Title: PSI 2015/2016 Quantum Field Theory II - Francois David - Lecture 8

Date: Nov 18, 2015 09:00 AM

URL: http://pirsa.org/15110034

Abstract:

1) Renormalization at 1 loop

2) Perturbative vs. Wilsonian Renormalization

malization

Massless ϕ^4

g_R Ren. Coupling $\quad g_B = C$ "Bare" coupling

μ Ren. Scale $\quad \Lambda$ UV. momentum cut-off

$$g_B = C = g_R + g_R^2 \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{\mu^2}\right)$$

β-function (for g)

$$\beta(g_R) = \mu \frac{\partial}{\partial \mu} g_R \Big|_{\text{Physics is fixed}}$$

Λ and g_B fixed $\Leftrightarrow$ $S_R[\phi]$ fixed

Λ fixed

$$\beta(g_R) = \lim_{\substack{\Lambda \to \infty \\ g_R \text{ fixed}}} \left[\mu \frac{\partial}{\partial \mu} g_R \Big|_{\substack{g_B \\ \Lambda}} \right]$$

coupling

momentum cut-off

$$d\mu \longrightarrow d g_R$$

$$\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$0 = d g_R \left(1 + 2 g_R \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda}{\mu}\right)^2 \right) - 2 \frac{d\mu}{\mu} g_R^2 \frac{3}{2} \frac{1}{(4\pi)^2}$$

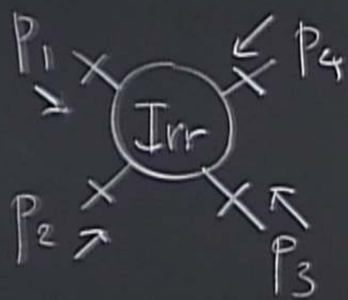
$S_R[\phi]$ fixed

Λ fixed

$$\underbrace{\boxed{\beta(g) = \mu \frac{d g_R}{d\mu} = 3 g_R^2 \frac{1}{(4\pi)^2}}}_{\text{1 loop}} + \underbrace{O(g_R^3)}_{\text{2 loop contribution}}$$

contains $\log \frac{\Lambda}{\mu}$

- $g_R(\mu)$ Effective coupling at scale μ



$= \Gamma^{(4)}(\{p_i/s\}, g_R; \mu) =$

How does this scale with energy $\quad p_i \to \lambda p_i$

$p_i = (iE, \vec{k})$ $\quad$ λ scaling factor

Euclidean space-time $\quad$ (pure number)

$\lambda \nearrow \infty \Leftrightarrow s \searrow 0$ small distances $\quad \lambda = 1/s$

coupling at scale μ

$$\Gamma^4(\{p\}, \ ) = g_R + g_R^2 \sum_{\text{channels}} \log\left(\frac{\mu}{p}\right)^2$$

dimensional analysis

Renormalization theory

$$\}, g_R; \mu) = \Gamma^{(4)}(\{p_i\}, g_R, \mu s) = \Gamma^{(4)}(\{p_i\}, g_R(s), \mu)$$

energy $p_i \to \lambda p_i$

work out

$$\boxed{s\frac{d}{ds} g_R(s) = -\beta(g_R(s))}$$

scaling factor

pure number)

$= 1/s$

μ

nalysis

$$\Gamma^4(\{p\}, \quad) = g_R + g_R^2 \sum_{\text{channels}} \log\left(\frac{\mu}{p}\right)^2$$

Renormalization theory

$$\Gamma(\{p_i\}, g_R, \mu s) = \Gamma^{(4)}(\{p_i\}, g_R(s), \mu)$$

work out

$$\boxed{s \frac{d}{ds} g_R(s) = -\beta(g_R(s))}$$

effective (running) coupling at energies rescaled by a factor s^{-1}

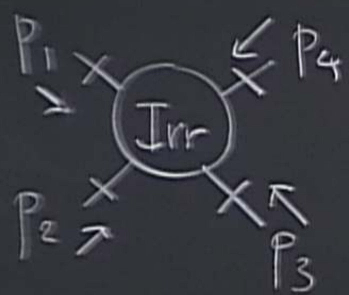
at distances rescaled by factor s

- $g_R(\mu)$ Effective coupling at scale μ

$\Gamma^4(\{p\},$

dimensional analysis

Re



$$= \Gamma^{(4)}(\{p_i/s\}, g_R; \mu) = \Gamma^{(4)}(\{p_i\}, g_R, \mu s)$$

How does this scale with energy $p_i \to \lambda p_i$

work ou

$p_i = (iE, \vec{k})$

λ scaling factor (pure number)

