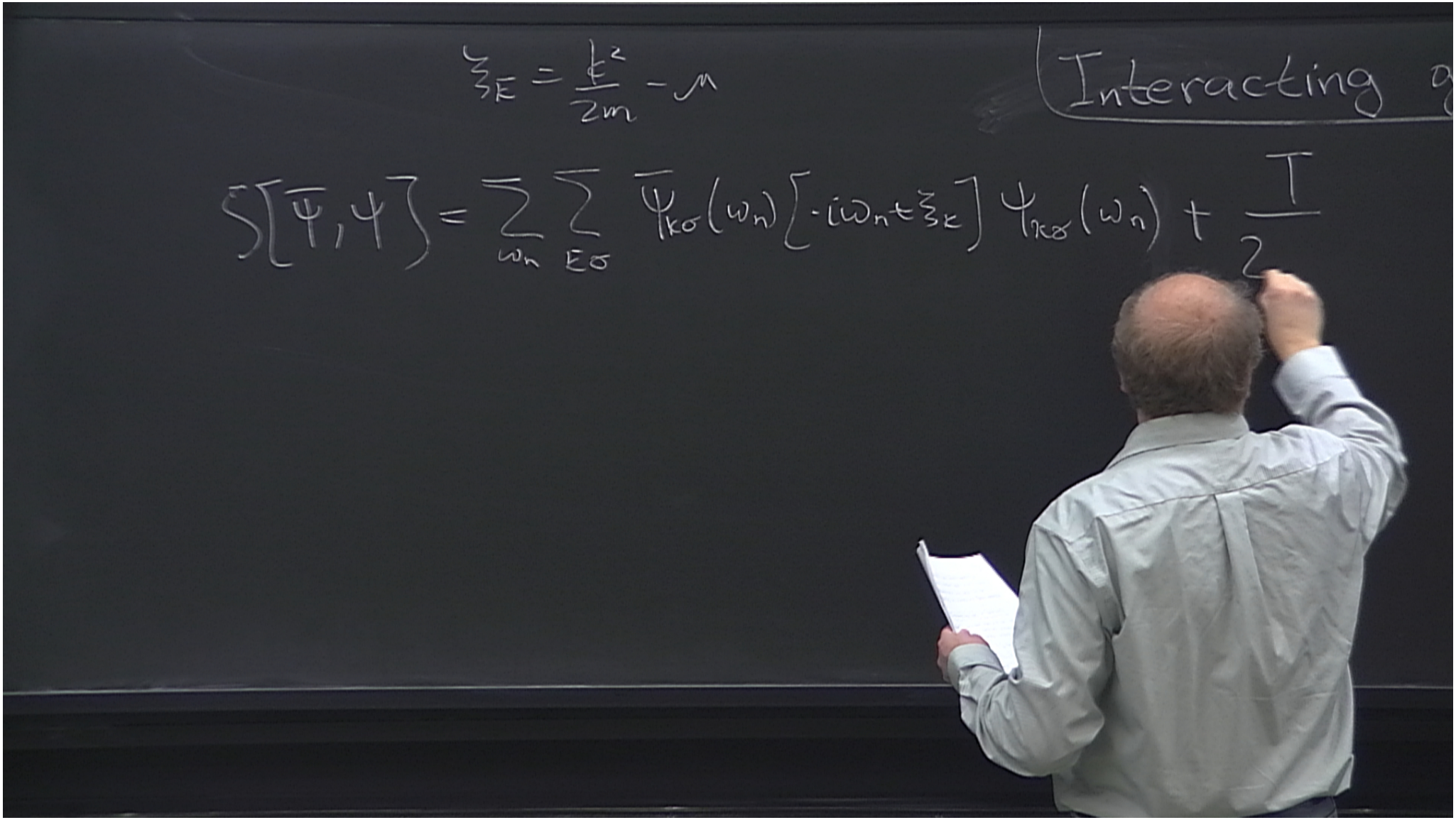
Title: PSI 2015/2016 Condensed Matter - Oleg Tchernyshyov - Lecture 9

Date: Nov 19, 2015 10:45 AM

URL: http://pirsa.org/15110050

Abstract:

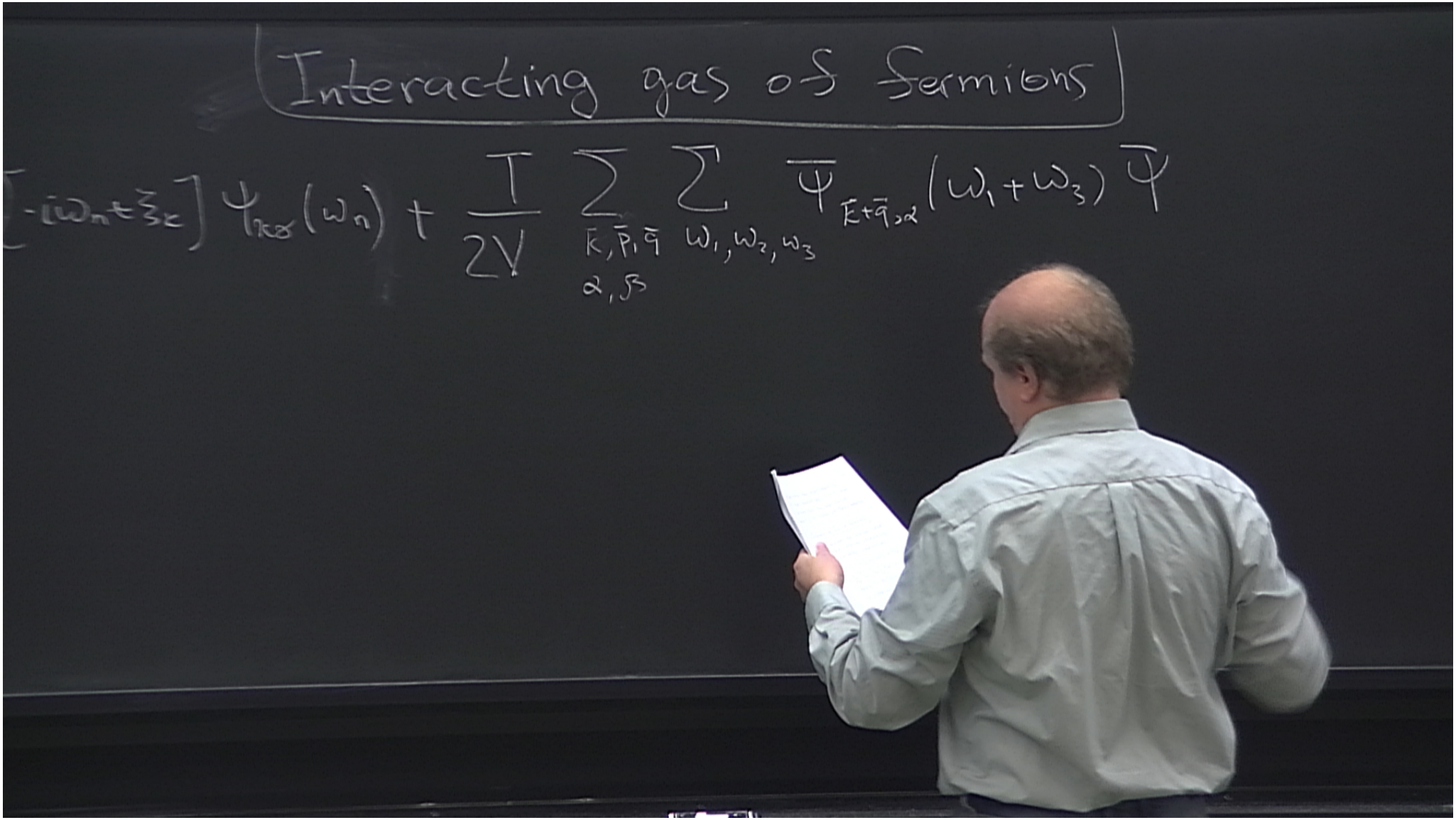

$\xi_k = \frac{k^2}{2m} - \mu$
Interacting
$S[\bar{\psi},\psi] = \sum_{\omega_n} \sum_{k\sigma} \bar{\psi}_{k\sigma}(\omega_n)\left[-i\omega_n + \xi_k\right]\psi_{k\sigma}(\omega_n) + \frac{I}{2}$

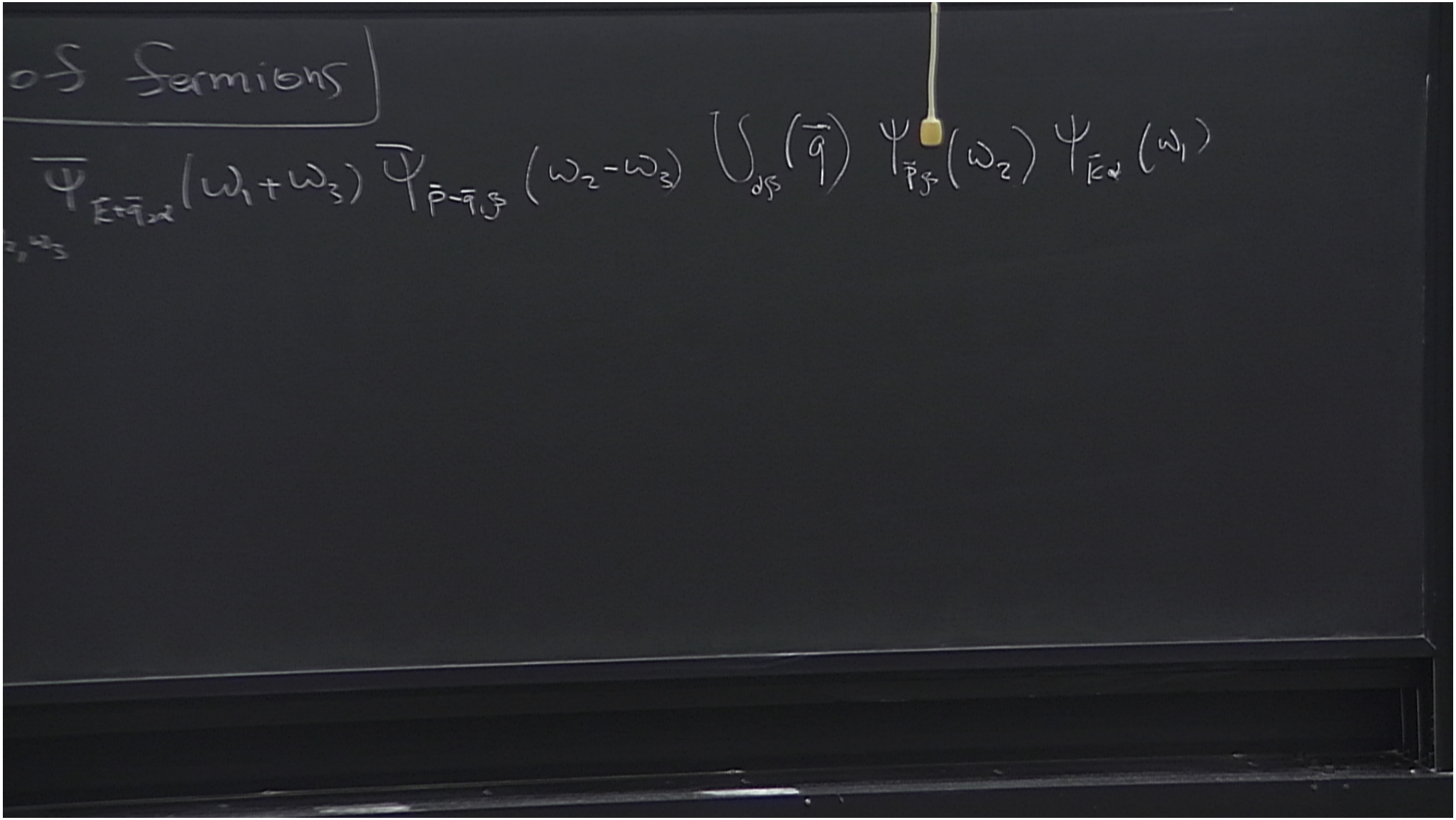
$$\xi_k = \frac{k^2}{2m} - \mu$$

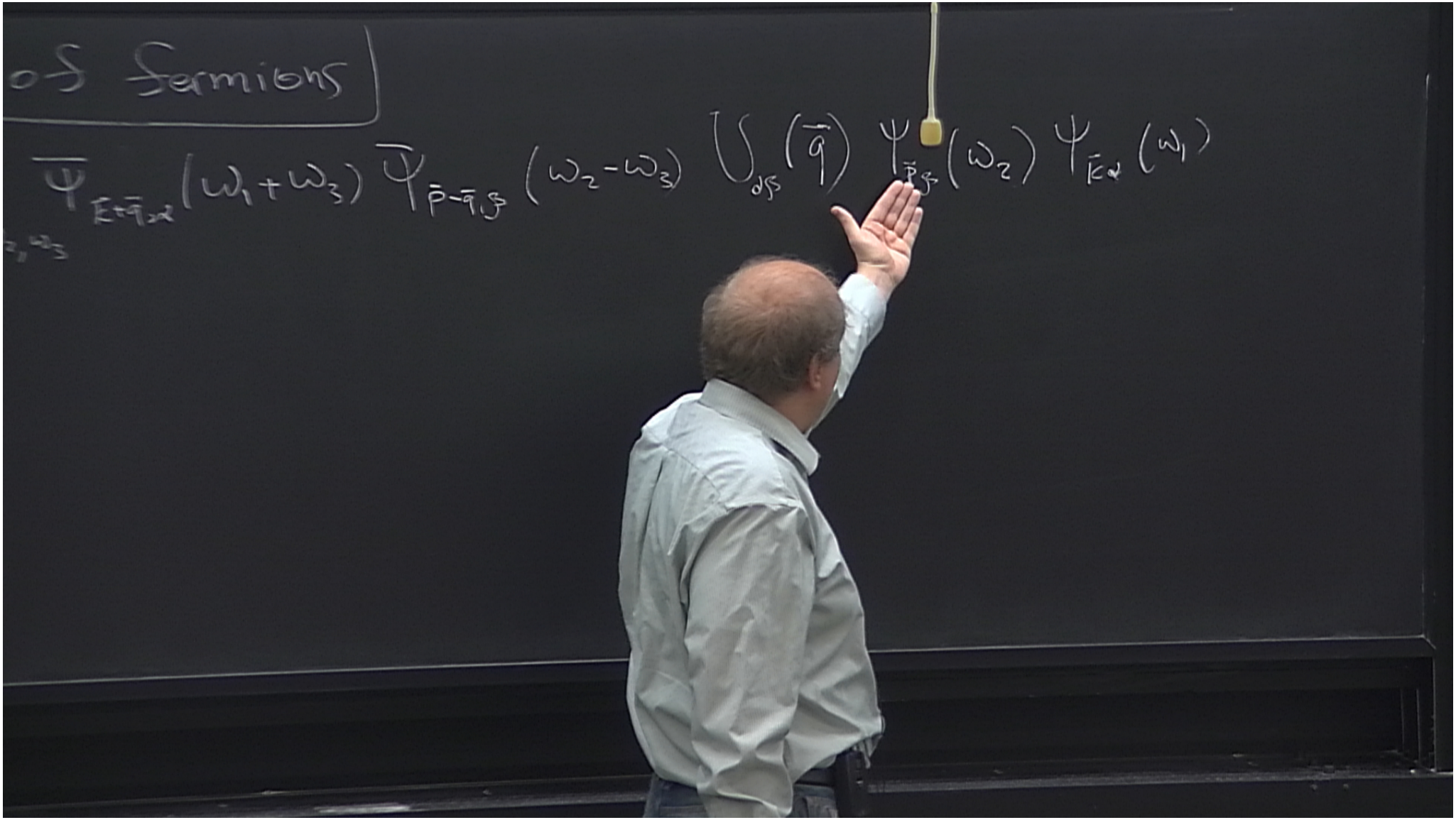
Interacting g

$$S[\bar{\psi},\psi] = \sum_{\omega_n} \sum_{k\sigma} \bar{\psi}_{k\sigma}(\omega_n)\left[-i\omega_n + \xi_k\right]\psi_{k\sigma}(\omega_n) + \frac{I}{2V} \sum_{\bar{k},\bar{p},\bar{q}}$$




Interacting gas of fermions
$\left[-i\omega_n + \xi_{\vec{k}}\right] \psi_{\vec{k}\alpha}(\omega_n) + \frac{T}{2V} \sum_{\vec{k},\vec{p},\vec{q}} \sum_{\omega_1,\omega_2,\omega_3} \bar{\psi}_{\vec{k}+\vec{q},\alpha}(\omega_1+\omega_3)\,\bar{\psi}$
α, β


of fermions
$\overline{\Psi}_{E+\bar{q},\alpha}(\omega_1+\omega_3)\,\overline{\Psi}_{\bar{p}-\bar{q},\beta}(\omega_2-\omega_3)\,U_{\alpha\beta}(\bar{q})\,\Psi_{\bar{p}\beta}(\omega_2)\,\Psi_{E\alpha}(\omega_1)$


of fermions

$$\xi_k = \frac{k^2}{2m} - \mu$$

Interacting gas of Fe

$$S[\bar{\psi},\psi] = \sum_{\omega_n} \sum_{k\sigma} \bar{\psi}_{k\sigma}(\omega_n)\left[-i\omega_n + \xi_k\right]\psi_{k\sigma}(\omega_n) + \frac{T}{2V} \sum_{\substack{\bar{k},\bar{p},\bar{q} \\ \alpha,\beta}} \sum_{\omega_1,\omega_2,\omega_3} \bar{\psi}_{k+\bar{q},\alpha}$$

