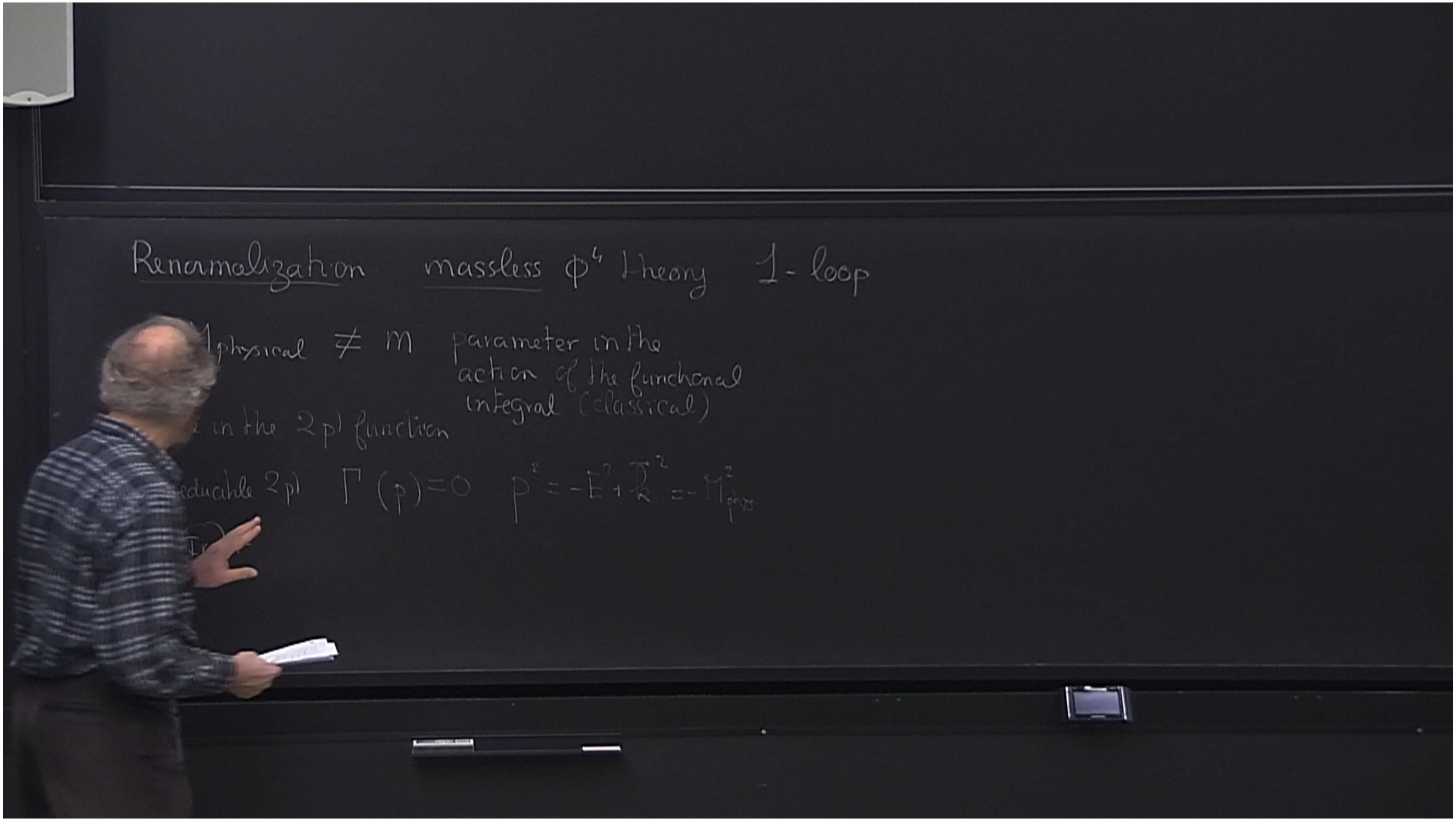
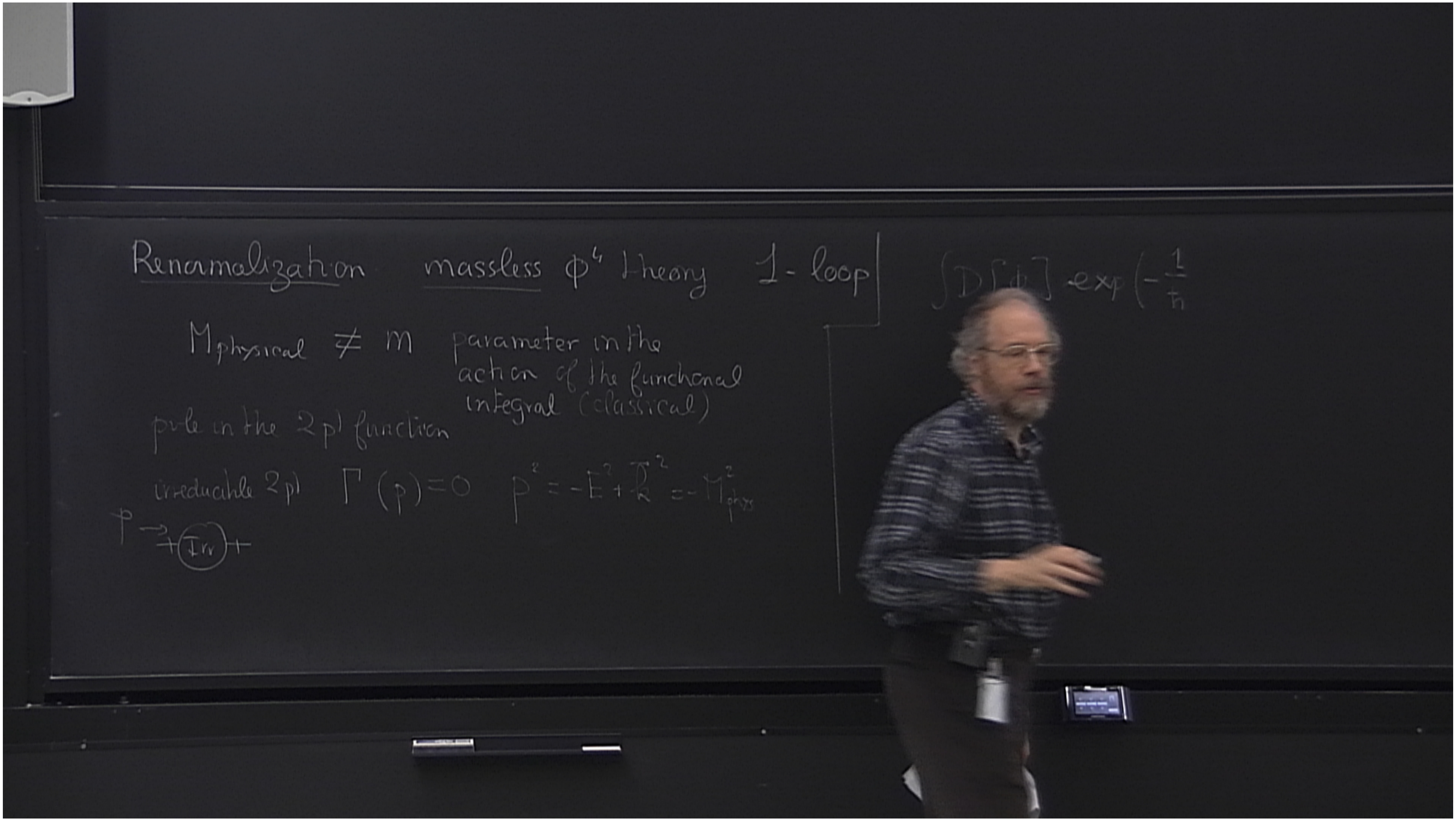
Title: PSI 2015/2016 Quantum Field Theory II - Francois David - Lecture 7

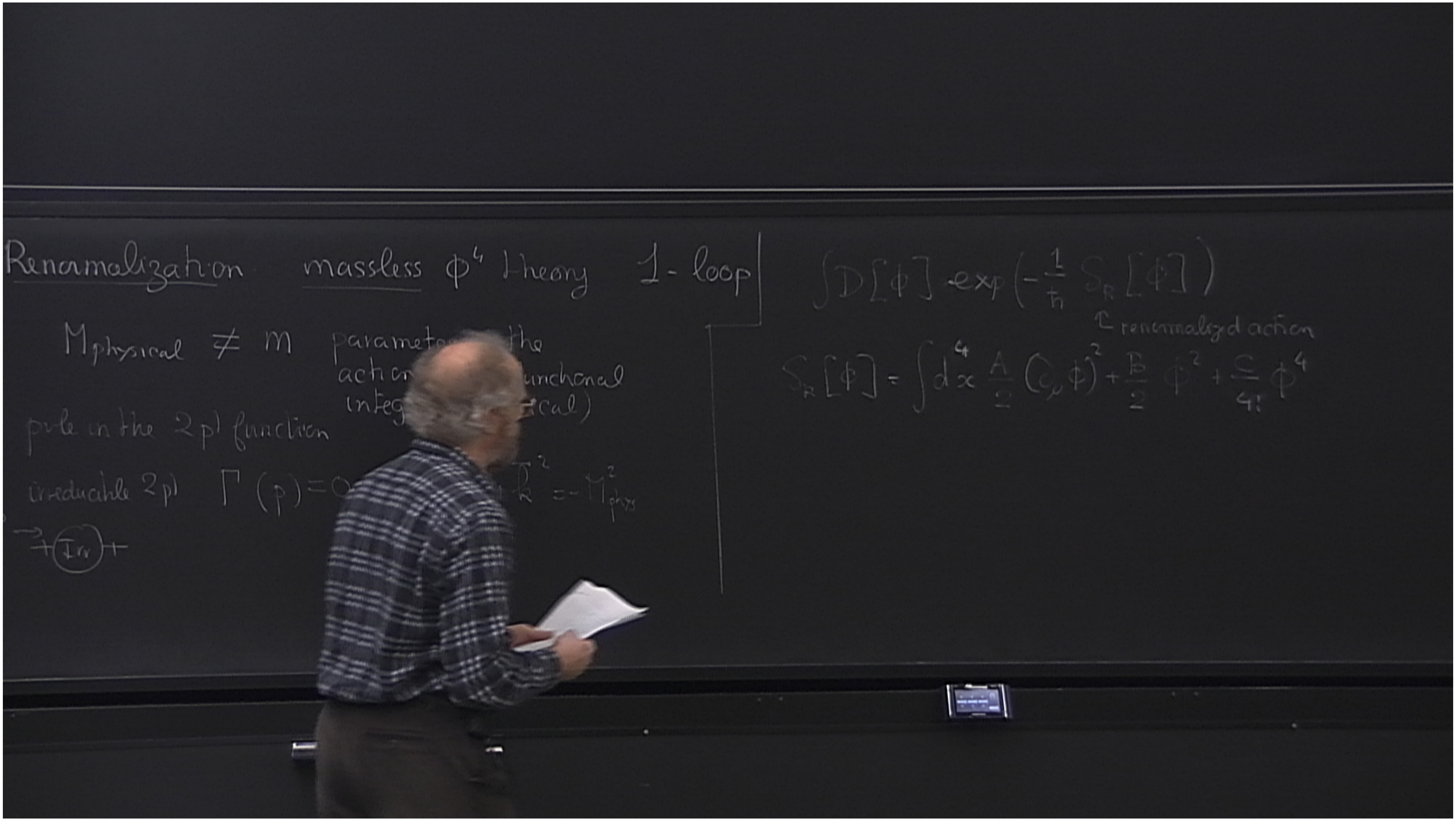
Date: Nov 17, 2015 09:00 AM

URL: http://pirsa.org/15110033

Abstract:


Renormalization massless ϕ^4 theory 1-loop
$M_{physical} \neq m$ parameter in the action of the functional integral (classical)
in the 2pt function
educible 2pt $\Gamma(p)=0$ $p^2=-E^2+\vec{k}^2=-M^2_{phys}$