Euclidean space-time

$$\boxed{s \frac{d}{ds} g_R(s}$$

$\lambda \nearrow \infty \Leftrightarrow s \searrow 0$ small distances

$\lambda = 1/s$

- $\beta(g) \neq 0$: scale invariance is anomalous
broken by quantum effets

Scale anomaly

Classical ϕ^4 $D=4$. scale invariant $\qquad \phi(x) \to \phi_s(x) = s\,\phi(sx)$

(symmetry) $\qquad S[\phi_s] = S[\phi]$

$\Downarrow$

current

$$J^\mu_{\text{scale}} = T^\mu{}_\nu x^\nu + \phi\,\partial^\mu\phi \qquad x \to s\,x \qquad T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - \delta_{\mu\nu}\mathcal{L}$$

space $\qquad$ field

$$\partial_\mu J^\mu_{\text{scale}} = 0 \quad \text{classical} \qquad \mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi\,\partial^\mu\phi\right) + \frac{g}{4!}\phi^4$$

factors

...omalous $D = 4$

...ffets

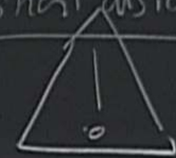
$\rightarrow \phi_s(x) = s\,\phi(s x)$

$] = S[\phi]$

$T_{\mu\nu} = \partial_\mu \phi\, \partial_\nu \phi - \delta_{\mu\nu} \mathcal{L}$

$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\, \partial^\mu \phi\right) + \frac{g}{4!}\phi^4$

1 loop : short distance singularities



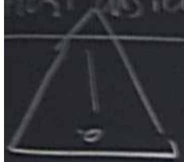
$$\partial_\mu J^\mu_{\text{scale}} = \underbrace{g_R^2 \frac{3}{(4\pi)^2}}_{\text{1 loop contribution}} \cdot \frac{\phi^4}{4!} \neq 0$$

$$= \beta(g_R)\, \frac{\phi^4}{4!} \left(\sim T^\mu_\mu\right)$$

β-function = scale anomaly

T^μ_μ dimensionless =

...al distance singularities



$T^{\mu}_{\ \mu}$ dimensionless $= g^{\mu\nu}\,T_{\mu\nu} \Rightarrow$ CFT & String Theory

$$\underbrace{g_R^2\,\frac{3}{(4\pi)^2}}_{\text{1 loop contribution}}\cdot\frac{\phi^4}{4!} \neq 0$$

$$= \beta(g_R)\cdot\frac{\phi^4}{4!}\left(\sim T^{\mu}_{\ \mu}\right)$$

... = scale anomaly

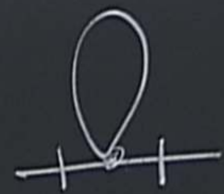
4] Massive theory : $M_{phys} \neq 0$

$$S_R[\phi] = \int d^4x \left[\frac{A}{2}(\partial_\nu \phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{4!}\phi^4\right]$$

$$A = 1$$

$$B = \left(-\Lambda^2 g_R \frac{1}{(4\pi)^2} \cdot \frac{1}{2}\right) + m_R^2 \Big($$

$$C = g_R + g_R^2 \frac{3}{(4\pi)^2} \frac{1}{2} \log\left(\frac{\Lambda^2}{\mu^2}\right)$$



$= T = \Lambda^2 \cdot + m^2 \cdot \log \Lambda$

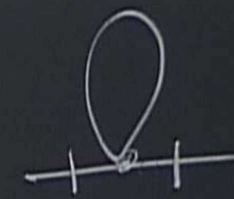
$m_{phys} \neq 0$

$(\partial_\mu \phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{4!}\phi^4 \Big]$

$+ m_R^2 \Big($

$\frac{\Lambda^2}{\mu^2}\Big)$

4] Massive theory : $M_{phys} \neq 0$

 =

$$S_R[\phi] = \int d^4x \left[\frac{A}{2}(\partial_\mu \phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{4!}\phi^4 \right]$$

$$A = 1$$

$$B = \boxed{\left(-\Lambda^2 g_R \frac{1}{(4\pi)^2} \cdot \frac{1}{2} \right)} + m_R^2 \boxed{+ \left(m_R^2 \, g_R \frac{1}{(4\pi)^2} \cdot \frac{1}{2} \log\left(\frac{\Lambda^2}{\mu^2}\right) \right)}$$

