Title: PSI 2015/2016 Condensed Matter - Oleg Tchernyshyov - Lecture 4

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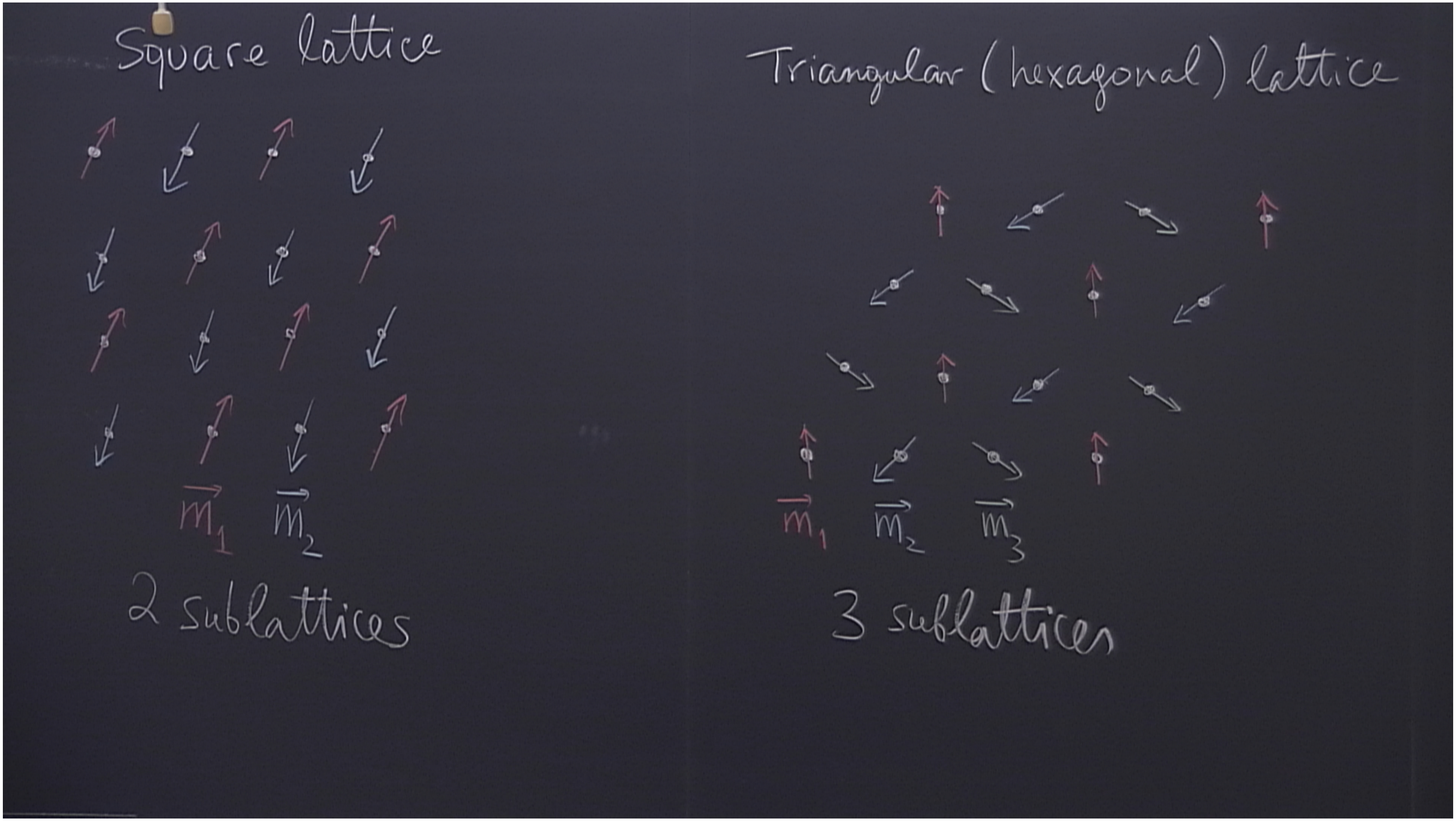
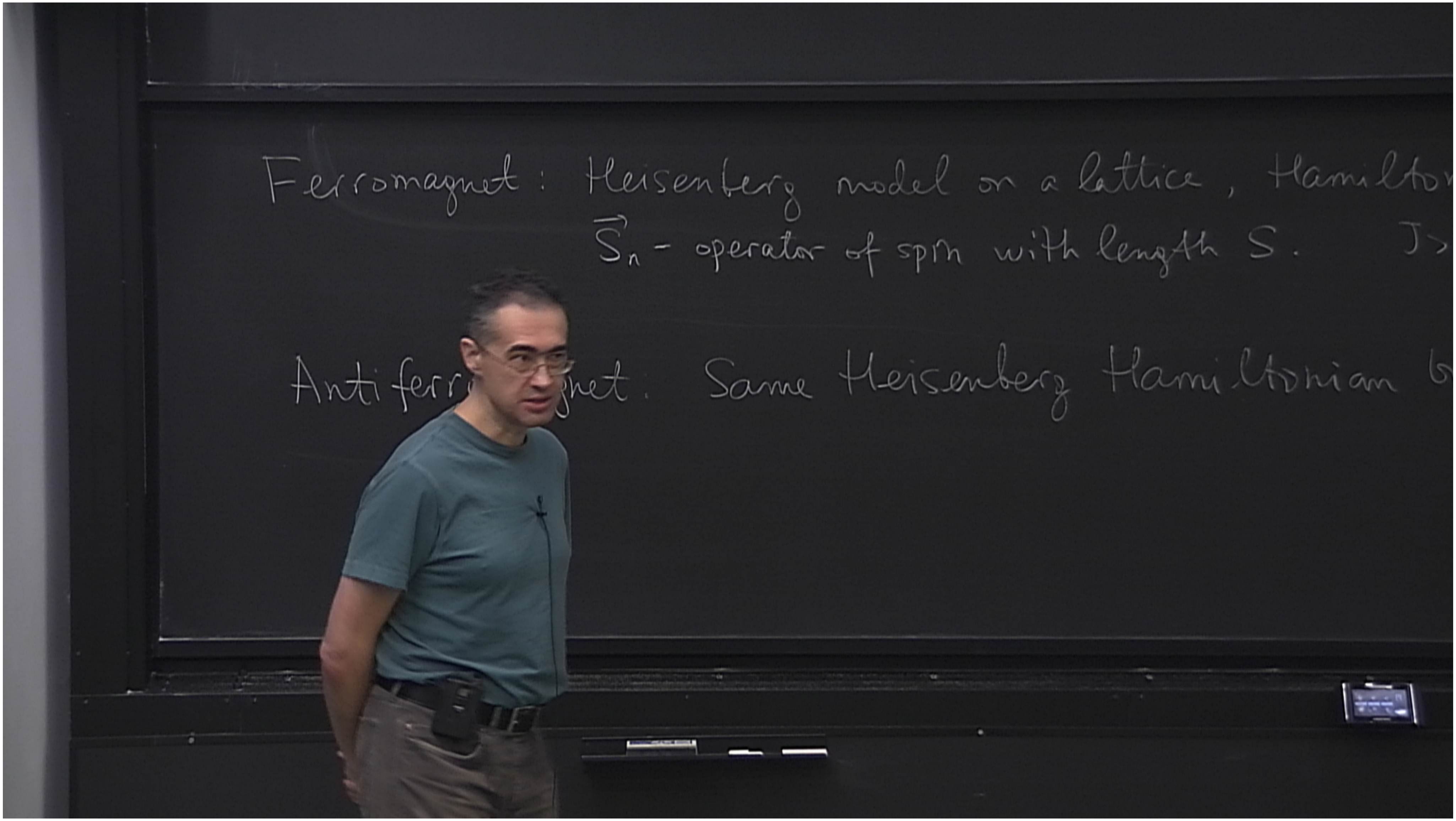