Renormalization massless φ⁴ theory 1-loop
M physical ≠ m parameter in the action of the functional integral (classical)
pole in the 2pt function
irreducible 2pt Γ(p)=0 p² = -E² + k² = -M²phys
Irr
∫D[φ] exp(-1/ħ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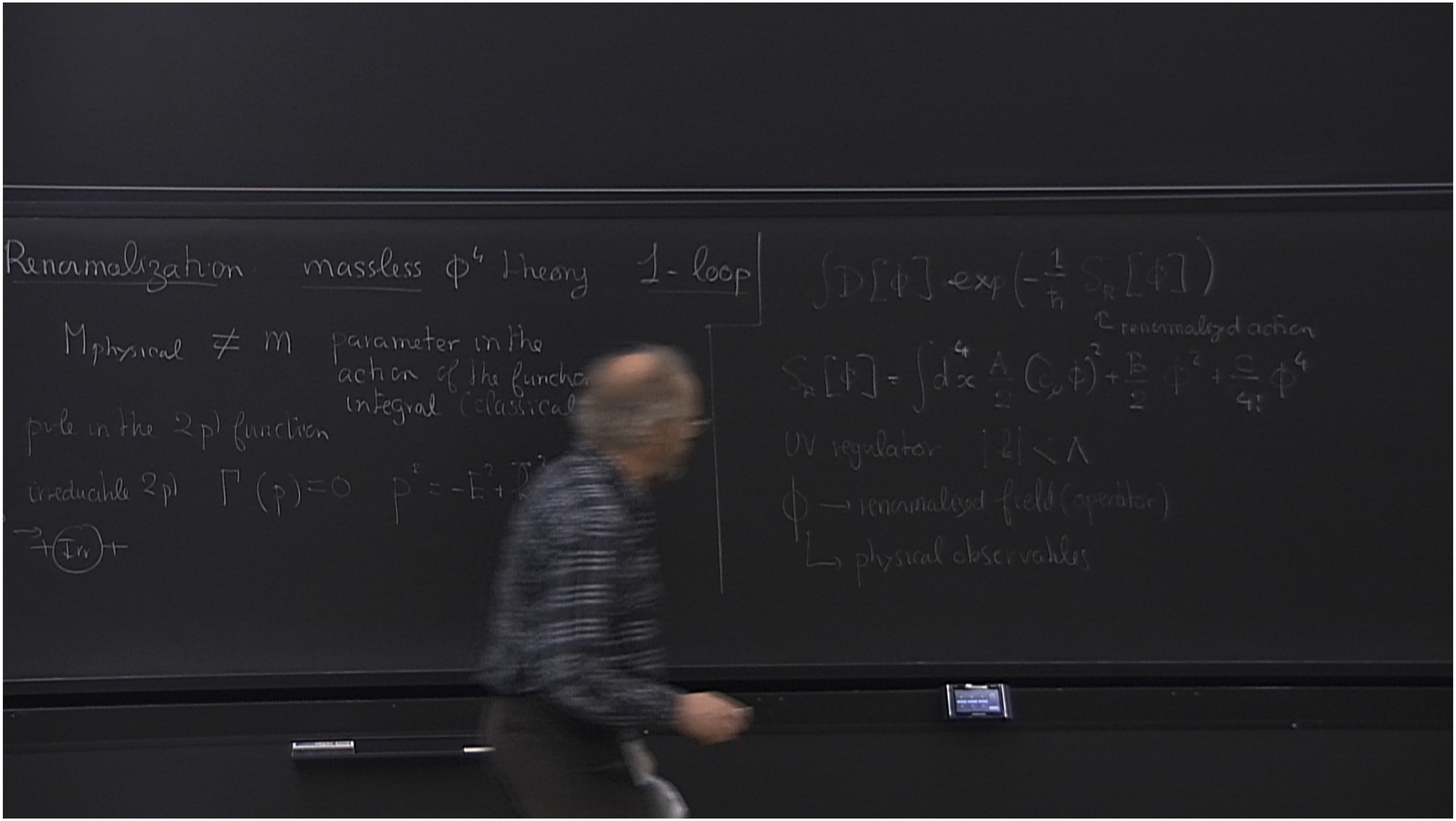
∫D[φ] exp(-1/ħ S_R[φ])
renormalized action
S_R[φ] = ∫d⁴x A/2 (∂_μ φ)² + B/2 φ² + C/4! φ⁴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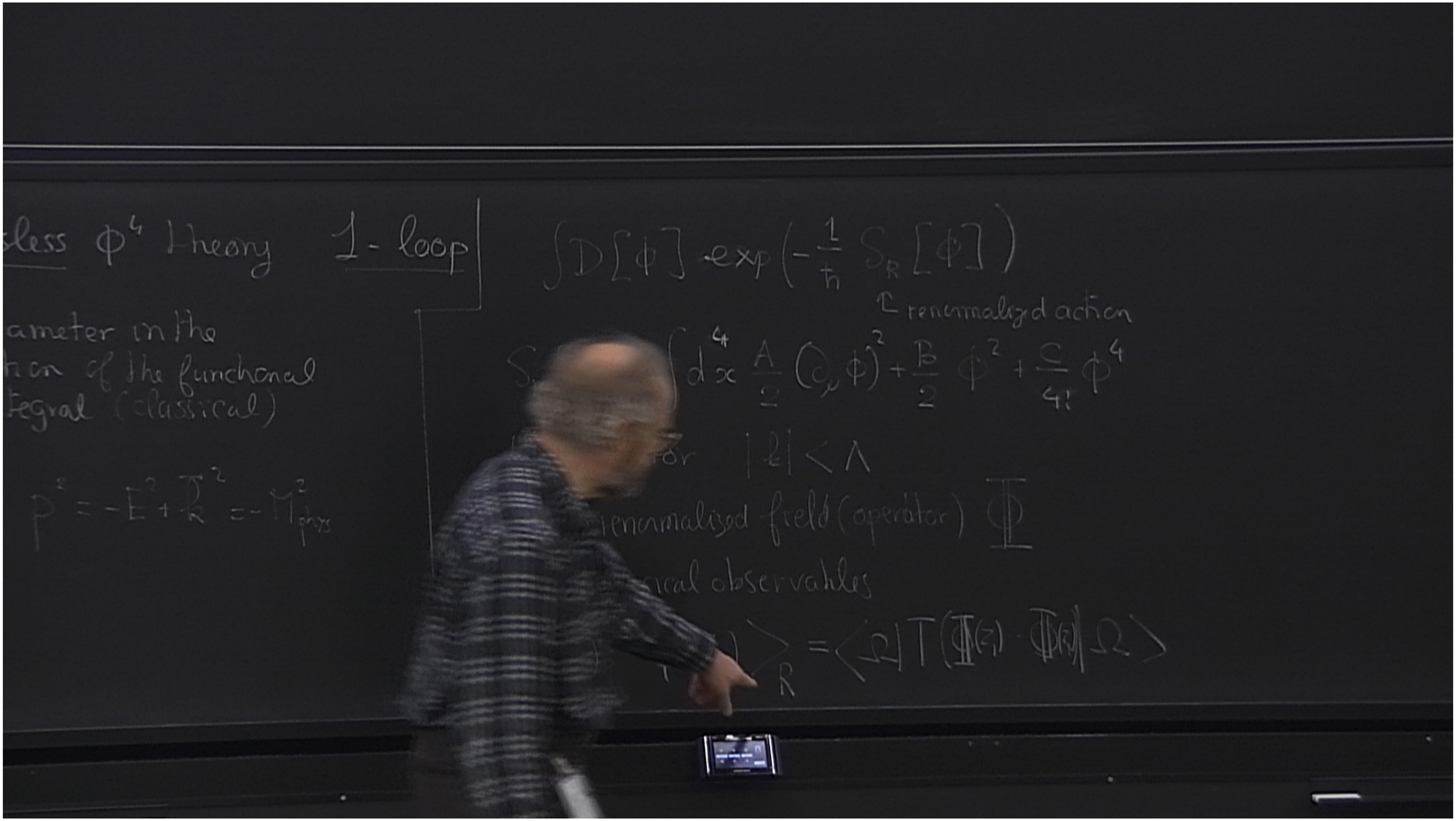
UV regulator |k| < Λ
φ → renormalized field (operator)
→ physical observables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for |k| < Λ
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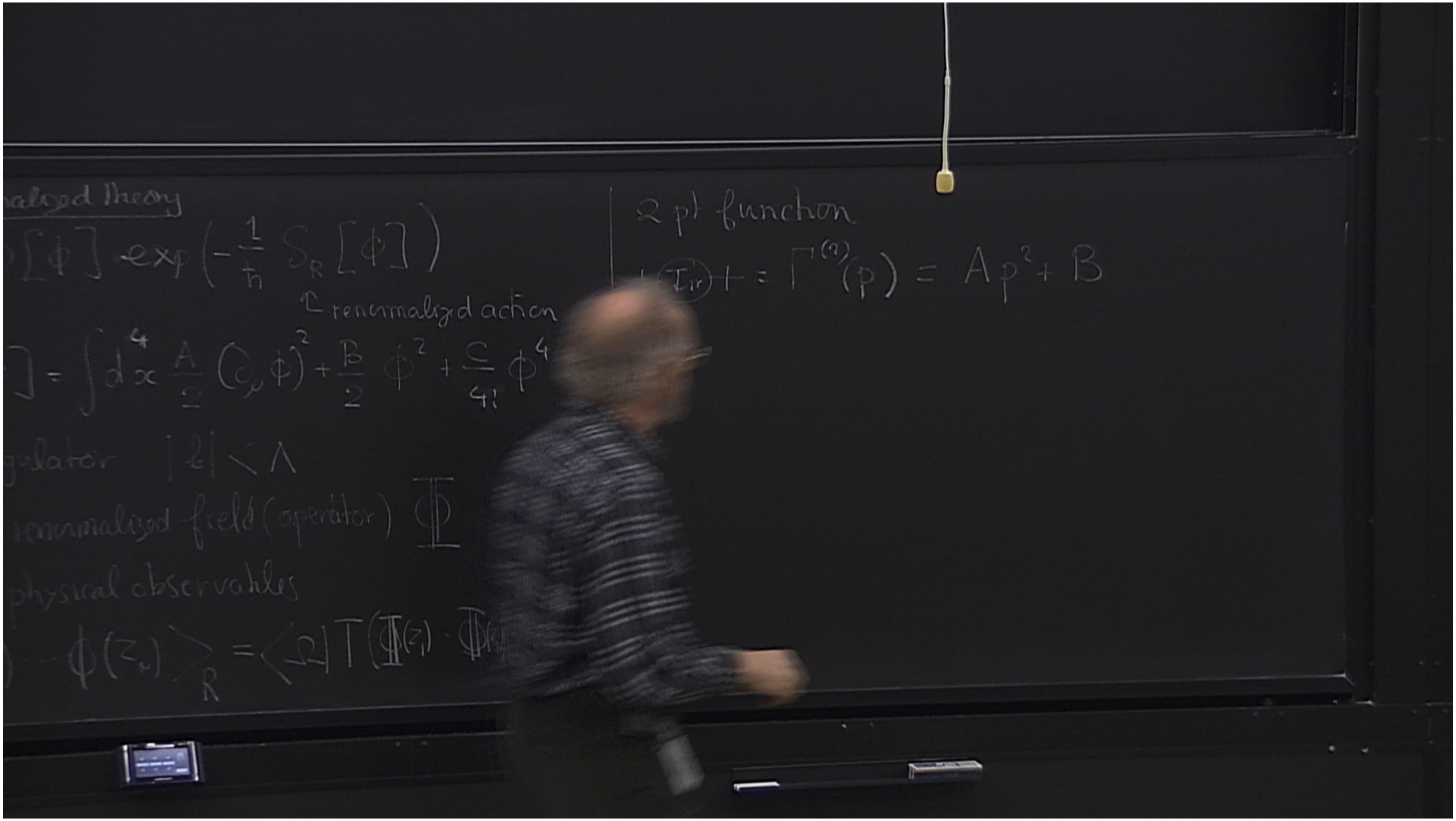
renormalised field (operator)
ical observables
R
= ⟨Ω| T(Φ(x₁) ⋯ Φ(x_n)) |Ω⟩


2 pt function
renormalizd action
gulator
renormalised field (operator)
physical observables

...nalized theory

$$\int \mathcal{D}[\phi] \exp\left(-\frac{1}{\hbar} S_R[\phi]\right)$$

renormalized action

$$S_R[\phi] = \int d^4x \, \frac{A}{2}(\partial_\mu \phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{4!}\phi^4$$

...gulator $|k| < \Lambda$

renormalized field (operator) Φ

physical observables

$$\langle \phi(z_1)\cdots\phi(z_n)\rangle_R = \langle \Omega | T(\Phi(z_1)\cdots\Phi(z_n)) | \Omega \rangle$$

2 pt function

$$-(\mathrm{1PI})- = \Gamma^{(2)}(p) = Ap^2 + B + \frac{1}{2}$$

$$— = \frac{1}{Ap^2 + B} \; ; \quad \times = C$$

...alized theory

$\ldots[\phi]\ \exp\left(-\frac{1}{\hbar} S_R[\phi]\right)$

renormalized action

$\ldots] = \int d^4x\ \frac{A}{2}(\partial_\mu\phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{4!}\phi^4$

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2 pt function

$\Gamma_{ir} = \Gamma^{(2)}(p) = Ap^2 + B + \frac{1}{2}\ \text{(loop)}$

$\text{—} = \frac{1}{Ap^2+B}\ ;\ \times = C$

$\text{(loop)} = T(m,\Lambda) = \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+m^2} = \frac{1}{(4\pi)^2}\Lambda^2 - \frac{1}{(4\pi)^2}\log\left(\frac{\Lambda}{m}\right)$

$\langle \phi(z_1) \cdots$

Massless. classical $\int \left[\frac{1}{2} (\partial \phi)^2 + \frac{g}{4!} \phi^4 \right] d^4x$