$$C = g_R + g_R^2 \frac{3}{(4\pi)^2} \frac{1}{2} \log\left(\frac{\Lambda^2}{\mu^2}\right)$$

new. counterterm

 $= T = \Lambda^2 \cdot + m^2 \cdot \log \Lambda$

$$\Gamma^{(2)}(p) = p^2 + M^2_{phys} + O(g_R^2)$$

2 loop

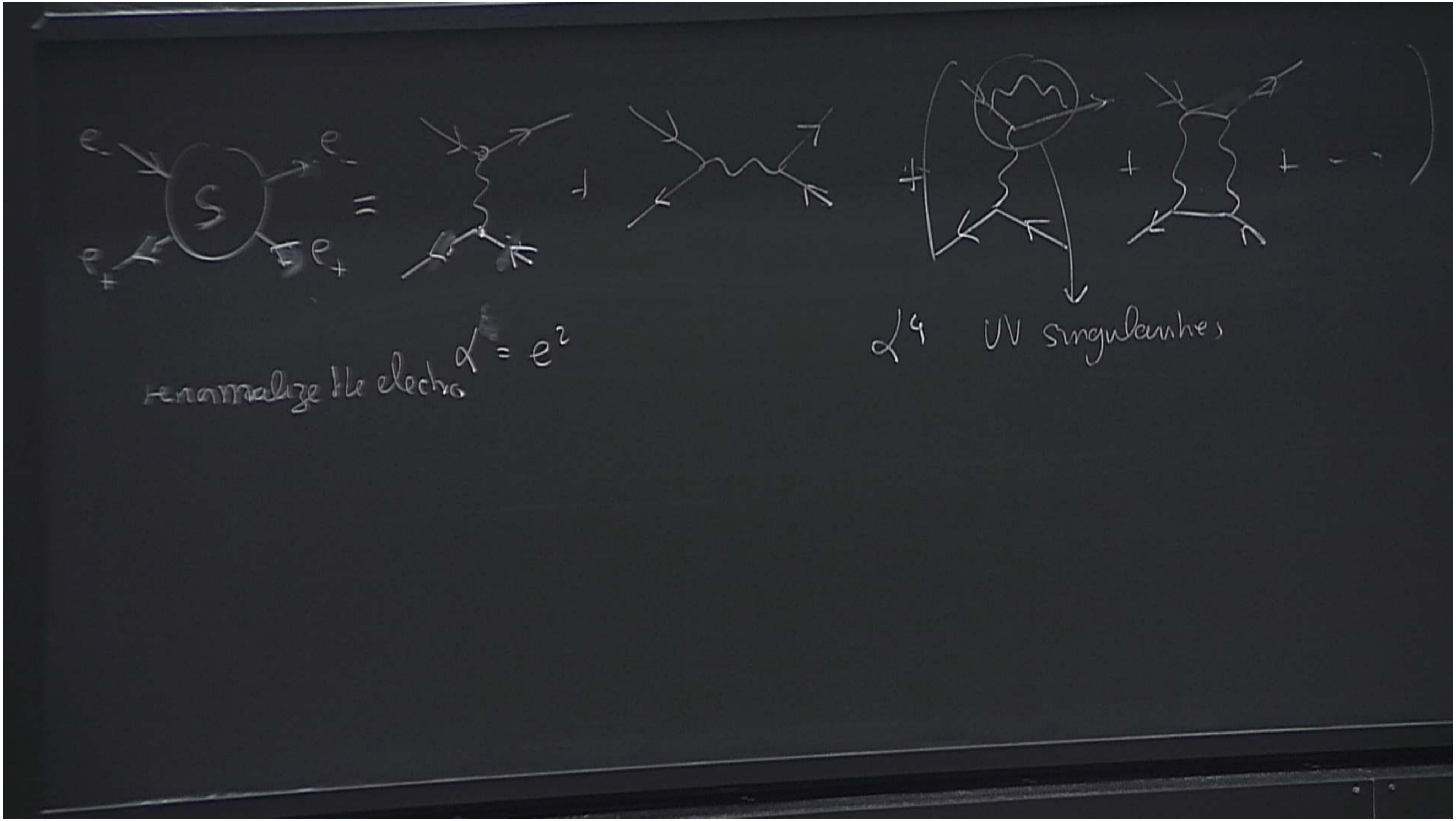
$$\boxed{M^2_{phys} = m_R^2 \left(1 + g_R \frac{1}{2}\frac{1}{(4\pi)^2} \log\left(\frac{m_R^2}{\mu^2}\right)\right)}$$

$\log\left(\frac{\Lambda^2}{\mu^2}\right)\Big)$

m_R renormalized mass

g_R " coupling

change in μ $\longleftrightarrow$ change in g_R & m_R


e-
e-
S
e+
e+
=
+
+
+
+ ...
α = e²
renormalize the electro
α⁴ UV singularities

Why do Renormalization Work?

