Title: PSI 2015/2016 Quantum Field Theory II - Francois David - Lecture 10

Date: Nov 20, 2015 09:00 AM

URL: http://pirsa.org/15110036

Abstract:

RG in the local potential approximation

Start from bare action for theory with cut-off $|k| < \Lambda$

$$\mathcal{A}_B(\phi_B) = \int d^4x_B\ \frac{1}{2}(\partial\phi_B)^2 + V_B(\phi_B)$$

Reexpress in dimensionless variables

$$\phi_B = \Lambda\phi\ ,\quad x_B = \Lambda^{-1}x\ ,\quad V_B = \Lambda^4 V$$

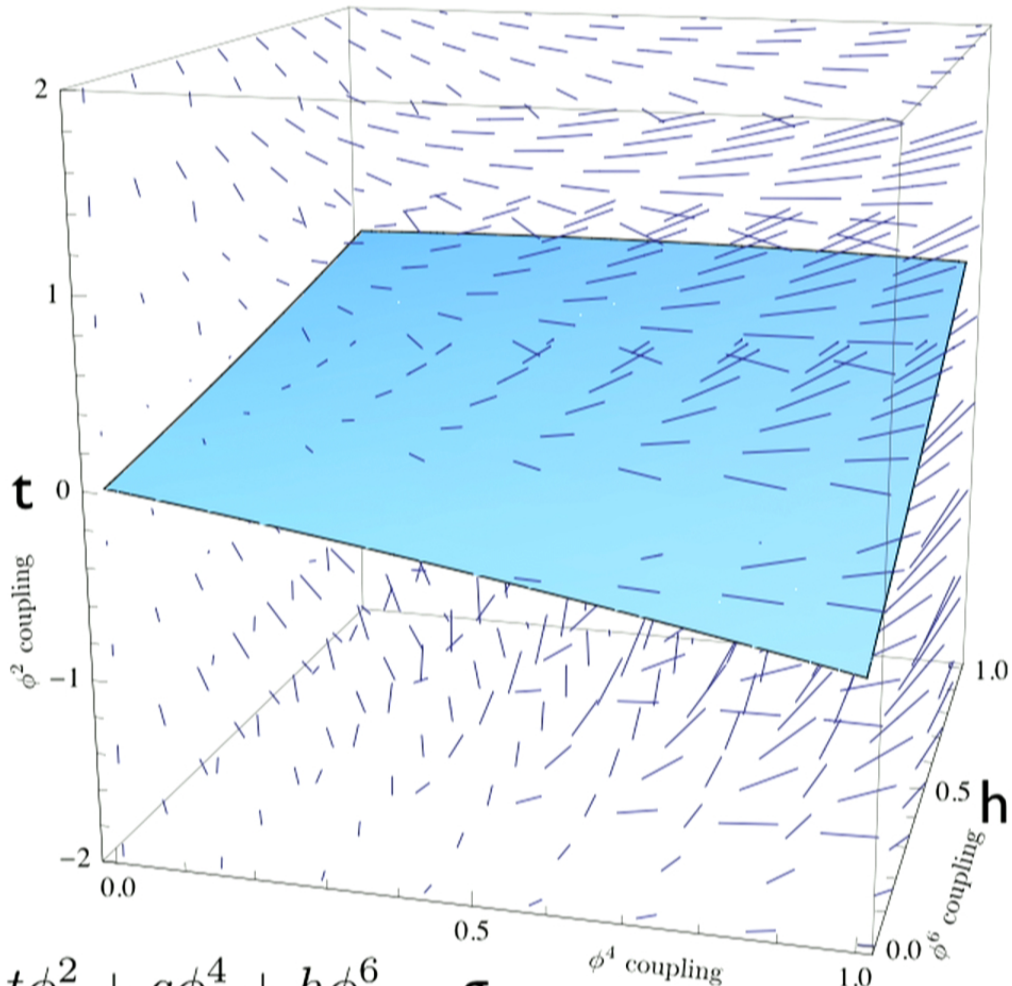
Perform renormalisation group transformation up to physical scale μ

$$S\frac{\partial}{\partial S}V_S = 4V_S - \phi\frac{\partial}{\partial\phi}S + \mathbb{A}\log\left(1 + \frac{\partial^2}{\partial\phi^2}V_S\right)$$

Reexpress in physical units to obtain the renormalized parameters

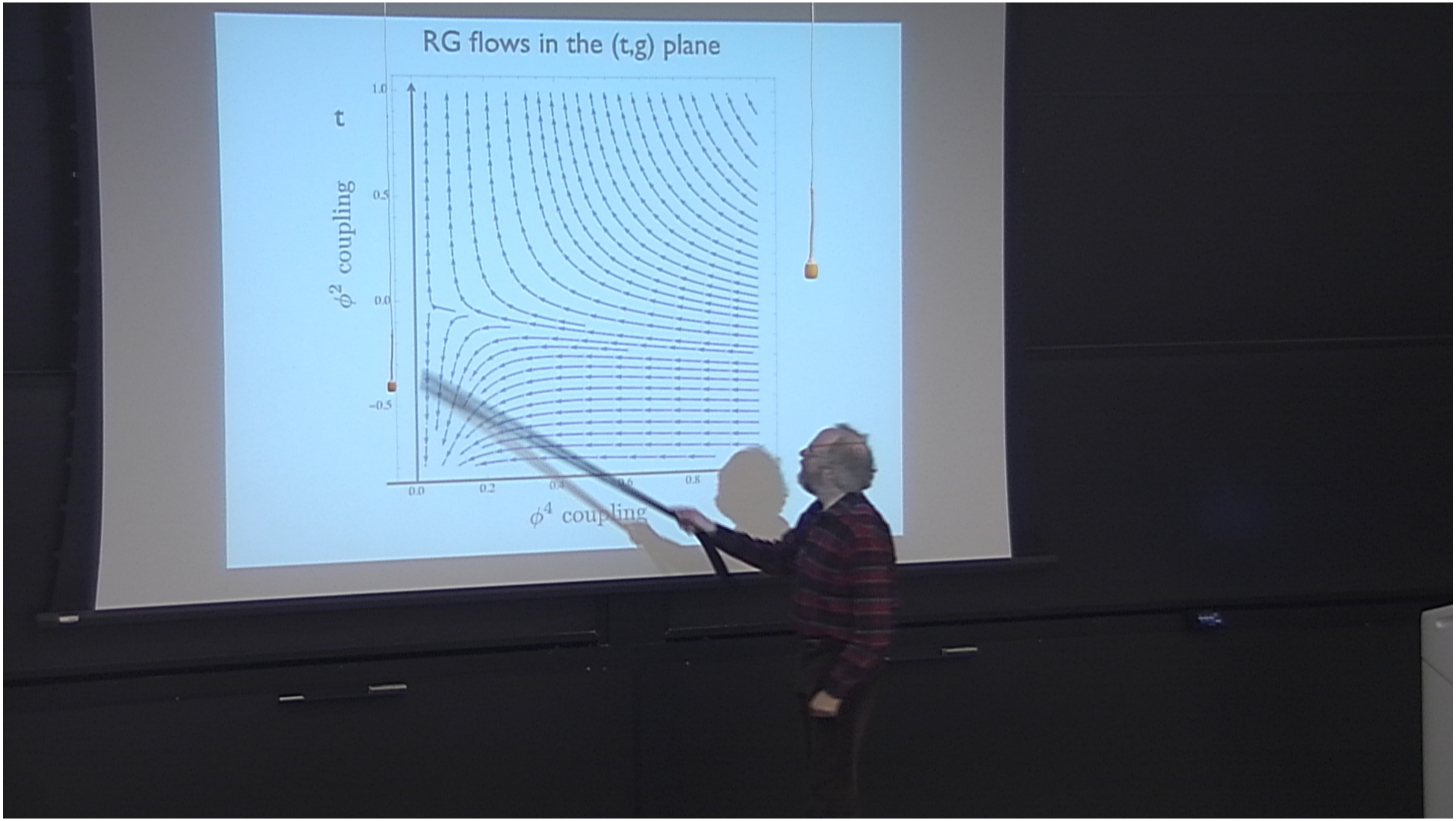
$$\mu = \Lambda/S\ ,\quad \phi = \mu^{-1}\phi_R\ ,\quad x = \mu x_R\ ,\quad V_S = \mu^{-4}V_R$$

RG flows in the (t, g, h) space



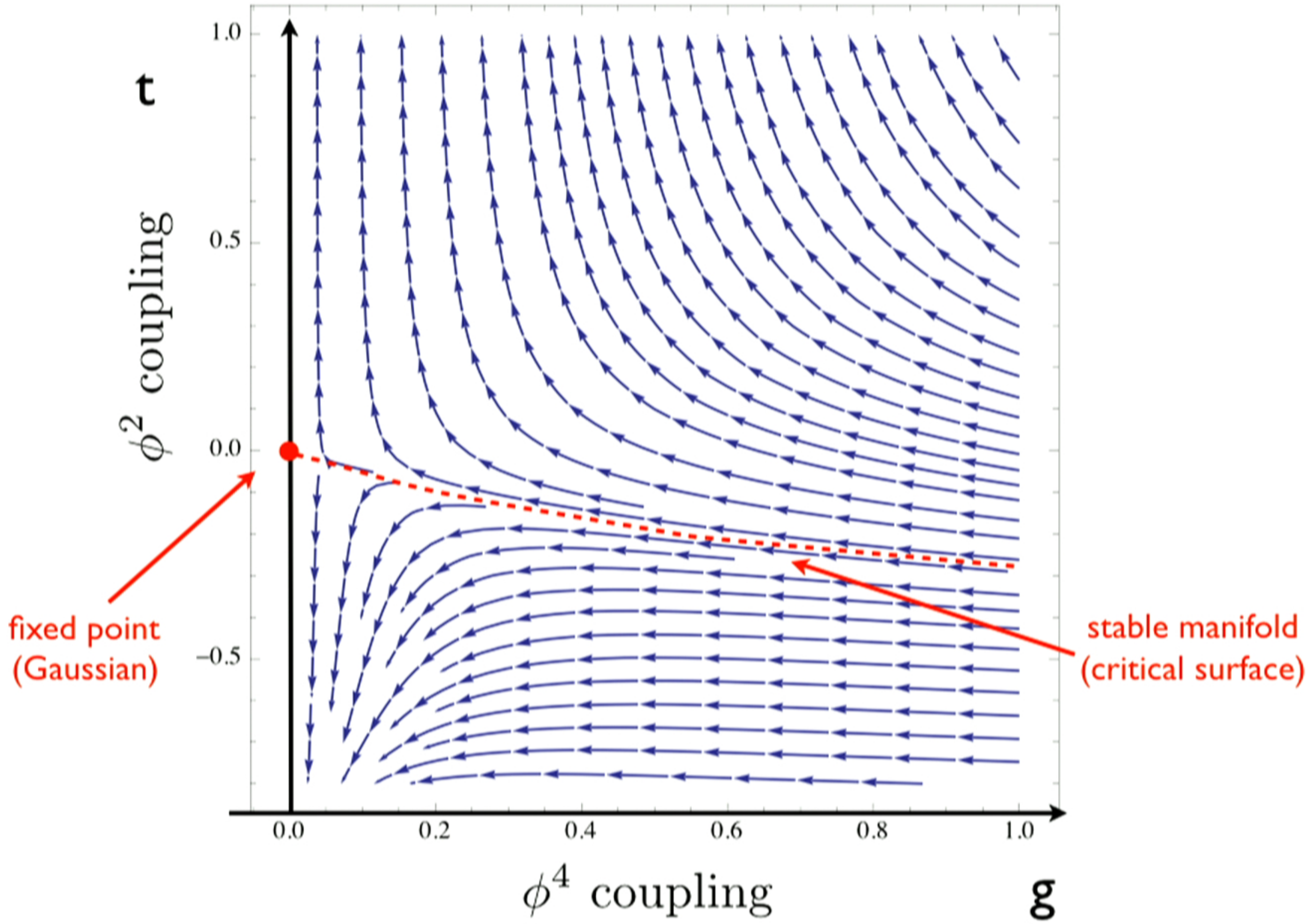
$$V(\phi) = t\phi^2 + g\phi^4 + h\phi^6$$

