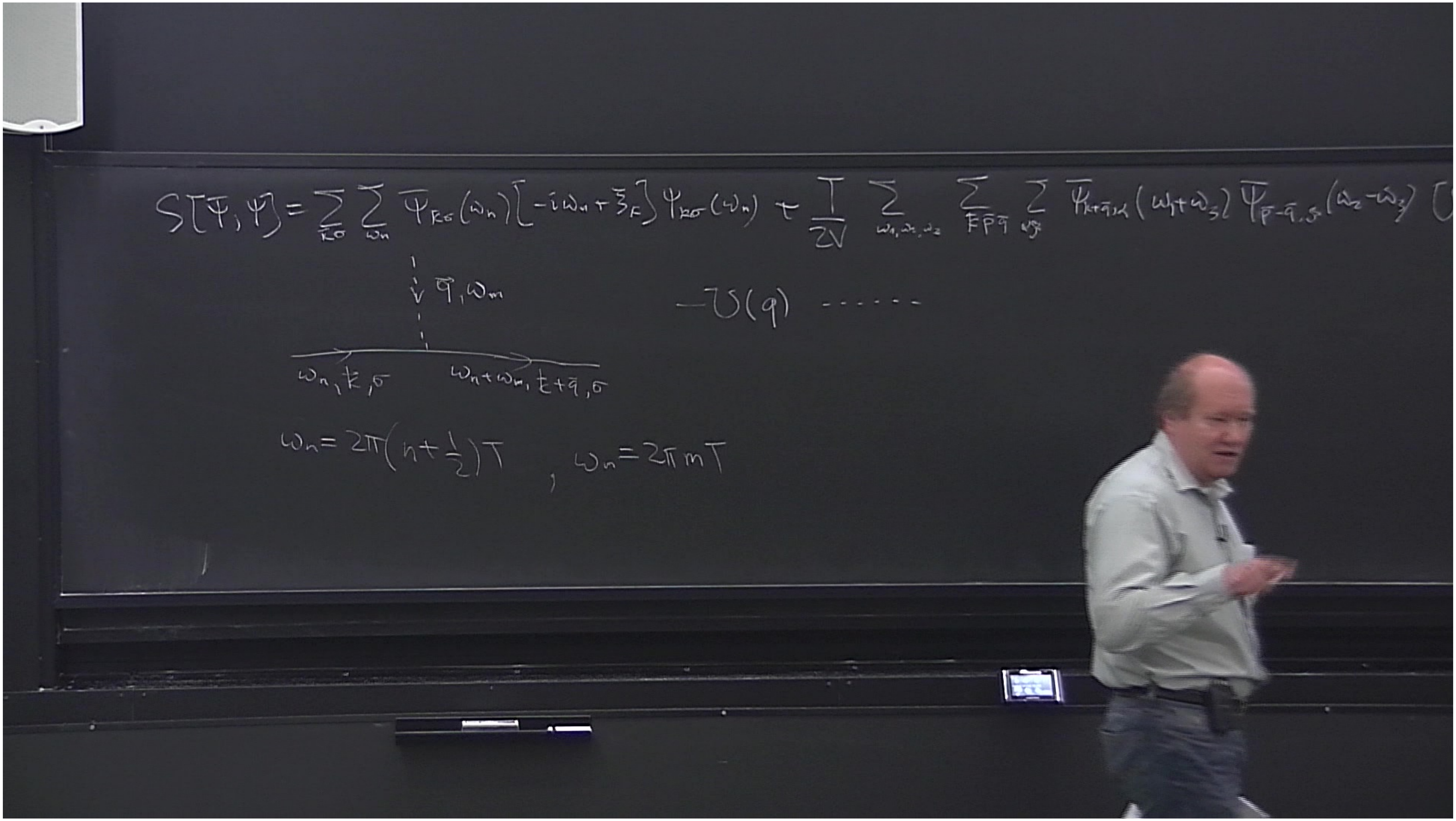
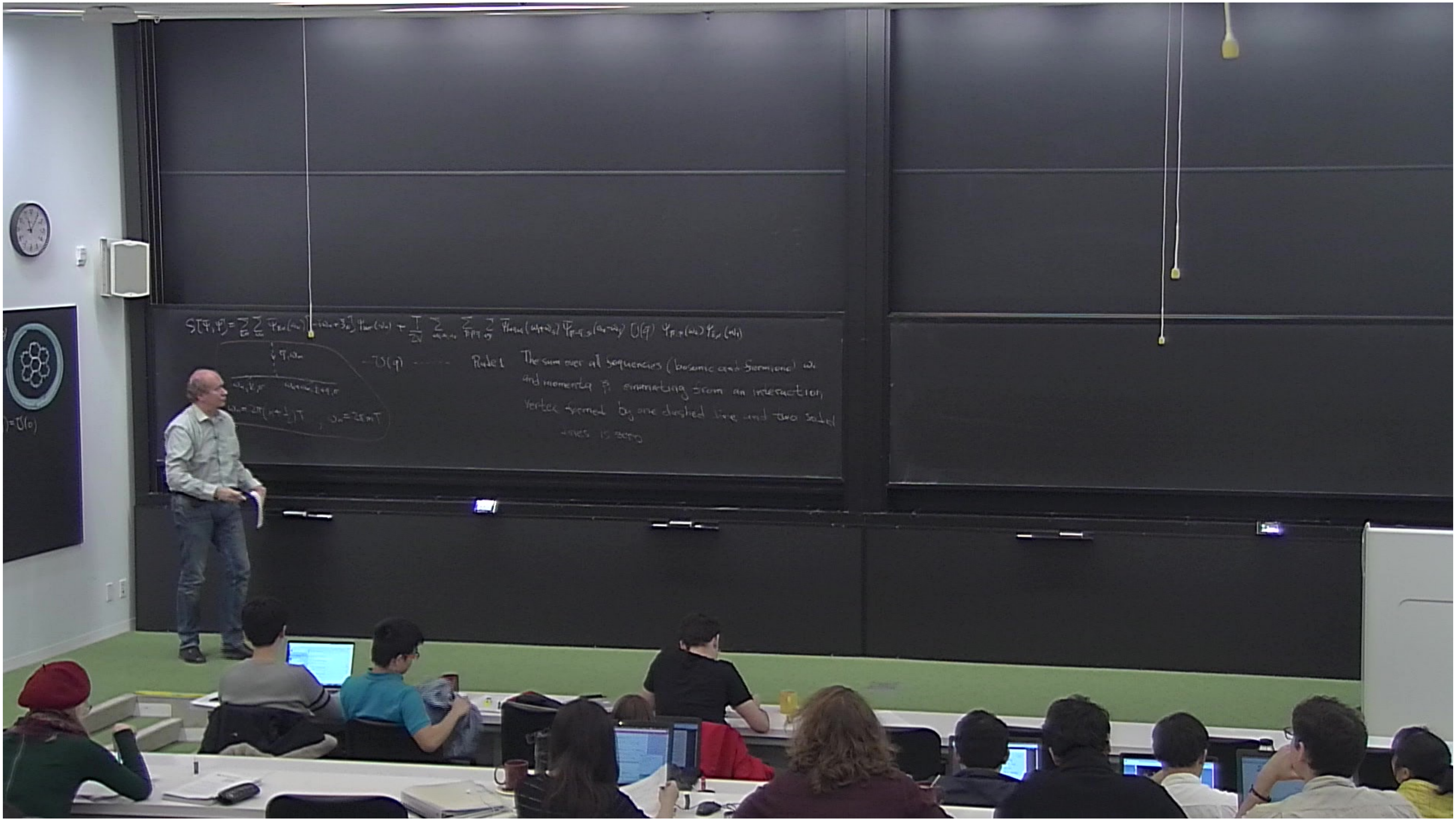
Title: PSI 2015/2016 Condensed Matter - Oleg Tchernyshyov - Lecture 10