Assume $U_{\alpha\beta}(\bar{q}) = U(\bar{q})\delta_{\alpha\beta} \longrightarrow U(0)\delta_{\bar{q},0}\delta_{\alpha\beta}$. For Coul

pointlike

ing gas of fermions

$$\sum_{\bar{k},\bar{p},\bar{q}} \sum_{\omega_1,\omega_2,\omega_3} \sum_{\alpha,\beta} \Psi^{\dagger}_{\bar{k}+\bar{q},\alpha}(\omega_1+\omega_3)\, \Psi^{\dagger}_{\bar{p}-\bar{q},\beta}(\omega_2-\omega_3)\, U_{\alpha\beta}(\bar{q})\, \Psi_{\bar{p}\beta}(\omega_2)\, \Psi_{\bar{k}\alpha}(\omega_1)$$

$\delta_{\alpha\beta}$. For Coulomb interaction $U(q) = \frac{4\pi e^2}{q^2}$

$\langle (p-q)\, U(q)\, \psi_\beta(p)\, \psi_\alpha(l) \rangle_0$

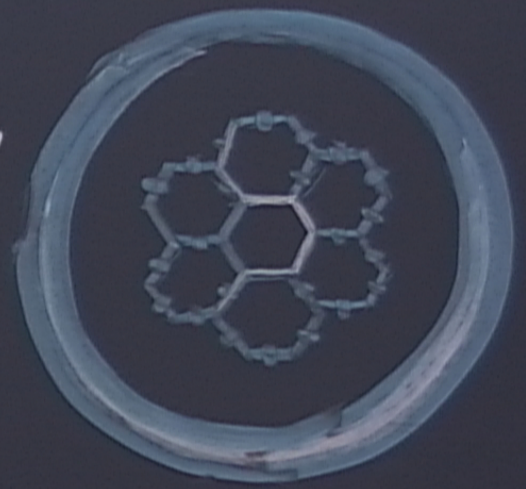
$U_{\alpha\beta}(q) = U_1(q)\,\delta_{\alpha\beta} +$

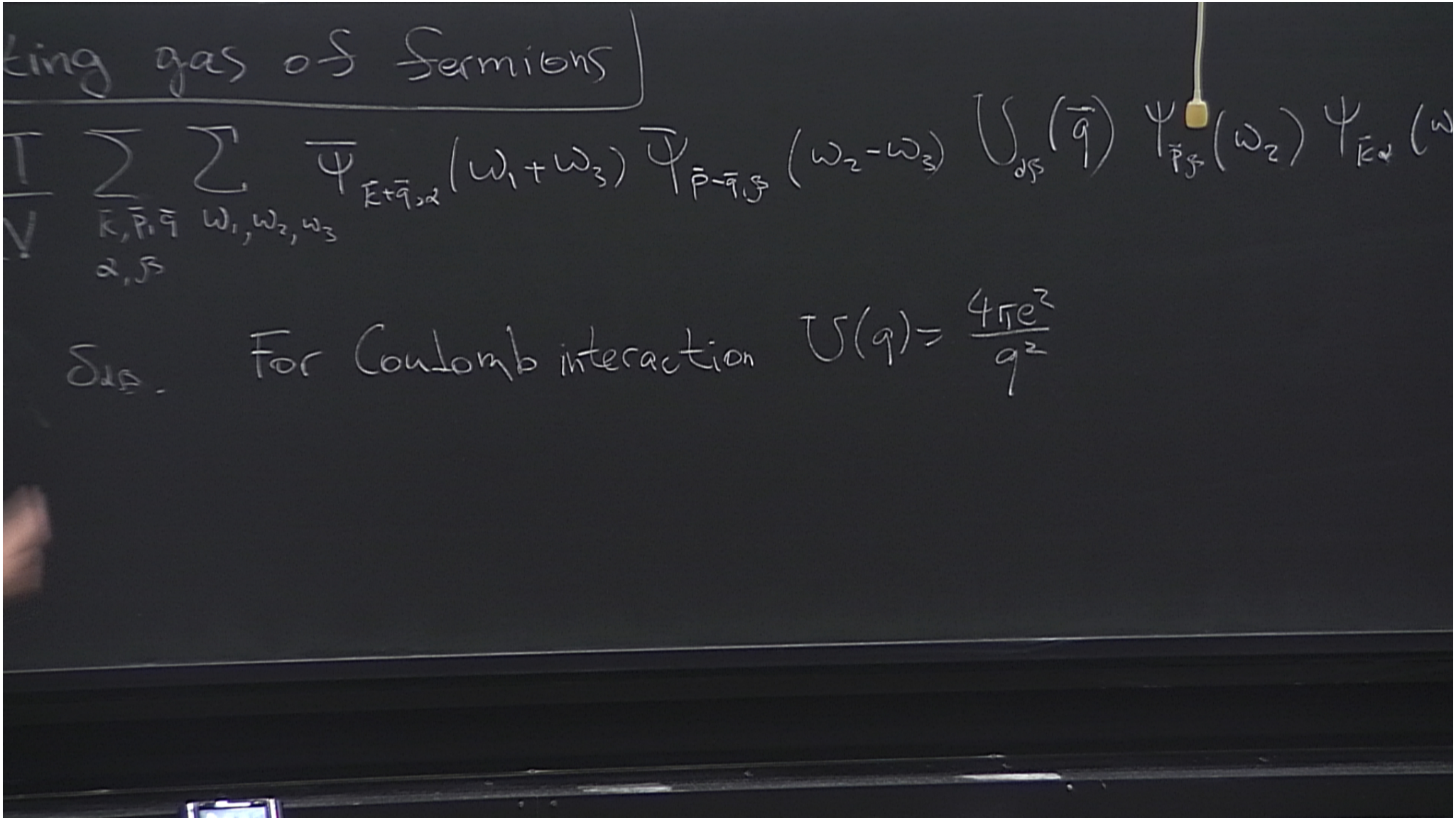
$+ U_2(q)(1-\delta_{\alpha\beta})$

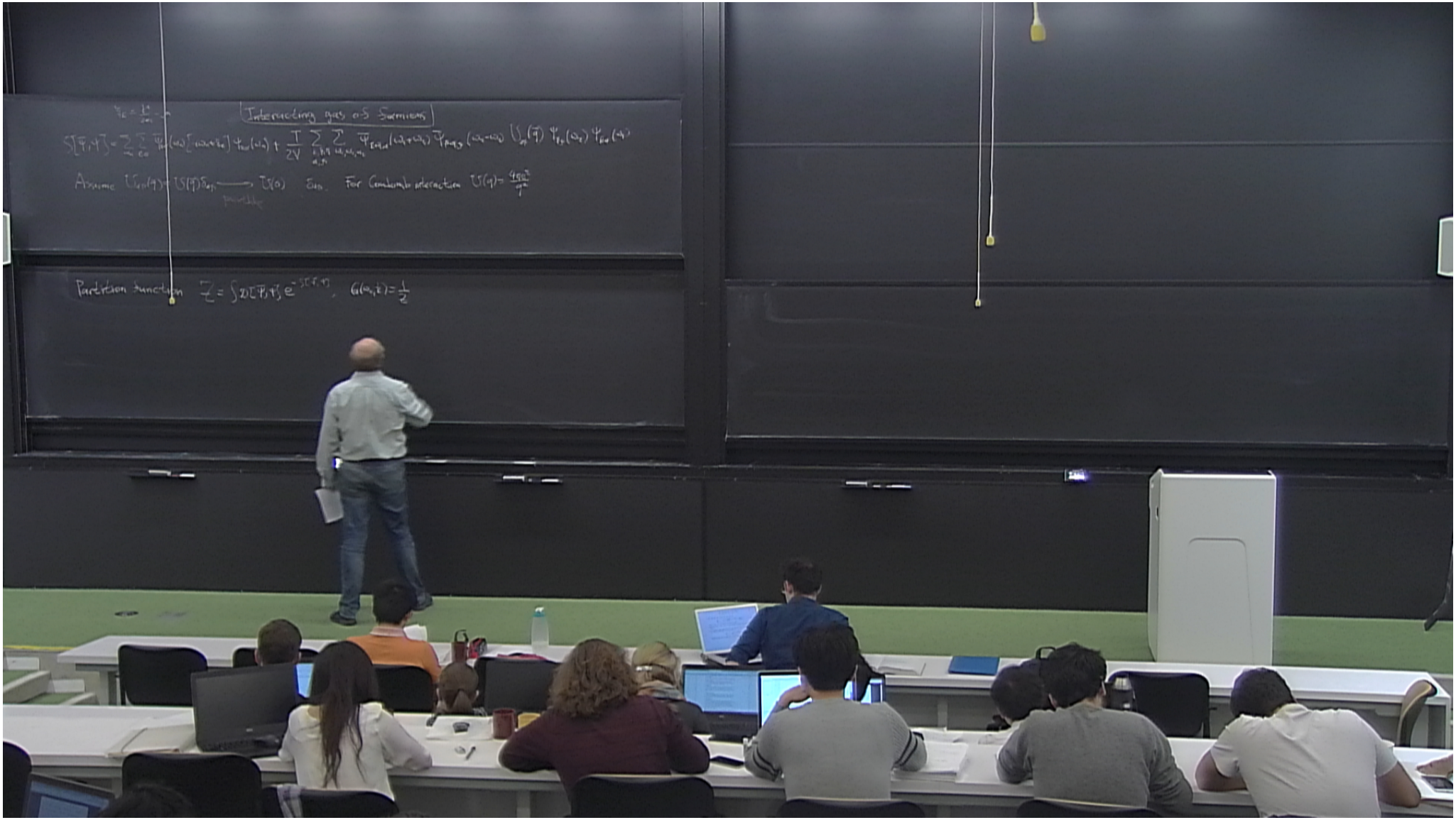
$\sigma = 0, \sigma = \alpha,$ β and β internal

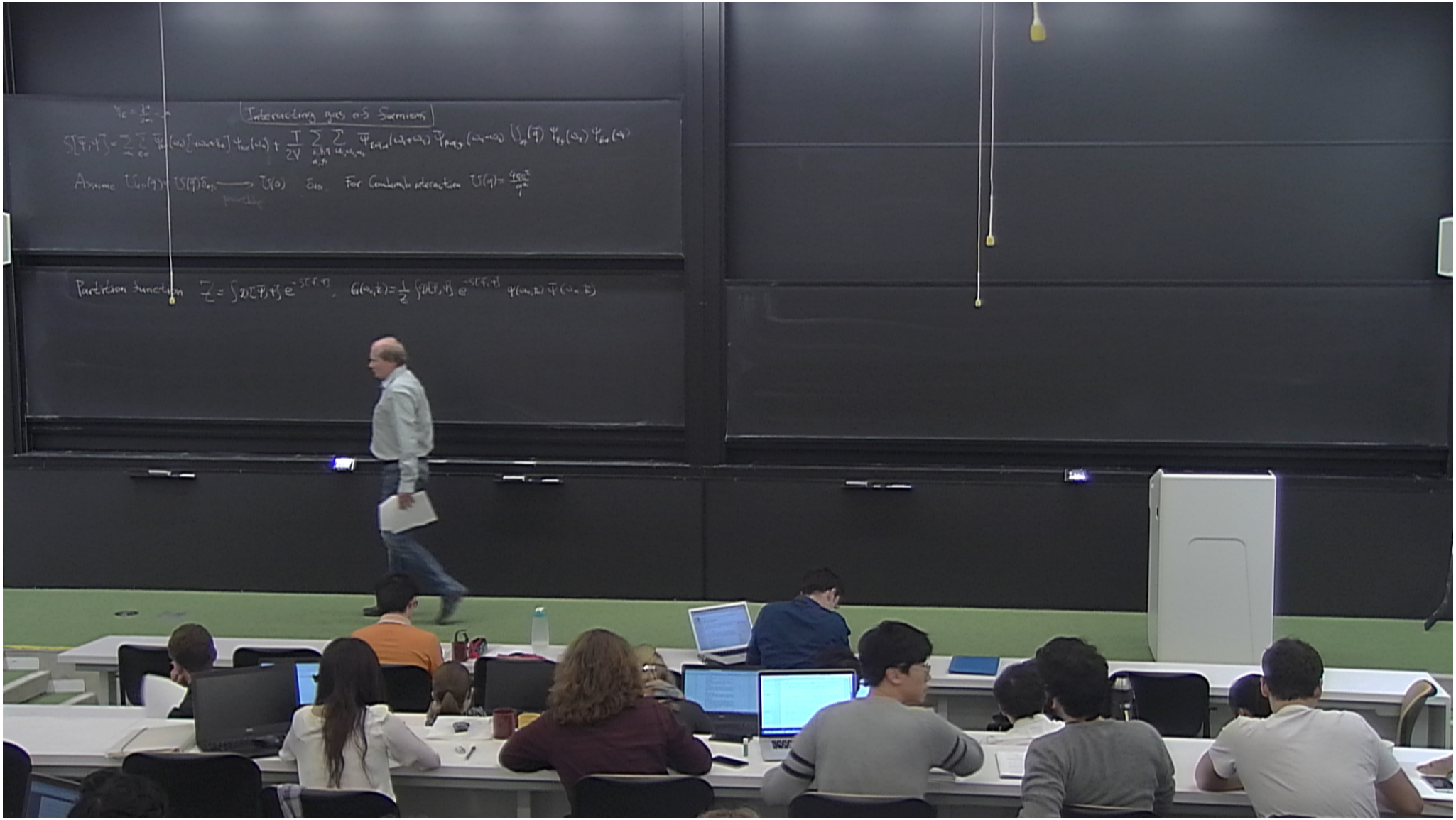
$\langle \psi_\beta(p-q)\, U(q)\, \psi_\beta(p')\, \psi_\alpha(l) \rangle_0$

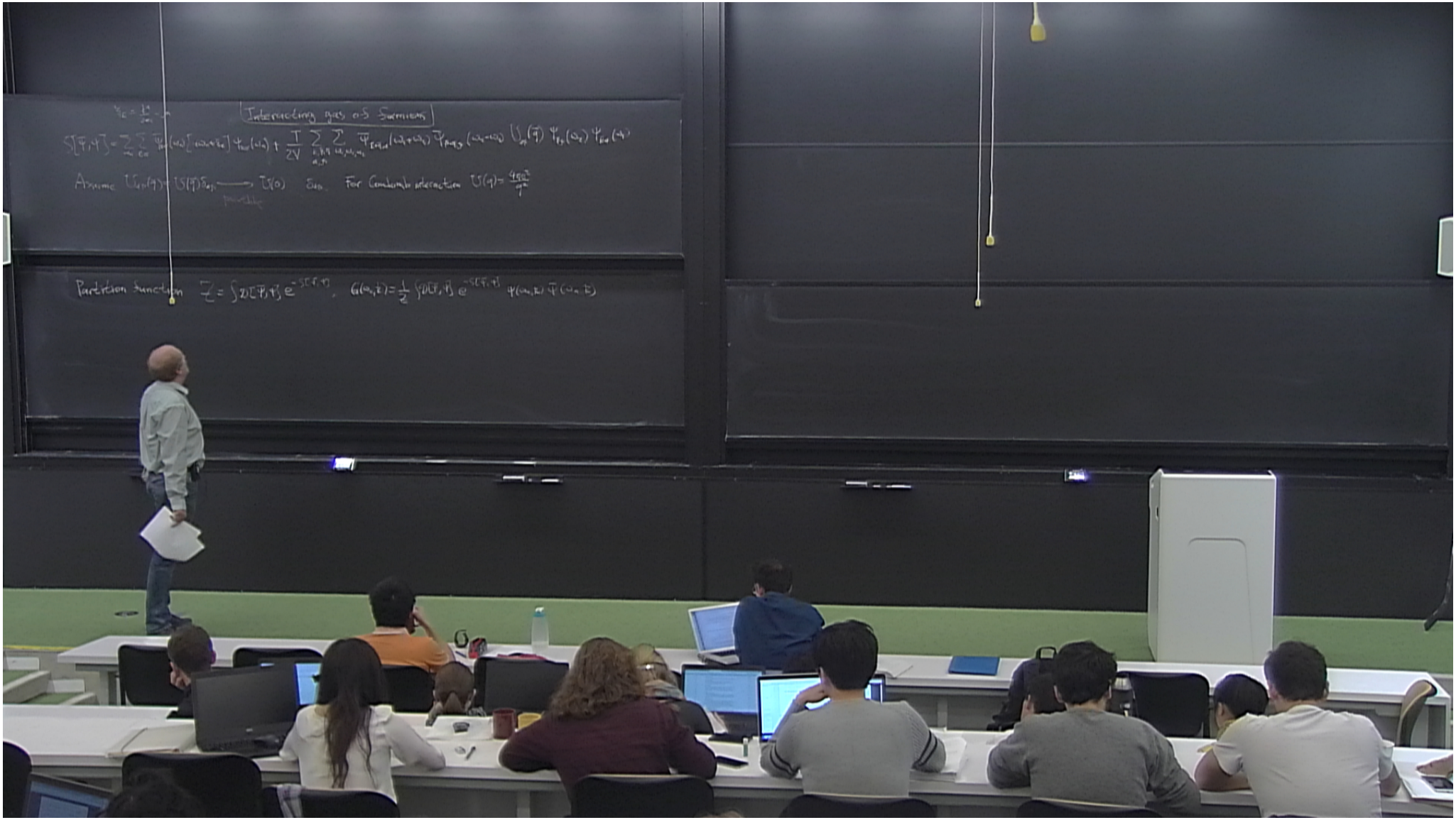
$\sigma = 0, \sigma = \beta,$ l and q internal




king gas of fermions
$\frac{1}{V}\sum_{\vec{k},\vec{p},\vec{q}}\sum_{\omega_1,\omega_2,\omega_3}\sum_{\alpha,\beta}\Psi_{\vec{k}+\vec{q},\alpha}(\omega_1+\omega_3)\Psi_{\vec{p}-\vec{q},\beta}(\omega_2-\omega_3)U_{\alpha\beta}(\vec{q})\Psi_{\vec{p},\beta}(\omega_2)\Psi_{\vec{k},\alpha}(\omega$
$\delta_{\alpha\beta}$. For Coulomb interaction $U(q)=\frac{4\pi e^2}{q^2}$