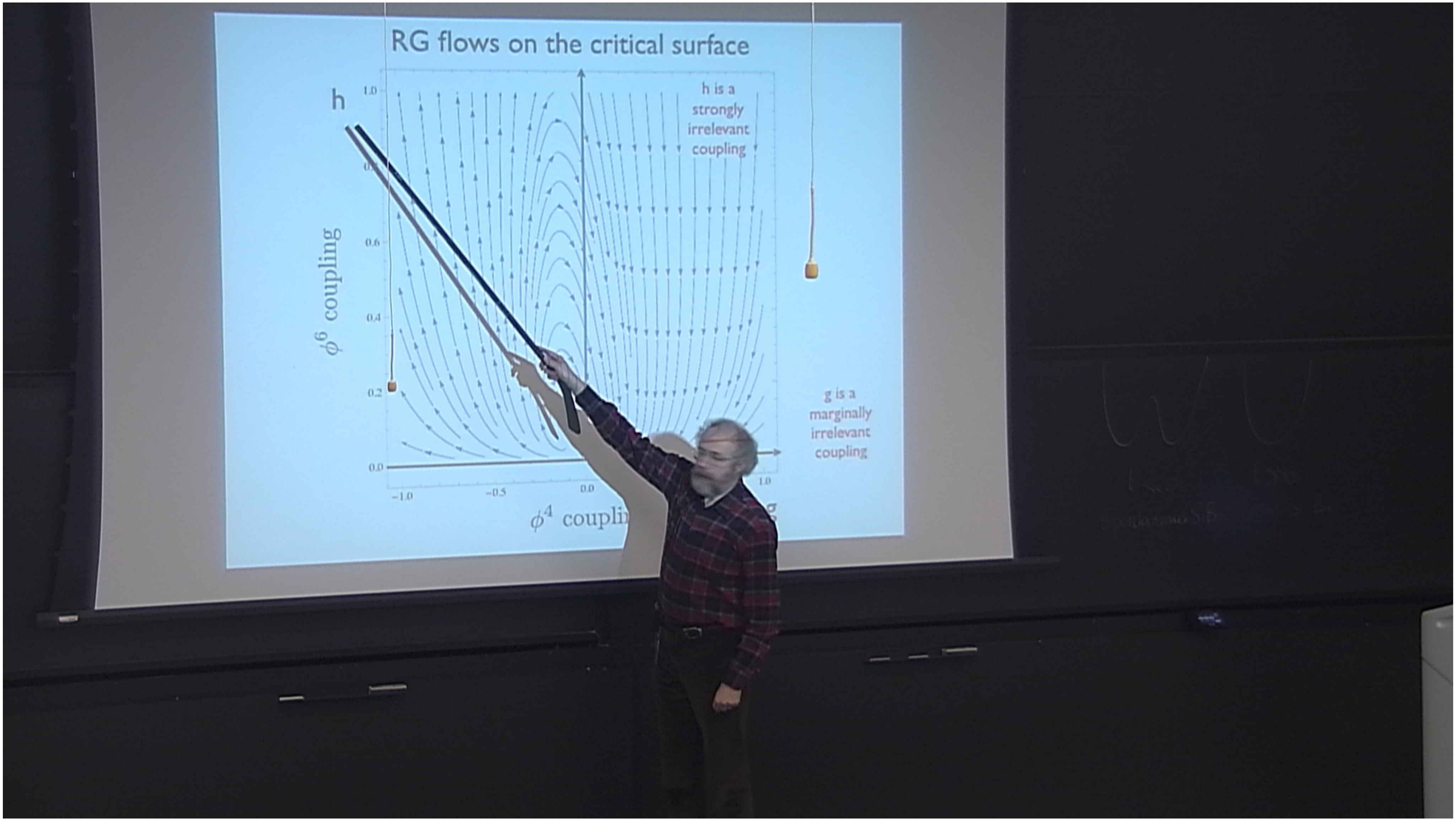
g

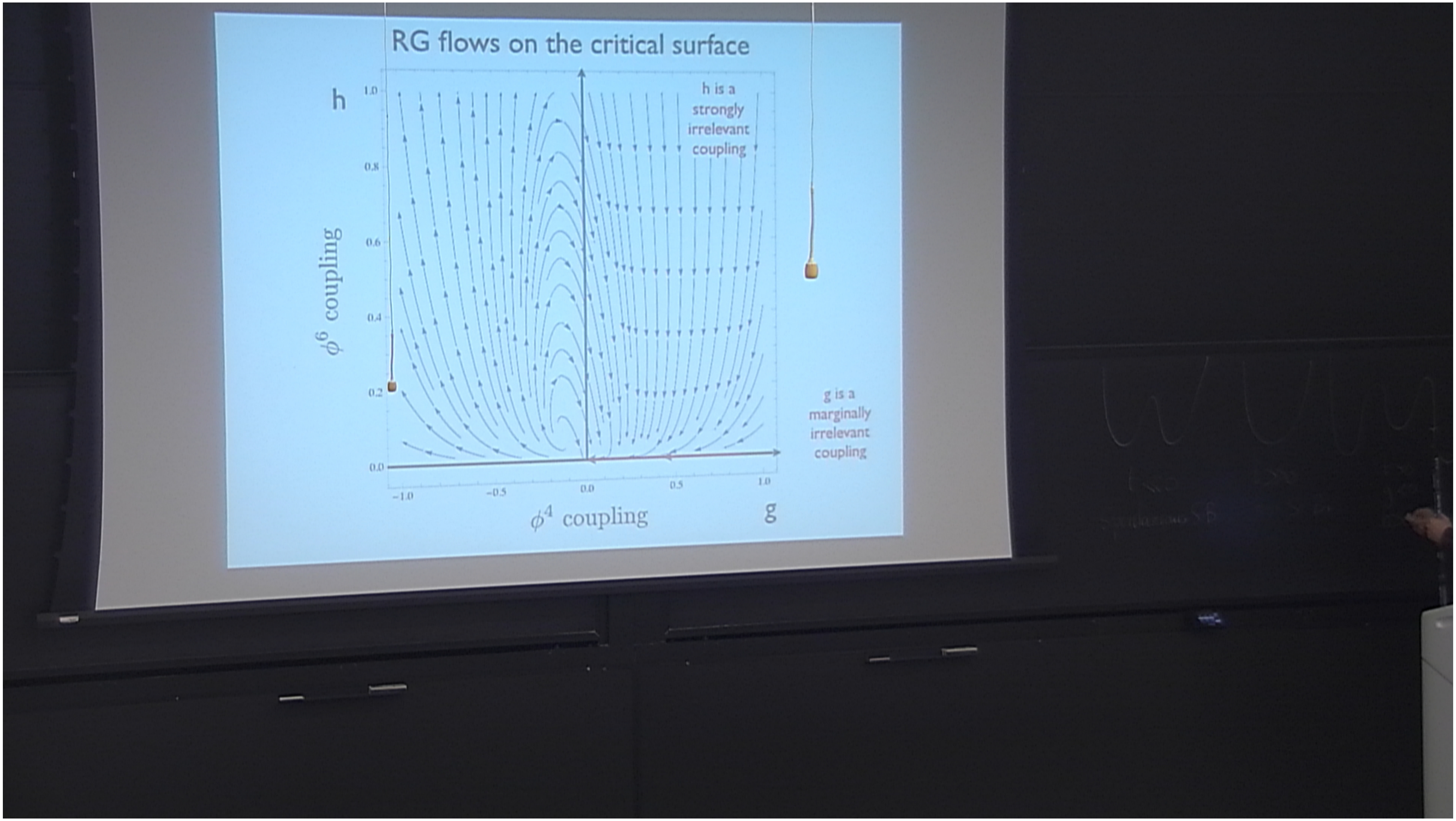


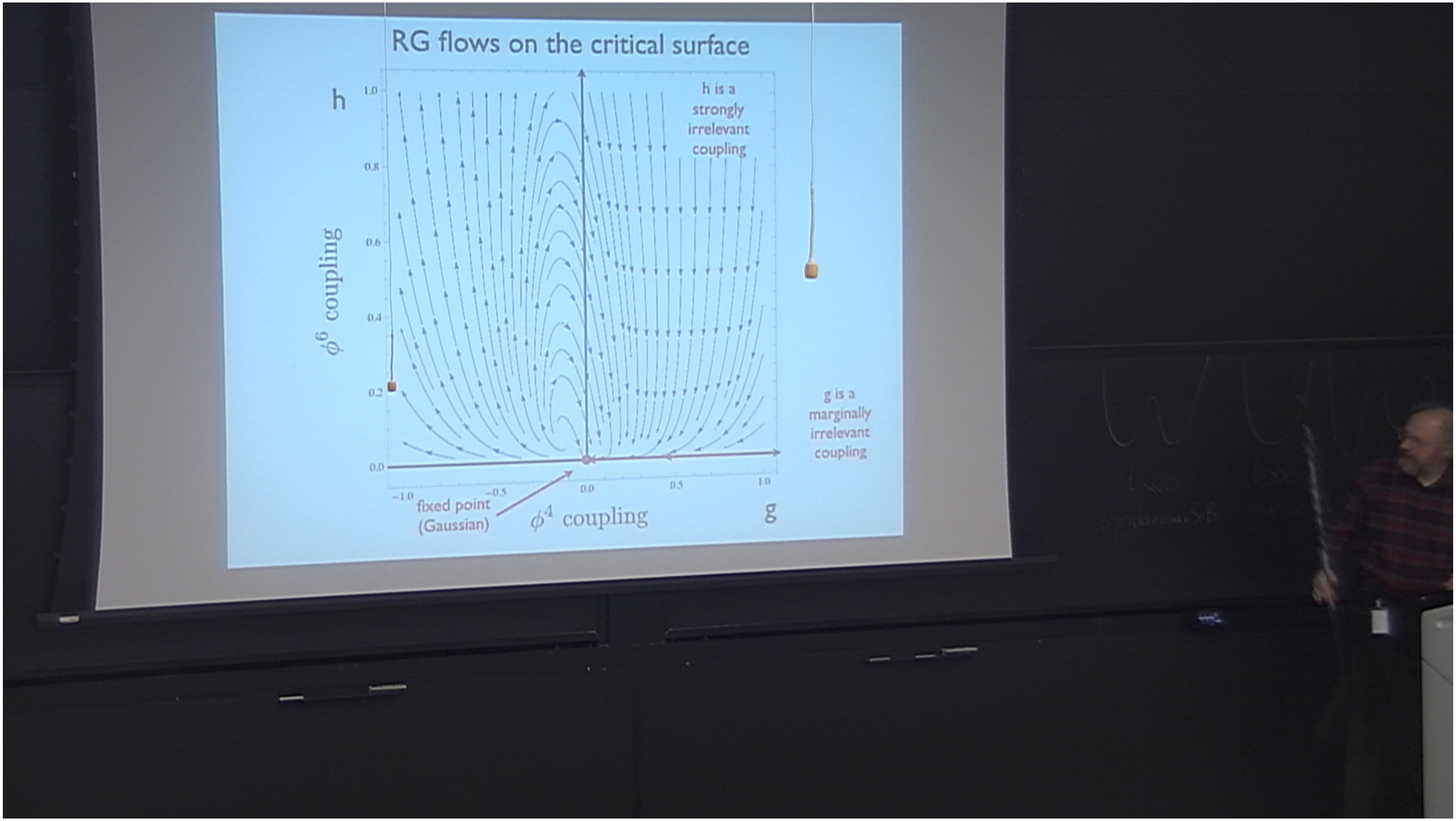
RG flows in the (t,g) plane
t
ϕ^2 coupling
ϕ^4 coupling
1.0
0.5
0.0
−0.5
0.0
0.2
0.4
0.8

RG flows in the (t,g) plane

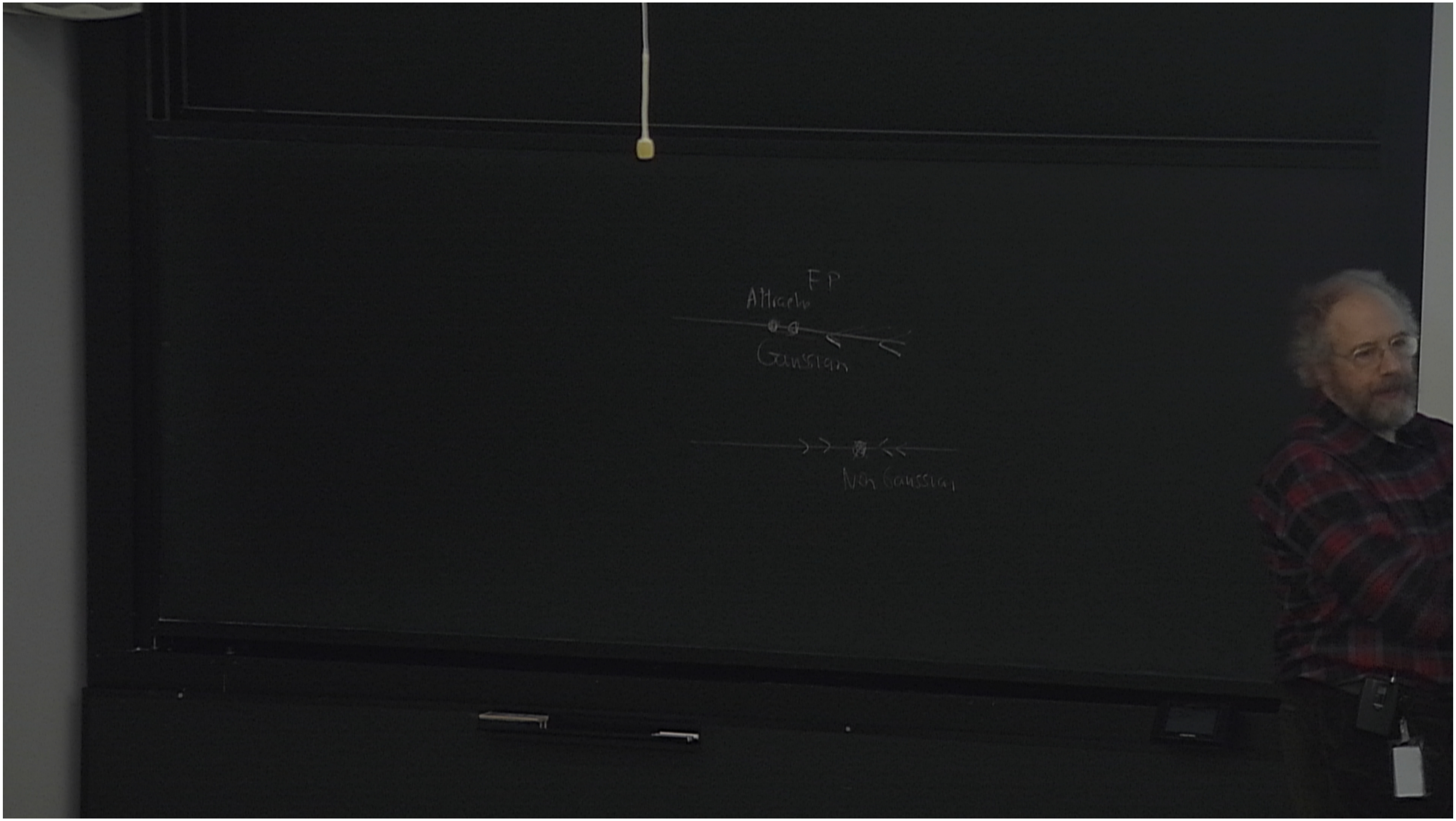


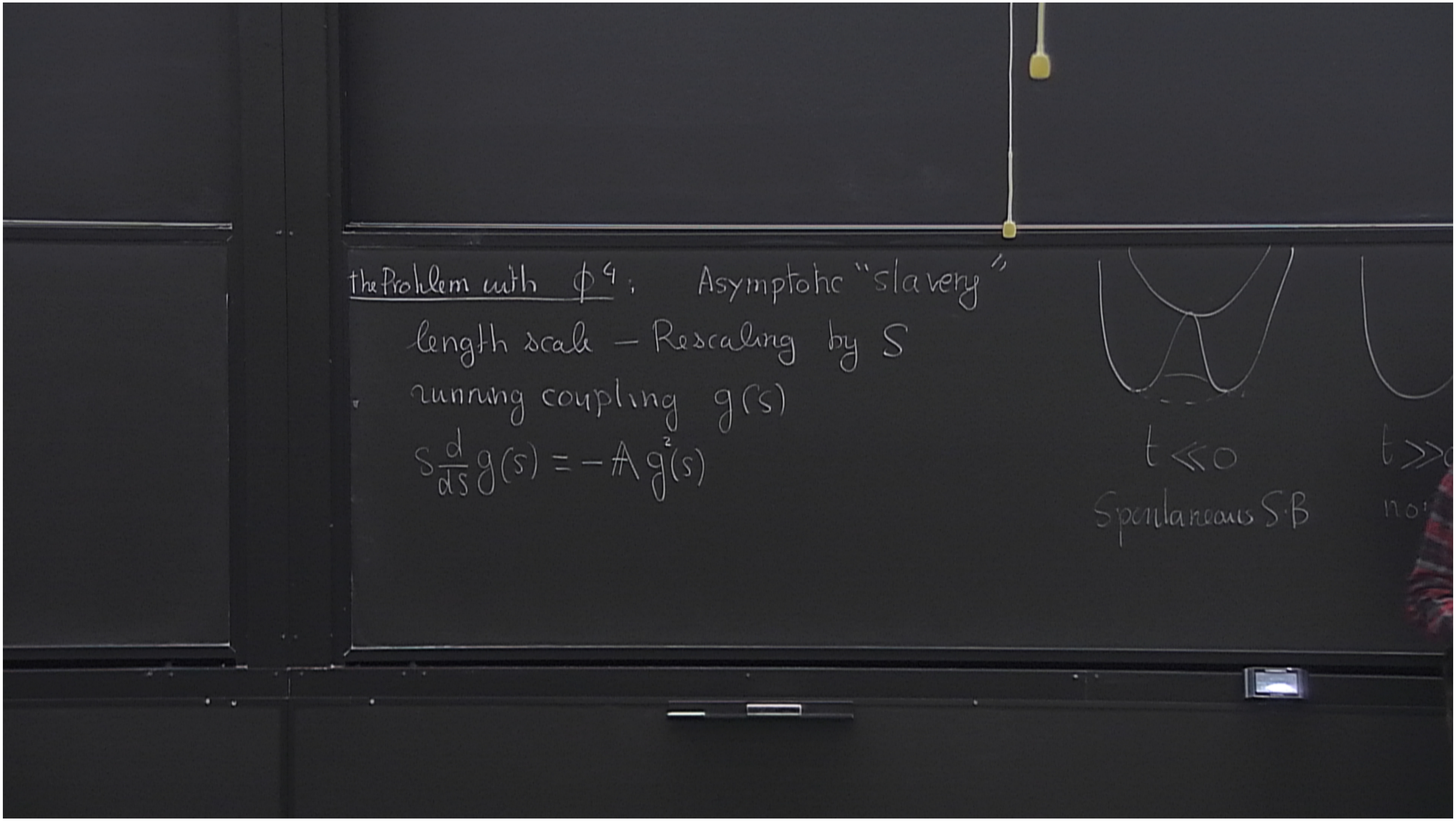

RG flows on the critical surface
h
h is a strongly irrelevant coupling
g is a marginally irrelevant coupling
ϕ^6 coupling
ϕ^4 coupli


RG flows on the critical surface
h
ϕ^6 coupling
h is a strongly irrelevant coupling
g is a marginally irrelevant coupling
ϕ^4 coupling
g
1.0
0.8
0.6
0.4
0.2
0.0
−1.0
−0.5
0.0
0.5
1.0



RG flows on the critical surface
h
ϕ^6 coupling
h is a strongly irrelevant coupling
g is a marginally irrelevant coupling
fixed point (Gaussian)
ϕ^4 coupling
g


FP
Gaussian
Non Gaussian


the Problem with ϕ⁴ : Asymptotic "slavery"
length scale – Rescaling by S
running coupling g(s)
s d/ds g(s) = −A g²(s)
t ≪ 0
Spontaneous S.B

the Problem with ϕ^4: Asymptotic "slavery"

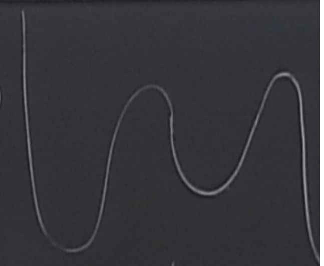
length scale — Rescaling by S $\quad x \to Sx$

running coupling $g(s)$

$$s\frac{d}{ds} g(s) = -A g^2(s) \qquad A = \frac{3}{(4\pi)^2} > 0$$

$$s = 1,\ g = g_0 > 0 \qquad g(s) = \frac{g_0}{1 + A g_0 \log s}$$

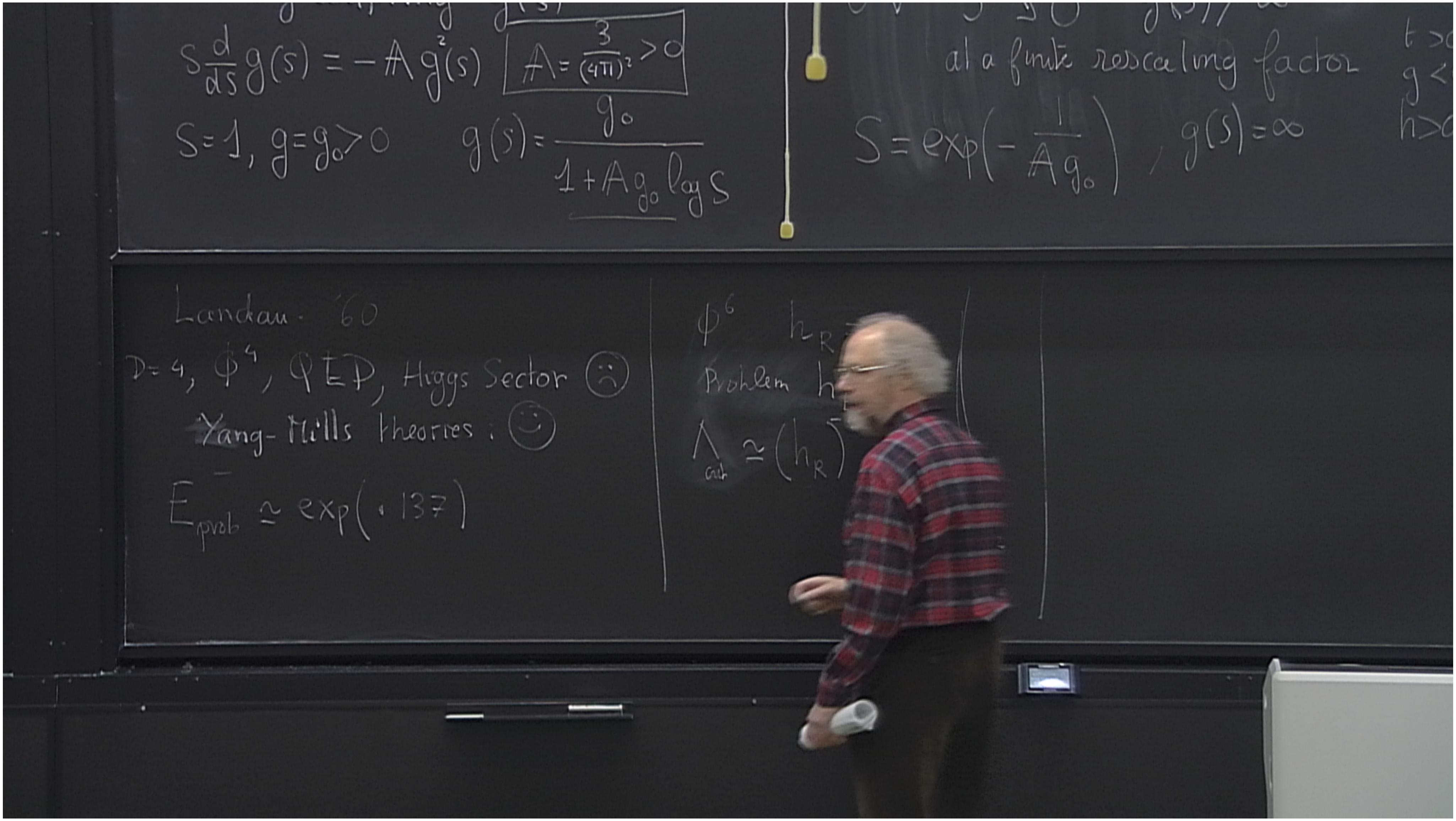
IR $\quad s \nearrow \infty \quad g(s) \searrow 0$

