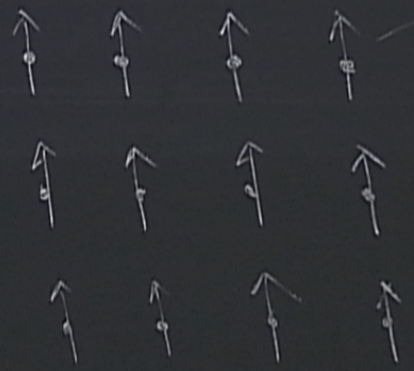
Title: PSI 2015/2016 Condensed Matter - Oleg Tchernyshyov - Lecture 2

Date: Nov 10, 2015 10:45 AM

URL: http://pirsa.org/15110043

Abstract:

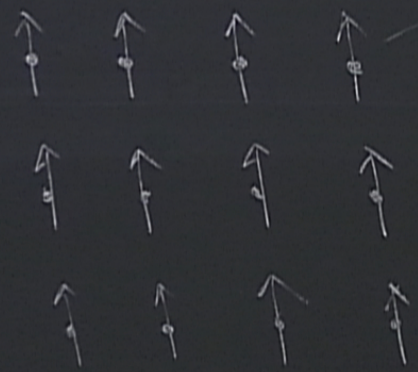
Heisenberg ferromagnet



Magnetic moment $\vec{\mu} = \gamma \vec{J}$

γ - gyromagnetic ration

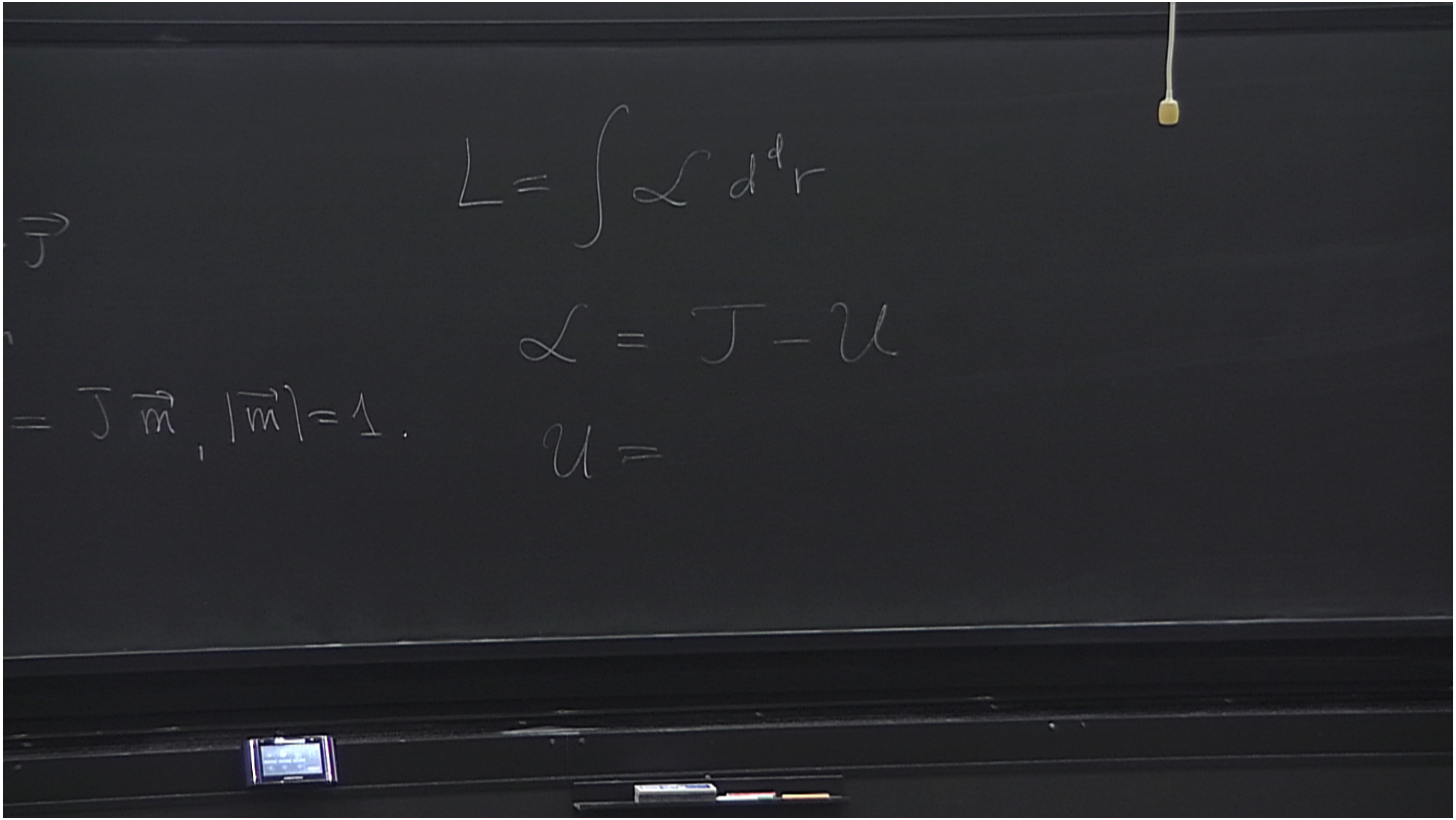
Heisenberg ferromagnet

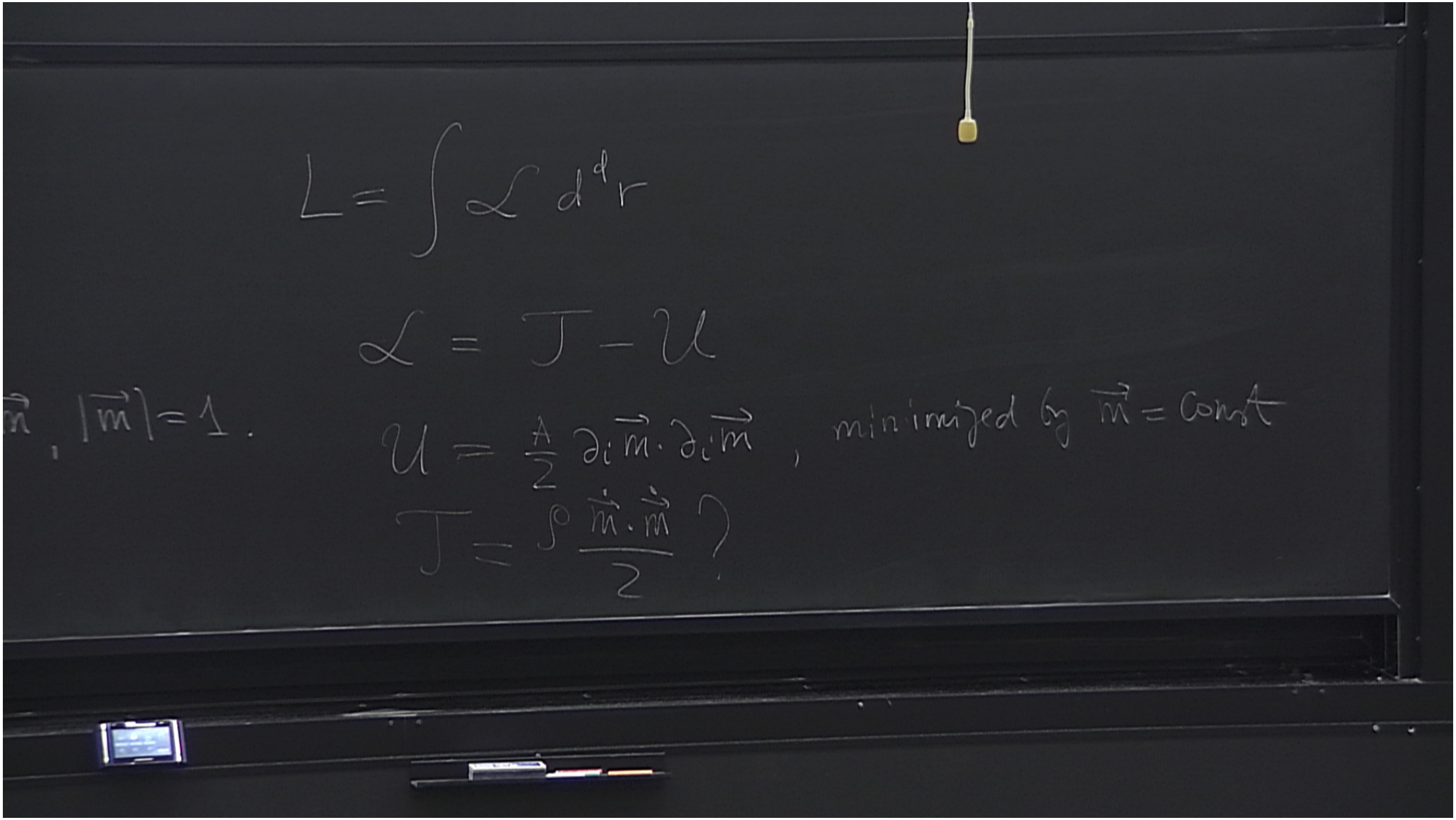


Magnetic moment $\vec{\mu} = \gamma \vec{J}$

γ - gyromagnetic ration

Classcal objects $\vec{J} = J\vec{n}$, $|\vec{n}| = 1$.


$L = \int \mathcal{L}\, d^q r$
$\mathcal{L} = T - U$
$= J\vec{m}, |\vec{m}| = 1.$
$U =$


$L = \int \mathcal{L}\, d^d r$
$\mathcal{L} = T - U$
$\vec{n}\,,\ |\vec{m}| = 1\,.$
$U = \frac{A}{2}\, \partial_i \vec{m} \cdot \partial_i \vec{m}$, minimized by $\vec{m} = \text{const}$
$T = \rho \frac{\dot{\vec{m}} \cdot \dot{\vec{m}}}{2}$?