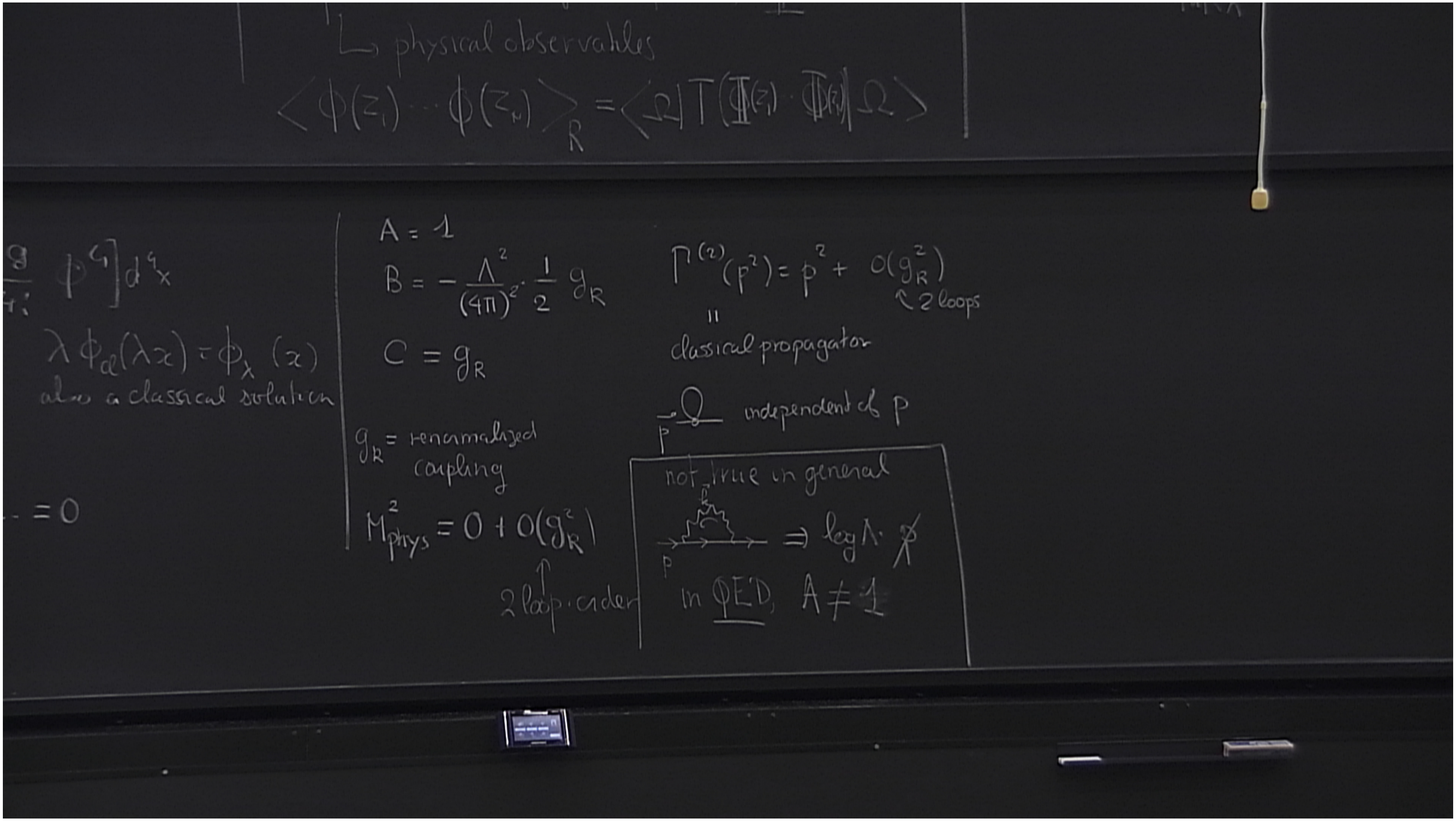
Scale invariant $\phi_{cl}(x)$ $\quad \lambda \phi_{cl}(\lambda x) = \phi_\lambda (x)$

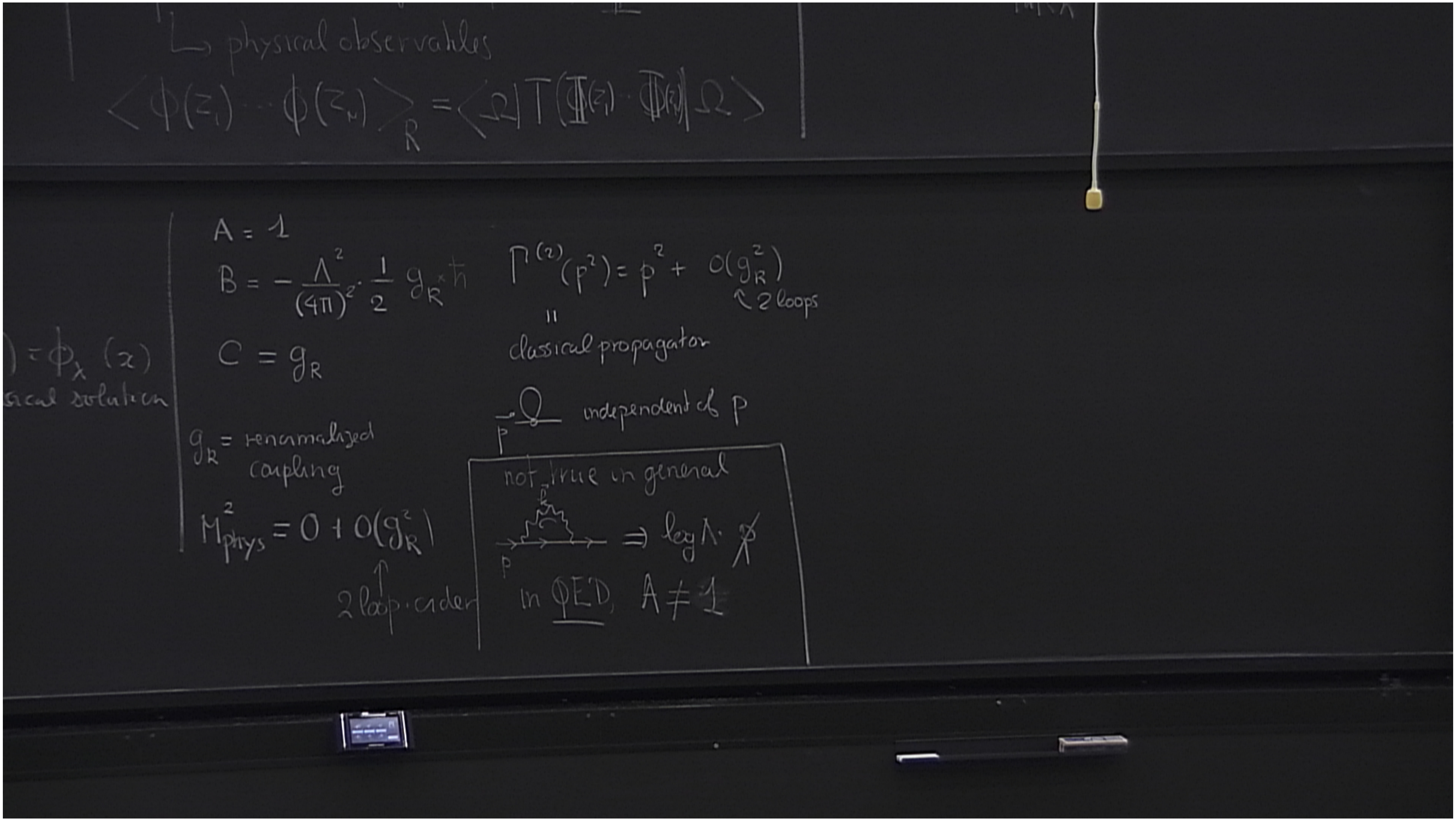
also a classical solution

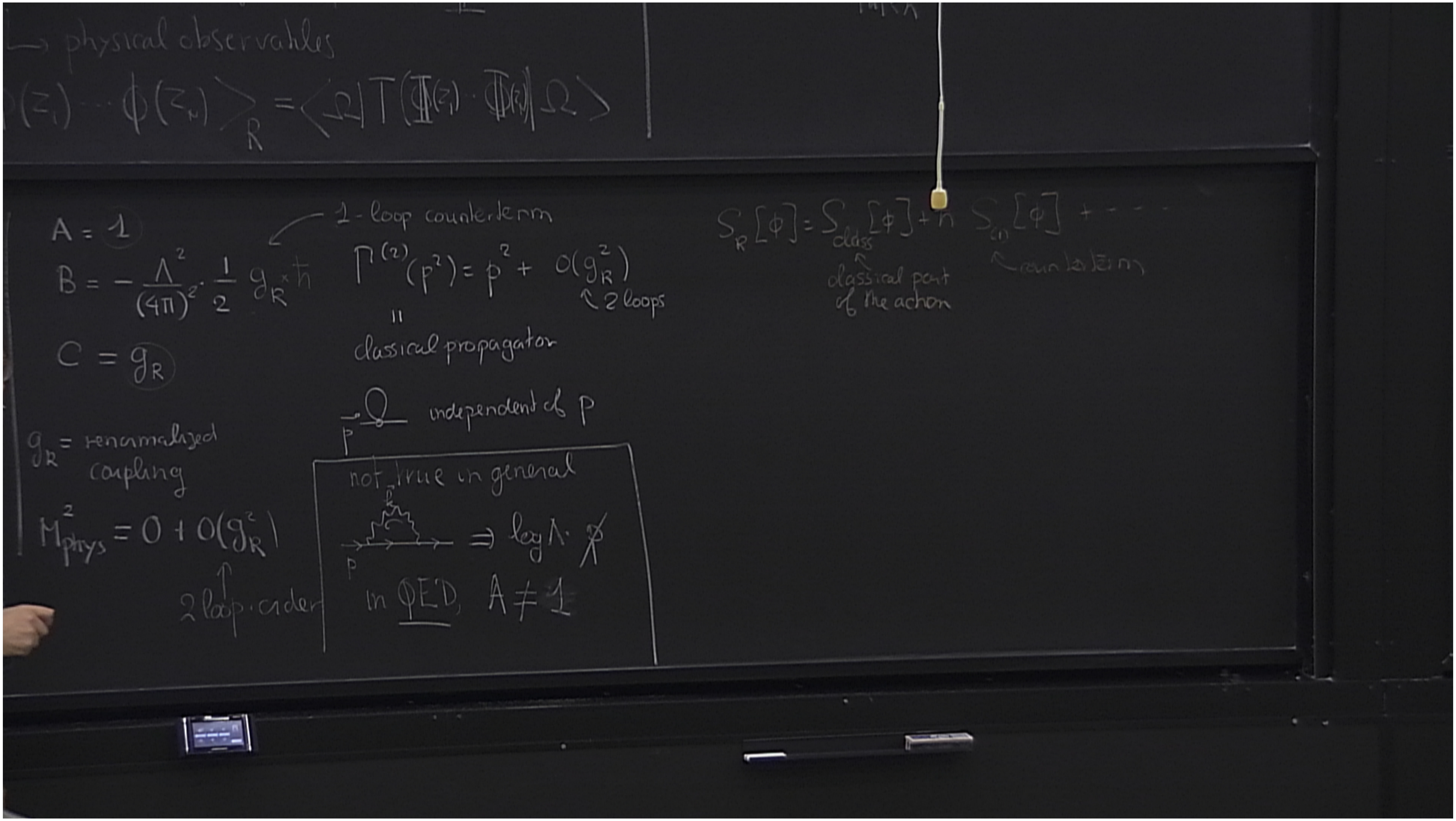
quantum. $M_{phys} = 0$

$\Gamma^{(2)}(p) = 0$ at $p^2 = 0$ $\quad B + \frac{1}{2} \quad + \dots = 0$

$B + \frac{1}{2} \frac{C}{A} T($


physical observables
A = 1
C = g_R
g_R = renormalized coupling
classical propagator
2 loops
independent of p
not true in general
in QED, A ≠ 1
2 loop order
also a classical solution


↳ physical observables
$\langle \phi(z_1) \cdots \phi(z_N) \rangle_R = \langle \Omega | T(\Phi(z_1) \cdots \Phi(z_N)) | \Omega \rangle$
$) = \phi_\chi(x)$
sical solution
$A = 1$
$B = -\frac{\Lambda^2}{(4\pi)^2} \cdot \frac{1}{2} g_R \times \hbar$
$C = g_R$
g_R = renormalized coupling
$M^2_{phys} = 0 + O(g_R^2)$
2 loop order
$\Gamma^{(2)}(p^2) = p^2 + O(g_R^2)$
2 loops
classical propagator
independent of p
not true in general
$\Rightarrow \log \Lambda$
in ϕED, $A \neq 1$



