Title: PSI 2015/2016 Quantum Field Theory II - Francois David - Lecture 12

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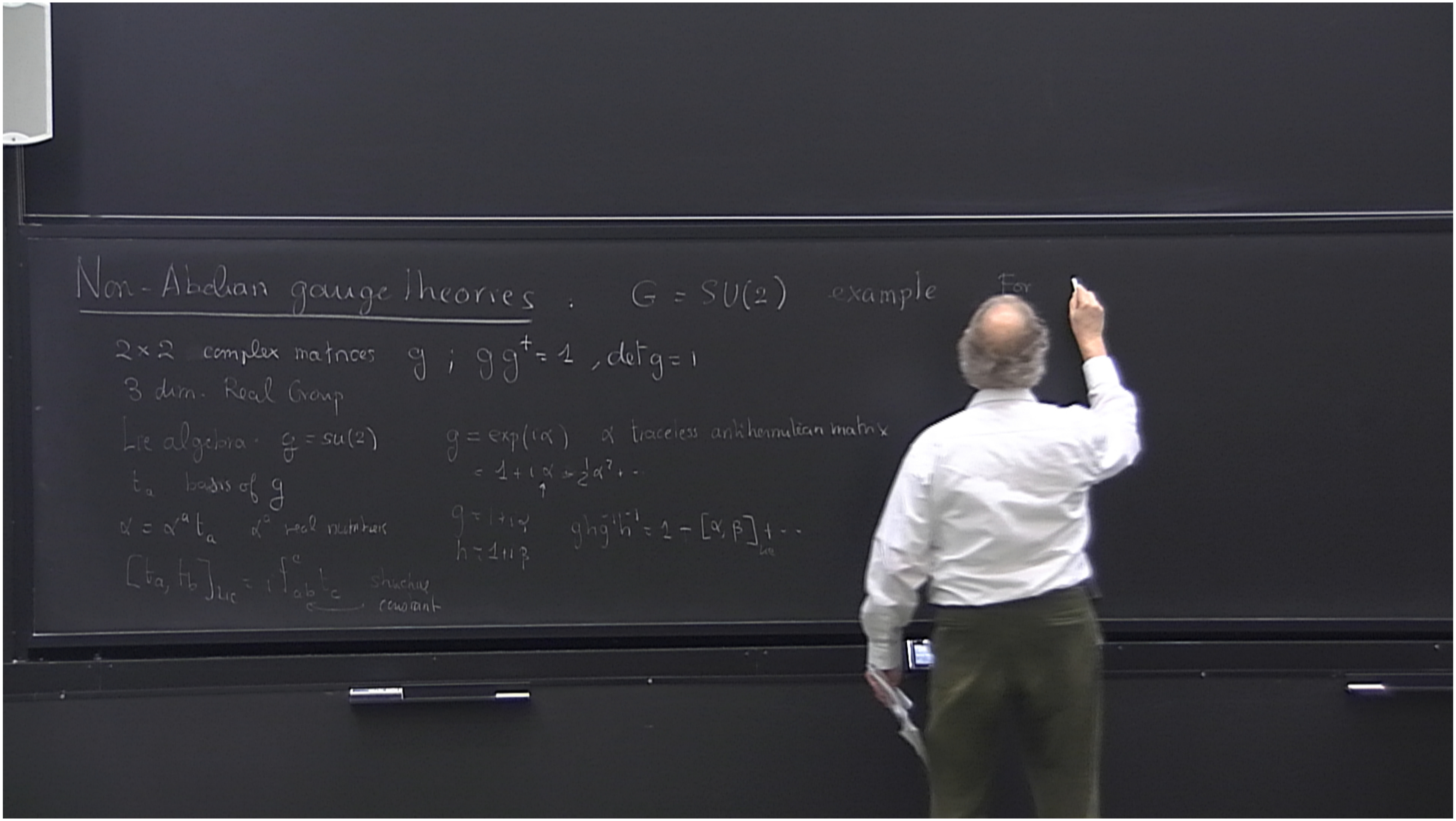
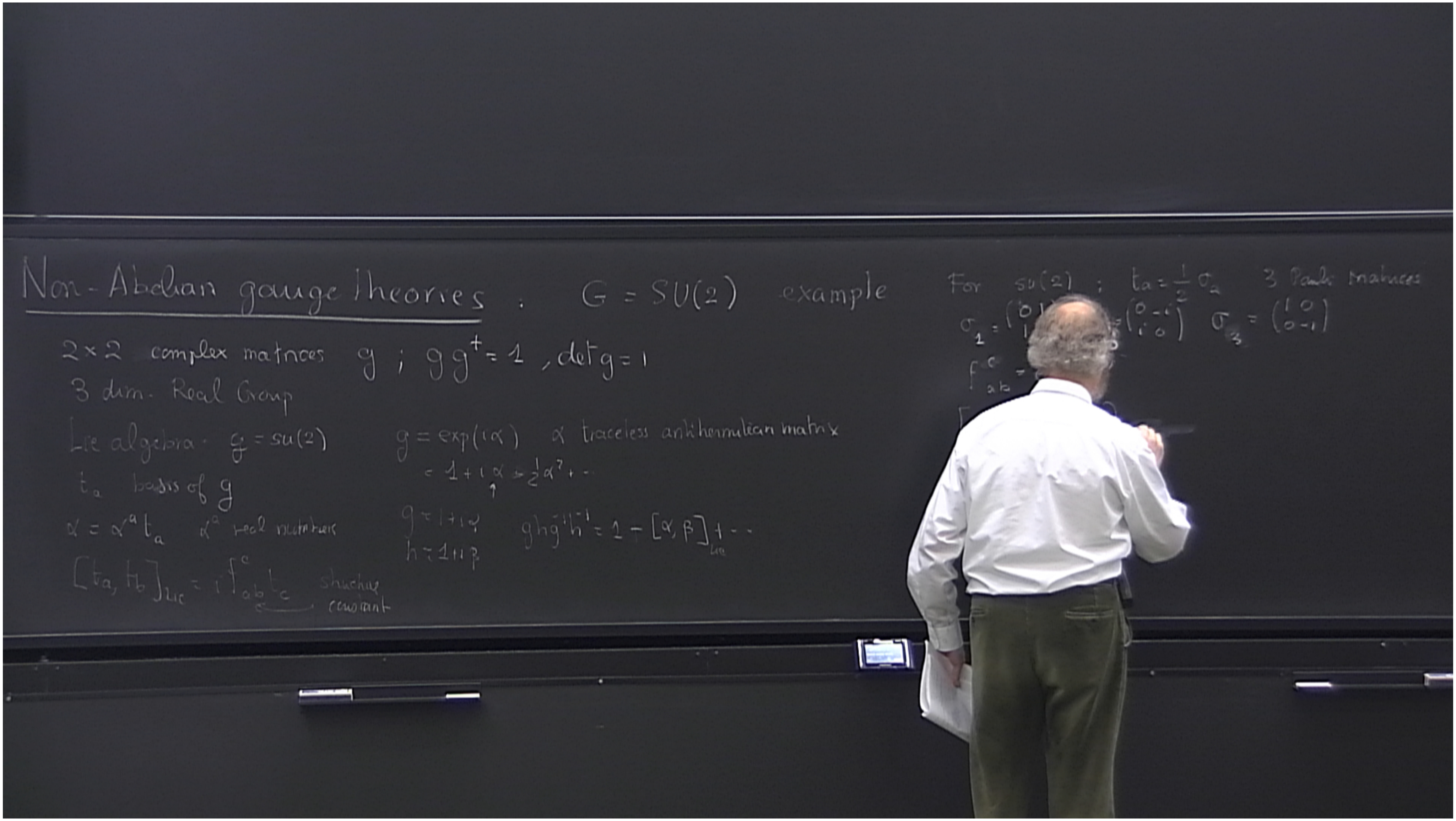
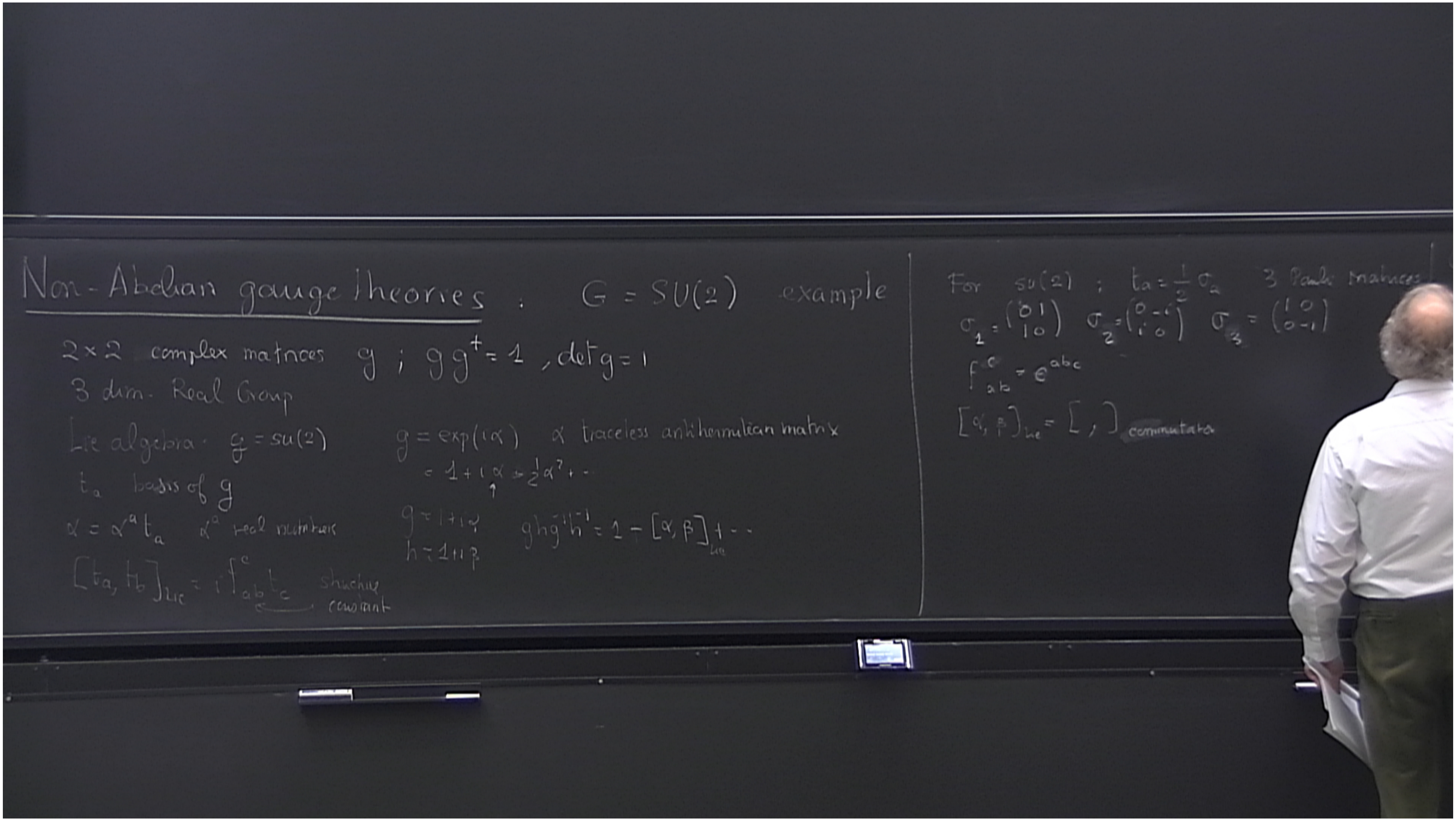
Abstract:


Non-Abelian gauge theories : G = SU(2) example For
2×2 complex matrices g ; gg† = 1 , det g = 1
3 dim. Real Group
Lie algebra. g = su(2)
g = exp(iα) α traceless antihermitian matrix
= 1 + iα + ½α² + ...
t_a basis of g
α = α^a t_a α^a real numbers
g = 1 + iα
h = 1 + iβ
ghg⁻¹h⁻¹ = 1 + [α, β] + ...
[t_a, t_b]_Lie = i f_ab^c t_c structure constant


Non-Abelian gauge theories : G = SU(2) example
2×2 complex matrices g ; gg† = 1 , det g = 1
3 dim. Real Group
Lie algebra 𝔤 = su(2)
t_a basis of 𝔤
α = α^a t_a α^a real numbers
[t_a, t_b]_Lie = i f_ab^c t_c structure constant
g = exp(iα) α traceless antihermitian matrix
= 1 + iα − ½α² + ...
g ≈ 1+iα
h ≈ 1+iβ
g h g⁻¹ h⁻¹ = 1 − [α, β]_Lie + ...
For su(2) ; t_a = ½ σ_a 3 Pauli matrices
σ_3 = (1 0 ; 0 −1)



Non-Abelian gauge theories : G = SU(2) example
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3 dim. Real Group
Lie algebra. g = su(2)
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g ≈ 1+iα
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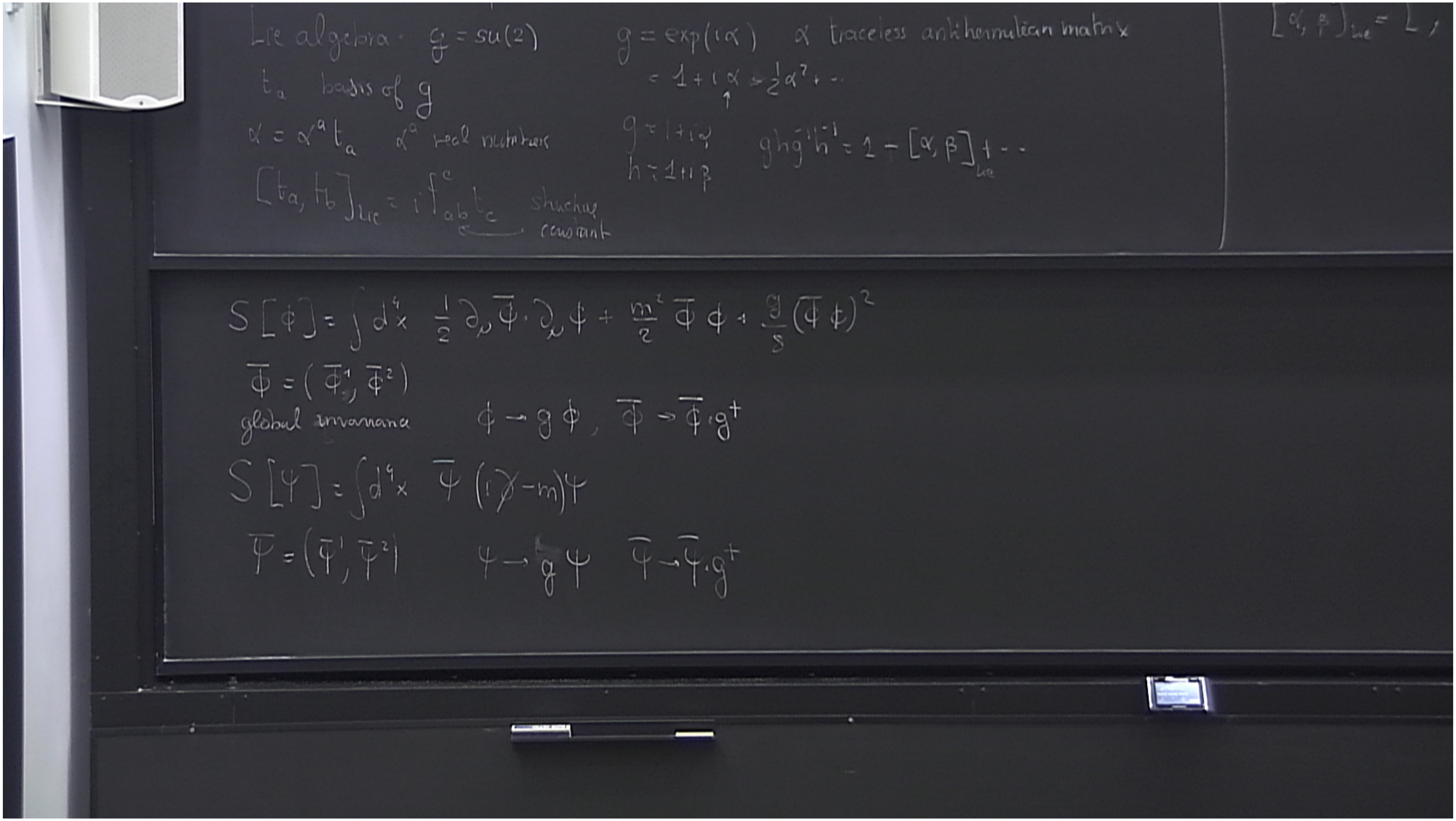
ghg⁻¹h⁻¹ = 1 − [α, β]_Lie + …
For su(2) ; t_a = ½ σ_a 3 Pauli matrices
σ_1 = (0 1; 1 0) σ_2 = (0 −i; i 0) σ_3 = (1 0; 0 −1)
f_ab^c = ε^abc
[α, β]_Lie = [,] commutator