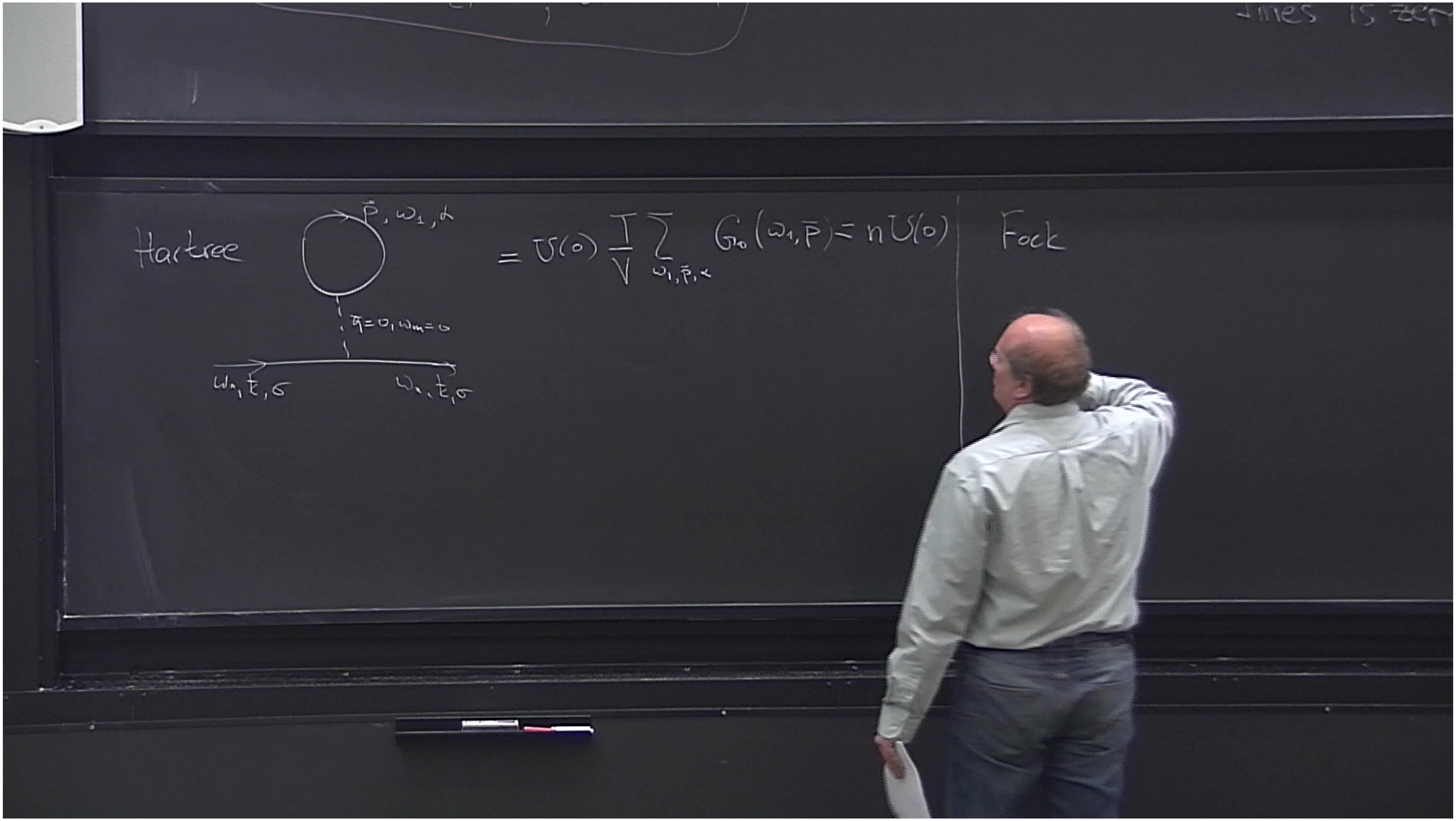
Date: Nov 20, 2015 10:45 AM

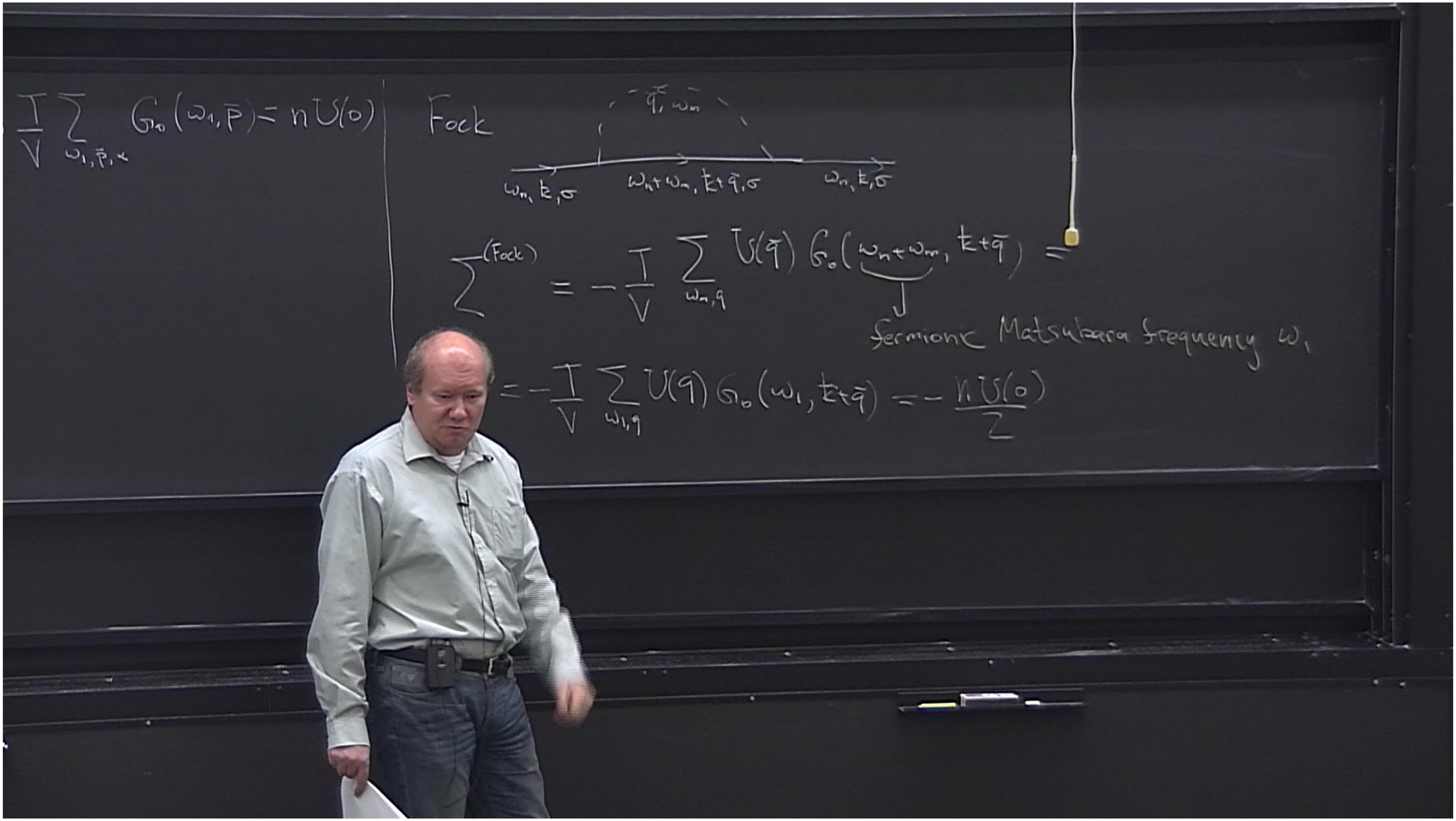
URL: http://pirsa.org/15110051

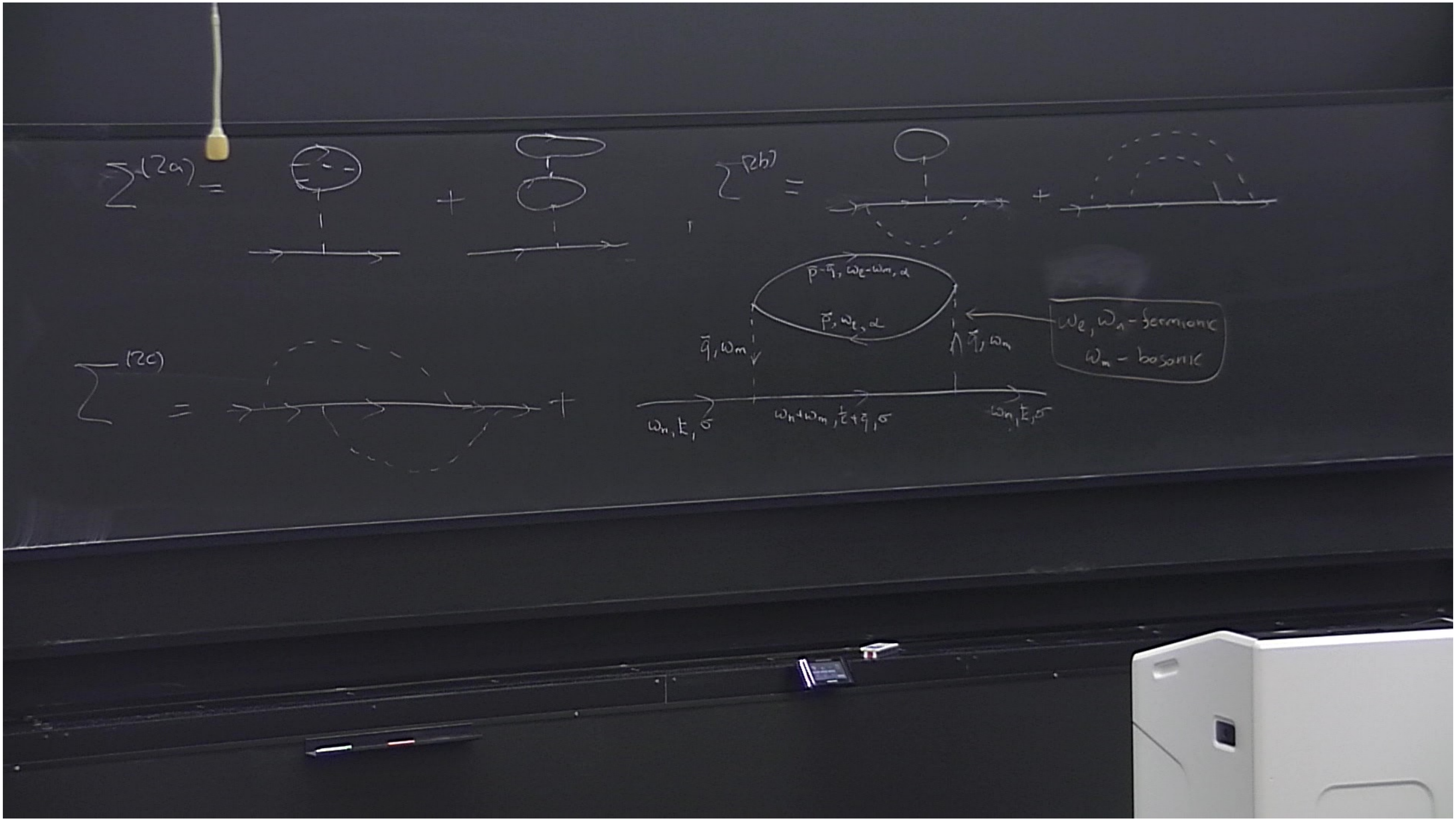
Abstract:

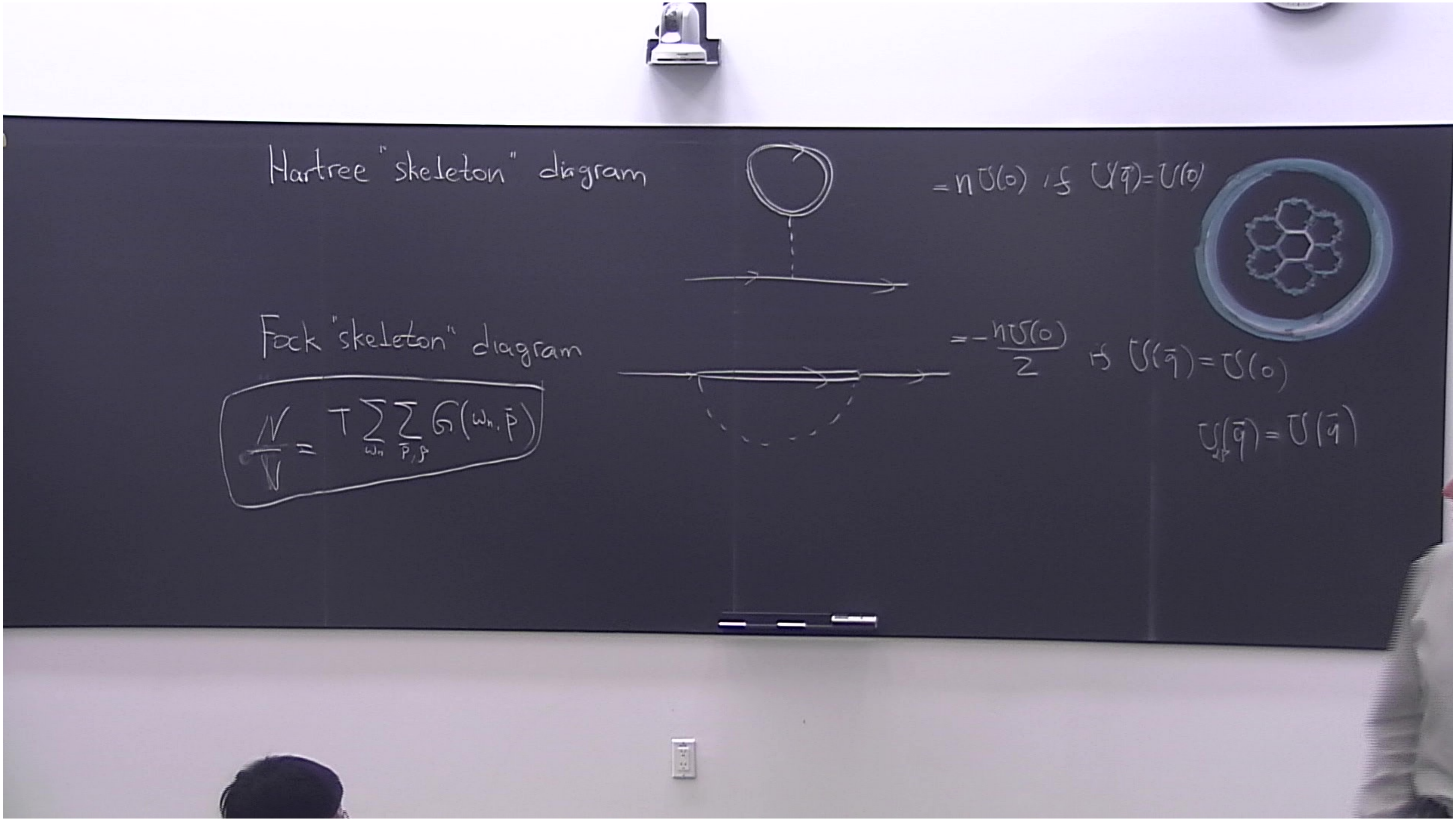

$S[\bar{\psi},\psi] = \sum_{\vec{k}\sigma} \sum_{\omega_n} \bar{\psi}_{\vec{k}\sigma}(\omega_n)\left[-i\omega_n + \xi_{\vec{k}}\right]\psi_{\vec{k}\sigma}(\omega_n) + \frac{T}{2V} \sum_{\omega_1,\omega_2,\omega_3} \sum_{\vec{k}\vec{p}\vec{q}} \sum_{\sigma\sigma'} \bar{\psi}_{\vec{k}+\vec{q},\sigma}(\omega_1+\omega_3)\, \bar{\psi}_{\vec{p}-\vec{q},\sigma'}(\omega_2-\omega_3)$
$\vec{q},\omega_m$
$-V(q)$
$\omega_n, \vec{k}, \sigma$
$\omega_n+\omega_m, \vec{k}+\vec{q}, \sigma$
$\omega_n = 2\pi(n+\frac{1}{2})T \quad , \quad \omega_m = 2\pi m T$


Rule 1
The sum over all sequencies (bosonic and fermionic) ω
and momenta p emanating from an interaction
vertex formed by one dashed line and two solid
lines is zero

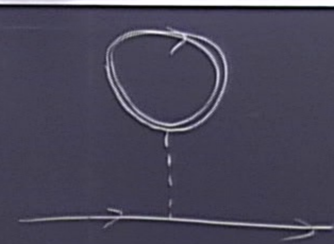

Hartree
p, ω1, α
q = 0, ωm = 0
ωn, k, σ
ωn, k, σ
= U(0) 1/V Σ ω1, p, α G0(ω1, p) = n U(0)
Fock


$\frac{T}{V}\sum_{\omega_1,\vec{p},\alpha} G_0(\omega_1,\vec{p}) = n\,U(0)$
Fock
$\vec{q}, \omega_m$
ω_n, k, σ
$\omega_n+\omega_m, k+\vec{q}, \sigma$
ω_n, k, σ
$\Sigma^{(Fock)} = -\frac{T}{V}\sum_{\omega_m, q} U(\vec{q})\, G_0(\underbrace{\omega_n+\omega_m}, k+\vec{q}) =$
fermionic Matsubara frequency ω_1
$= -\frac{T}{V}\sum_{\omega_1, q} U(q)\, G_0(\omega_1, k+\vec{q}) = -\frac{n\,U(0)}{2}$


Σ(2a) =
+
Σ(2b) =
+
p - q, ωℓ - ωm, α
p, ωℓ, α
q, ωm
q, ωm
ωℓ, ωn - fermionic
ωm - bosonic
Σ(2c) =
+
ωn, k, σ
ωn+ωm, k+q, σ
ωn, k, σ


Hartree "skeleton" diagram
$= n U(0)$ if $U(\bar{q}) = U(0)$
Fock "skeleton" diagram
$= -\frac{n U(0)}{2}$ if $U(\bar{q}) = U(0)$
$U_{\sigma\sigma'}(\bar{q}) = U(\bar{q})$
$\boxed{\frac{N}{V} = T \sum_{\omega_n} \sum_{\bar{p},\sigma} G(\omega_n, \bar{p})}$

