Title: PSI 2015/2016 Quantum Field Theory II - Francois David - Lecture 11

Date: Nov 23, 2015 09:00 AM

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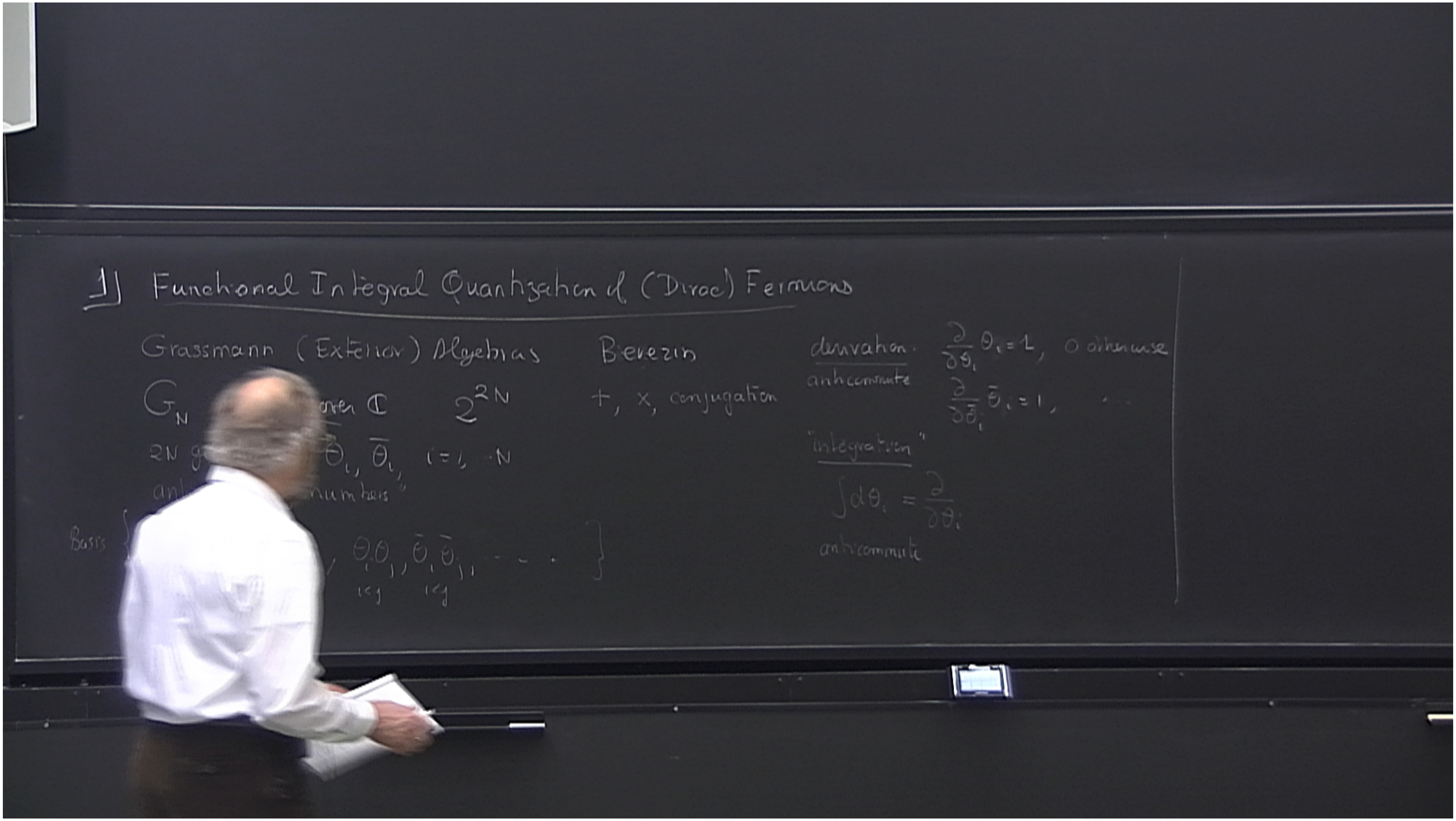
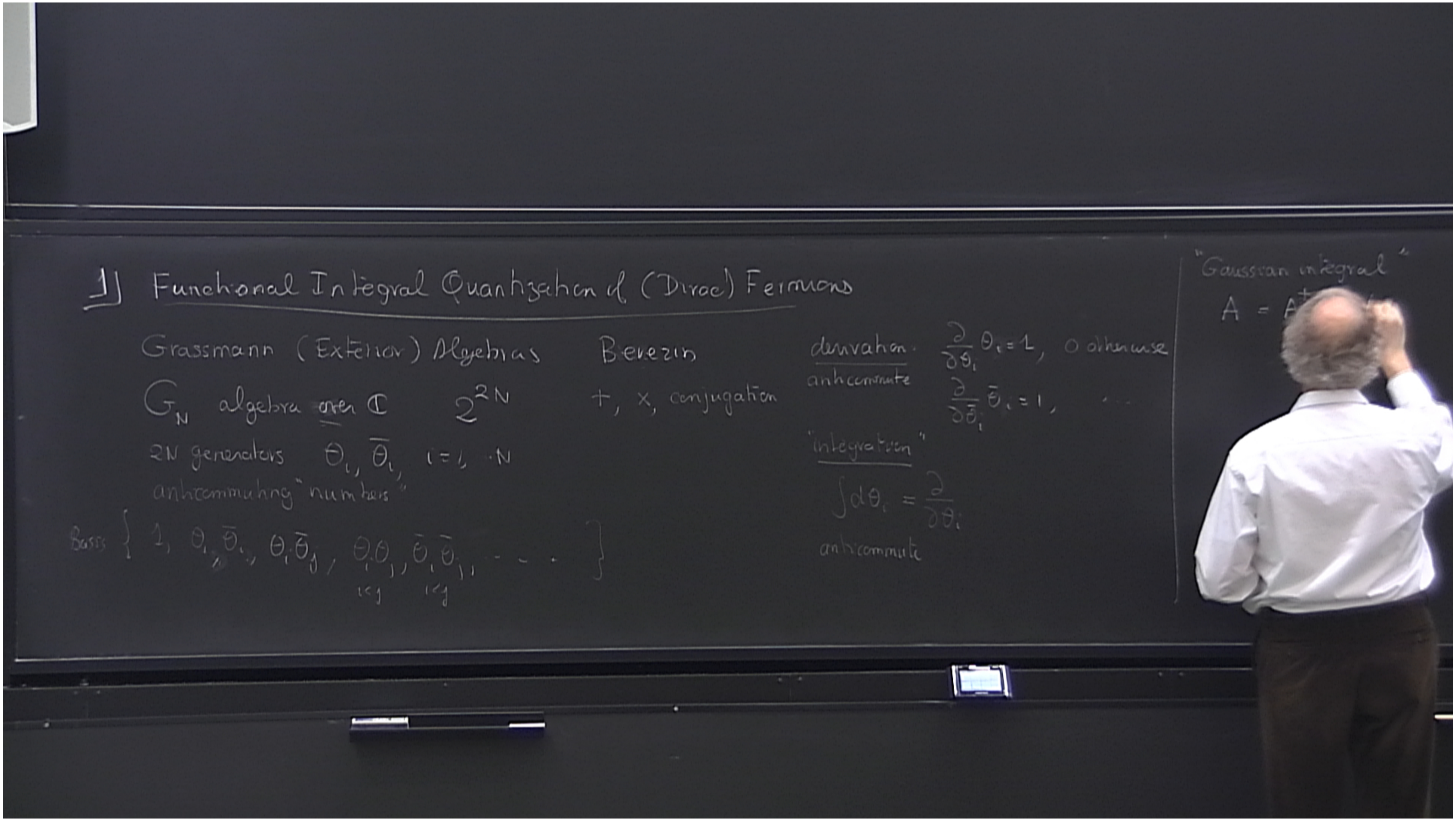