Lie algebra: g = su(2)
g = exp(iα) α traceless antihermitian matrix
= 1 + iα + ½α² + ..
t_a basis of g
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g ≈ 1 + iα
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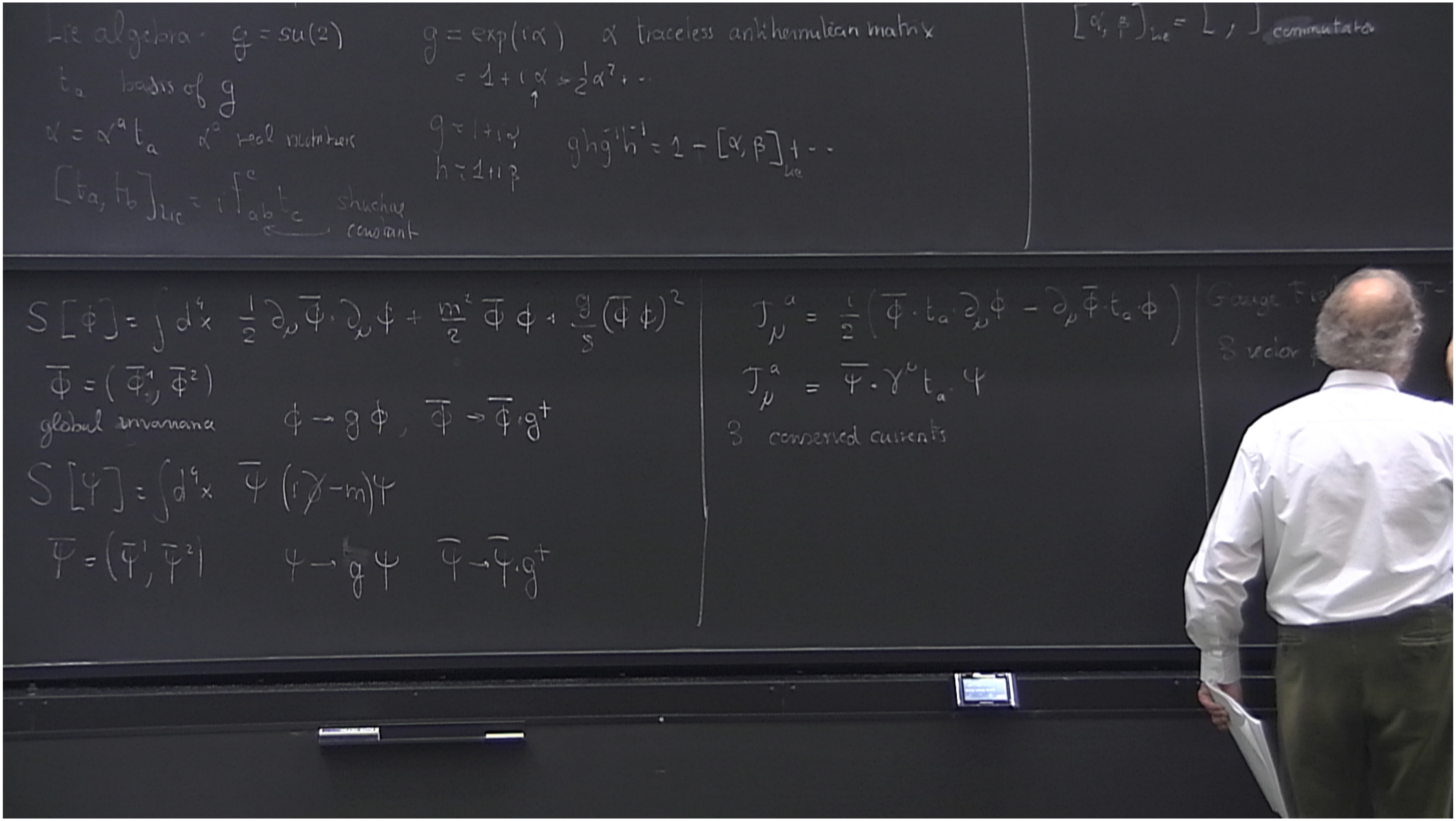
g h g⁻¹ h⁻¹ = 1 + [α, β]_Lie + ..
[t_a, t_b]_Lie = i f^c_ab t_c structure constant
S[φ] = ∫ d⁴x ½ ∂_μ φ̄ · ∂_μ φ + m²/2 φ̄ φ + g/8 (φ̄ φ)²
φ̄ = (φ̄¹, φ̄²)
global invariance φ → g φ, φ̄ → φ̄ · g†
S[ψ] = ∫ d⁴x ψ̄ (i∂̸ − m) ψ
ψ̄ = (ψ̄¹, ψ̄²) ψ → g ψ ψ̄ → ψ̄ · g†


Lie algebra $\mathfrak{g} = su(2)$
t_a basis of $\mathfrak{g}$
$\alpha = \alpha^a t_a$ α^a real numbers
$[t_a, t_b]_{Lie} = i f^c_{ab} t_c$ structure constant
$g = \exp(i\alpha)$ α traceless antihermitian matrix
$= 1 + i\alpha + \frac{1}{2}\alpha^2 + \cdots$
$g = 1 + i\alpha$
$h = 1 + i\beta$
$g h g^{-1} h^{-1} = 1 + [\alpha, \beta]_{Lie} + \cdots$
$[\alpha, \beta]_{Lie} = [\ ,\ ]$ commutator
$S[\phi] = \int d^4x \ \frac{1}{2}\partial_\mu \bar{\Phi} \cdot \partial_\nu \phi + \frac{m^2}{2} \bar{\Phi}\phi + \frac{g}{8}(\bar{\Phi}\phi)^2$
$\bar{\Phi} = (\bar{\Phi}^1, \bar{\Phi}^2)$
global invariance $\phi \to g\phi$, $\bar{\Phi} \to \bar{\Phi} \cdot g^\dagger$
$S[\psi] = \int d^4x \ \bar{\Psi}(i\partial\!\!\!/ - m)\psi$
$\bar{\Psi} = (\bar{\Psi}^1, \bar{\Psi}^2)$ $\psi \to g\psi$ $\bar{\Psi} \to \bar{\Psi} \cdot g^\dagger$
$J^a_\mu = \frac{i}{2}(\bar{\Phi} \cdot t_a \partial_\mu \phi - \partial_\mu \bar{\Phi} \cdot t_a \phi)$
$J^a_\mu = \bar{\Psi} \cdot \gamma^\mu t_a \cdot \psi$
3 conserved currents
Gauge Fie
3 vector

traceless antihermitian matrix

...

$n^{-1} = 1 - [\alpha, \beta]_{Lie} + \dots$

$[\alpha, \beta]_{Lie} = [\ ,\ ]$ commutator

$\Phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix} = (\phi^i)$ $i = 1, 2$ group indices of F

2 complex fields ← 4 real fields

Spin $\frac{1}{2}$ Dirac in F

$\Psi = \begin{pmatrix} \psi^1 \\ \psi^2 \end{pmatrix}$ ← Dirac fields $\psi^i = (\psi^i_\alpha)$

$(\bar\Phi \Phi)^2$

$J^a_\mu = \frac{1}{2}\left(\bar\Phi \cdot t_a \partial_\mu \phi - \partial_\mu \bar\Phi \cdot t_a \phi\right)$