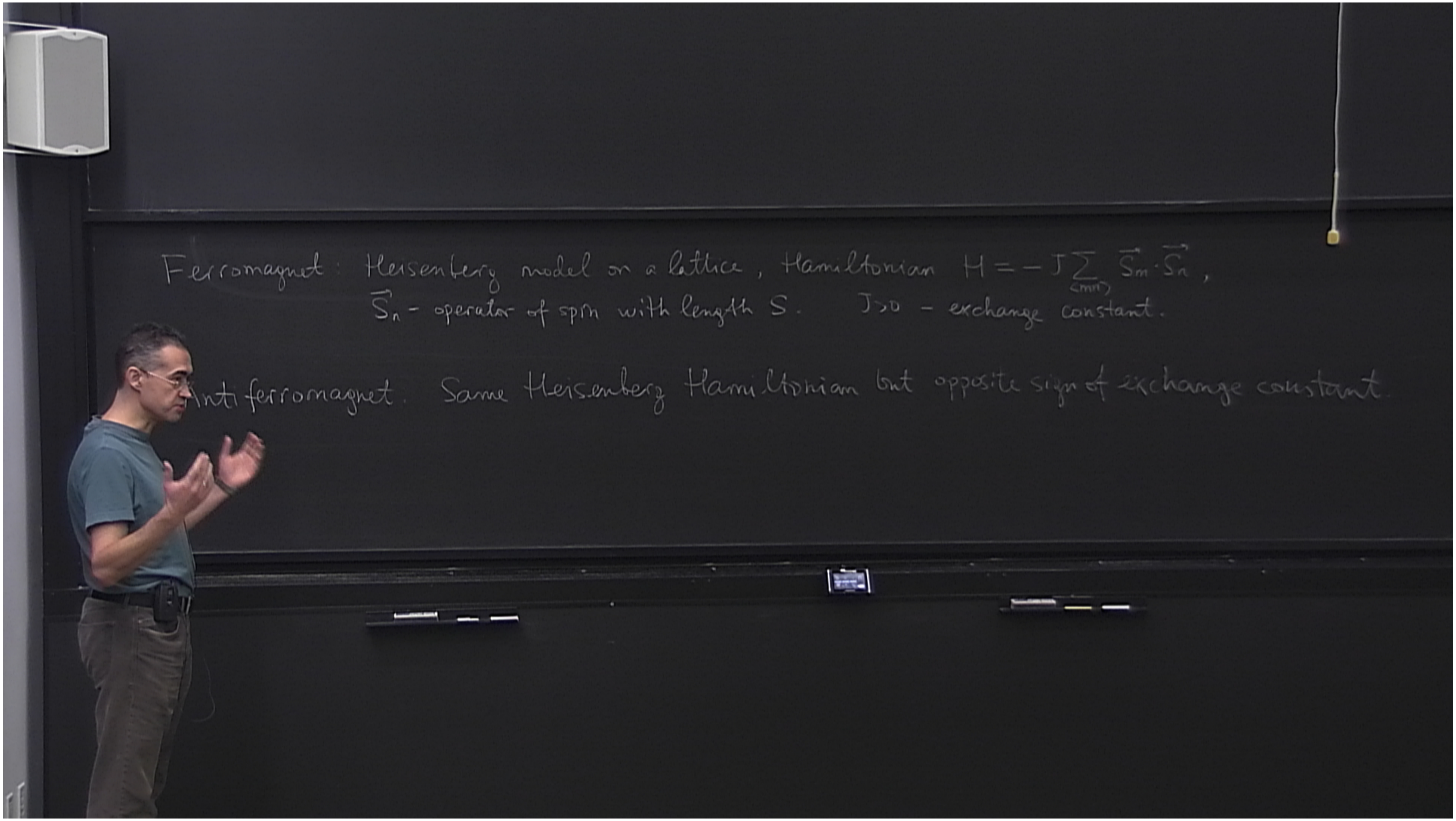
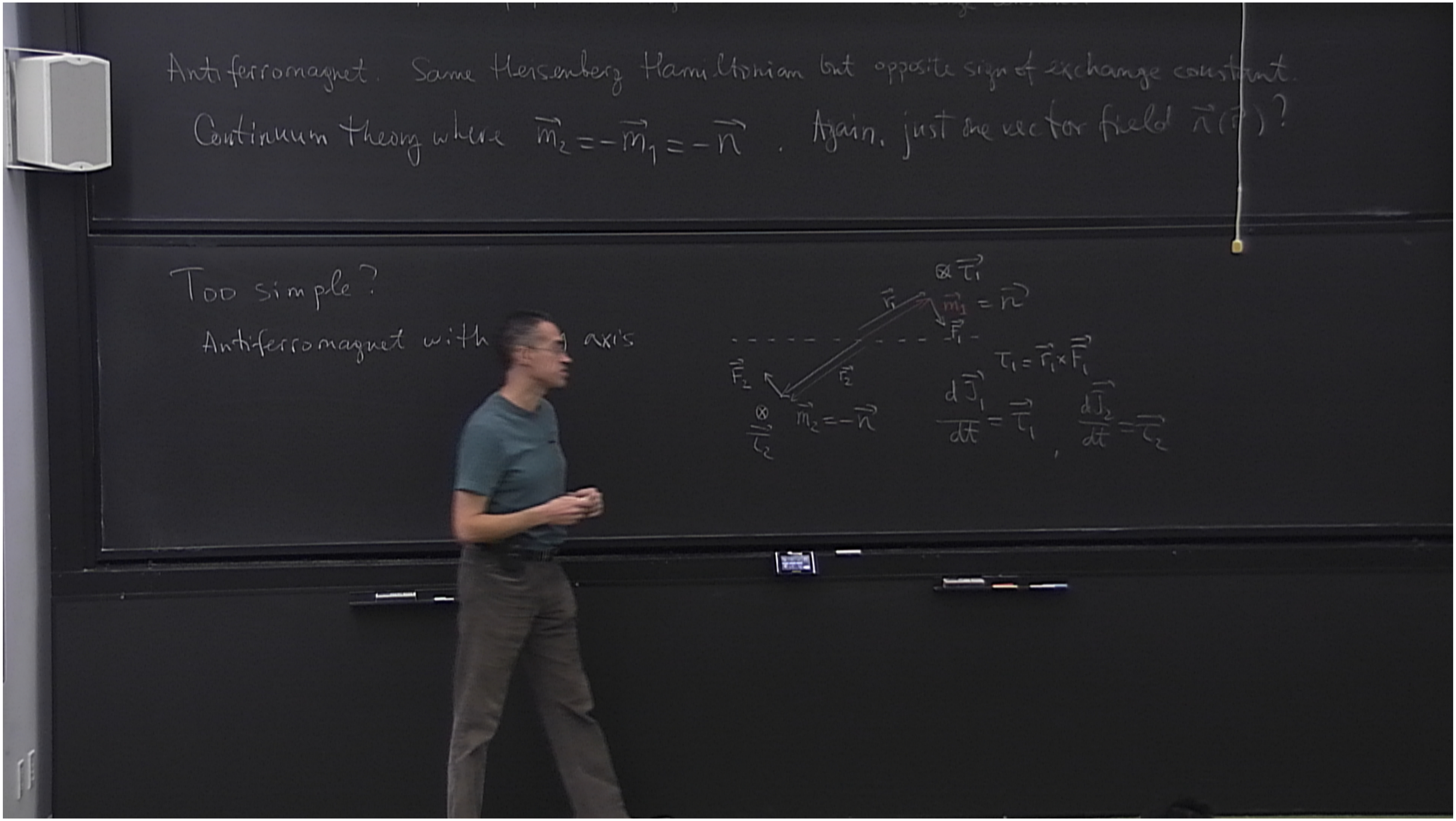
Abstract:

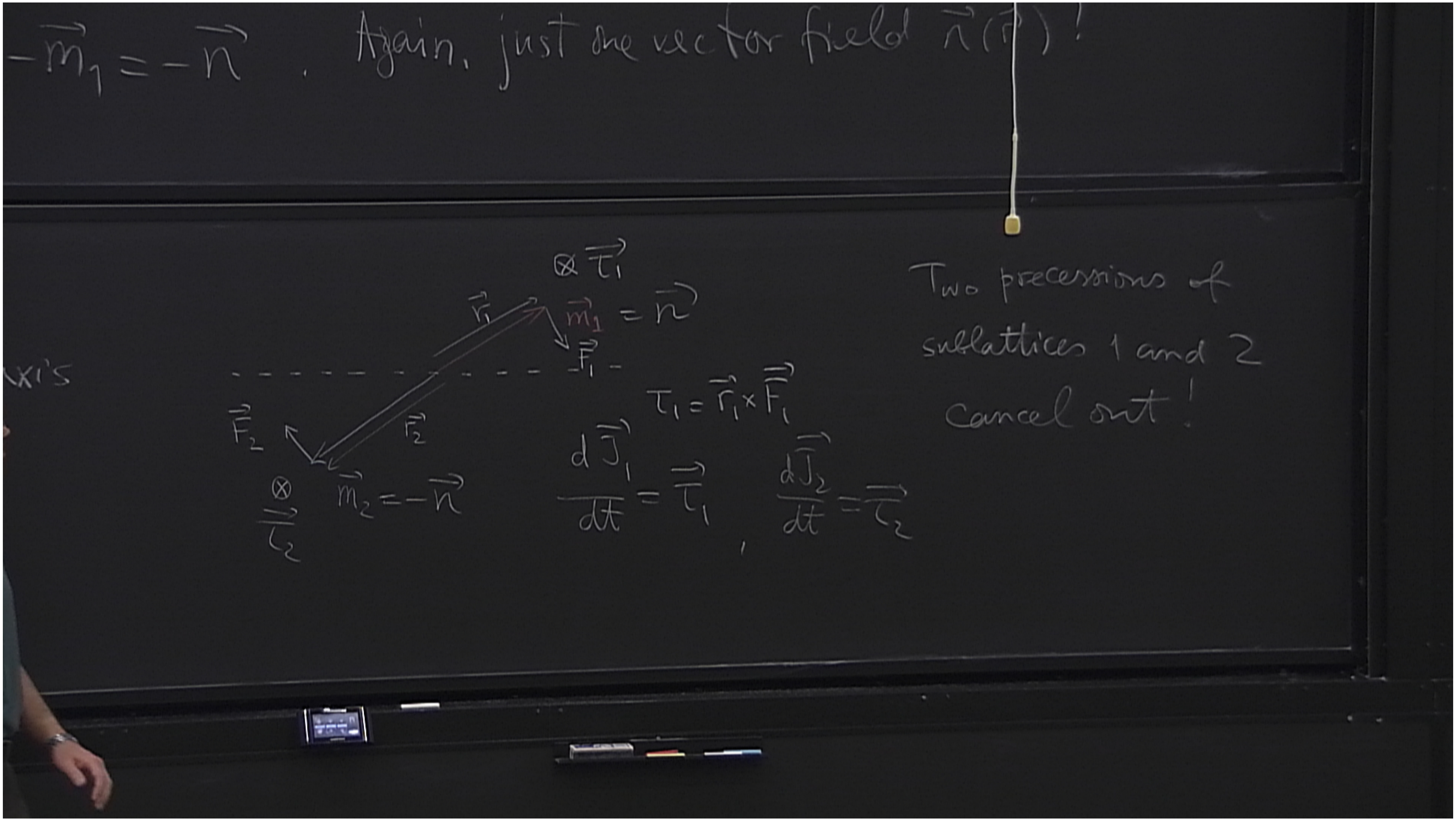

Square lattice
$\vec{m}_1$ $\vec{m}_2$
2 sublattices
Triangular (hexagonal) lattice
$\vec{m}_1$ $\vec{m}_2$ $\vec{m}_3$
3 sublattices


Ferromagnet: Heisenberg model on a lattice, Hamilto
$\vec{S}_n$ - operator of spin with length S. J>
Antiferr gnet: Same Heisenberg Hamiltonian


Ferromagnet: Heisenberg model on a lattice, Hamiltonian $H = -J \sum_{\langle mn \rangle} \vec{S}_m \cdot \vec{S}_n$,
$\vec{S}_n$ - operator of spin with length S. $J>0$ - exchange constant.
Antiferromagnet. Same Heisenberg Hamiltonian but opposite sign of exchange constant.