Interacting gas of Fermions
Partition function


Interacting gas of S Fermions
Partition function

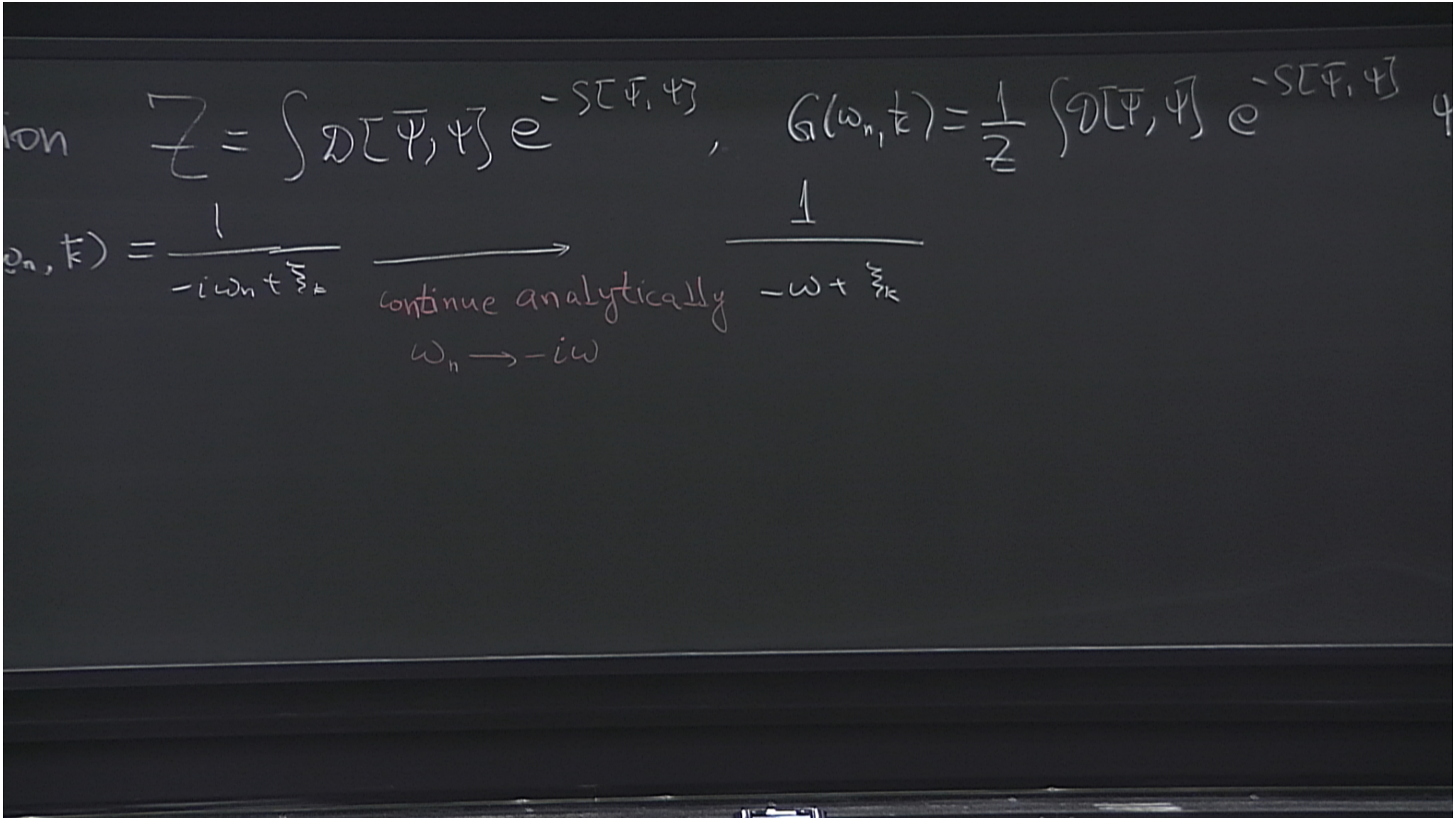

Interacting gas, o-S Summers
Partition function

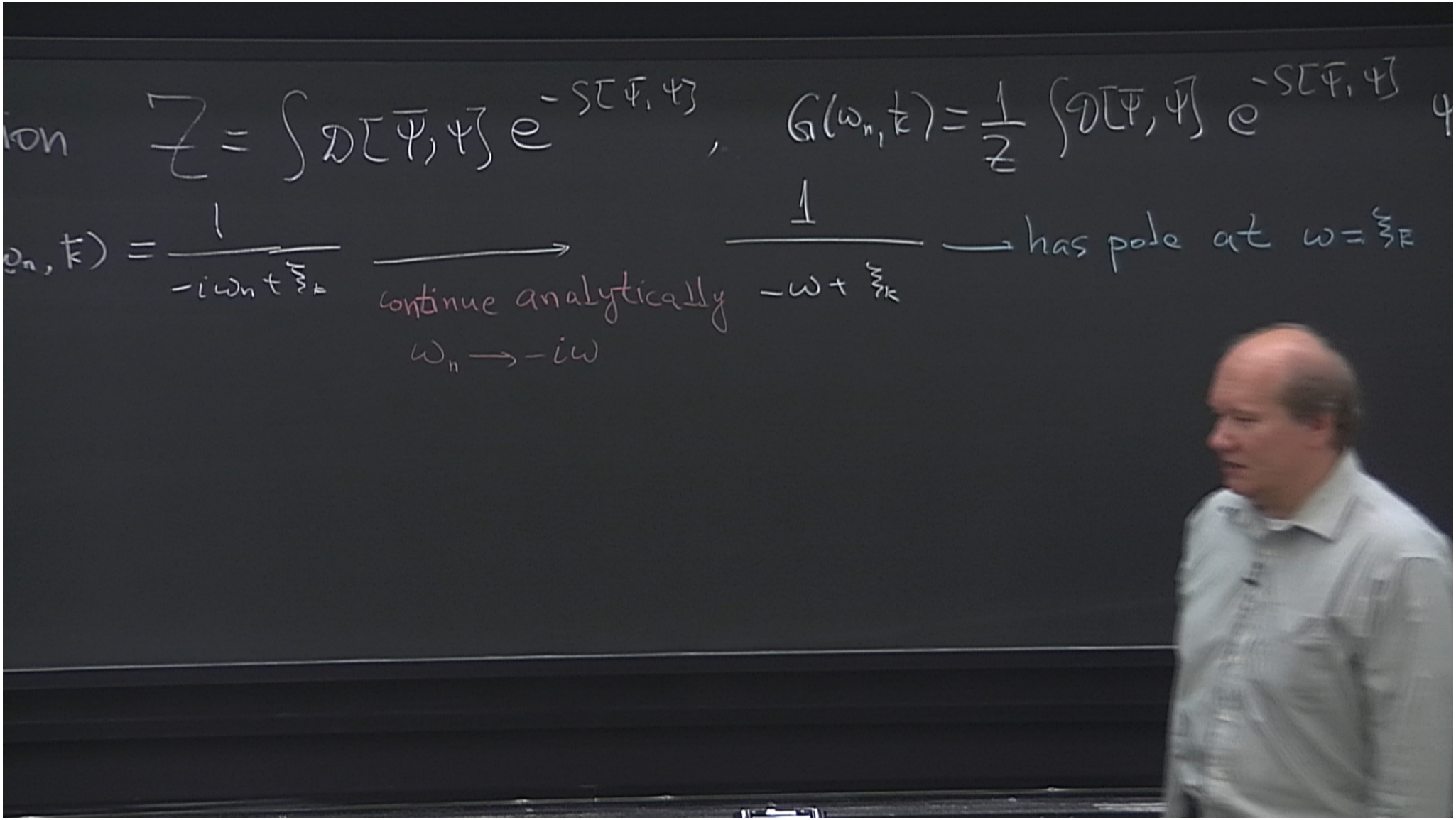
Partition function $Z = \int \mathcal{D}[\bar{\psi}, \psi]\, e^{-S[\bar{\psi}, \psi]}$, $G(\omega_n, k) =$

$$G_0(\omega_n, k) = \frac{1}{-i\omega_n + \xi_k} \longrightarrow \frac{1}{-\omega + \xi_k}$$

continue analytically

$\omega_n \to -i\omega$

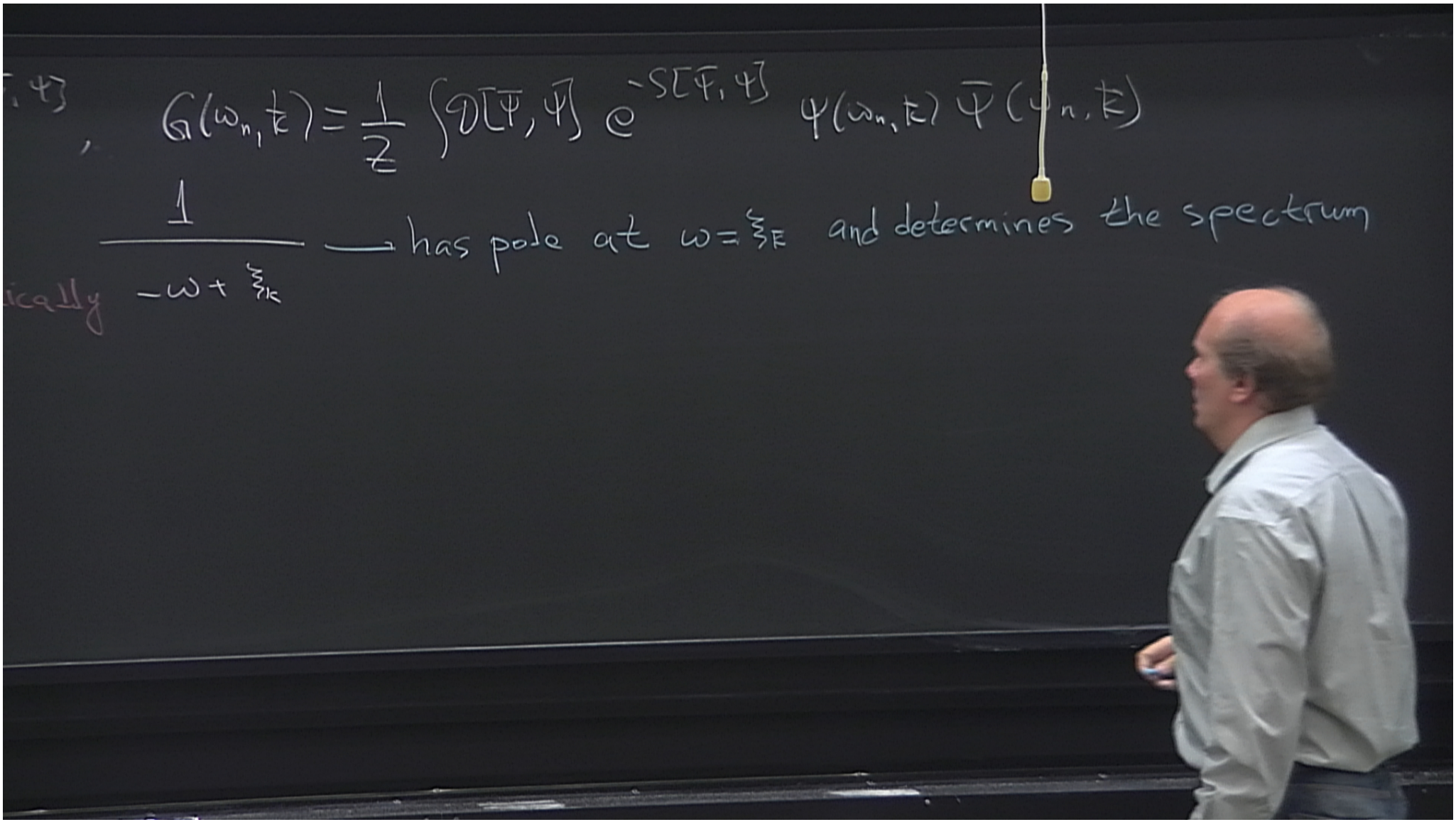

$Z = \int \mathcal{D}[\bar{\psi}, \psi]\, e^{-S[\bar{\psi}, \psi]}$, $G(\omega_n, k) = \frac{1}{Z} \int \mathcal{D}[\bar{\psi}, \psi]\, e^{-S[\bar{\psi}, \psi]}$
$\frac{1}{-i\omega_n + \xi_k}$
continue analytically
$\omega_n \to -i\omega$
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$G(\omega_n, k) = \frac{1}{-i\omega_n + \xi_k}$
continue analytically
$\omega_n \to -i\omega$
$\frac{1}{-\omega + \xi_k}$
has pole at $\omega = \xi_k$

$\bar{\psi}, \psi]$, $G(\omega_n, k) = \frac{1}{Z} \int \mathcal{D}[\bar{\psi}, \psi]\, e^{-S[\bar{\psi}, \psi]}\, \psi(\omega_n, k)\, \bar{\psi}(\omega_n, k)$

$\frac{1}{-\omega + \xi_k}$ ⟶ has pole at $\omega = \xi_k$ and determines the spectrum

ically



$\bar{\psi}, \psi]$, $G(\omega_n, k) = \frac{1}{Z} \int \mathcal{D}[\bar{\psi}, \psi]\, e^{-S[\bar{\psi}, \psi]}\, \psi(\omega_n, k)\, \bar{\psi}(\omega_n, k)$

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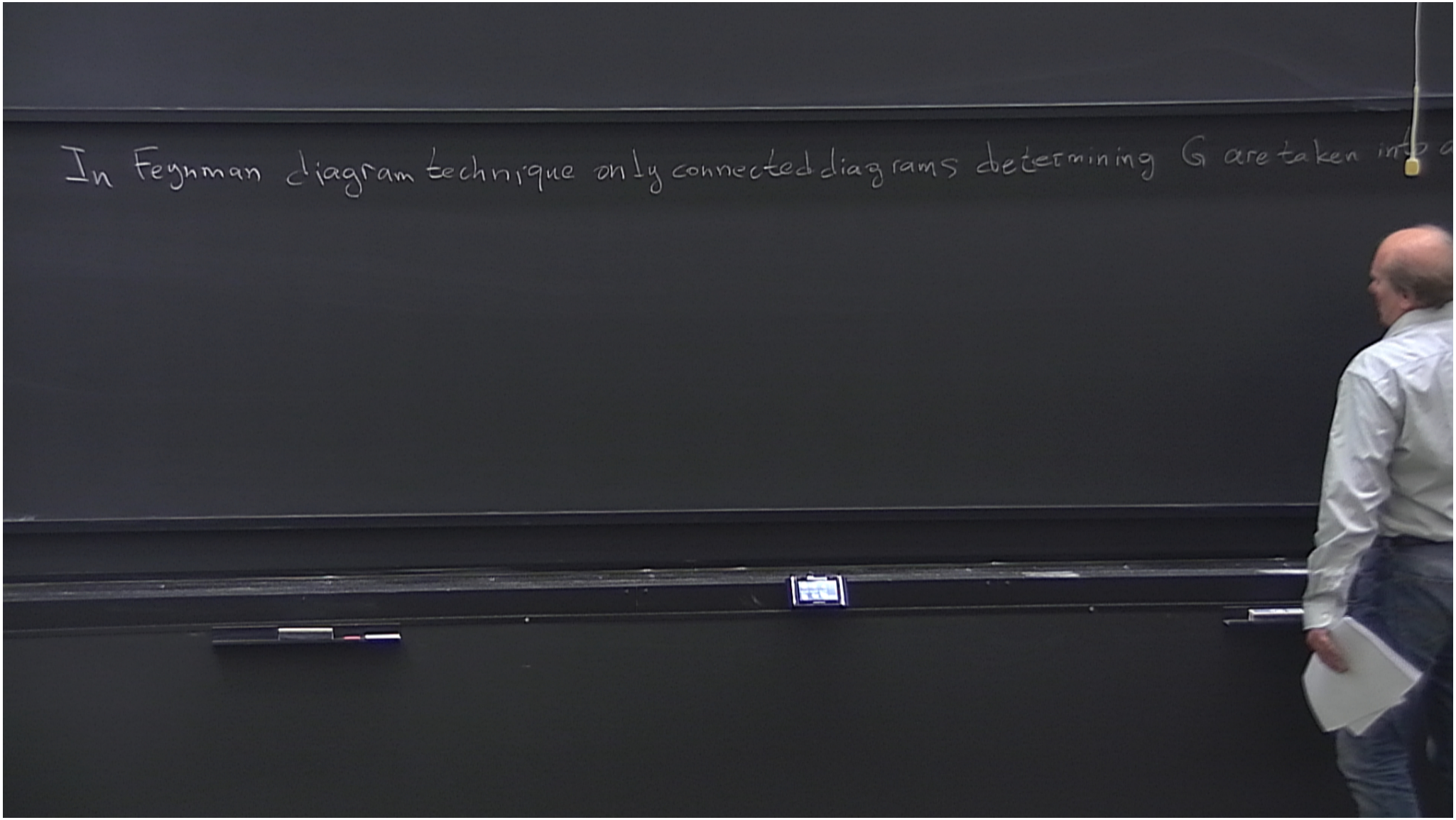
by substitution $\omega = \varepsilon_k - \mu$