$J^a_\mu = \bar\Psi \cdot \gamma^\mu t_a \Psi$

3 conserved currents

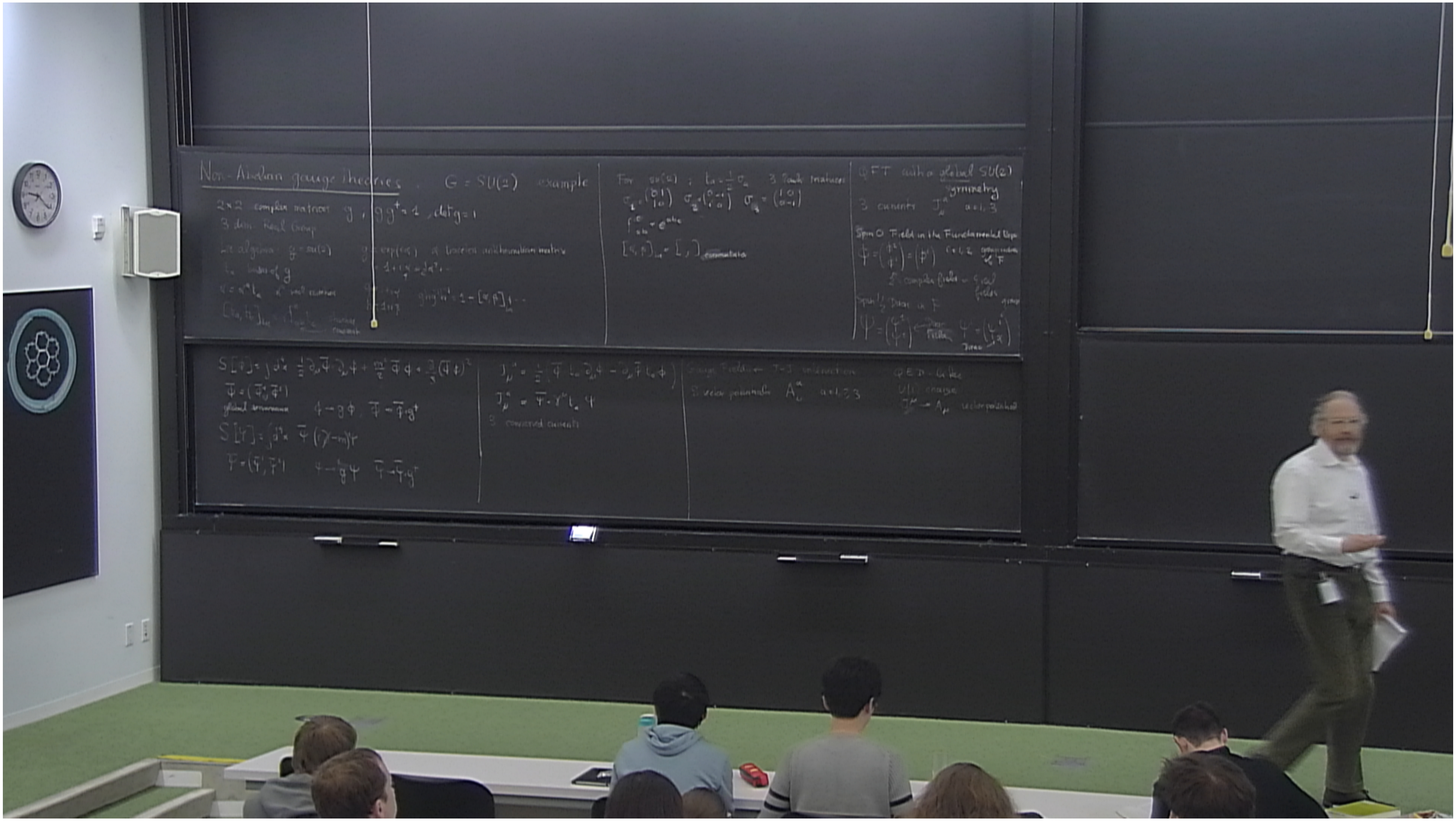
Gauge Fields ← J-J interaction

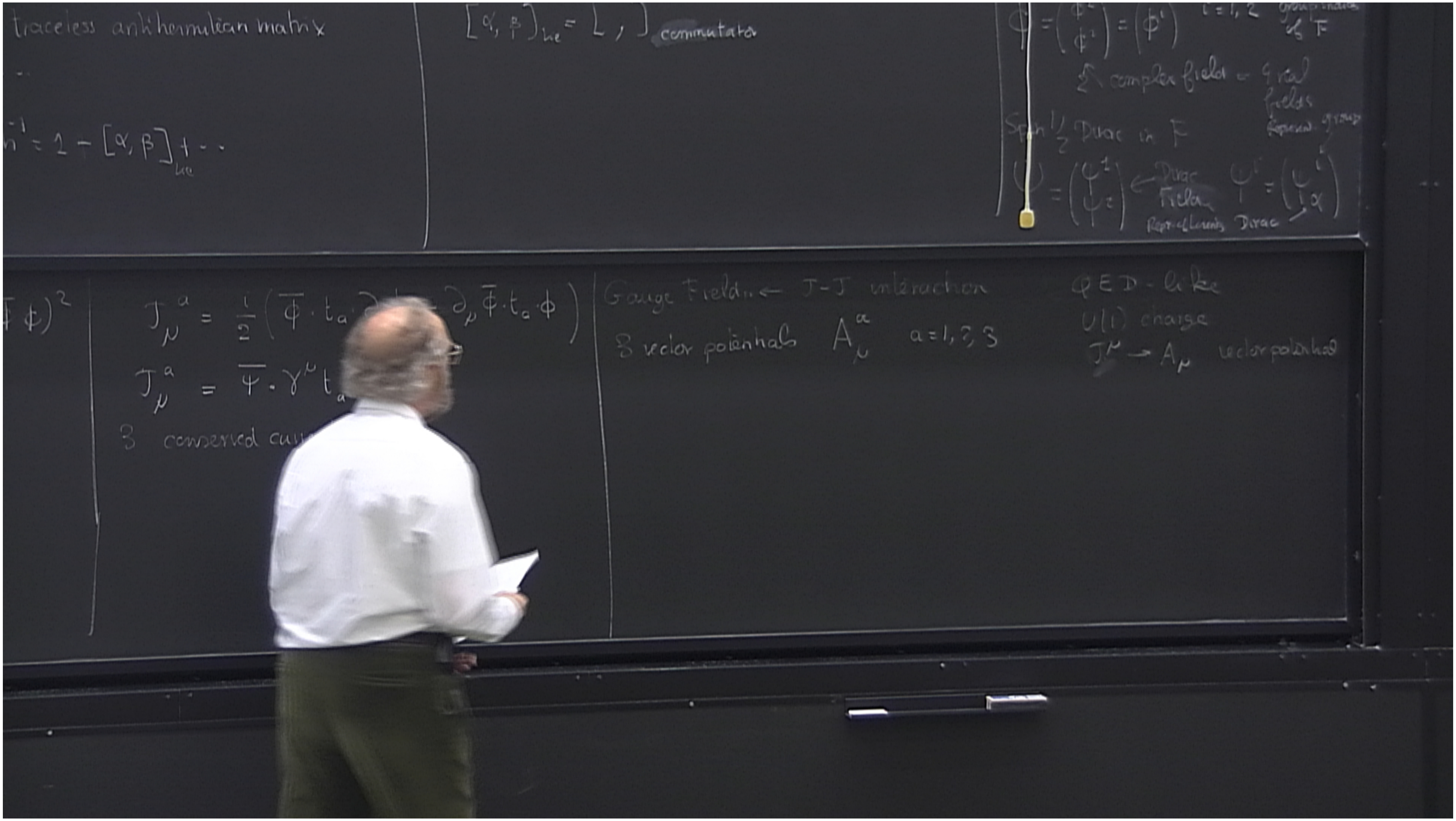
3 vector potentials A^a_μ $a = 1, 2, 3$

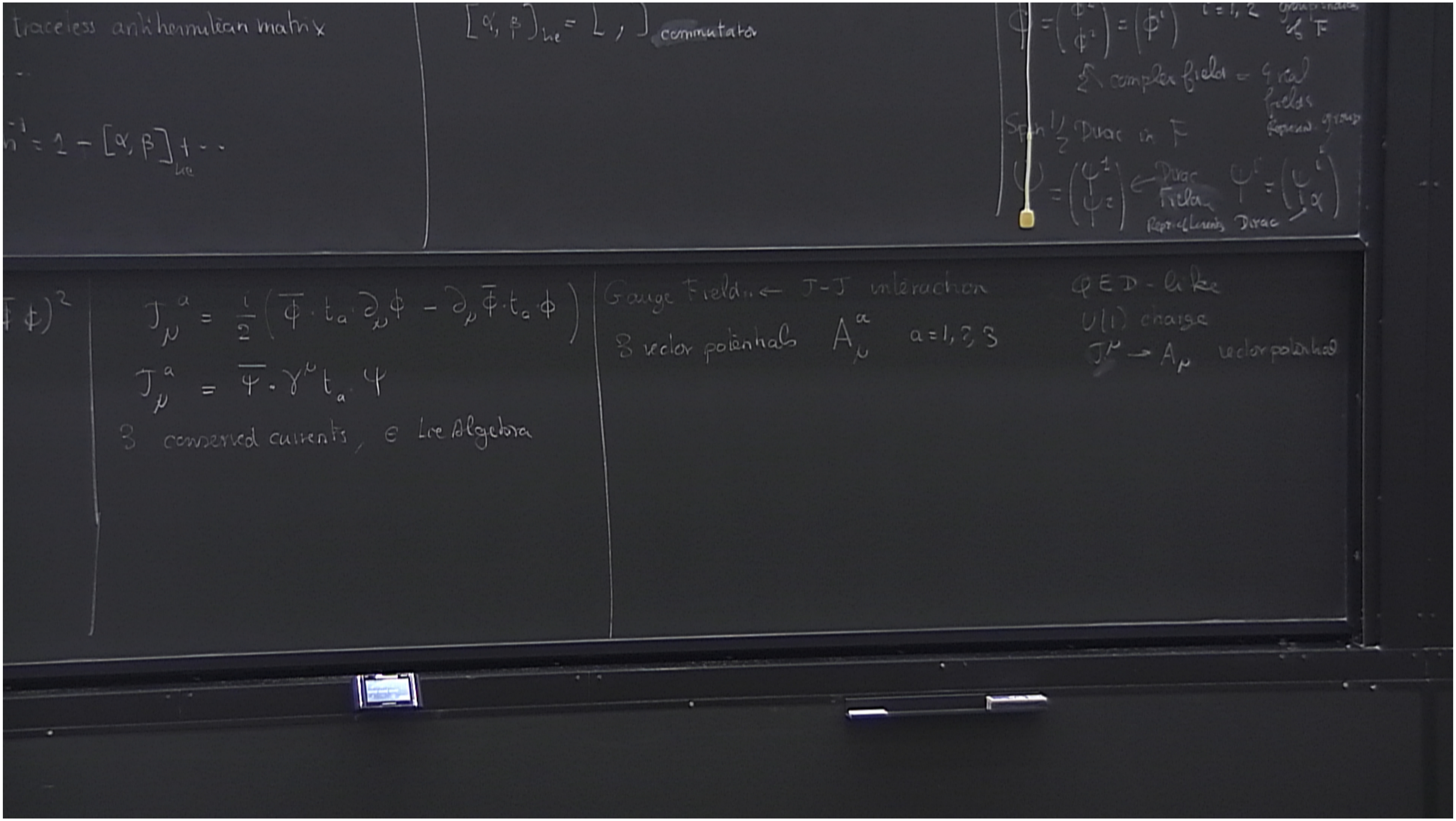
QED-like

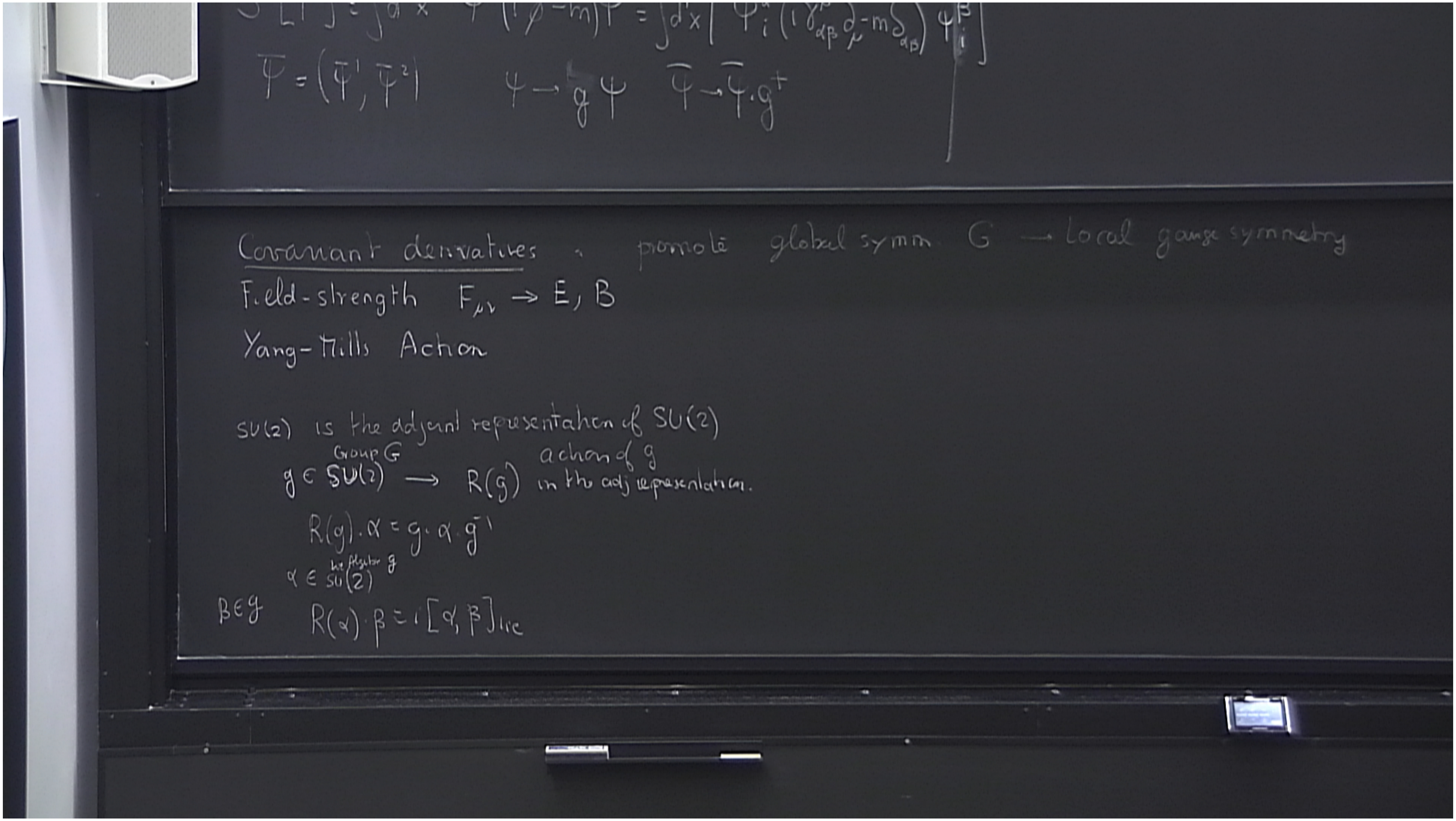
U(1) charge

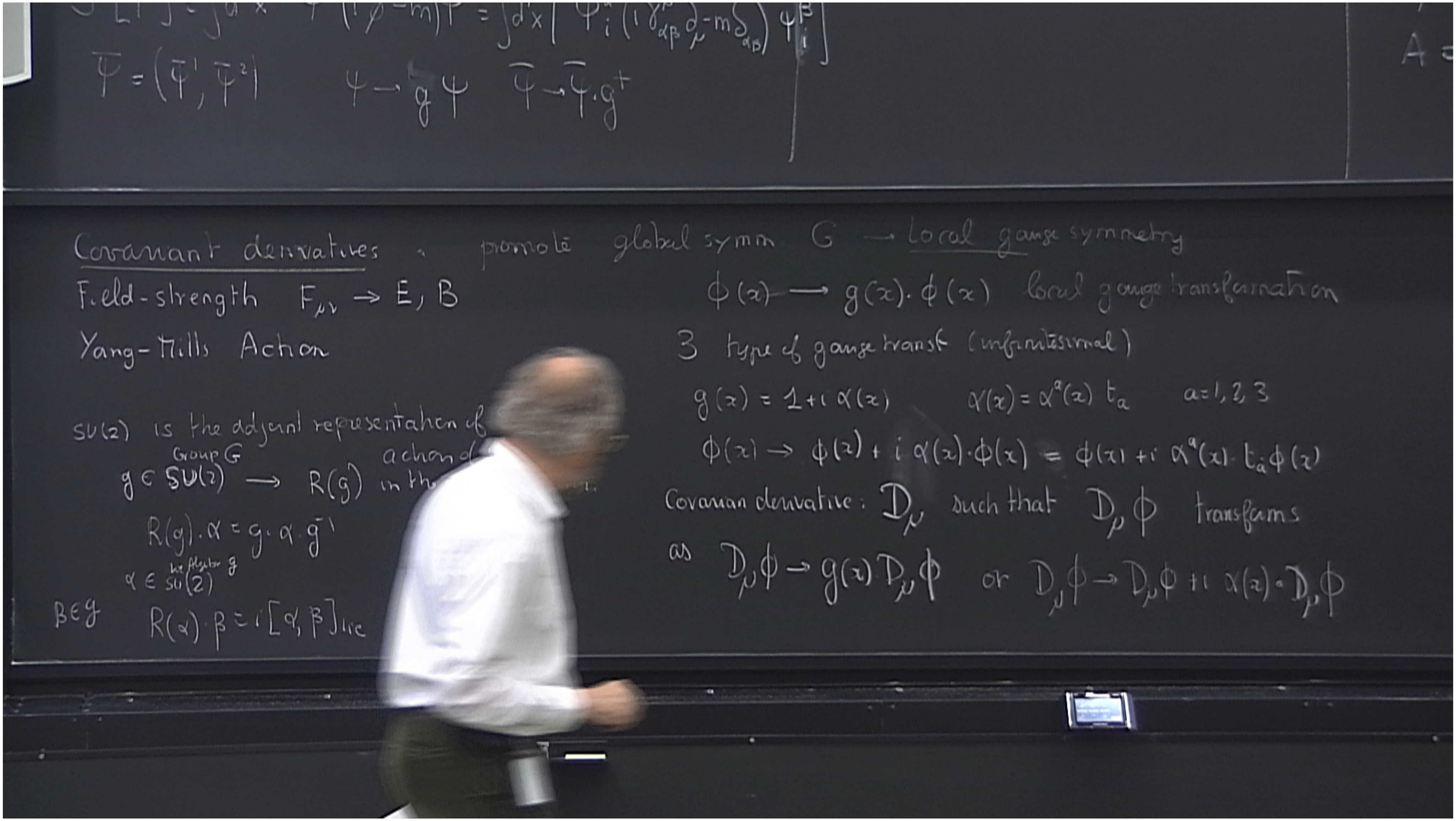
$J^\mu \to A_\mu$ vector potential

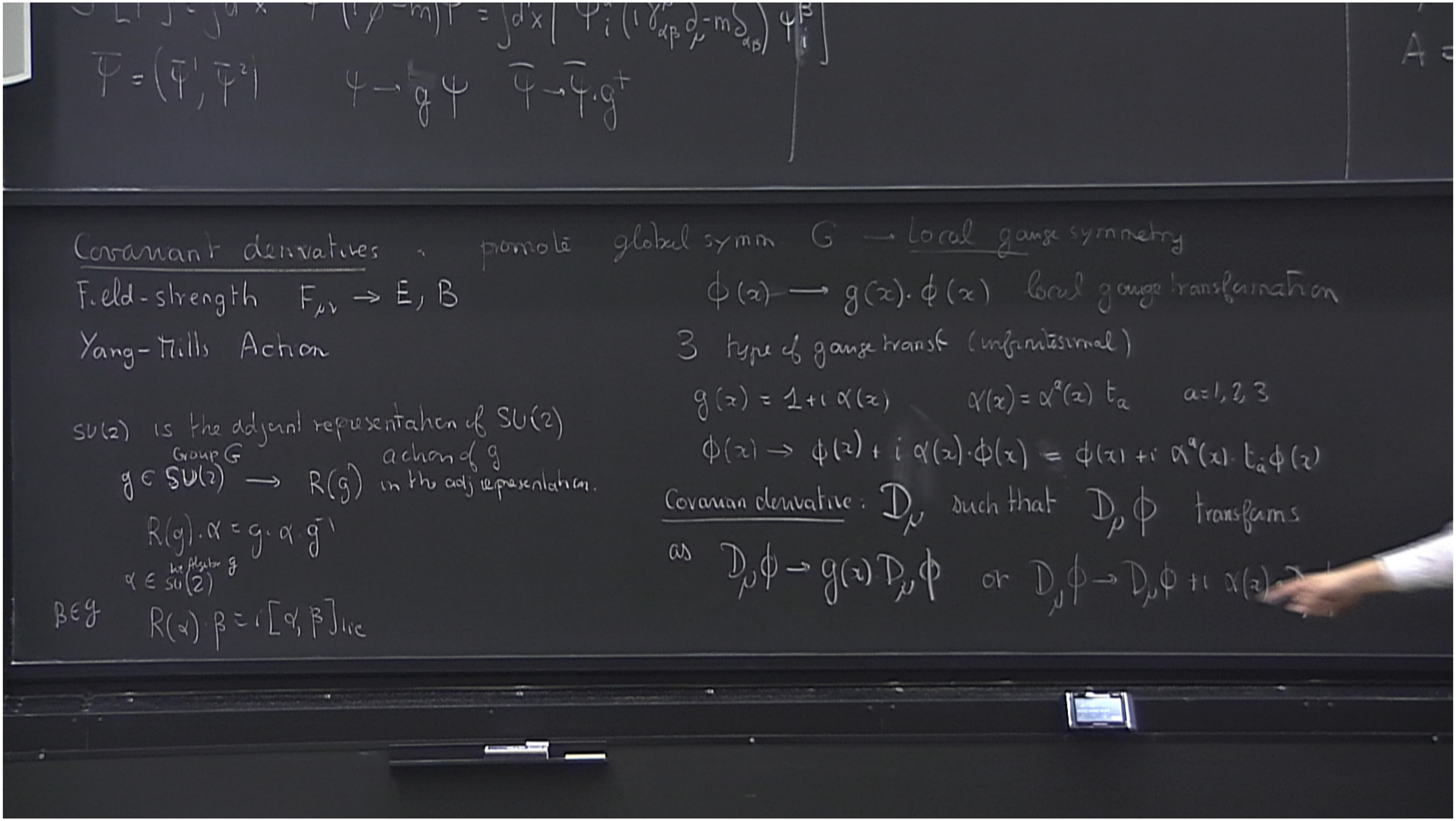

Non-Abelian gauge theories G = SU(2) example


traceless antihermitian matrix
commutator
complex field
Dirac
Gauge Field ← J-J interaction
3 vector potentials
a = 1, 2, 3
QED - like
U(1) charge
vector potential

