Title: PSI 2015/2016 Condensed Matter - Oleg Tchernyshyov - Lecture 3

Date: Nov 11, 2015 10:45 AM

URL: http://pirsa.org/15110044

Abstract:

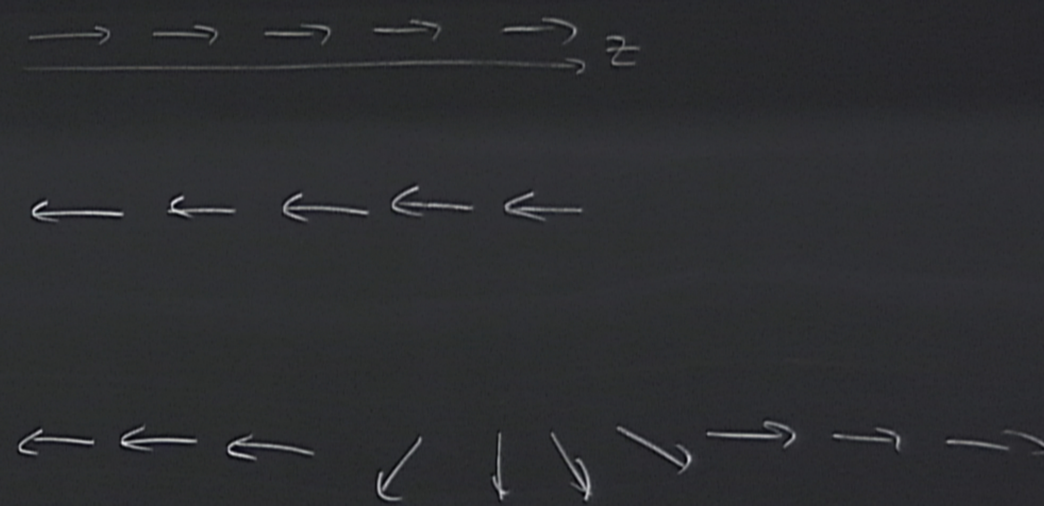
$d=1$ ferromagnet with an easy axis

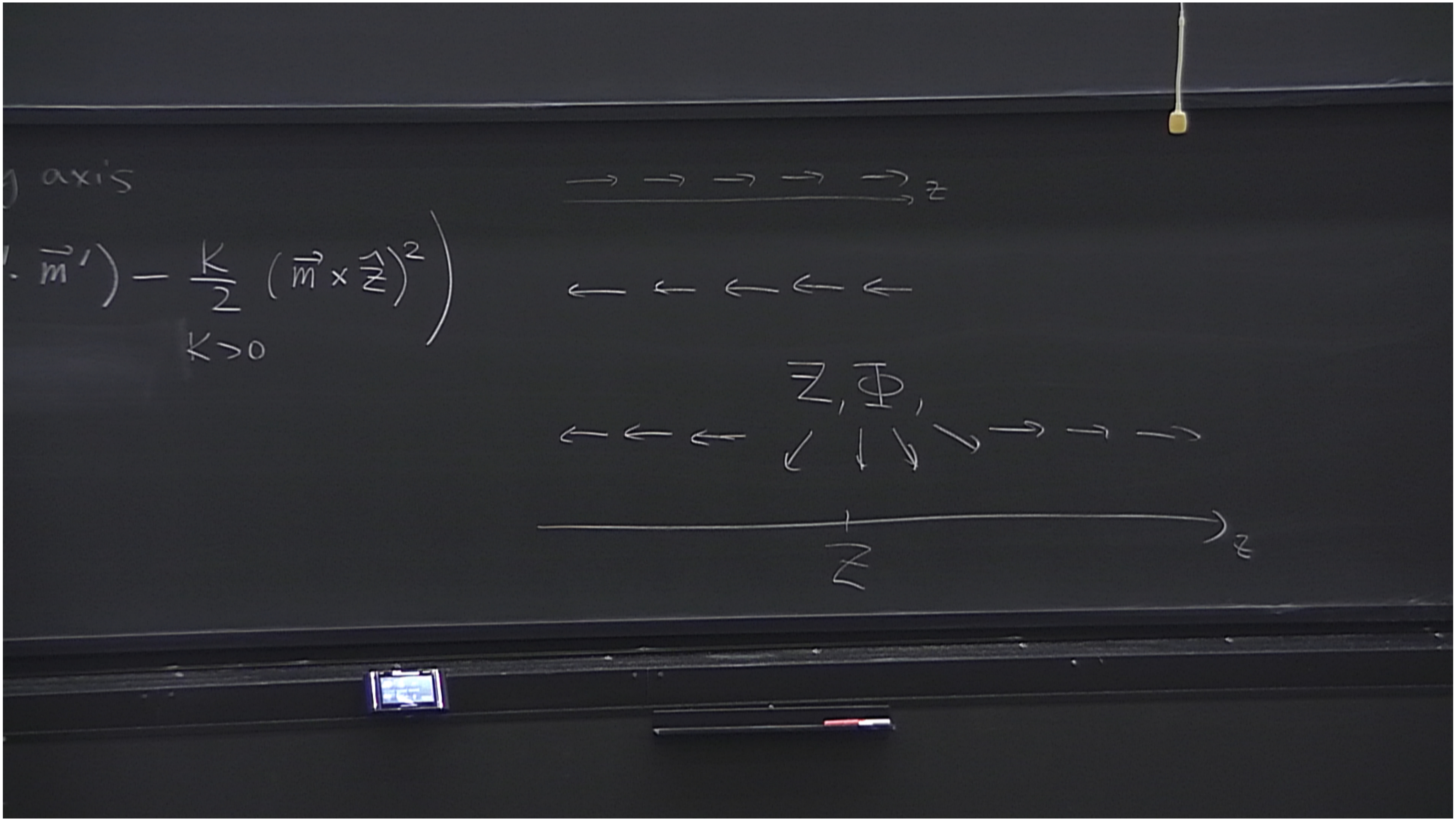
$$L = \int_{-\infty}^{\infty} dz \left(\vec{a}(\vec{m})\, \dot{\vec{m}} - \frac{A}{2}\left(\vec{m}' \cdot \vec{m}'\right) - \frac{K}{2}\left(\vec{m} \times \hat{z}\right)^2 \right)$$

y axis

$$' \cdot \vec{m}'\big) - \frac{K}{2}\left(\vec{m}\times\hat{z}\right)^2\Big)$$

$K>0$




axis
$\cdot\,\vec{m}'\big) - \frac{K}{2}\left(\vec{m}\times\hat{z}\right)^2\Big)$
$K>0$
z
$Z, \Phi,$
Z
z

$$U = \int_{-\infty}^{\infty} dz \left(\frac{A}{2}\left(\theta'^2 + \sin^2\theta\, \phi'^2\right) + \frac{K}{2}\sin^2\theta \right)$$