Antiferromagnet. Same Heisenberg Hamiltonian but opposite sign of exchange constant.
Continuum theory where m2 = -m1 = -n. Again, just one vector field n(r)?
Too simple?
Antiferromagnet with
axis
τ1
r1
m1 = n
F1
τ1 = r1 × F1
F2
r2
m2 = -n
τ2
dJ1/dt = τ1, dJ2/dt = τ2


$-\vec{m}_1 = -\vec{n}$. Again, just one vector field $\vec{n}(\vec{r})$!
$\otimes \vec{\tau}_1$
$\vec{r}_1$
$\vec{m}_1 = \vec{n}$
$\vec{F}_1$
$\vec{F}_2$
$\vec{r}_2$
$\otimes$
$\vec{\tau}_2$
$\vec{m}_2 = -\vec{n}$
$\tau_1 = \vec{r}_1 \times \vec{F}_1$
$\frac{d\vec{J}_1}{dt} = \vec{\tau}_1 , \quad \frac{d\vec{J}_2}{dt} = \vec{\tau}_2$
Two precessions of
sublattices 1 and 2
cancel out!

S_n - operator of spin with length S. $J>0$ - exchange constant.

Antiferromagnet. Same Heisenberg Hamiltonian but opposite sign of exchange constant.

Continuum theory where $\vec{m}_2 = -\vec{m}_1 = -\vec{n}$. Again, just one vector field $\vec{n}(\vec{r})$?

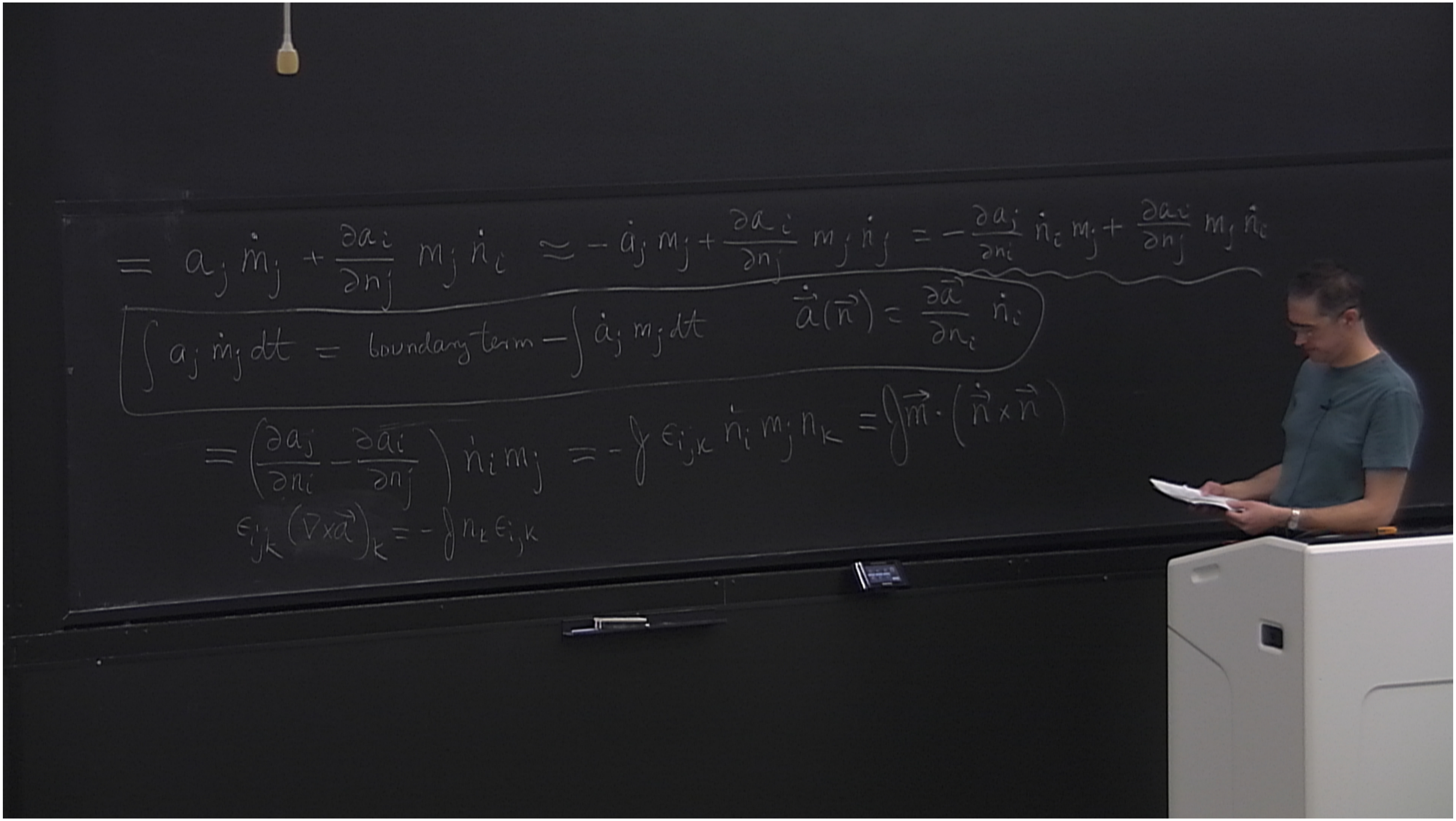
$$\vec{m}_1 = \vec{n} + \frac{\vec{m}}{2} \qquad n \approx 1, \quad m \ll 1$$