$\bar{\psi}, \psi]$, $G(\omega_n, k) = \frac{1}{Z} \int \mathcal{D}[\bar{\psi}, \psi]\, e^{-S[\bar{\psi}, \psi]}\, \psi(\omega_n, k)\, \bar{\psi}(\omega_n, k)$

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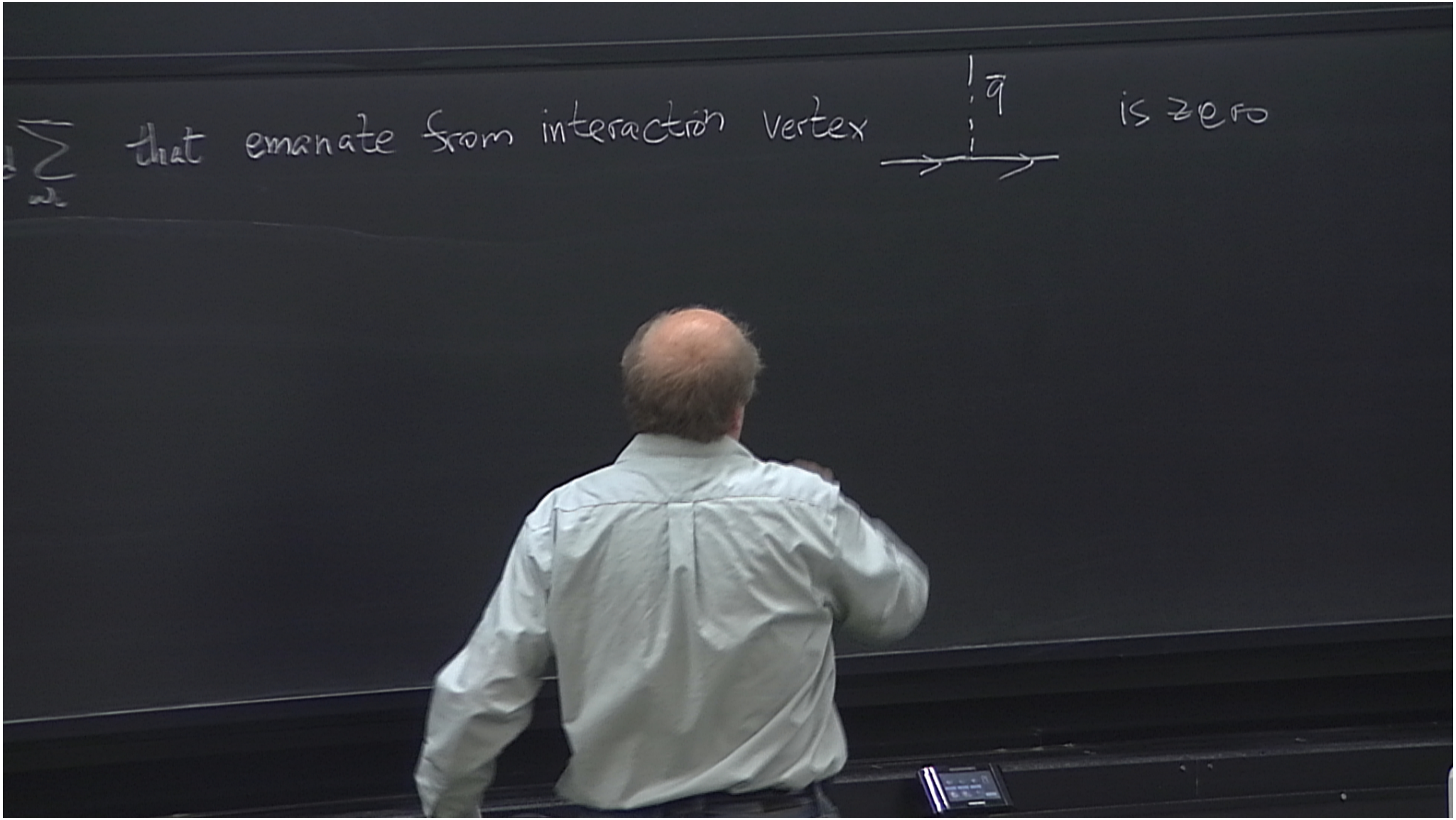
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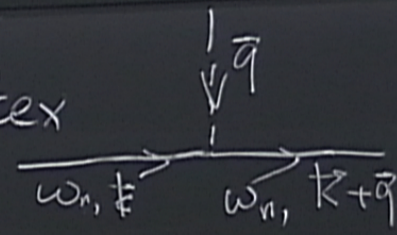

In Feynman diagram technique only connected diagrams determining G are taken into

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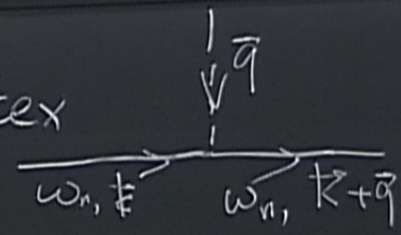
$\Rightarrow$ only $\langle \dots \rangle_0 = \frac{1}{Z_0} \int \mathcal{D}[\bar{\psi}, \psi] e^{-S_0[\bar{\psi}, \psi]} (\dots)$ are considered

(1) $\longrightarrow$ $G_0(\omega_n, k) = \frac{1}{-i\omega_n + \xi_k}$, ------ $-U(9)$

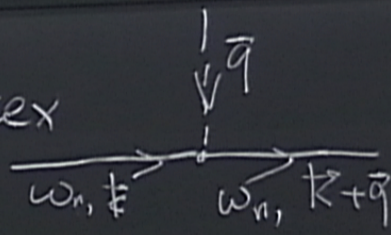

that emanate from interaction vertex
q
is zero

$\sum_{\omega_n}$ that emanate from interaction vertex 

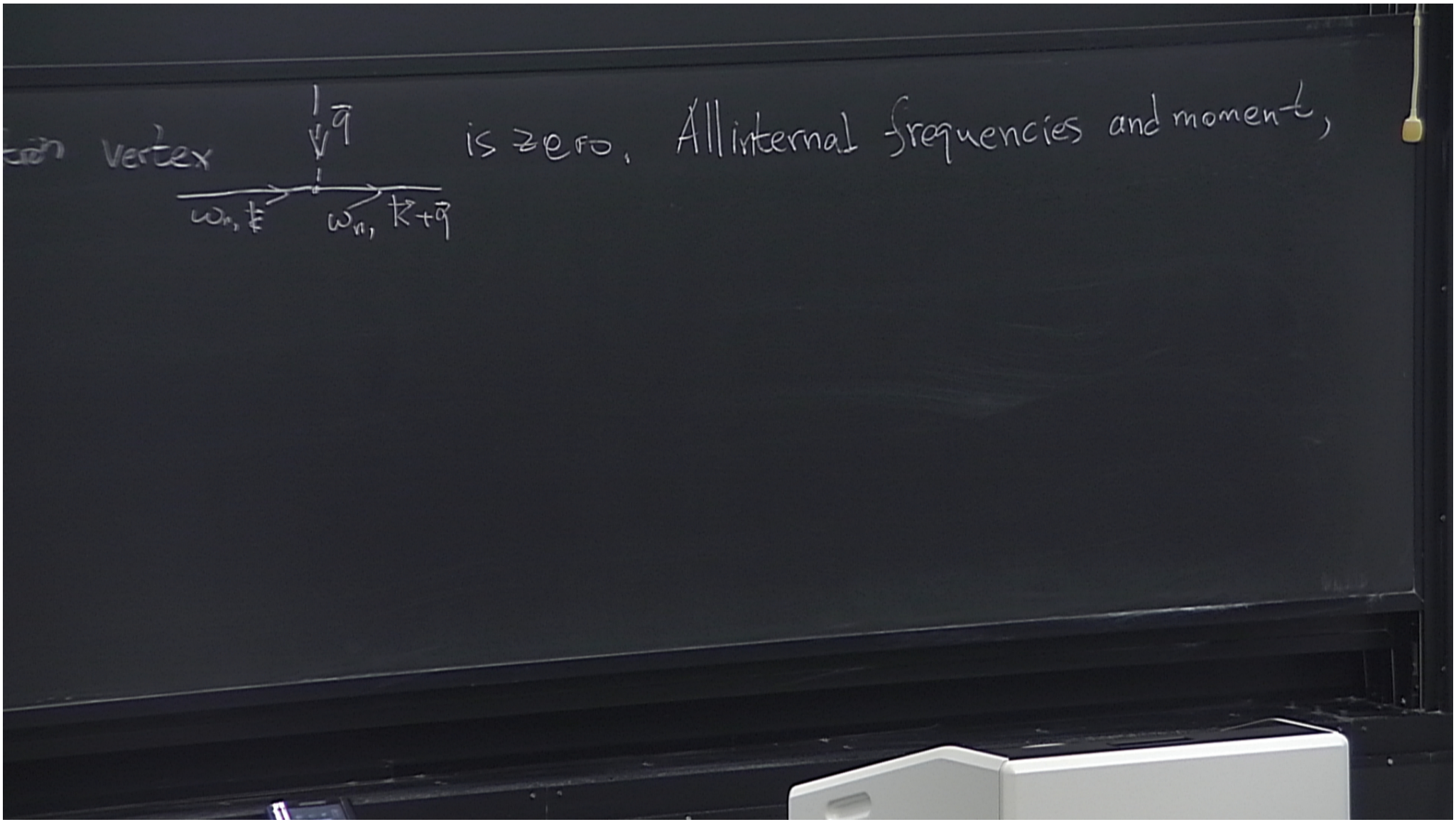
is zero.

$\sum_{\omega_n}$ that emanate from interaction vertex 

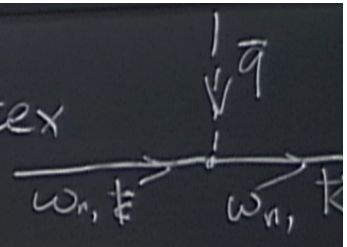
is zero.

$\sum_{\omega_n}$ that emanate from interaction vertex 

is zero. All

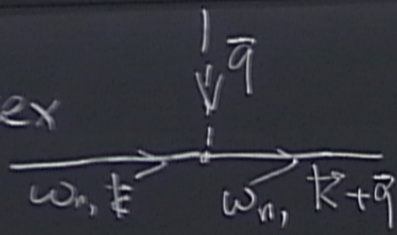

tion vertex
$\vec{q}$
$\omega_n, \vec{k}$
$\omega_n, \vec{k}+\vec{q}$
is zero. All internal frequencies and momenta,

(2) $\sum_{\vec{p}_i}$ and $\sum_{\omega_i}$ that emanate from interaction vertex



as well as spins are summed over

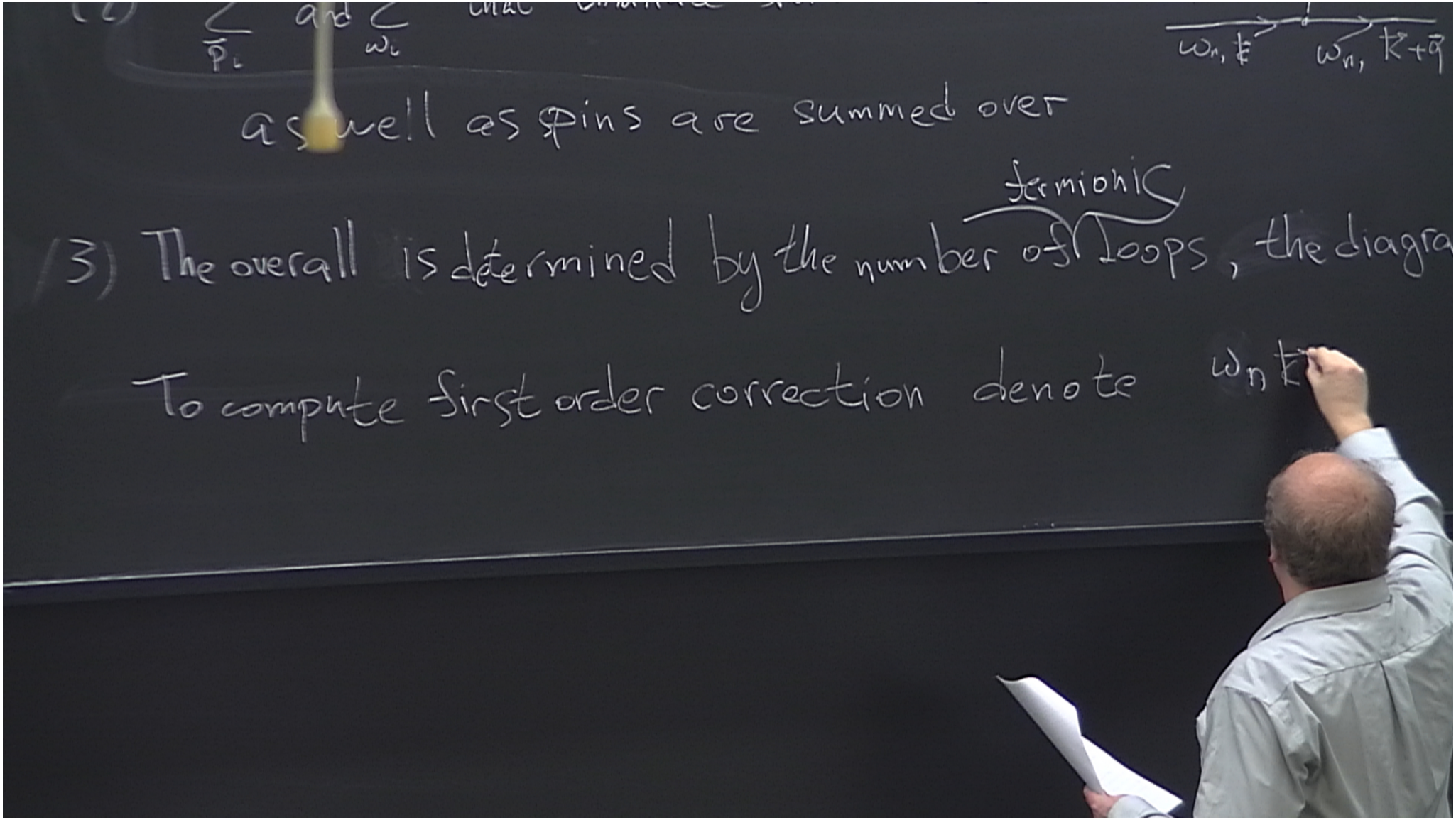
(3) The overall is determined by the number of loops, the dia