$$\vec{m} = (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$

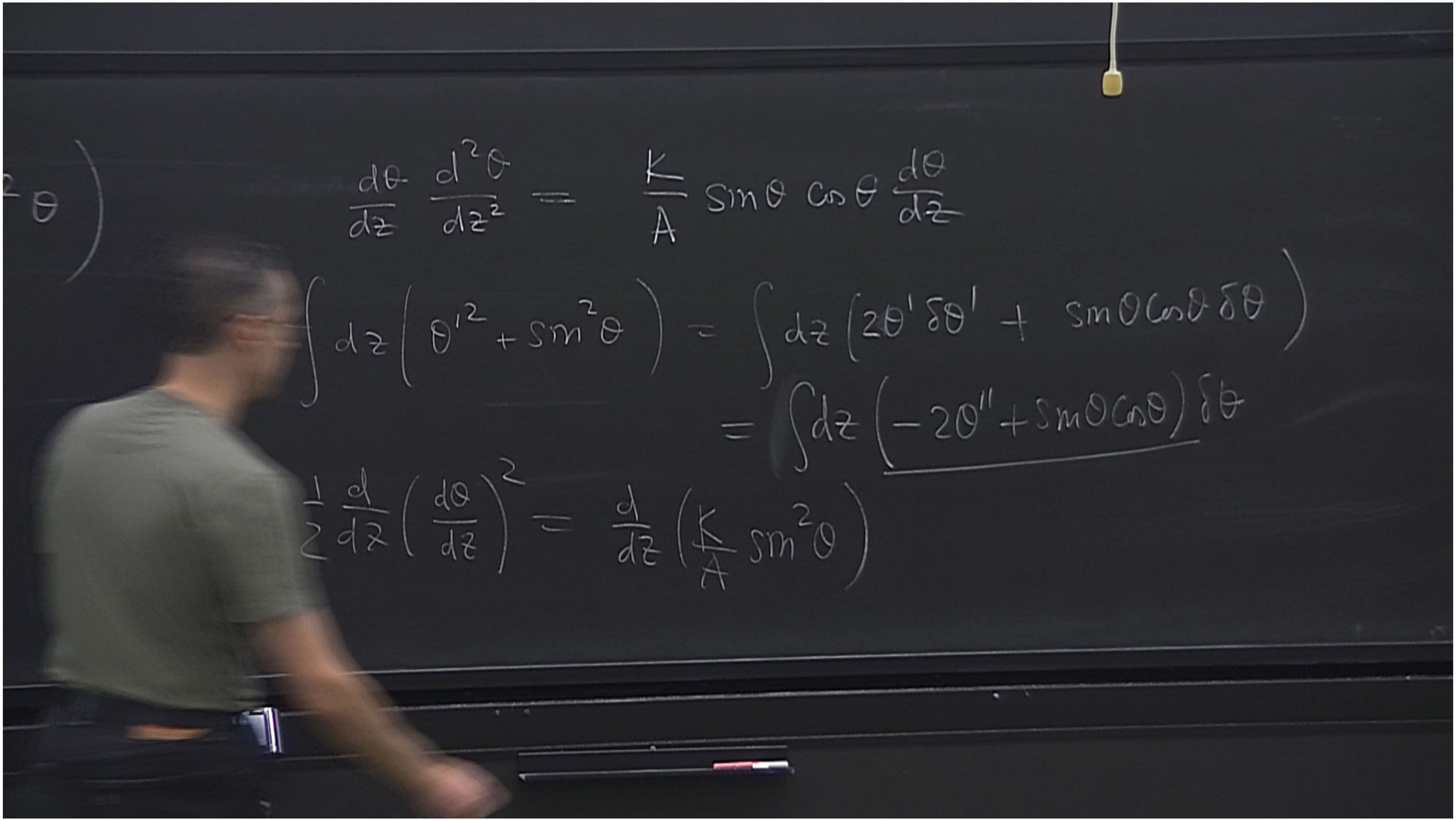
$$0 = \frac{\delta U}{\delta \phi} = -\frac{d}{dz}\left(A\sin^2\theta\,\frac{d\phi}{dz}\right) \qquad \phi = \Phi = \text{const}$$

$$0 = \frac{\delta U}{\delta \theta} = -A\frac{d^2\theta}{dz^2} + A\sin\theta\cos\theta\,\cancel{\phi'^2}\to 0 + K\sin\theta\cos\theta$$

$$\frac{d^2\theta}{dz^2} = \frac{K}{A}\sin\theta\cos\theta$$

$$\delta\int dz\left(\theta'^2+\sin^2\theta\right) = \int dz\left(2\theta'\delta\theta' + \sin\theta\cos\theta\,\delta\theta\right)$$

$$= \int dz\,\underline{\left(-2\theta''+\sin\theta\cos\theta\right)}\,\delta\theta$$



$\frac{d\theta}{dz}\frac{d^2\theta}{dz^2} = \frac{K}{A}\sin\theta\cos\theta\frac{d\theta}{dz}$
$\int dz\left(\theta'^2 + \sin^2\theta\right) = \int dz\left(2\theta'\delta\theta' + \sin\theta\cos\theta\,\delta\theta\right)$
$= \int dz\left(-2\theta'' + \sin\theta\cos\theta\right)\delta\theta$
$\frac{1}{2}\frac{d}{dz}\left(\frac{d\theta}{dz}\right)^2 = \frac{d}{dz}\left(\frac{K}{A}\sin^2\theta\right)$

$$^2\theta \Big)$$

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$$\frac{1}{2}\frac{d}{dz}\left(\frac{d\theta}{dz}\right)^2 = \frac{d}{dz}\left(\frac{K}{2A}\sin^2\theta\right)$$