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classical propagator
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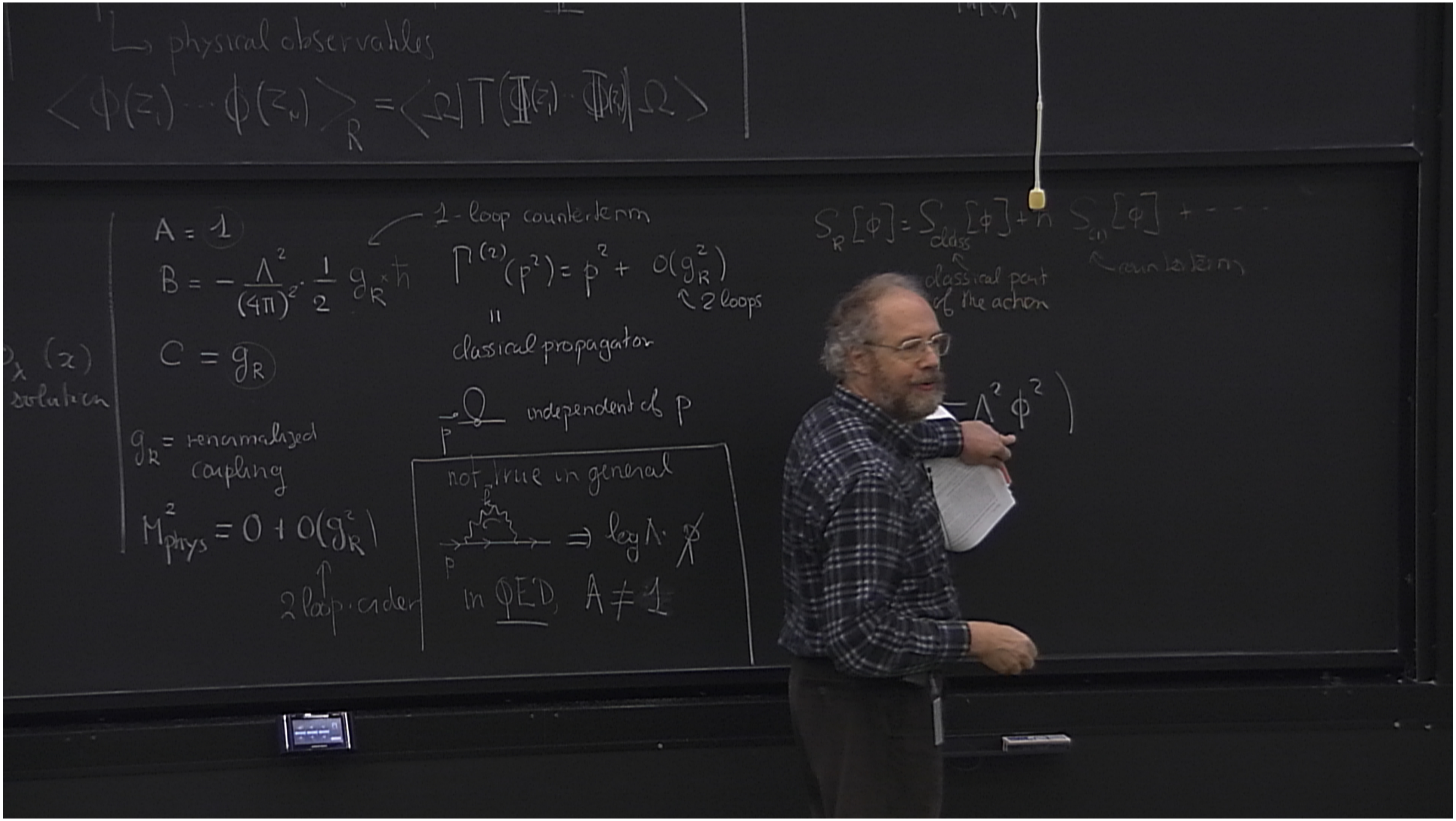
in QED, $A \neq 1$
$S_R[\phi] = S_{class}[\phi] + \hbar\, S_{(1)}[\phi] + \cdots$
classical part of the action
counterterm



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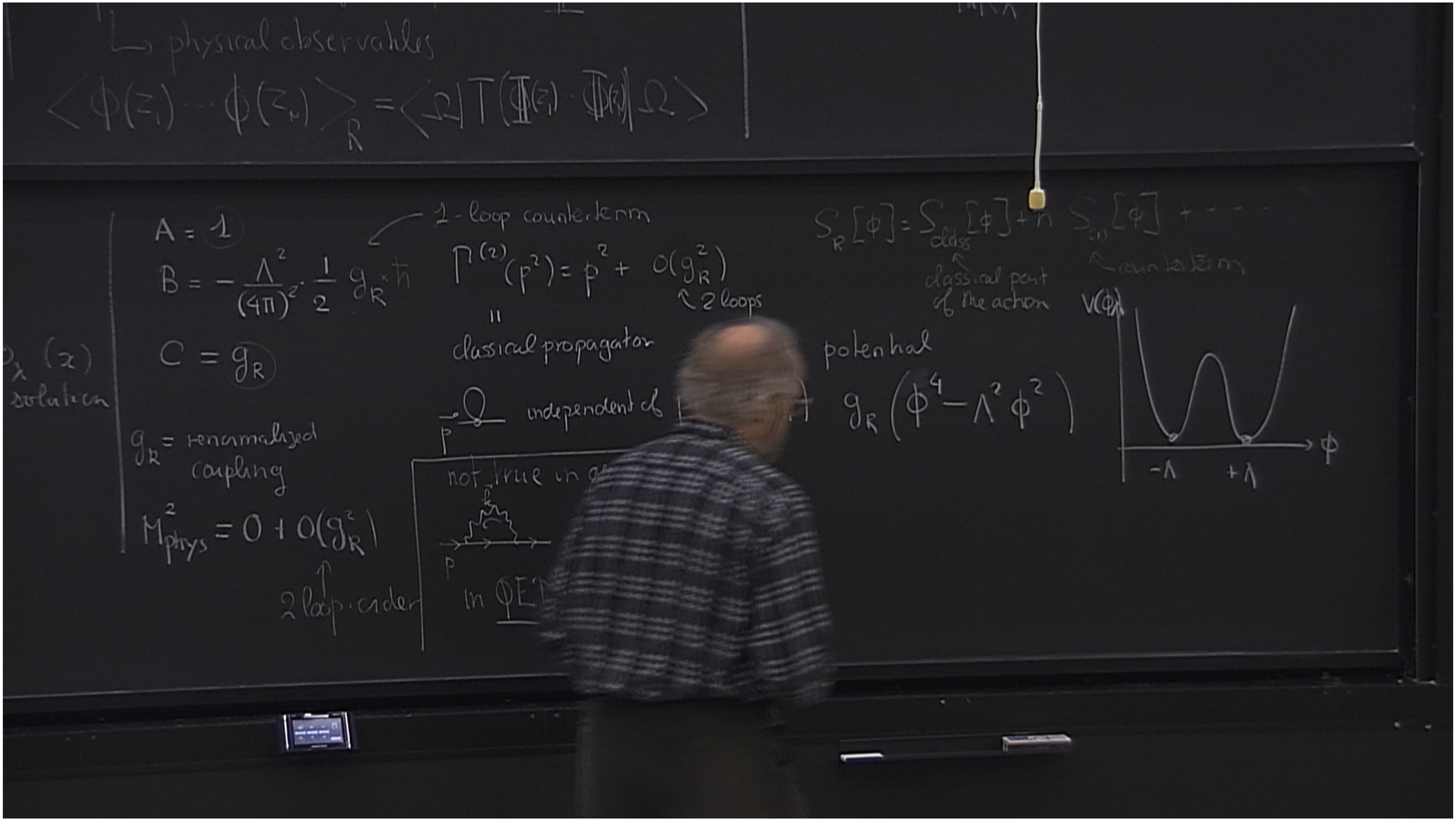

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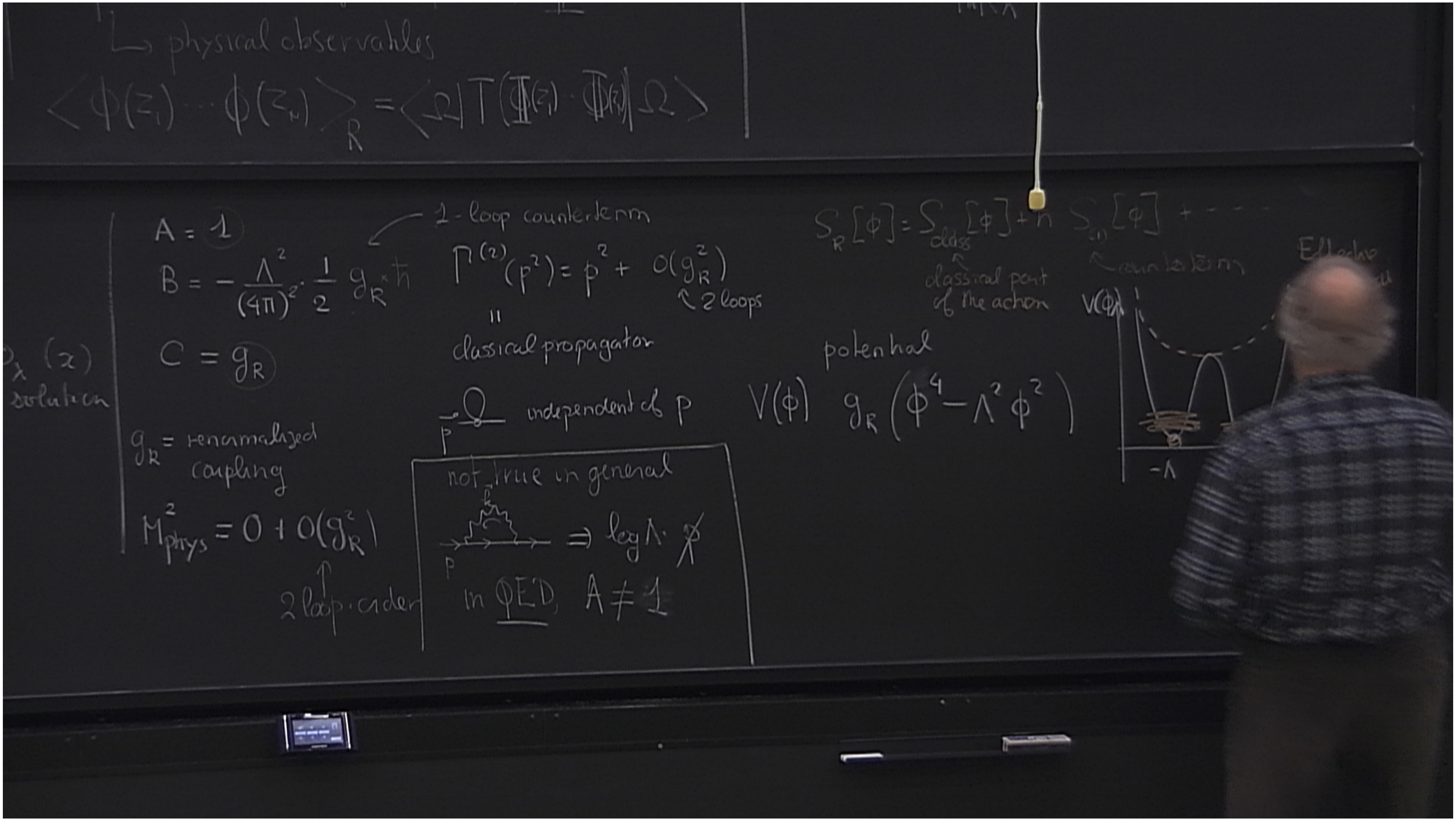
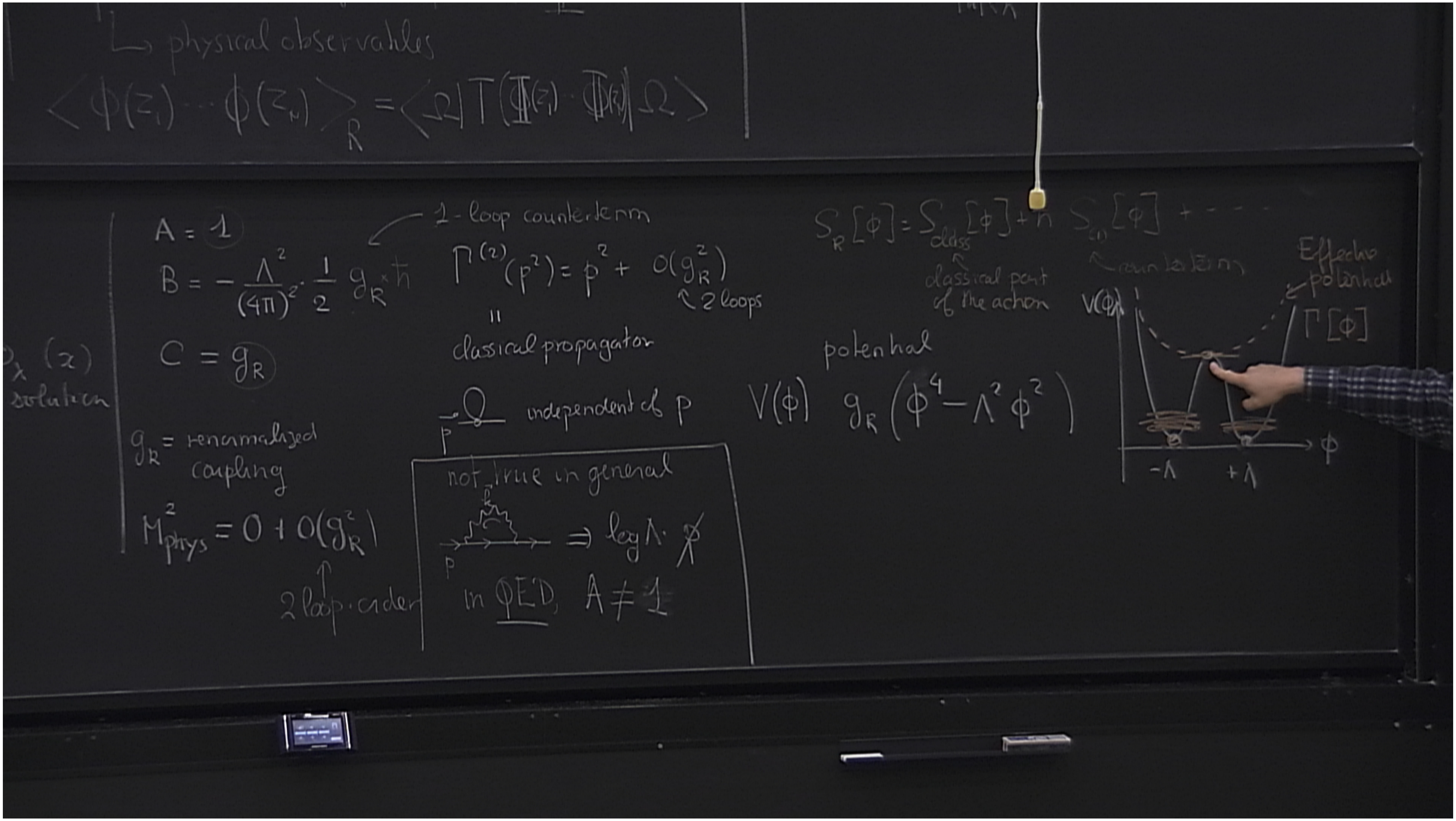
$S_R[\phi] = S_{class}[\phi] + \hbar S_{(1)}[\phi] + \cdots$
classical part of the action
counterterm
potential
$g_R(\phi^4 - \Lambda^2 \phi^2)$
$V(\phi)$
ϕ
$-\Lambda$
$+\Lambda$