vertex 

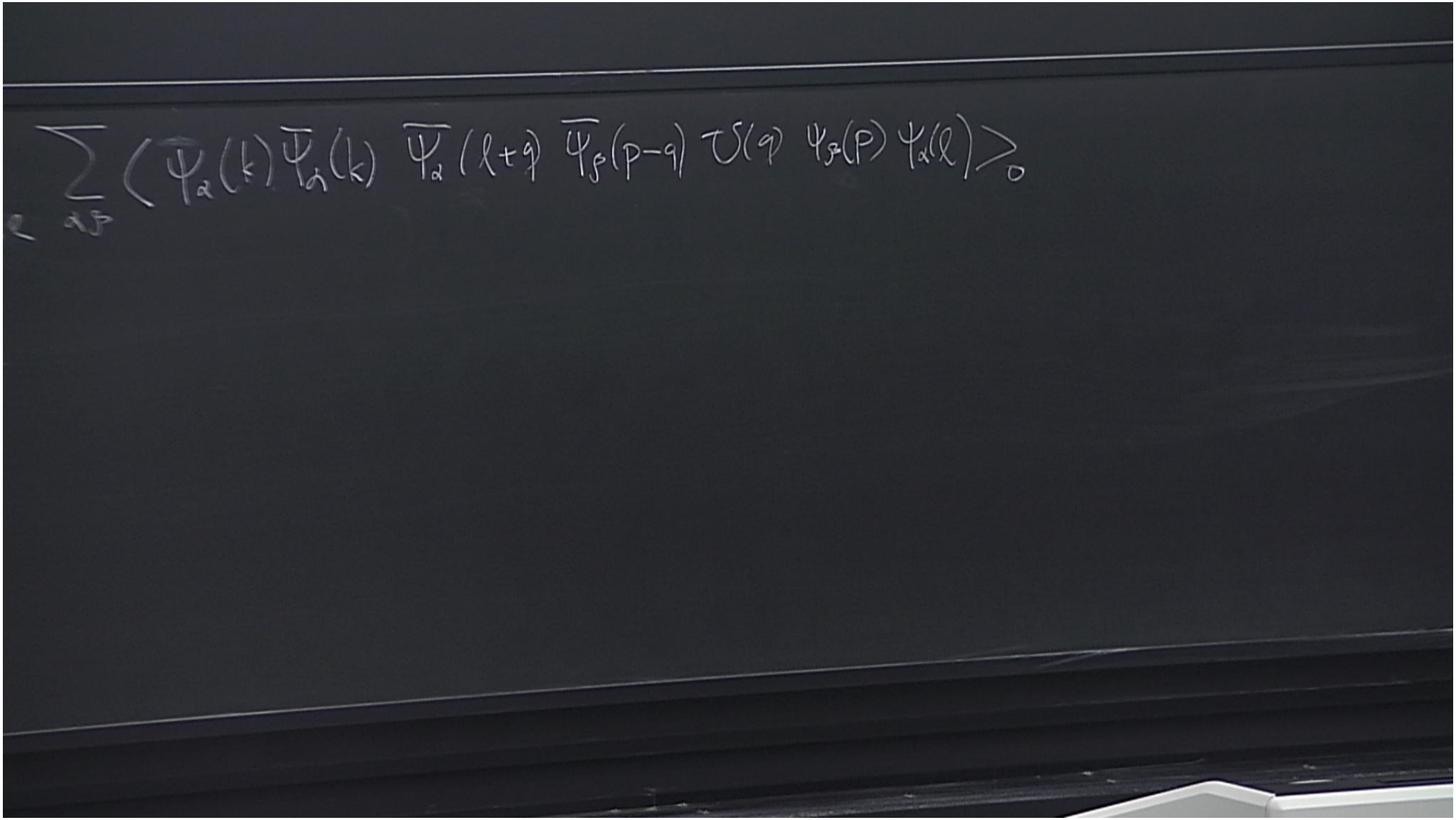
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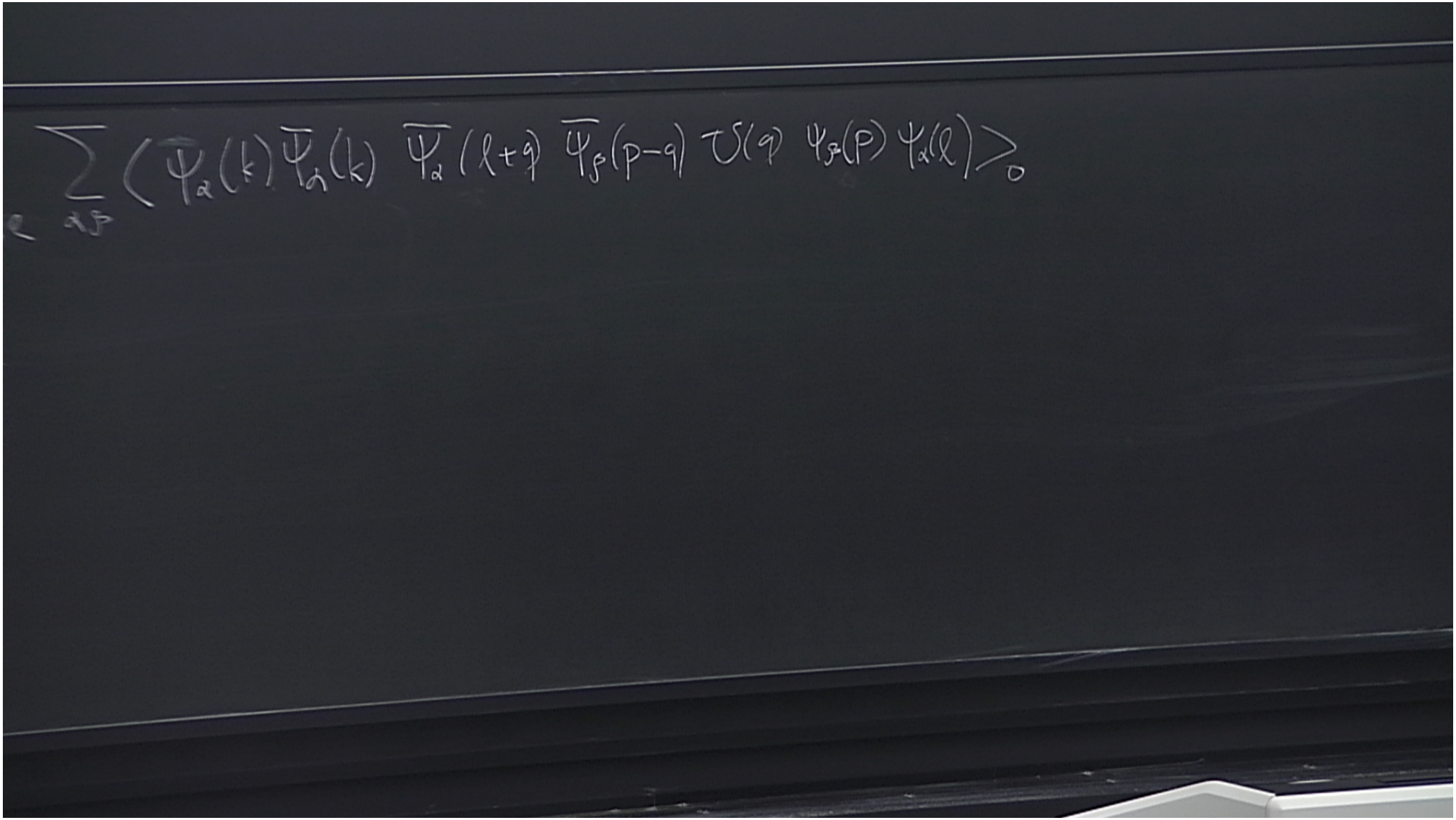
over

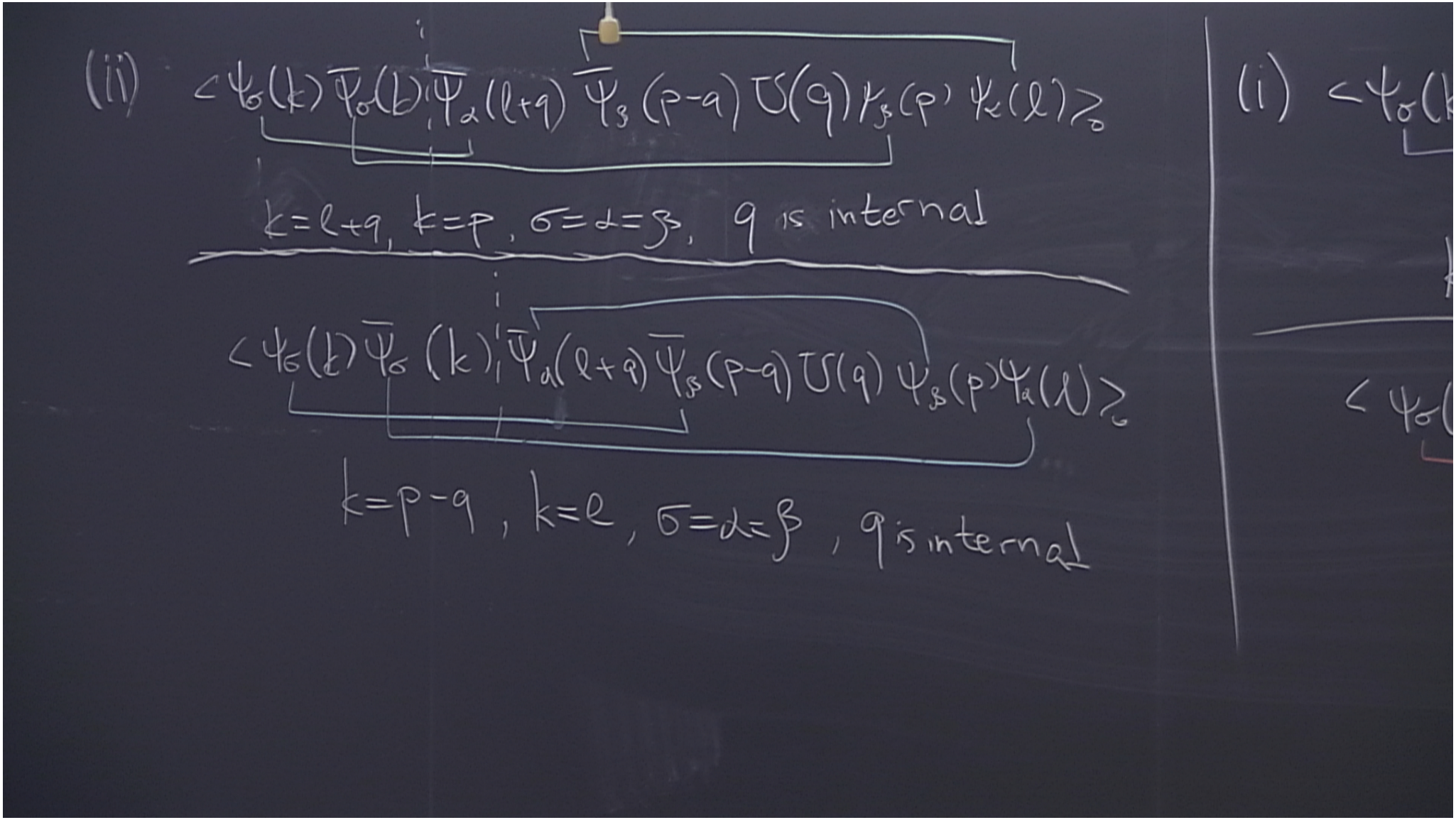
of loops, the diagram contains


as well as spins are summed over
fermionic
3) The overall is determined by the number of loops, the diagra
To compute first order correction denote

$$G^{(1)}(\omega_n, \vec{k}) = G_0(\omega_n, \vec{k}) - \frac{T}{2V} \sum_{p,q,\ell} \sum_{\alpha\beta} \left(\psi_\alpha(k) \bar{\psi}_\alpha(k) \; \bar{\psi}_\alpha(\ell+q) \; \bar{\psi}_\beta(p-q) \; \tau \right.$$


$\sum_{k\,\alpha\beta} \left(\bar{\psi}_\alpha(k)\, \psi_\alpha(k)\, \bar{\psi}_\alpha(l+q)\, \bar{\psi}_\beta(p-q)\, U(q)\, \psi_\beta(p)\, \psi_\alpha(l) \right) > 0$


$\sum_{\ell\ \alpha\beta} \left(\bar{\psi}_\alpha(k)\, \psi_\alpha(k)\, \bar{\psi}_\alpha(\ell+q)\, \bar{\psi}_\beta(p-q)\, U(q)\, \psi_\beta(p)\, \psi_\alpha(\ell) \right) > 0$


(ii) $\langle \psi_b(k)\, \bar{\psi}_b(b)\, \bar{\psi}_a(l+q)\, \bar{\psi}_s(p-q)\, U(q)\, \psi_s(p)\, \psi_e(l) \rangle_0$
$k = l+q,\ k = p,\ \sigma = \alpha = \beta,\ q$ is internal
$\langle \psi_b(k)\, \bar{\psi}_b(k)\, \bar{\psi}_a(l+q)\, \bar{\psi}_s(p-q)\, U(q)\, \psi_s(p)\, \psi_a(l) \rangle_0$
$k = p-q,\ k = l,\ \sigma = \alpha = \beta,\ q$ is internal
(i)

$$(\omega_n, \vec{k}) - \frac{T}{2V} \sum_{p,q,\alpha\beta} \left(\psi_\alpha(k) \bar{\psi}_\alpha(k) \; \bar{\psi}_\beta(l+q) \; \bar{\psi}_\beta(p-q) \; U(q) \; \psi_\beta(p) \; \psi_\alpha(l) \right) > 0$$

p, β

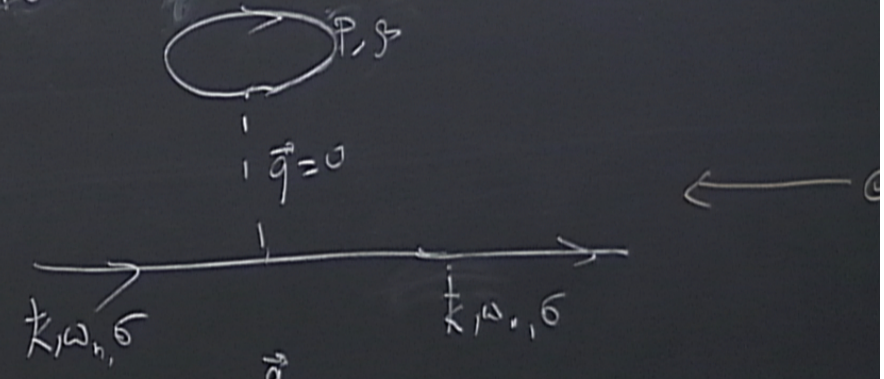
$\vec{q} = 0$

es Hartree diagrams

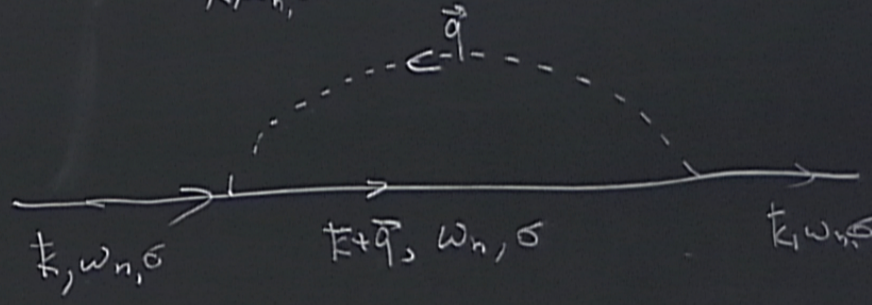
k, ω_n, σ $\quad$ k, ω_n, σ

$$G^{(4)}(\omega_n, \vec{k}) = G_0(\omega_n, \vec{k}) - \frac{T}{2V} \sum_{p,q,\ell} \sum_{\alpha\beta} \left(\psi_\sigma(k) \bar{\psi}_\sigma(k) \bar{\psi}_\alpha(\ell+q) \bar{\psi}_\beta(p-q) U(q) \right.$$

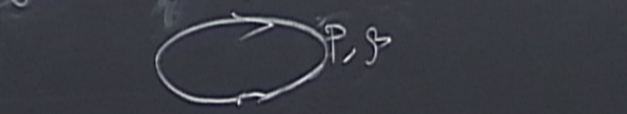
Group (i) gives Hartree diagram



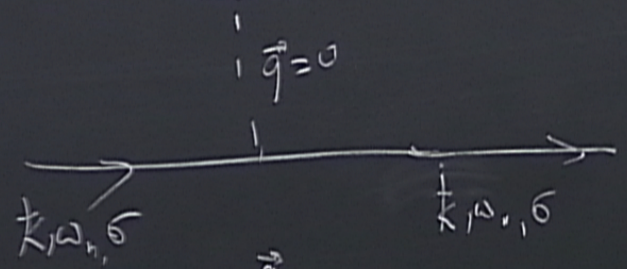
Group (ii) gives Fock diagram



$$\sum_{p,q,\ell} \sum_{\alpha\beta} \left(\Psi_\sigma(k)\, \bar{\Psi}_\sigma(k)\, \bar{\Psi}_\alpha(\ell+q)\, \bar{\Psi}_\beta(p-q)\, U(q)\, \Psi_\beta(p)\, \Psi_\alpha(\ell) \right)_0$$