Hartree "skeleton" diagram



$= n U(0)$ if $U(\bar{q}) = U(0)$

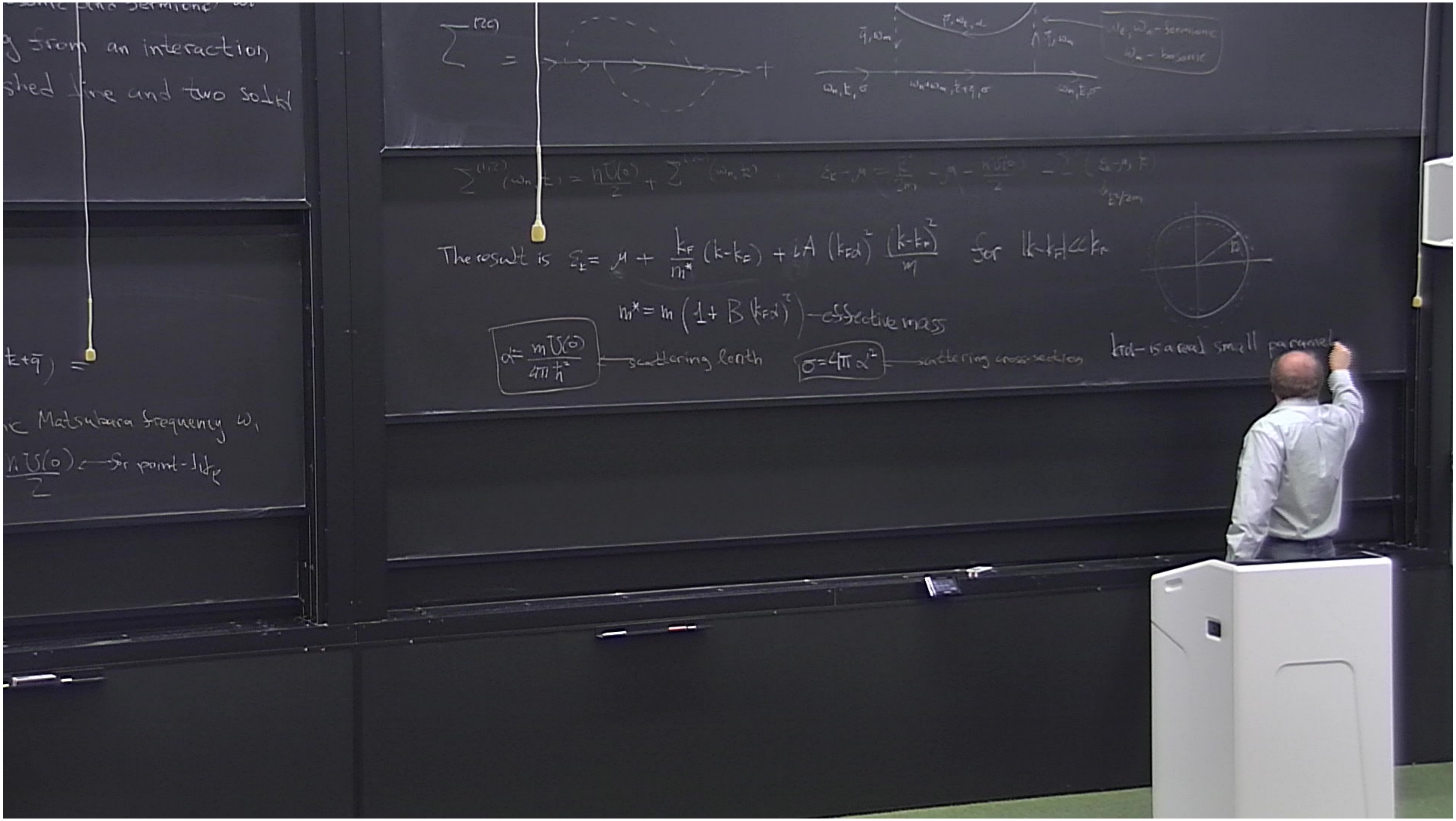
Fock "skeleton" diagram

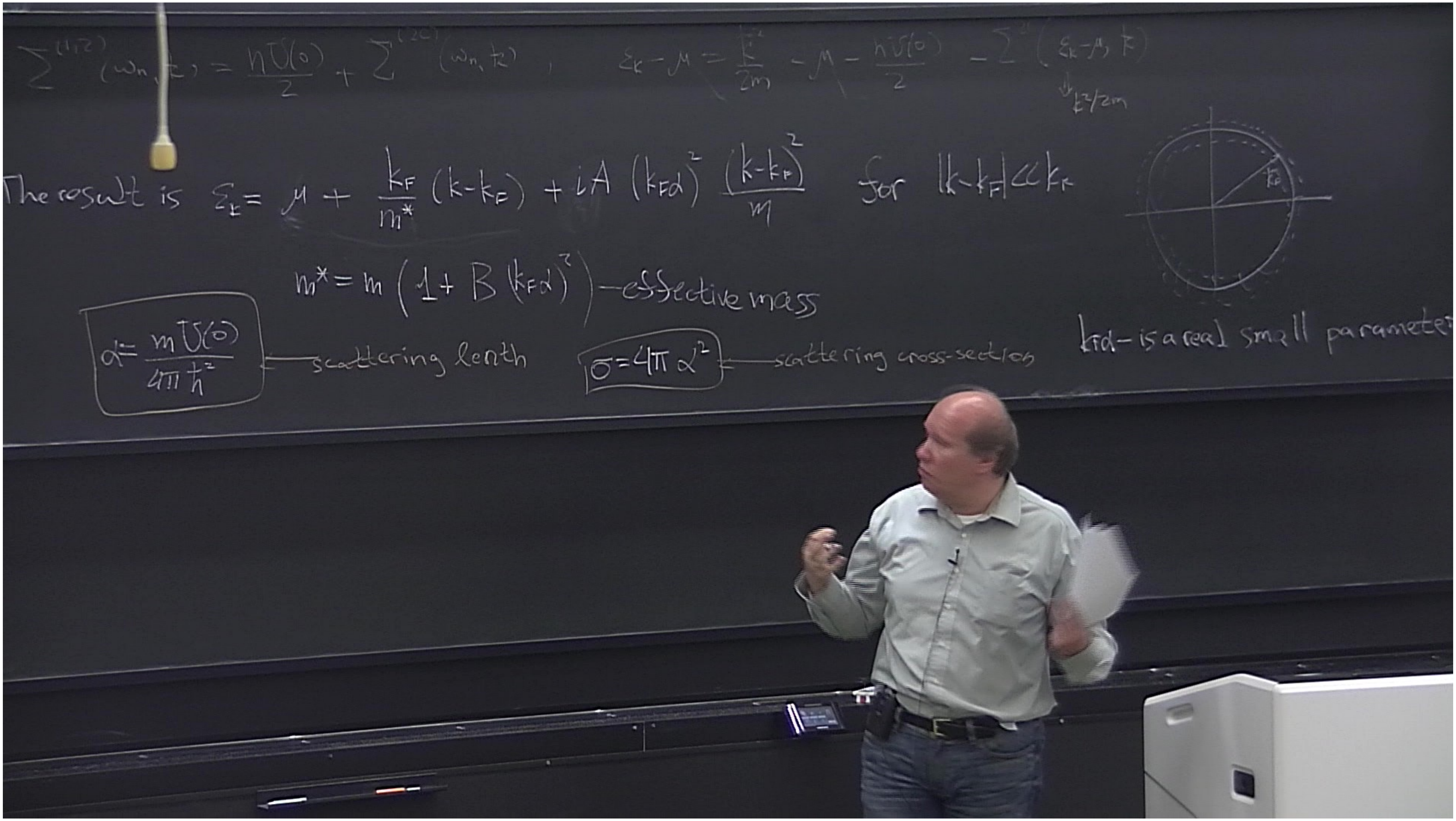
$= -\frac{n U(0)}{2}$ if $U(\bar{q}) = U(0)$

for point-like only

$U_{\sigma\sigma'}(\bar{q}) = U(\bar{q})$

$$\boxed{\frac{N}{V} = T \sum_{\omega_n} \sum_{\bar{p},\sigma} G(\omega_n, \bar{p})}$$


g from an interaction
shed line and two solid
ω_n - fermionic
Ω_m - bosonic
The result is ε_k = μ + k_F/m* (k - k_F) + iA (k_F a)^2 (k - k_F)^2/m for |k - k_F| << k_F
m* = m (1 + B (k_F a)^2) - effective mass
α = m U(0)/4π ħ^2 ← scattering length
σ = 4π α^2 ← scattering cross-section
k_F a - is a real small parameter
Matsubara frequency ω,
for point-like

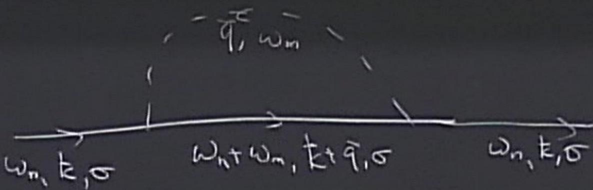


$\Sigma^{(1,2)}(\omega_n, k) = \frac{nU(0)}{2} + \Sigma^{(2c)}(\omega_n, k)$, $\varepsilon_k - \mu = \frac{k^2}{2m} - \mu - \frac{nU(0)}{2} - \Sigma^{(2)}(\varepsilon_k - \mu, k)$
$k^2/2m$
The result is $\varepsilon_k = \mu + \frac{k_F}{m^*}(k-k_F) + iA\,(k_F a)^2\,\frac{(k-k_F)^2}{m}$ for $|k-k_F| \ll k_F$
k_F
$m^* = m\left(1 + B\,(k_F a)^2\right)$ – effective mass
$\boxed{a = \frac{mU(0)}{4\pi\hbar^2}}$ ← scattering lenth
$\boxed{\sigma = 4\pi a^2}$ ← scattering cross-section
$k_F a$ – is a real small paramete

lines is zero

$(\omega_n, \bar{p}) = n U(0)$

Fock

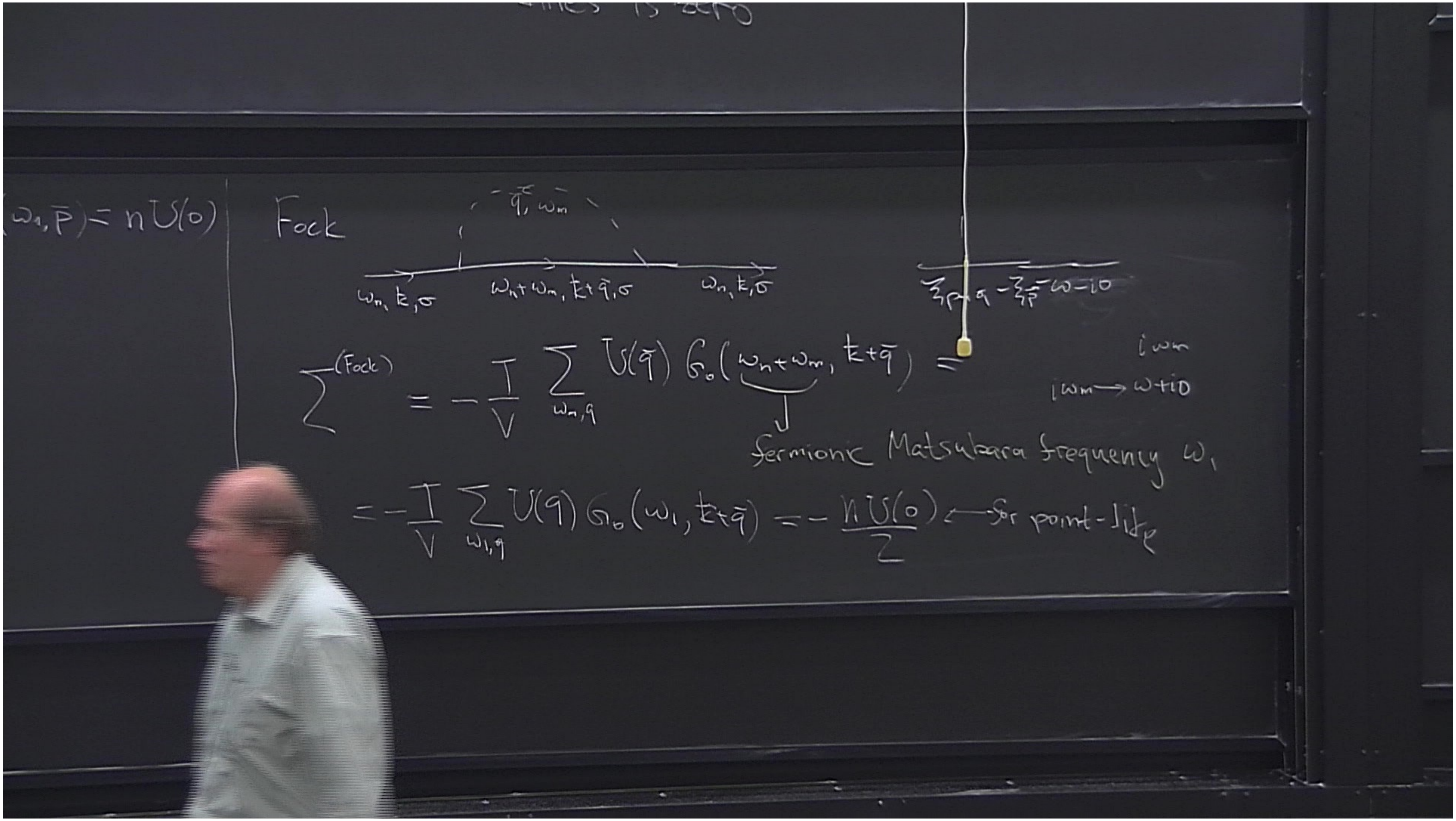


$$\Sigma^{(\text{Fock})} = -\frac{T}{V} \sum_{\omega_m, q} U(\bar{q})\, G_0(\underbrace{\omega_n + \omega_m}_{\text{fermionic Matsubara frequency } \omega_1}, \bar{k}+\bar{q}) =$$

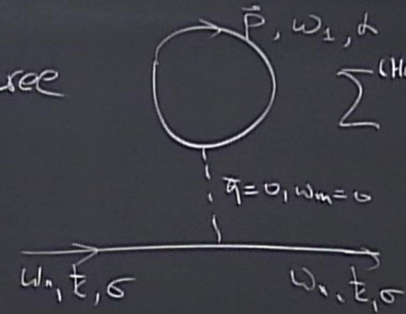
$i\omega_m$

$i\omega_m \to \omega + i0$

$$= -\frac{T}{V} \sum_{\omega_1, q} U(q)\, G_0(\omega_1, \bar{k}+\bar{q}) = -\frac{n U(0)}{2} \longleftrightarrow \text{for point-like}$$