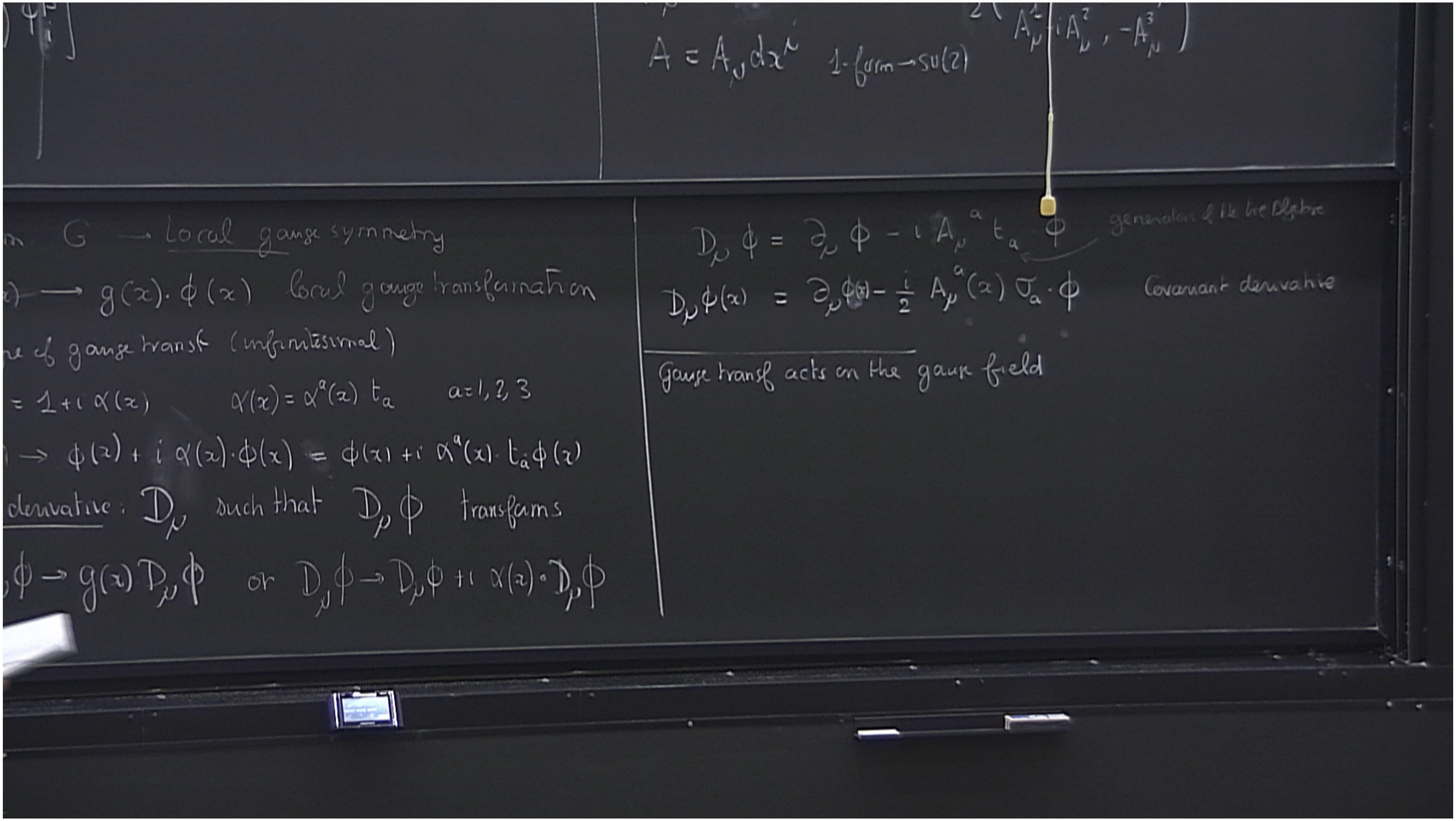

traceless antihermitian matrix
[α, β]_Lie = [,] commutator
= 1 − [α, β]_Lie + ...
complex field = 4 real fields
Spin 1/2 Dirac in F
Dirac field
Represents Dirac
J_μ^a = i/2 (Φ̄ · t_a ∂_μ φ − ∂_μ Φ̄ · t_a · φ)
J_μ^a = Ψ̄ · γ^μ t_a · Ψ
3 conserved currents, ∈ Lie Algebra
Gauge Field ← J-J interaction
3 vector potentials A_μ^a a = 1, 2, 3
QED-like
U(1) charge
J^μ → A_μ vector potential

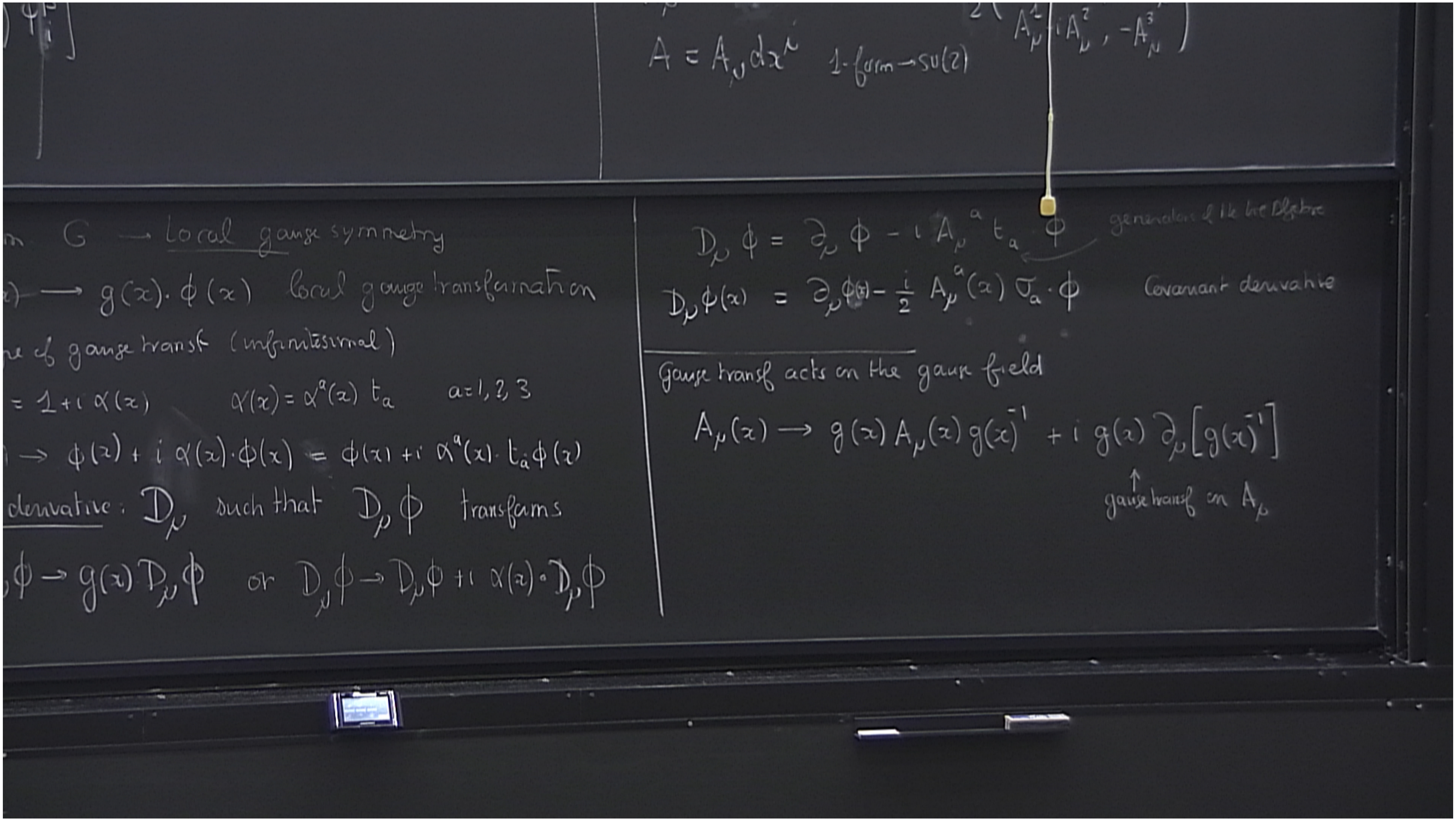

$\bar{\Psi} = (\bar{\Psi}^1, \bar{\Psi}^2)$ $\Psi \to g\Psi$ $\bar{\Psi} \to \bar{\Psi} \cdot g^+$
Covariant derivatives : promote global symm. G → local gauge symmetry
Field-strength $F_{\mu\nu} \to E, B$
Yang-Mills Action
su(2) is the adjoint representation of SU(2)
Group G
$g \in SU(2) \longrightarrow R(g)$
action of g
in the adj representation.
$R(g) \cdot \alpha = g \cdot \alpha \cdot g^{-1}$
Lie Algebra g
$\alpha \in su(2)$
$\beta \in g$
$R(\alpha) \cdot \beta = i[\alpha, \beta]_{Lie}$


Covariant derivatives
promote global symm G → Local gauge symmetry
Field-strength $F_{\mu\nu} \to E, B$
Yang-Mills Action
$\phi(x) \longrightarrow g(x)\cdot\phi(x)$ local gauge transformation
3 type of gauge transf (infinitesimal)
$g(x) = 1 + i\,\alpha(x)$ $\alpha(x) = \alpha^a(x)\, t_a$ $a = 1, 2, 3$
$\phi(x) \to \phi(x) + i\,\alpha(x)\cdot\phi(x) = \phi(x) + i\,\alpha^a(x)\, t_a \phi(x)$
SU(2) is the adjoint representation of
Group G
$g \in SU(2) \longrightarrow R(g)$
Covariant derivative: D_μ such that $D_\mu\phi$ transforms
as $D_\mu\phi \to g(x) D_\mu\phi$ or $D_\mu\phi \to D_\mu\phi + i\,\alpha(x)\cdot D_\mu\phi$
$R(g).\alpha = g\cdot\alpha\cdot g^{-1}$
$\alpha \in su(2)$
$\beta \in g$ $R(\alpha)\cdot\beta = i[\alpha, \beta]_{Lie}$
$\bar\psi = (\bar\psi^1, \bar\psi^2)$ $\psi \to g\psi$ $\bar\psi \to \bar\psi\cdot g^+$



Covariant derivatives : promote global symm G → Local gauge symmetry
Field-strength F_{μν} → E, B
Yang-Mills Action
φ(x) → g(x)·φ(x) local gauge transformation
3 type of gauge transf (infinitesimal)
g(x) = 1 + i α(x)   α(x) = α^a(x) t_a   a = 1, 2, 3
φ(x) → φ(x) + i α(x)·φ(x) = φ(x) + i α^a(x)·t_a φ(x)
Covarian derivative : D_μ such that D_μ φ transforms
as D_μ φ → g(x) D_μ φ or D_μ φ → D_μ φ + i α(x)
SU(2) is the adjoint representation of SU(2)
Group G
action of g
g ∈ SU(2) → R(g) in the adj representation.
R(g)·α = g·α·g^{-1}
Lie Algebra g
α ∈ su(2)
β ∈ g   R(α)·β = i[α, β]_{Lie}
Ψ̄ = (Ψ̄^1, Ψ̄^2)   Ψ → gΨ   Ψ̄ → Ψ̄·g^+


A = A_ν dx^ν 1-form → su(2)
A_ν^1 + iA_ν^2 , −A_ν^3
G → Local gauge symmetry
→ g(x)·φ(x) local gauge transformation
of gauge transf (infinitesimal)
= 1 + i α(x) α(x) = α^a(x) t_a a = 1, 2, 3
→ φ(x) + i α(x)·φ(x) = φ(x) + i α^a(x)·t_a φ(x)
derivative: D_μ such that D_μ φ transforms
φ → g(x) D_μ φ or D_μ φ → D_μ φ + i α(x)·D_μ φ
D_μ φ = ∂_μ φ − i A_μ^a t_a φ
generators of the Lie Algebra
D_μ φ(x) = ∂_μ φ(x) − i/2 A_μ^a(x) σ_a · φ
Covariant derivative
Gauge transf acts on the gauge field


$A = A_\nu dx^\nu$ 1-form → su(2)
$(A^1_\nu \; iA^2_\nu, \; -A^3_\nu)$
G → local gauge symmetry
→ $g(x) \cdot \phi(x)$ local gauge transformation
of gauge transf (infinitesimal)
$= 1 + i\alpha(x)$ $\alpha(x) = \alpha^a(x)\, t_a$ a = 1, 2, 3
→ $\phi(x) + i\alpha(x)\cdot\phi(x) = \phi(x) + i\alpha^a(x)\cdot t_a\phi(x)$
derivative: D_μ such that $D_\mu\phi$ transforms
$\phi \to g(x) D_\mu\phi$ or $D_\mu\phi \to D_\mu\phi + i\alpha(x)\cdot D_\mu\phi$
$D_\mu\phi = \partial_\mu\phi - iA_\mu^a t_a \phi$ generators of the Lie Algebra
$D_\mu\phi(x) = \partial_\mu\phi(x) - \frac{i}{2}A_\mu^a(x)\sigma_a\cdot\phi$ Covariant derivative
Gauge transf acts on the gauge field
$A_\mu(x) \to g(x)A_\mu(x)g(x)^{-1} + i\,g(x)\,\partial_\mu[g(x)^{-1}]$
gauge transf on A_μ

$A = A_\mu dx^\mu$ 1-form → su(2)

$\ldots (A^1_\mu + i A^2_\mu , -A^3_\mu)$

... G → Local gauge symmetry

$\ldots \longrightarrow g(x) \cdot \phi(x)$ local gauge transformation

... of gauge transf (infinitesimal)

$\ldots = 1 + i\,\alpha(x) \qquad \alpha(x) = \alpha^a(x)\, t_a \qquad a = 1,2,3$

$\ldots \rightarrow \phi(x) + i\,\alpha(x)\cdot\phi(x) = \phi(x) + i\,\alpha^a(x)\cdot t_a \phi(x)$

... derivative: D_μ such that $D_\mu \phi$ transforms

$\ldots \phi \rightarrow g(x) D_\mu \phi$ or $D_\mu \phi \rightarrow D_\mu \phi + i\,\alpha(x) \cdot D_\mu \phi$