$$\left(\frac{d\theta}{dz}\right)^2 = \frac{K}{A}\sin^2\theta + C$$

$$\lambda = \sqrt{\frac{A}{K} - w}$$

$z \to \infty$, $\theta \to 0$ or π, $\theta' = 0$

$0 = 0 + C$, $C = 0$

$$\frac{d\theta}{dz} = \pm\sqrt{\frac{K}{A}}\,\sin\theta$$

$1/\lambda$

$+ C$

$, \theta' = 0$

$$\lambda = \sqrt{\frac{A}{K}} \text{ - width of domain wall}$$

$$\frac{\sin\theta \, d\theta}{\sin^2\theta} = \pm \frac{dz}{\lambda}$$

$$-\frac{d\cos\theta}{1-\cos^2\theta} = \pm \frac{dz}{\lambda}$$

$$\frac{1}{2}\ln\frac{1-\cos\theta}{1+\cos\theta} = \frac{z-Z}{\lambda}$$

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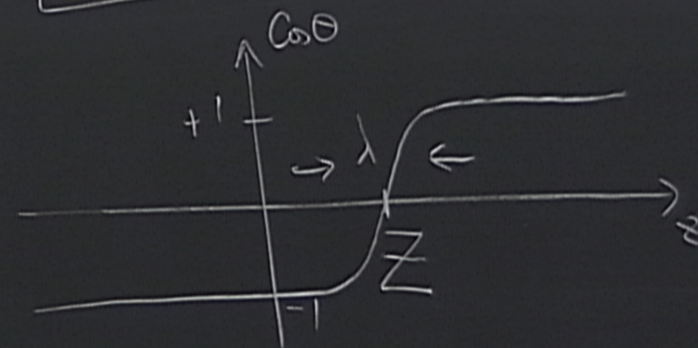
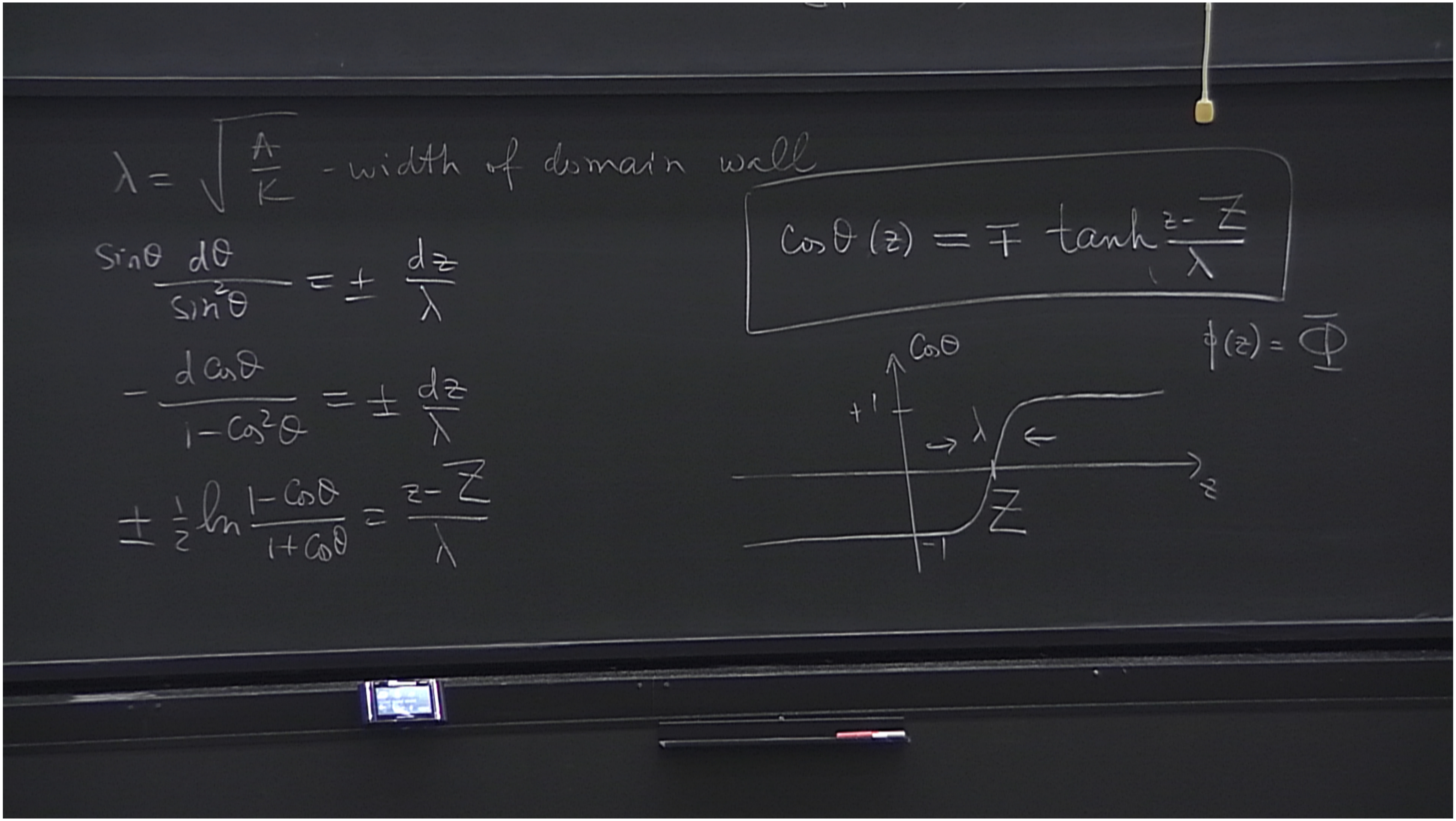
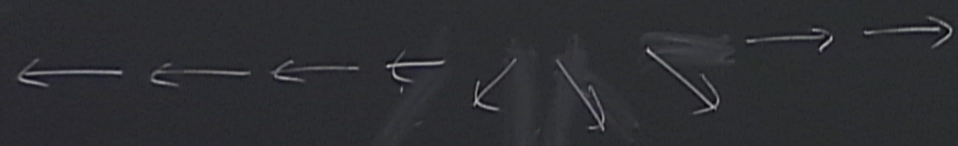
$$\boxed{\cos\theta(z) = \mp \tanh \frac{z-Z}{\lambda}}$$




$\lambda = \sqrt{\frac{A}{K}}$ - width of domain wall
$\frac{\sin\theta \, d\theta}{\sin^2\theta} = \pm \frac{dz}{\lambda}$
$-\frac{d\cos\theta}{1-\cos^2\theta} = \pm \frac{dz}{\lambda}$
$\pm \frac{1}{2} \ln \frac{1-\cos\theta}{1+\cos\theta} = \frac{z-Z}{\lambda}$
$\boxed{\cos\theta(z) = \mp \tanh \frac{z-Z}{\lambda}}$
$\phi(z) = \Phi$
$\cos\theta$
+1
λ
z
Z
-1

What are linear and angular momenta of this soliton?

Angular momentum $J_z = CZ$, $C = -2\mathcal{J}$, $\mathcal{J}$ - density of spin



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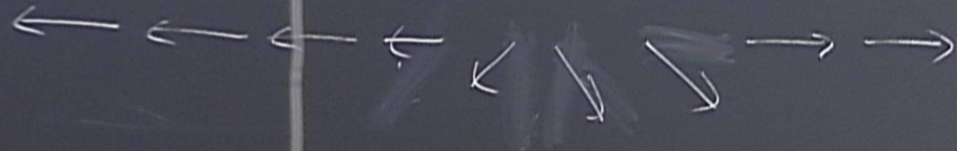

$$\boxed{\tau = \frac{dJ_z}{dt} = -2\mathcal{J}\frac{dZ}{dt}}$$

$$(Z, \Phi) \simeq (P_\phi, \phi) \qquad \Delta P_\phi \, \Delta\phi \geq \frac{\hbar}{2} \qquad 2J \, \Delta Z \, \Delta\Phi \geq \frac{\hbar}{2},$$

$$\Delta Z \, \Delta\Phi \geq \frac{\hbar}{4J}.$$

In a model with spins of length $\hbar S$ and lattice const a, $J = \frac{\hbar S}{a}$