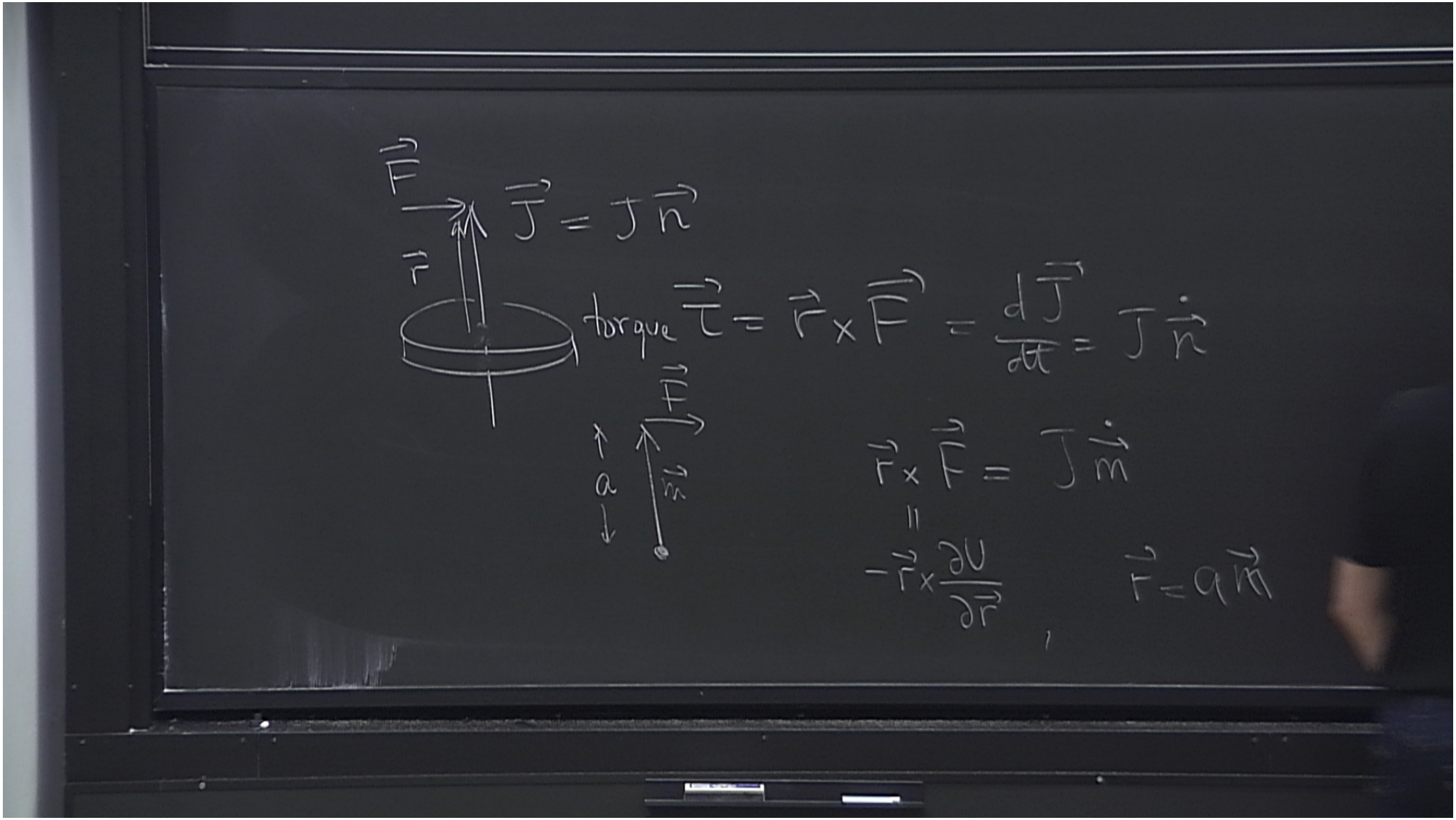
$$L = \int \mathcal{L}\, d^d r$$

$$\mathcal{L} = T - U$$

$\vec{n}$, $|\vec{m}| = 1$.

$$U = \frac{A}{2}\, \partial_i \vec{m} \cdot \partial_i \vec{m}\,, \quad \text{minimized by } \vec{m} = \text{Const}$$

$$T = \frac{\rho}{2}\, \dot{\vec{m}} \cdot \dot{\vec{m}} \; ?$$


$\vec{F}$
$\vec{J} = J\vec{n}$
$\vec{r}$
torque $\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{J}}{dt} = J\dot{\vec{n}}$
$\vec{F}$
a
$\vec{n}$
$\vec{r} \times \vec{F} = J\dot{\vec{n}}$
$= -\vec{r} \times \frac{\partial V}{\partial \vec{r}}$,
$\vec{r} = q\vec{n}$

$$\vec{m} \times \left(J \dot{\vec{m}} = - \vec{m} \times \frac{\partial U}{\partial \vec{m}} \right)$$

$$\vec{m} \times \left(\vec{m} \times \frac{\partial U}{\partial \vec{m}} \right) + J \, \vec{m} \times \vec{m} = 0$$

$$- \frac{\partial U}{\partial \vec{m}} + \vec{m} \left(\vec{m} \cdot \frac{\partial U}{\partial \vec{m}} \right) - J \dot{\vec{m}} \times \vec{m} = 0$$

tangential to sphere

gyroscopic

dissipative

$$-\frac{\partial V}{\partial \vec{m}} - J\dot{\vec{m}} \times \vec{m} - \alpha J \dot{\vec{m}} = 0$$

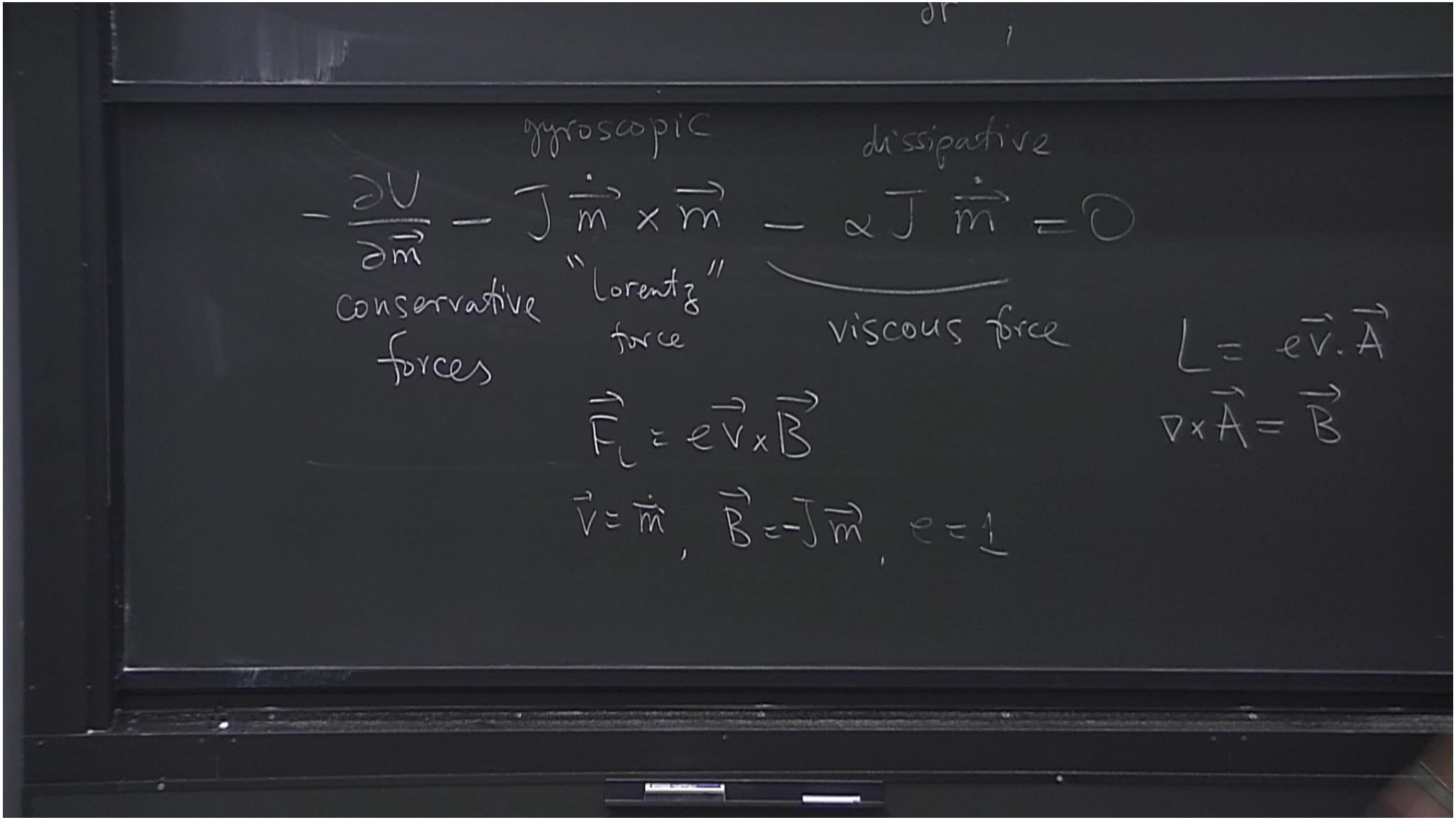
conservative forces

"Lorentz" force

viscous force

$$\vec{F}_L = e\vec{v} \times \vec{B}$$

$$\vec{v} = \dot{\vec{m}}, \quad \vec{B} = -J\vec{m}, \quad e = 1$$

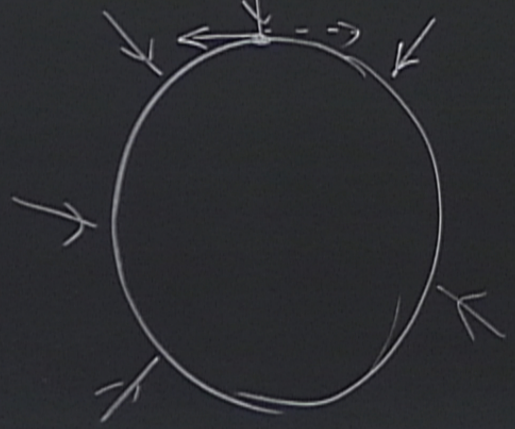

gyroscopic
dissipative
$-\frac{\partial U}{\partial \vec{m}} - J\dot{\vec{m}} \times \vec{m} - \alpha J \dot{\vec{m}} = 0$
conservative forces
"Lorentz" force
viscous force
$L = e\vec{v}.\vec{A}$
$\vec{F}_L = e\vec{v} \times \vec{B}$
$\nabla \times \vec{A} = \vec{B}$
$\vec{v} = \dot{\vec{m}}, \quad \vec{B} = -J\vec{m}, \quad e = 1$

$L = -U + \vec{a}(\vec{m}) \cdot \dot{\vec{m}}$

$\vec{\nabla}_m \times \vec{a} = -J\vec{m}$

$= e\vec{V} \cdot \vec{A}$

$\vec{A} = \vec{B}$



$\vec{F}_{ext} + \vec{F}_L = 0$

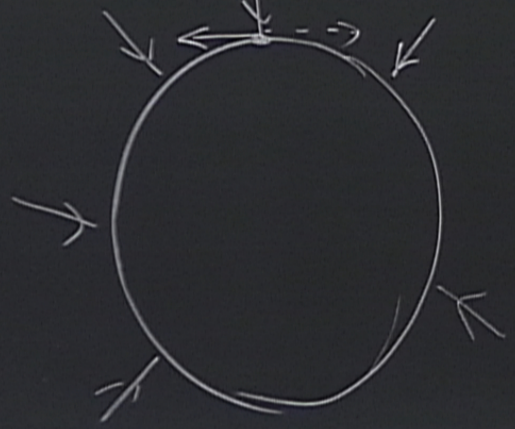
$$L = -U + \vec{a}(\vec{m}) \cdot \dot{\vec{m}}$$

$$\boxed{\vec{\nabla}_m \times \vec{a} = -J\vec{m}}$$

$$= e\vec{v} \cdot \vec{A}$$

$$\vec{A} = \vec{B}$$

$$\vec{F}_{ext} + \vec{F}_L = 0$$



$$L = -U + \vec{a}(\vec{m}) \cdot \dot{\vec{m}}$$

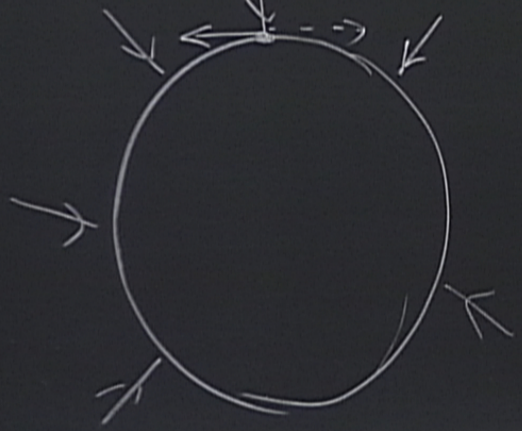
$$\boxed{\vec{\nabla}_m \times \vec{a} = -\vec{J}\vec{m}}$$