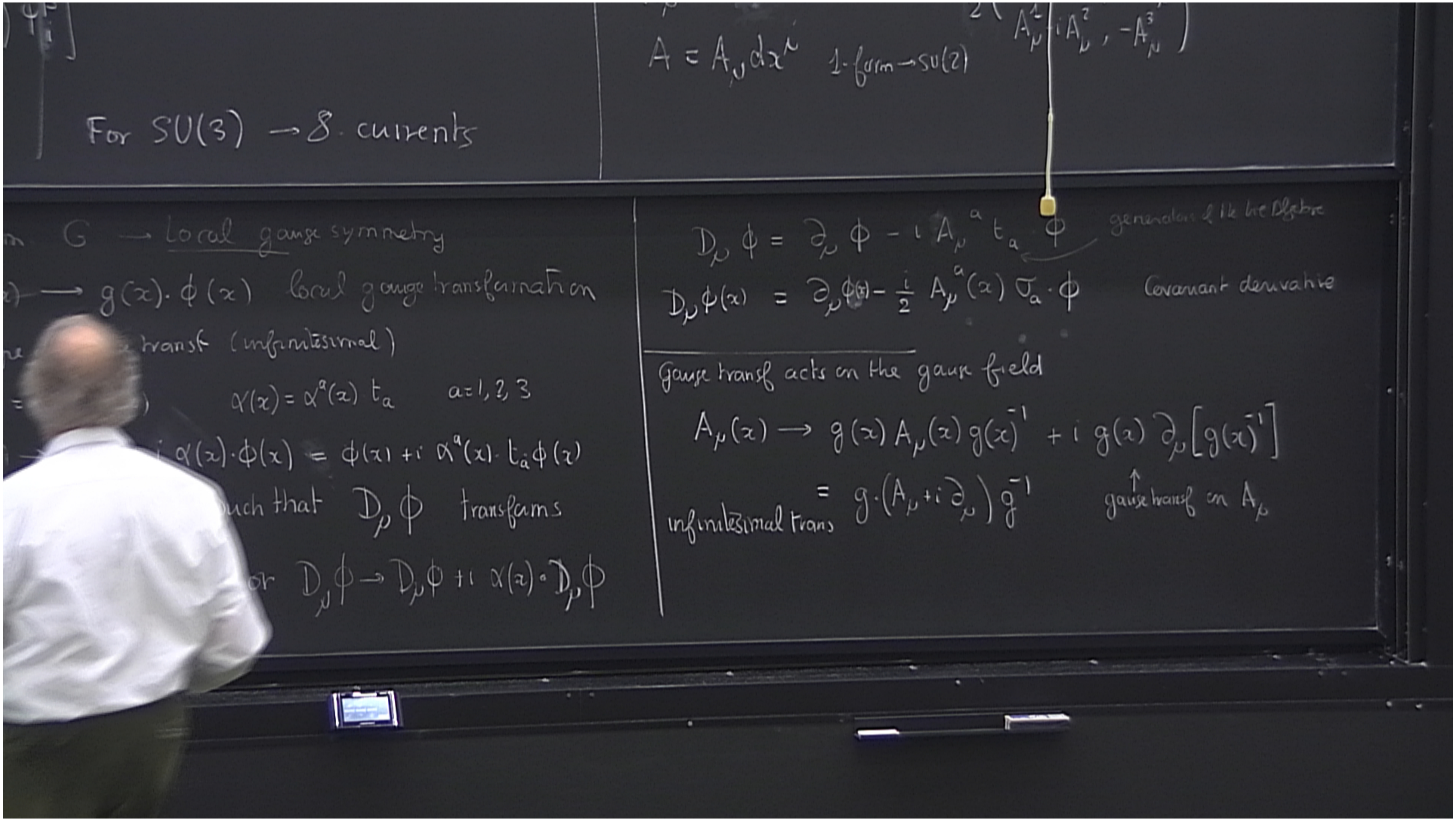
$D_\mu \phi = \partial_\mu \phi - i A_\mu^{\ a}\, t_a\, \phi$ generators of the Lie algebra

$D_\mu \phi(x) = \partial_\mu \phi(x) - \frac{i}{2} A_\mu^{\ a}(x)\, \sigma_a \cdot \phi$ Covariant derivative

Gauge transf acts on the gauge field

$A_\mu(x) \longrightarrow g(x) A_\mu(x) g(x)^{-1} + i\, g(x)\, \partial_\mu \left[g(x)^{-1} \right]$

↑ gauge transf on A_μ


A = A_μ dx^μ
1-form → su(2)
A_μ^1 + iA_μ^2 , −A_μ^3)
For SU(3) → 8 currents
G → Local gauge symmetry
→ g(x)·φ(x) local gauge transformation
transf (infinitesimal)
α(x) = α^a(x) t_a
a=1,2,3
α(x)·φ(x) = φ(x) + i α^a(x)·t_a φ(x)
uch that D_μ φ transforms
or D_μφ → D_μφ + i α(x)·D_μφ
D_μ φ = ∂_μ φ − i A_μ^a t_a φ
generators of the Lie Algebra
D_μφ(x) = ∂_μφ(x) − i/2 A_μ^a(x) σ_a · φ
Covariant derivative
Gauge transf acts on the gauge field
A_μ(x) ⟶ g(x) A_μ(x) g(x)^{-1} + i g(x) ∂_μ[g(x)^{-1}]
= g·(A_μ + i∂_μ) g^{-1}
infinitesimal trans
gauge transf on A_μ

$A = A_\mu dx^\mu$ 1-form $\to su(2)$

$\left(A^1_\mu + i A^2_\mu ,\ -A^3_\mu \right)$

For $SU(3) \to 8$ currents

$G \to$ Local gauge symmetry

$\phi(x) \longrightarrow g(x) \cdot \phi(x)$ local gauge transformation

of gauge transf (infinitesimal)

$g = 1 + i\,\alpha(x) \qquad \alpha(x) = \alpha^a(x)\, t_a \qquad a = 1,2,3$

$\phi \to \phi(x) + i\,\alpha(x)\cdot\phi(x) = \phi(x) + i\,\alpha^a(x)\cdot t_a \phi(x)$

derivative: D_μ such that $D_\mu \phi$ transforms

$D_\mu \phi \to g(x) D_\mu \phi$ or $D_\mu \phi \to D_\mu \phi + i\,\alpha(x) \cdot D_\mu \phi$

$D_\mu \phi = \partial_\mu \phi - i A_\mu^a t_a \phi$ generators of the Lie Algebra

$$D_\mu \phi(x) = \partial_\mu \phi(x) - \frac{i}{2} A_\mu^a(x)\, \sigma_a \cdot \phi$$

Covariant derivative acting in the Fund Repr

Gauge transf acts on the gauge field

$$A_\mu(x) \longrightarrow g(x) A_\mu(x) g(x)^{-1} + i\, g(x)\, \partial_\mu \left[g(x)^{-1} \right]$$

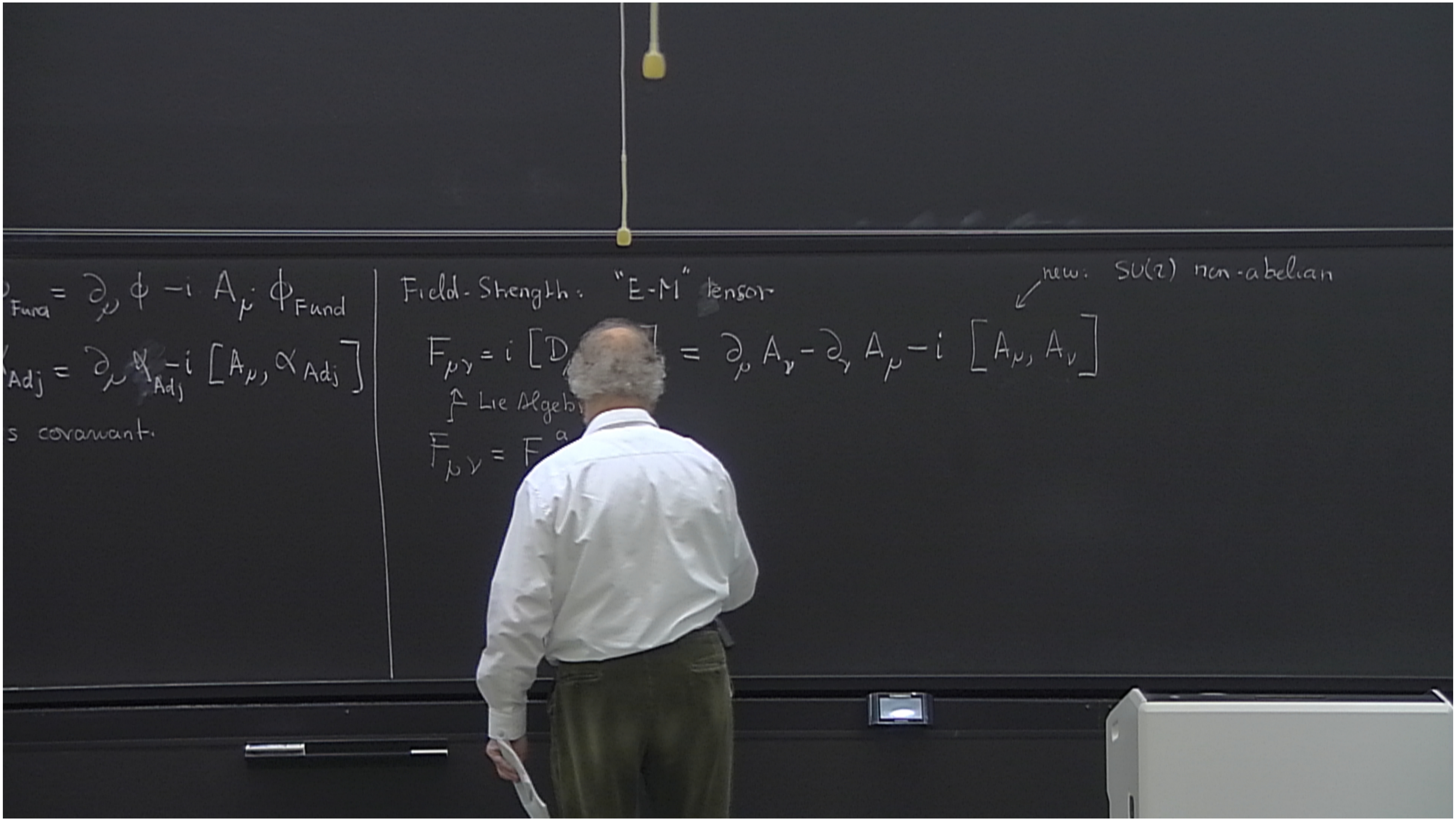
Cov Deriv Adj Repr

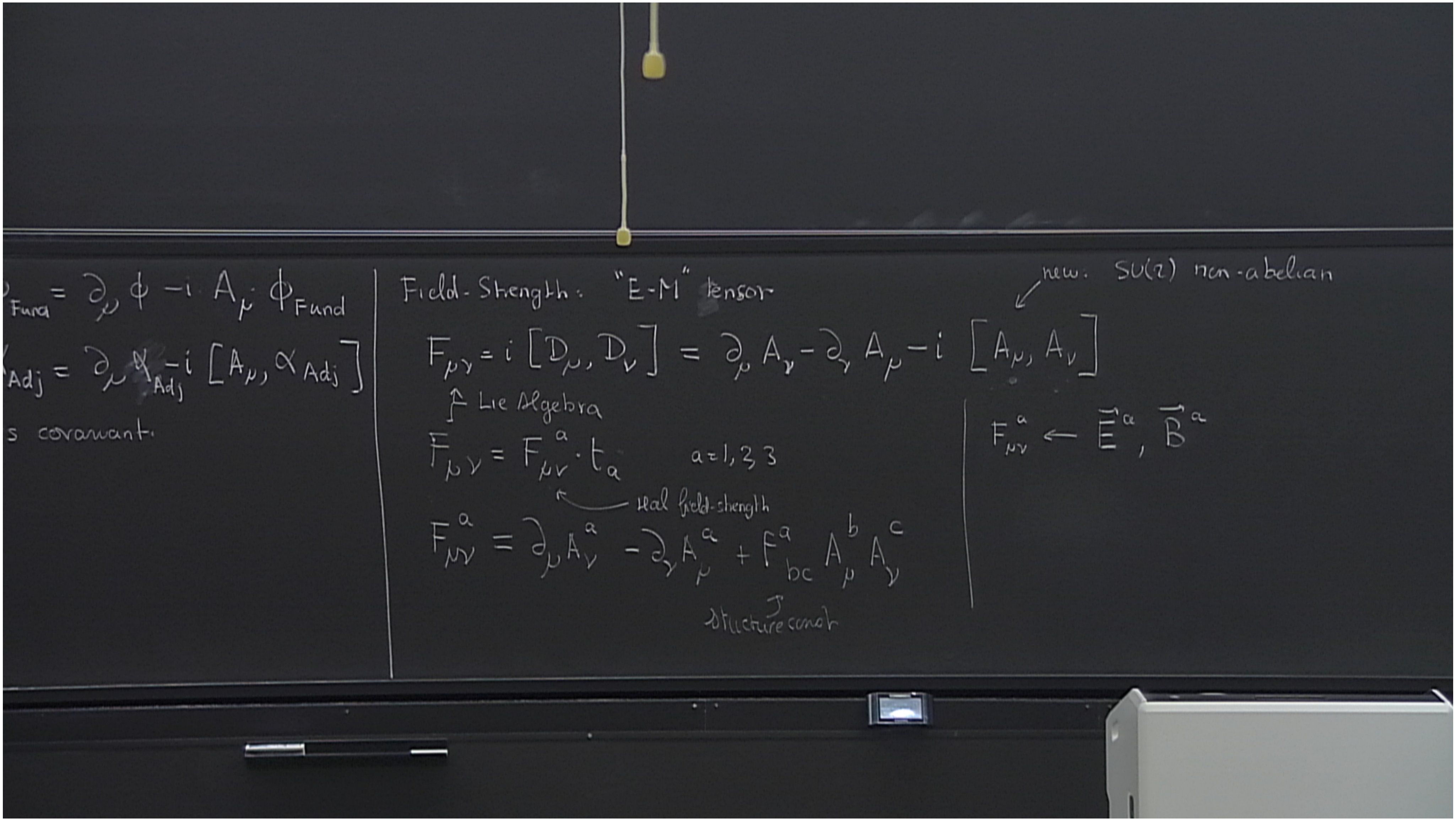
$$= g \cdot (A_\mu + i\partial_\mu)\, g^{-1}$$

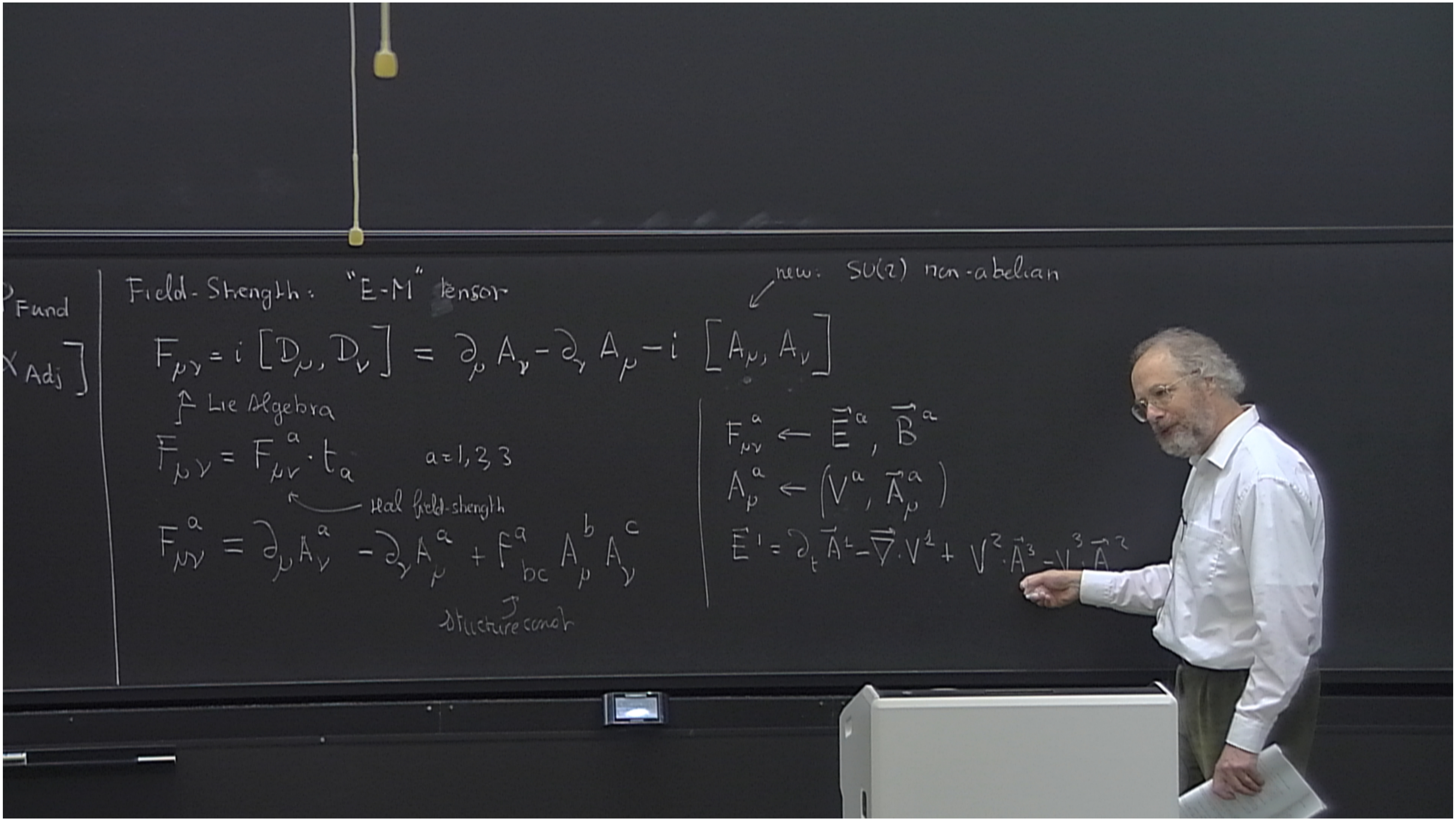
gauge transf on A_μ

infinitesimal trans

$$A_\mu(x) \longrightarrow A_\mu(x) + D_\mu \alpha(x) ; \quad D_\mu \alpha(x) = \partial_\mu \alpha(x) - i\left[A_\mu , \alpha \right]$$


Field-Strength: "E-M" tensor
new: SU(2) non-abelian
$= \partial_\mu A_\nu - \partial_\nu A_\mu - i\,[A_\mu, A_\nu]$
covariant.