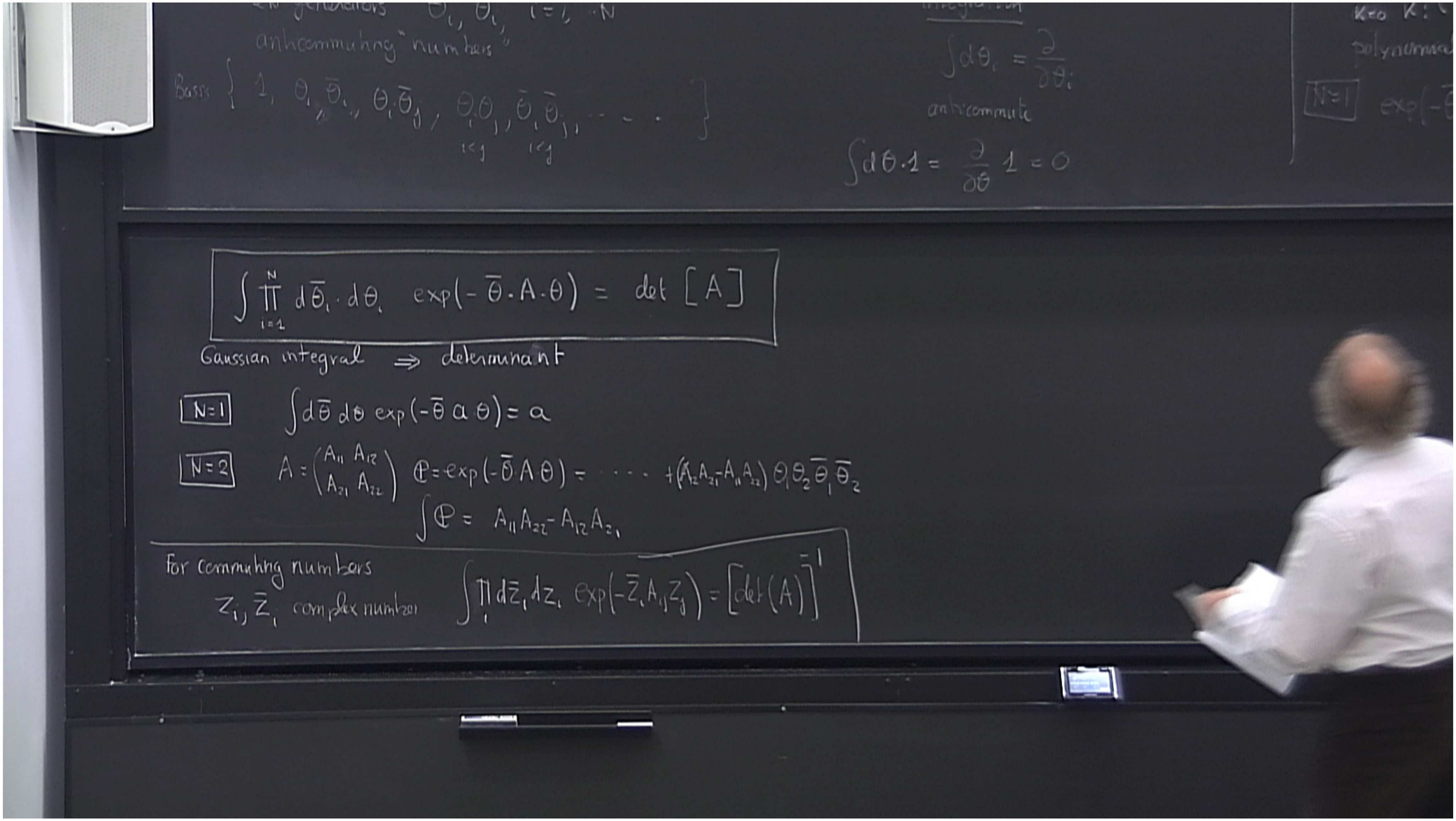
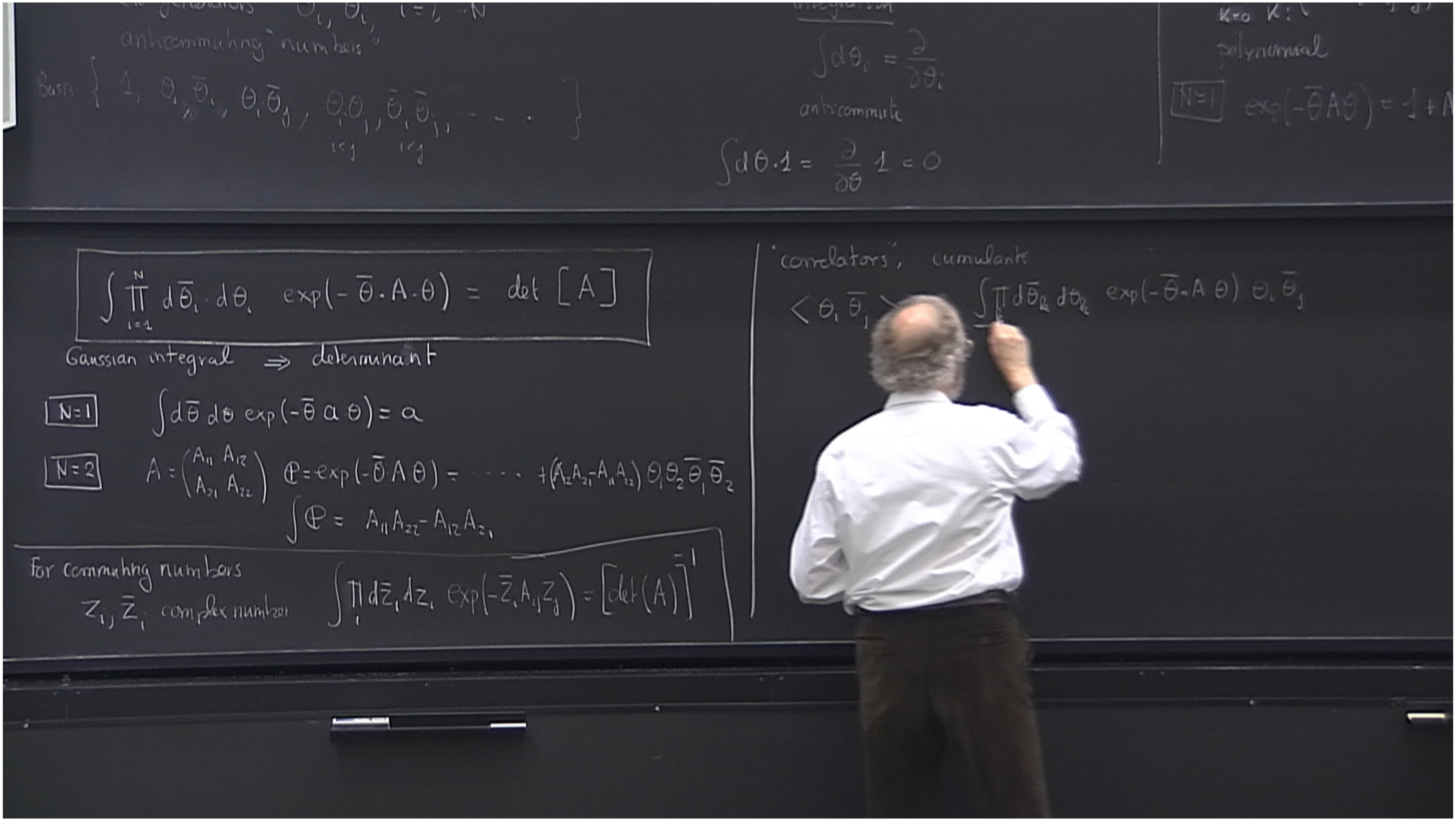
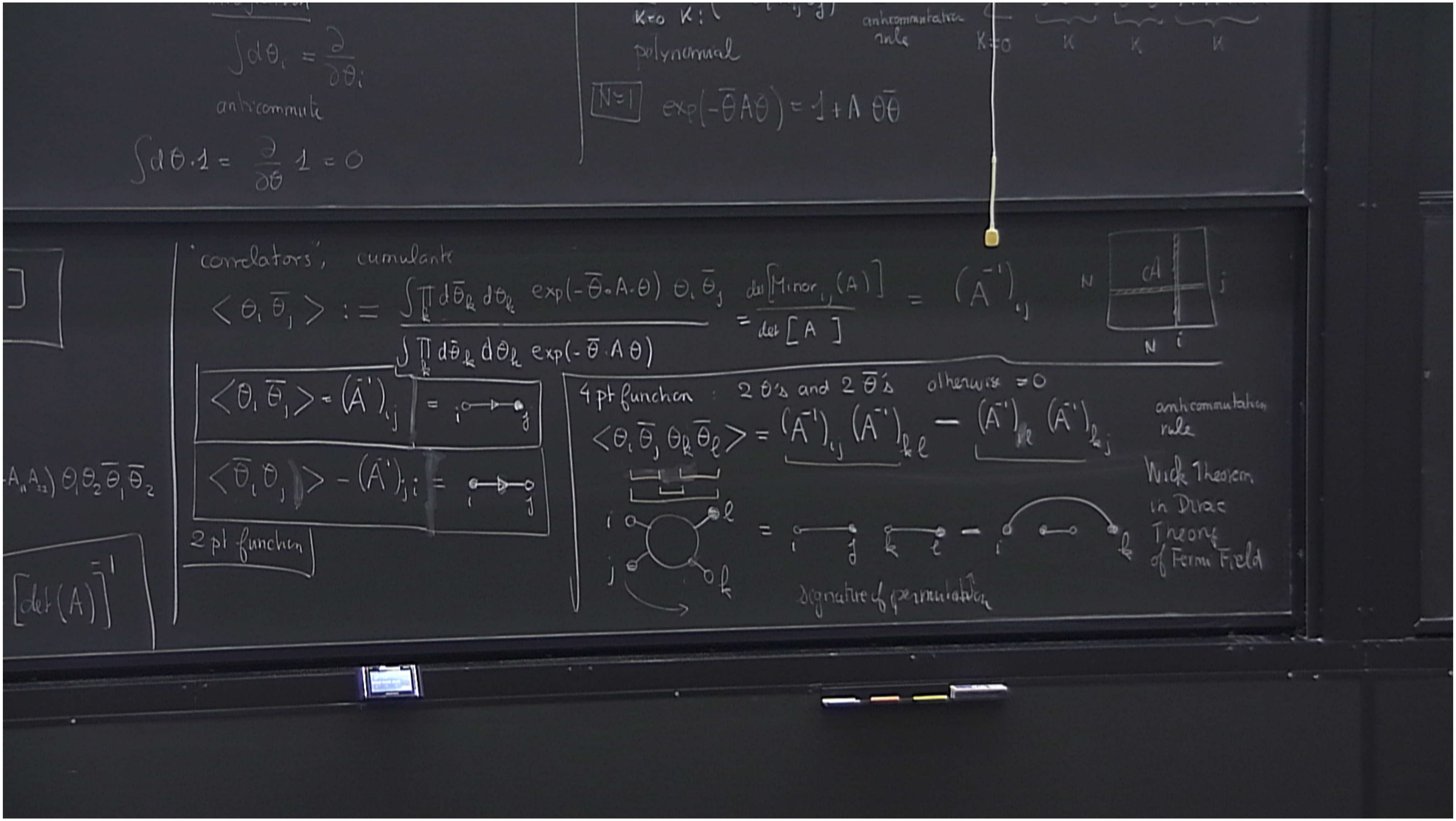
Abstract:

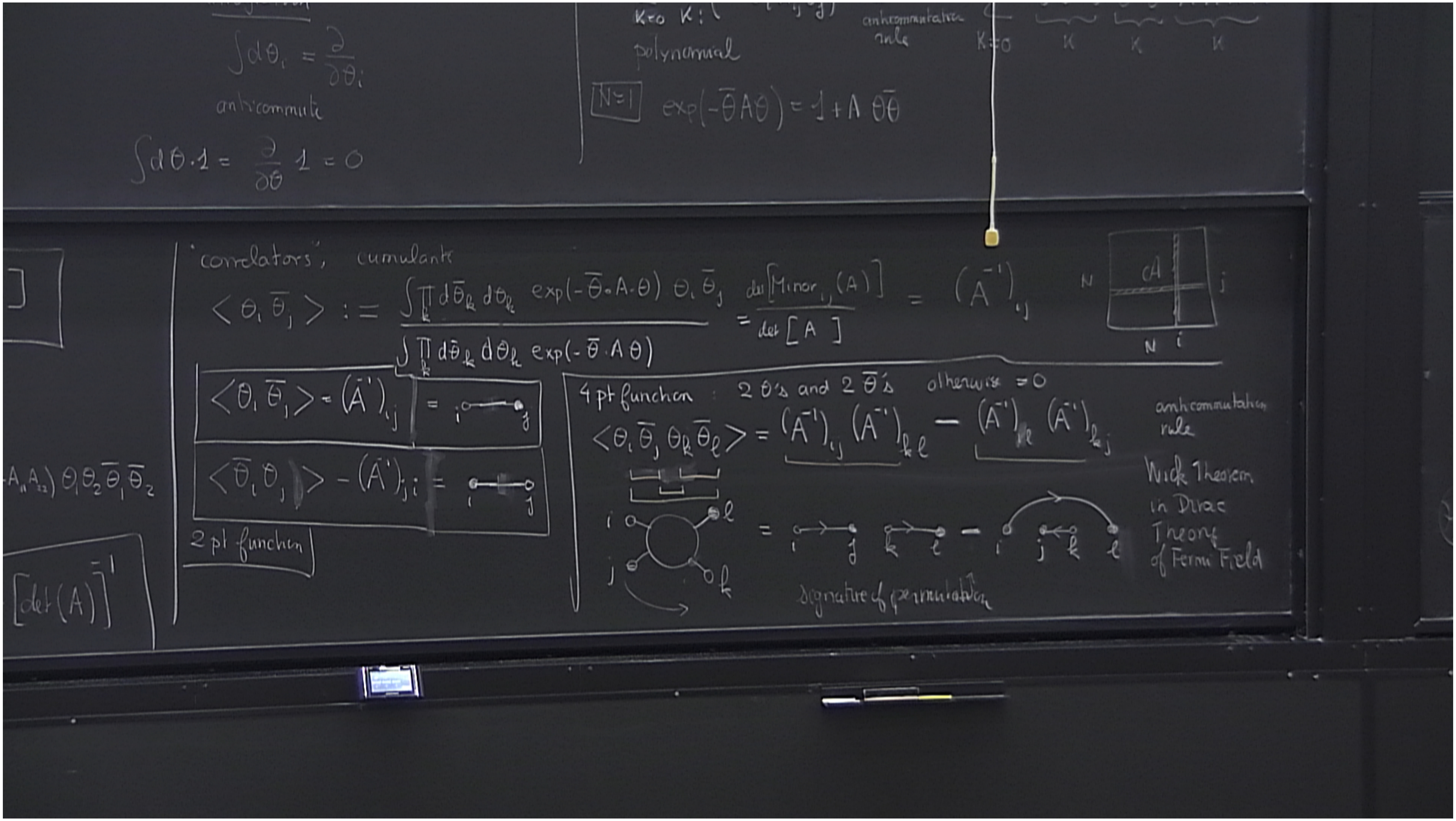

1) Functional Integral Quantization of (Dirac) Fermions
Grassmann (Exterior) Algebras
Berezin
G_N over ℂ
2^2N
+, x, conjugation
θ_i, θ̄_i i = 1 .. N
numbers
Basis
θ_iθ_j, θ̄_iθ̄_j
i<j
derivation
anticommute
∂/∂θ_i θ_i = 1, 0 otherwise
∂/∂θ̄_i θ̄_i = 1
"integration"
∫dθ_i = ∂/∂θ_i
anticommute