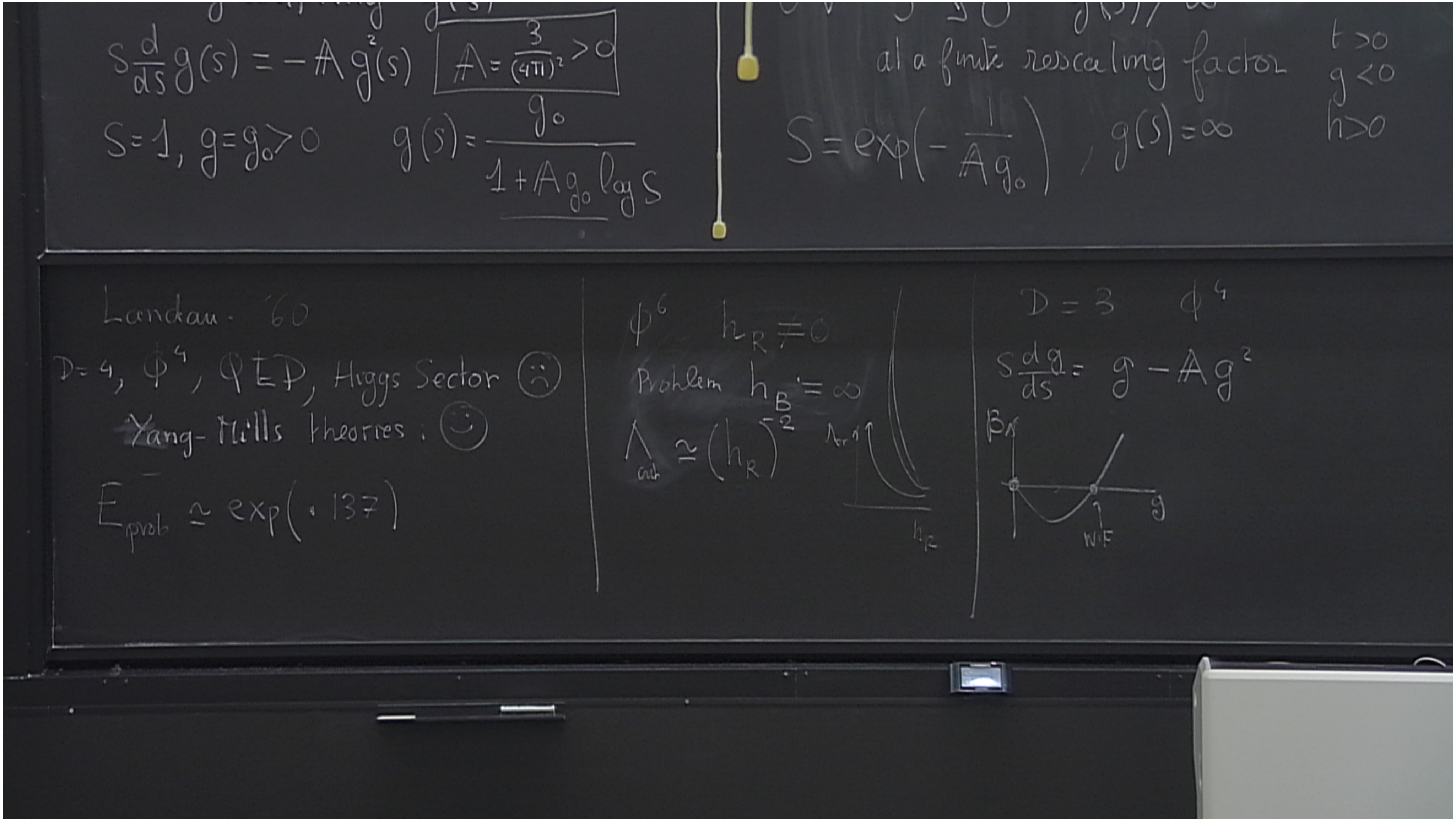


$t > 0$

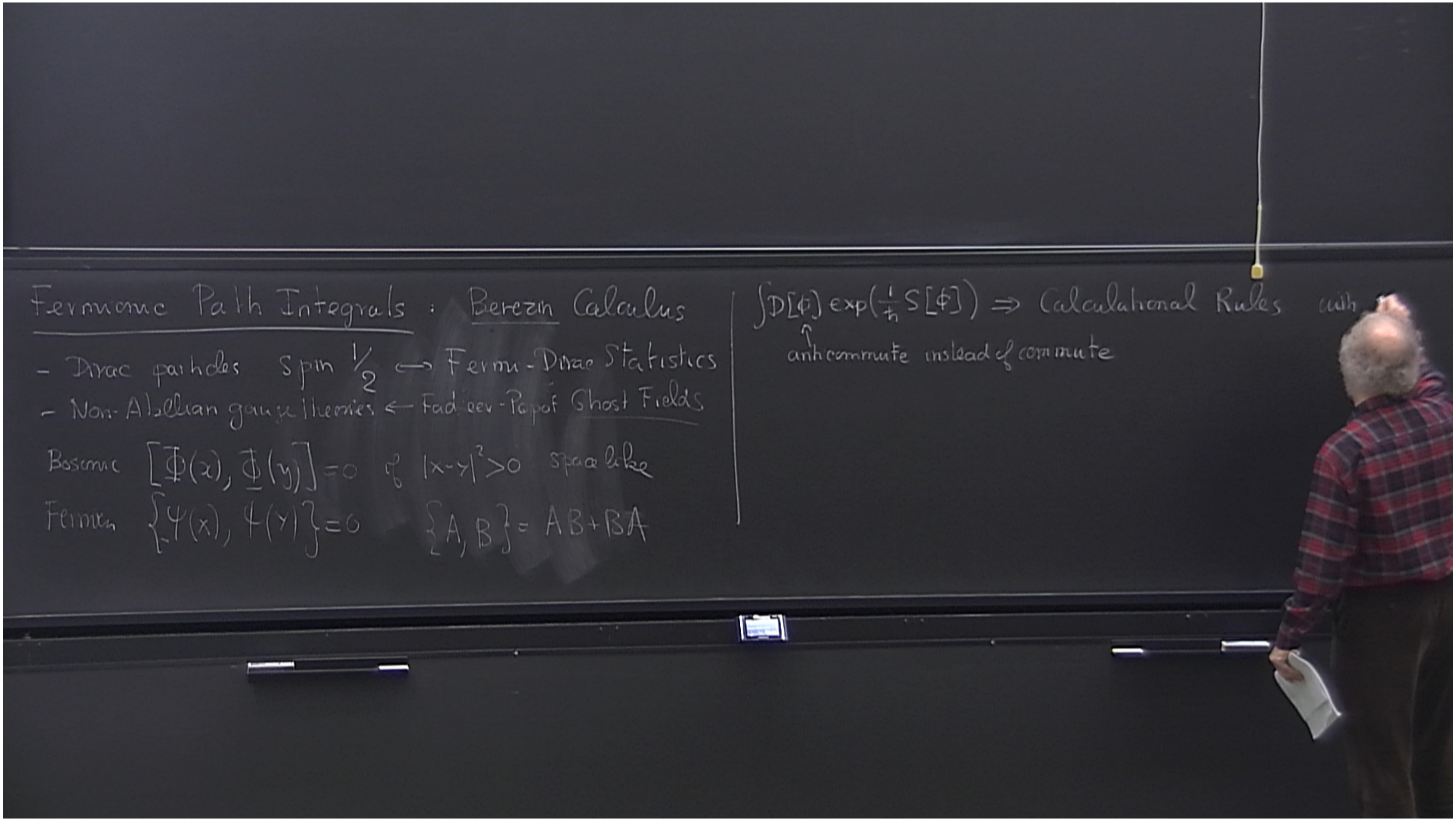
$g < 0$

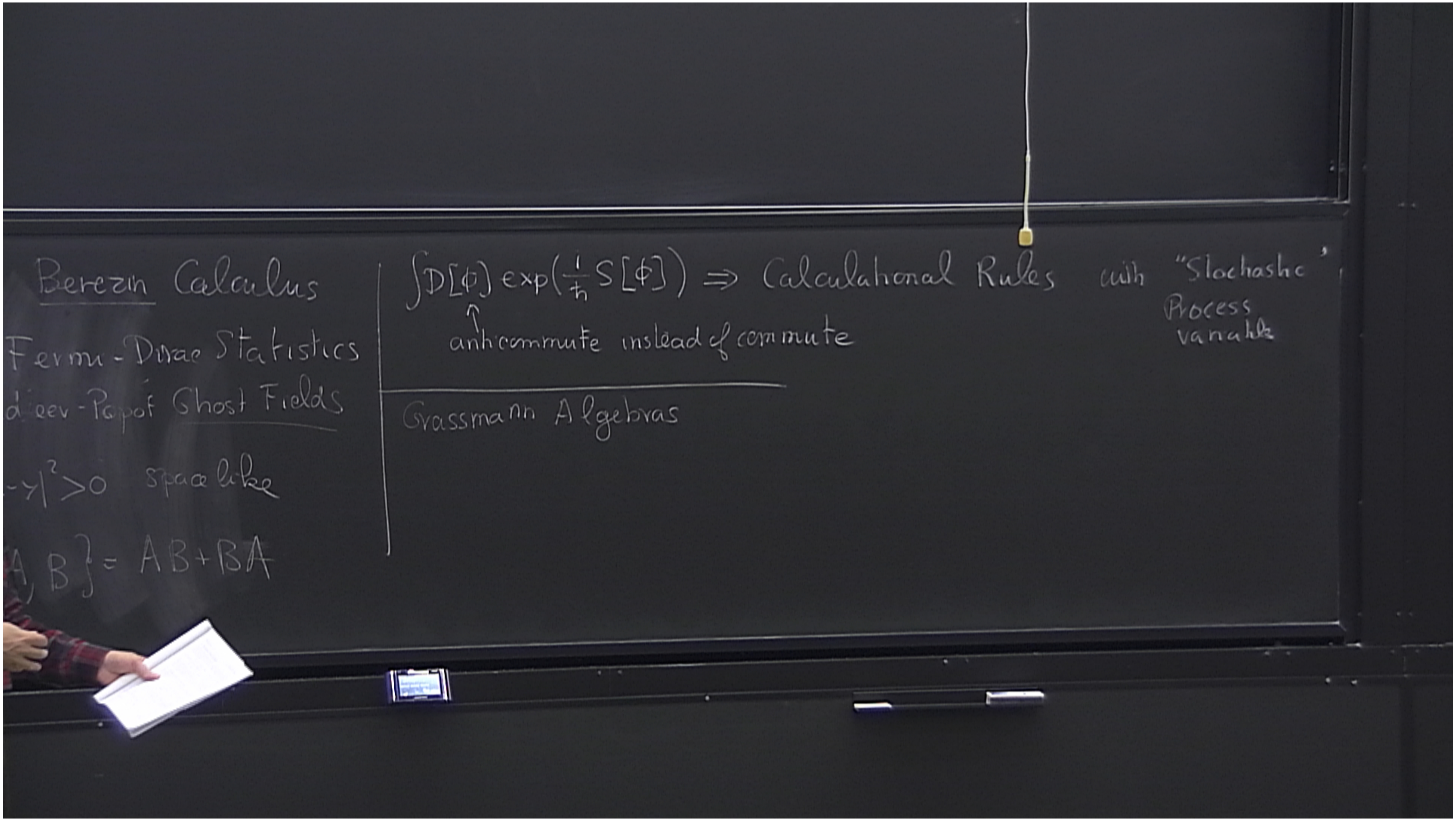
$h > 0$

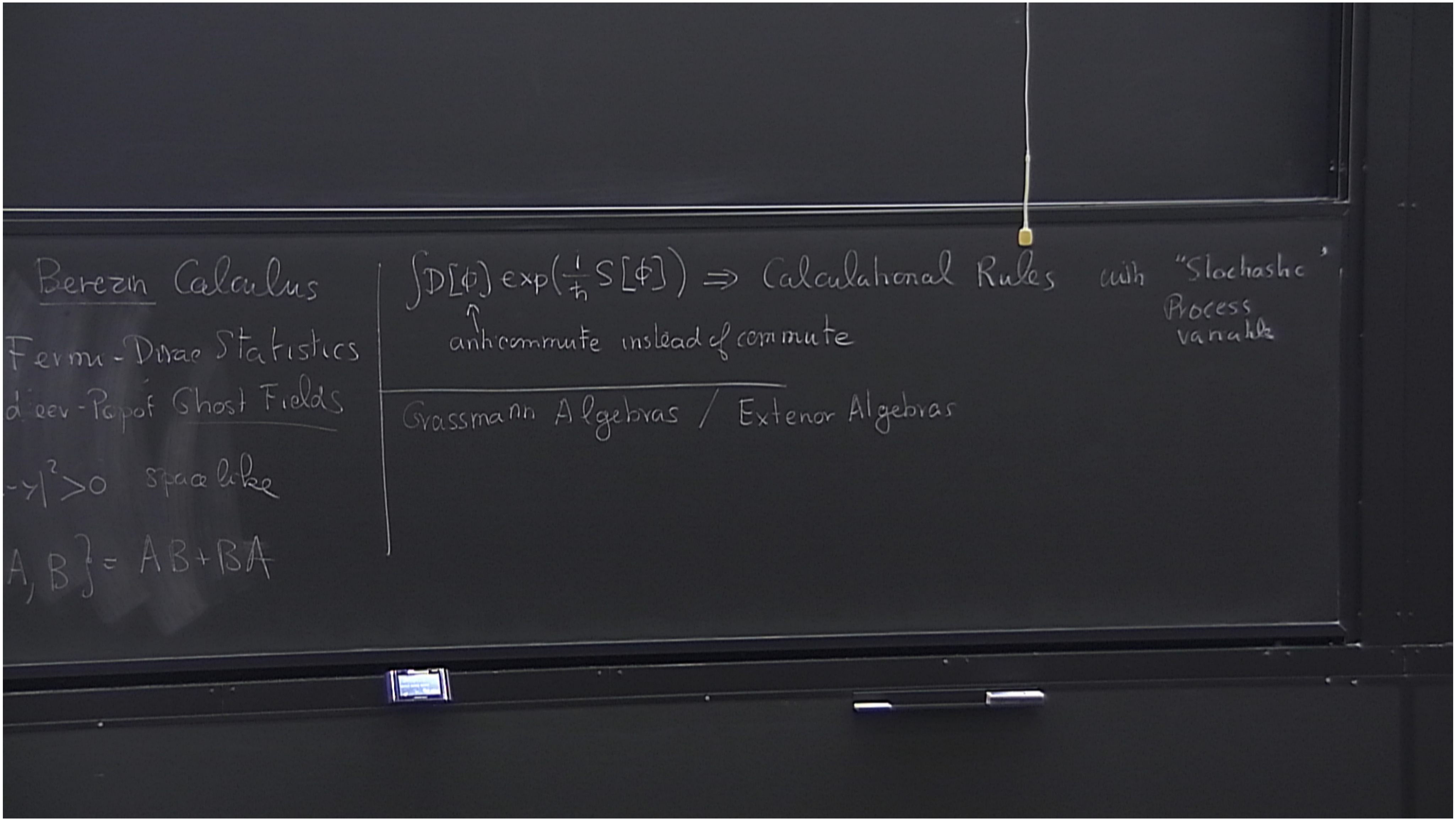

s d/ds g(s) = −A g²(s)
A = 3/(4π)² > 0
s = 1, g = g₀ > 0
g(s) = g₀ / (1 + A g₀ log s)
at a finite rescaling factor
s = exp(− 1/(A g₀)), g(s) = ∞
Landau '60
D = 4, φ⁴, QED, Higgs Sector
Yang-Mills theories :
E_prob ≃ exp(·137)
φ⁶
Problem