$(\omega_1, \bar{p}) = nU(0)$
Fock
$\bar{q}, \omega_m$
ω_n, k, σ
$\omega_n + \omega_m, k+\bar{q}, \sigma$
ω_n, k, σ
$\xi_{p+q} - \xi_{\bar{p}} - \omega - i0$
$\Sigma^{(Fock)} = -\frac{T}{V} \sum_{\omega_m, q} U(\bar{q})\, G_0(\omega_n + \omega_m, k+\bar{q}) =$
$i\omega_m$
$i\omega_m \to \omega + i0$
fermionic Matsubara frequency ω_1
$= -\frac{T}{V} \sum_{\omega_1, q} U(q)\, G_0(\omega_1, k+\bar{q}) = -\frac{nU(0)}{2}$ ← for point-like

Hartree



$$\Sigma^{(\text{Hartree})} = U(0)\,\frac{T}{V}\sum_{\omega_1,\vec p,\alpha} G_0(\omega_1,\vec p) = n\,U(0)$$

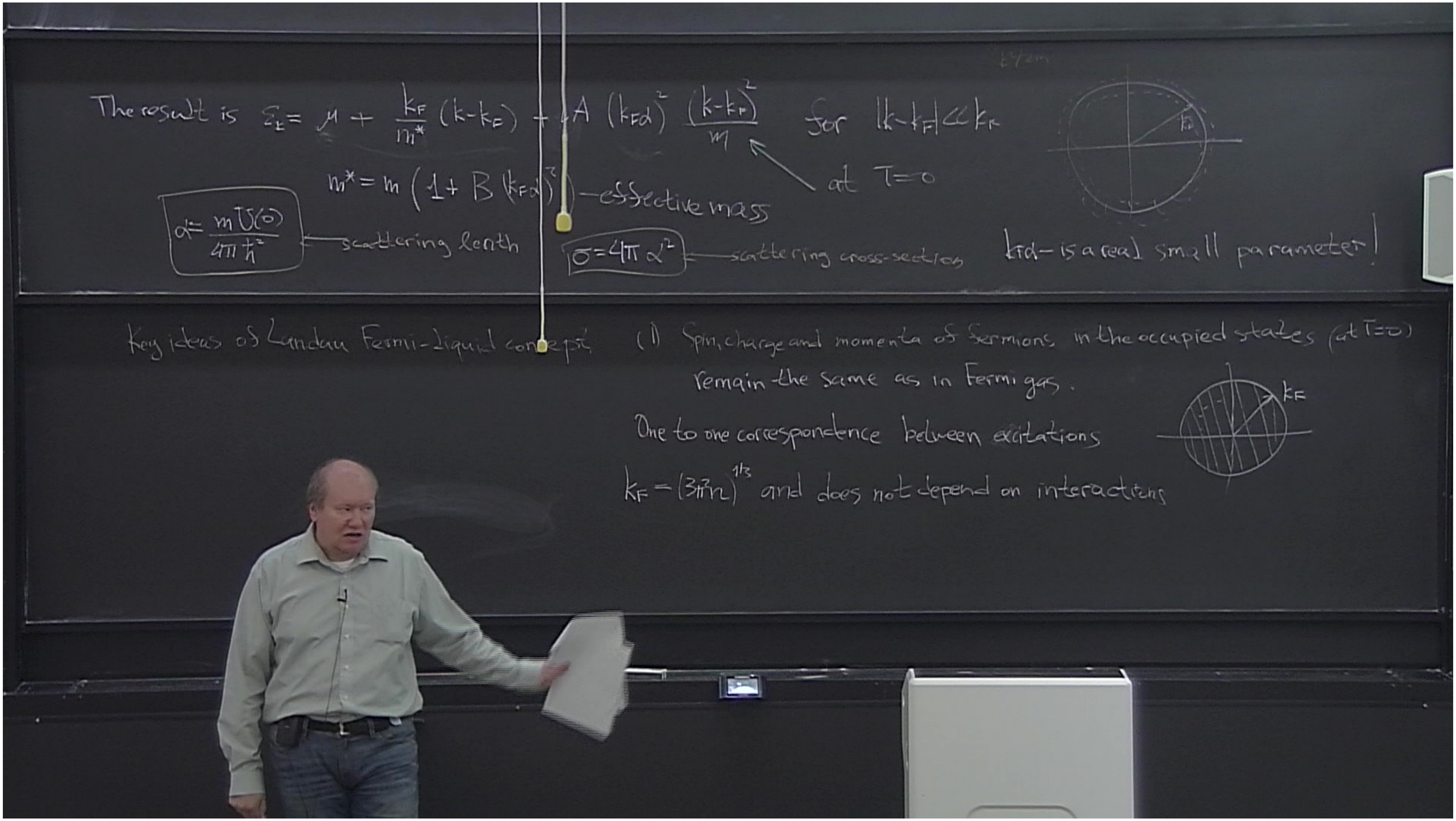
$$\int \frac{1}{x-a-i0}(\dots) = ⨍ \frac{(\dots)}{x-a} + i\pi\delta(x-a)$$

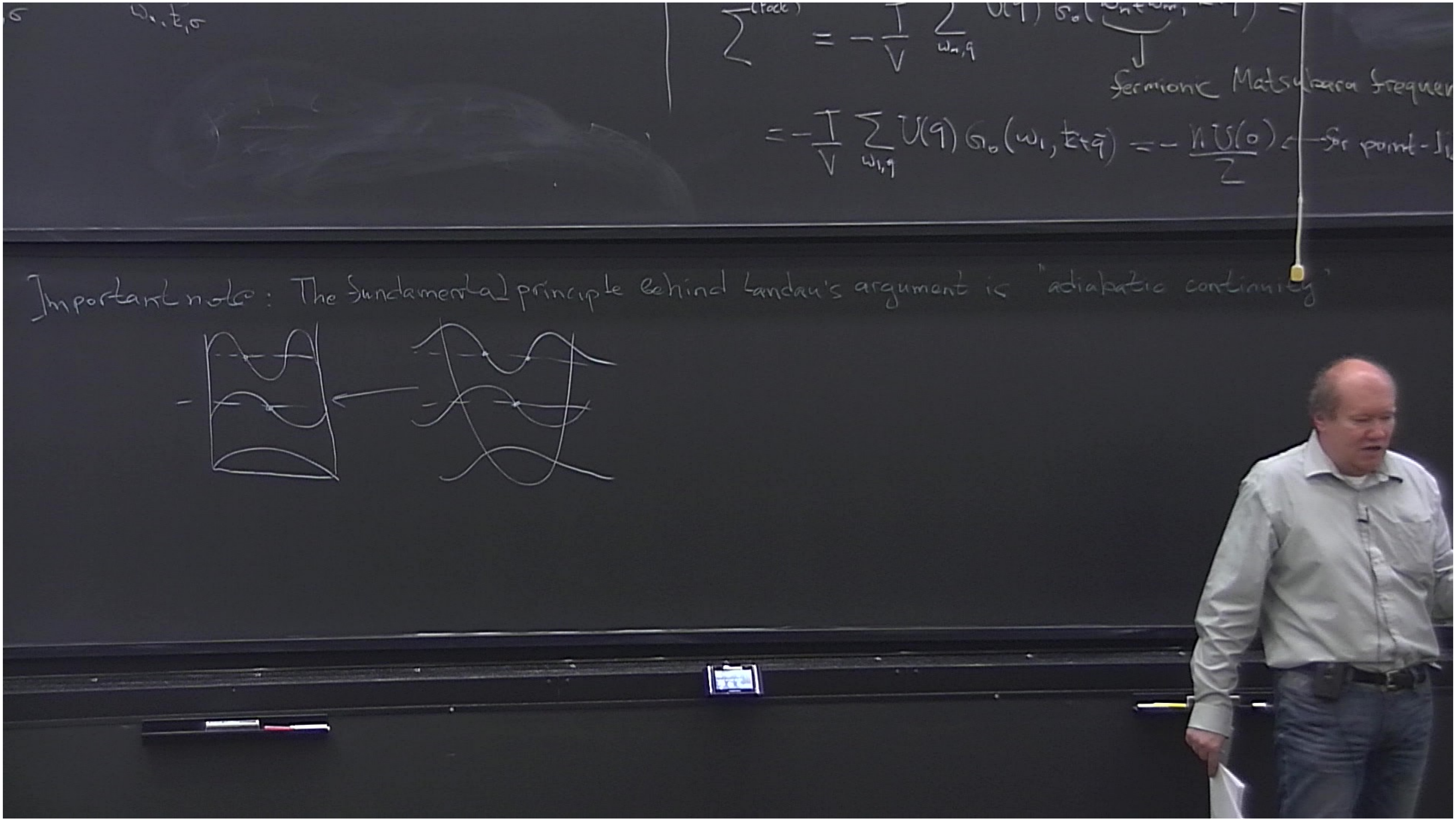
Fock

$\omega_n, \vec k, \sigma$

$$\Sigma^{(\text{Fock})} = -$$

$$= -\frac{T}{V}\sum$$


The result is $\varepsilon_k = \mu + \frac{k_F}{m^*}(k-k_F) + A(k_F a)^2 \frac{(k-k_F)^2}{m}$ for $|k-k_F| \ll k_F$
at $T=0$
$m^* = m\left(1 + B(k_F a)^2\right)$ – effective mass
$a = \frac{mU(0)}{4\pi\hbar^2}$ scattering lenth
$\sigma = 4\pi a^2$ scattering cross-section
$k_F a$ – is a real small parameter!
k_F
Key ideas of Landau Fermi-liquid concept
(1) Spin, charge and momenta of fermions in the occupied states (at T=0)
remain the same as in Fermi gas.
One to one correspondence between excitations
$k_F = (3\pi^2 n)^{1/3}$ and does not depend on interactions
k_F


fermionic Matsubara frequen
$= -\frac{T}{V} \sum_{\omega_1, q} U(q)\, G_0(\omega_1, k+q) = -\frac{n\, U(0)}{2}$
for point-li
Important note: The fundamental principle behind Landau's argument is "adiabatic continuity"

effective mass

$\boxed{\alpha = \frac{m U(0)}{4\pi \hbar^2}}$ ← scattering lenth

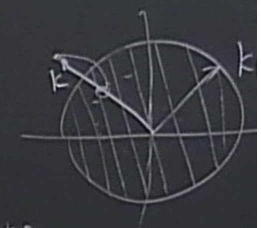
$\boxed{\sigma = 4\pi \alpha^2}$ ← scattering cross-section