$$\vec{B} = (0, 0, B)$$

$$\vec{A} = (-yB, 0, 0)$$

$$(0, xB, 0)$$

$$\vec{F}_{ext} + \vec{F}_L = 0$$



$$L = -U + \vec{a}(\vec{m}) \cdot \dot{\vec{m}}$$

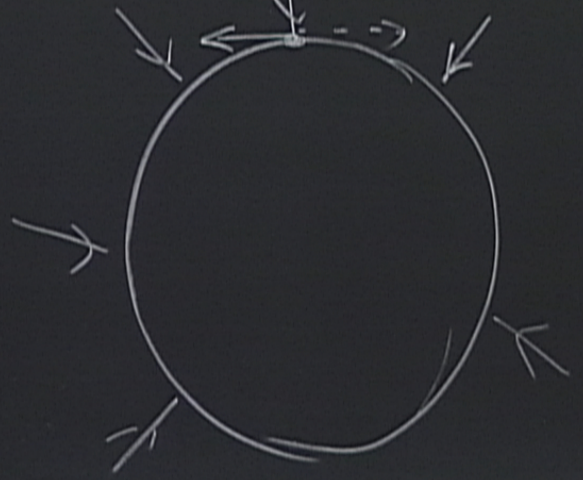
$$\boxed{\vec{\nabla}_m \times \vec{a} = -\mathbb{J}\vec{m}}$$

$$\vec{B} = (0, 0, B)$$

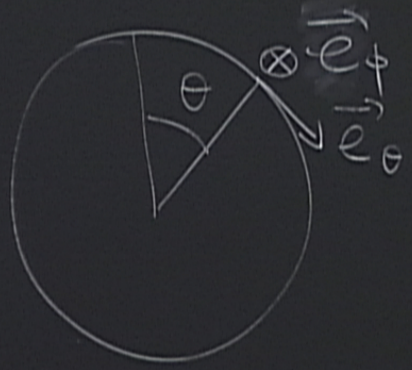
$$\vec{A} = (-yB, 0, 0$$

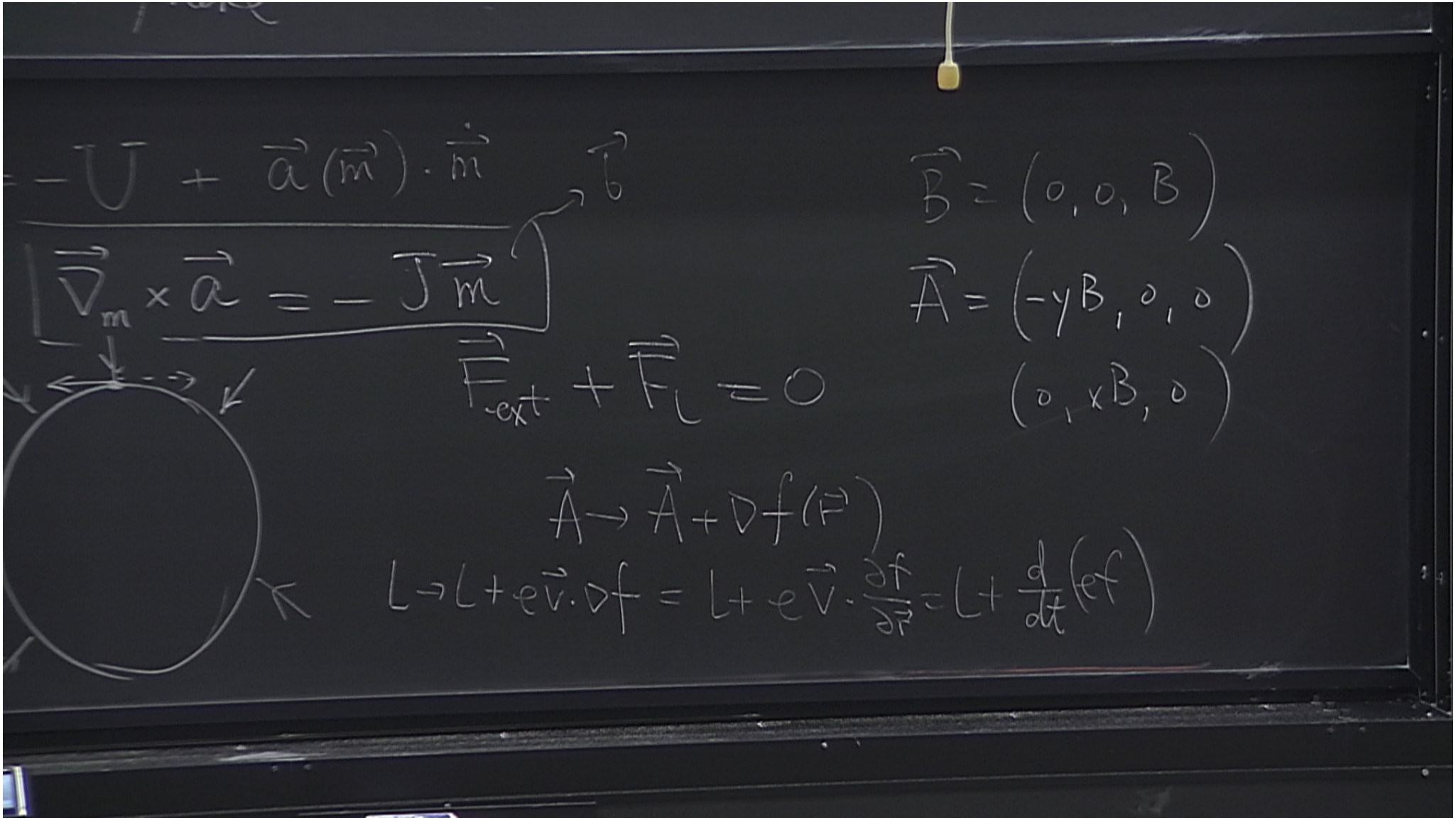
$$(0, xB, 0$$

$$\vec{F}_{ext} + \vec{F}_L = 0$$

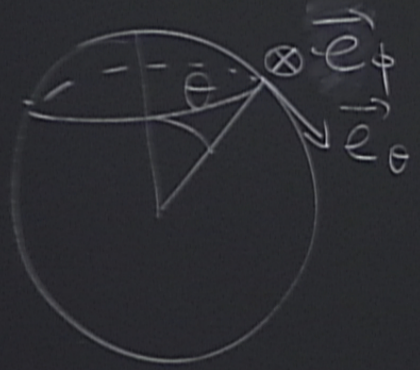


$$\vec{a} = a_\theta \vec{e}_\theta + a_\phi \vec{e}_\phi , \quad a_\theta = 0, \quad a_\phi = \frac{J(\cos\theta - 1)}{\sin\theta}$$




$= -U + \vec{a}(\vec{m}) \cdot \dot{\vec{m}}$
$\vec{\nabla}_m \times \vec{a} = -\vec{J}\vec{m}$
$\vec{b}$
$\vec{B} = (0, 0, B)$
$\vec{A} = (-yB, 0, 0)$
$(0, xB, 0)$
$\vec{F}_{ext} + \vec{F}_L = 0$
$\vec{A} \to \vec{A} + \nabla f(\vec{r})$
$L \to L + e\vec{v} \cdot \nabla f = L + e\vec{v} \cdot \frac{\partial f}{\partial \vec{r}} = L + \frac{d}{dt}(ef)$

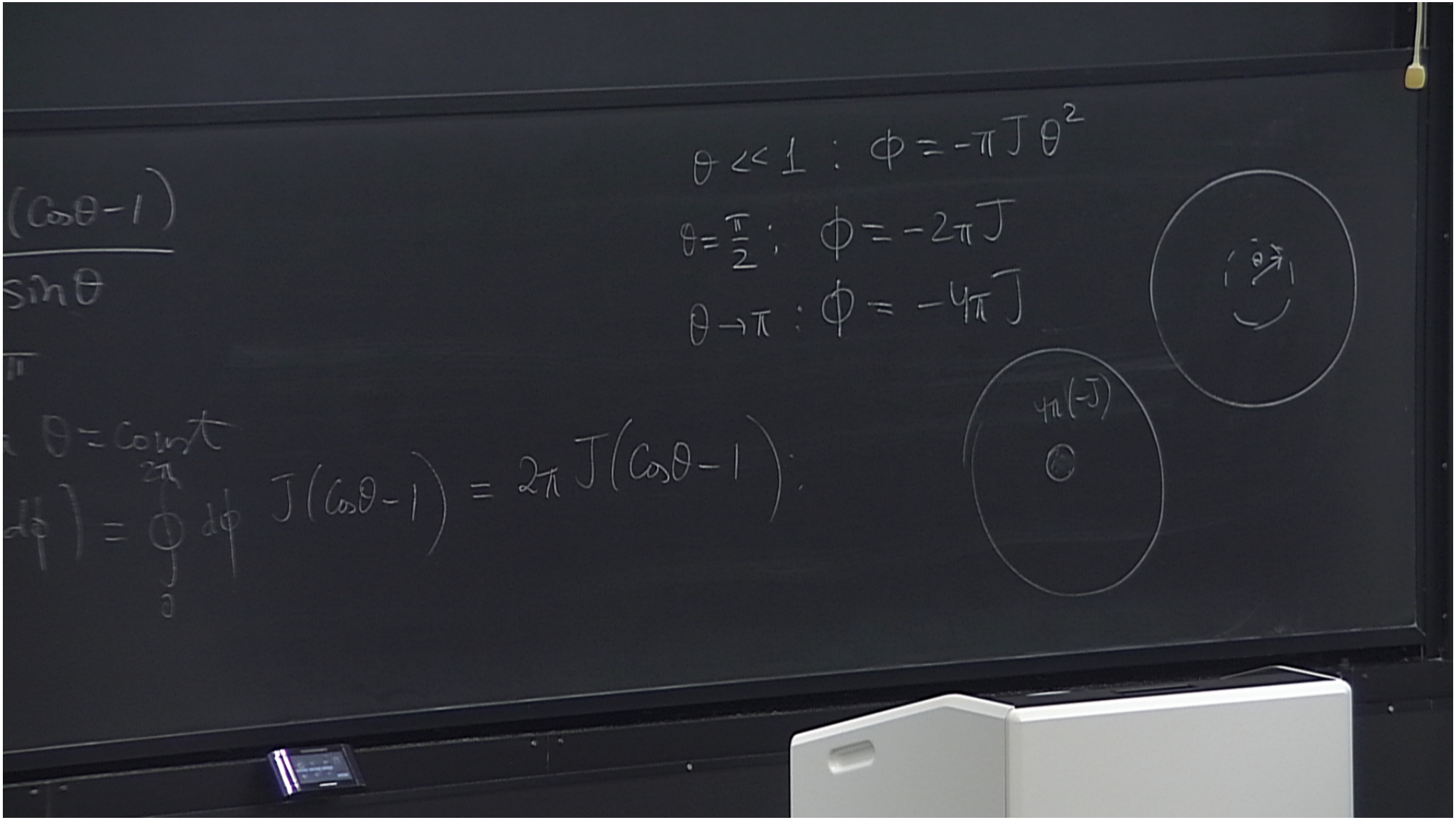
$$\vec{a} = a_\theta \vec{e}_\theta + a_\phi \vec{e}_\phi , \quad a_\theta = 0 , \quad a_\phi = \frac{J(\cos\theta - 1)}{\sin\theta}$$