$_{Fund} = \partial_\mu \phi - i A_\mu \phi_{Fund}$
$_{Adj} = \partial_\mu \alpha_{Adj} - i [A_\mu, \alpha_{Adj}]$
s covariant.
Field-Strength: "E-M" tensor
new: SU(2) non-abelian
$F_{\mu\nu} = i[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$
Lie Algebra
$F_{\mu\nu} = F^a_{\mu\nu} \cdot t_a$
$a=1,2,3$
real field-strength
$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^a_{bc} A^b_\mu A^c_\nu$
structure const
$F^a_{\mu\nu} \leftarrow \vec{E}^a, \vec{B}^a$


Fund
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Field-Strength: "E-M" tensor
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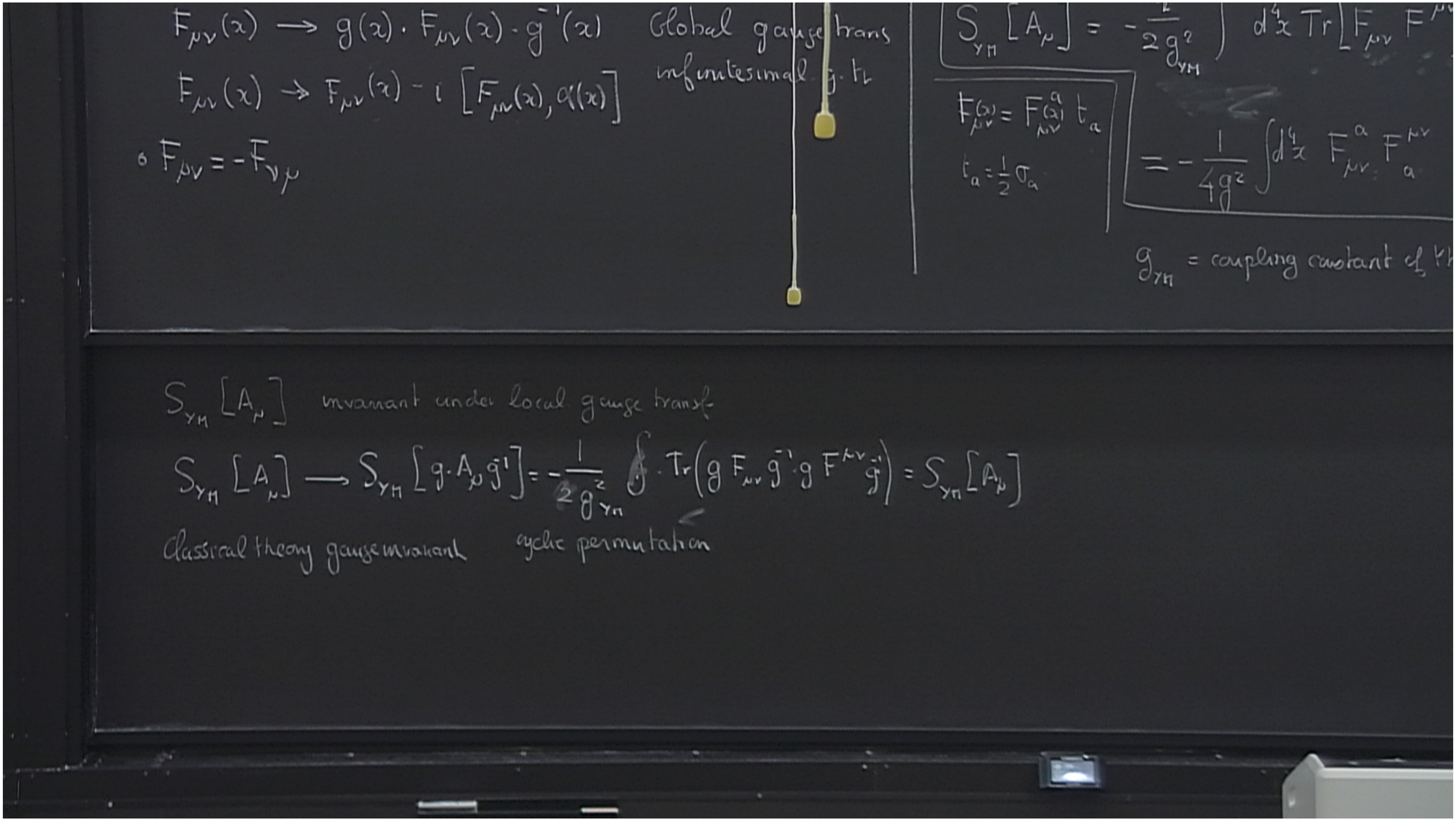
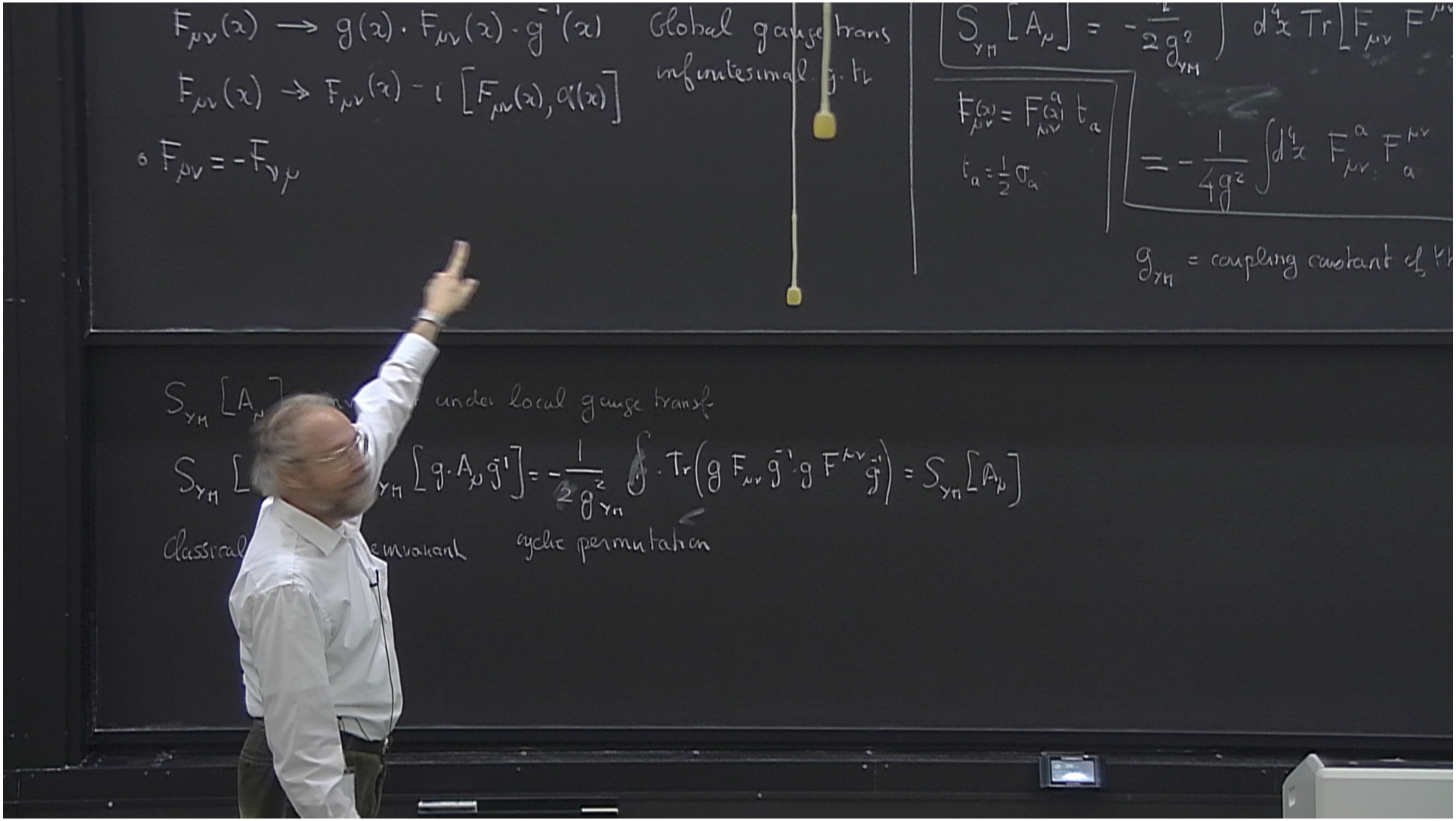
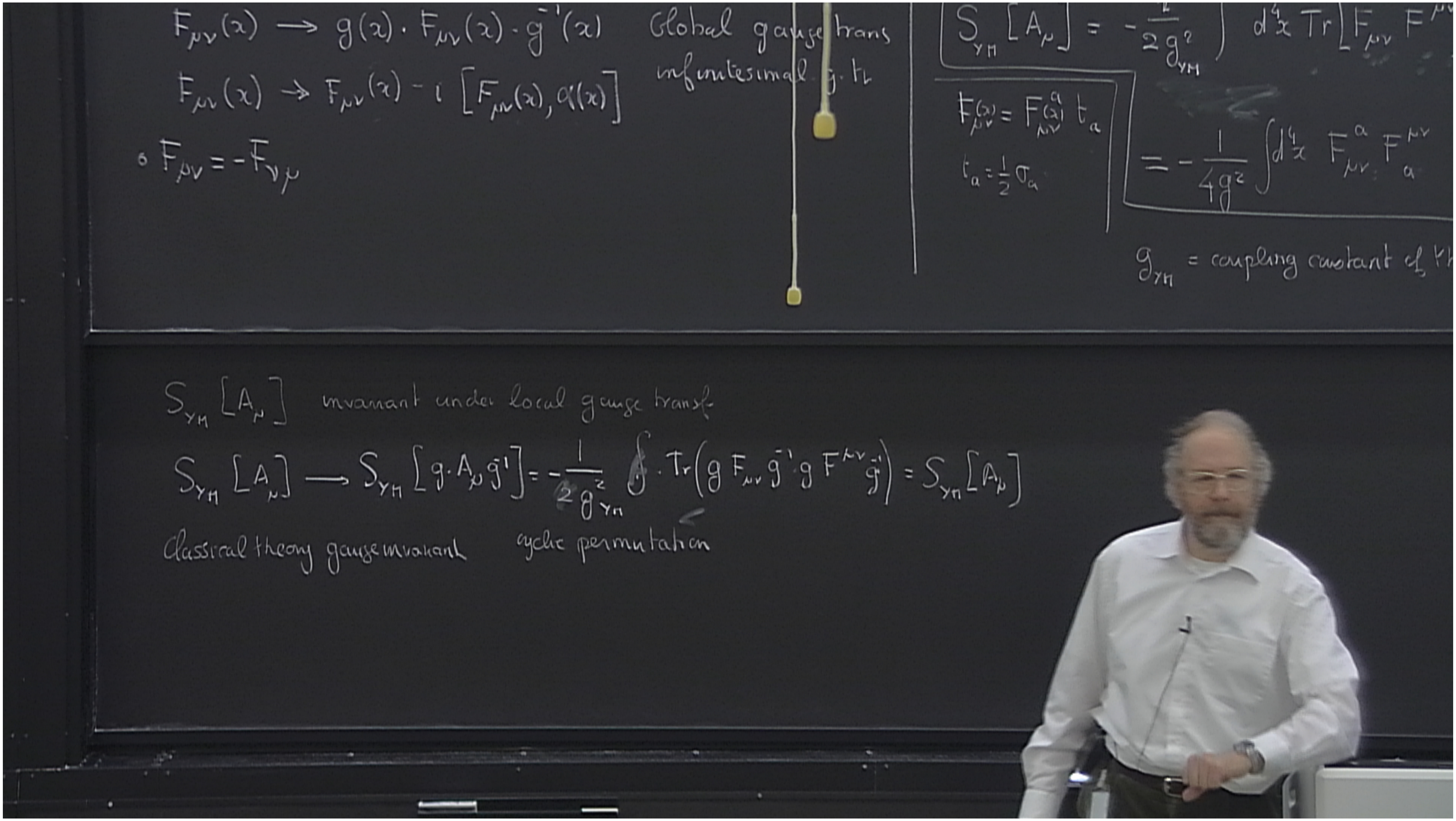
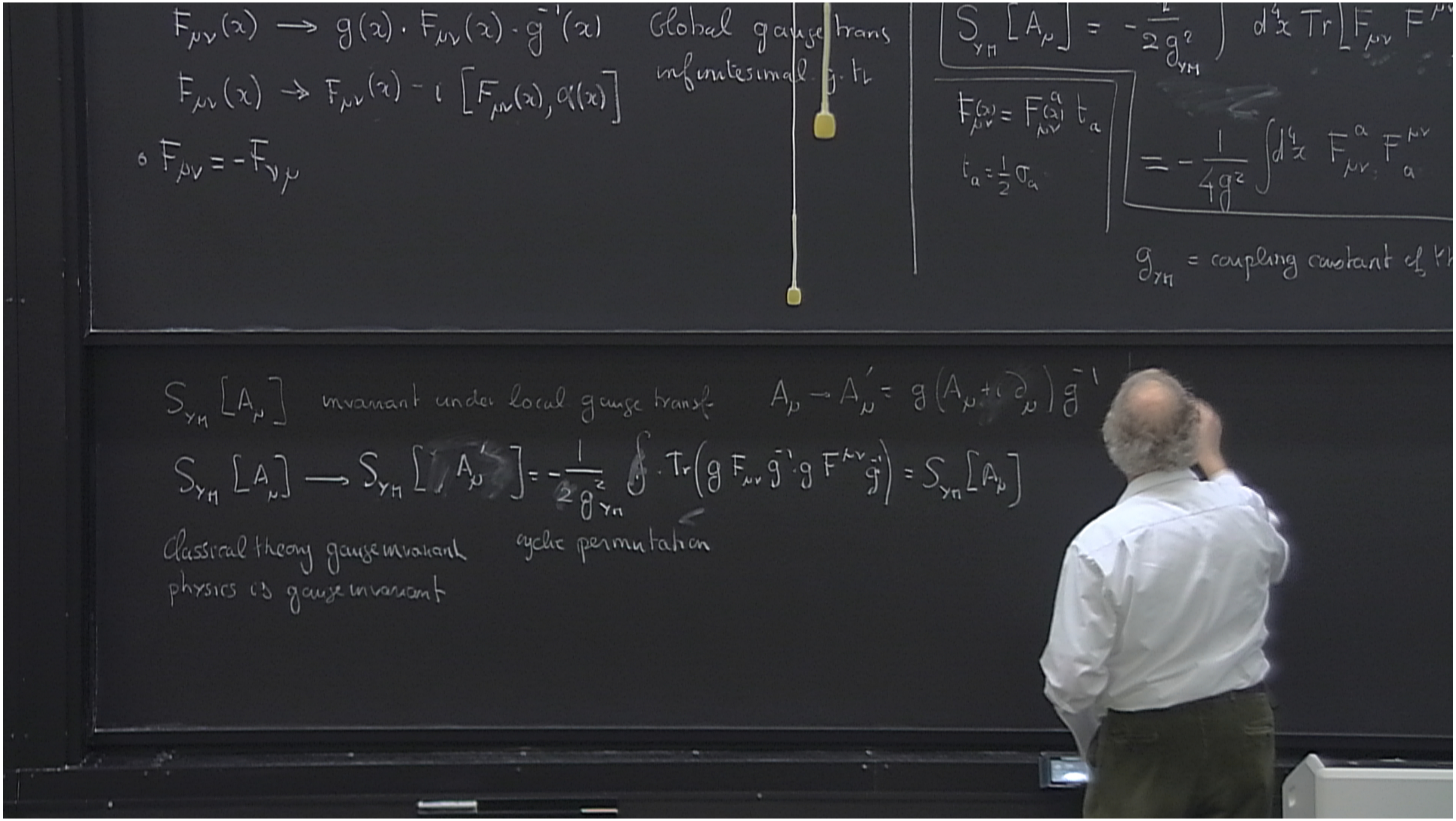
$F_{\mu\nu} = i [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu]$
Lie Algebra
$F_{\mu\nu} = F^a_{\mu\nu} \cdot t_a$
a=1,2,3
real field-strength
$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^a{}_{bc} A^b_\mu A^c_\nu$
structure const
$F^a_{\mu\nu} \leftarrow \vec{E}^a, \vec{B}^a$
$A^a_\mu \leftarrow (V^a, \vec{A}^a_\mu)$
$\vec{E}^1 = \partial_t \vec{A}^1 - \vec{\nabla} \cdot V^1 + V^2 \cdot \vec{A}^3 - V^3 \cdot \vec{A}^2$