$s\frac{d}{ds}g(s) = -A g^2(s)$ $\boxed{A = \frac{3}{(4\pi)^2} > 0}$
$s=1,\ g=g_0>0$ $g(s) = \frac{g_0}{1 + A g_0 \log s}$
at a finite rescaling factor
$s = \exp\left(-\frac{1}{A g_0}\right),\ g(s)=\infty$
$t>0$
$g<0$
$h>0$
Landau '60
$D=4,\ \phi^4$, QED, Higgs Sector
Yang-Mills theories :
$E_{prob} \simeq \exp(\cdot 137)$
ϕ^6 $h_R \neq 0$
Problem $h_B = \infty$
$\Lambda_{cut} \simeq (h_R)^{-2}$
h_R
$D=3$ ϕ^4
$s\frac{dg}{ds} = g - A g^2$
β
g
W.F


Fermionic Path Integrals : Berezin Calculus
- Dirac particles spin 1/2 ⟷ Fermi-Dirac Statistics
- Non-Abelian gauge theories ← Faddeev-Popov Ghost Fields
Bosonic [Φ(x), Φ(y)] = 0 if |x-y|² > 0 spacelike
Fermion {Ψ(x), Ψ(y)} = 0
{A, B} = AB + BA
∫D[Φ] exp(i/ħ S[Φ]) ⇒ Calculational Rules with
anticommute instead of commute


Berezin Calculus
Fermi-Dirac Statistics
deev-Popof Ghost Fields
$-y|^2 > 0$ spacelike
$A, B\} = AB + BA$
$\int D[\phi] \exp(\frac{i}{\hbar} S[\phi]) \Rightarrow$ Calculational Rules with "Stochastic" Process variable
anticommute instead of commute
Grassmann Algebras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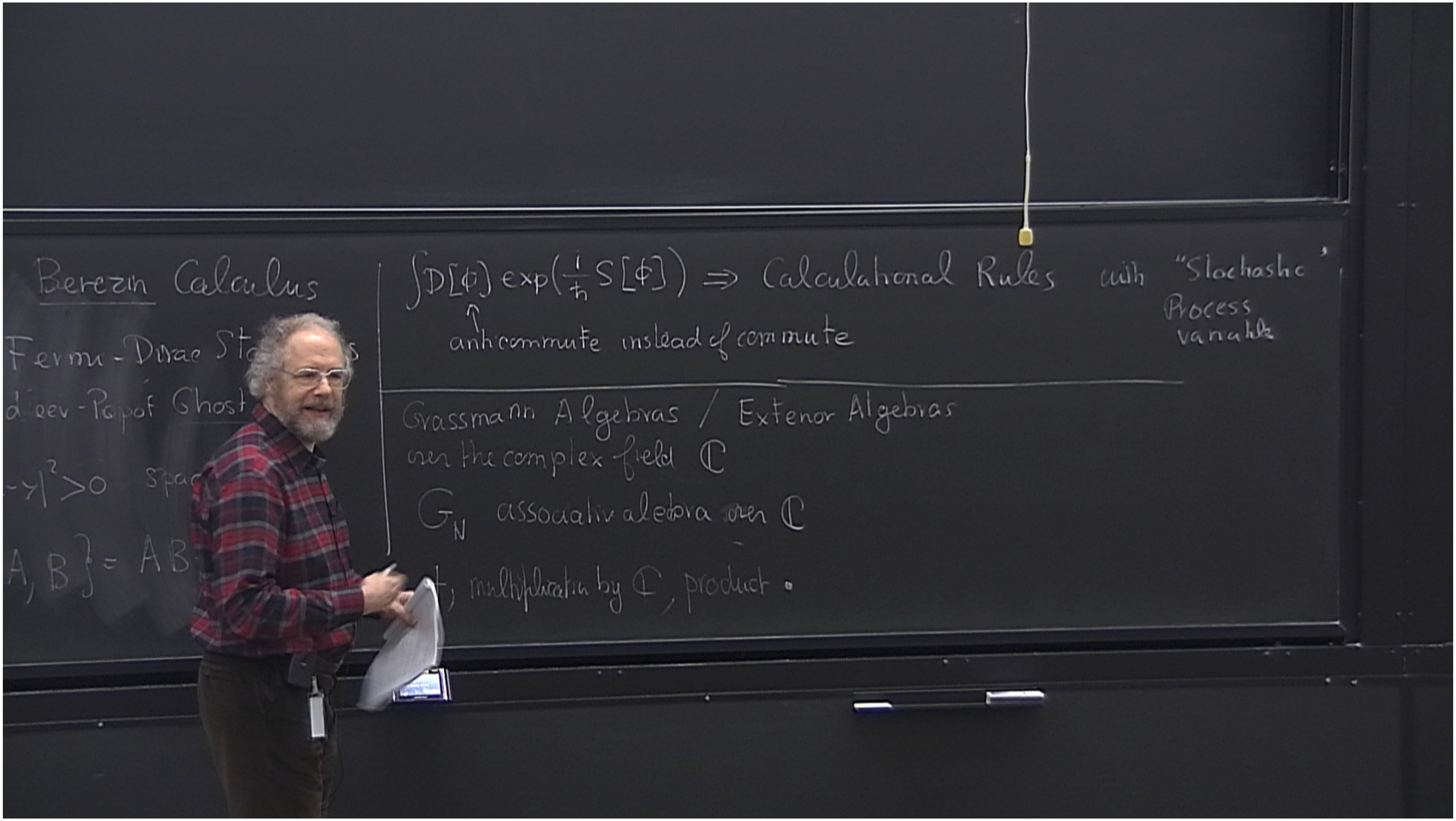
Grassmann Algebras / Exterior Algebras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∫D[φ] exp(i/ħ S[φ]) ⇒ Calculational Rules with "Stochastic" Process variable
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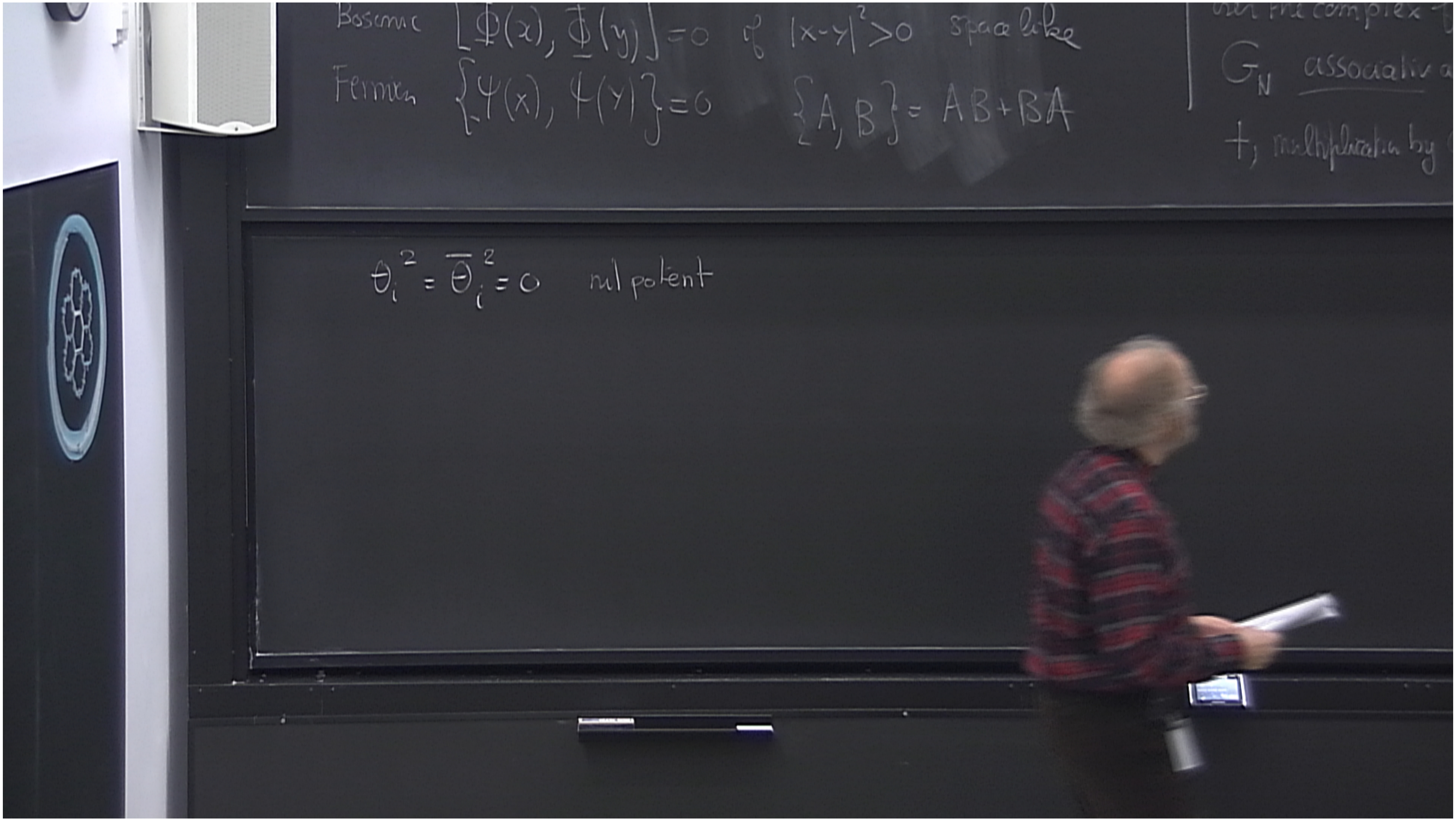
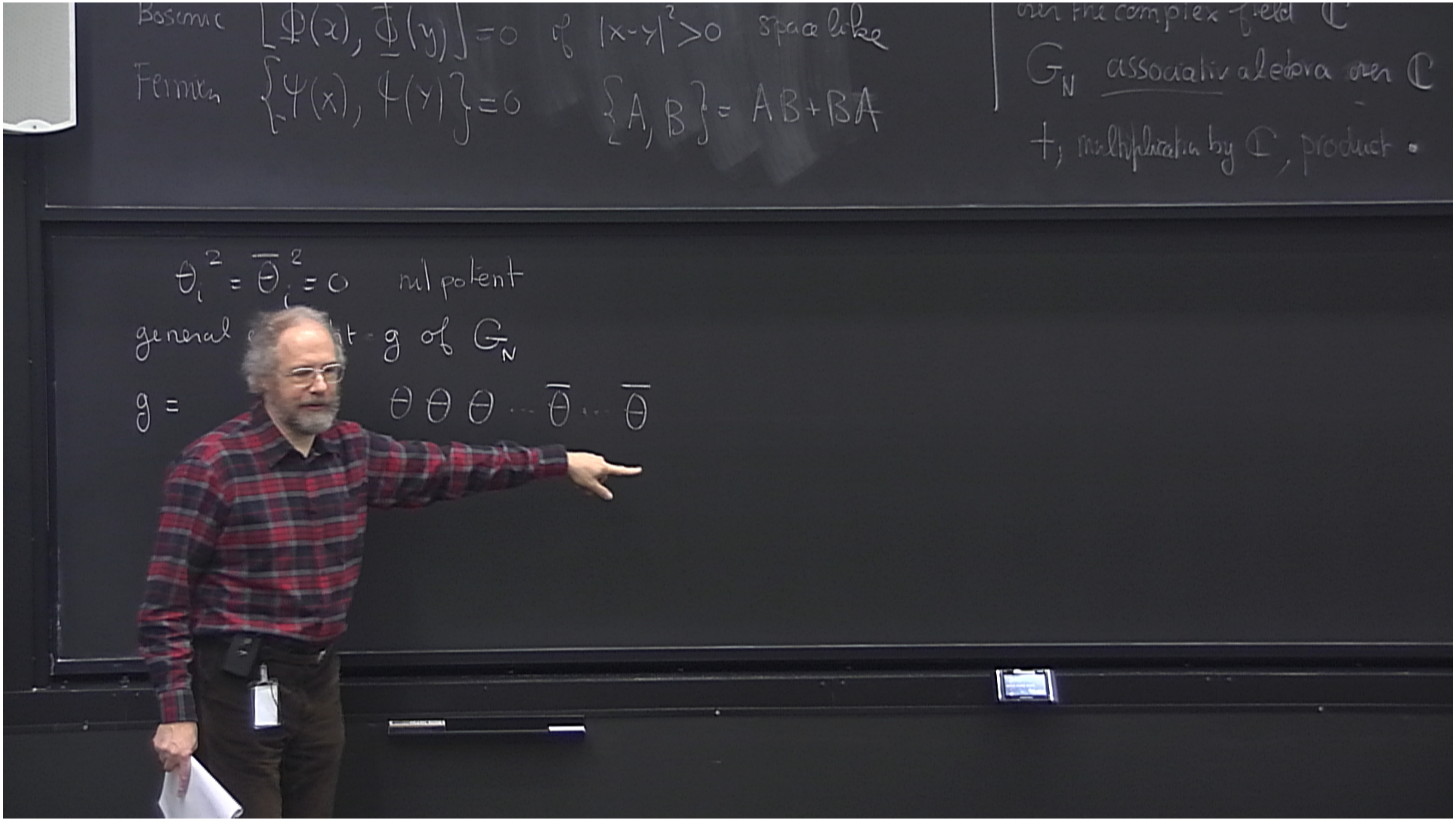
Grassmann Algebras / Exterior Algebras
over the complex field ℂ
G_N associative algebra over ℂ
+, multiplication by ℂ, product ∘