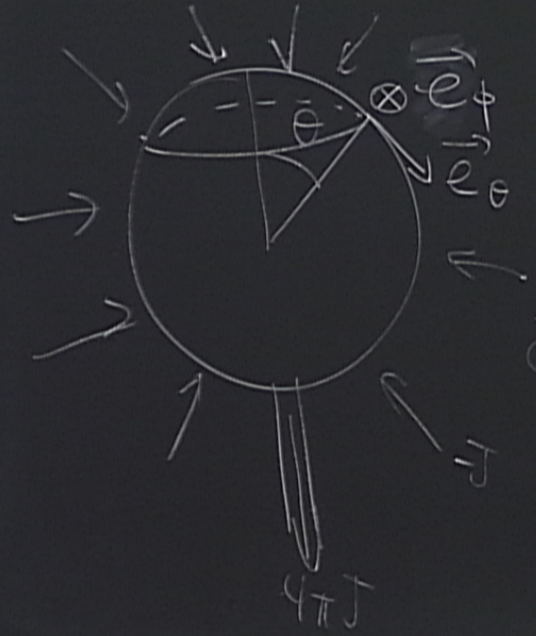
Divergence at $\theta = \pi$

Magnetic flux through a $\theta = \text{const}$

$$\Phi = \oint \vec{a} \cdot d\vec{m} = \oint \left(a_\theta \underbrace{d\theta}_{0} + a_\phi \sin\theta \, d\phi \right) = \int_0^{2\pi} d\phi \; J(\cos\theta - 1) =$$


$\theta \ll 1 : \Phi = -\pi J \theta^2$
$\theta = \frac{\pi}{2} ; \Phi = -2\pi J$
$\theta \to \pi : \Phi = -4\pi J$
$\theta \to 0$
$4\pi(-J)$
$\frac{(\cos\theta - 1)}{\sin\theta}$
π
$\theta = \text{Const}$
2π
$\int_0 d\phi \, J(\cos\theta - 1) = 2\pi J(\cos\theta - 1)$

$$\vec{a} = a_\theta \vec{e}_\theta + a_\phi \vec{e}_\phi , \quad a_\theta = 0, \quad a_\phi = \frac{J(\cos\theta - 1)}{\sin\theta}$$



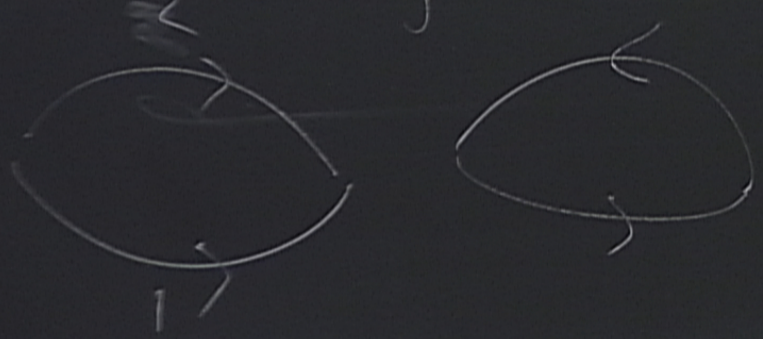
Divergence at $\theta = \pi$

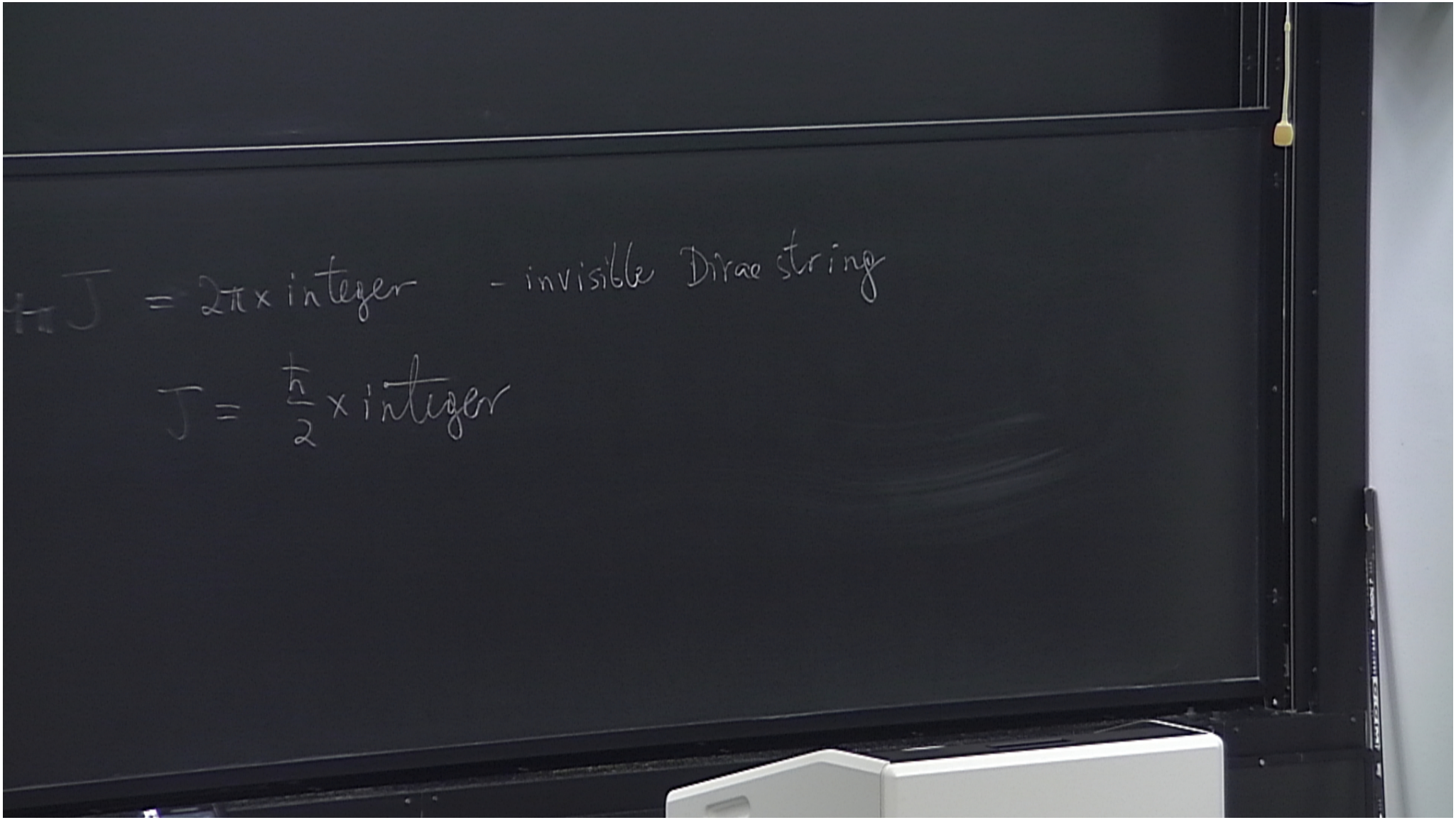
Magnetic flux through a $\theta = \text{const}$

$$\Phi = \oint \vec{a} \cdot d\vec{m} = \int_0^{2\pi} \left(a_\theta \cancel{d\theta}^{0} + a_\phi \sin\theta \, d\phi \right) = \int_0^{2\pi} d\phi$$

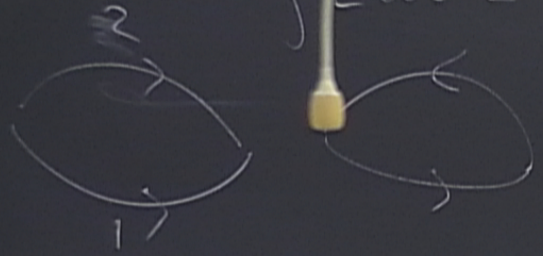
$$\Delta\phi = \frac{\Delta S}{\hbar} = \frac{1}{\hbar}\oint \vec{a}\cdot d\vec{m} = \frac{1}{\hbar}\Phi = \frac{1}{\hbar}\,4\pi J$$

$$S = \int L\,dt = \int \vec{a}\cdot\dot{\vec{m}}\,dt = \int \vec{a}\cdot d\vec{m}$$




$\oint \vec{J} = 2\pi \times \text{integer}$ - invisible Dirac string
$J = \frac{\hbar}{2} \times \text{integer}$

$S = \int L\,dt = \int \vec{a}\cdot\vec{m}\,dt = \int a\cdot d\vec{m}$



Lagrangian of a ferromagnet with magnetization field $\vec{m}(\vec{r})$ - Non-linear σ-model.

$$L = \int \mathcal{L}\, d^d r, \quad \mathcal{L} = -\frac{A}{2}(\partial_i \vec{m})\cdot(\partial_i \vec{m}) + \vec{a}(\vec{m})\cdot\partial_t \vec{m}, \quad \nabla_{\vec{m}} \times \vec{a}(\vec{m}) = -J\vec{m}, \quad J = \frac{J}{a^d}$$

$= \int \vec{a} \cdot \vec{m}\, dt = \int a \cdot d\vec{m}$

ferromagnet with magnetization field $\vec{m}(\vec{r})$ - Non-linear σ-model.

$|\vec{m}| = 1$.

$$\mathcal{L} = -\frac{A}{2}(\partial_i \vec{m}) \cdot (\partial_i \vec{m}) + \vec{a}(\vec{m}) \cdot \partial_t \vec{m}, \qquad \nabla_{\vec{m}} \times \vec{a}(\vec{m}) = -\mathcal{J}\vec{m}, \quad \mathcal{J} = \frac{J}{a^d}$$

- density of angular momentum

Lagrangian of a ferromagnet with magnetization field $\vec{m}(\vec{r})$

$$|\vec{m}| = 1.$$

$$L = \int \mathcal{L}\, d^d r, \quad \mathcal{L} = -\frac{A}{2}(\partial_i \vec{m})\cdot(\partial_i \vec{m}) + \vec{a}(\vec{m})\cdot\partial_t \vec{m}, \quad \nabla_m \times \vec{a}(\vec{m})$$

Microscopic basis $H = -K \sum_{\langle mn \rangle} \vec{S}_n \cdot \vec{S}_m$ (Heisenberg)

$$-\vec{S}_m \cdot \vec{S}_n = \frac{(\vec{S}_m - \vec{S}_n)^2}{2} - S_m^2 - S_n^2$$

$$\vec{n} = (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$

$$\mathcal{L} = \int(\cos\theta - 1)\dot{\phi} - \frac{A}{2}\left[(\nabla\theta)^2 + \sin^2\theta\,(\nabla\phi)^2\right]$$

Euler-Lagrange eqns:

$$\frac{\partial}{\partial t}\left[\int(\cos\theta - 1)\right] - \nabla\left[A\sin^2\theta\,\nabla\phi\right] = 0$$