physical observables
A = 1
B = − Λ² / (4π)² · 1/2 g_R ħ
C = g_R
g_R = renormalized coupling
M²_phys = 0 + O(g_R²)
2 loop order
1-loop counterterm
Γ^(2)(p²) = p² + O(g_R²)
2 loops
classical propagator
independent of p
not true in general
⇒ log Λ
in φED, A ≠ 1
S_R[φ] = S_class[φ] + ħ S_(1)[φ] + ...
classical part of the action
counterterm
potential
V(φ) g_R(φ⁴ − Λ² φ²)
V(φ)
−Λ
solution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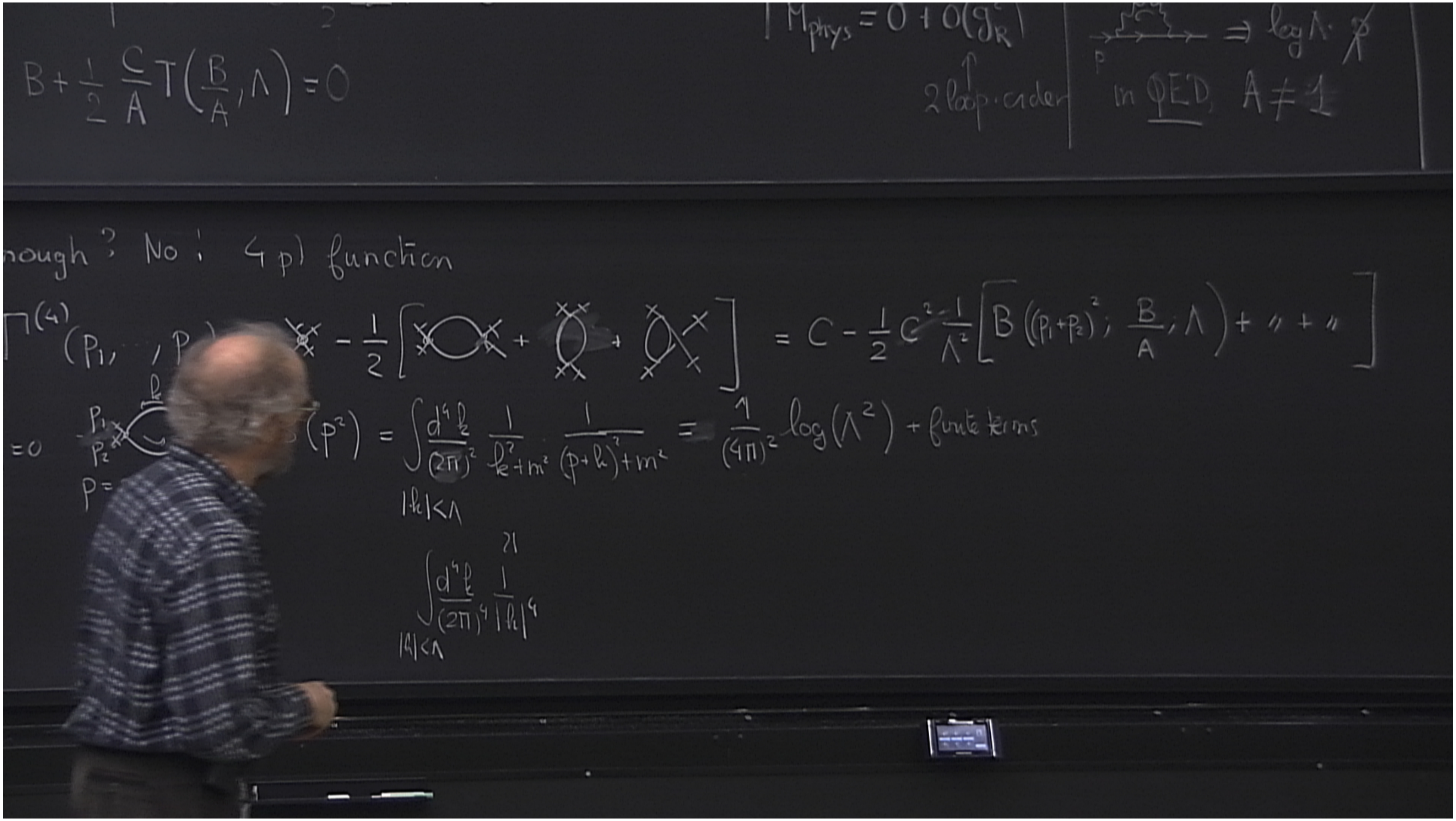
S_R[φ] = S_class[φ] + ħ S_(1)[φ] + ...
classical part of the action
counterterm
potential
V(φ) g_R (φ⁴ − Λ² φ²)
Effective potential
Γ[φ]
V(φ)
φ
−Λ
+Λ



B + 1/2 C/A T(B/A, Λ) = 0
M_phys = 0 + O(g_R²)
2 loop order
⇒ log Λ
in QED, A ≠ 1
nough? No: 4 pt function
Γ⁽⁴⁾(p₁, …, p
= C − 1/2 C² 1/Λ² [B((p₁+p₂)²; B/A, Λ) + 〃 + 〃]
(p²) = ∫ d⁴k/(2π)⁴ 1/(k²+m²) 1/((p+k)²+m²) = 1/(4π)² log(Λ²) + finite terms
|k|<Λ
∫ d⁴k/(2π)⁴ 1/|k|⁴
|k|<Λ

$$B + \frac{1}{2}\frac{C}{A}T\left(\frac{B}{A}, \Lambda\right) = 0$$

$M_{phys} = 0 + O(g_R^2)$

2 loop order

$\Rightarrow \log\Lambda \cdot \cancel{p}$

in QED, $A \neq 1$

…nough? No: 4 pt function

$$\Gamma^{(4)}(p_1, \ldots, p_4) = \quad - \frac{1}{2}\left[\quad + \quad + \quad\right] = C - \frac{1}{2}C^2\frac{1}{\Lambda^2}\left[B\left((p_1+p_2)^2; \frac{B}{A}, \Lambda\right) + " + "\right]$$

$$= B(p^2) = \int_{|k|<\Lambda}\frac{d^4k}{(2\pi)^4}\frac{1}{k^2+m^2}\,\frac{1}{(p+k)^2+m^2} = \frac{1}{(4\pi)^2}\log(\Lambda^2) + \text{finite terms}$$

$p = p_1 + p_2$

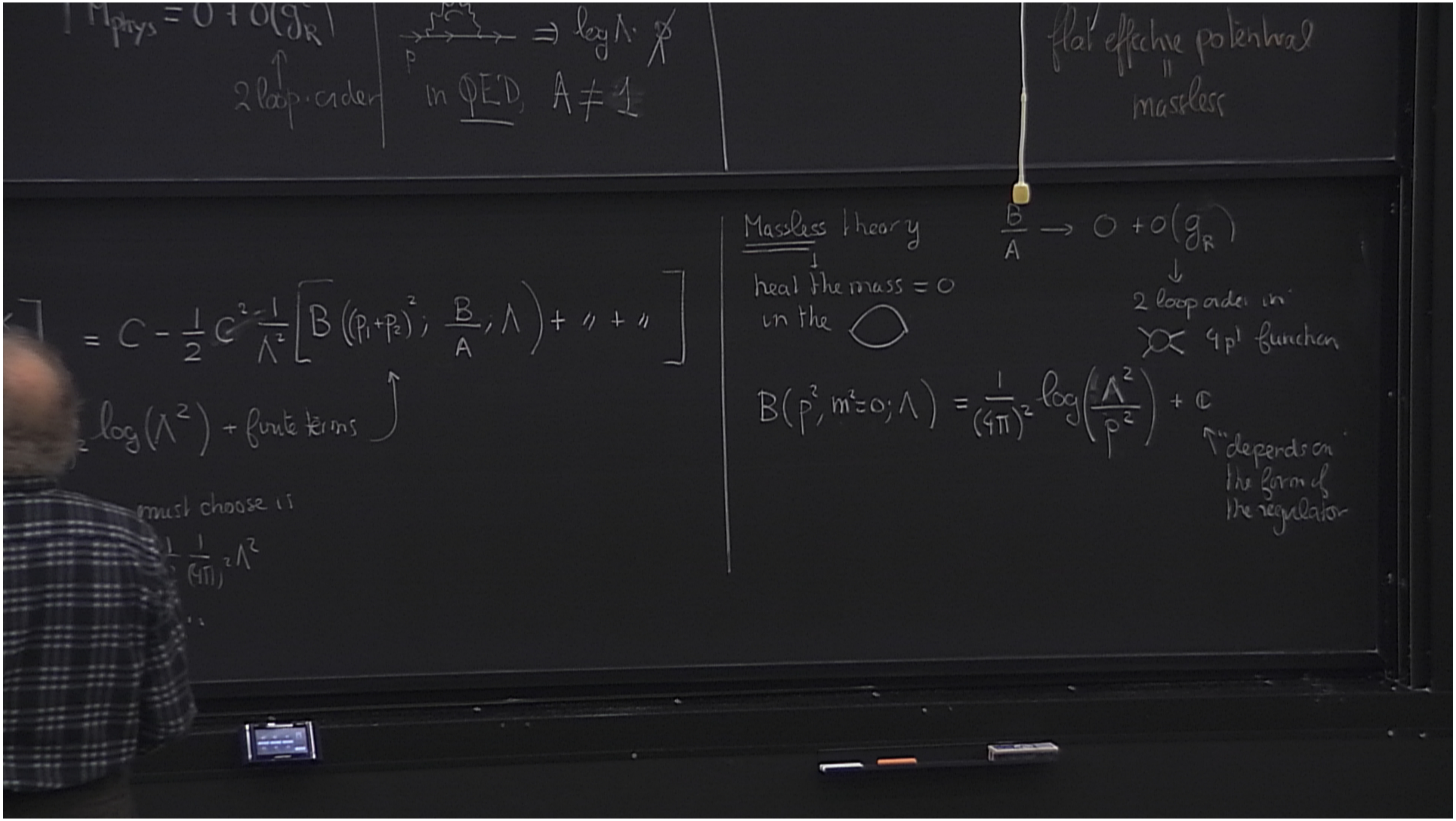
?!