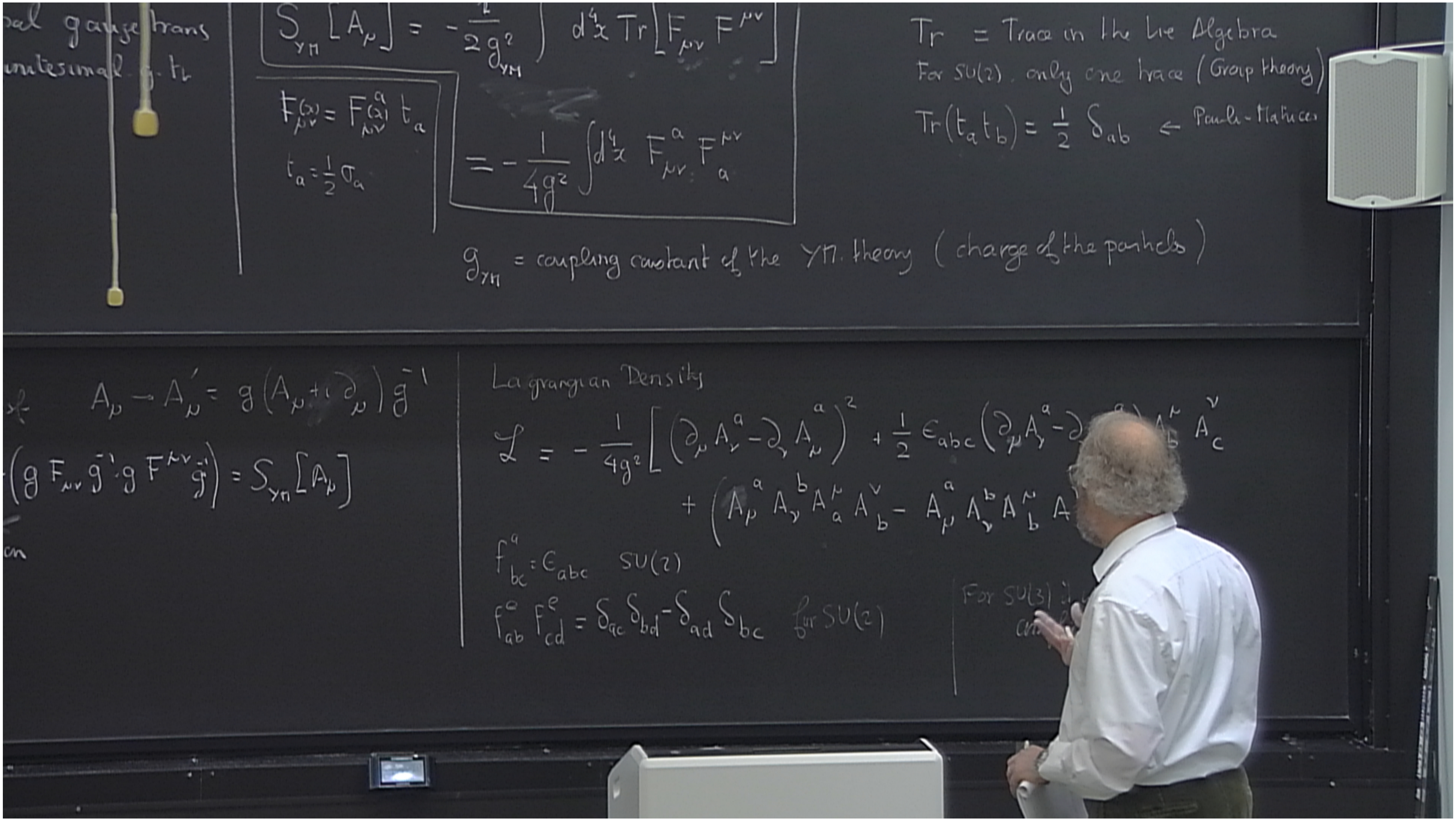


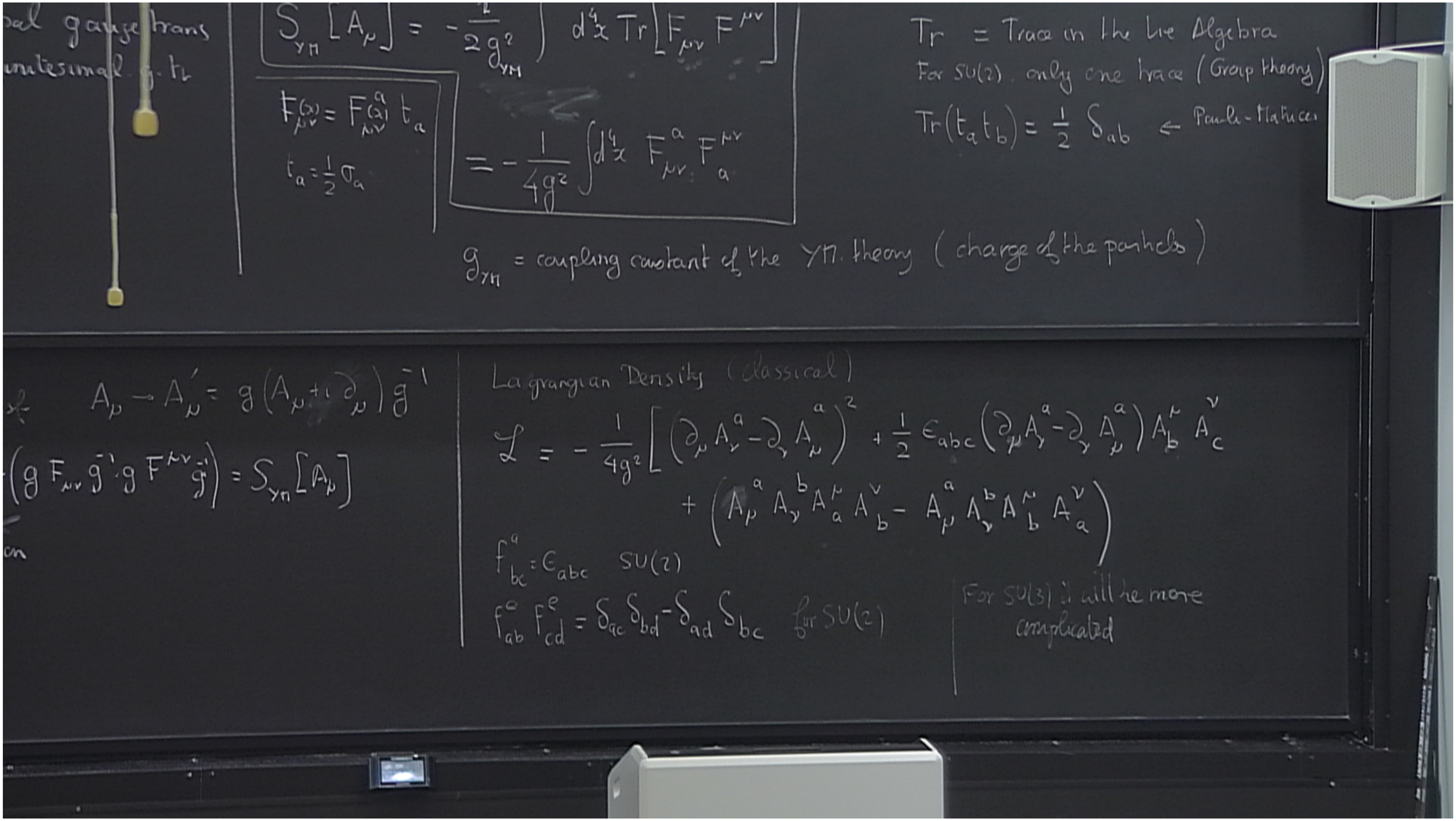
$F_{\mu\nu}(x) \longrightarrow g(x) \cdot F_{\mu\nu}(x) \cdot g^{-1}(x)$ Global gauge trans
$F_{\mu\nu}(x) \to F_{\mu\nu}(x) - i\,[F_{\mu\nu}(x), \alpha(x)]$ infinitesimal g. tr
$F_{\mu\nu} = -F_{\nu\mu}$
$S_{YM}[A_\mu] = -\frac{1}{2g^2_{YM}} \int d^4x\, Tr[F_{\mu\nu} F^{\mu\nu}$
$F_{\mu\nu}(x) = F^a_{\mu\nu}(x)\, t_a$
$t_a = \frac{1}{2}\sigma_a$
$= -\frac{1}{4g^2}\int d^4x\, F^a_{\mu\nu} F^{\mu\nu}_a$
g_{YM} = coupling constant of YM
$S_{YM}[A_\mu]$ invariant under local gauge transf.
$S_{YM}[A_\mu] \longrightarrow S_{YM}[g \cdot A_\mu g^{-1}] = -\frac{1}{2g^2_{YM}} \int Tr(g F_{\mu\nu} g^{-1} g F^{\mu\nu} g^{-1}) = S_{YM}[A_\mu]$
Classical theory gauge invariant
cyclic permutation

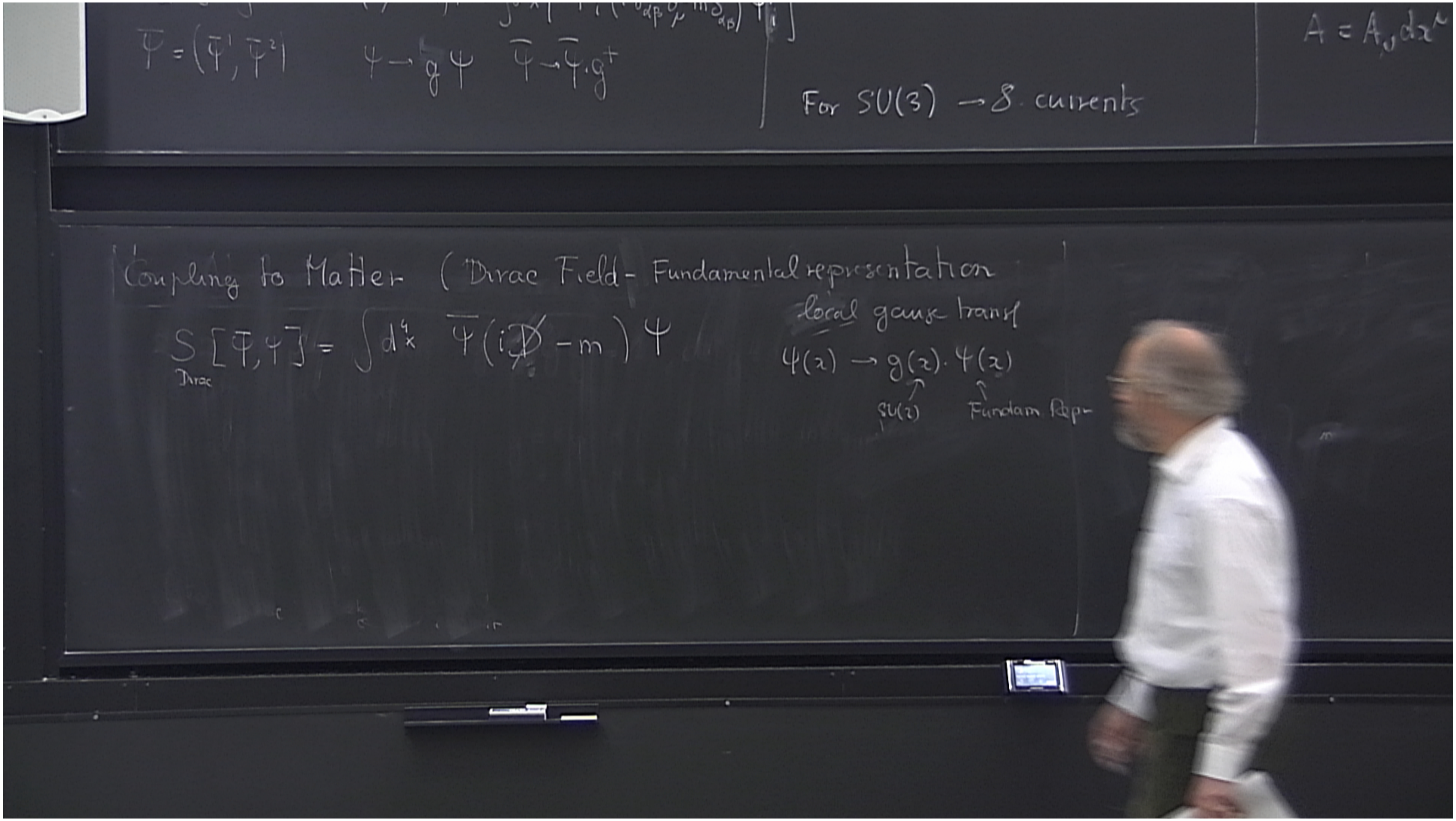

Global gauge trans
infinitesimal
g_{YM} = coupling constant of YM
under local gauge transf.
Classical
invariant
cyclic permutation