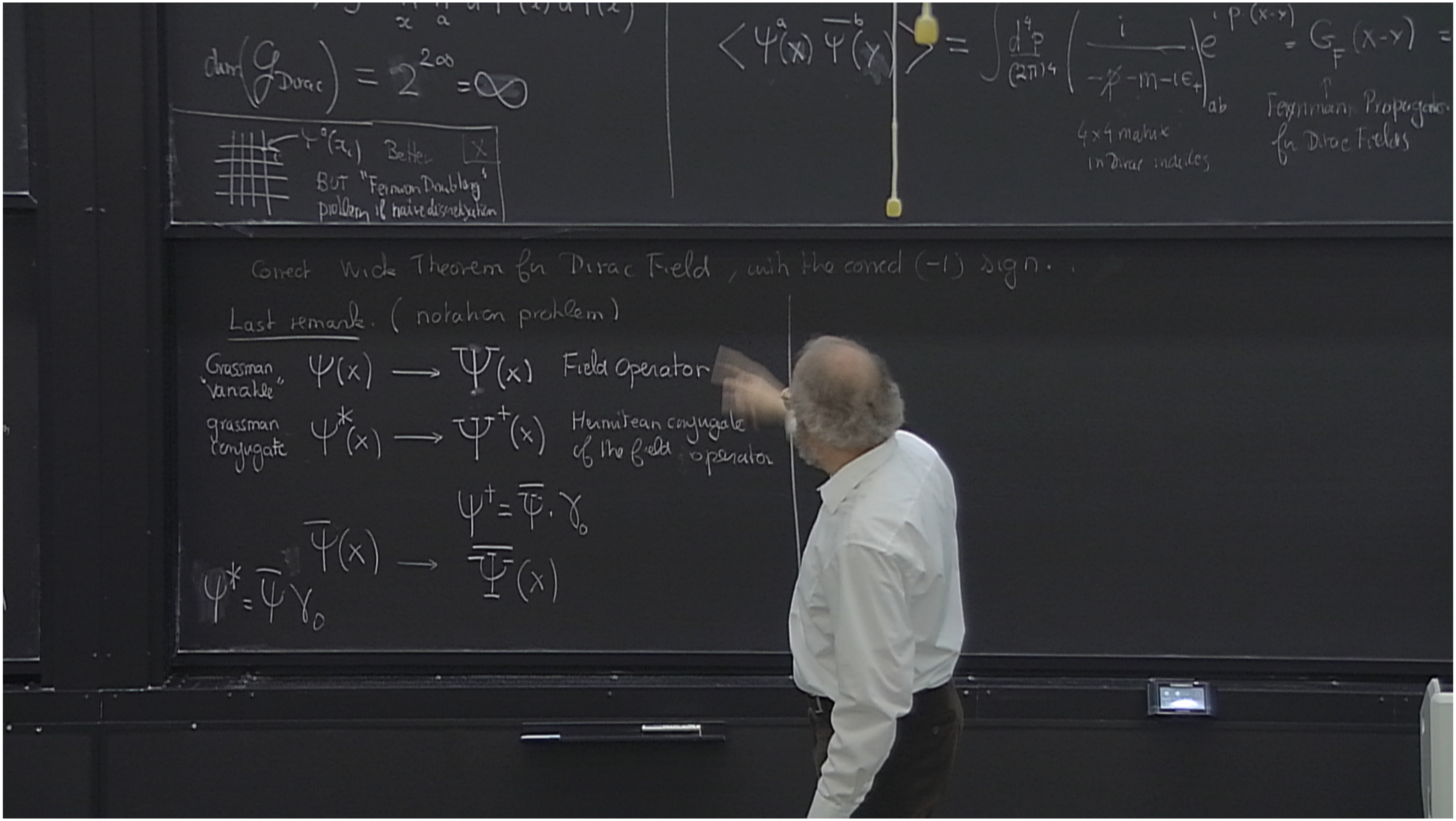

1) Functional Integral Quantization of (Dirac) Fermions
Grassmann (Exterior) Algebras
Berezin
G_N algebra over $\mathbb{C}$
2^{2N}
+, ×, conjugation
2N generators $\theta_i, \bar\theta_i$ $i = 1, \cdots N$
anticommuting "numbers"
Basis $\{ 1, \theta_i, \bar\theta_i, \theta_i\bar\theta_j, \theta_i\theta_j, \bar\theta_i\bar\theta_j, \ldots \}$ $i<j$ $i<j$
derivation
anticommute
$\frac{\partial}{\partial\theta_i}\theta_i = 1$, 0 otherwise
$\frac{\partial}{\partial\bar\theta_i}\bar\theta_i = 1$, ...
"integration"
$\int d\theta_i = \frac{\partial}{\partial\theta_i}$
anticommute
"Gaussian integral"
A = A