What if $D \neq 4$

Short singularities in QFT are proportional to local operators

(QFT are "local" theories, locality $\Rightarrow$ Causality, Lorentz invariance)

no FTL effects (physical)

$$\langle \phi \cdots \phi \rangle_{\text{interaction}} = \langle \phi \cdots \phi \quad \phi^4(x_1)\phi^4(x_2) \cdots \rangle_0$$

$\uparrow$ Free

p

$x_1 \quad x_2$

$$\propto \left(|x_1-x_2|^{-2}\right)^2 d^4x_1\, d^4x_2 \simeq \int \frac{d^4y}{|y|^4} \times \int d^4x_1$$

$$= \phi^4(x_1)$$

$x_1 \to x_2 \qquad y = x_1 - x_2$

$\Lambda \sim \dfrac{1}{\text{minimal distance}} \qquad \log \Lambda$

$$\phi^4(x_1)\,\phi^4(x_2) \underset{x_1\to x_2}{=} |x_1-x_2|^{-4}\,\phi^4(x_1)$$

$$\propto \left(|x_1-x_2|^{-2}\right)^2 d^4x_1\, d^4x_2 \simeq \underbrace{\int \frac{d^4y}{|y|^4}}_{\log\Lambda} \times \int d^4x_1$$

$= \phi^4(x_1)$

$x_1 \to x_2$ $\quad y = x_1 - x_2$ $\quad \Lambda \sim \frac{1}{\text{minimal distance}}$

e)

cal)

$$\boxed{\phi^4(x_1)\,\phi^4(x_2) \underset{x_1\to x_2}{=} |x_1-x_2|^{-6}\,\phi^2(x_1) + |x_1-x_2|^{-4}\,\phi^4(x_1) + |x_1-x_2|^{-8}\,\mathbb{1}(x_1) + |x_1-x_2|^{-4}\,(\partial\phi)^2}$$

$\phi^2(\partial\phi)^2$ $\quad \phi^4$

$$\propto \left(|x_1-x_2|^{-2}\right)^2 d^4x_1\, d^4x_2 \simeq \int \frac{d^4y}{|y|^4} \times \int d^4x_1$$

$$x_1 \to x_2 \qquad y = x_1 - x_2 \qquad \Lambda \sim \frac{1}{\text{minimal distance}} \qquad \log\Lambda \qquad \phi^4(x_1)$$

$$\boxed{\phi^4(x_1)\,\phi^4(x_2) \underset{x_1\to x_2}{=} |x_1-x_2|^{-6}\,\phi^2(x_1) + |x_1-x_2|^{-4}\,\phi^4(x_1) + |x_1-x_2|^{-8}\,\mathbb{1}(x_1) + |x_1-x_2|^{-4}(\partial\phi)^2}$$

k ?

D=4	$\phi^4 \times \phi^4 = \lvert y \rvert^{-4}\, \phi^4$	Renormalizable
D<4	$\phi^4 \times \phi^4 = \lvert y \rvert^{-2(D-2)}\, \phi^4$	super. R
		non-renormalizable
D>4		

are proportional to local operators

locality $\Rightarrow$ (Causality, Lorentz invariance)

no FTL effects (physical)

$\phi^4(x_1)\phi^4(x_2) \cdots \sum_{O}$

↑ Free

ion (Wilson)

$\underset{\phi^4}{x} \; \underset{\phi^4}{x} = \cdots$

$\mathbb{1} \quad \phi^2 (\partial\phi)^2 \quad \phi^4$

$x_1 \quad x_2$

$x_1 \to x_2$

$\phi^4(x_1)\,\phi^4(x_2$

QFT renormalization $\longleftrightarrow$ Wilson Ren. Group
(Gell-Mann - Low, Callan - Zymanzik)

BPHZ "Theorem" $\Leftarrow$ all orders

Renormalization
Scale μ

Energy
Scale

QFT renormalization $\longleftrightarrow$ Wilson Ren. Group

(Gell-Mann-Low, Callan-Zymazik)