$$\int_{|k|<\Lambda}\frac{d^4k}{(2\pi)^4}\frac{1}{|k|^4}$$

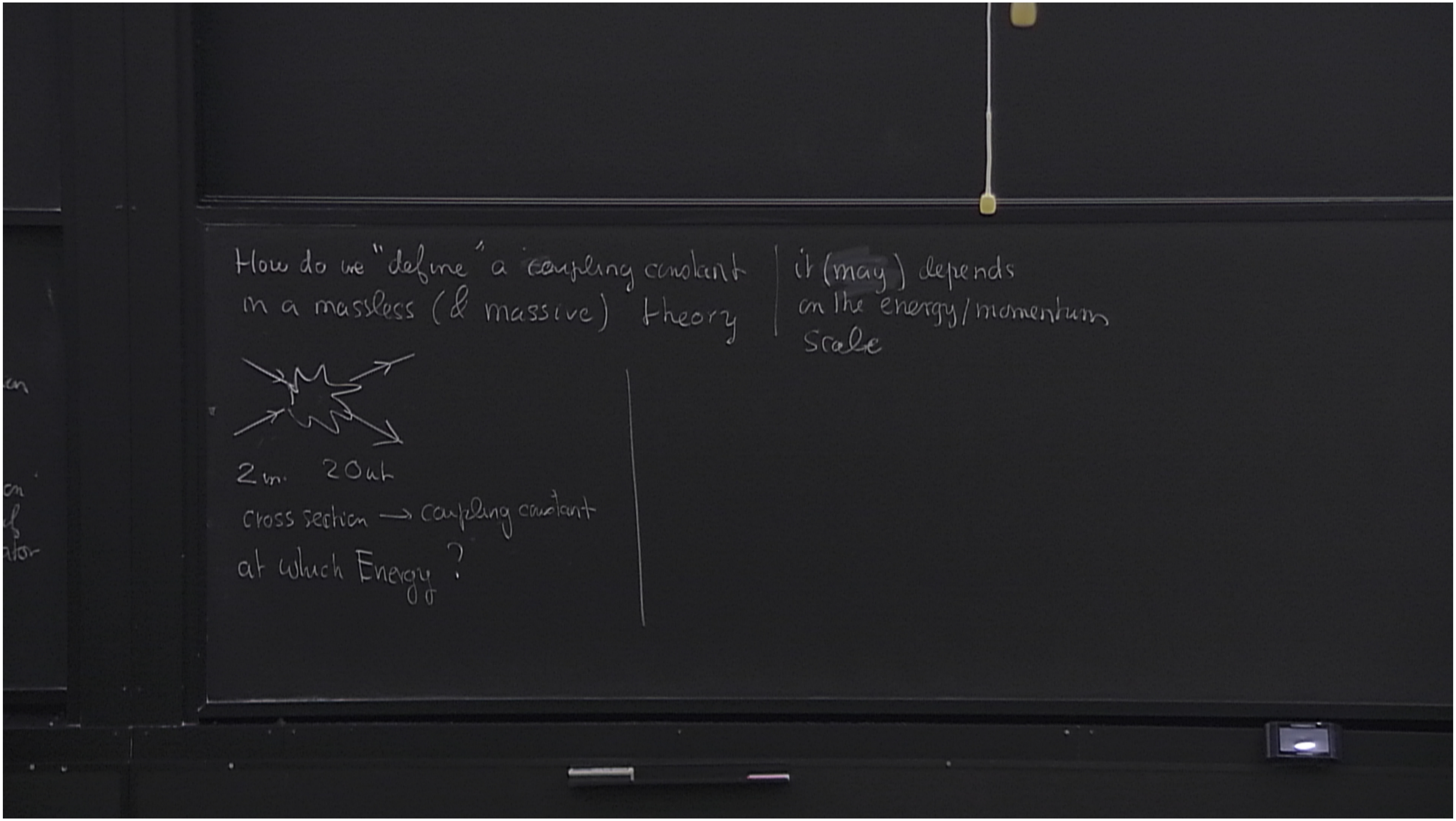
in fact we must choose is

$A = 1$, $B = -\frac{1}{2}\frac{1}{(4\pi)^2}\Lambda^2$

$C = g_R + \cdots$

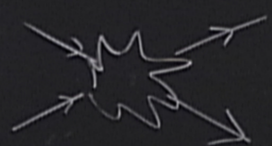


$M_{phys} = 0 + O(g_R)$
2 loop order
$\Rightarrow \log\Lambda$
in QED, $A \neq 1$
flat effective potential
massless
Massless theory
heal the mass = 0
in the
$\frac{B}{A} \longrightarrow 0 + O(g_R)$
2 loop order in
4 pt function
$= C - \frac{1}{2} C^2 \frac{1}{\Lambda^2} \left[B\left((p_1+p_2)^2; \frac{B}{A}; \Lambda\right) + \text{"} + \text{"} \right]$
$\log(\Lambda^2)$ + finite terms
$B(p^2, m^2=0; \Lambda) = \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{p^2}\right) + C$
"depends on the form of the regulator
must choose is
$\frac{1}{(4\pi)^2}\Lambda^2$


How do we "define" a coupling constant
in a massless (& massive) theory
it (may) depends
on the energy/momentum
scale
2 in 2 Out
cross section → coupling constant
at which Energy ?

How do we "define" a coupling constant in a massless (& massive) theory

it (may) depends on the energy/momentum scale



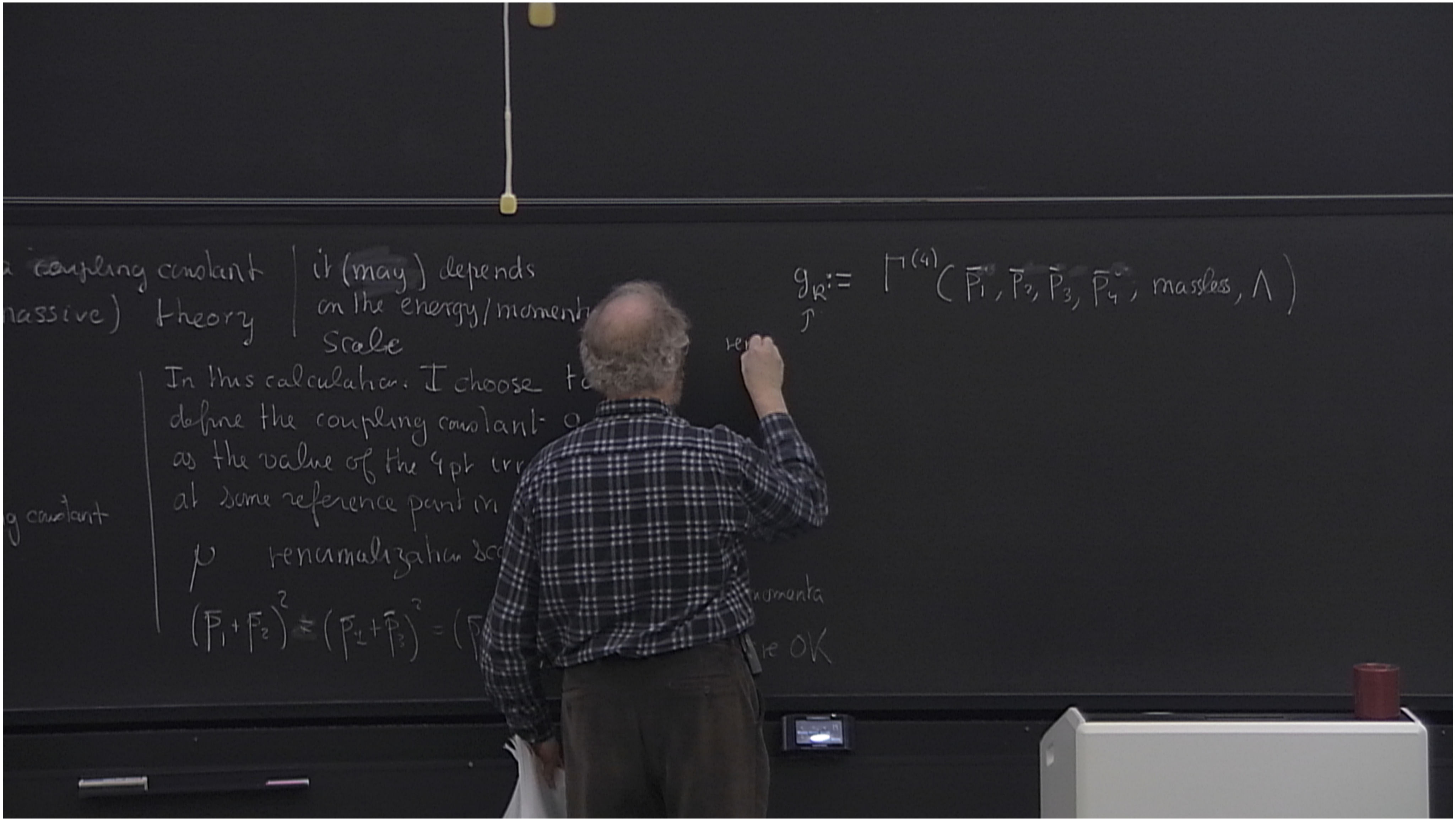
2 in 2 out

cross section $\rightarrow$ coupling constant

at which Energy?

In this calculation. I choose to define the coupling constant g_R as the value of the 4pt irr. fuction at some reference point in momentum space

μ renormalization scale


coupling constant
(massive) theory
it (may) depends
on the energy/momentum
Scale
In this calculation. I choose to
define the coupling constant
as the value of the 4pt
at some reference point in
constant
renormalization
momenta
OK
$g_R := \Gamma^{(4)}(\bar{p}_1, \bar{p}_2, \bar{p}_3, \bar{p}_4; \text{massless}, \Lambda)$
$(\bar{p}_1+\bar{p}_2)^2 = (\bar{p}_2+\bar{p}_3)^2 =$

a coupling constant | it (may) depends on the energy/momentum scale

massive) theory

$$g_R := \Gamma^{(4)}(\bar{p}_1, \bar{p}_2, \bar{p}_3, \bar{p}_4; \text{masses}, \Lambda)$$

renormalized coupling

In this calculation I choose to define the coupling constant g_R as the value of the 4pt irr. function at some reference point in momentum space

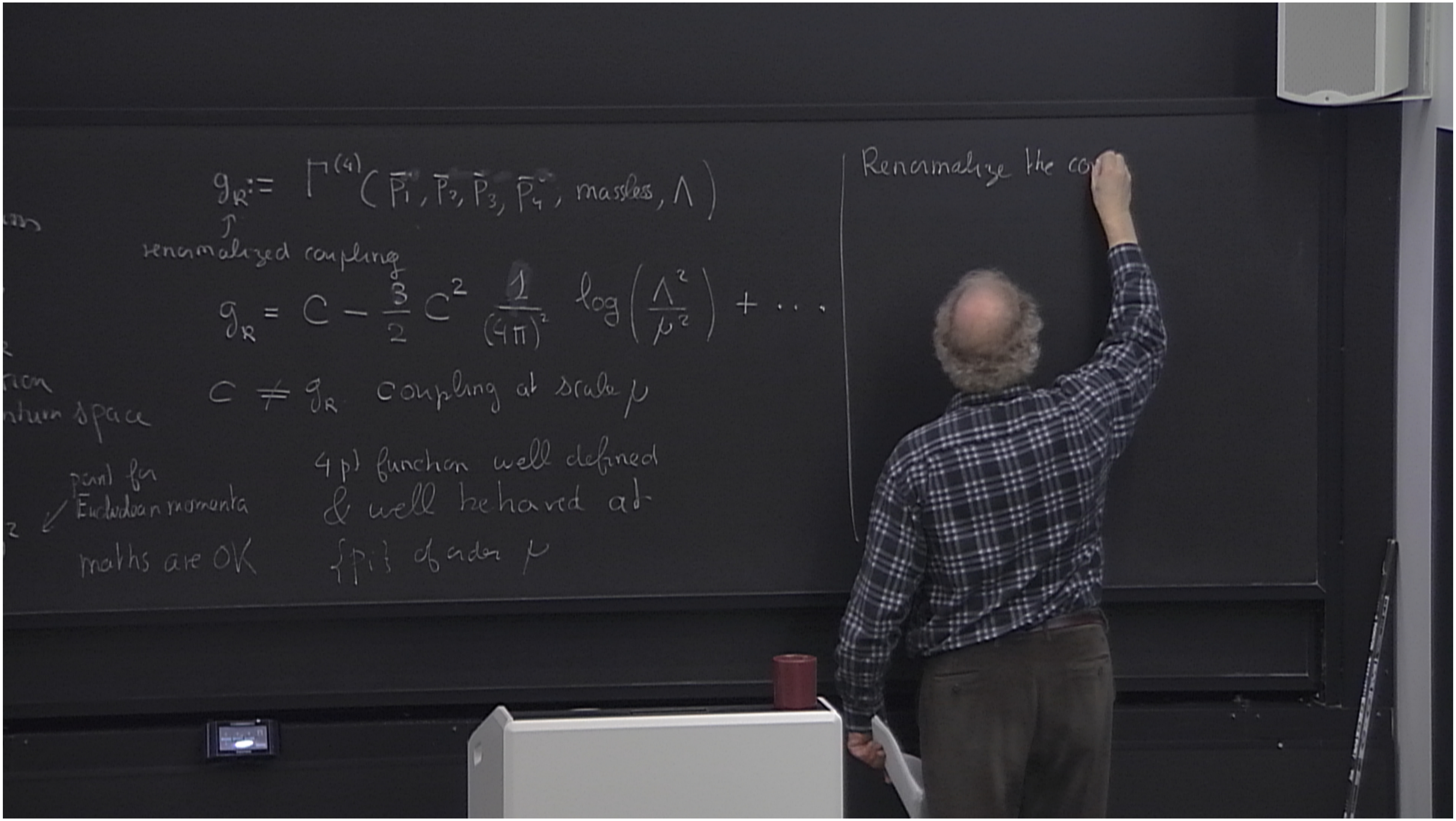
g constant

μ renormalization scale

$$(\bar{p}_1+\bar{p}_2)^2 = (\bar{p}_2+\bar{p}_3)^2 = (\bar{p}_1+\bar{p}_4)^2 = \mu^2$$

point for Euclidean momenta

maths are OK


$g_R := \Gamma^{(4)}(\bar{p}_1, \bar{p}_2, \bar{p}_3, \bar{p}_4; \text{massless}, \Lambda)$
renormalized coupling
$g_R = C - \frac{3}{2} C^2 \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \ldots$
$C \neq g_R$ coupling at scale μ
4 pt function well defined
& well behaved at
$\{p_i\}$ of order μ
Euclidean momenta
maths are OK
Renormalize the co

$$g_R := \Gamma^{(4)}(\bar{p}_1, \bar{p}_2, \bar{p}_3, \bar{p}_4;\ \text{massless}, \Lambda)$$

renormalized coupling

$$g_R = C - \frac{3}{2} C^2 \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \dots$$