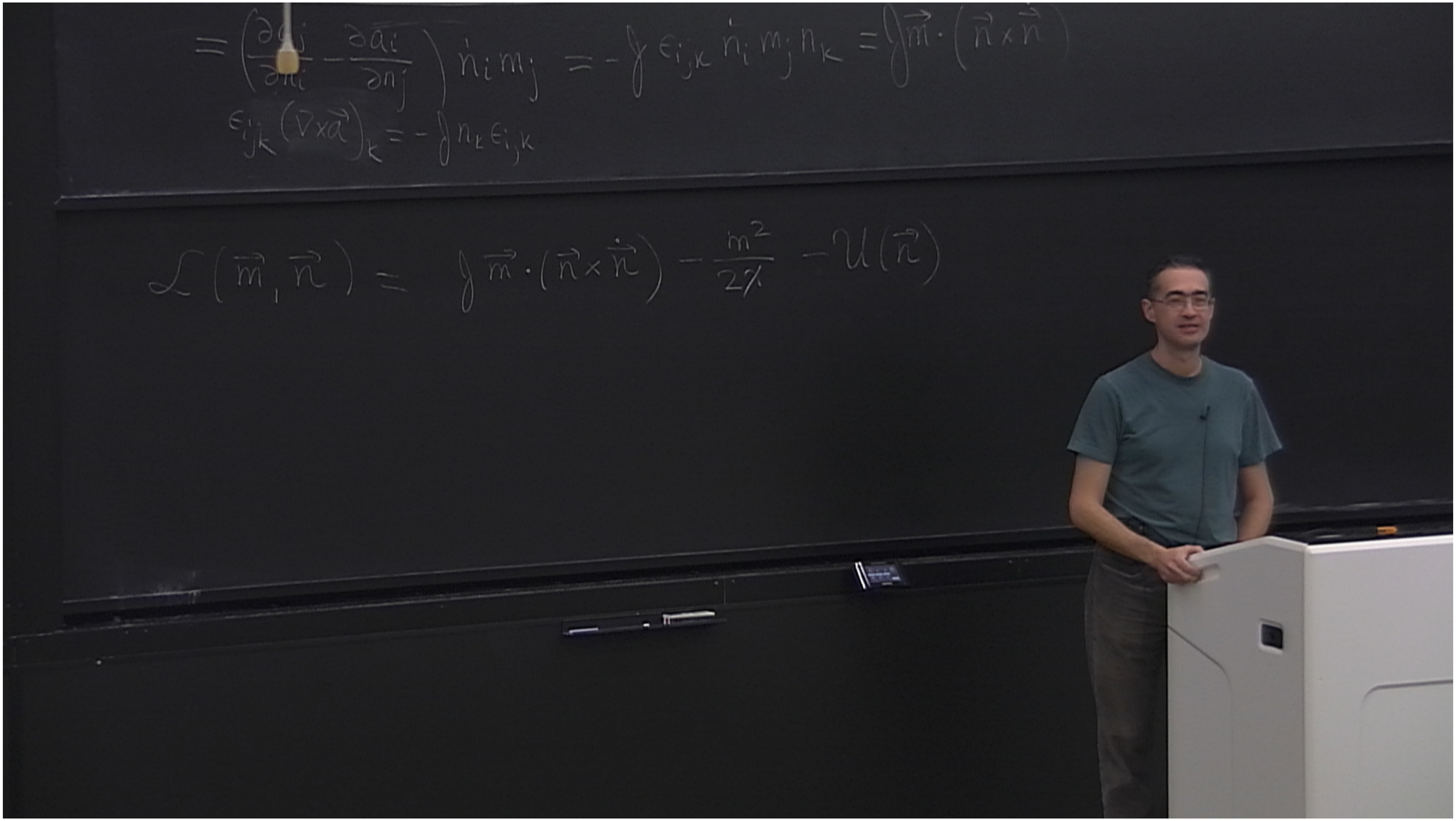
$$\vec{m}_2 = -\vec{n} + \frac{\vec{m}}{2} \qquad \mathcal{L}(\vec{m}_1, \vec{m}_2) = \mathcal{L}(\vec{n}, \vec{m}) = \vec{a}_1(\vec{m}_1)\cdot\dot{\vec{m}}_1 + \vec{a}_2(\vec{m}_2)\cdot\dot{\vec{m}}_2 - \mathcal{U}(\vec{m}, \vec{n})$$