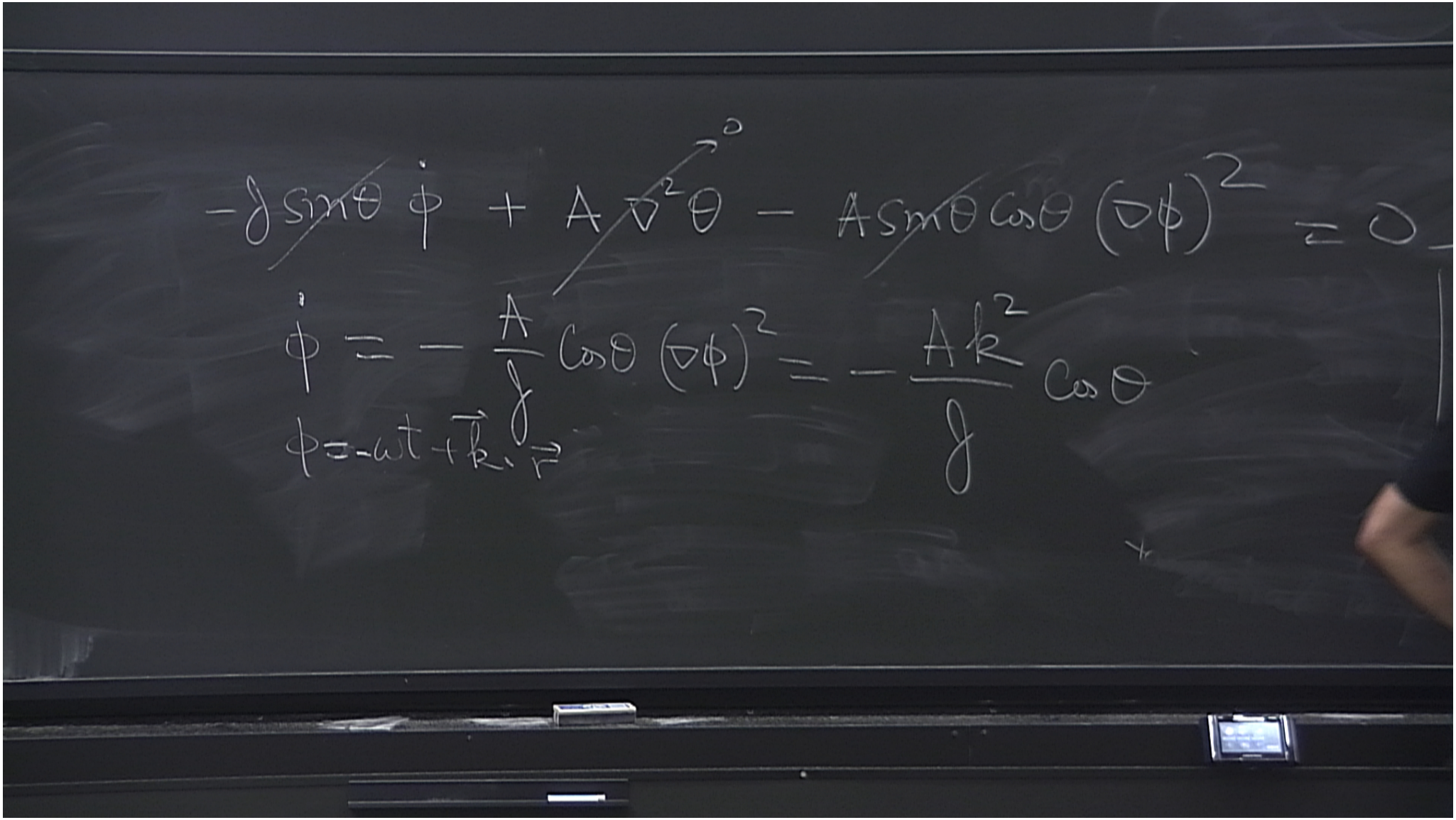
$$= 0$$

$\theta = \text{Const}$, $\phi = \phi(t) + \vec{k} \cdot \vec{r}$

From North Pole:

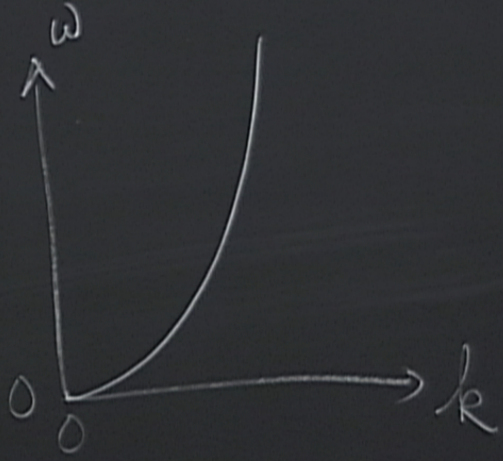


$\phi = \phi(t) + \vec{k} \cdot \vec{r}$


$-\gamma \sin\theta\, \dot{\phi} + A\nabla^2\theta - A\sin\theta\cos\theta\,(\nabla\phi)^2 = 0$
$\dot{\phi} = -\frac{A}{\gamma}\cos\theta\,(\nabla\phi)^2 = -\frac{Ak^2}{\gamma}\cos\theta$
$\phi = -\omega t + \vec{k}\cdot\vec{r}$

$= 0$

$$\omega = \frac{A \cos\theta}{f} k^2$$

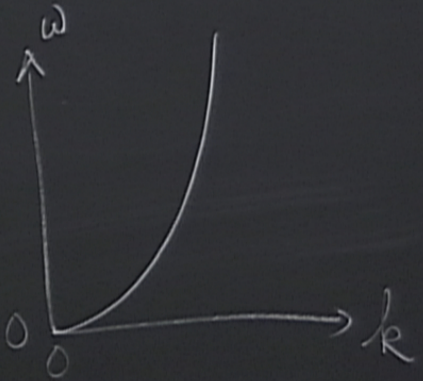


$\phi = \phi(t) + \vec{k}\cdot\vec{r}$

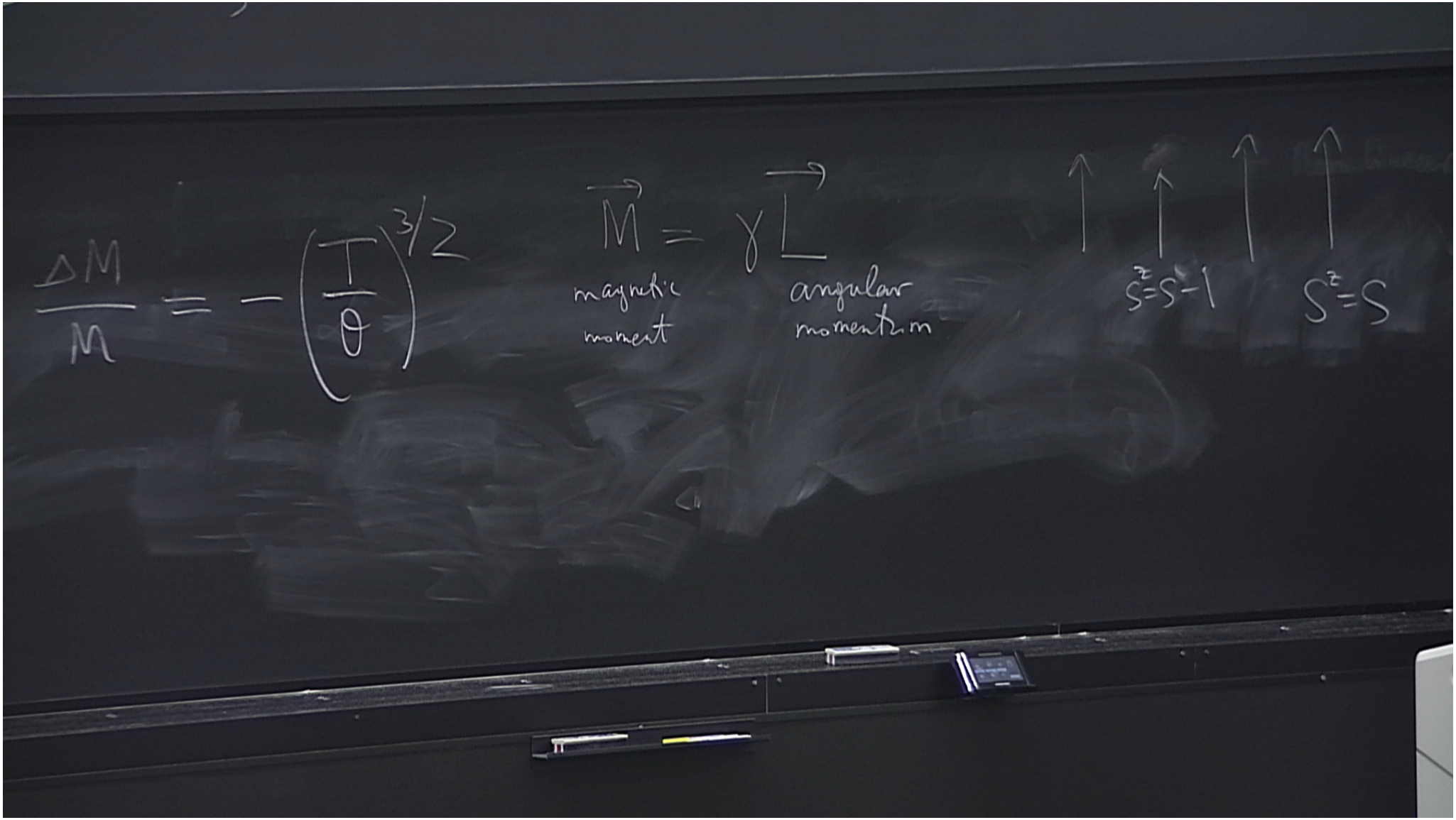
$\theta \cos\theta \, (\nabla\phi)^2 = 0$