$C \neq g_R$ coupling at scale μ

4 pt function well defined & well behaved at $\{p_i\}$ of order μ

space

point for Euclidean momenta

maths are OK

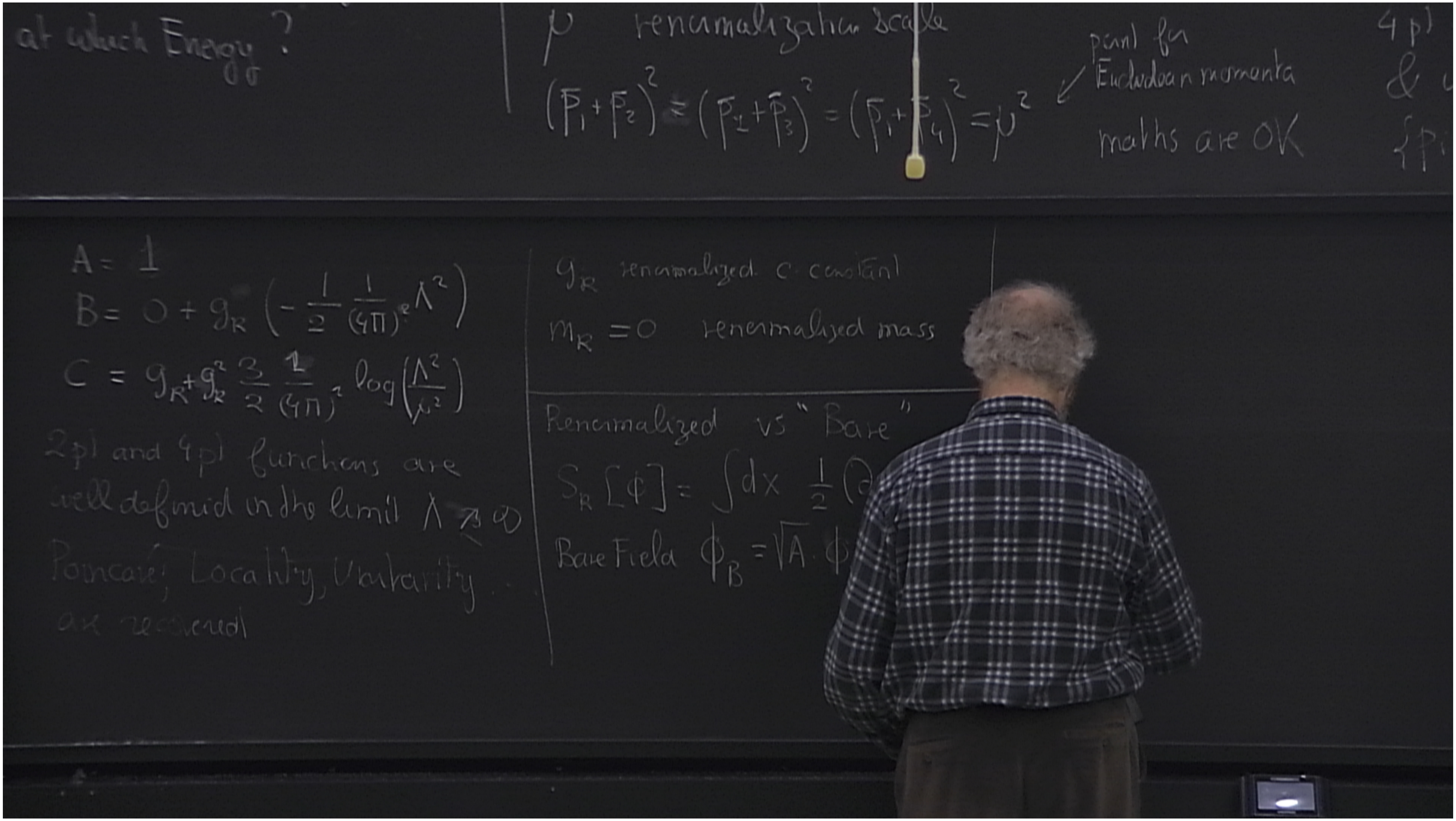
Renormalize the coupling constant

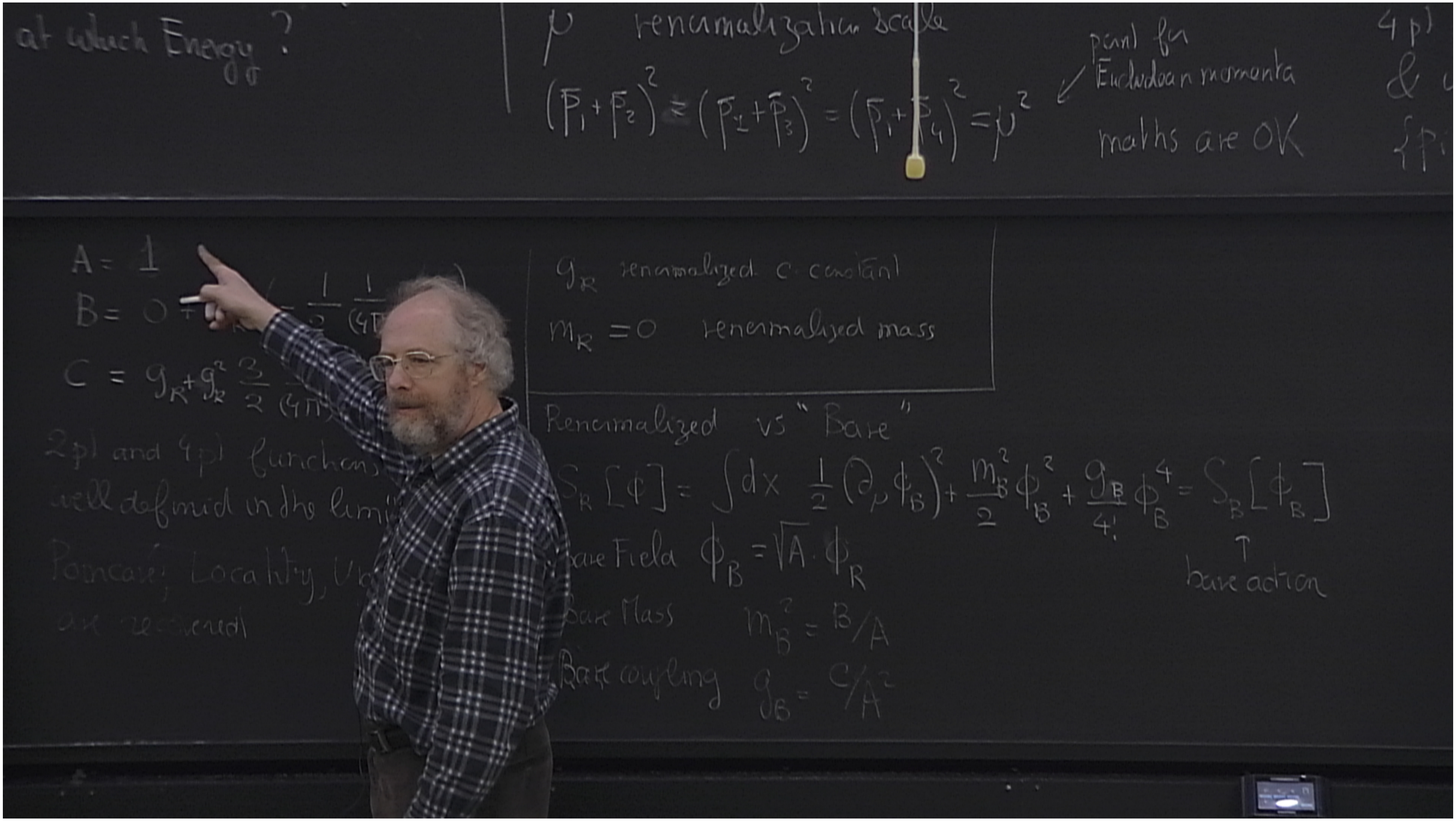
$$C = g_R + g_R^2 \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{\mu^2}\right)$$

4pt Irreducible Function

$$\Gamma^{(4)}(p_i) = g_R - \frac{3}{2} g_R^2 \frac{1}{(4\pi)^2} \times \left[\log\left(\frac{\mu^2}{(p_1+p_2)^2}\right) + \log\left(\frac{\mu^2}{(p_1+p_3)^2}\right) + \log\left(\frac{\mu^2}{(p_1+p_4)^2}\right)\right]$$

[+ higher order terms in g_R contain $\log\Lambda$ divergences]