Bosonic $[\Phi(x), \Phi(y)] = 0$ if $|x-y|^2 > 0$ space like
Fermion $\{\Psi(x), \Psi(y)\} = 0$
$\{A, B\} = AB + BA$
$\theta_i^2 = \bar{\theta}_i^2 = 0$ nilpotent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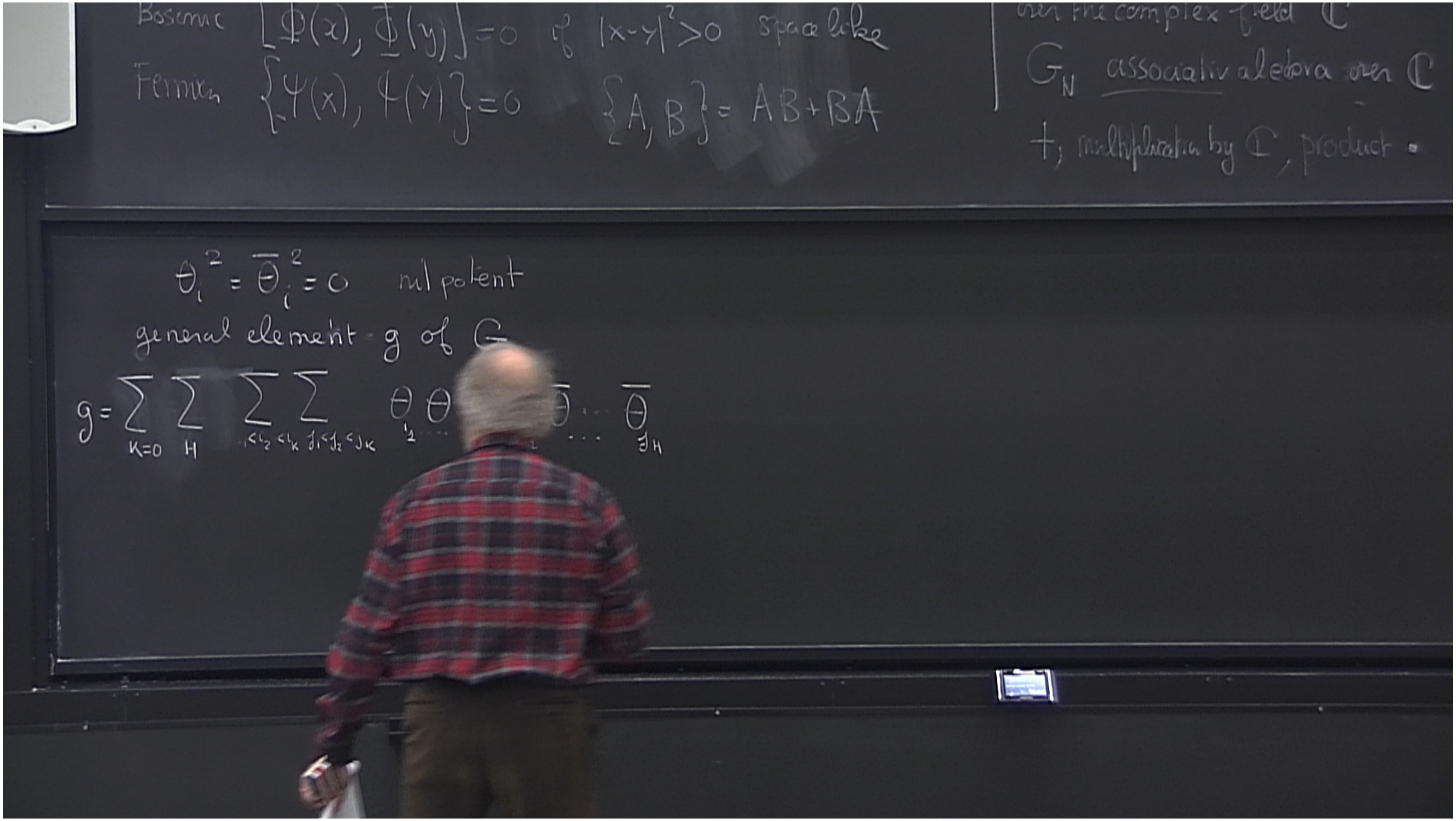
over the complex field $\mathbb{C}$
G_N associative algebra over $\mathbb{C}$
+, multiplication by $\mathbb{C}$, product •
$\theta_i^2 = \bar{\theta}_i^2 = 0$ nilpotent
general element g of G_N
$g =$ $\theta\theta\theta \cdots \bar{\theta} \cdots \bar{\theta}$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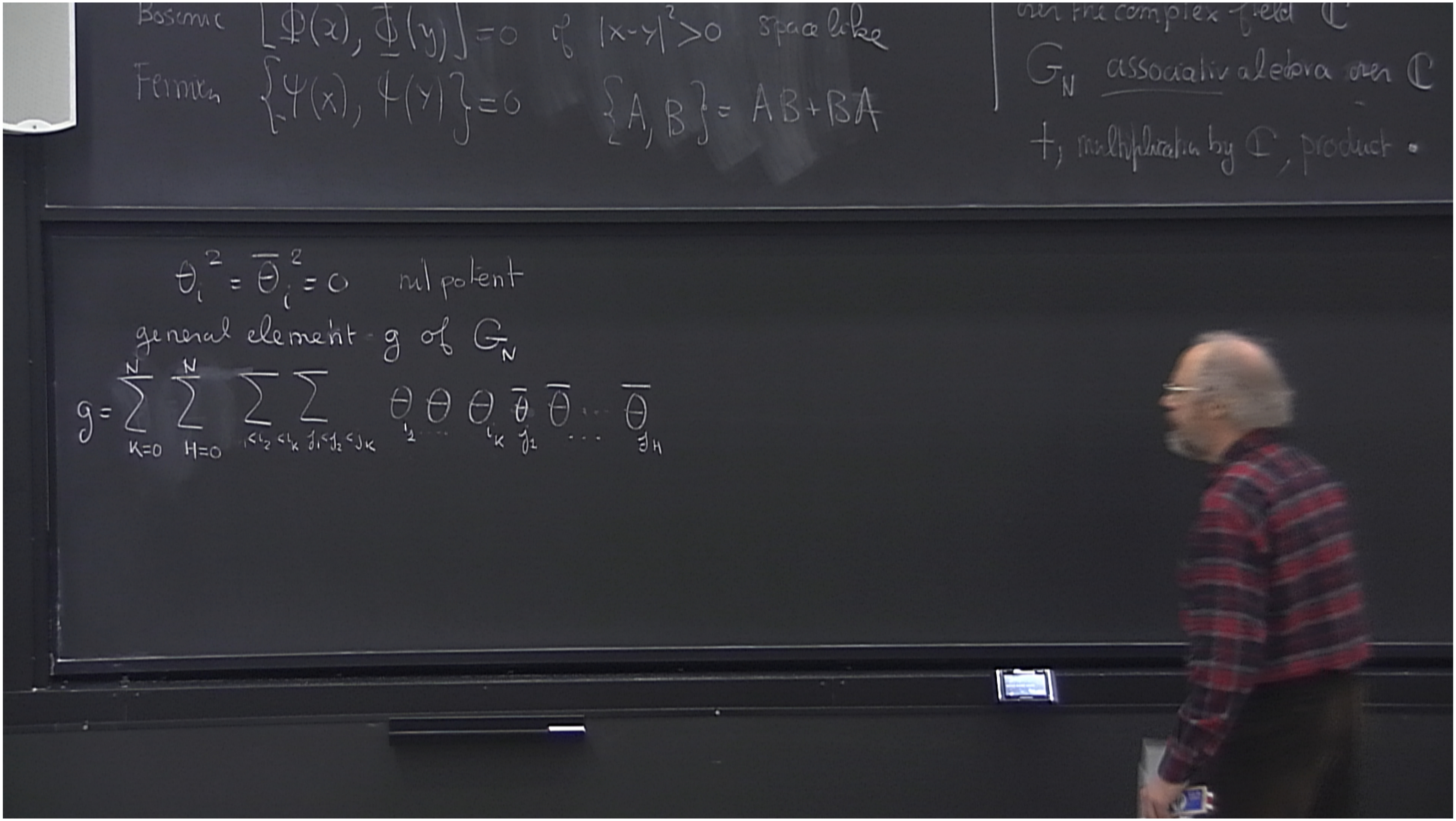
θ_i² = θ̄_i² = 0 nilpotent
general element g of G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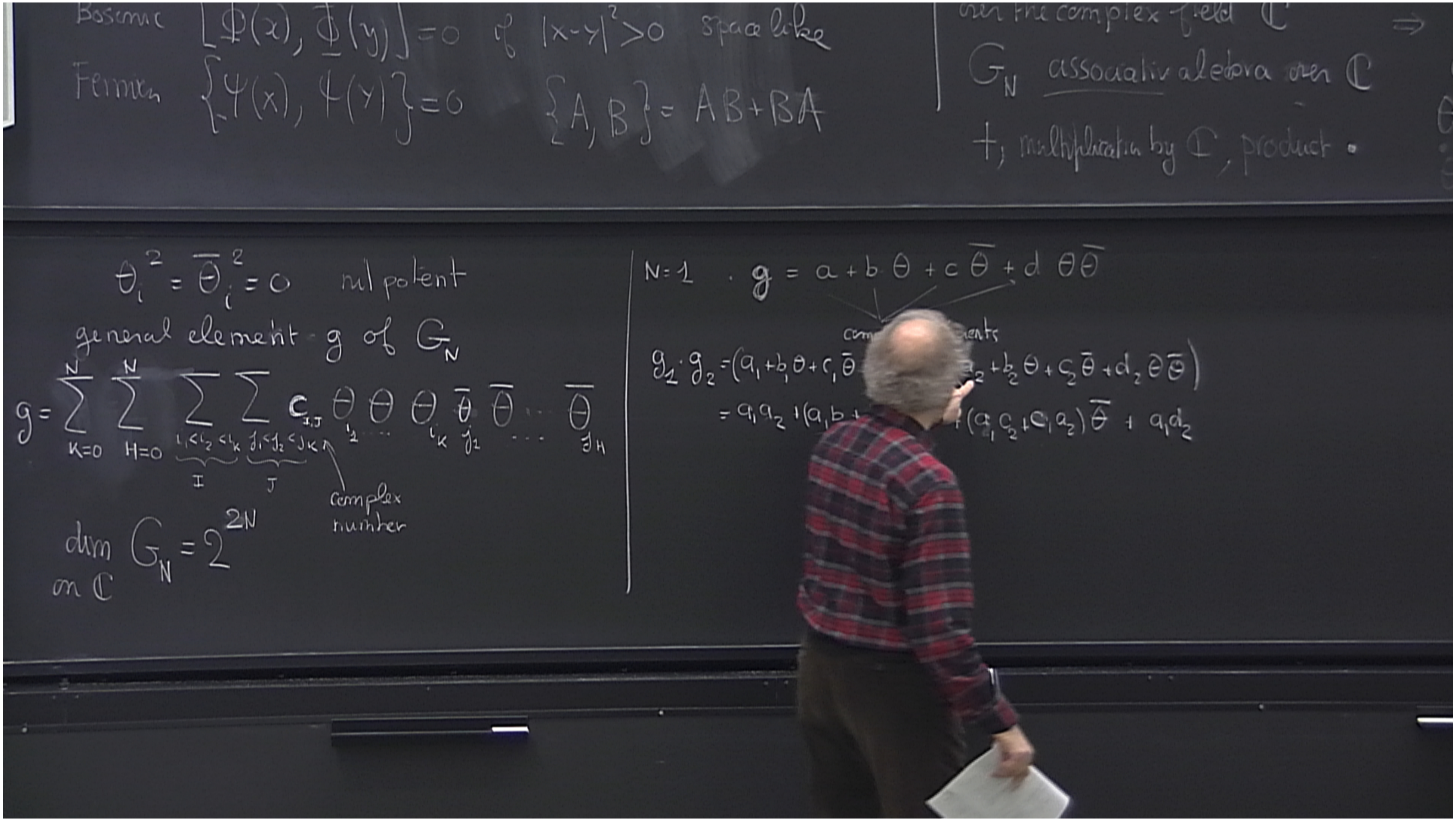
general element g of G_N
$g = \sum_{K=0}^{N} \sum_{H=0}^{N} \sum_{i_1<i_2<i_K} \sum_{j_1<j_2<j_K} \theta_{i_1} \theta \cdots \theta_{i_K} \bar{\theta}_{j_1} \bar{\theta} \cdots \bar{\theta}_{j_H}$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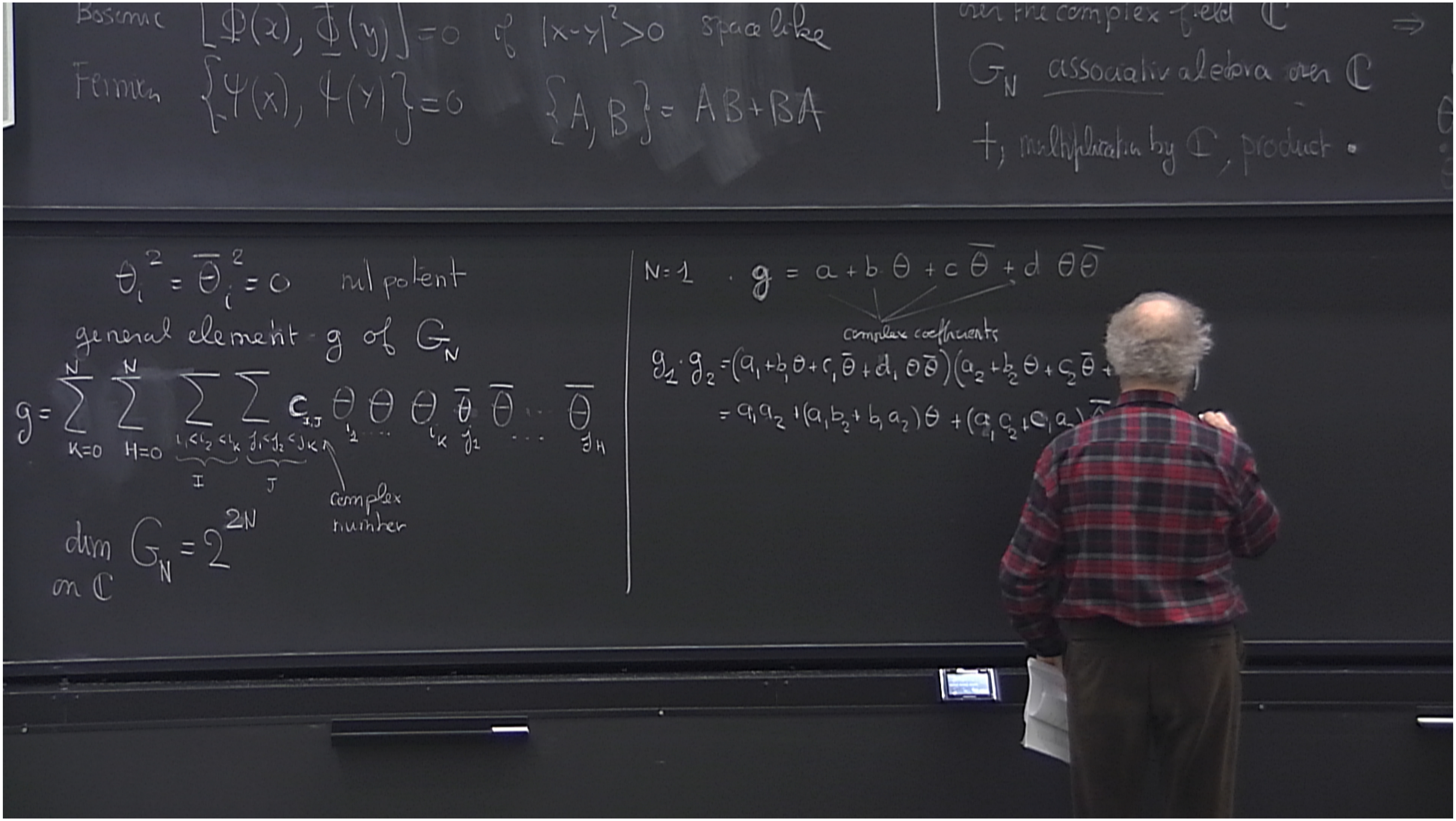
complex number
dim $G_N = 2^{2N}$
on $\mathbb{C}$
N = 1 $g = a + b\,\theta + c\,\bar{\theta} + d\,\theta\bar{\theta}$
$g_1 \cdot g_2 = (a_1 + b_1\theta + c_1\bar{\theta}$ … $+ b_2\theta + c_2\bar{\theta} + d_2\theta\bar{\theta})$
$= a_1 a_2 + (a_1 b$ … $+ (a_1 c_2 + c_1 a_2)\bar{\theta} + a_1 d_2$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θ_i² = θ̄_i² = 0 nilpotent
general element g of G_N
complex number
dim G_N = 2^2N
on ℂ
N = 1
g = a + b θ + c θ̄ + d θ θ̄
complex coefficients

Bosonic $[\Phi(x), \Phi(y)] = 0$ if $|x-y|^2 > 0$ space like

Fermion $\{\Psi(x), \Psi(y)\} = 0$ $\{A, B\} = AB + BA$

over the complex field $\mathbb{C}$ $\Rightarrow$ 2N g...

G_N associative algebra over $\mathbb{C}$

$+$, multiplication by $\mathbb{C}$, product $\circ$

antico...

$\theta_i \theta_j + \theta$...

$\theta_i \bar{\theta}_j + \bar{\theta}$...

$\theta_i^2 = \bar{\theta}_i^2 = 0$ nil potent

general element g of G_N

$$\sum_{K=0}^{N} \sum_{H=0} \sum_{\underbrace{i_1 < i_2 < i_K}_{I}} \sum_{\underbrace{j_1 < j_2 < j_K}_{J}} c_{I,J}\, \theta_{i_1} \cdots \theta_{i_K} \bar{\theta}_{j_1} \bar{\theta} \cdots \bar{\theta}_{j_H}$$

complex number

dim $G_N = 2^{2N}$

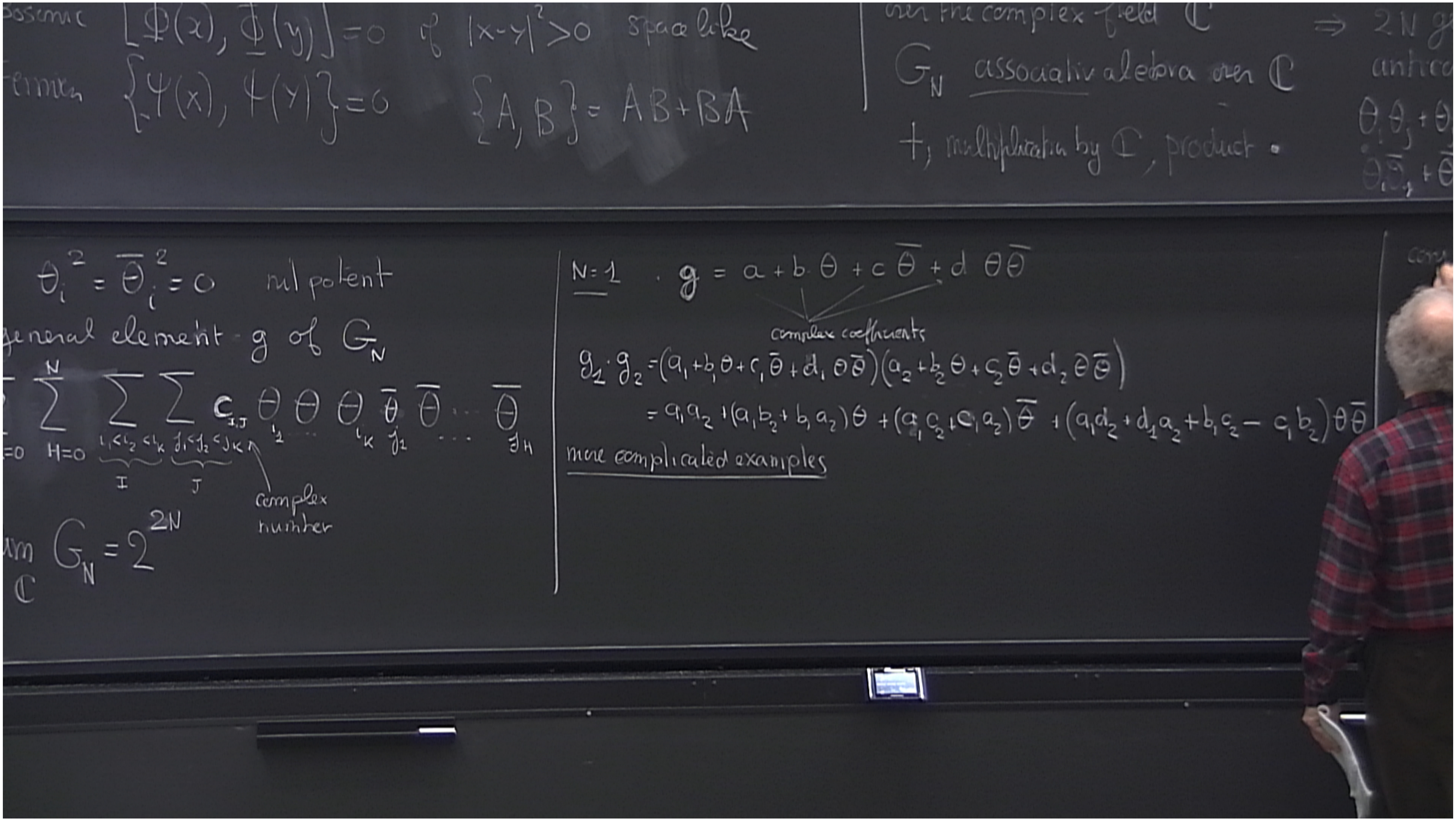
$\mathbb{C}$

$\underline{N=1}$ $\cdot$ $g = a + b\,\theta + c\,\bar{\theta} + d\,\theta\bar{\theta}$