$\vec{m}$ - uniform magnetization $\quad \mathcal{U}(\vec{m}, \vec{n}) = \frac{m^2}{2\chi} - U(\vec{n}), \quad U(\vec{n}) = \frac{A}{2}(\partial_i \vec{n}\cdot\partial_i \vec{n})$

$\vec{n}$ – staggered magnetization

χ - magnetic susceptibility


$= a_j \dot{m}_j + \frac{\partial a_i}{\partial n_j} m_j \dot{n}_i \simeq -\dot{a}_j m_j + \frac{\partial a_i}{\partial n_j} m_j \dot{n}_j = -\frac{\partial a_j}{\partial n_i} \dot{n}_i m_j + \frac{\partial a_i}{\partial n_j} m_j \dot{n}_i$
$\int a_j \dot{m}_j dt =$ boundary term $- \int \dot{a}_j m_j dt$
$\dot{\vec{a}}(\vec{n}) = \frac{\partial \vec{a}}{\partial n_i} \dot{n}_i$
$= \left(\frac{\partial a_j}{\partial n_i} - \frac{\partial a_i}{\partial n_j} \right) \dot{n}_i m_j = -\int \epsilon_{ijk} \dot{n}_i m_j n_k = \int \vec{m} \cdot (\dot{\vec{n}} \times \vec{n})$
$\epsilon_{ijk} (\nabla \times \vec{a})_k = -\int n_k \epsilon_{ijk}$


$= \left(\frac{\partial a_j}{\partial n_i} - \frac{\partial a_i}{\partial n_j}\right)\dot{n}_i m_j = -g\,\epsilon_{ijk}\,\dot{n}_i m_j n_k = g\,\vec{m}\cdot(\vec{n}\times\dot{\vec{n}})$
$\epsilon_{ijk}(\nabla\times\vec{a})_k = -g\,n_k\,\epsilon_{ijk}$
$\mathcal{L}(\vec{m},\vec{n}) = g\,\vec{m}\cdot(\vec{n}\times\dot{\vec{n}}) - \frac{m^2}{2\chi} - \mathcal{U}(\vec{n})$

$$= \left(\frac{\partial a_j}{\partial n_i} - \frac{\partial a_i}{\partial n_j}\right)\dot{n}_i m_j = -g\,\epsilon_{ijk}\,\dot{n}_i m_j n_k = g\,\vec{m}\cdot(\vec{n}\times\dot{\vec{n}})$$

$$\epsilon_{ijk}(\nabla\times\vec{a})_k = -g\,n_k\,\epsilon_{ijk}$$

$$\mathcal{L}(\vec{m},\vec{n}) = g\,\vec{m}\cdot(\vec{n}\times\dot{\vec{n}}) - \frac{m^2}{2\chi} - \mathcal{U}(\vec{n})$$

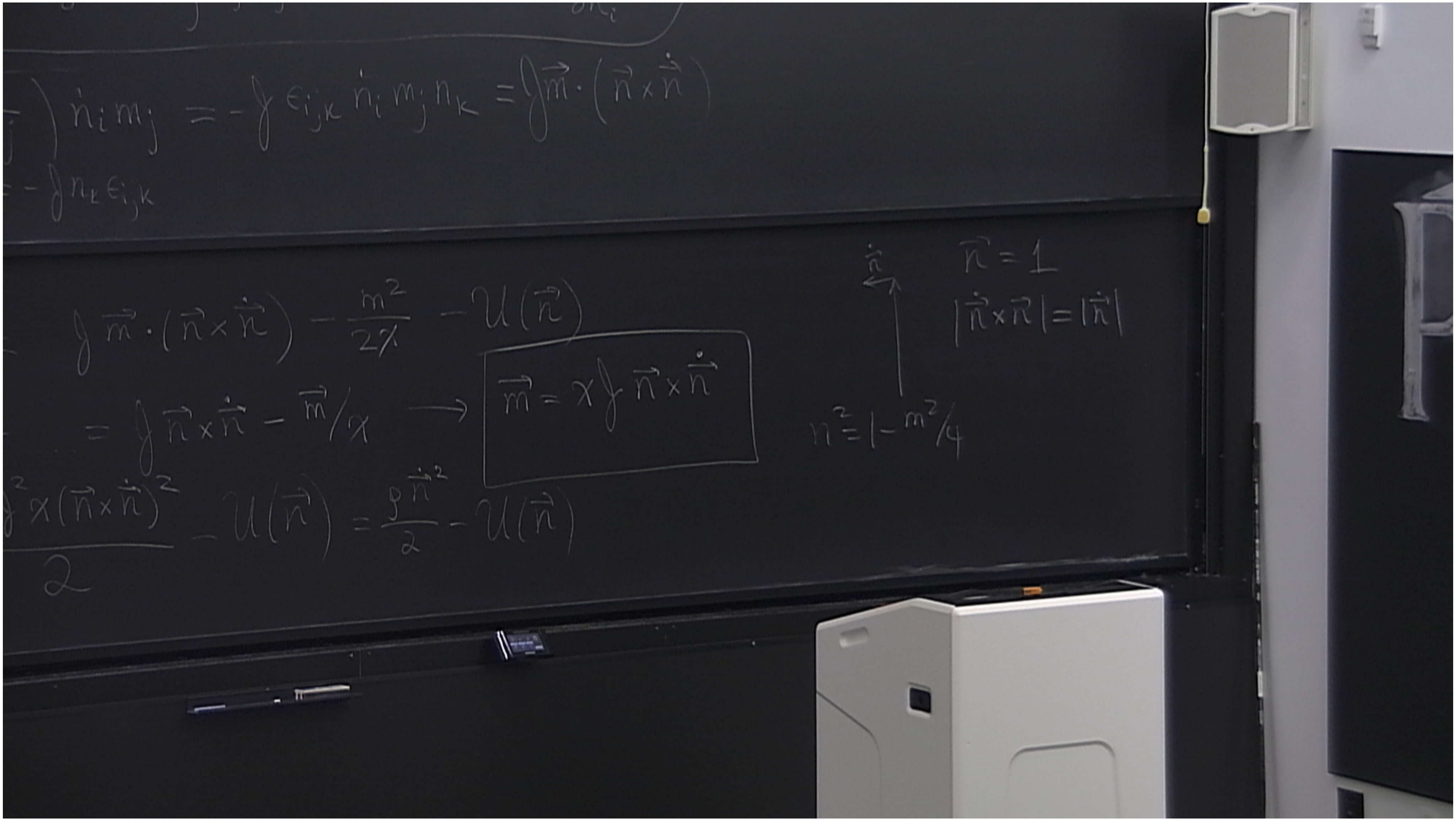
$$0 = \frac{\delta S}{\delta\vec{m}} = \frac{\partial\mathcal{L}}{\partial\vec{m}} - \ldots = g\,\vec{n}\times\dot{\vec{n}} - \vec{m}/\chi \longrightarrow \boxed{\vec{m} = \chi g\,\vec{n}\times\dot{\vec{n}}}$$