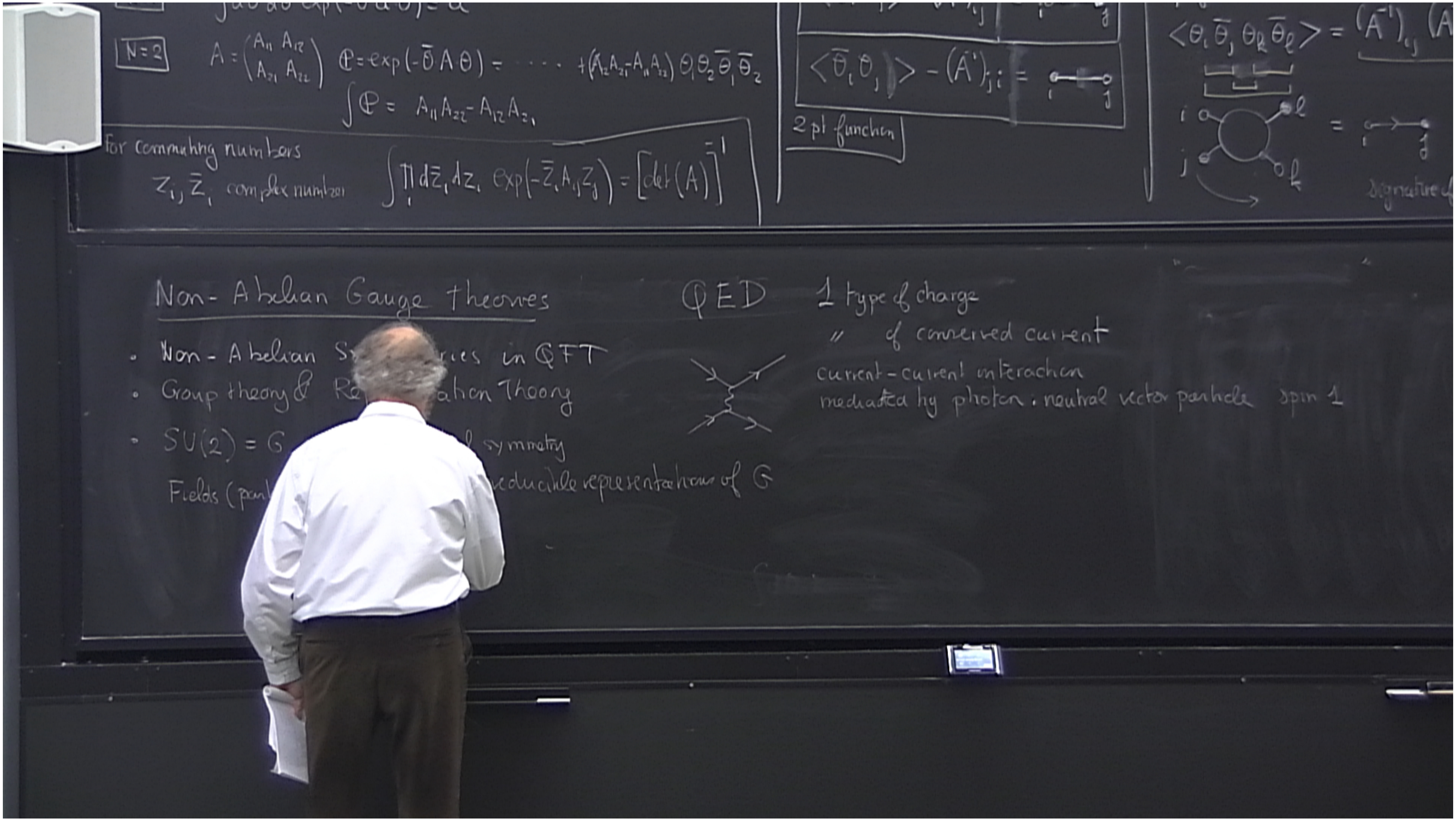

anticommuting "numbers"
Basis $\{ 1, \theta_i, \bar\theta_i, \theta_i\bar\theta_j, \theta_i\theta_j, \bar\theta_i\bar\theta_j, \ldots \}$ $i<j$ $i<j$
$\int d\theta_i = \frac{\partial}{\partial\theta_i}$
anticommute
$\int d\theta \cdot 1 = \frac{\partial}{\partial\theta} 1 = 0$
polynomial
N=1 $\exp(-\bar\theta$
$\int \prod_{i=1}^{N} d\bar\theta_i \cdot d\theta_i \ \exp(-\bar\theta \cdot A \cdot \theta) = \det[A]$
Gaussian integral ⇒ determinant
N=1 $\int d\bar\theta \, d\theta \, \exp(-\bar\theta \, a \, \theta) = a$
N=2 $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$ $\Phi = \exp(-\bar\theta A \theta) = \cdots + (A_{12}A_{21} - A_{11}A_{22})\theta_1\theta_2\bar\theta_1\bar\theta_2$
$\int \Phi = A_{11}A_{22} - A_{12}A_{21}$
For commuting numbers
$z_i, \bar z_i$ complex number $\int \prod_i d\bar z_i \, dz_i \ \exp(-\bar z_i A_{ij} z_j) = [\det(A)]^{-1}$


anticommuting "numbers"
Basis $\{1, \theta_i, \bar\theta_i, \theta_i\bar\theta_j, \theta_i\theta_j, \bar\theta_i\bar\theta_j, \ldots\}$
$i<j$ $i<j$
$\int d\theta_i = \frac{\partial}{\partial\theta_i}$
anticommute
$\int d\theta \cdot 1 = \frac{\partial}{\partial\theta} 1 = 0$
polynomial
N=1 $\exp(-\bar\theta A \theta) = 1 + A$
$\int \prod_{i=1}^{N} d\bar\theta_i \cdot d\theta_i \ \exp(-\bar\theta \cdot A \cdot \theta) = \det[A]$
Gaussian integral ⇒ determinant
N=1 $\int d\bar\theta\, d\theta\, \exp(-\bar\theta\, a\, \theta) = a$
N=2 $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$ $\Phi = \exp(-\bar\theta A \theta) = \cdots + (A_{12}A_{21} - A_{11}A_{22})\,\theta_1\theta_2\bar\theta_1\bar\theta_2$
$\int \Phi = A_{11}A_{22} - A_{12}A_{21}$
For commuting numbers
$z_i, \bar z_i$ complex number
$\int \prod_i d\bar z_i\, dz_i\, \exp(-\bar z_i A_{ij} z_j) = [\det(A)]^{-1}$
"correlators", cumulants
$\langle \theta_i \bar\theta_j \rangle$
$\int \prod d\bar\theta_k\, d\theta_k\, \exp(-\bar\theta \cdot A\, \theta)\, \theta_i \bar\theta_j$


∫dθ_i = ∂/∂θ_i
anticommute
∫dθ·1 = ∂/∂θ 1 = 0
polynomial
N=1 exp(−θ̄Aθ) = 1 + A θθ̄
anticommutation rule
'correlators', cumulants
<θ_i θ̄_j> := ∫∏_k dθ̄_k dθ_k exp(−θ̄·A·θ) θ_i θ̄_j / ∫∏_k dθ̄_k dθ_k exp(−θ̄·A·θ) = det[Minor_ij(A)] / det[A] = (A^{-1})_ij
<θ_i θ̄_j> = (A^{-1})_ij = i → j
<θ̄_i θ_j> = −(A^{-1})_ji = i → j
2 pt function
4 pt function : 2 θ's and 2 θ̄'s otherwise = 0
<θ_i θ̄_j θ_k θ̄_l> = (A^{-1})_ij (A^{-1})_kl − (A^{-1})_il (A^{-1})_kj
anticommutation rule
signature of permutation
Wick Theorem in Dirac Theory of Fermi Field
[det(A)]^{-1}


∫dθ_i = ∂/∂θ_i
anticommute
∫dθ·1 = ∂/∂θ 1 = 0
polynomial
N=1 exp(−θ̄Aθ) = 1 + A θθ̄
anticommutation rule
'correlators', cumulants
<θ_i θ̄_j> := ∫∏_k dθ̄_k dθ_k exp(−θ̄·A·θ) θ_i θ̄_j / ∫∏_k dθ̄_k dθ_k exp(−θ̄·A·θ) = det[Minor_ij(A)] / det[A] = (A^{-1})_ij
<θ_i θ̄_j> = (A^{-1})_ij
<θ̄_i θ_j> = −(A^{-1})_ji
2 pt function
4 pt function : 2 θ's and 2 θ̄'s otherwise = 0
<θ_i θ̄_j θ_k θ̄_l> = (A^{-1})_ij (A^{-1})_kl − (A^{-1})_il (A^{-1})_kj
anticommutation rule
Wick Theorem in Dirac Theory of Fermi Field
signature of permutation
[det(A)]^{-1}


dim(G Dirac) = 2^{2∞} = ∞
Better
BUT "Fermion Doubling" problem if naive discretization
⟨Ψ^a(x) Ψ̄^b(y)⟩ = ∫ d^4p/(2π)^4 (i / (-p̸ - m - iε+))_ab e^{ip·(x-y)} = G_F(x-y) =
4×4 matrix in Dirac indices
Feynman Propagator for Dirac Fields
Correct Wick Theorem for Dirac Field, with the correct (-1) sign.
Last remark. (notation problem)
Grassman "variable" ψ(x) ⟶ Ψ(x) Field Operator
grassman conjugate ψ*(x) ⟶ Ψ⁺(x) Hermitean conjugate of the field operator
Ψ⁺ = Ψ̄ · γ0
ψ̄(x) ⟶ Ψ̄(x)
ψ* = ψ̄ γ0


N=2
2 pt function
for commuting numbers
complex number
Non-Abelian Gauge theories
Non-Abelian
in QFT
Group theory &
Theory
symmetry
Fields
irreducible representations of G
QED
1 type of charge
of conserved current
current-current interaction
mediated by photon. neutral vector particle spin 1

$N=2$ $\quad A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$ $\quad \Phi = \exp(-\bar\theta A \theta) = \cdots + (A_{12}A_{21} - A_{11}A_{22})\,\theta_1\theta_2\bar\theta_1\bar\theta_2$

$$\int \Phi = A_{11}A_{22} - A_{12}A_{21}$$

for commuting numbers

$z_i, \bar z_i$ complex number $\quad \int \prod_i d\bar z_i\, dz_i \, \exp(-\bar z_i A_{ij} z_j) = [\det(A)]^{-1}$