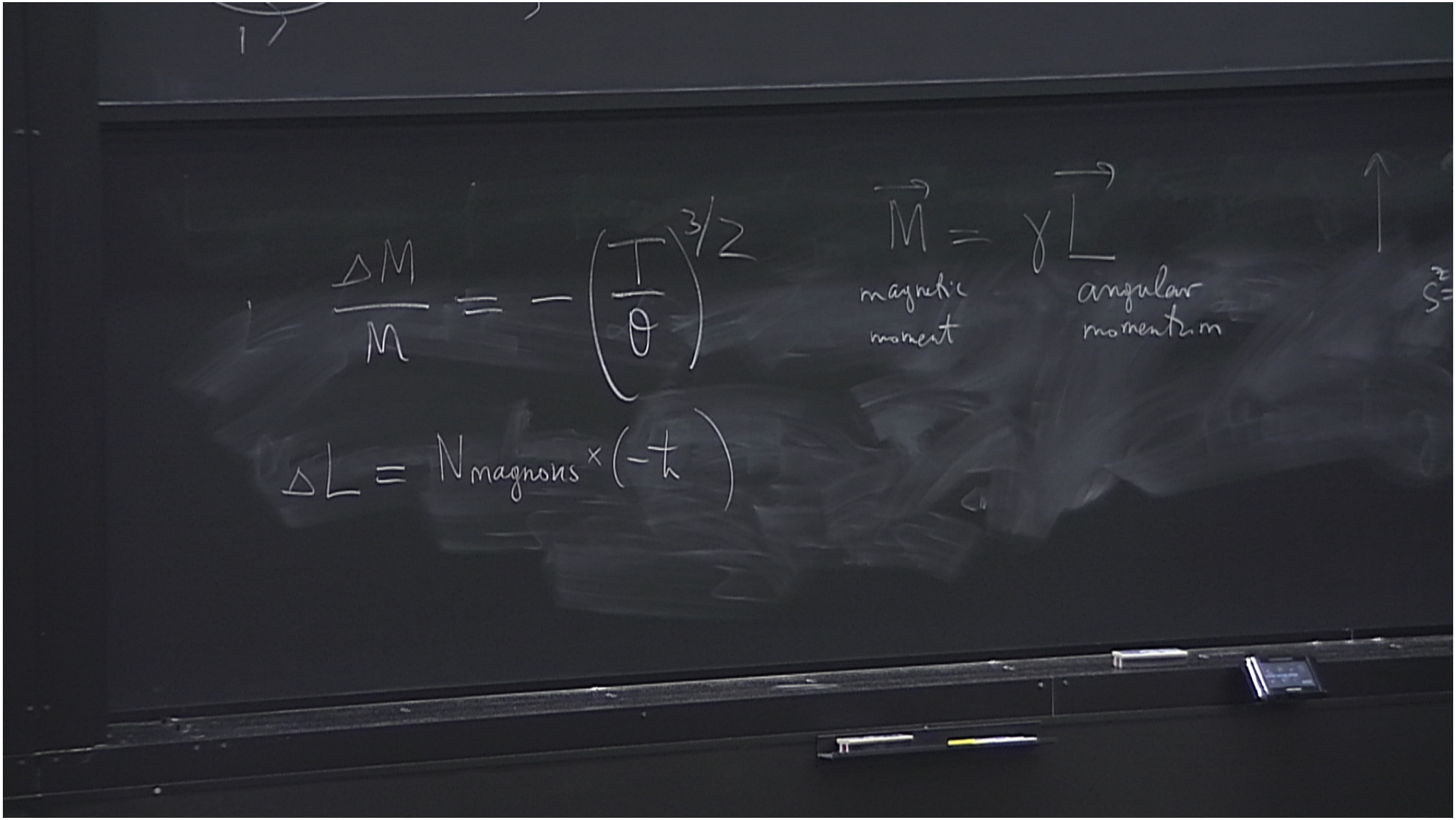
$\omega = \frac{A\cos\theta}{J} k^2$

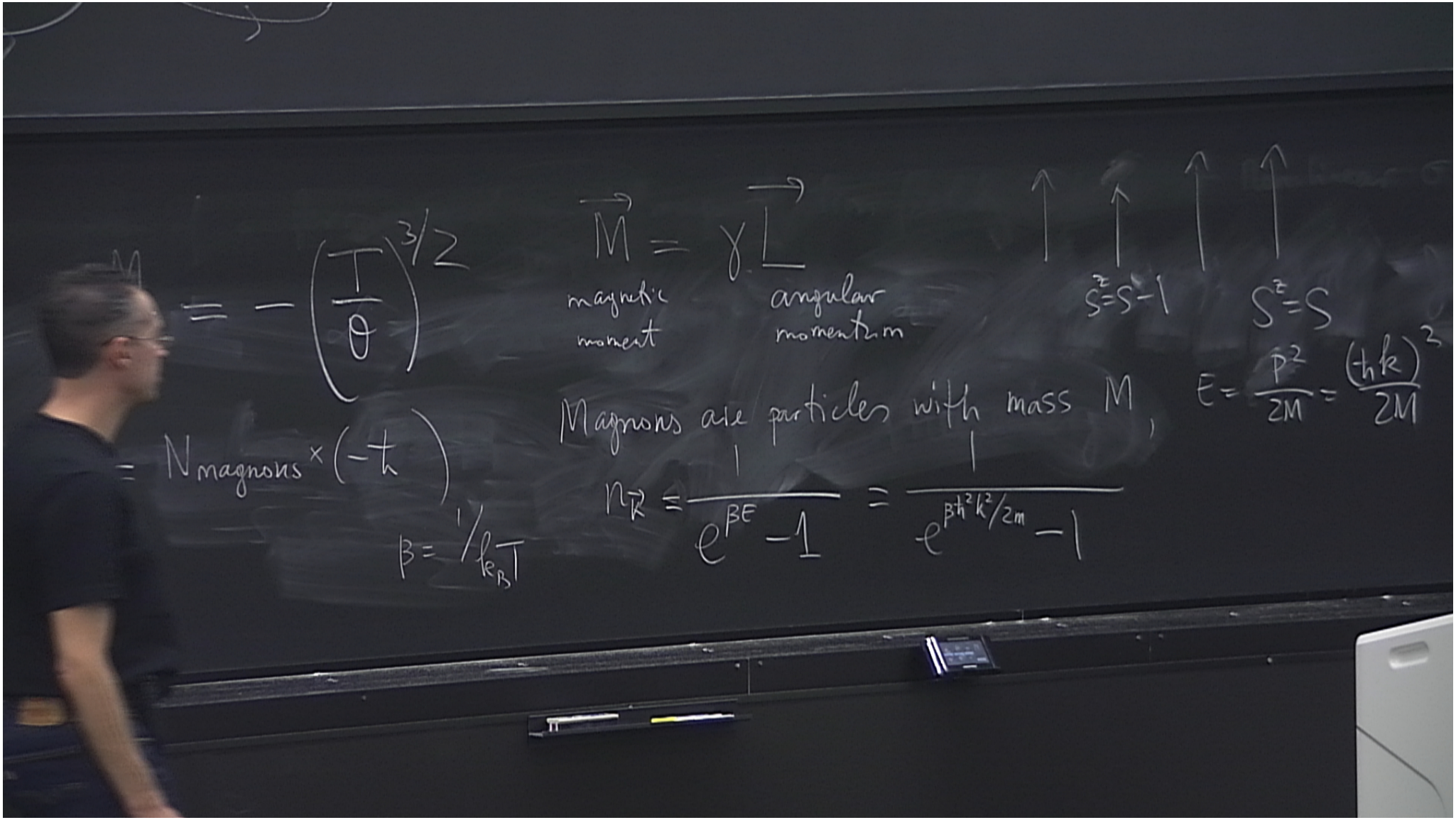
$\frac{k^2}{J}\cos\theta$

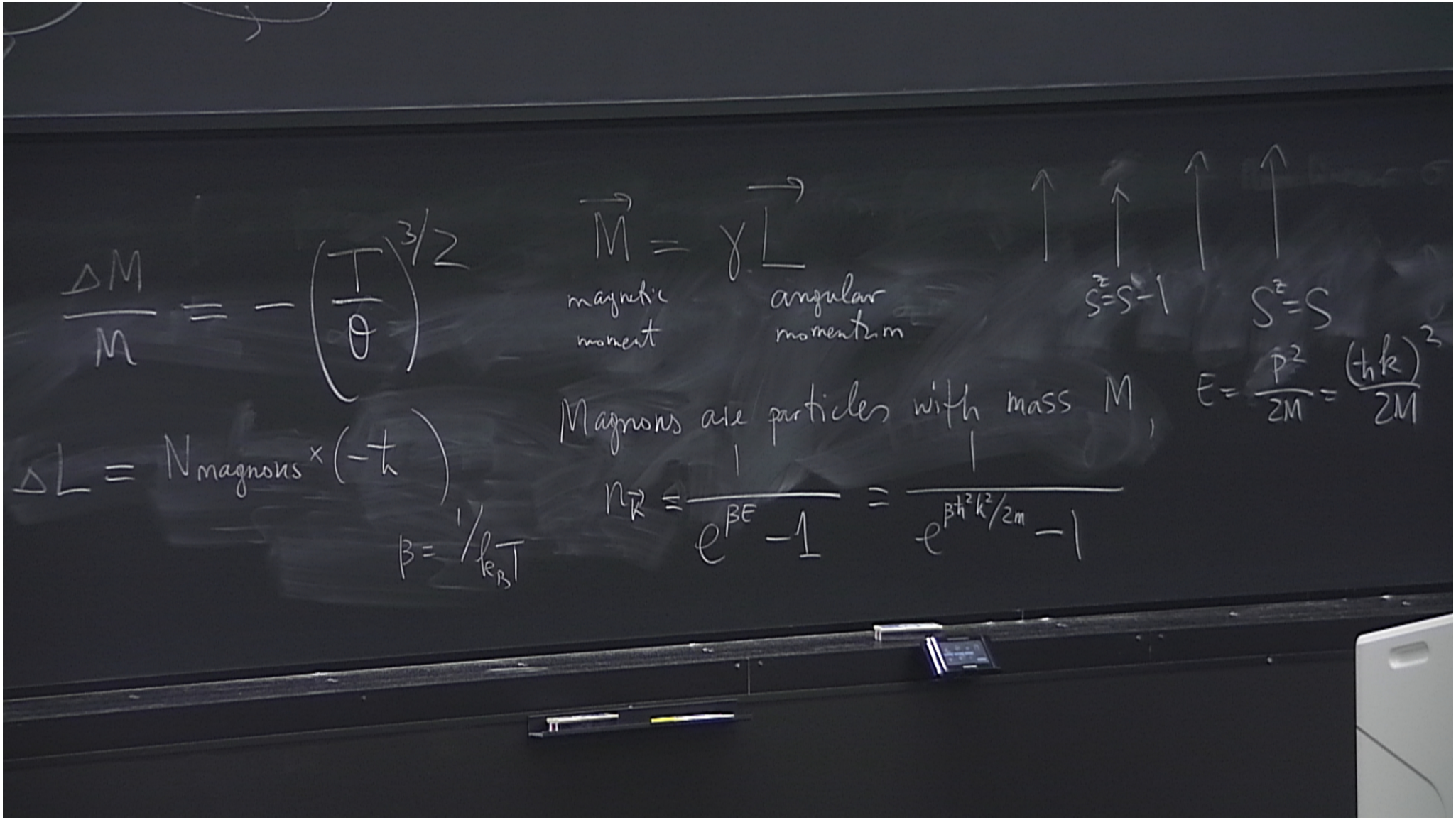


Magnon $E = \hbar\omega = \frac{A}{J}\hbar k^2 = \frac{(\hbar k)^2}{2M}$ $E = \frac{p^2}{2M}$


$\frac{\Delta M}{M} = -\left(\frac{T}{\theta}\right)^{3/2}$
$\vec{M} = \gamma \vec{L}$
magnetic moment
angular momentum
$S^z = S - 1$
$S^z = S$


$\frac{\Delta M}{M} = -\left(\frac{T}{\theta}\right)^{3/2}$
$\vec{M} = \gamma \vec{L}$
magnetic moment
angular momentum
$\Delta L = N_{magnons} \times (-\hbar$


$= -\left(\frac{T}{\theta}\right)^{3/2}$
$\vec{M} = \gamma \vec{L}$
magnetic moment
angular momentum
$S^z = S - 1$
$S^z = S$
$= N_{magnons} \times (-t$
Magnons are particles with mass M,
$E = \frac{p^2}{2M} = \frac{(\hbar k)^2}{2M}$
$\beta = 1/k_B T$
$n_{\vec{k}} = \frac{1}{e^{\beta E} - 1} = \frac{1}{e^{\beta \hbar^2 k^2 / 2m} - 1}$