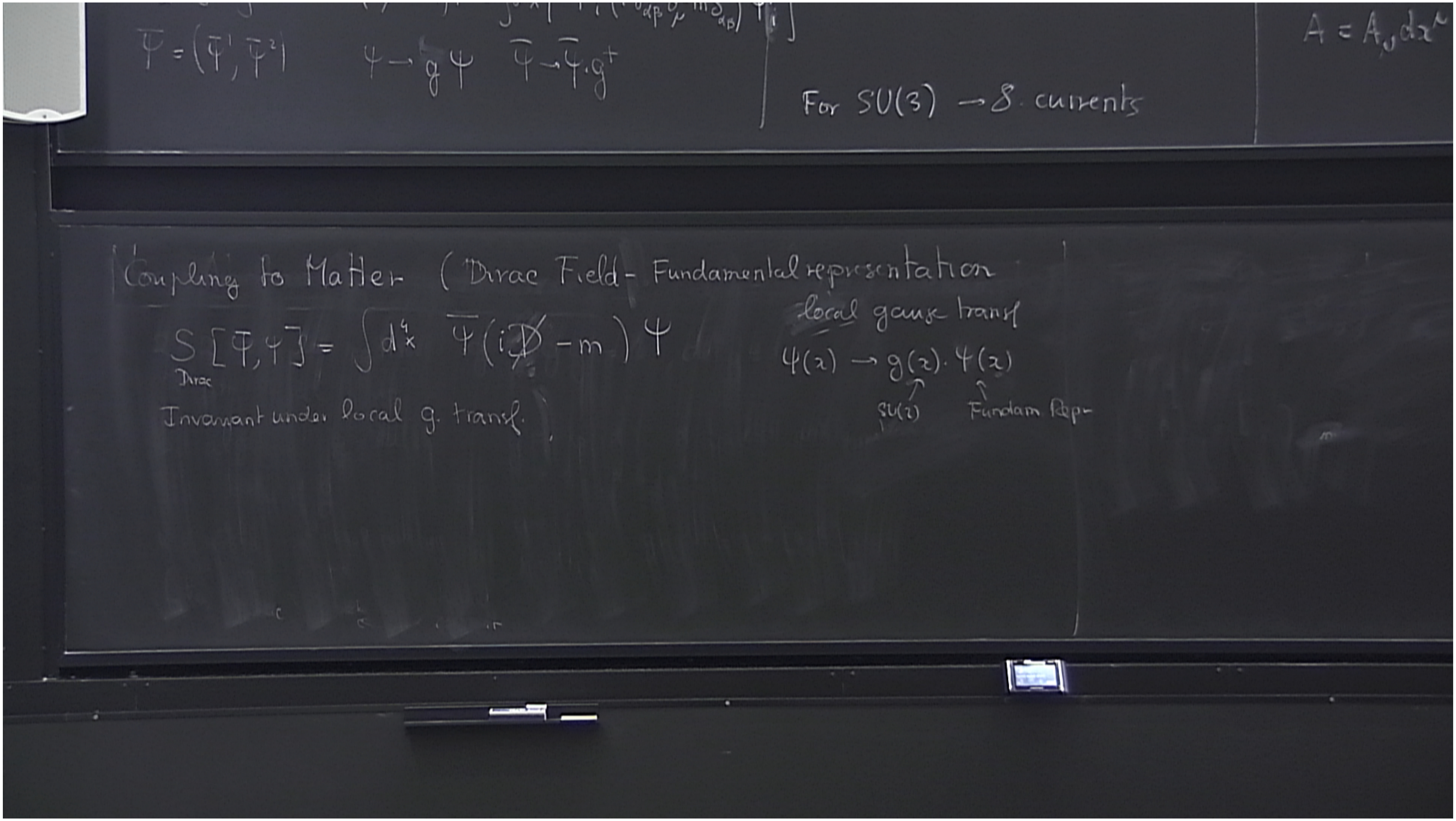

Global gauge trans
infinitesimal
$F_{\mu\nu}(x) \longrightarrow g(x)\cdot F_{\mu\nu}(x)\cdot g^{-1}(x)$
$F_{\mu\nu}(x) \to F_{\mu\nu}(x) - i\,[F_{\mu\nu}(x), \alpha(x)]$
$F_{\mu\nu} = -F_{\nu\mu}$
$F_{\mu\nu}(x) = F^a_{\mu\nu}(x)\, t_a$
$t_a = \frac{1}{2}\sigma_a$
$= -\frac{1}{4g^2}\int d^4x\, F^a_{\mu\nu} F^{\mu\nu}_a$
g_{YM} = coupling constant of YM
$S_{YM}[A_\mu]$ invariant under local gauge transf.
$S_{YM}[A_\mu] \longrightarrow S_{YM}[g\cdot A_\mu g^{-1}] = -\frac{1}{2g^2_{YM}}\int \cdot \mathrm{Tr}(g F_{\mu\nu} g^{-1} g F^{\mu\nu} g^{-1}) = S_{YM}[A_\mu]$
Classical theory gauge invariant
cyclic permutation


F_μν(x) → g(x) · F_μν(x) · g^-1(x)
Global gauge trans
infinitesimal g · t_L
F_μν(x) → F_μν(x) − i [F_μν(x), α(x)]
F_μν = −F_νμ
S_YM [A_μ] = − 1/(2g²_YM) ∫ d⁴x Tr[F_μν F^μν]
F_μν(x) = F^a_μν(x) t_a
t_a = ½ σ_a
= − 1/(4g²) ∫ d⁴x F^a_μν F^μν_a
g_YM = coupling constant of YM
S_YM [A_μ] invariant under local gauge transf.
A_μ → A'_μ = g (A_μ + i ∂_μ) g^-1
S_YM [A_μ] ⟶ S_YM [A'_μ] = − 1/(2g²_YM) ∫ Tr(g F_μν g^-1 g F^μν g^-1) = S_YM [A_μ]
cyclic permutation
Classical theory gauge invariant
physics is gauge invariant

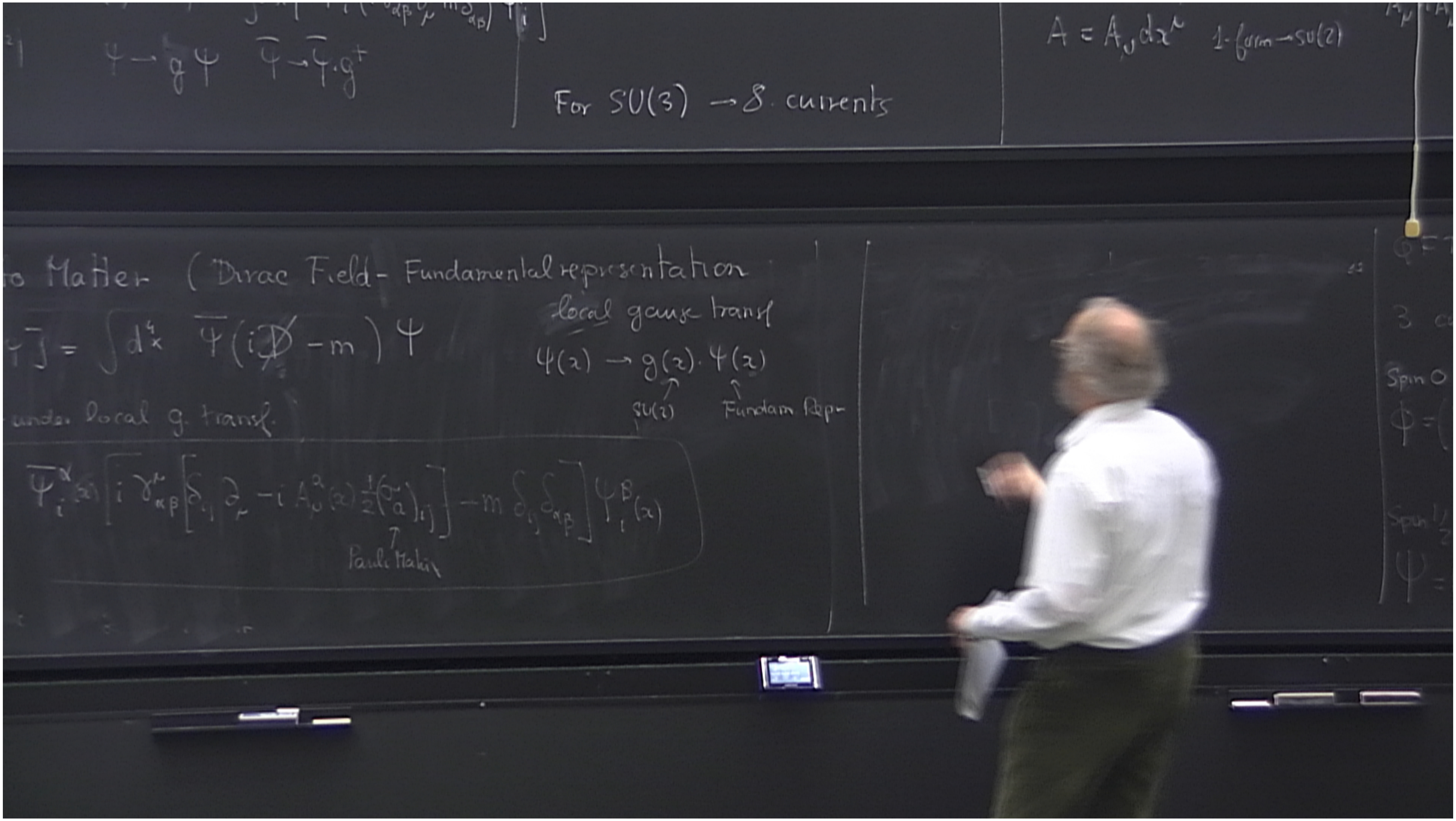

gauge trans
infinitesimal g. tr
S_YM [A_μ] = - 1/(2g²_YM) ∫ d⁴x Tr[F_μν F^μν]
F_μν(x) = F^a_μν(x) t_a
t_a = ½ σ_a
= - 1/(4g²) ∫ d⁴x F^a_μν F^μν_a
Tr = Trace in the Lie Algebra
For SU(2), only one trace (Group theory)
Tr(t_a t_b) = ½ δ_ab ← Pauli-Matrices
g_YM = coupling constant of the YM theory (charge of the particles)
A_μ → A′_μ = g (A_μ + i ∂_μ) g⁻¹
(g F_μν g⁻¹ g F^μν g⁻¹) = S_YM [A_μ]
Lagrangian Density
𝓛 = - 1/(4g²) [(∂_μ A^a_ν - ∂_ν A^a_μ)² + ½ ε_abc (∂_μ A^a_ν - ∂ ...) A^μ_b A^ν_c
+ (A^a_μ A^b_ν A^μ_a A^ν_b - A^a_μ A^b_ν A^μ_b A ...
f^a_bc = ε_abc SU(2)
f^e_ab f^e_cd = δ_ac δ_bd - δ_ad δ_bc for SU(2)
For SU(3)

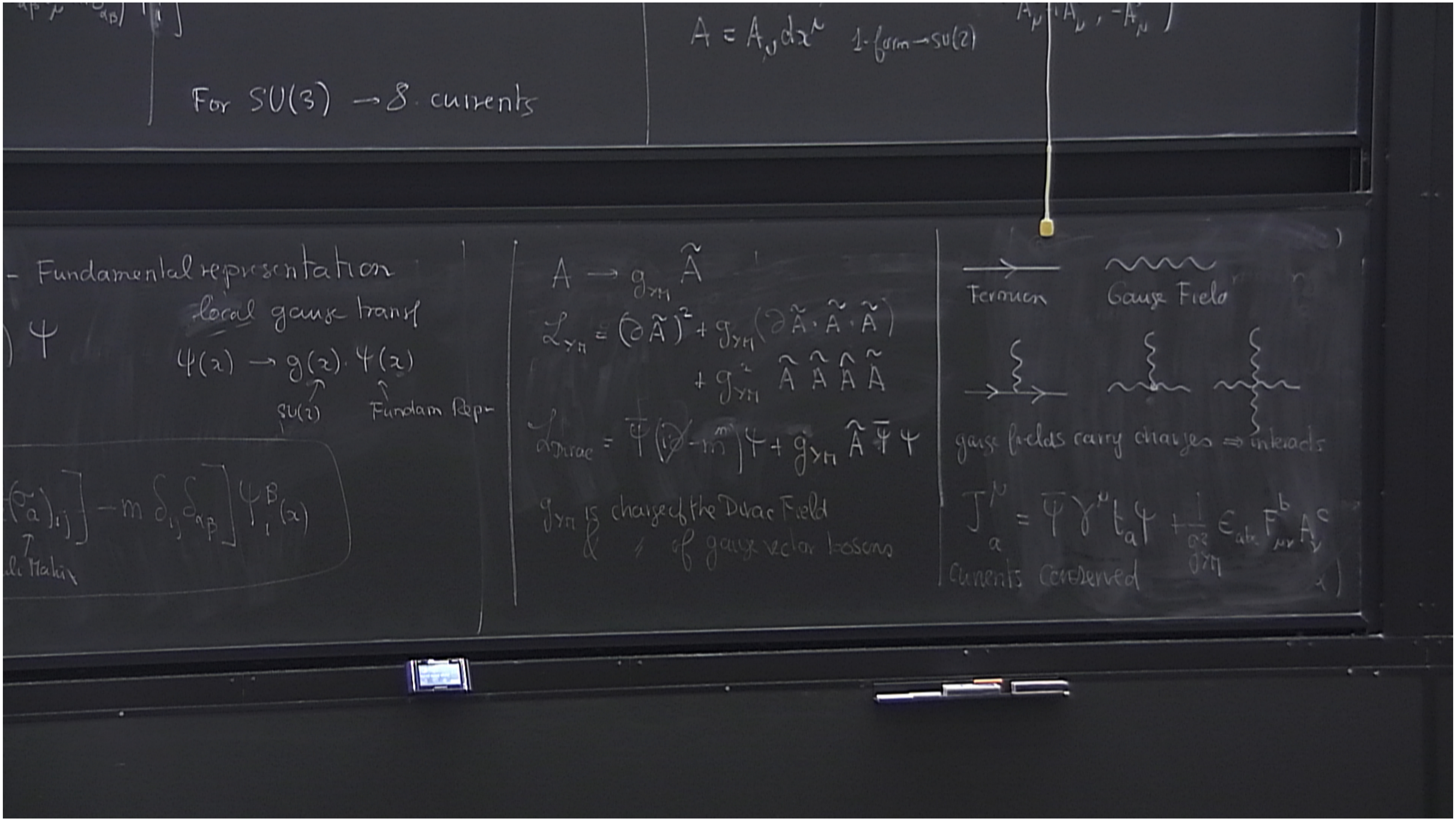

gauge trans
S_YM [A_μ] = - 1/(2 g²_YM) ∫ d⁴x Tr[F_μν F^μν]
F_μν(x) = F^a_μν(x) t_a
t_a = 1/2 σ_a
= - 1/(4g²) ∫ d⁴x F^a_μν F^μν_a
Tr = Trace in the Lie Algebra
For SU(2), only one trace (Group theory)
Tr(t_a t_b) = 1/2 δ_ab ← Pauli-Matrices
g_YM = coupling constant of the YM theory (charge of the particles)
A_μ → A'_μ = g (A_μ + i ∂_μ) g⁻¹
(g F_μν g⁻¹ · g F^μν g⁻¹) = S_YM [A_μ]
Lagrangian Density (classical)
ℒ = - 1/(4g²) [(∂_μ A^a_ν - ∂_ν A^a_μ)² + 1/2 ε_abc (∂_μ A^a_ν - ∂_ν A^a_μ) A^μ_b A^ν_c
+ (A^a_μ A^b_ν A^μ_a A^ν_b - A^a_μ A^b_ν A^μ_b A^ν_a)]
f^a_bc = ε_abc SU(2)
f^e_ab f^e_cd = δ_ac δ_bd - δ_ad δ_bc for SU(2)
For SU(3) it will be more complicated


For SU(3) → 8 currents
Coupling to Matter (Dirac Field - Fundamental representation
local gauge transf
Fundam Repr



$\bar{\Psi} = (\bar{\Psi}^1, \bar{\Psi}^2)$ $\Psi \to g\Psi$ $\bar{\Psi} \to \bar{\Psi} \cdot g^\dagger$
For SU(3) → 8 currents
$A = A_\mu dx^\mu$
Coupling to Matter (Dirac Field - Fundamental representation
$S_{Dirac}[\bar{\Psi}, \Psi] = \int d^4x \; \bar{\Psi}(i\not{D} - m)\Psi$
local gauge transf
$\Psi(x) \to g(x) \cdot \Psi(x)$
SU(2)
Fundam Rep
Invariant under local g. transf.


ψ → g ψ   ψ̄ → ψ̄ · g†
For SU(3) → 8 currents
A = A_μ dx^μ   1-form → SU(2)
Matter (Dirac Field - Fundamental representation
local gauge transf
ψ(x) → g(x) · ψ(x)
SU(2)
Fundam Repr
under local g. transf.
Pauli Matrix


For SU(3) → 8 currents
A = A_ν dx^ν 1-form → su(2)
Fundamental representation
local gauge transf
ψ(x) → g(x)·ψ(x)
SU(2)
Fundam Rep
A → g_YM Ã
Fermion
Gauge Field
gauge fields carry charges ⇒ interacts
g_YM is charge of the Dirac Field
& of gauge vector bosons
Currents Conserved