BPHZ "Theorem" $\Leftarrow$ all orders

QFT

$\int D[\phi]\, e^{-S[\phi]}$

ϕ^4 theory

HE

$\boxed{\text{Renormalization Scale } \mu}$

UV cut off

$\Lambda \longrightarrow \infty$

Energy Scale

Stat. Physics

Wilson. RG

$\mathcal{H}_{\text{renormalized}}$

$\mathcal{H}_{\text{microscopic}}$

Length Scale

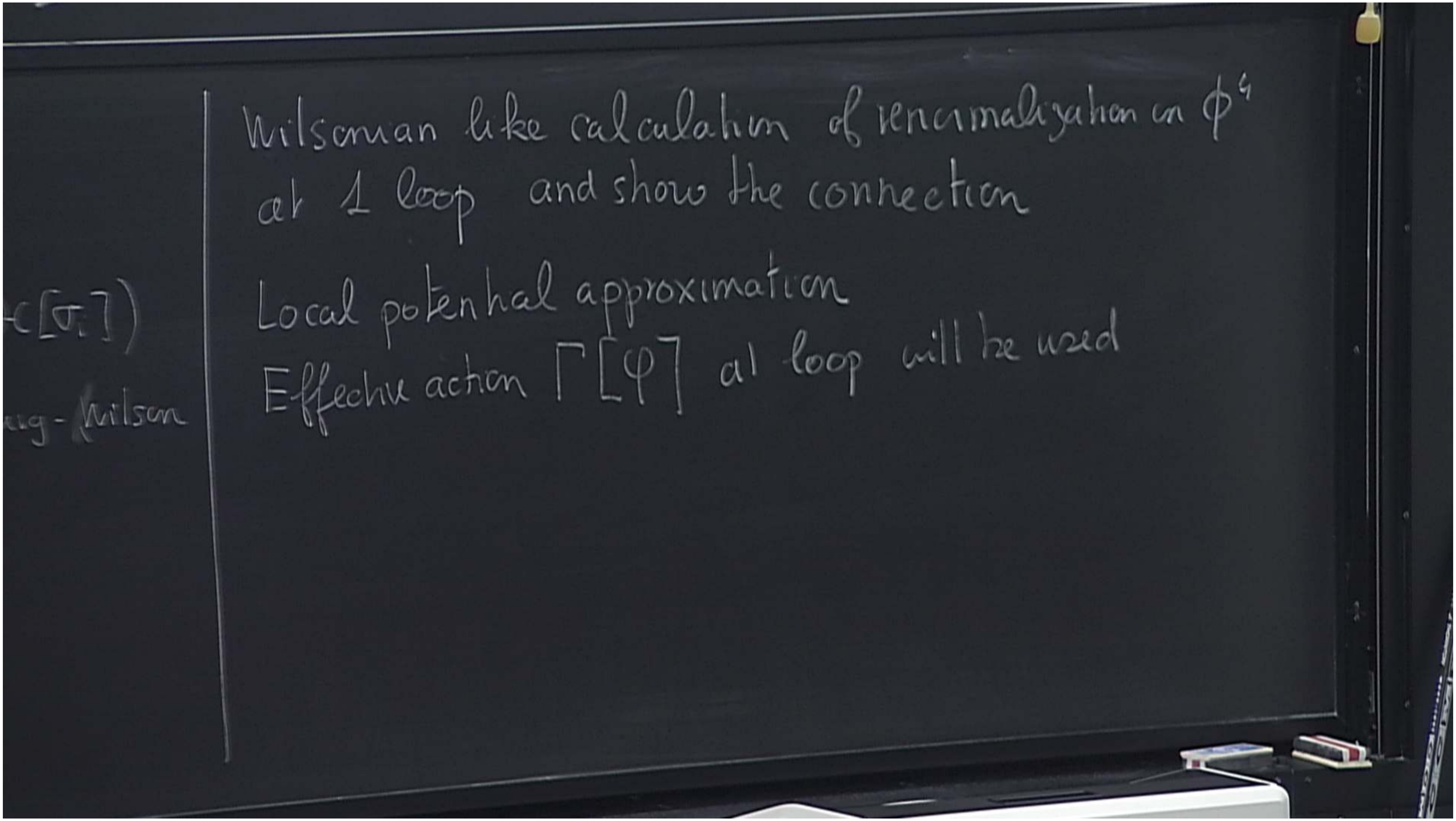
large scales

$\ell \gg a$

$\ell \simeq \xi$ correlation length

a

$\boxed{\text{microscopic scale}}$


Wilsonian like calculation of renormalization in ϕ^4
at 1 loop and show the connection
Local potential approximation
Effective action $\Gamma[\varphi]$ at loop will be used
$c[\sigma_i])$
-g- Wilson