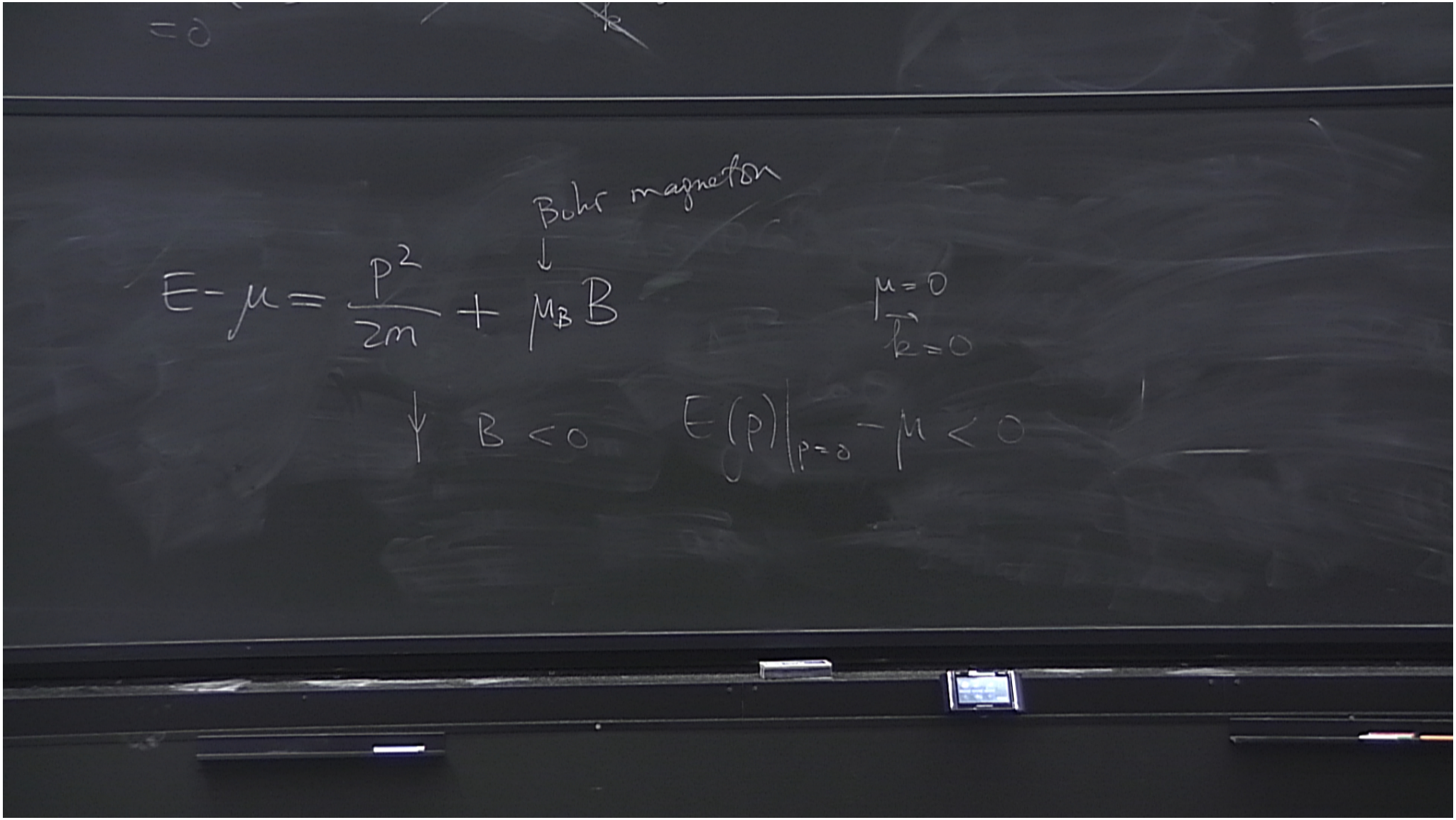

$\frac{\Delta M}{M} = -\left(\frac{T}{\theta}\right)^{3/2}$
$\vec{M} = \gamma \vec{L}$
magnetic moment
angular momentum
$S^z = S - 1$
$S^z = S$
$\Delta L = N_{magnons} \times (-\hbar)$
Magnons are particles with mass M, $E = \frac{p^2}{2M} = \frac{(\hbar k)^2}{2M}$
$\beta = 1/k_B T$
$n_{\vec{k}} = \frac{1}{e^{\beta E} - 1} = \frac{1}{e^{\beta \hbar^2 k^2 / 2m} - 1}$

$= 0$

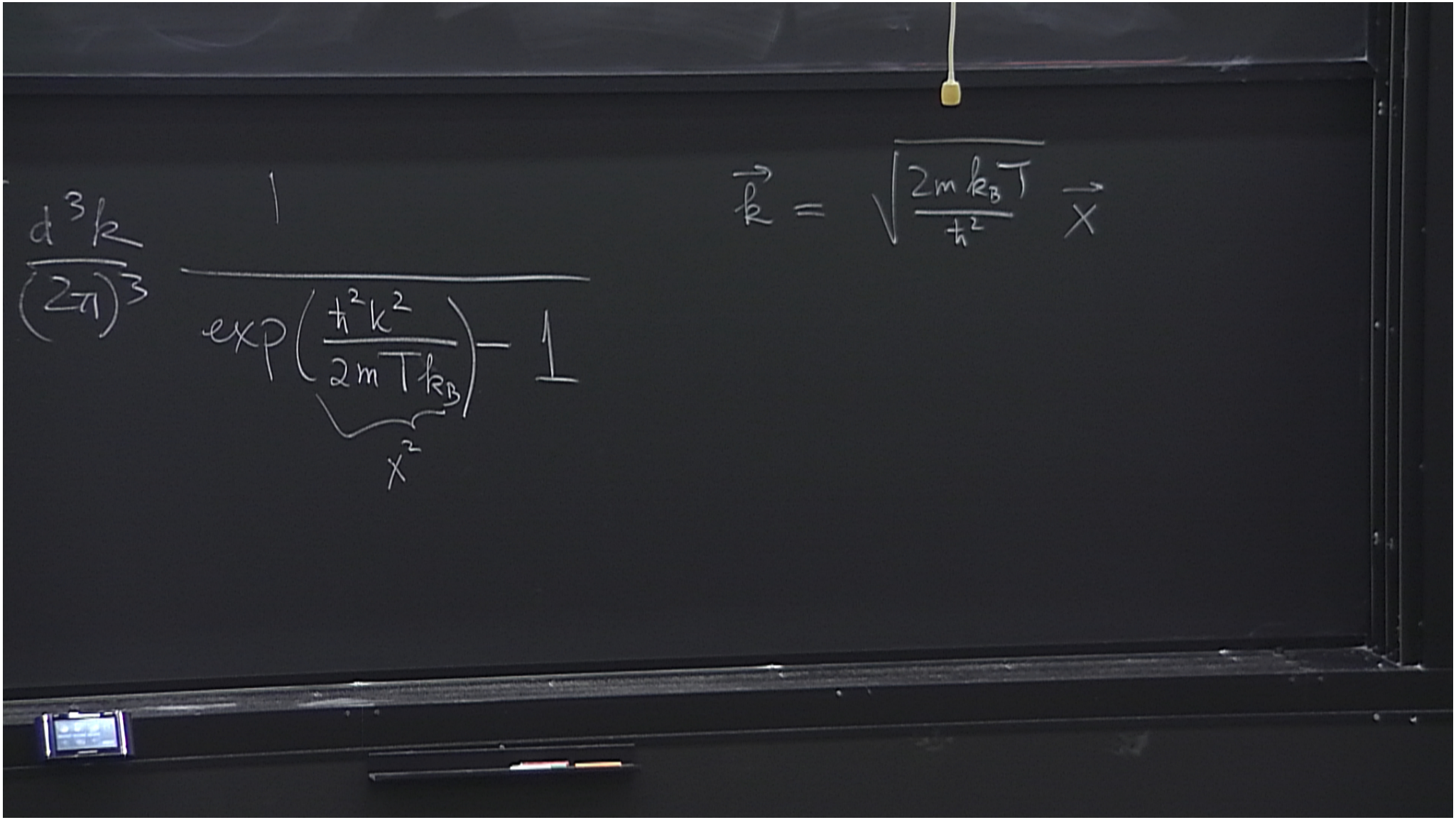
Bohr magneton

$$E - \mu = \frac{p^2}{2m} + \mu_B B$$

$B < 0$ $\quad E(p)\big|_{p=0} - \mu < 0$


Bohr magneton
$E - \mu = \frac{p^2}{2m} + \mu_B B$
$\mu = 0$
$\vec{k} = 0$
$B < 0$
$E(p)\big|_{p=0} - \mu < 0$

$$N_{magnons} = \sum_{\vec{k}} n_{\vec{k}} = V \int \frac{d^3k}{(2\pi)^3} n_{\vec{k}} = V \int \frac{d^3k}{(2\pi)^3} \frac{1}{\exp\left(\frac{\hbar^2 k^2}{2m T k_B}\right) - 1}$$

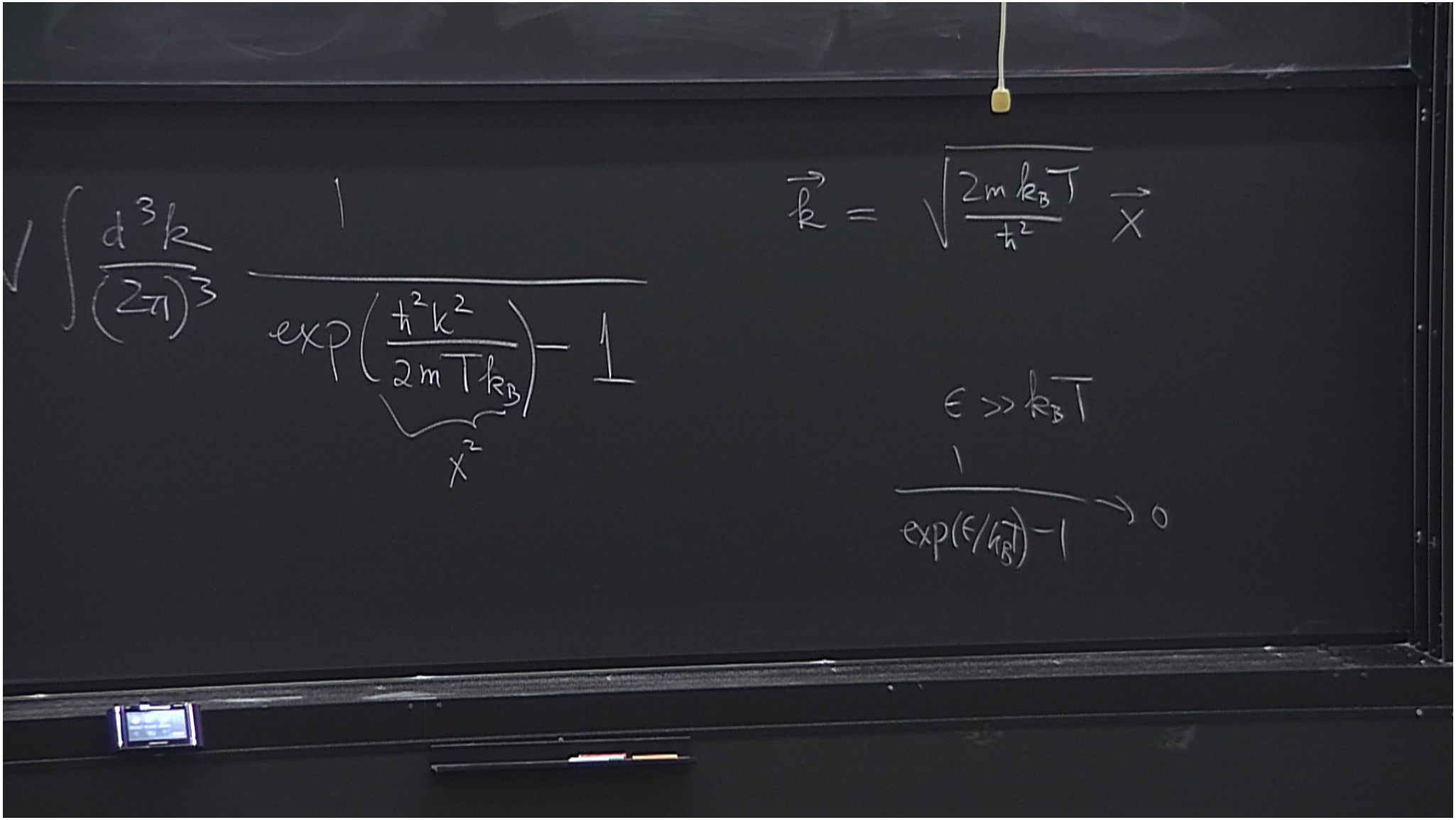

$\frac{d^3k}{(2\pi)^3} \frac{1}{\exp\left(\frac{\hbar^2 k^2}{2mTk_B}\right) - 1}$
x^2
$\vec{k} = \sqrt{\frac{2mk_BT}{\hbar^2}}\,\vec{x}$

$$N_{magnons} = \sum_{\vec{k}} n_{\vec{k}} = V \int \frac{d^3k}{(2\pi)^3} n_{\vec{k}} = V \int \frac{d^3k}{(2\pi)^3} \frac{1}{\exp\left(\underbrace{\frac{\hbar^2 k^2}{2mTk_B}}_{x^2}\right) - 1}$$

$$-\frac{\pi}{a} < k_x < \frac{\pi}{a}$$

$$< k_y <$$

$$k_z$$


$V \int \frac{d^3k}{(2\pi)^3} \frac{1}{\exp\left(\frac{\hbar^2 k^2}{2mTk_B}\right) - 1}$
x^2
$\vec{k} = \sqrt{\frac{2mk_BT}{\hbar^2}}\,\vec{x}$
$\epsilon \gg k_BT$
$\frac{1}{\exp(\epsilon/k_BT) - 1} \to 0$

$$\vec{k} = V \int \frac{d^3k}{(2\pi)^3} \qquad n_{\vec{k}} = V \int \frac{d^3k}{(2\pi)^3} \frac{1}{\exp\left(\frac{\hbar^2 k^2}{2mTk_B}\right) - 1}$$

$$-\frac{\pi}{a} < k_x < \frac{\pi}{a}$$

$$< k_y <$$

$$k_z$$

$$d^3x \; T^{3/2} \qquad x^2$$

$$= V T^{3/2} \int \frac{d^3x}{(2\pi)^3} \frac{1}{e^x - 1}$$

$$N_{magnons} = \sum_{\vec{k}} n_{\vec{k}} = V \int \frac{d^3k}{(2\pi)^3} n_{\vec{k}} =$$