$$\Delta Z \, \Delta\Phi \geq \frac{a}{4S}$$

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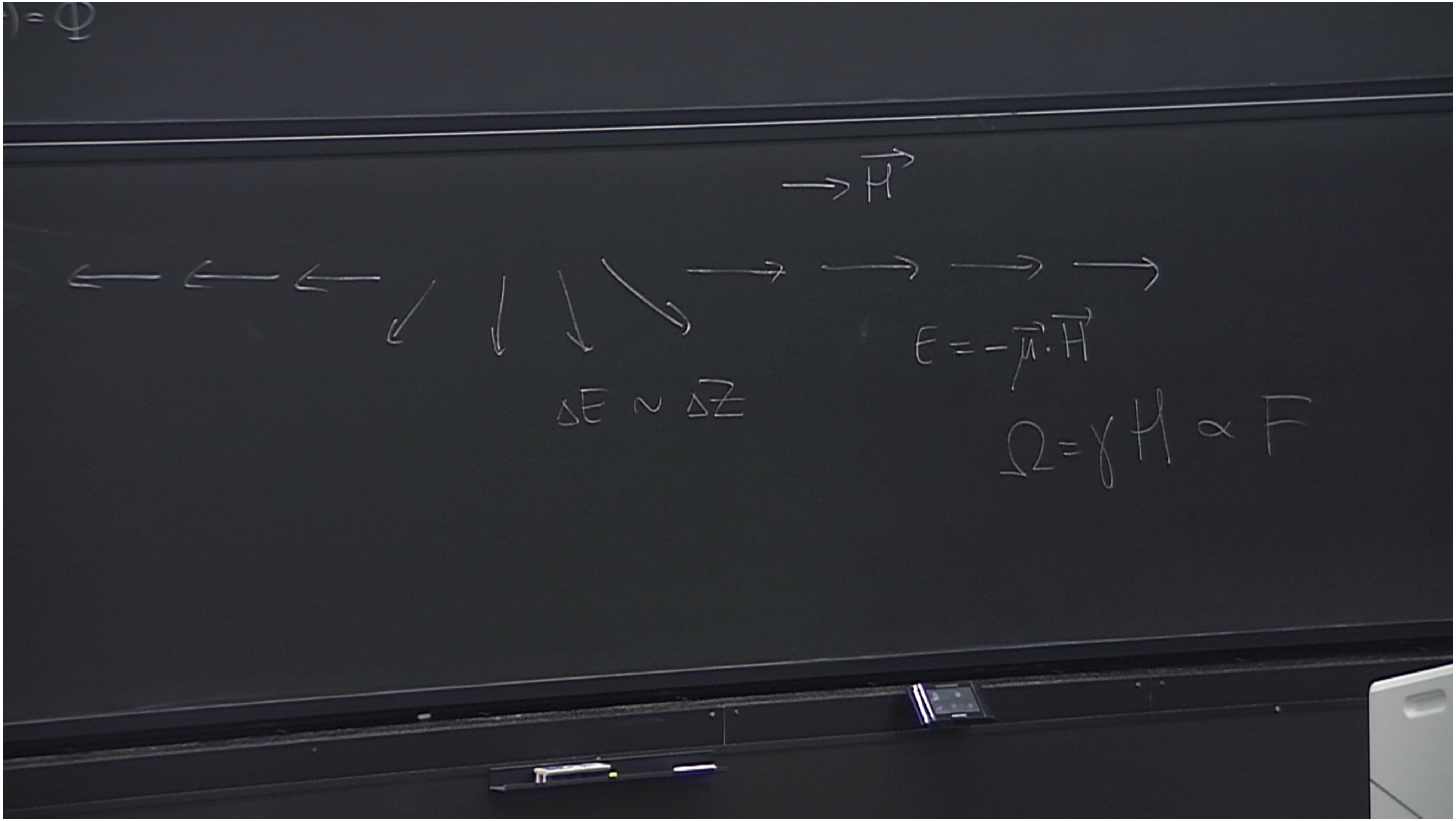
$$\Delta Z \, \Delta \Phi \ge \frac{a}{4S}$$

$$(Z, \phi) \simeq (Z, P_Z), \qquad \underline{P_Z = 2J\Phi}$$


$\to \vec{H}$
$\Delta E \sim \Delta Z$
$E = -\vec{\mu} \cdot \vec{H}$
$\Omega = \gamma H \propto F$

$\phi(z) = \Phi$

$$L = \int dz \left(\vec{a}(\vec{m}) \cdot \dot{\vec{m}} \right) - U_0 - U_1$$

main part — perturbations

$$\vec{m}(\vec{r},t) \longrightarrow \vec{m}\left(\{q(t)\}\right) \quad , \quad \{q\} = \{Z, \phi, \lambda, \ldots\}$$

$$\dot{\vec{m}} = \frac{\partial \vec{m}}{\partial q_i} \dot{q}_i$$

$$L = -U_0(\{q\}) - U_1(\{q\}) + \int \vec{a}(\vec{m}) \cdot \frac{\partial \vec{m}}{\partial q_i} dz \, \dot{q}_i$$

$\phi(z) = \Phi$

$$L = \int dz \left(\vec{a}(\vec{m}) \cdot \dot{\vec{m}} \right) - \underset{\text{main part}}{U_0} - \underset{\text{perturbations}}{U_1}$$

$$L = \dot{q}_i A_i - U$$

$$\vec{m}(\vec{r}, t) \longrightarrow \vec{m}\left(\{q(t)\}\right) \quad , \quad \{q\} = \{Z, \phi, \lambda, \dots\}$$

A_i

$$\dot{\vec{m}} = \frac{\partial \vec{m}}{\partial q_i} \dot{q}_i$$

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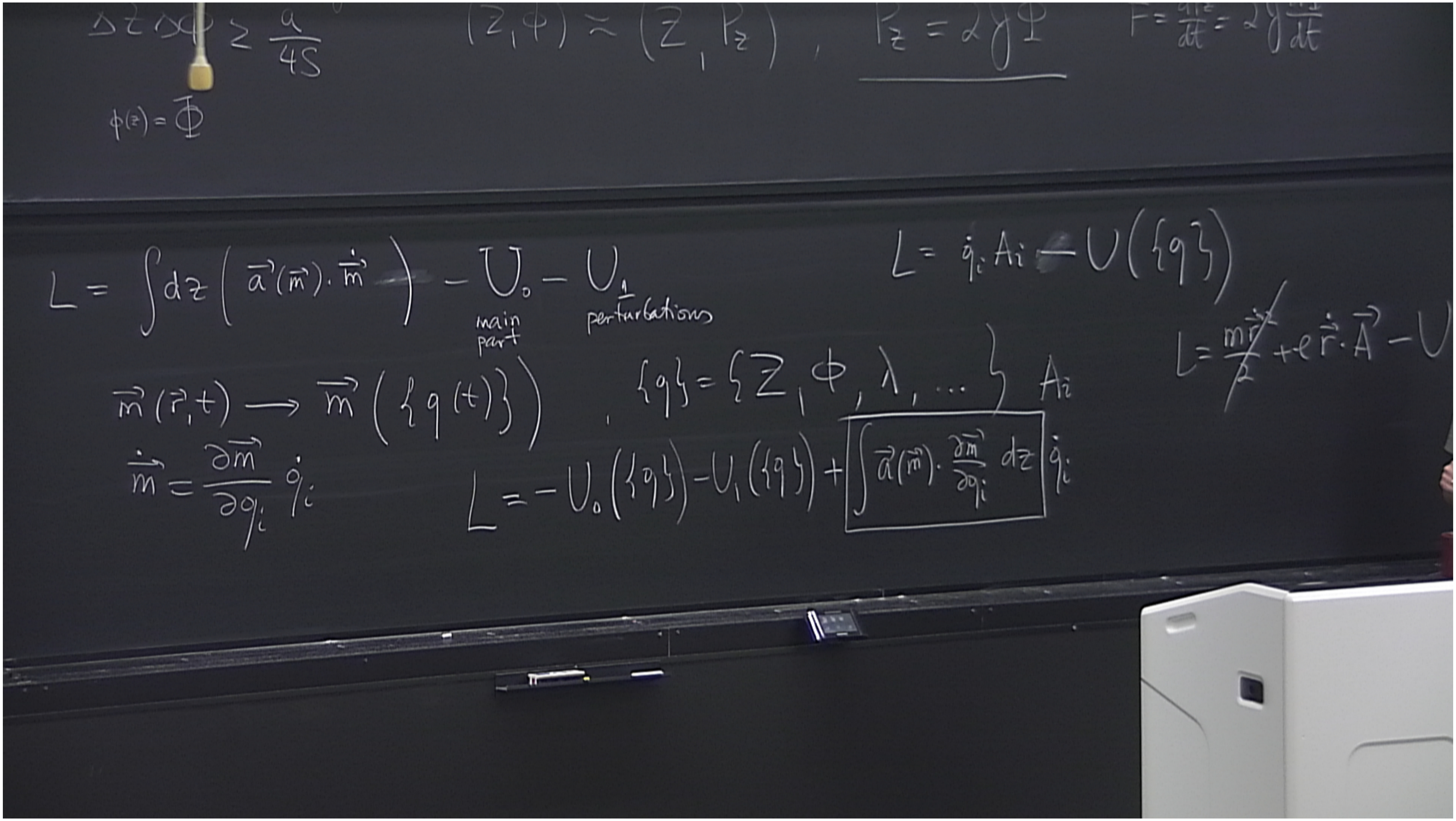