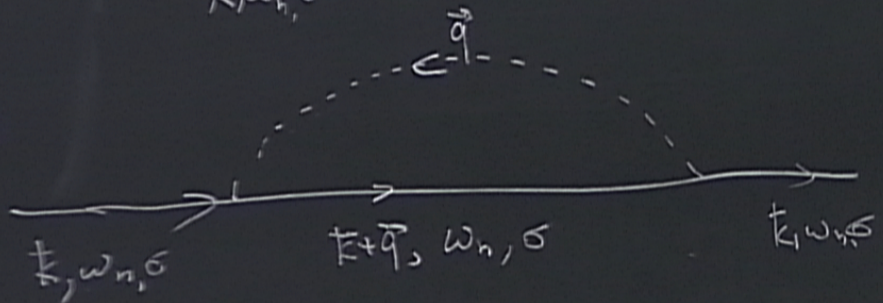


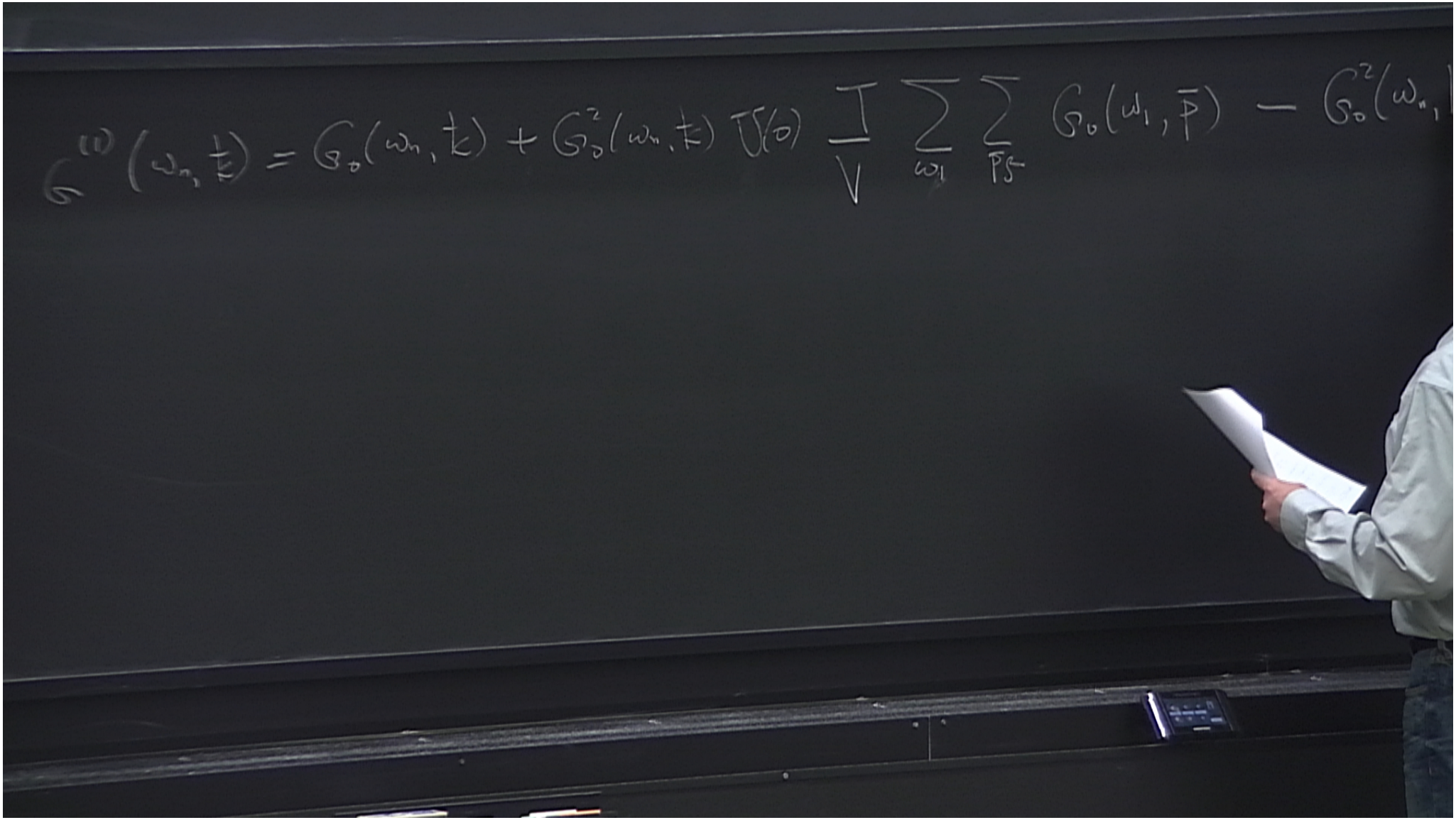
diagram

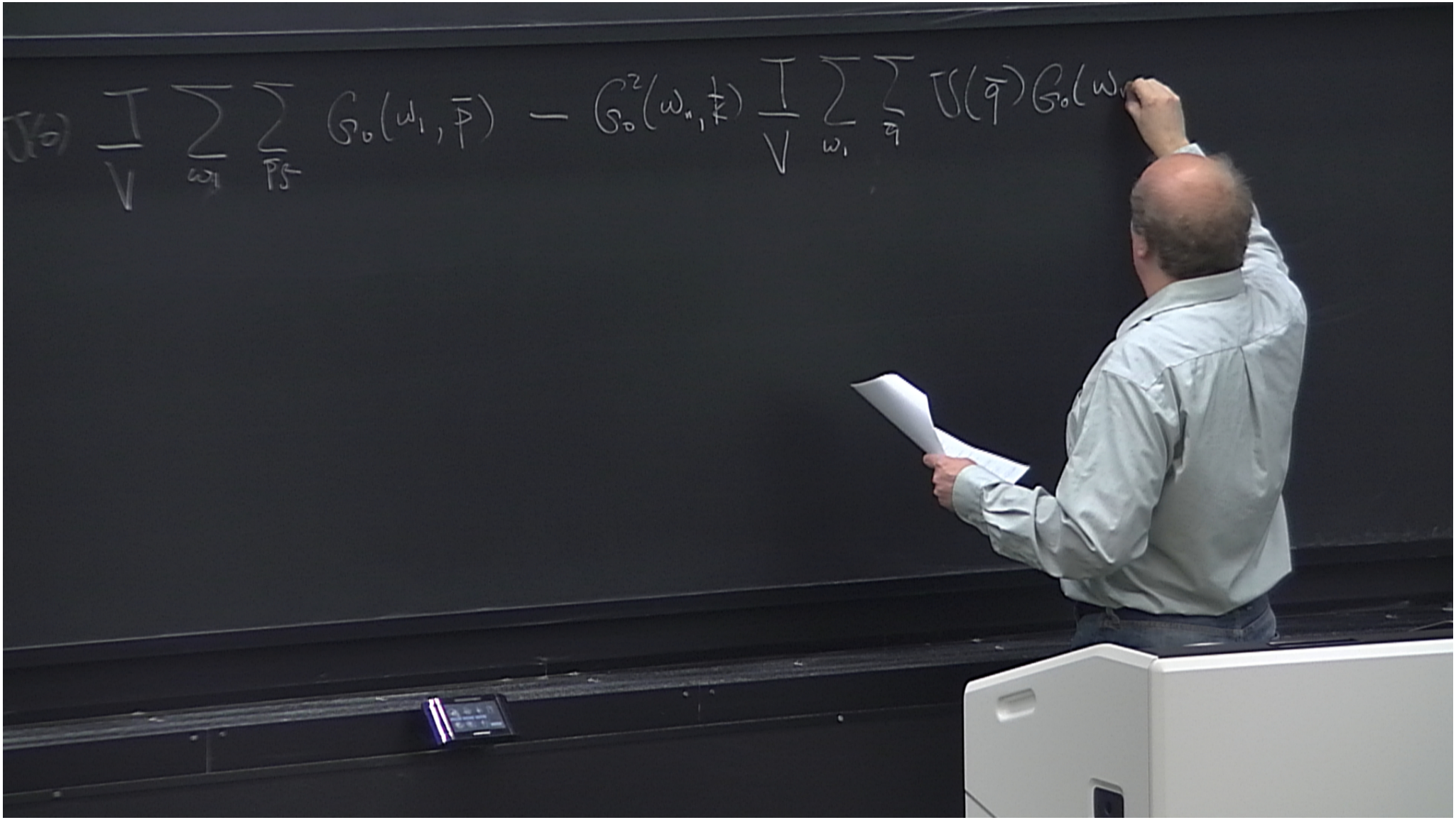


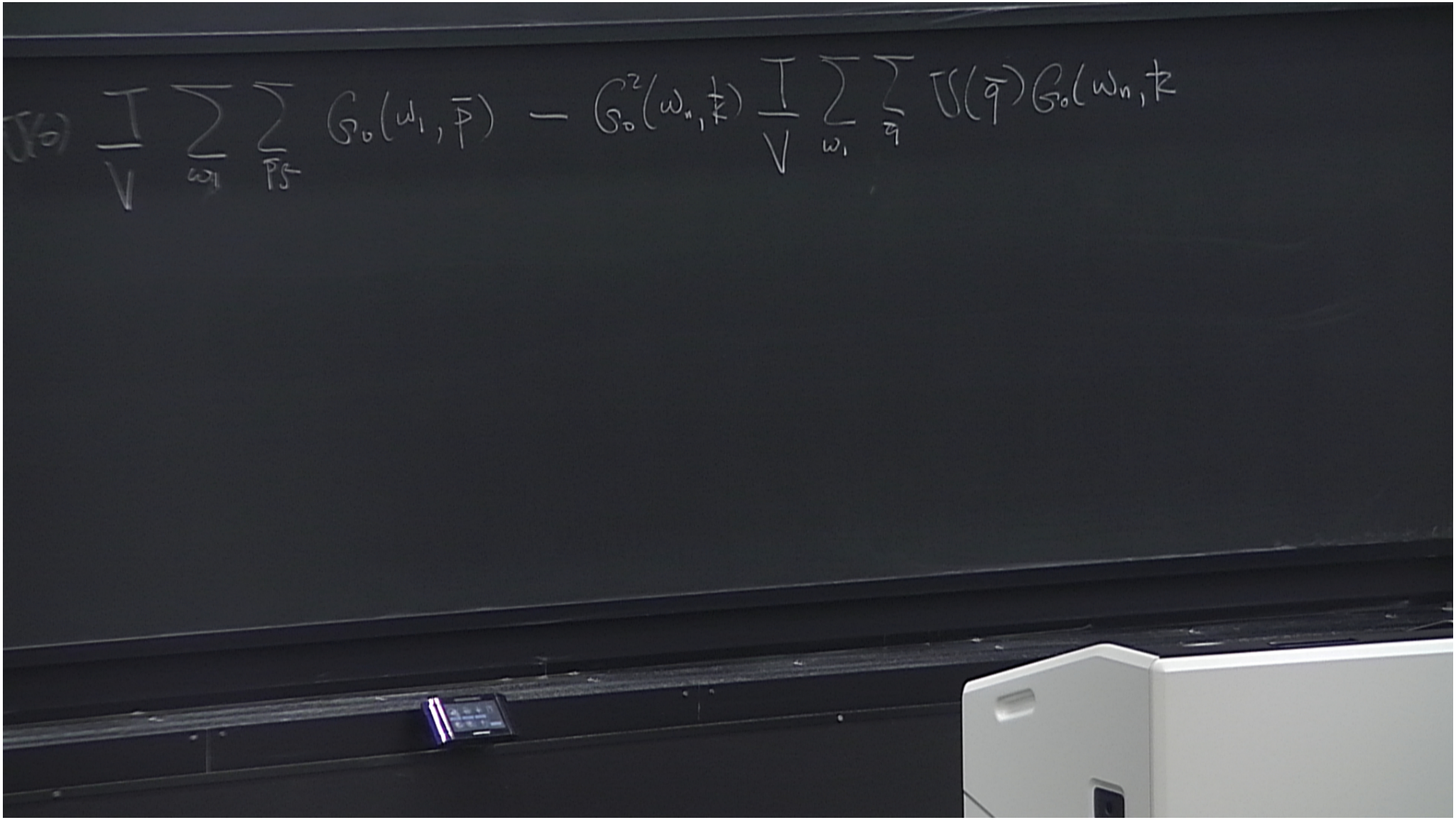
← enters with additional − sign

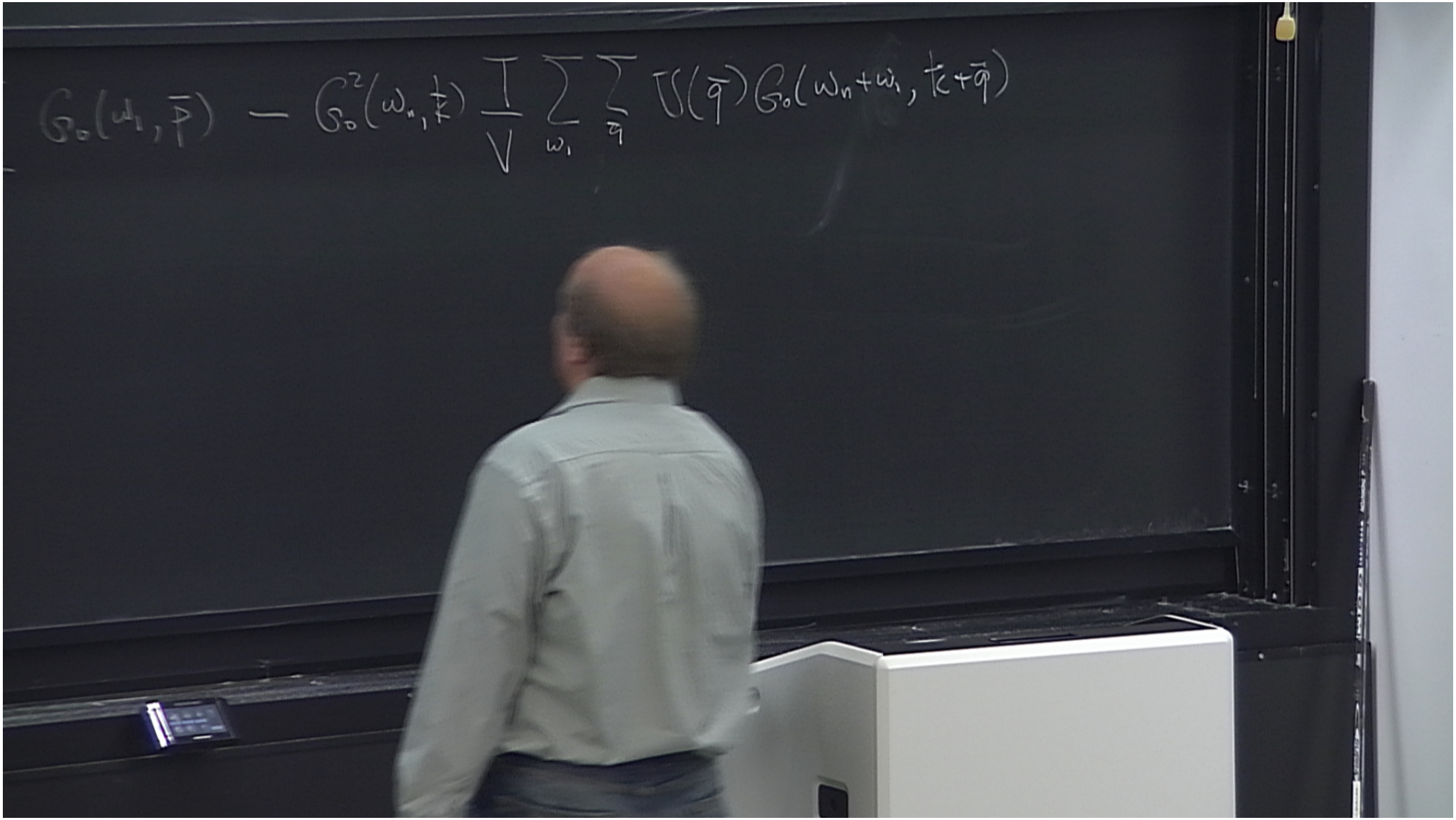
diagram

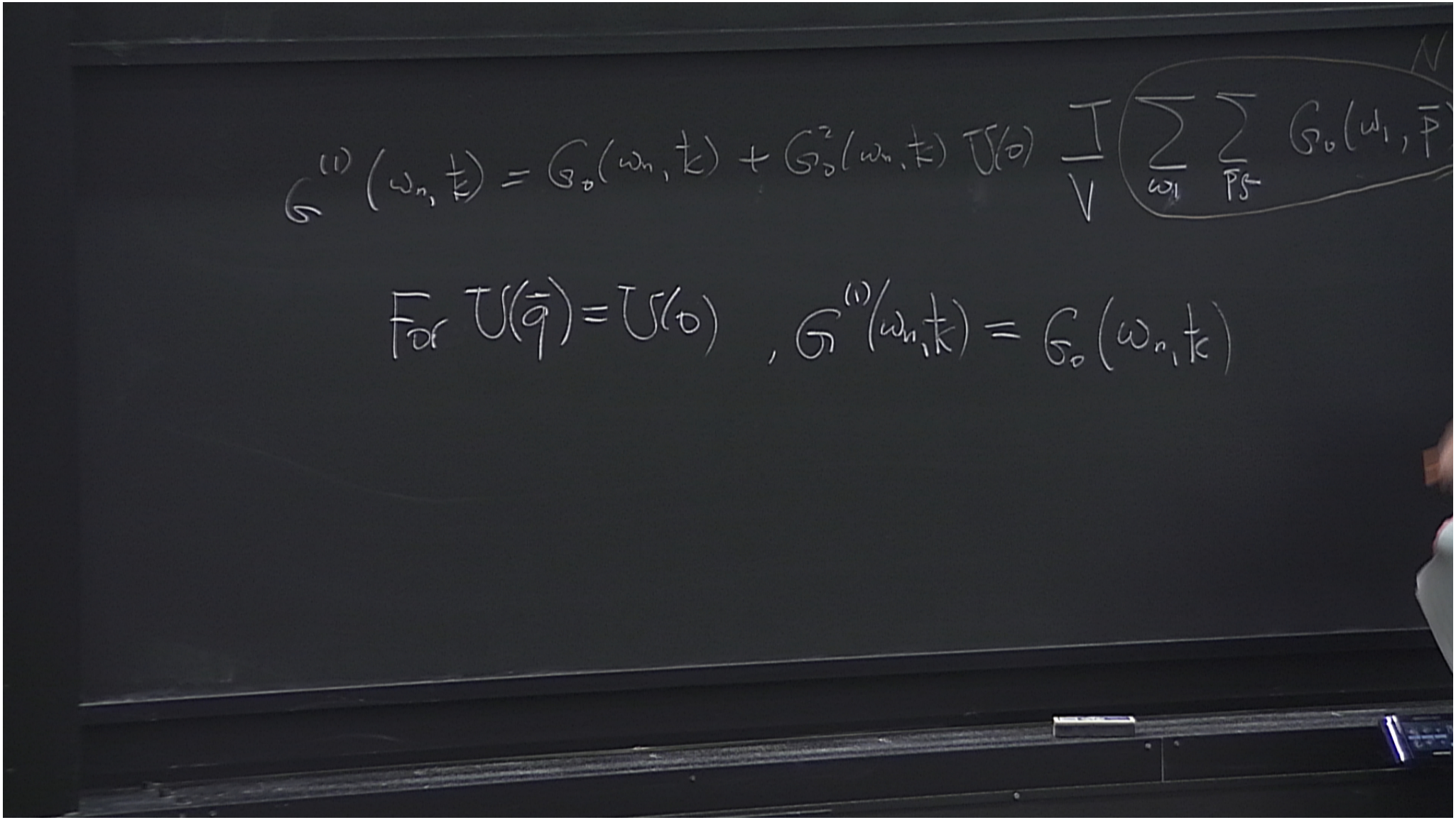


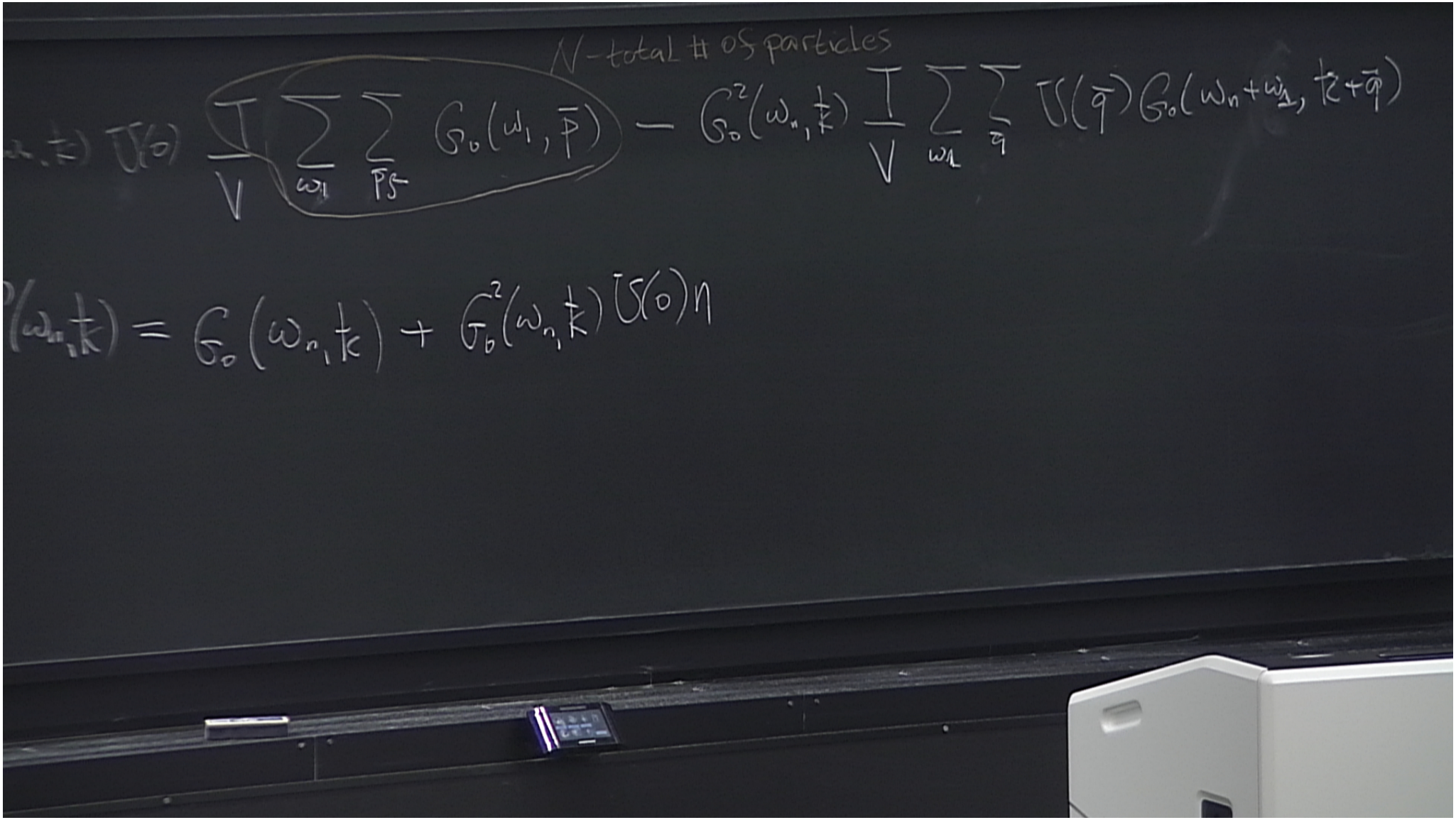

$G^{(1)}(\omega_n, \vec{k}) = G_0(\omega_n, \vec{k}) + G_0^2(\omega_n, \vec{k})\, U(0)\, \frac{1}{V} \sum_{\omega_1} \sum_{\vec{p}} G_0(\omega_1, \vec{p}) - G_0^2(\omega_n,$

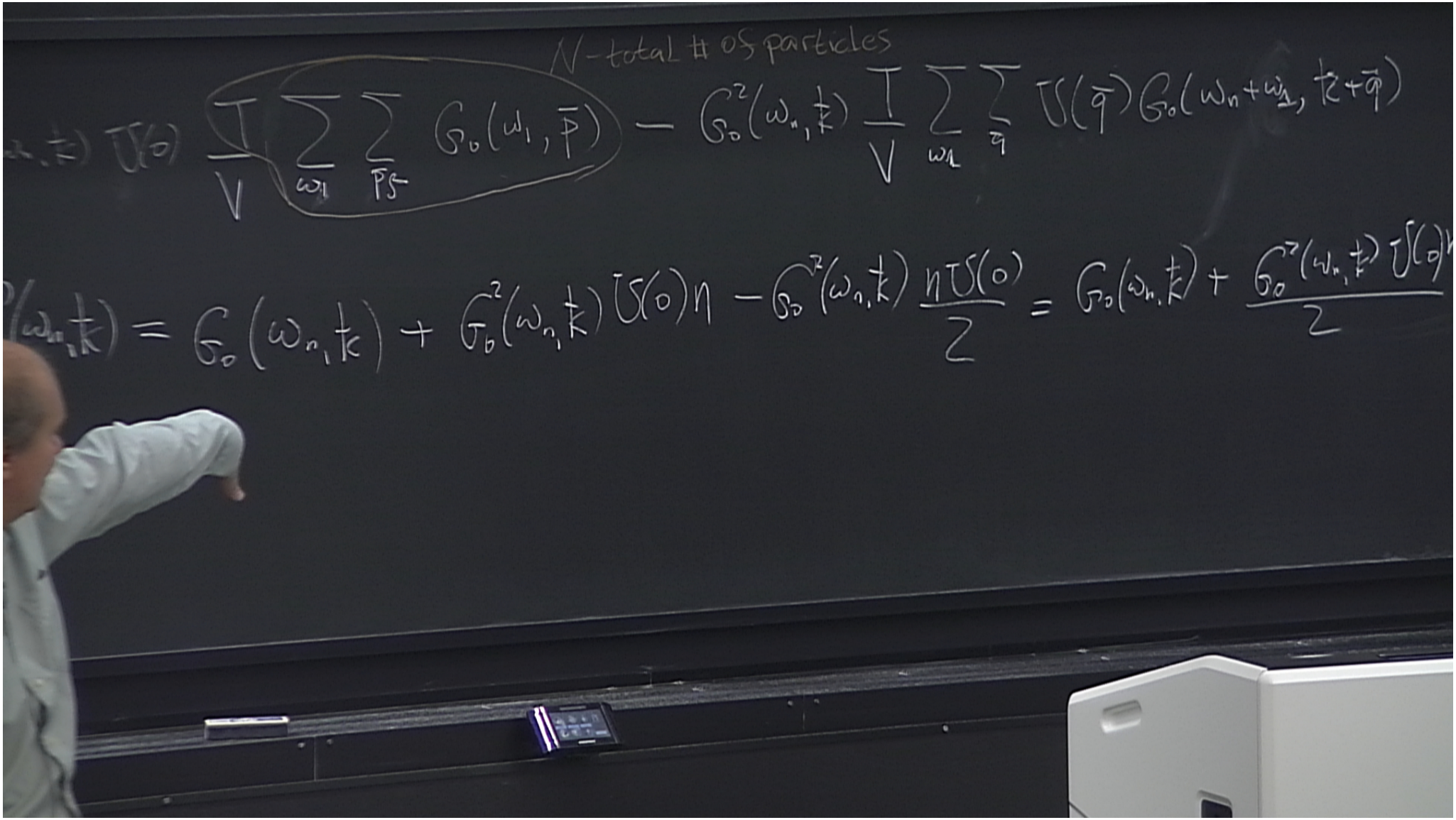