at which Energy ?
μ renormalization scale
$(\bar{p}_1+\bar{p}_2)^2 = (\bar{p}_2+\bar{p}_3)^2 = (\bar{p}_1+\bar{p}_4)^2 = \mu^2$
p.s.l for Euclidean momenta
maths are OK
4 pt
&
$\{p$
$A = 1$
$B = 0 + g_R\left(-\frac{1}{2}\frac{1}{(4\pi)^2}\Lambda^2\right)$
$C = g_R + g_R^2\,\frac{3}{2}\,\frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{\mu^2}\right)$
2pt and 4pt functions are well defined in the limit $\Lambda \to \infty$
Poincaré, Locality, Unitarity ... are recovered
g_R renormalized c. constant
$m_R = 0$ renormalized mass
Renormalized vs "Bare"
$S_R[\phi] = \int dx\ \frac{1}{2}($
Bare Field $\phi_B = \sqrt{A}\cdot\phi$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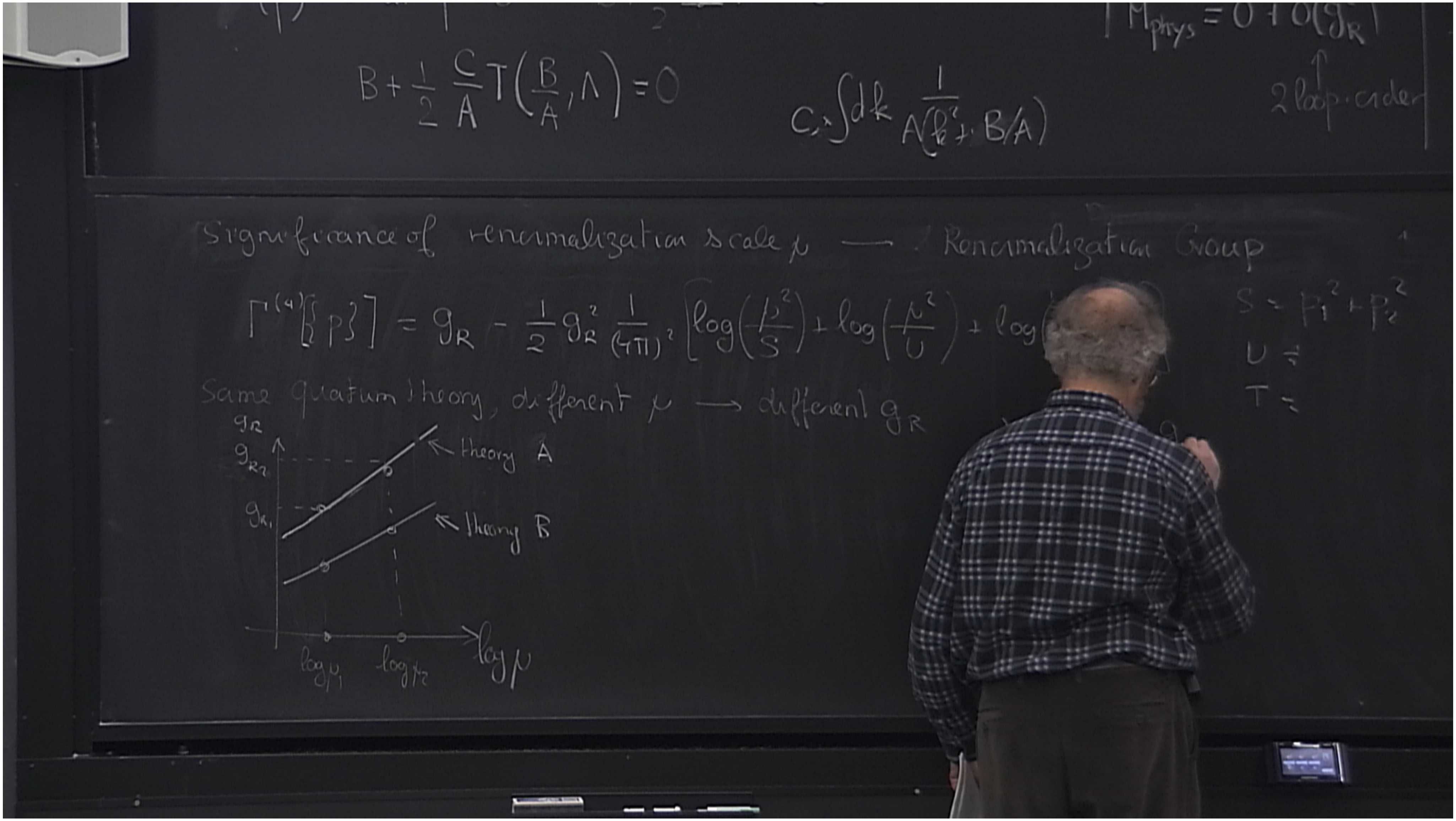
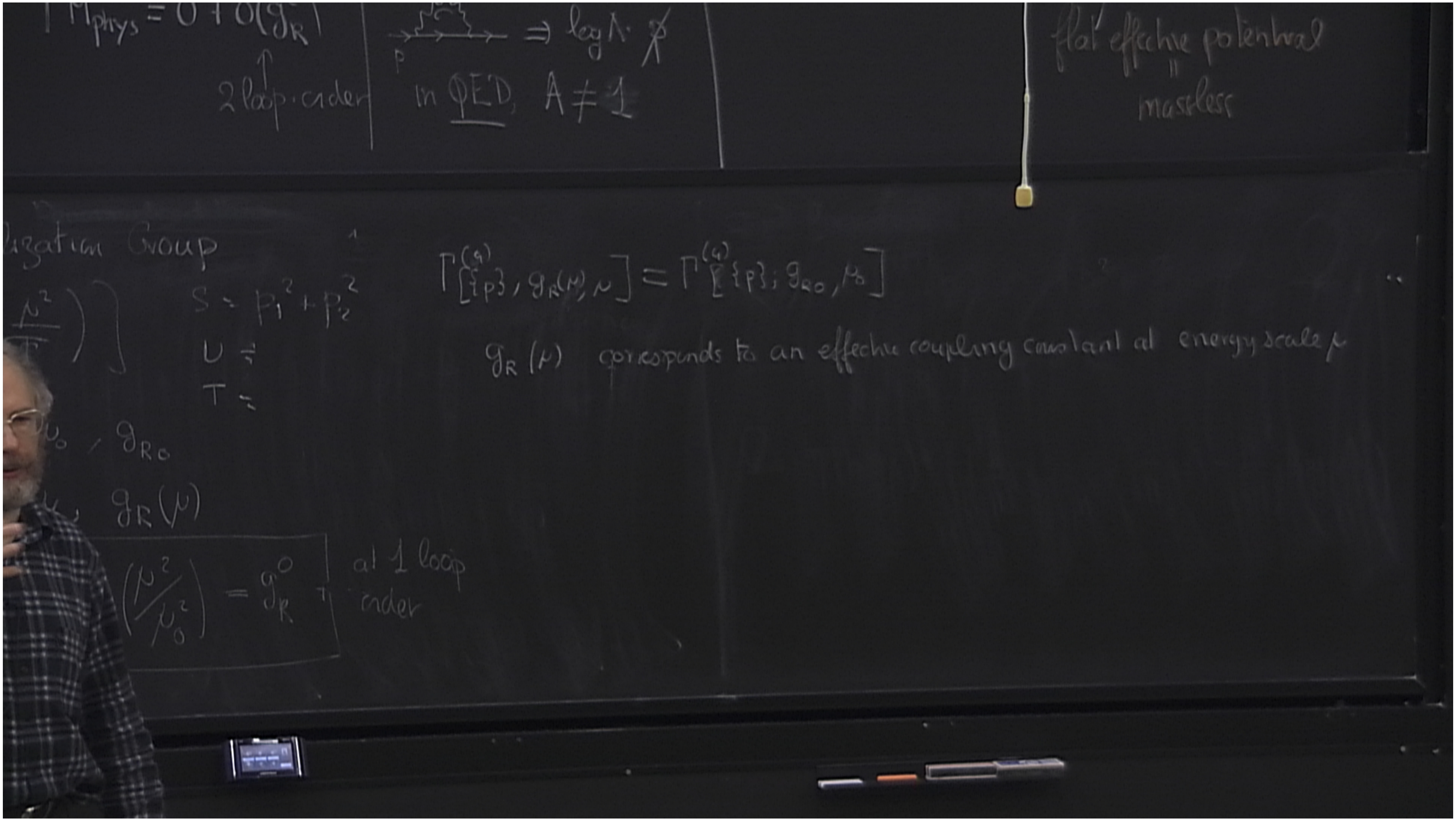
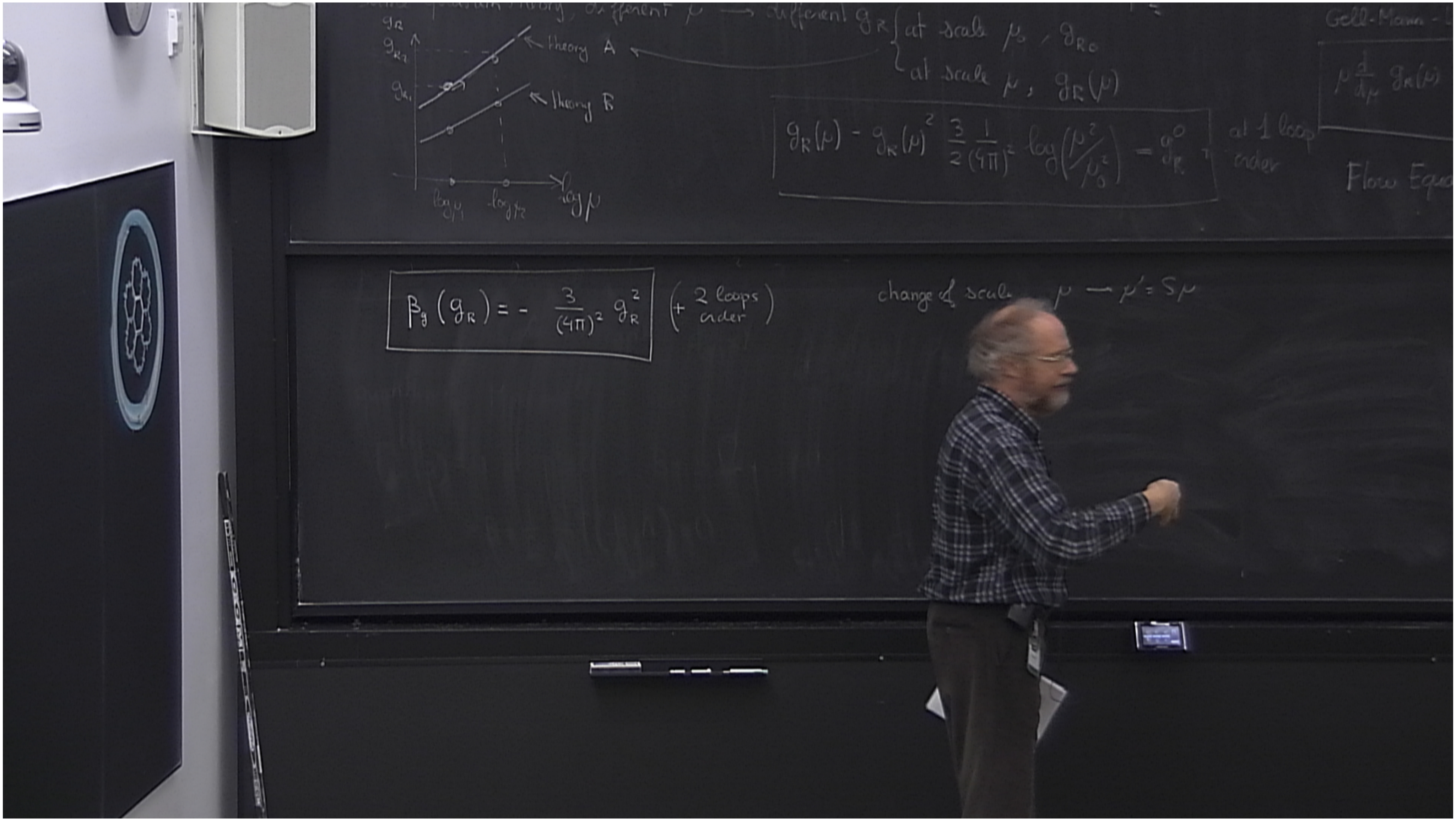
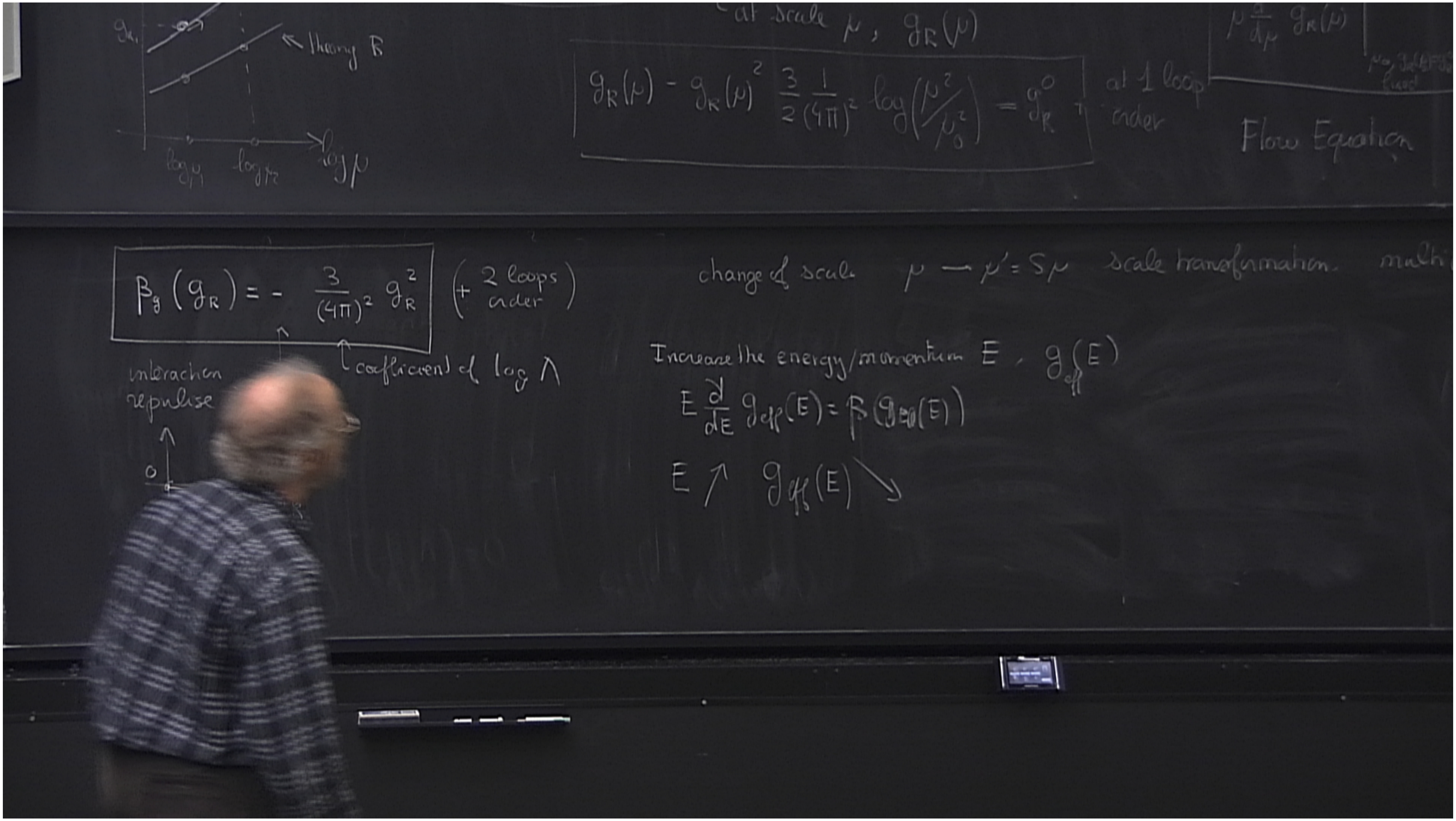
μ renormalization scale
$(\bar{p}_1+\bar{p}_2)^2 = (\bar{p}_2+\bar{p}_3)^2 = (\bar{p}_1+\bar{p}_4)^2 = \mu^2$
Euclidean momenta
maths are OK
A = 1
B = 0
$C = g_R + g_R^2 \frac{3}{2} \frac{1}{(4\pi)}$
2pt and 4pt functions
well defined in the limit
Poincaré, Locality, U
are recovered
g_R renormalized c. constant
$m_R = 0$ renormalized mass
Renormalized vs "Bare"
$S_R[\phi] = \int dx \; \frac{1}{2}(\partial_\mu \phi_B)^2 + \frac{m_B^2}{2}\phi_B^2 + \frac{g_B}{4!}\phi_B^4 = S_B[\phi_B]$
bare action
Bare Field $\phi_B = \sqrt{A}\cdot\phi_R$
Bare Mass $m_B^2 = B/A$
Bare coupling $g_B = C/A^2$



B + 1/2 C/A T(B/A, Λ) = 0
C, ∫dk 1/A(k² + B/A)
M_phys = 0 + O(g_R)
2 loop order
Significance of renormalization scale μ — Renormalization Group
Γ^(4)[{p}] = g_R − 1/2 g_R² 1/(4π)² [log(μ²/S) + log(μ²/U) + log(
S = p₁² + p₂²
U =
T =
Same quantum theory, different μ → different g_R
g_R
g_R2
g_R1
theory A
theory B
log μ1
log μ2
log μ


2 loop order
in QED, A ≠ 1
flat effective potential
massless
S = p1² + p2²
Γ(4)[{p}, gR(μ), μ] = Γ(4)[{p}; gR0, μ0]
gR(μ) corresponds to an effective coupling constant at energy scale μ
at 1 loop order


different $\mu \to$ different g_R
at scale μ_0, g_{R0}
at scale μ, $g_R(\mu)$
theory A
theory B
$\log \mu$
$g_R(\mu) - g_R(\mu)^2 \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\mu^2/\mu_0^2\right) = g_R^0$
at 1 loop order
Gell-Mann -
Flow Equ
$\beta_g(g_R) = - \frac{3}{(4\pi)^2} g_R^2$
(+ 2 loops order)
change of scale $\mu \to \mu' = s\mu$


at scale μ, g_R(μ)
theory B
log μ₁ log μ₂ log μ
g_R(μ) - g_R(μ)² 3/2 1/(4π)² log(μ²/μ₀²) = g⁰_R
at 1 loop order
Flow Equation
β_g(g_R) = - 3/(4π)² g_R²
(+ 2 loops order)
coefficient of log Λ
interaction repulsive
change of scale μ → μ' = sμ scale transformation
Increase the energy/momentum E, g_eff(E)
E d/dE g_eff(E) = β(g_eff(E))
E ↗ g_eff(E) ↘