complex coefficients

$$g_1 \cdot g_2 = (a_1 + b_1\theta + c_1\bar{\theta} + d_1\theta\bar{\theta})(a_2 + b_2\theta + c_2\bar{\theta} + d_2\theta\bar{\theta})$$

$$= a_1 a_2 + (a_1 b_2 + b_1 a_2)\theta + (a_1 c_2 + c_1 a_2)\bar{\theta} + (a_1 d_2 + d_1 a_2 + b_1 c_2 - c_1 b_2)\theta\bar{\theta}$$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$G_N = 2^{2N}$
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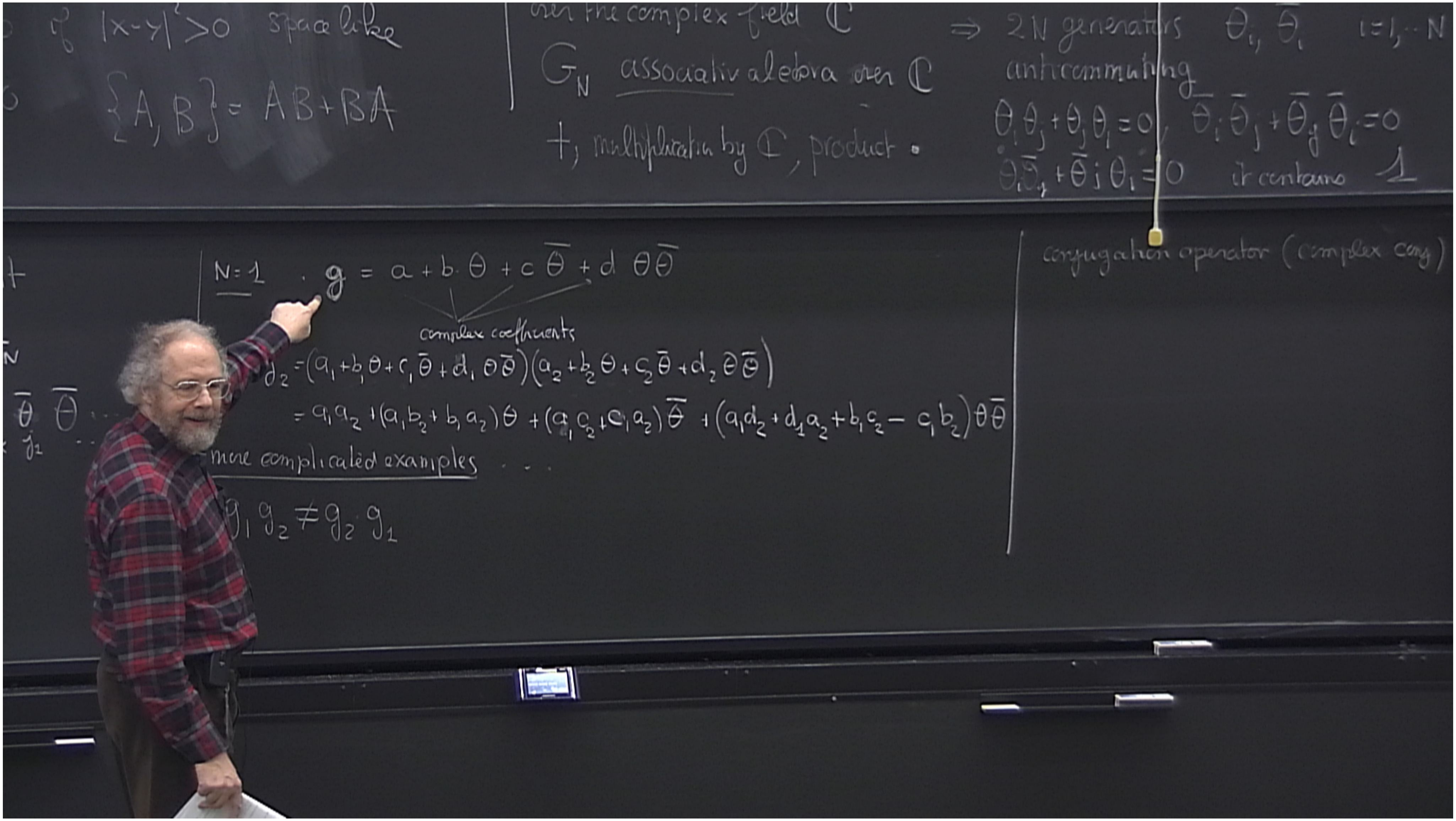
$= a_1a_2 + (a_1b_2 + b_1a_2)\theta + (a_1c_2 + c_1a_2)\bar{\theta} + (a_1d_2 + d_1a_2 + b_1c_2 - c_1b_2)\theta\bar{\theta}$
more complicated examples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⇒ 2N generators θ_i, θ̄_i i = 1, .. N
anticommuting
θ_i θ_j + θ_j θ_i = 0, θ̄_i θ̄_j + θ̄_j θ̄_i = 0
θ_i θ̄_j + θ̄_j θ_i = 0 it contains 1
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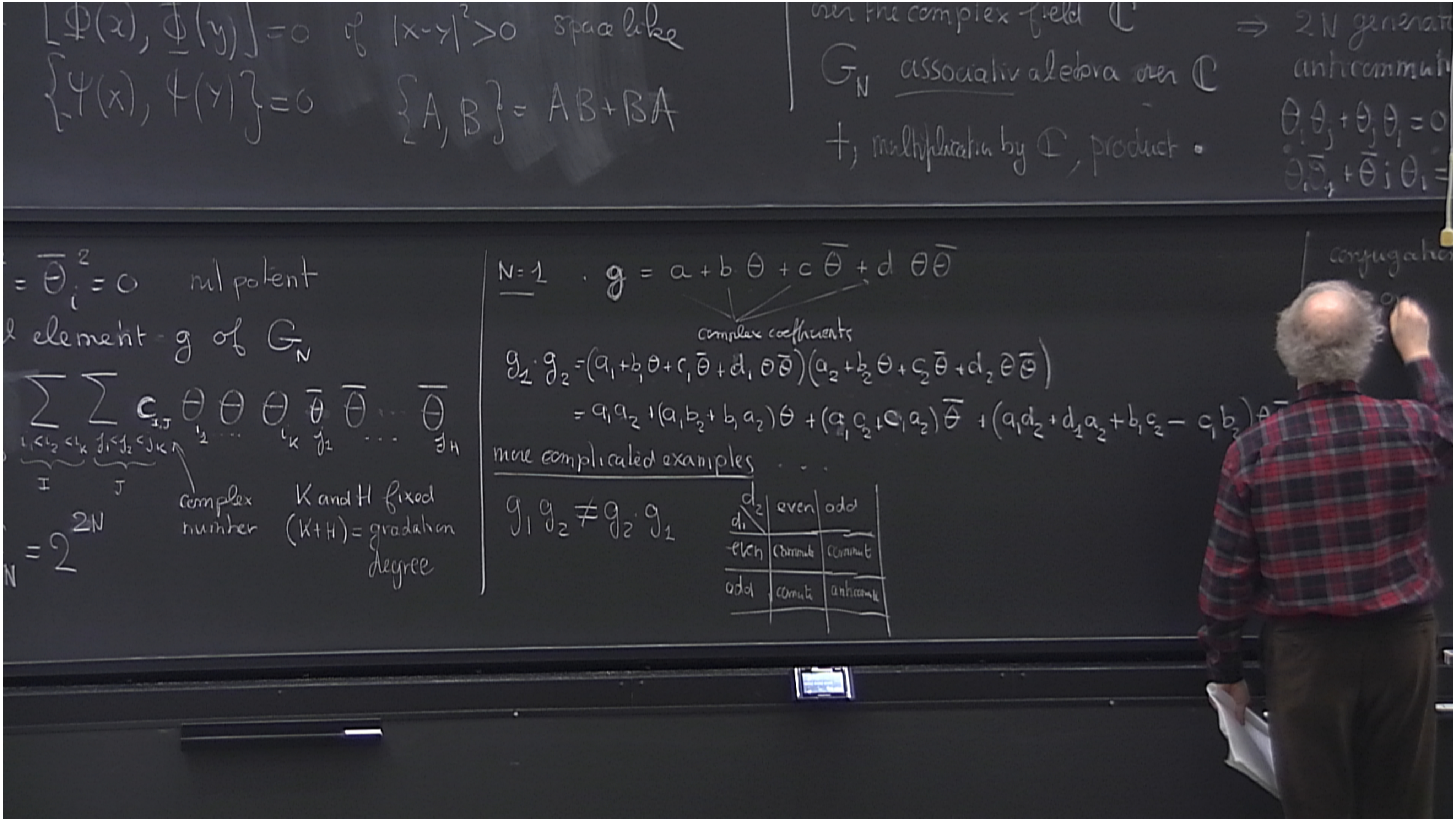
g_1 g_2 = (a_1 + b_1 θ + c_1 θ̄ + d_1 θθ̄)(a_2 + b_2 θ + c_2 θ̄ + d_2 θθ̄)
= a_1 a_2 + (a_1 b_2 + b_1 a_2) θ + (a_1 c_2 + c_1 a_2) θ̄ + (a_1 d_2 + d_1 a_2 + b_1 c_2 − c_1 b_2) θθ̄
more complicated examples . . .
g_1 g_2 ≠ g_2 g_1
conjugation operator (complex conj)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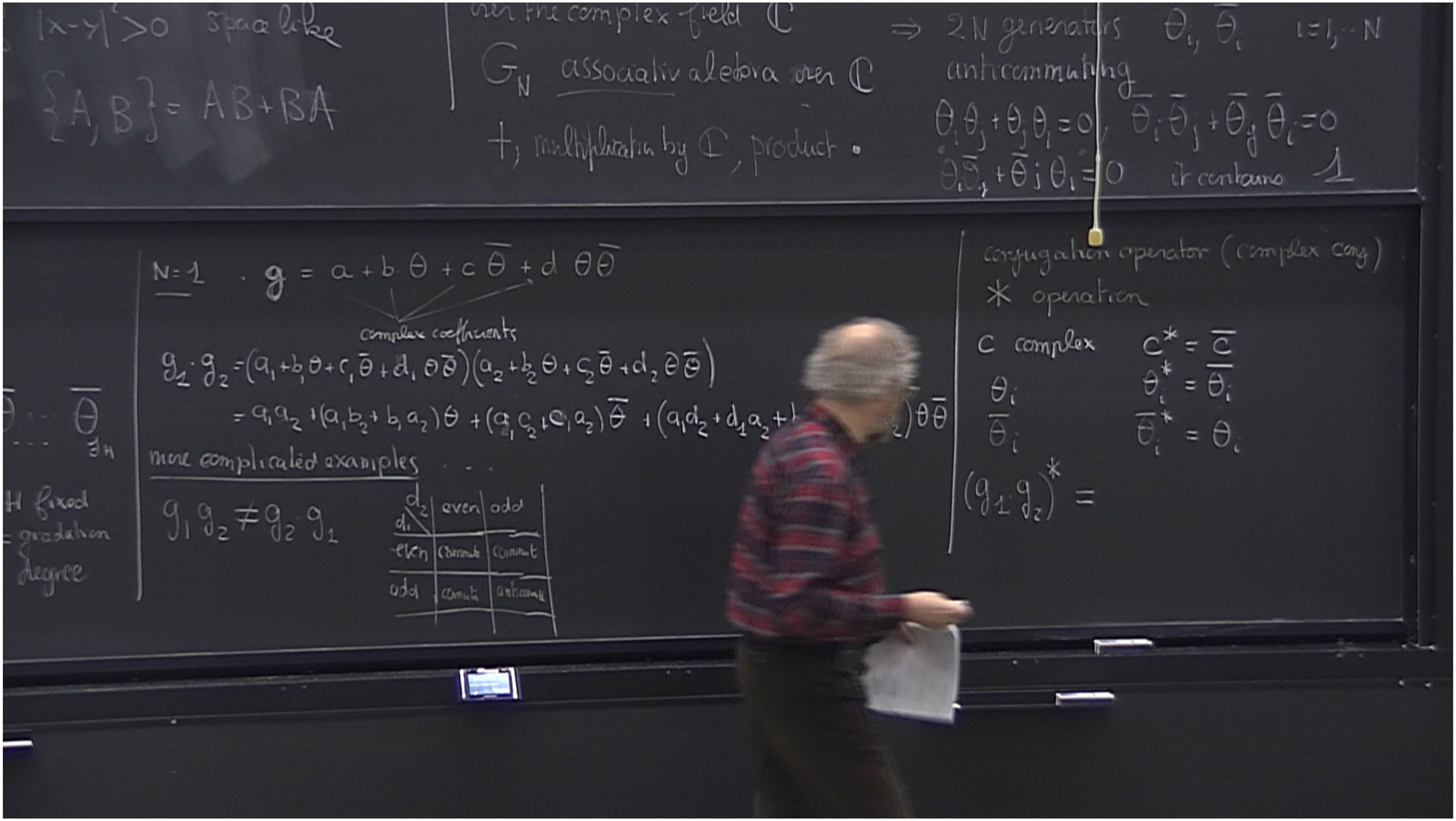
= a_1a_2 + (a_1b_2 + b_1a_2)θ + (a_1c_2 + c_1a_2)θ̄ + (a_1d_2 + d_1a_2 + b_1c_2 − c_1b_2)
more complicated examples . . .
g_1 g_2 ≠ g_2 · g_1
even odd
even commute commut
odd comute anticomm
conjugation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more complicated examples . . .
g_1 g_2 ≠ g_2 g_1
even odd
even commute commute
odd commute anticommute
H fixed
= gradation
degree
conjugation operator (complex conj)
* operation
c complex c* = c̄
θ_i θ_i* = θ̄_i
θ̄_i θ̄_i* = θ_i
(g_1 · g_2)* =

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$\{A, B\} = AB + BA$

over the complex field $\mathbb{C}$

G_N associative algebra over $\mathbb{C}$

$+$, multiplication by $\mathbb{C}$, product $\cdot$

$\Rightarrow$ $2N$ generators $\theta_i, \bar{\theta}_i$ $i = 1, \cdots N$

anticommuting

$\theta_i \theta_j + \theta_j \theta_i = 0$, $\bar{\theta}_i \bar{\theta}_j + \bar{\theta}_j \bar{\theta}_i = 0$

$\theta_i \bar{\theta}_j + \bar{\theta}_j \theta_i = 0$ it contains $\mathbb{1}$

$\bar{\theta} \cdots \bar{\theta}_{j_H}$

$\ldots$

H fixed

= gradation

degree

$N = 1$. $g = a + b\,\theta + c\,\bar{\theta} + d\,\theta\bar{\theta}$

complex coefficients

$$g_1 \cdot g_2 = (a_1 + b_1\theta + c_1\bar{\theta} + d_1\,\theta\bar{\theta})(a_2 + b_2\theta + c_2\bar{\theta} + d_2\,\theta\bar{\theta})$$

$$= a_1 a_2 + (a_1 b_2 + b_1 a_2)\theta + (a_1 c_2 + c_1 a_2)\bar{\theta} + (a_1 d_2 + d_1 a_2 + b_1 c_2 - c_1 b_2)\theta\bar{\theta}$$

more complicated examples . . .