$\Delta z \Delta\phi \geq \frac{a}{4S}$
$(z,\phi) \simeq (Z, P_z)$, $P_z = \alpha y \Phi$
$\phi(z) = \Phi$
$L = \int dz \left(\vec{a}(\vec{m}) \cdot \dot{\vec{m}} \right) - U_0 - U_1$
main part
perturbations
$L = \dot{q}_i A_i - U(\{q\})$
$L = \frac{m\dot{r}^2}{2} + e\dot{\vec{r}}\cdot\vec{A} - U$
$\vec{m}(\vec{r},t) \longrightarrow \vec{m}(\{q(t)\})$, $\{q\} = \{Z, \phi, \lambda, \ldots\}$ A_i
$\dot{\vec{m}} = \frac{\partial \vec{m}}{\partial q_i}\dot{q}_i$
$L = -U_0(\{q\}) - U_1(\{q\}) + \boxed{\int \vec{a}(\vec{m}) \cdot \frac{\partial \vec{m}}{\partial q_i} dz}\,\dot{q}_i$

$$\frac{\delta}{\delta\theta} = -A\frac{d^2\theta}{dz^2} + A\sin\theta\cos\theta\,\cancel{\phi'^2} + K\sin\theta\cos\theta \qquad \frac{d}{2\,dz}\left(\frac{d\theta}{dz}\right)^2 = \frac{d}{dz}\left(\frac{K}{2A}\sin^2\theta\right)$$

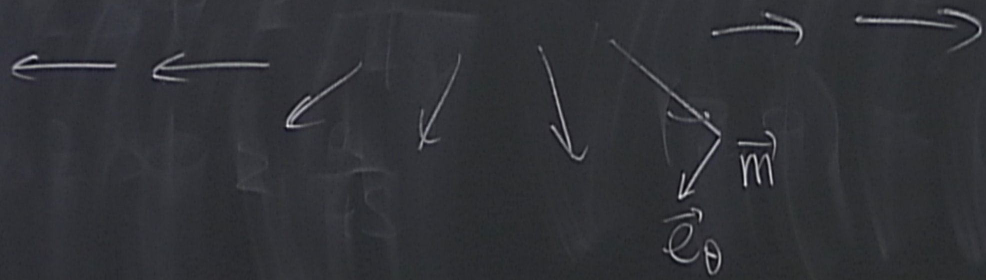
Keep Z and Φ as collective coordinates and neglect all other features

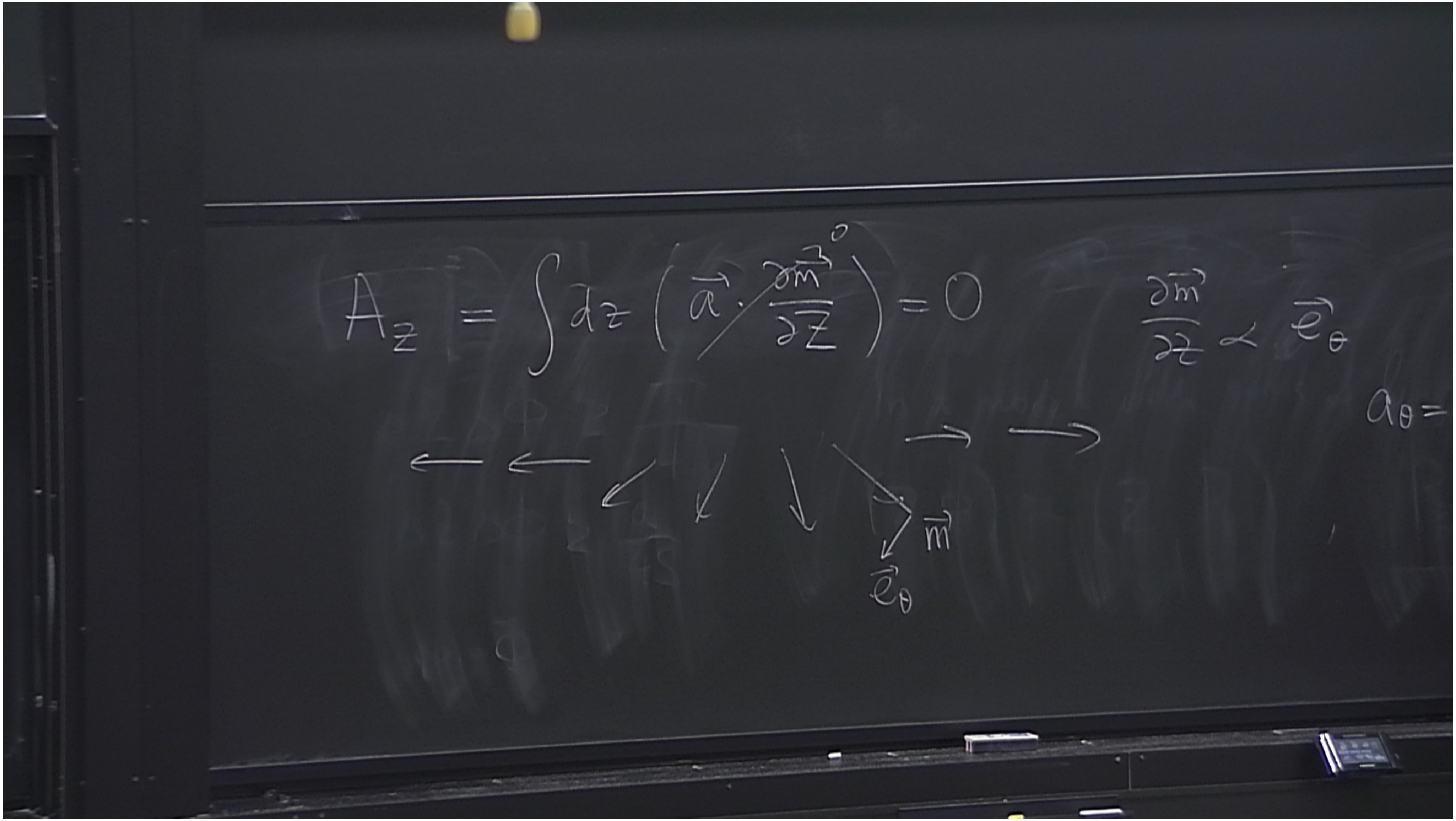
$$L = A_Z\dot{Z} + A_\Phi\dot{\Phi} - \underbrace{\cancel{U_0(Z,\Phi)}}_{\substack{\text{easy-axis}\\ \text{ferromagnet}}} - \underbrace{U_1(Z,\Phi)}_{\substack{\text{external}\\ \text{perturbations}}}$$