Ground: ↑ ↑ ↑ ↑ ↑ ↑

↑ ↑ ↑ ↓ ↑ ↑

1 2 3 4 5 6

$|4\rangle$

$-\frac{\pi}{a} < k_x < \frac{\pi}{a}$

$< k_y <$

k_z

$= VT^{3/2}$

$$N_{magnons} = \sum_{\vec{k}} n_{\vec{k}} = V \int \frac{d^3k}{(2\pi)^3}$$

Ground: ↑ ↑ ↑ ↑ ↑ ↑

$-\frac{\pi}{a} < k_x < \frac{\pi}{a}$

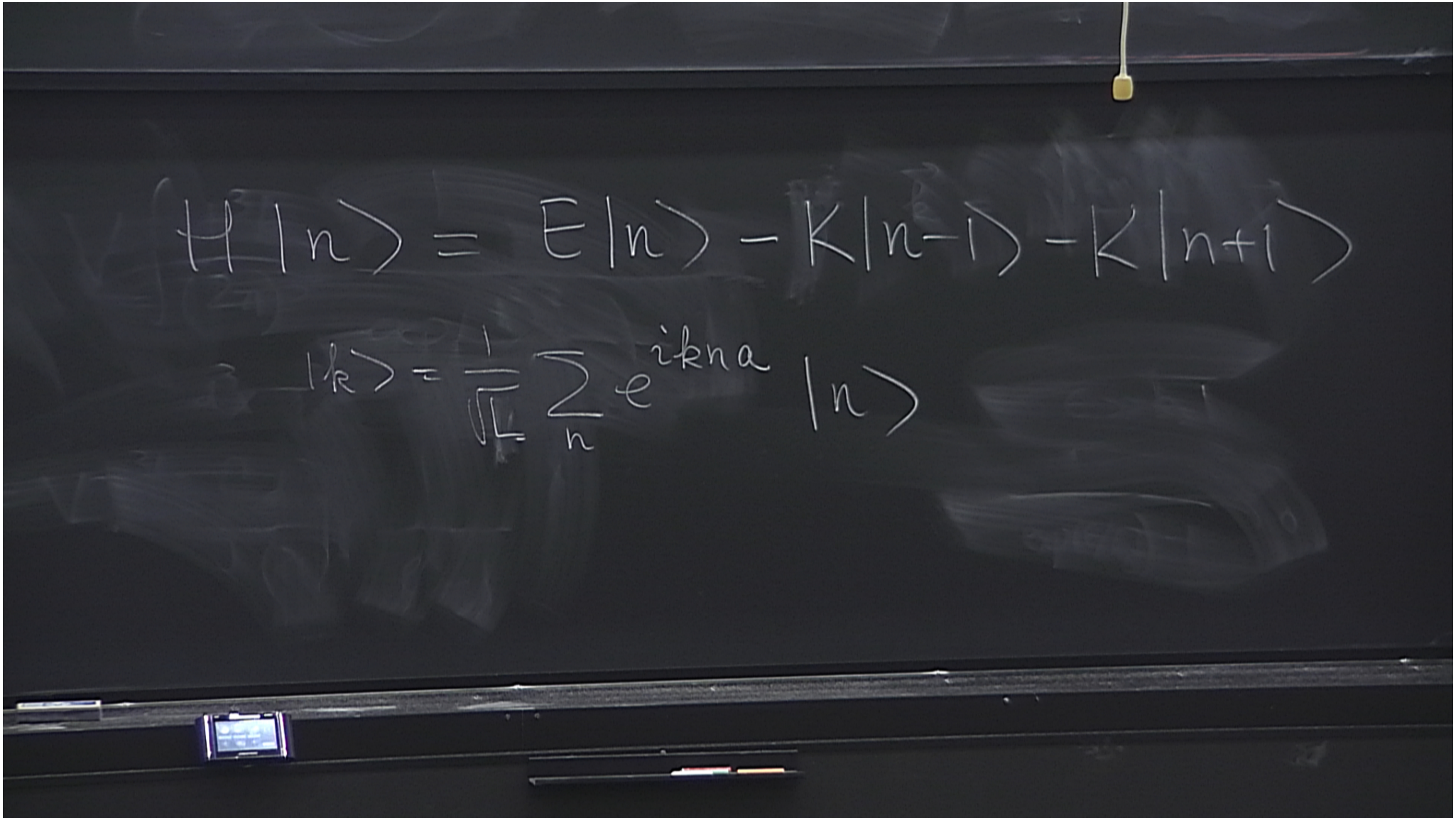
$< k_y <$

k_z

↑ ↑ ↑ ↓ ↑ ↑

1 2 3 4 5 6

$|4\rangle$


H|n⟩ = E|n⟩ − K|n−1⟩ − K|n+1⟩
|k⟩ = 1/√L Σ_n e^{ikna} |n⟩

$$N_{magnons} = \sum_{\vec{k}} n_{\vec{k}} = V \int \frac{d^3k}{(2\pi)^3}$$

Ground: ↑ ↑ ↑ ↑ ↑ ↑

$S = N/2, \quad S^z = N/2$

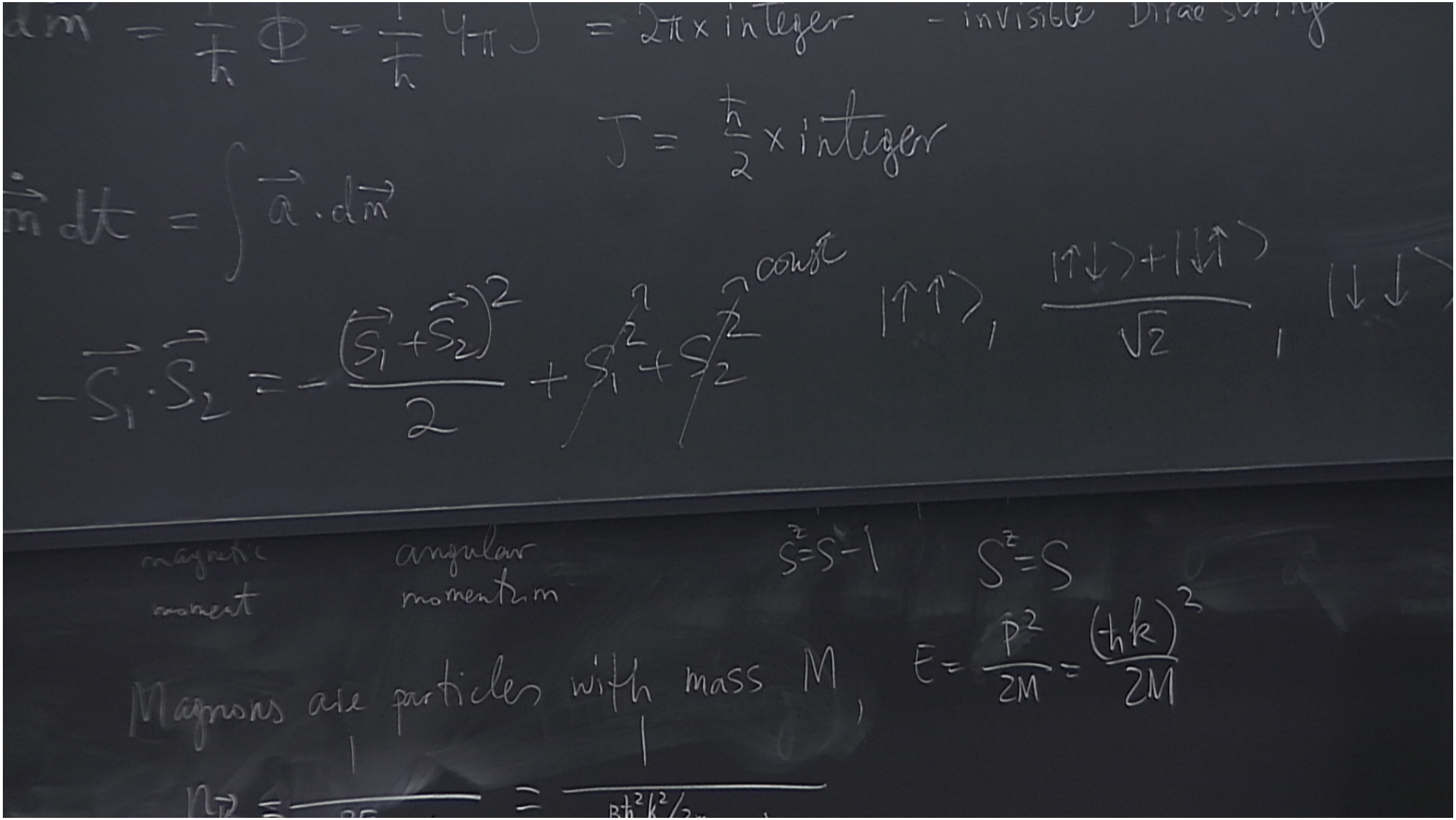
$S = N/2, \quad S^z = \frac{N}{2} - 1$

$-\frac{\pi}{a} < k_x < \frac{\pi}{a}$

$< k_y <$

k_z

$k = 0$


$\frac{1}{\hbar}\Phi = \frac{1}{\hbar}4\pi J = 2\pi \times$ integer
- invisible Dirac string
$J = \frac{\hbar}{2} \times$ integer
$\int \vec{m}\,dt = \int \vec{a}\cdot d\vec{m}$
$-\vec{S}_1\cdot\vec{S}_2 = -\frac{(\vec{S}_1+\vec{S}_2)^2}{2} + S_1^2 + S_2^2$
$\vec{S}^2$ const
$|\uparrow\uparrow\rangle$, $\frac{|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle}{\sqrt{2}}$, $|\downarrow\downarrow\rangle$
magnetic moment
angular momentum
$S^z = S-1$
$S^z = S$
Magnons are particles with mass M,
$E = \frac{p^2}{2M} = \frac{(\hbar k)^2}{2M}$