$\frac{1}{V}\sum_{\omega_1}\sum_{\vec{p}}$
$G_0(\omega_1, \vec{p})$
$- G_0^2(\omega_1, \vec{p})$
$\frac{1}{V}\sum_{\omega_1}\sum_{\vec{q}}$
$V(\vec{q})\, G_0(\omega$

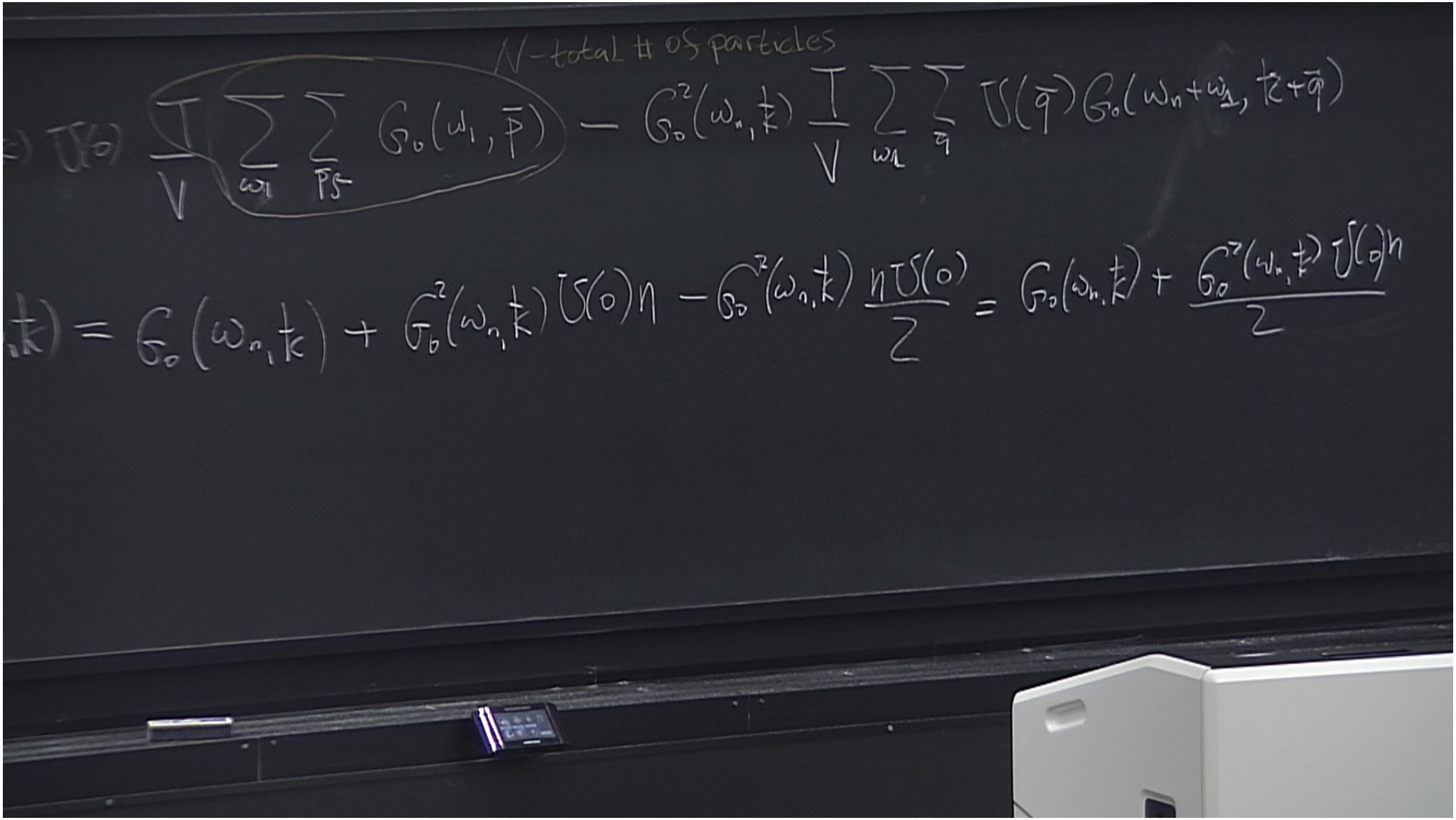

$\frac{1}{V}\sum_{\omega_1}\sum_{\vec{p}}$ $G_0(\omega_1, \vec{p})$ $- G_0^2(\omega_n, \vec{k})$ $\frac{1}{V}\sum_{\omega_1}\sum_{\vec{q}}$ $V(\vec{q})\, G_0(\omega_n, k$

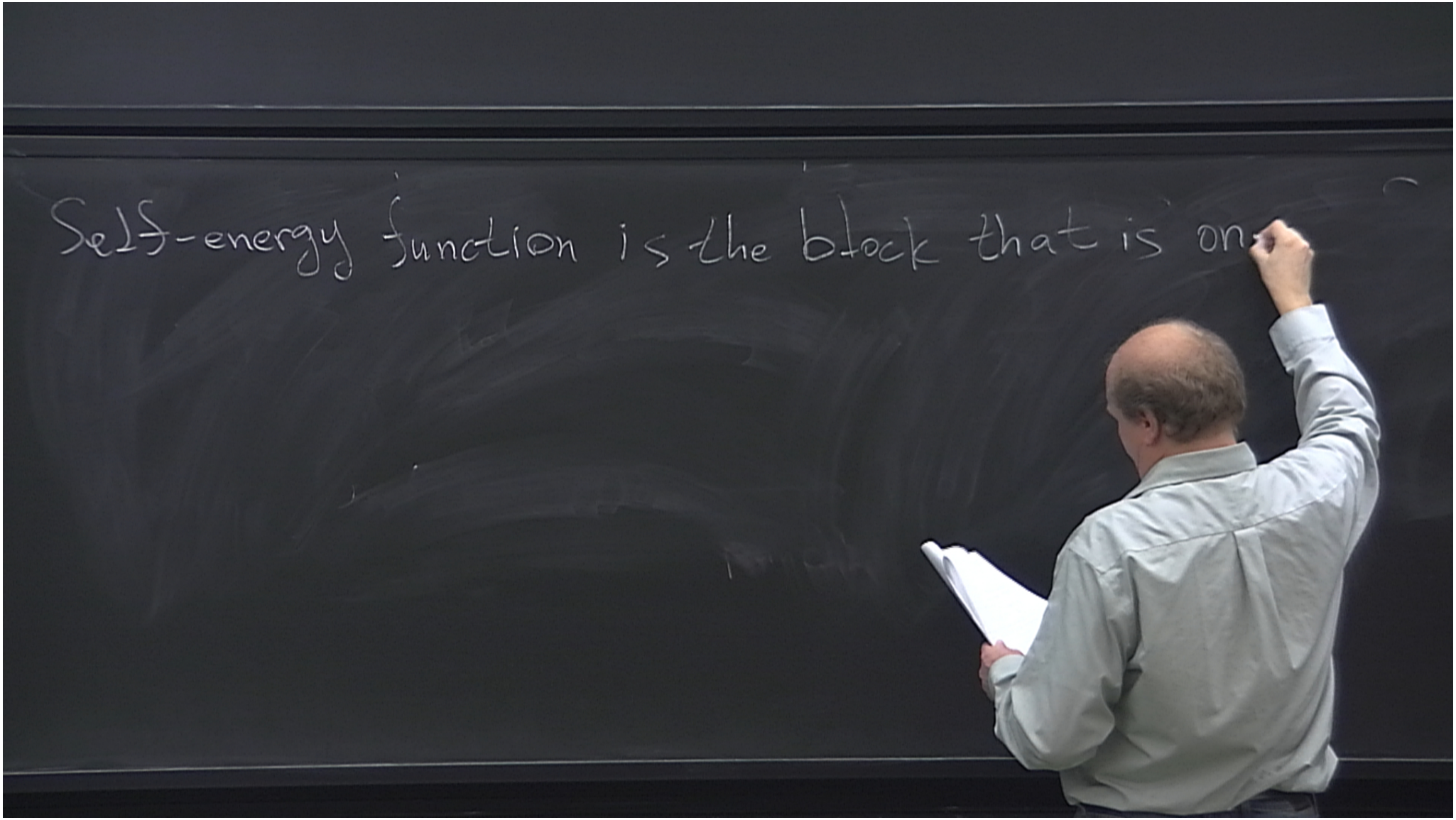

$G_0(\omega_1, \vec{p}) - G_0^2(\omega_n, \vec{k}) \frac{1}{V} \sum_{\omega_1} \sum_{\vec{q}} U(\vec{q}) G_0(\omega_n + \omega_1, \vec{k} + \vec{q})$