$\langle \bar\theta_i \theta_j \rangle - (A^{-1})_{ji}$

2 pt function

$\langle \theta_i \bar\theta_j \theta_k \bar\theta_\ell \rangle = (A^{-1})_{ij} \ldots$

signature of

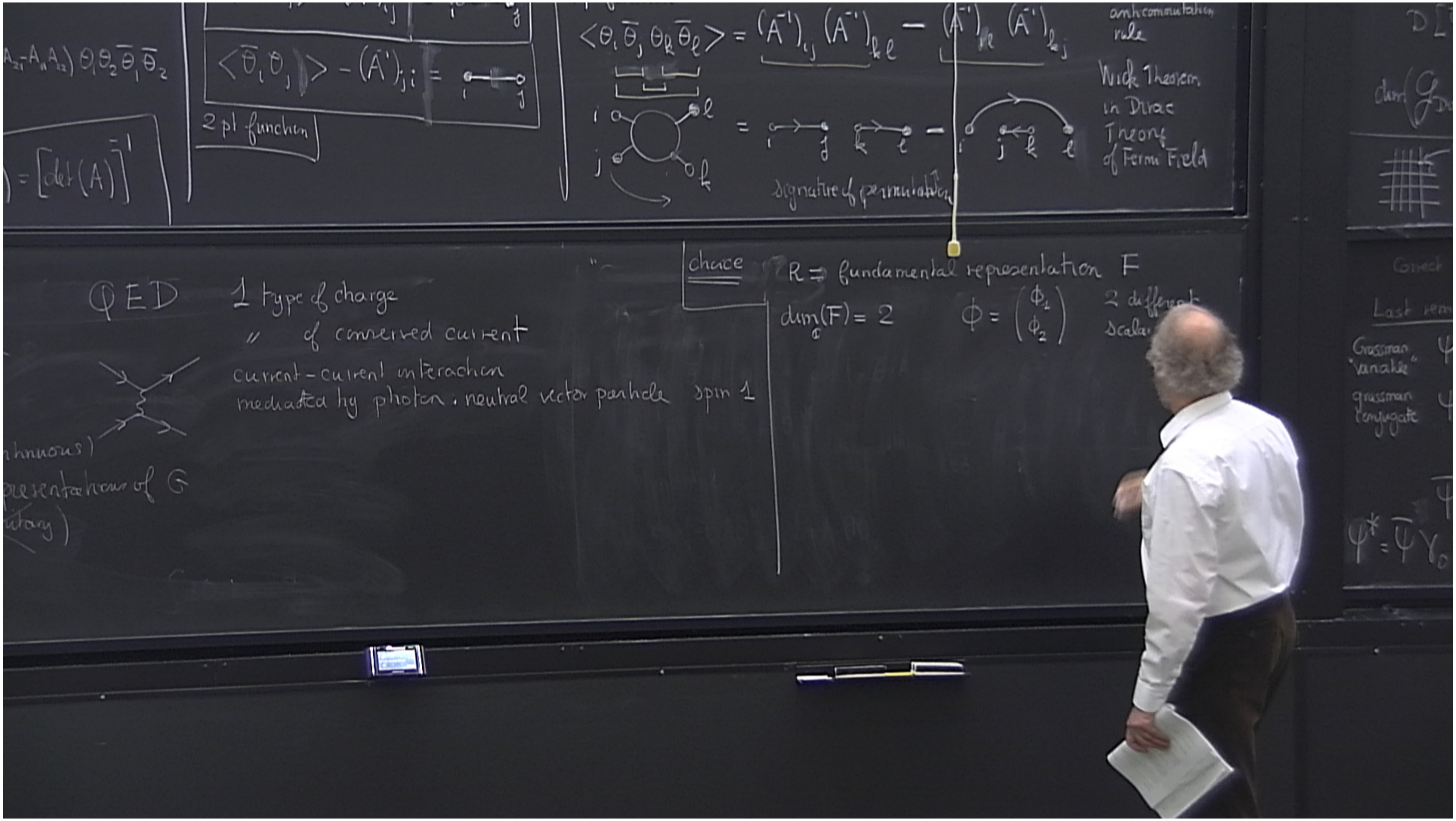
Non-Abelian Gauge theories

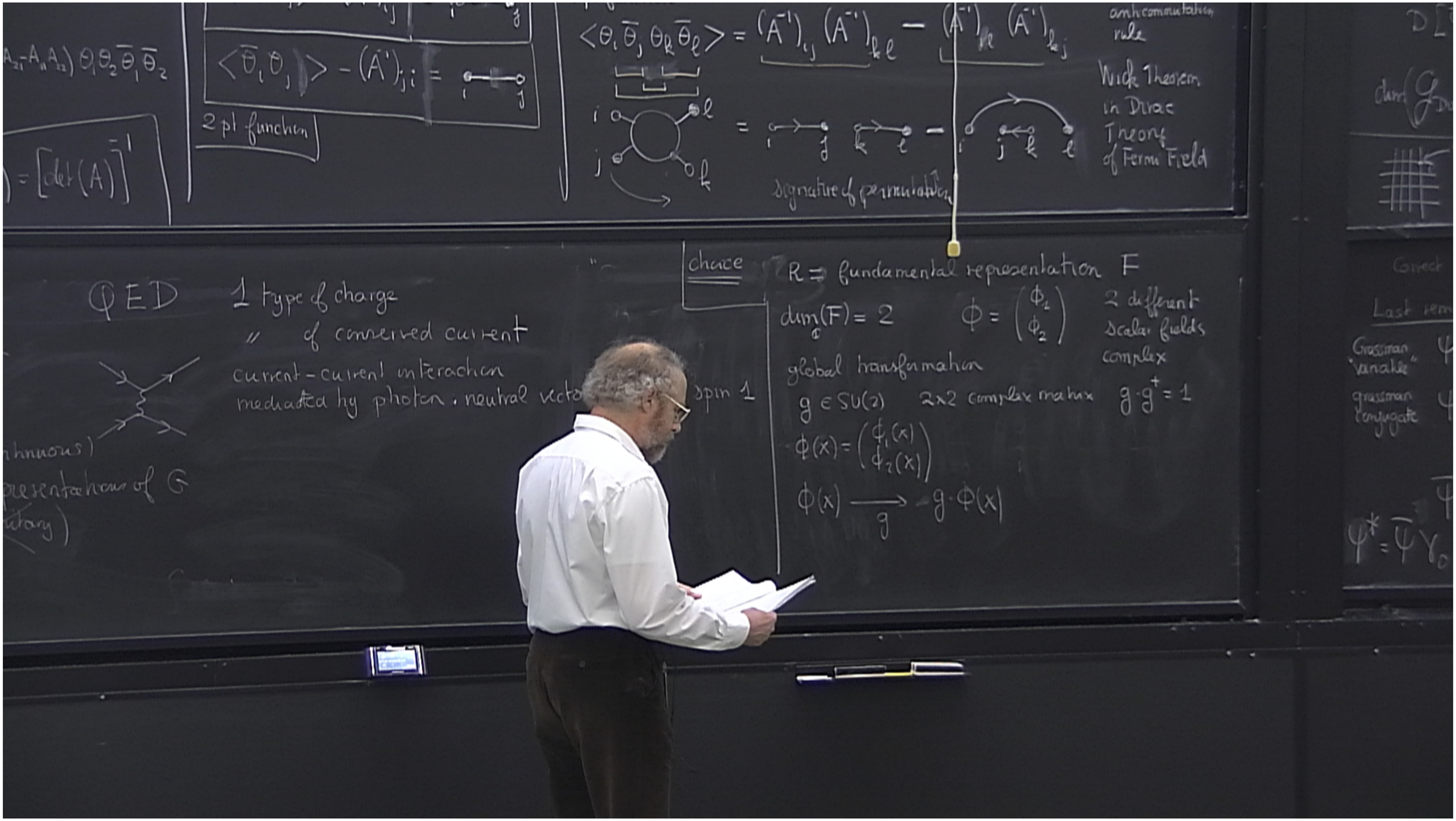
- Non-Abelian Symmetries in QFT
- Group theory & Representation Theory
- SU(2) = G group $\quad$ global symmetry continuous)