$g_1 g_2 \neq g_2 g_1$

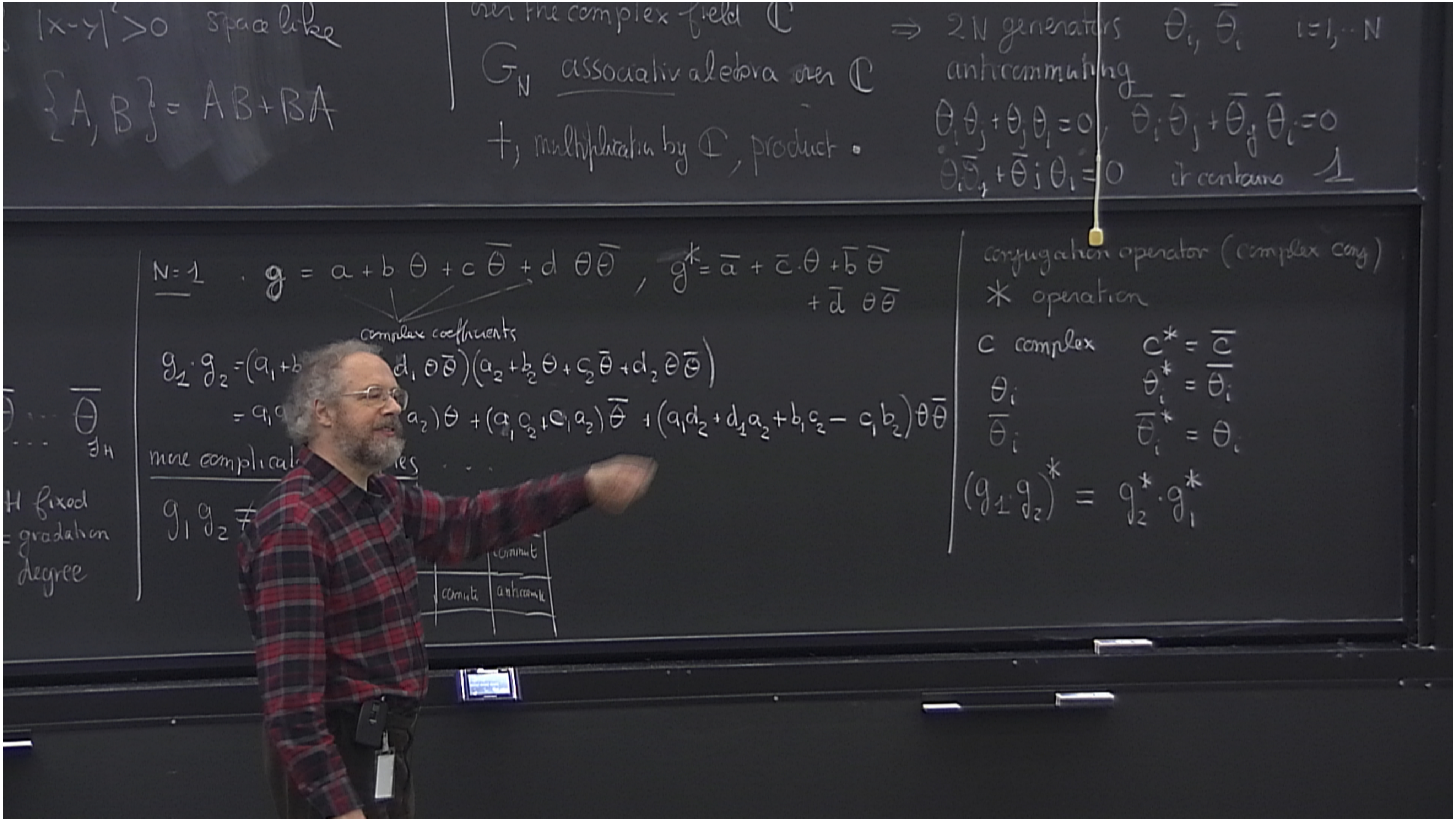
$d_1 \backslash d_2$	even	odd
even	commute	commut
odd	commute	anticommute

conjugation operator (complex conj)

$*$ operation

c complex	$c^* = \bar{c}$
θ_i	$\theta_i^* = \bar{\theta}_i$
$\bar{\theta}_i$	$\bar{\theta}_i^* = \theta_i$

$$(g_1 \cdot g_2)^* = g_2^* \cdot g_1^*$$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+, multiplication by ℂ, product ·
⇒ 2N generators θ_i, θ̄_i i = 1, .. N
anticommuting
θ_i θ_j + θ_j θ_i = 0, θ̄_i θ̄_j + θ̄_j θ̄_i = 0
θ_i θ̄_j + θ̄_j θ_i = 0 it contains 1
N = 1 · g = a + b θ + c θ̄ + d θθ̄ , g* = ā + c̄ θ + b̄ θ̄ + d̄ θθ̄
complex coefficients
more complicated
H fixed
= gradation
degree
commut
commut
anticommut
conjugation operator (complex conj)
* operation
c complex c* = c̄
θ_i θ_i* = θ̄_i
θ̄_i θ̄_i* = θ_i
(g_1 · g_2)* = g_2* · g_1*

$|x-y|^2 > 0$ space like

$\{A, B\} = AB + BA$

over the complex field $\mathbb{C}$

G_N associative algebra over $\mathbb{C}$

$+$, multiplication by $\mathbb{C}$, product $\cdot$

$\Rightarrow$ 2N generators $\theta_i, \bar\theta_i$ $i = 1, \dots N$

anticommuting

$\theta_i\theta_j + \theta_j\theta_i = 0$, $\bar\theta_i\bar\theta_j + \bar\theta_j\bar\theta_i = 0$

$\theta_i\bar\theta_j + \bar\theta_j\theta_i = 0$ it contains $\mathbb{1}$

$\bar\theta \dots \bar\theta_{J_H}$

H fixed

= gradation degree

N=1 . $g = a + b\,\theta + c\,\bar\theta + d\,\theta\bar\theta$, $g^* = \bar a + \bar c\,\theta + \bar b\,\bar\theta + \bar d\,\theta\bar\theta$

complex coefficients

$$g_1 \cdot g_2 = (a_1 + b_1\theta + c_1\bar\theta + d_1\theta\bar\theta)(a_2 + b_2\theta + c_2\bar\theta + d_2\theta\bar\theta)$$

$$= a_1a_2 + (a_1b_2 + b_1a_2)\theta + (a_1c_2 + c_1a_2)\bar\theta + (a_1d_2 + d_1a_2 + b_1c_2 - c_1b_2)\theta\bar\theta$$

more complicated examples . . .

$g_1 g_2 \neq g_2 g_1$

$d_1 \backslash d_2$	even	odd
even	commute	commut
odd	commute	anticommute

conjugation operator (complex conj)

$*$ operation

c complex	$c^* = \bar c$
θ_i	$\theta_i^* = \bar\theta_i$
$\bar\theta_i$	$\bar\theta_i^* = \theta_i$

$(g_1 \cdot g_2)^* = g_2^* \cdot g_1^*$

("good basis for my algebra)

$K=0 \quad H=0 \quad \underbrace{i_1<i_2<i_K}_{I} \; \underbrace{j_1<j_2<j_K}_{J}$ ... $i_K \; j_1 \; \ldots \; j_H$

complex number

$\dim_{\text{on } \mathbb{C}} G_N = 2^{2N}$

K and H fixed

$(K+H)=$ gradation degree

more complicated examples ...

$$g_1 g_2 \neq g_2 \cdot g_1$$

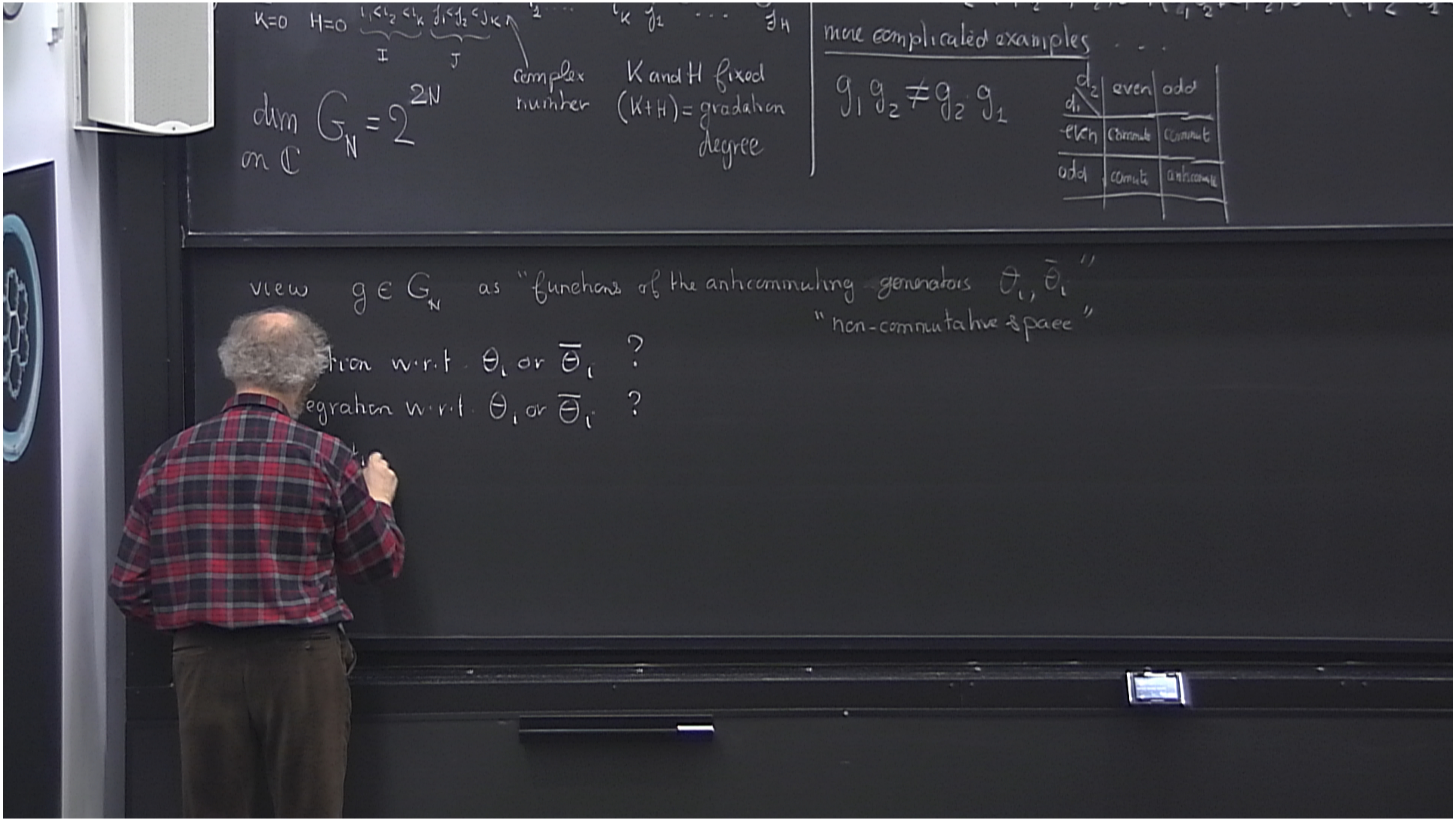
$d_1 \backslash d_2$	even	odd
even	commute	commute
odd	commute	anticommute

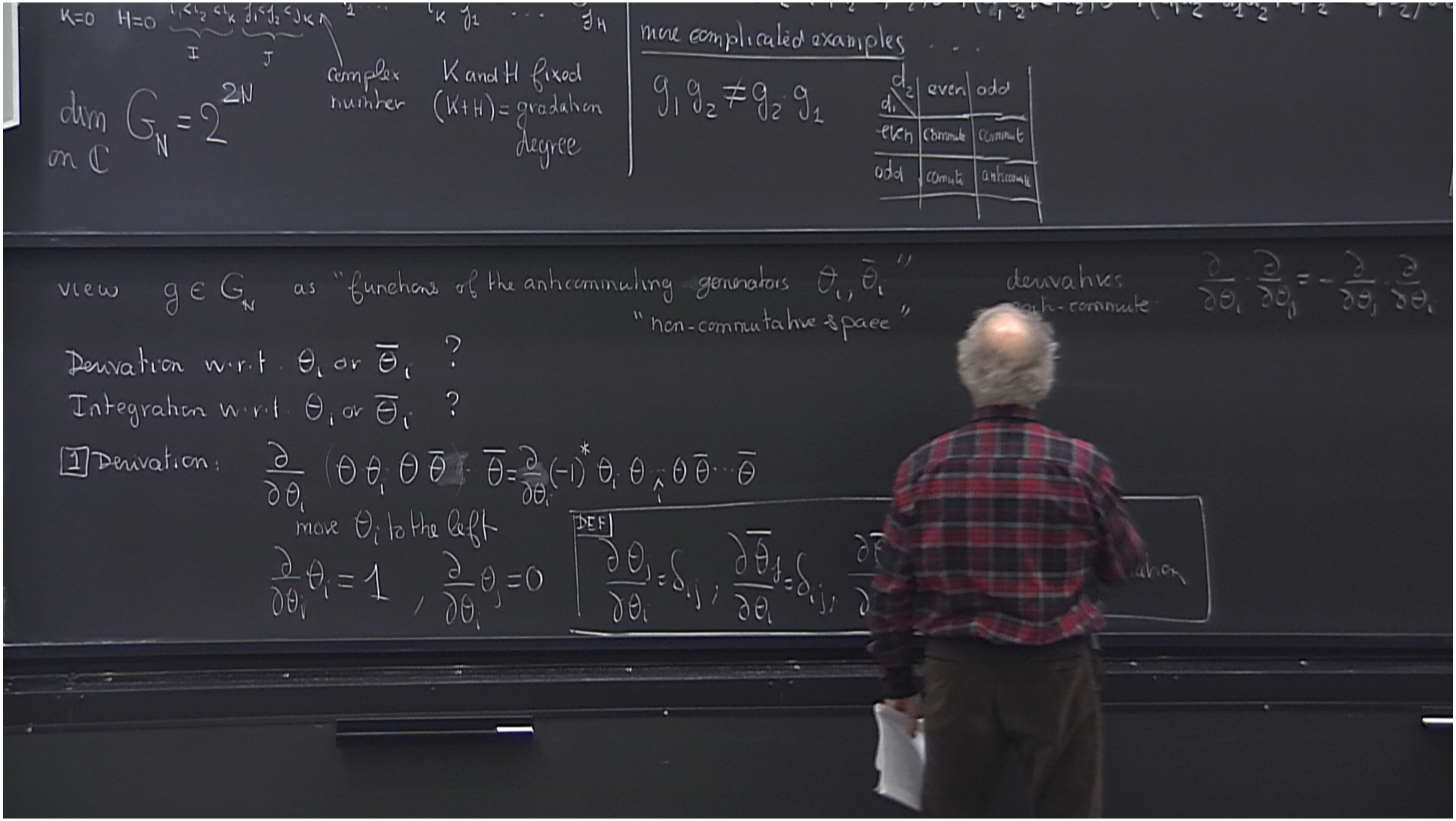
view $g \in G_N$ as "functions of the anticommuting generators $\theta_i, \bar{\theta}_i$"

"non-commutative space"

Derivation w.r.t. θ_i or $\bar{\theta}_i$?

Integration w.r.t. θ_i or $\bar{\theta}_i$


K=0 H=0
complex number
K and H fixed
(K+H) = gradation degree
dim $G_N = 2^{2N}$ on $\mathbb{C}$
more complicated examples ...
$g_1 g_2 \neq g_2 g_1$
even odd
even commute commute
odd commute anticommute
view $g \in G_N$ as "functions of the anticommuting generators $\theta_i, \bar{\theta}_i$"
"non-commutative space"
...tion w.r.t. θ_i or $\bar{\theta}_i$?
...egration w.r.t. θ_i or $\bar{\theta}_i$?


K=0 H=0
I
J
complex number
K and H fixed
(K+H) = gradation degree
dim on ℂ G_N = 2^2N
more complicated examples . . .
g1 g2 ≠ g2 g1
d2 d1 | even | odd
even | commute | commute
odd | commute | anticommute
view g ∈ G_N as "functions of the anticommuting generators θi, θ̄i"
"non-commutative space"
derivatives anti-commute
Derivation w.r.t. θi or θ̄i ?
Integration w.r.t. θi or θ̄i ?
1 Derivation:
move θi to the left
∂/∂θi θi = 1 , ∂/∂θi θj = 0
DEF
∂θj/∂θi = δij , ∂θ̄j/∂θi = δij ,

$K=0 \quad H=0 \quad \underbrace{i_1<i_2<i_k}_{I} \; \underbrace{j_1<j_2<j_K}_{J} \quad \ldots \; i_K \; j_1 \; \ldots \; j_H$

complex number

K and H fixed
$(K+H)$ = gradation degree

$\dim_{\mathbb{C}} G_N = 2^{2N}$

more complicated examples . . .

$g_1 g_2 \neq g_2 \cdot g_1$

$d_1 \backslash d_2$	even	odd
even	commute	commute
odd	commute	anticommute

view $g \in G_N$ as "functions of the anticommuting generators $\theta_i, \bar\theta_i$"
"non-commutative space"

Derivation w.r.t. θ_i or $\bar\theta_i$?

Integration w.r.t. θ_i or $\bar\theta_i$?