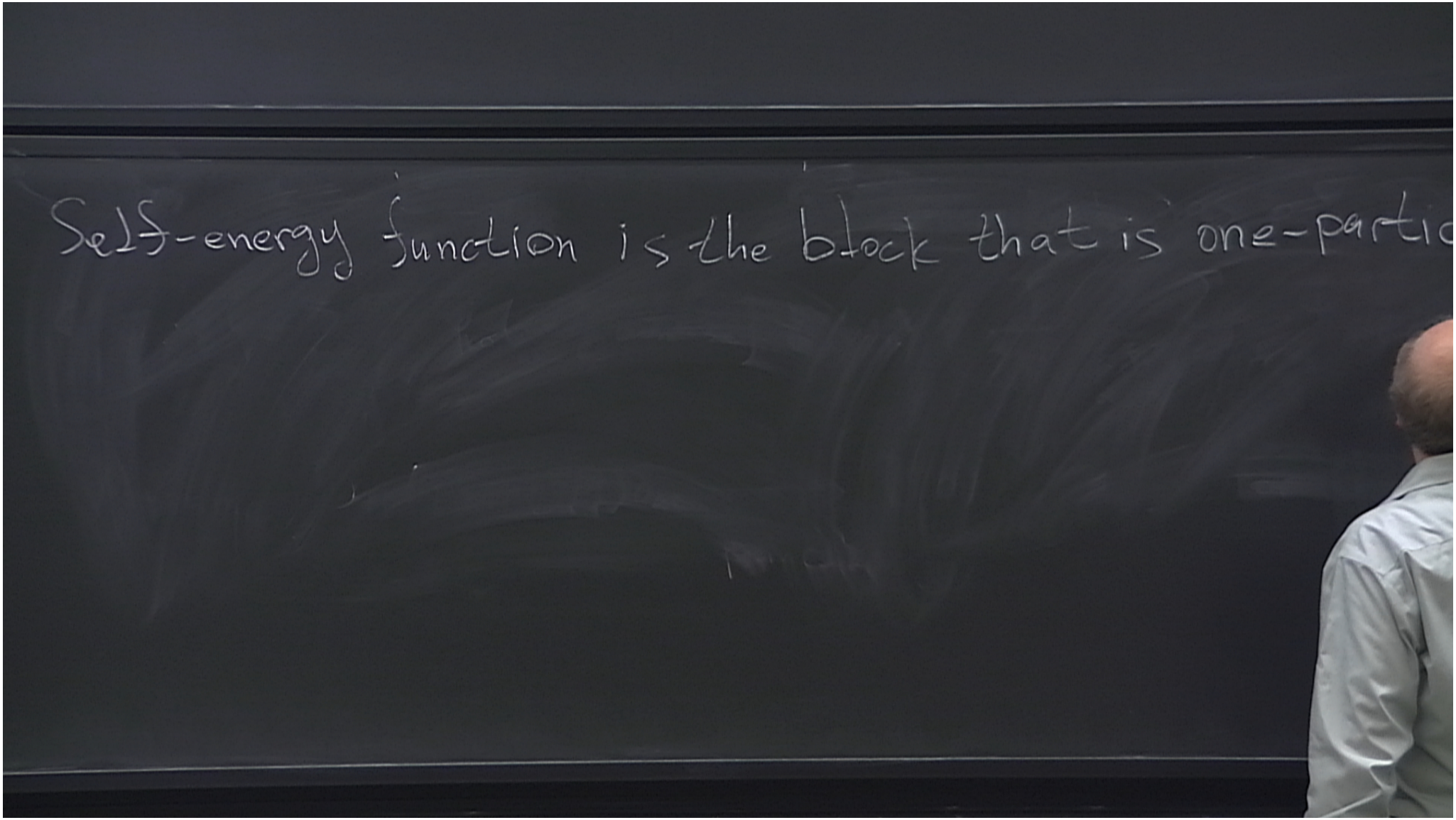

$G^{(1)}(\omega_n, \vec{k}) = G_0(\omega_n, \vec{k}) + G_0^2(\omega_n, \vec{k})\, U(0)\, \frac{I}{V} \sum_{\omega_1} \sum_{PS} G_0(\omega_1, \vec{p}$
N
For $U(\vec{q}) = U(0)$, $G^{(1)}(\omega_n, \vec{k}) = G_0(\omega_n, \vec{k})$

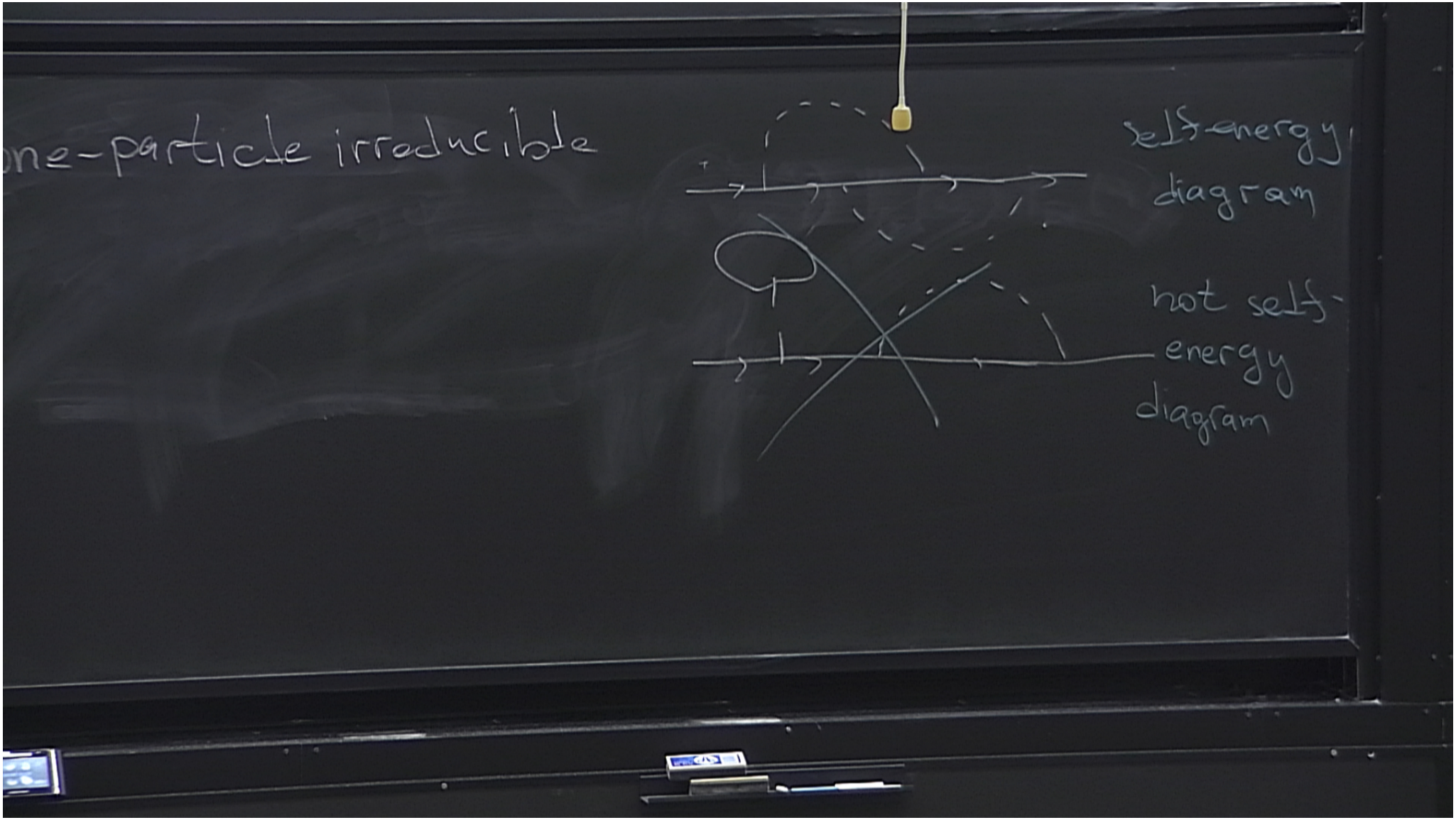

N - total # of particles
$\frac{1}{V}\sum_{\omega_1}\sum_{\vec{p}} G_0(\omega_1, \vec{p})$
$- G_0^2(\omega_n, \vec{k}) \frac{1}{V}\sum_{\omega_1}\sum_{\vec{q}} U(\vec{q}) G_0(\omega_n + \omega_1, \vec{k}+\vec{q})$
$G(\omega_n, \vec{k}) = G_0(\omega_n, \vec{k}) + G_0^2(\omega_n, \vec{k}) U(0) n$


N - total # of particles
$\frac{1}{V}\sum_{\omega_1}\sum_{\vec{p}} G_0(\omega_1, \vec{p})$
$- G_0^2(\omega_n, \vec{k}) \frac{1}{V}\sum_{\omega_1}\sum_{\vec{q}} U(\vec{q}) G_0(\omega_n + \omega_1, \vec{k} + \vec{q})$
$G(\omega_n, \vec{k}) = G_0(\omega_n, \vec{k}) + G_0^2(\omega_n, \vec{k}) U(0) n - G_0^2(\omega_n, \vec{k}) \frac{n U(0)}{2} = G_0(\omega_n, \vec{k}) + \frac{G_0^2(\omega_n, \vec{k}) U(0)}{2}$

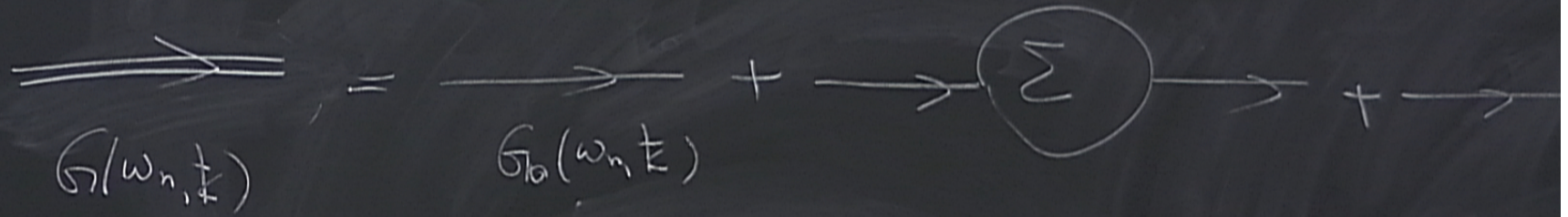

N - total # of particles
$\frac{1}{V}\sum_{\omega_1}\sum_{\vec{p}} G_0(\omega_1, \vec{p}) - G_0^2(\omega_n, \vec{k})\frac{1}{V}\sum_{\omega_1}\sum_{\vec{q}} U(\vec{q}) G_0(\omega_n+\omega_1, \vec{k}+\vec{q})$
$G_0(\omega_n, \vec{k}) = G_0(\omega_n, \vec{k}) + G_0^2(\omega_n, \vec{k}) U(0) n - G_0^2(\omega_n, \vec{k})\frac{nU(0)}{2} = G_0(\omega_n, \vec{k}) + \frac{G_0^2(\omega_n, \vec{k}) U(0) n}{2}$


Self-energy function is the block that is on

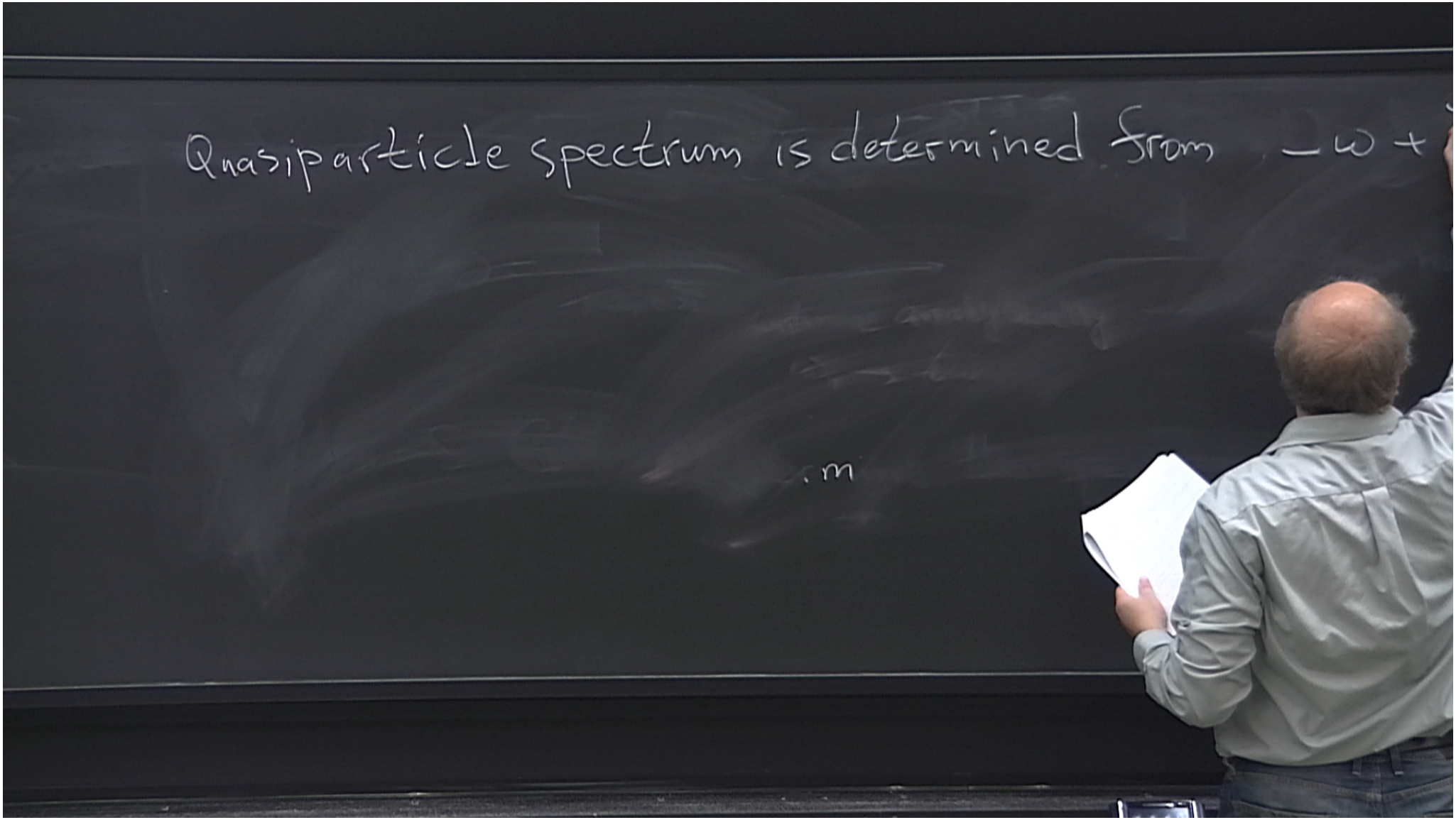

Self-energy function is the block that is one-partic

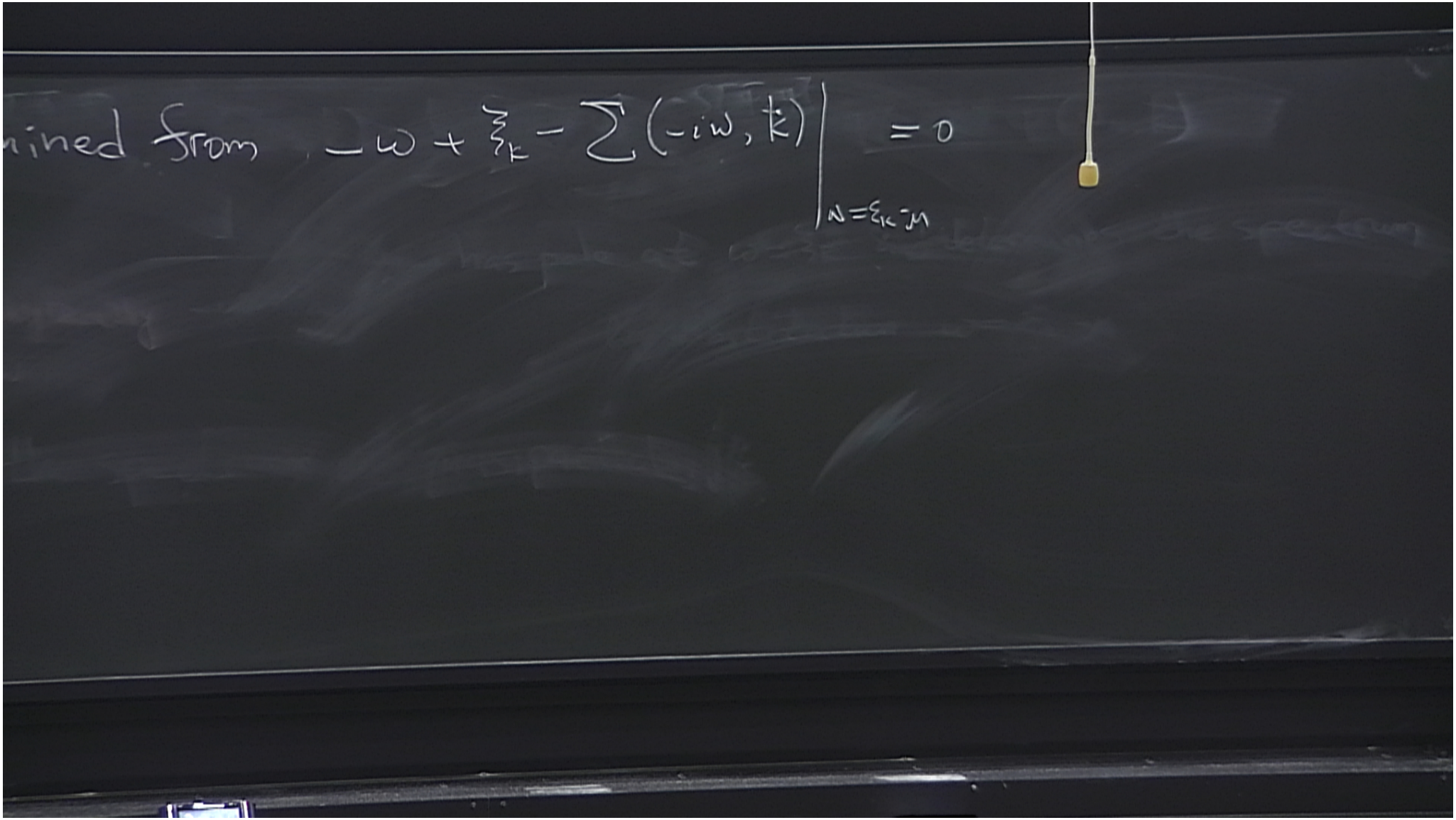

one-particle irreducible
self-energy diagram
hot self-energy diagram

Self-energy function is the block that is



$G(\omega_n, k)$ $G_0(\omega_n, k)$


Quasiparticle spectrum is determined from
$-\omega +$
$\cdot m$


mined from $-\omega + \xi_k - \Sigma(-i\omega, k)\Big|_{\omega=\xi_k-\mu} = 0$

Quasiparticle spectrum is determined from $-\omega + \xi_k - \Sigma$

$$\Sigma^{(1)}(\omega_n, k) = n U(0) - \frac{1}{V} \sum_{\omega_1} \sum_{\vec{q}} U(\vec{q})\, G_b(\omega_n + \omega_1, k - q)$$