$k_F a$ — is a real small para

Key ideas of Landau Fermi-Liquid concept

(i) Spin, charge and momenta of fermions in the occupied stat

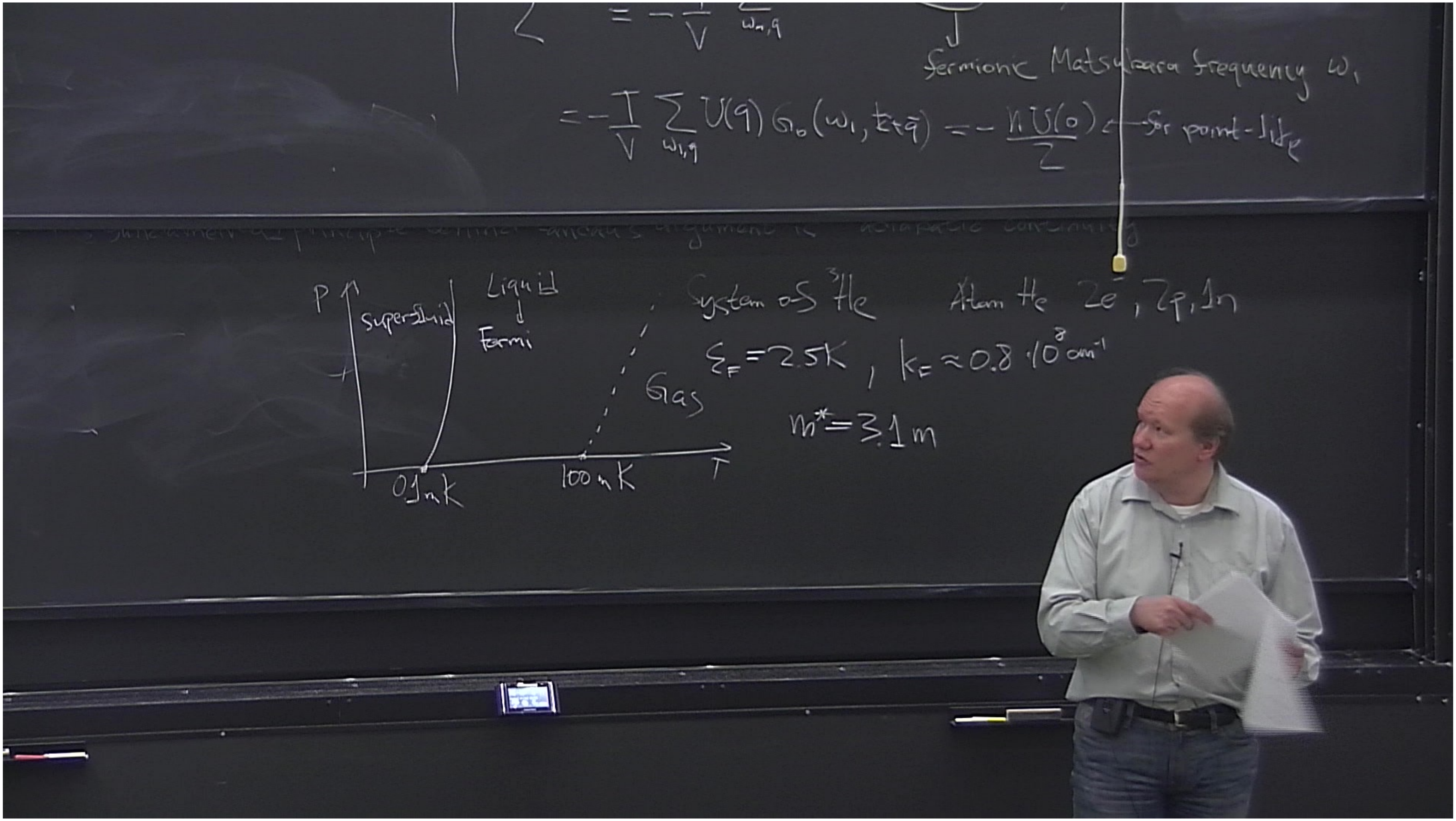
remain the same as in Fermi gas.

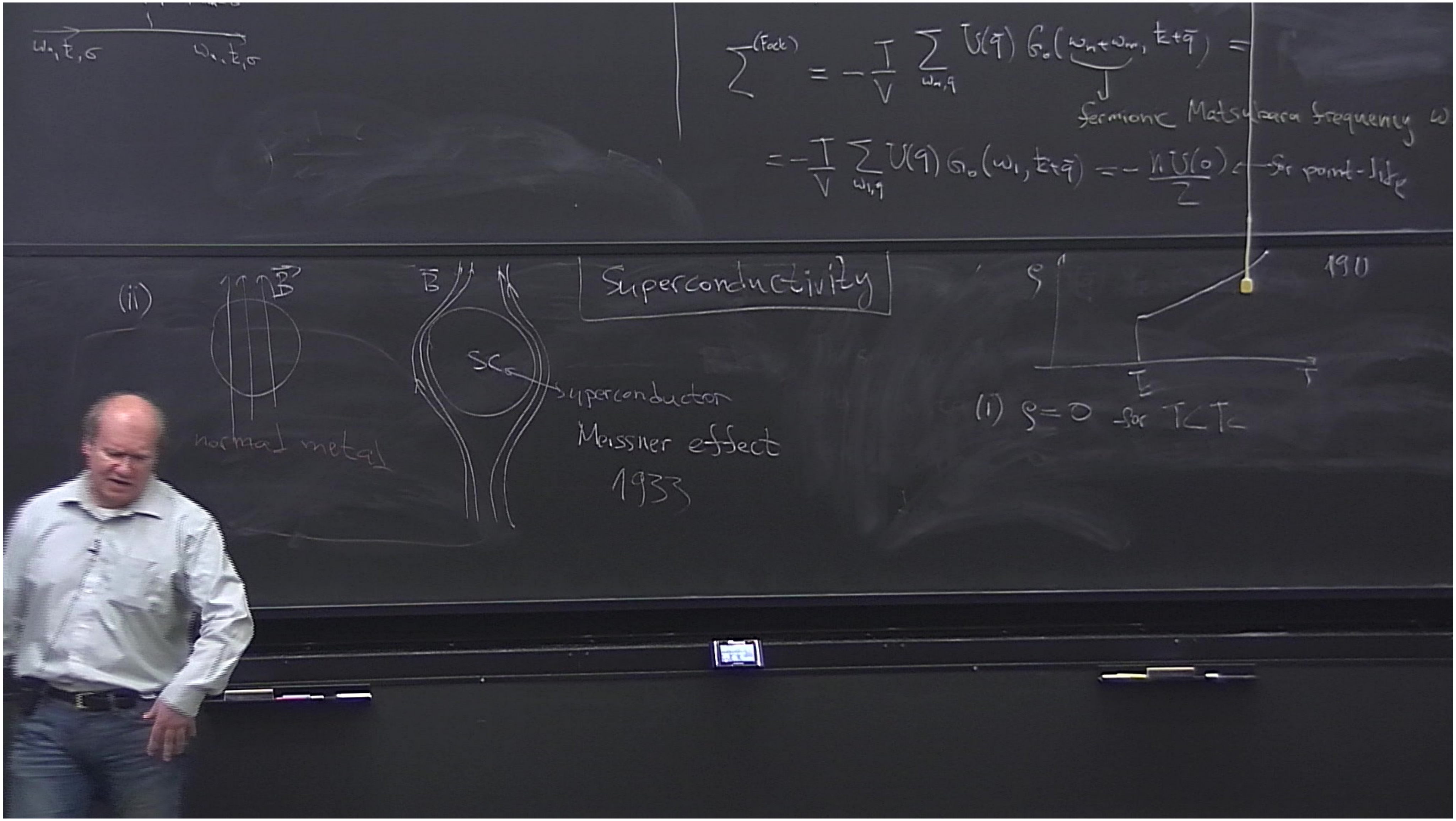


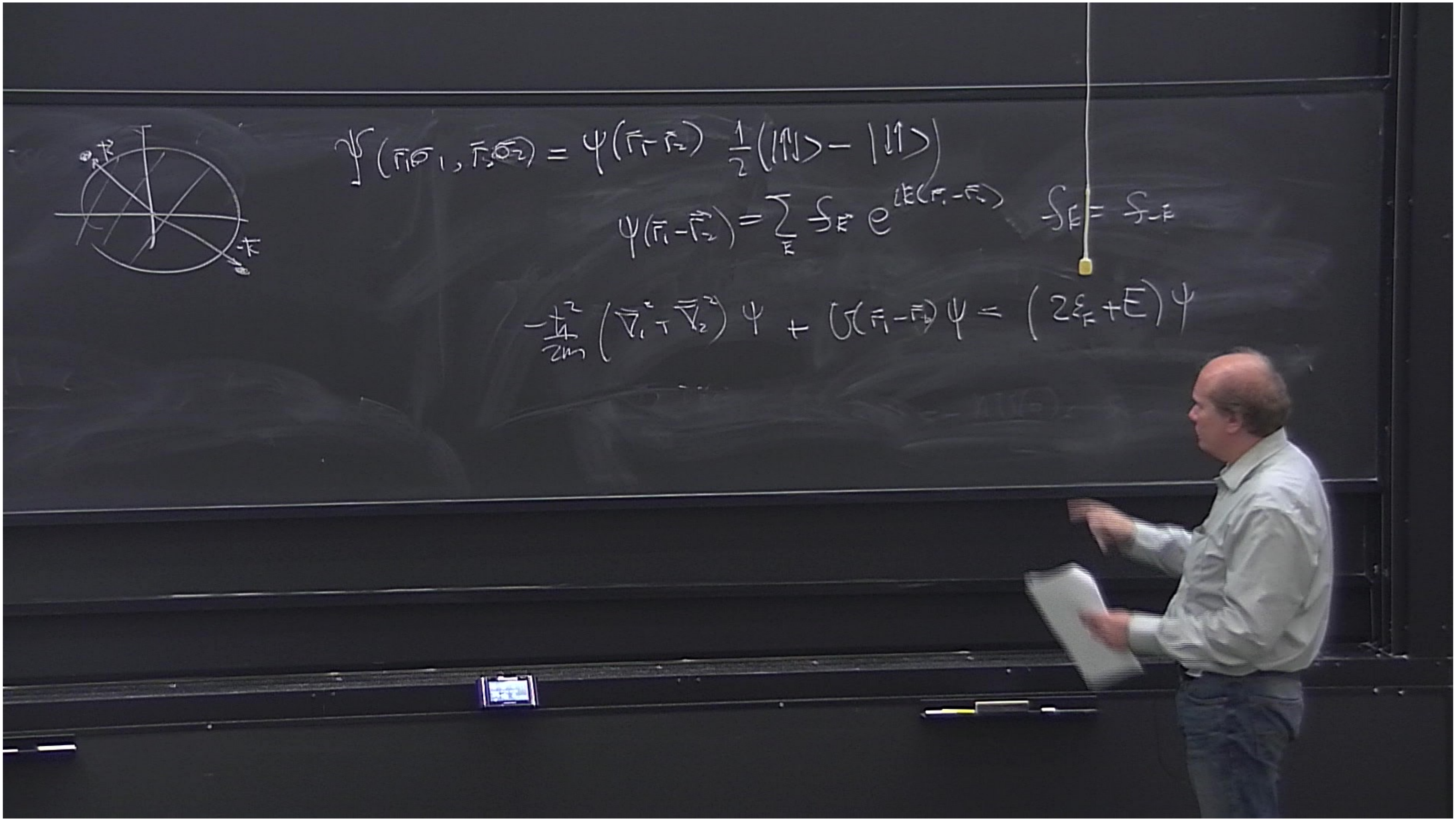
One to one correspondence between excitations