const

$$A_z = \int dz \left(\vec{a} \cdot \frac{\partial \vec{m}}{\partial z} \right)$$

$$\frac{\partial \vec{m}}{\partial z} \propto \vec{e}_\theta \qquad \vec{a} \cdot \vec{e}_\theta = a_\theta$$



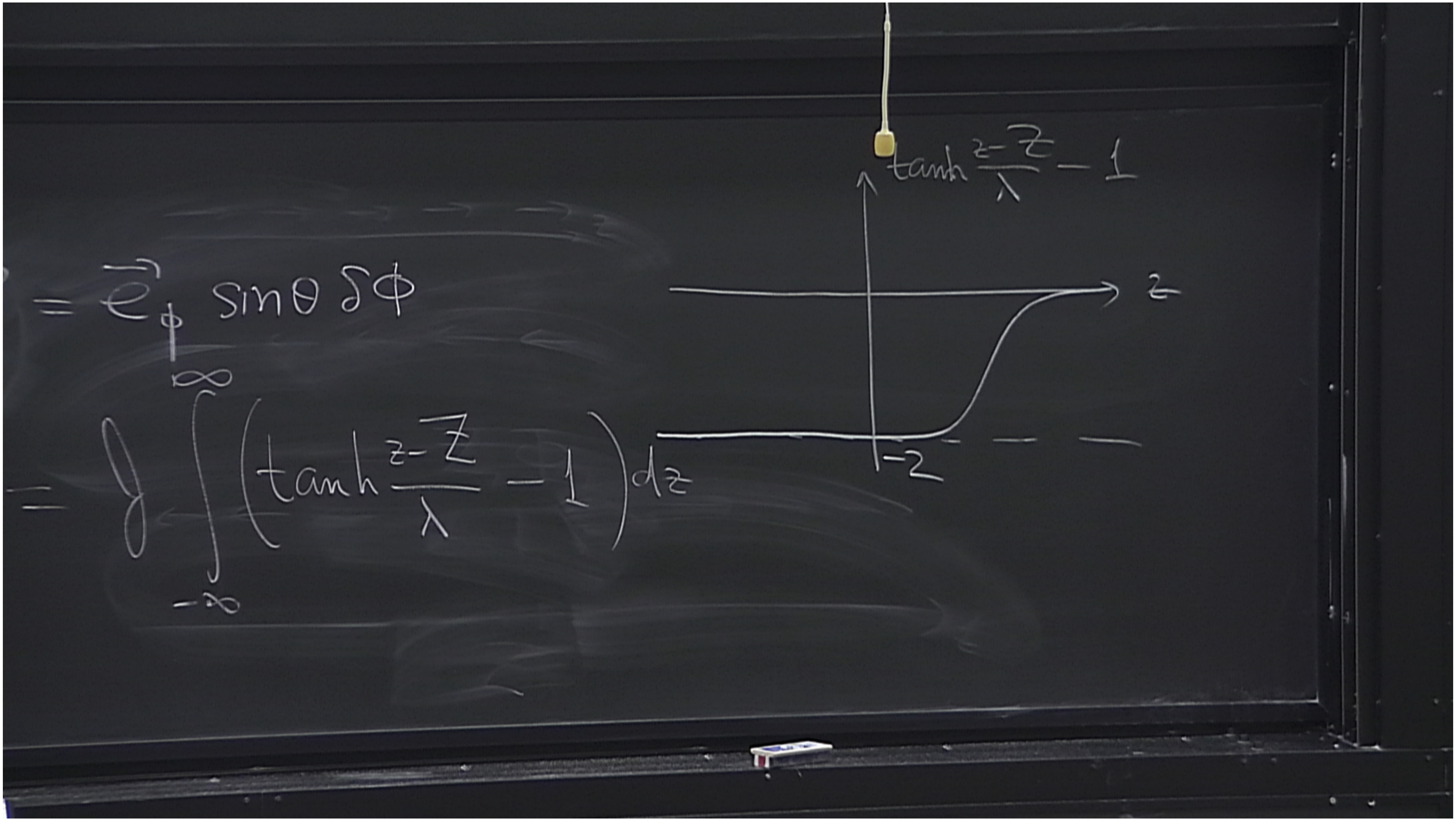

$A_z = \int dz \left(\cancel{\vec{a} \cdot \frac{\partial \vec{m}}{\partial z}}^{\,0} \right) = 0$
$\frac{\partial \vec{m}}{\partial z} \propto \vec{e}_\theta$
$a_\theta =$
$\vec{m}$
$\vec{e}_\theta$

$$\frac{\partial \vec{m}}{\partial z} \propto \vec{e}_\theta \qquad \vec{a}\cdot\vec{e}_\theta = a_\theta$$

$$a_\theta = 0 \,, \quad a_\phi = \frac{g(\cos\theta - 1)}{\sin\theta}$$

$$A_\Phi = \int \vec{a} \cdot \frac{\partial \vec{m}}{\partial \Phi}\, dz \qquad \delta\Phi \longrightarrow \delta\vec{m} = \vec{e}_\phi \sin\theta\, \delta\phi$$

$$= \int_{-\infty}^{\infty} a_\phi \sin\theta\, dz = g \int \frac{\cos\theta - 1}{\cancel{\sin\theta}} \cancel{\sin\theta}\, dz = g \int_{-\infty}^{\infty} \left(\tanh\frac{z - Z}{\lambda} - 1 \right) dz$$