$\boxed{1}$ Derivation: w.r.to θ_i and $\bar\theta_i$

$\frac{\partial}{\partial\theta_i}(\theta\,\theta_i\,\theta\,\bar\theta \cdots \bar\theta) = \frac{\partial}{\partial\theta_i}(-1)^{*}\theta_i\,\theta\,\hat{\ }\,\theta\,\bar\theta\cdots\bar\theta$

move θ_i to the left

$\frac{\partial}{\partial\theta_i}\theta_i = 1 \; , \; \frac{\partial}{\partial\theta_i}\theta_j = 0$

DEF

$$\boxed{\frac{\partial\theta_j}{\partial\theta_i} = \delta_{ij} \, , \; \frac{\partial\bar\theta_j}{\partial\bar\theta_i} = \delta_{ij} \, , \; \frac{\partial\bar\theta_j}{\partial\theta_i} = 0 \, , \; \frac{\partial\theta_j}{\partial\bar\theta_i} = 0}$$

+ anti commutation

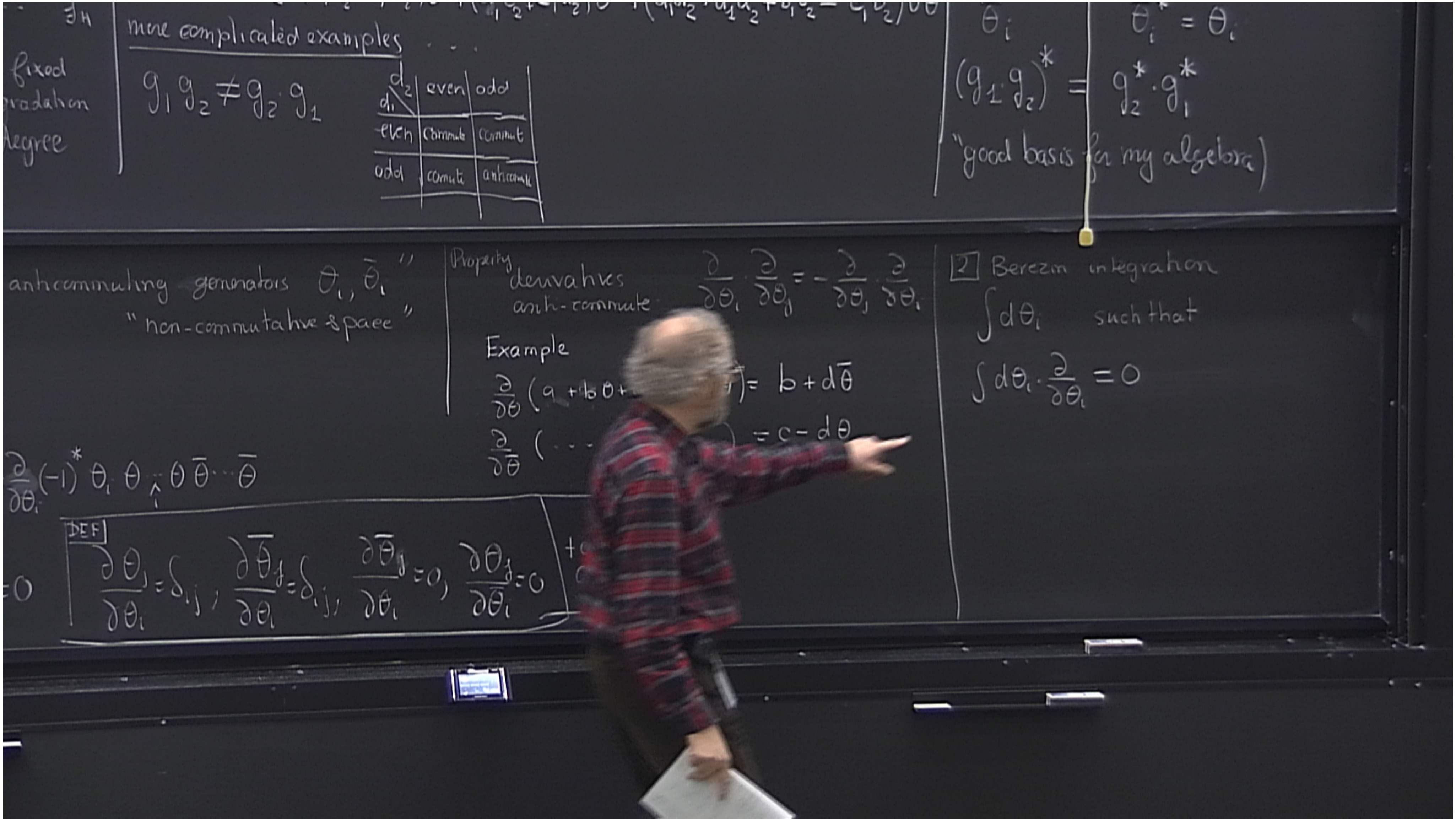
Property derivatives anti-commute

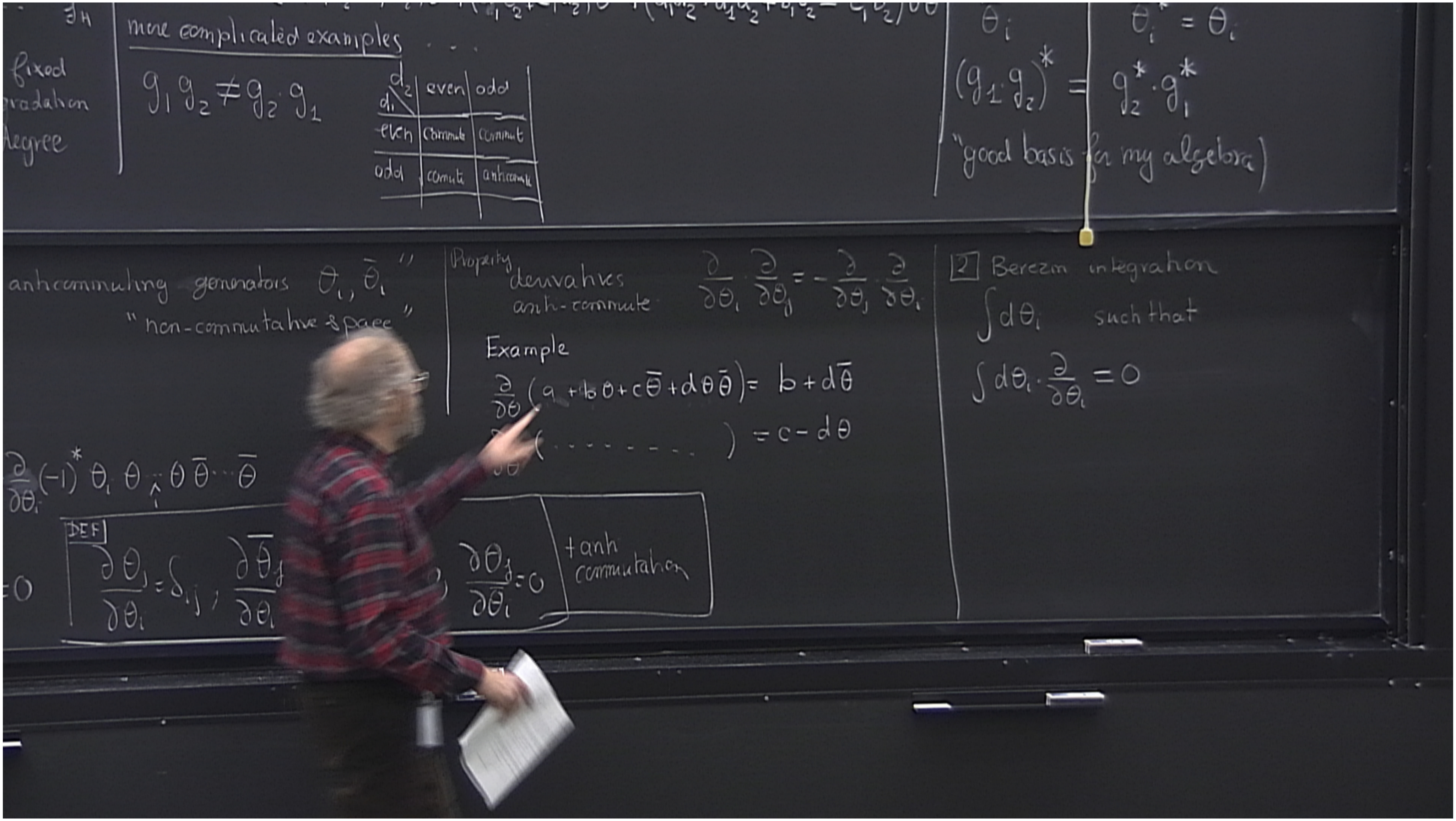
$\frac{\partial}{\partial\theta_i}\frac{\partial}{\partial\theta_j} = -\frac{\partial}{\partial\theta_j}\frac{\partial}{\partial\theta_i}$

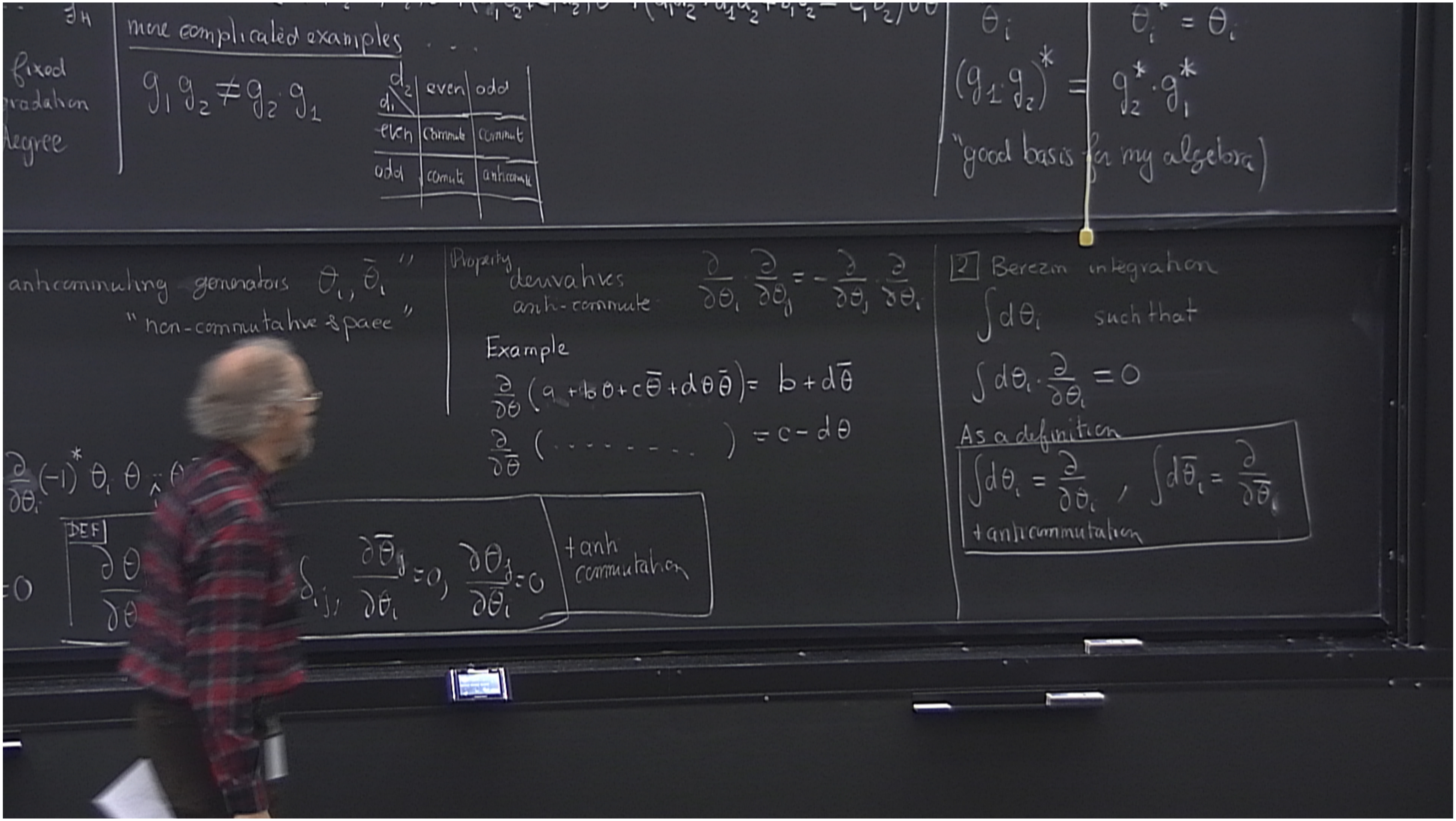
Example

$\frac{\partial}{\partial\theta}(a + b\theta + c\bar\theta + d\theta\bar\theta) = b + d\bar\theta$

$\frac{\partial}{\partial\bar\theta}(\ldots\ldots\ldots) = c - d\theta$


more complicated examples
fixed gradation degree
$g_1 g_2 \neq g_2 \cdot g_1$
even odd
commute commute
commute anticommute
$(g_1 \cdot g_2)^* = g_2^* \cdot g_1^*$
"good basis for my algebra)
anticommuting generators $\theta_i, \bar{\theta}_i$
" non-commutative space "
Property derivatives anti-commute
$\frac{\partial}{\partial\theta_i}\cdot\frac{\partial}{\partial\theta_j} = -\frac{\partial}{\partial\theta_j}\cdot\frac{\partial}{\partial\theta_i}$
Example
$= b + d\bar{\theta}$
$= c - d\theta$
DEF
$\frac{\partial\theta_j}{\partial\theta_i} = \delta_{ij}, \frac{\partial\bar{\theta}_j}{\partial\bar{\theta}_i} = \delta_{ij}, \frac{\partial\bar{\theta}_j}{\partial\theta_i} = 0, \frac{\partial\theta_j}{\partial\bar{\theta}_i} = 0$
2 Berezin integration
$\int d\theta_i$ such that
$\int d\theta_i \cdot \frac{\partial}{\partial\theta_i} = 0$


more complicated examples
fixed gradation degree
g1 g2 ≠ g2 · g1
even odd
even commute commute
odd commute anticommute
(g1 · g2)* = g2* · g1*
θi* = θi
"good basis for my algebra)
anticommuting generators θi, θ̄i
"non-commutative space"
Property derivatives anti-commute
Example
= b + dθ̄
= c − dθ
DEF
+ anti commutation
2 Berezin integration
∫dθi such that
∫dθi · ∂/∂θi = 0


more complicated examples . . .
fixed
gradation
degree
g1 g2 ≠ g2 · g1
d1 \ d2 | even | odd
even | commute | commute
odd | commute | anticommute
(g1 · g2)* = g2* · g1*
θi* = θi
"good basis for my algebra)
anticommuting generators θi, θ̄i
" non-commutative space "
Property
derivatives anti-commute
∂/∂θi · ∂/∂θj = − ∂/∂θj · ∂/∂θi
Example
∂/∂θ (a + bθ + cθ̄ + dθθ̄) = b + dθ̄
∂/∂θ̄ (. . . . .) = c − dθ
DEF
+ anti commutation
[2] Berezin integration
∫dθi such that
∫dθi · ∂/∂θi = 0
As a definition
∫dθi = ∂/∂θi , ∫dθ̄i = ∂/∂θ̄i
+ anticommutation

fixed
gradation
degree

more complicated examples . . .

$$g_1 g_2 \neq g_2 g_1$$

$d_1 \backslash d_2$	even	odd
even	commute	commute
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$\theta_i \qquad \theta_i^* = \theta_i$

$$(g_1 g_2)^* = g_2^* \cdot g_1^*$$

"good basis for my algebra)

anticommuting generators $\theta_i, \bar\theta_i$ "
" non-commutative space "

Property
derivatives anti-commute $\dfrac{\partial}{\partial\theta_i}\dfrac{\partial}{\partial\theta_j} = -\dfrac{\partial}{\partial\theta_j}\dfrac{\partial}{\partial\theta_i}$

Example

$$\frac{\partial}{\partial\theta}\left(a + b\theta + c\bar\theta + d\theta\bar\theta\right) = b + d\bar\theta$$

$$\frac{\partial}{\partial\bar\theta}\left(\cdots\cdots \right) = c - d\theta$$

$\boxed{2}$ Berezin integration = derivation.

$\int d\theta_i$ such that

$$\int d\theta_i \cdot \frac{\partial}{\partial\theta_i} = 0$$

As a definition

$$\boxed{\int d\theta_i = \frac{\partial}{\partial\theta_i}, \quad \int d\bar\theta_i = \frac{\partial}{\partial\bar\theta_i} \quad \text{+ anticommutation}}$$

$$\frac{\partial}{\partial\theta_i}(-1)^* \theta_i\, \theta \cdots \theta\, \bar\theta \cdots \bar\theta$$

$= 0$

DEF

$$\boxed{\frac{\partial\theta_j}{\partial\theta_i} = \delta_{ij}, \quad \frac{\partial\bar\theta_j}{\partial\bar\theta_i} = \delta_{ij}, \quad \frac{\partial\bar\theta_j}{\partial\theta_i} = 0, \quad \frac{\partial\theta_j}{\partial\bar\theta_i} = 0 \quad \text{+ anti commutation}}$$

$K=0 \quad H=0 \quad \underbrace{i_1<i_2<i_K}_{I} \; \underbrace{j_1<j_2<j_K}_{J}$

complex number

K and H fixed

$(K+H)$ = gradation degree

$\dim G_N = 2^{2N}$ on $\mathbb{C}$

more complicated examples . . .

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Integration w.r.t. θ_i or $\bar{\theta}_i$?

$\boxed{1}$ Derivation: w.r.to θ_1 and $\bar{\theta}_1$

$$\frac{\partial}{\partial\theta_i}(\theta \cdot \theta_i \, \theta \, \bar{\theta} \cdots \bar{\theta}) = \frac{\partial}{\partial\theta_i}(-1)^{*} \theta_i \, \theta \, \hat{\ } \, \theta \, \bar{\theta} \cdots \bar{\theta}$$

move θ_i to the left

$$\frac{\partial}{\partial\theta_i}\theta_i = 1 \;, \quad \frac{\partial}{\partial\theta_i}\theta_j = 0$$

DEF

$$\frac{\partial\theta_j}{\partial\theta_i} = \delta_{ij} \;, \quad \frac{\partial\bar{\theta}_j}{\partial\bar{\theta}_i} = \delta_{ij} \;, \quad \frac{\partial\bar{\theta}_j}{\partial\theta_i} = 0 \;, \quad \frac{\partial\theta_j}{\partial\bar{\theta}_i} = 0$$

+ anti commutation

Property: derivatives anti-commute $\frac{\partial}{\partial\theta_i}\frac{\partial}{\partial\theta_j} =$

Example

$$\frac{\partial}{\partial\theta}(a + b\theta + c\bar{\theta} + d\theta\bar{\theta}) = b +$$

$$\frac{\partial}{\partial\bar{\theta}}(\ldots\ldots\ldots) = c -$$