$$\epsilon_{ijk}(\nabla\times\vec{a})_k = -g\, n_k \epsilon_{ijk}$$

$$\mathcal{L}(\vec{m},\vec{n}) = g\,\vec{m}\cdot(\vec{n}\times\dot{\vec{n}}) - \frac{m^2}{2\chi} - U(\vec{n})$$

$$0 = \frac{\delta S}{\delta \vec{m}} = \frac{\partial \mathcal{L}}{\partial \vec{m}} - \ldots = g\,\vec{n}\times\dot{\vec{n}} - \vec{m}/\chi \longrightarrow \boxed{\vec{m} = \chi g\, \vec{n}\times\dot{\vec{n}}}$$

$$\mathcal{L}(\vec{n}) = \frac{g^2\chi(\vec{n}\times\dot{\vec{n}})^2}{2} - U(\vec{n}) = \frac{\rho\,\dot{\vec{n}}^2}{2} - U(\vec{n})$$


$\dot{n}_i m_j = -g\,\epsilon_{ijk}\,\dot{n}_i m_j n_k = g\vec{m}\cdot(\vec{n}\times\dot{\vec{n}})$
$= -g\, n_k \epsilon_{ijk}$
$g\,\vec{m}\cdot(\vec{n}\times\dot{\vec{n}}) - \frac{m^2}{2\chi} - U(\vec{n})$
$= g\,\vec{n}\times\dot{\vec{n}} - \vec{m}/\chi \longrightarrow \boxed{\vec{m} = \chi g\,\vec{n}\times\dot{\vec{n}}}$
$\frac{g^2\chi(\vec{n}\times\dot{\vec{n}})^2}{2} - U(\vec{n}) = \frac{\rho\dot{\vec{n}}^2}{2} - U(\vec{n})$
$\vec{n} = 1$
$|\dot{\vec{n}}\times\vec{n}| = |\dot{\vec{n}}|$
$n^2 = 1 - m^2/4$

Antiferromagnet. Same Heisenberg Hamiltonian but opposite sign of exchange constant

Continuum theory where $\vec{m}_2 = -\vec{m}_1 = -\vec{n}$. Again, just one vector field $\vec{n}(\vec{r})$?

Isotropic antiferromagnet: $\mathcal{U}(\vec{n}) = \frac{A}{2}\, \partial_i \vec{n} \cdot \partial_i \vec{n}$

$$\mathcal{L} = \frac{\rho}{2}\, \partial_0 \vec{n} \cdot \partial_0 \vec{n} - \frac{A}{2}\, \partial_i \vec{n} \cdot \partial_i \vec{n} \quad , \quad c^2 = \frac{A}{\rho}$$

Spin waves. Start with ground state $\vec{n}_0 = (0,0,1)$ $\quad -(\delta n_x^2 + \delta n_y^2)/2$

Small deviations: $\vec{n}(\vec{r}) = \vec{n}_0 + \delta\vec{n}(\vec{r})$, $\delta\vec{n} \perp \vec{n}_0 = (\delta n_x, \delta n_y, 0)$