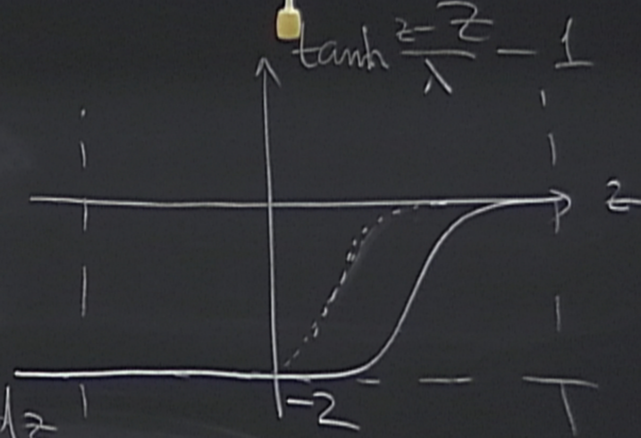

$= \vec{e}_\phi \sin\theta \, \delta\phi$
$= g \int_{-\infty}^{\infty} \left(\tanh \frac{z-Z}{\lambda} - 1\right) dz$
$\tanh \frac{z-Z}{\lambda} - 1$
z
-2

$$-L < z < L$$

$$\to \quad \delta\vec{m} = \vec{e}_\phi \sin\theta\, \delta\phi$$

$$\theta\, dz = \delta \int_{-\infty}^{\infty} \left(\tanh\frac{z-Z}{\lambda} - 1 \right) dz$$

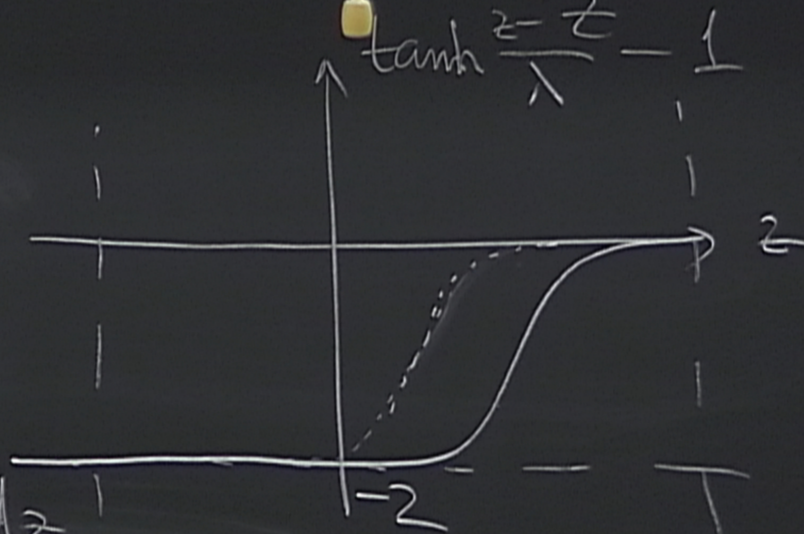
$$= \mathrm{Const}(L,\lambda) - 2\delta Z$$



$$-L < z < L$$

$$\vec{e}_\phi \sin\theta \, \delta\phi$$

$$\hbar \int_{-\infty}^{\infty} \left(\tanh\frac{z-Z}{\lambda} - 1 \right) dz = \mathrm{Const}(L,\lambda) - 2\hbar Z + O\left(e^{-L/\lambda}\right)$$



$$L = -2J Z \dot{\phi} - U, \qquad = -2J Z \dot{\phi} + F Z + \tau \phi$$

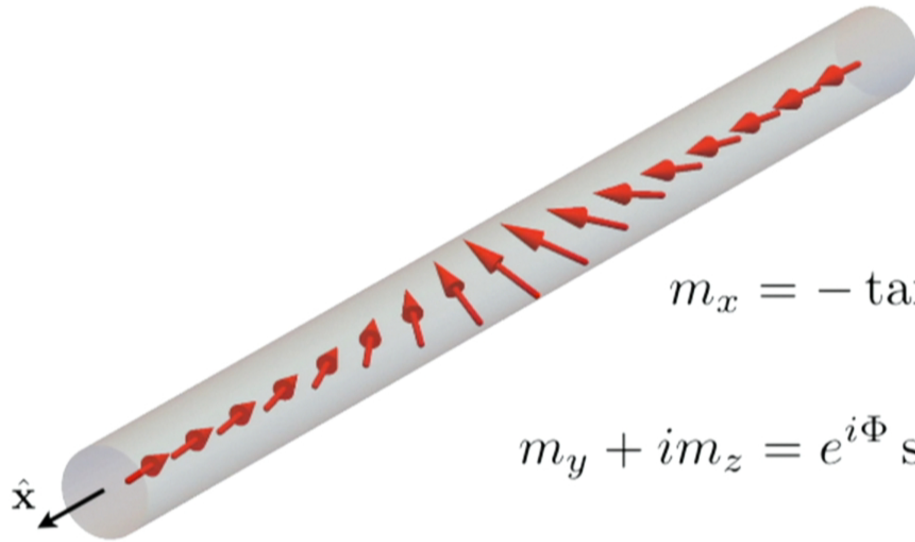
$$Z: \quad \frac{d}{dt}\frac{\partial L}{\partial \dot{Z}} - \frac{\partial L}{\partial Z} = 0 \qquad -F - 2J \dot{\phi} = 0$$

$$\phi: \quad 2J \dot{Z} - \tau = 0$$

Toy model of a thin cylindrical ferromagnetic wire:
Potential energy

$$U = \int dx\ \big[\underbrace{A(d\mathbf{m}/dx)^2}_{\text{exchange}} + \underbrace{K(\mathbf{m}\times\hat{\mathbf{x}})^2}_{\text{anisotropy}}\big]$$

Magnetization is represented by a unit-vector field $\mathbf{m}(x)$



$$m_x = -\tanh\frac{x-X}{\lambda}$$

$$m_y + i m_z = e^{i\Phi}\,\mathrm{sech}\frac{x-X}{\lambda}$$

Soft mode: azimuthal angle Φ. Does not affect energy.

Toy model of a thin cylindrical ferromagnetic wire:
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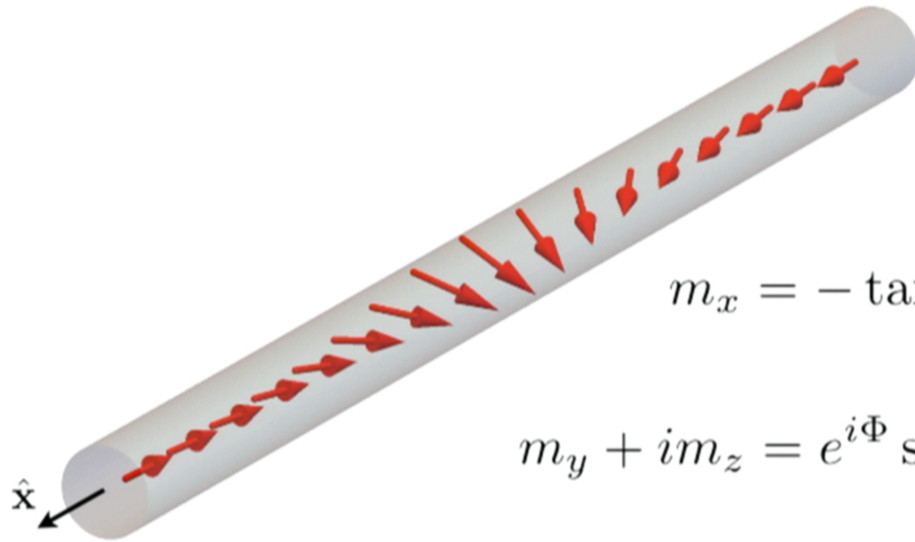