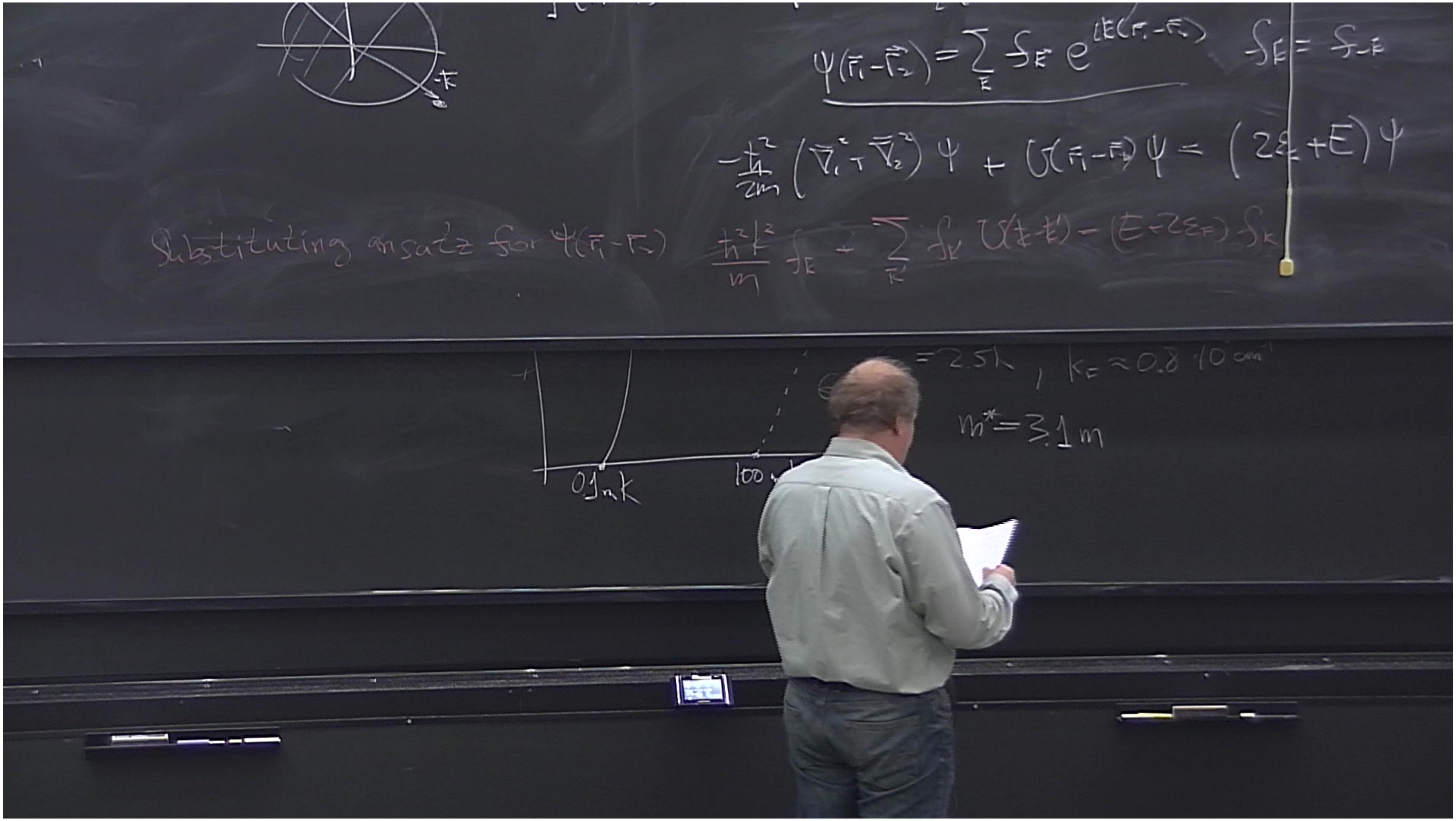
$k_F = (3\pi^2 n)^{1/3}$ and does not depend on interactions

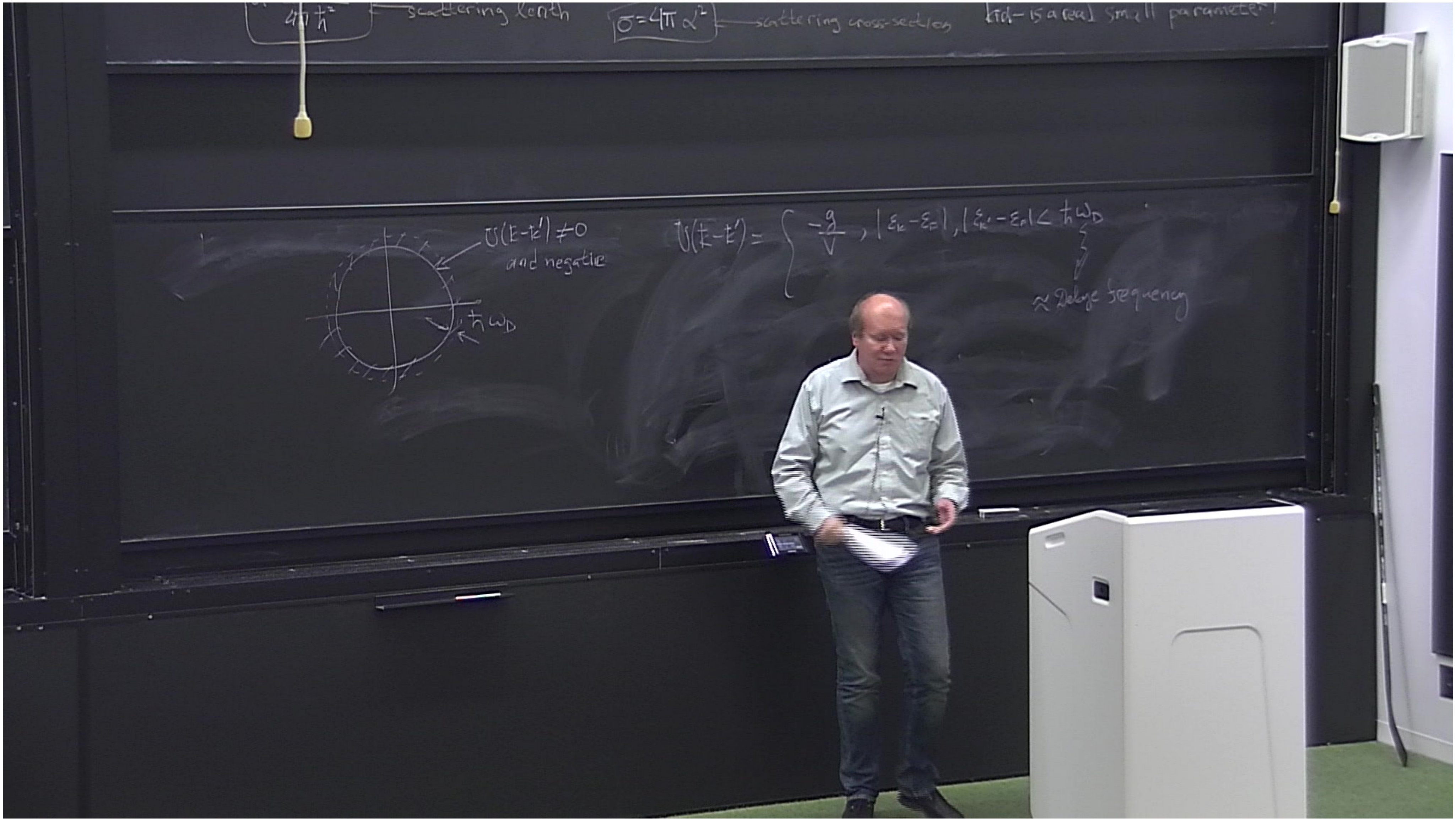
(ii) $\varepsilon_k = \varepsilon_F + \frac{k_F}{m^*}(k - k_F)$ specific heat is $\frac{k_F m^*}{3} T$