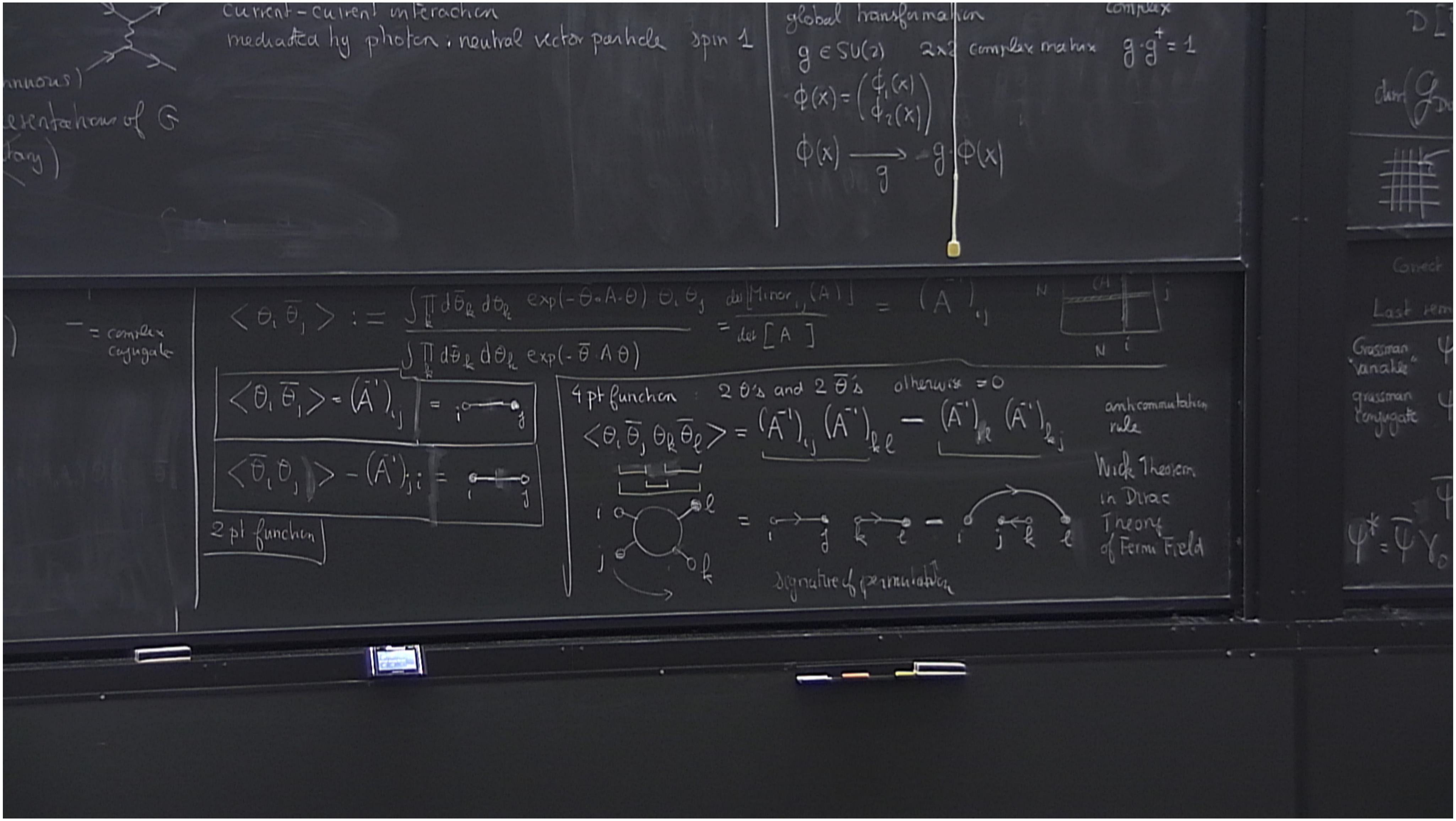
Fields (particle) belongs to some irreducible representations of G
(unitary or ~~antiunitary~~)

QED $\quad$ 1 type of charge
" $\quad$ of conserved current

current-current interaction
mediated by photon: neutral vector particle spin 1


QED
1 type of charge
" of conserved current
current-current interaction
mediated by photon. neutral vector particle spin 1
R ⇒ fundamental representation F
dim(F) = 2
2 pt function
Wick Theorem in Dirac Theory of Fermi Field
signature of permutation
anticommutation rule
Last term
Grassman "variable"
grassman conjugate


QED
1 type of charge
" of conserved current
current-current interaction
mediated by photon, neutral vector spin 1
choice
R = fundamental representation F
dim(F) = 2
2 different scalar fields complex
global transformation
g ∈ SU(2) 2x2 complex matrix
Wick Theorem in Dirac Theory of Fermi Field
2 pt function
signature of permutation
anticommutation rule


Current-current interaction
mediated by photon: neutral vector particle spin 1
global transformation
complex
2x2 complex matrix
= complex conjugate
2 pt function
4 pt function
2 θ's and 2 θ̄'s otherwise = 0
anticommutation rule
Wick Theorem in Dirac Theory of Fermi Field
signature of permutation
Grassman variable
grassman conjugate