$$m_x = -\tanh\frac{x-X}{\lambda}$$

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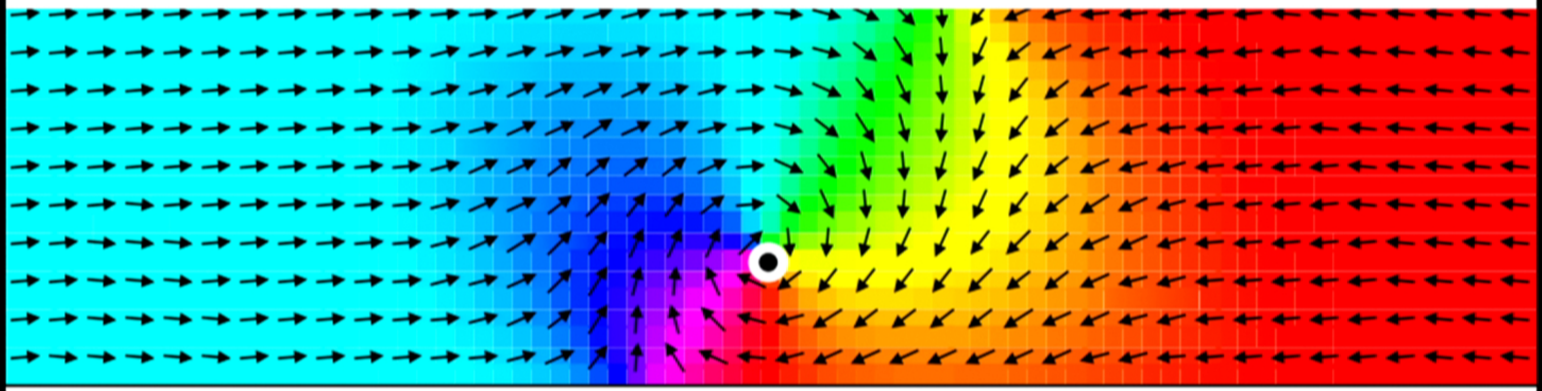
Soft mode: azimuthal angle Φ. Does not affect energy.

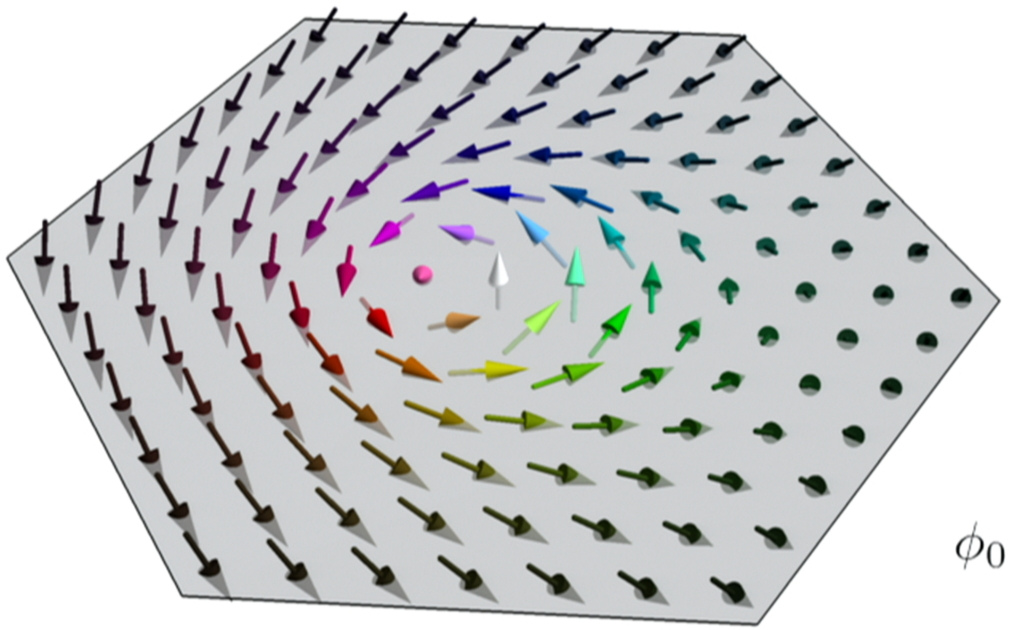
Width 300 nm, thickness 20 nm. Permalloy.



Shape anisotropy, induced by long-range dipolar forces, keeps the magnetization in the plane of the wire almost everywhere, except at vortex defects.

At the edge, the same dipolar forces align magnetization with the edge almost everywhere, except at edge solitons.

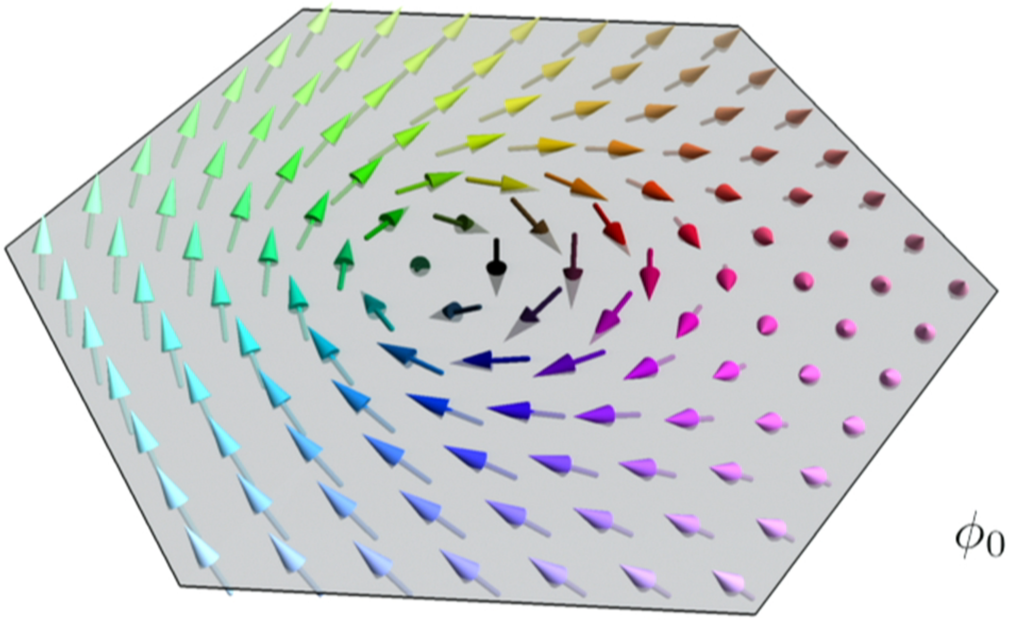
$$q = 1$$



$$\phi_0 = \frac{\pi}{2}$$

Belavin-Polyakov skyrmion: $\cos\theta = \dfrac{r_0 - r}{r_0 + r}, \quad \phi = \alpha + \phi_0.$

$$q = -1$$



$$\phi_0 = \frac{3\pi}{2}$$

BP anti-skyrmion: $\cos\theta = \dfrac{r - r_0}{r_0 + r}, \quad \phi = \alpha + \phi_0.$