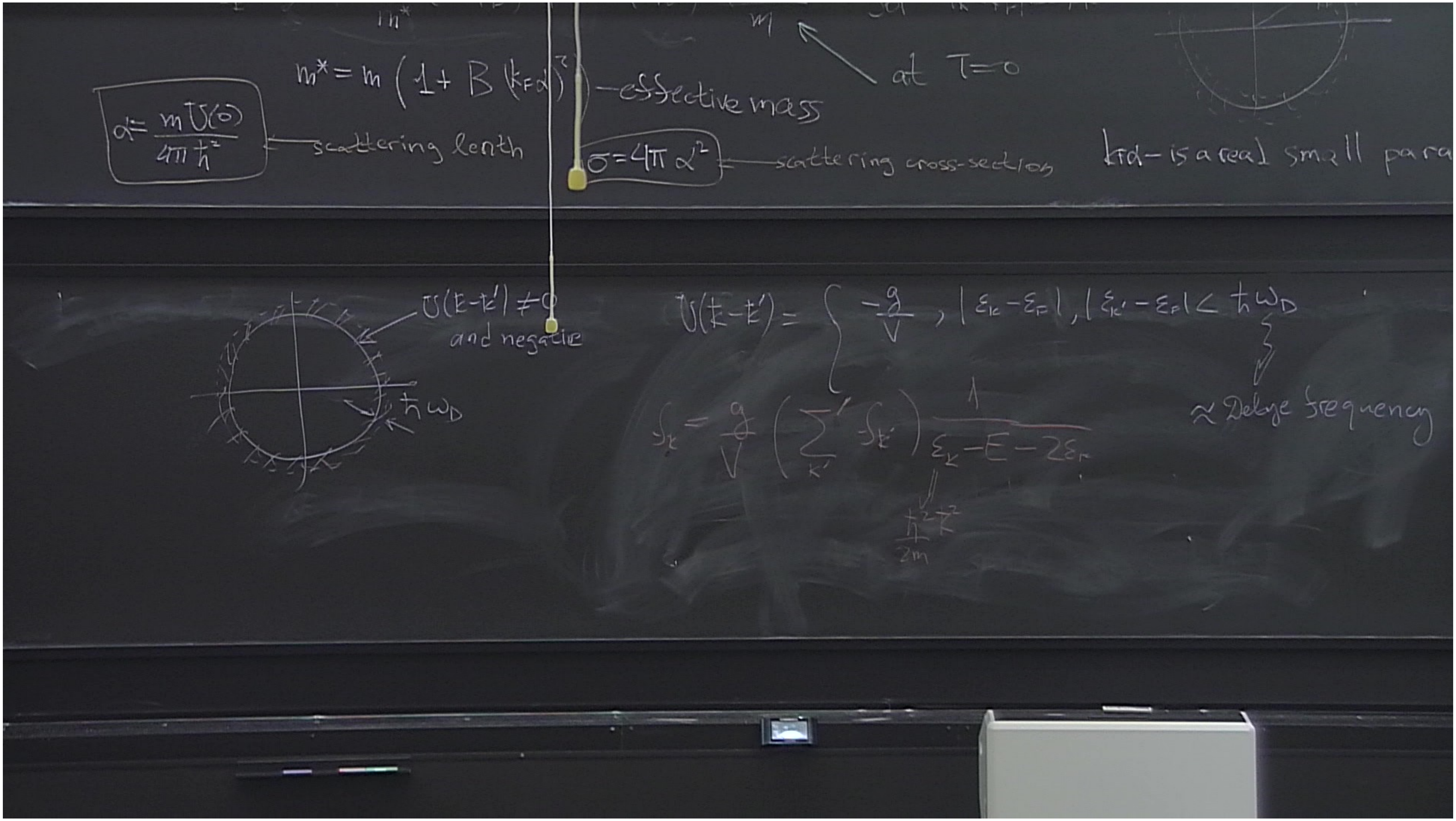

fermionic Matsubara frequency ω₁
= − T/V Σ_{ω₁,q} U(q) G₀(ω₁, k+q) = − n U(0)/2 ← for point-like
P
superfluid
Liquid
Fermi
Gas
01 mK
100 mK
T
System of ³He
Atom He 2e⁻, 2p, 1n
ε_F = 2.5 K , k_F ≈ 0.8 · 10⁸ cm⁻¹
m* = 3.1 m

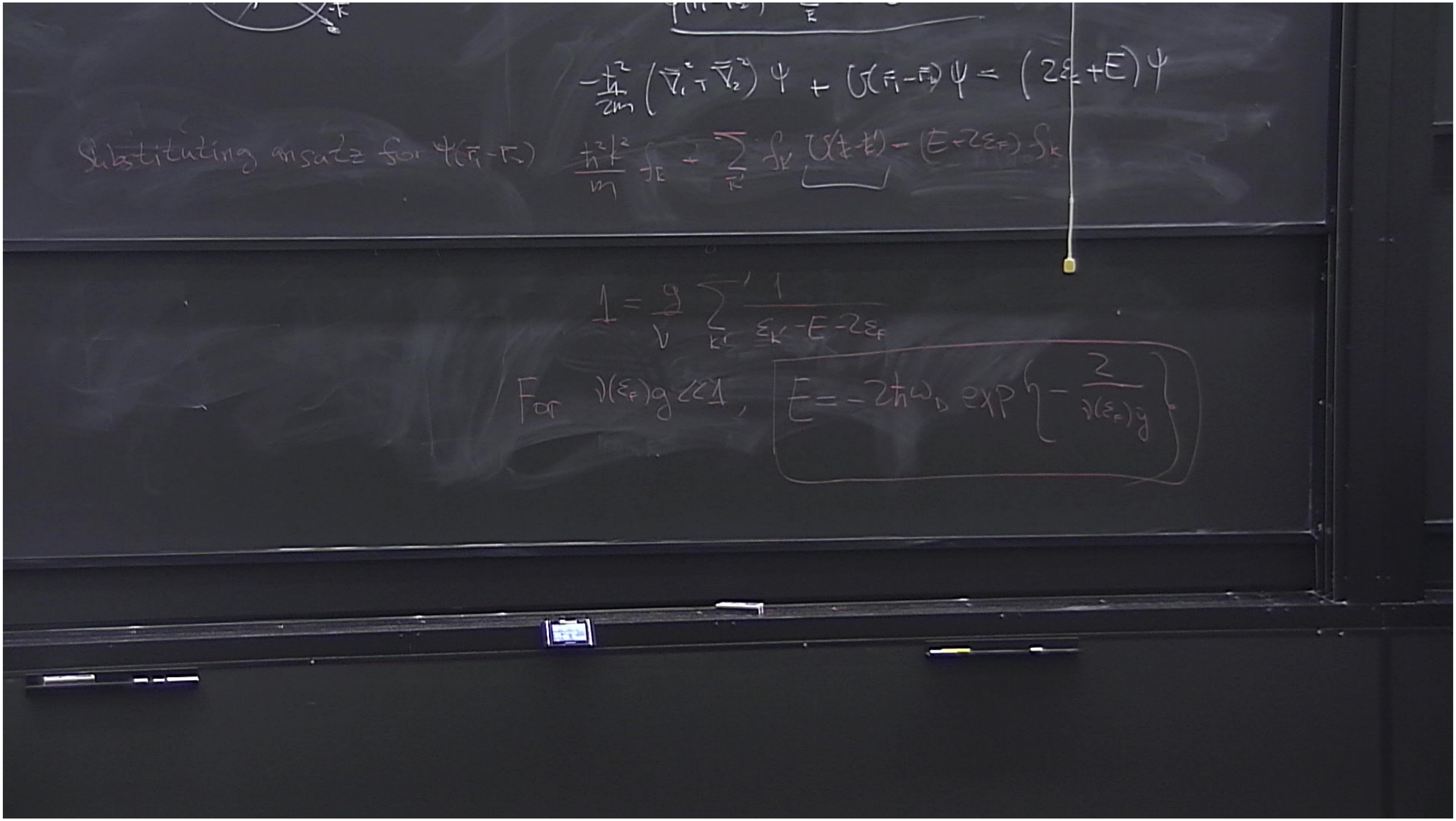

Supercondnctivity
(ii)
B
SC
superconductor
normal metal
Meissner effect
1933
(i) ρ = 0 for T < Tc
fermionic Matsubara frequency ω
for point-like


$\psi(\vec{r}_1\sigma_1, \vec{r}_2\sigma_2) = \psi(\vec{r}_1 - \vec{r}_2)\ \frac{1}{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$
$\psi(\vec{r}_1 - \vec{r}_2) = \sum_{\vec{k}} f_{\vec{k}}\, e^{i\vec{k}(\vec{r}_1 - \vec{r}_2)}$
$f_{\vec{k}} = f_{-\vec{k}}$
$-\frac{\hbar^2}{2m}(\nabla_1^2 + \nabla_2^2)\psi + U(\vec{r}_1 - \vec{r}_2)\psi = (2\varepsilon_F + E)\psi$
$\vec{k}$
$-\vec{k}$


$\psi(\vec r_1-\vec r_2)=\sum_k f_k e^{ik(\vec r_1-\vec r_2)}$
$f_k = f_{-k}$
Substituting ansatz for $\psi(\vec r_1-\vec r_2)$
$m^* = 3.1\,m$


scattering lenth
$\boxed{\sigma = 4\pi a^2}$ scattering cross-section
$k_F a$ – is a real small parameter
$U(k-k') \neq 0$
and negative
$\hbar\omega_D$
$U(k-k') = \begin{cases} -\frac{g}{V}, \; |\varepsilon_k - \varepsilon_F|, |\varepsilon_{k'} - \varepsilon_F| < \hbar\omega_D \end{cases}$
≈ Debye frequency


m* = m (1 + B (kFα)²) - effective mass
at T=0
α = mU(0) / 4πħ²
scattering lenth
σ = 4πα²
scattering cross-section
kFα - is a real small para
U(k-k') ≠ 0
and negatie
ħωD
U(k-k') = { -g/V , |εk - εF|, |εk' - εF| < ħωD
≈ Debye frequency
ψk = g/V (Σ'k' ψk') 1/(εk - E - 2εF)
ħ²k²/2m


$-\frac{\hbar^2}{2m}(\nabla_1^2+\nabla_2^2)\psi + U(\vec{r}_1-\vec{r}_2)\psi = (2\varepsilon_F+E)\psi$
Substituting ansatz for $\psi(\vec{r}_1-\vec{r}_2)$
$\frac{\hbar^2k^2}{m} f_k + \sum_{k'} f_{k'} U(k-k') = (E+2\varepsilon_F) f_k$
$1 = \frac{g}{V}\sum_{k'}' \frac{1}{\varepsilon_{k'}-E-2\varepsilon_F}$
For $\nu(\varepsilon_F) g \ll 1$,
$E = -2\hbar\omega_D \exp\left\{-\frac{2}{\nu(\varepsilon_F)g}\right\}$