Title: PSI 2015/2016 Quantum Field Theory II - Francois David - Lecture 6

Date: Nov 16, 2015 09:00 AM

URL: http://pirsa.org/15110032

Abstract:

$\boxed{1}$ Effective Action : $\longleftrightarrow$ 1 particle irreducible - vertex diagrams

$\tilde{\Phi} \cdot S''[$

$\boxed{2}$ Renormalization

$j \cdot \phi = \int d^d x \, j(x) \, \phi(x)$

$$Z[j] = \int D[\phi] \exp\left(-\frac{1}{\hbar}\left[S[\phi] - j \cdot \phi\right]\right)$$

$$W[j] = \frac{1}{\hbar} \log Z[j] \quad \text{connected}$$

$$\Gamma[\varphi] = j \cdot \varphi - W[j] \qquad \varphi = \langle \phi \rangle_j = \frac{\delta W[j]}{\delta j} \quad \text{Background Field}$$

1 - loop calculation : 1^{st} order in $\hbar$

$\hbar \to 0$, dominates saddle point $S[\phi]$ is minimal $\quad \phi_c : \dfrac{\delta S[\phi]}{\delta \phi} - j = 0$

ϕ_c is a "functional" of j

$$\phi = \phi_c + \sqrt{\hbar}\, \tilde{\phi}$$

expand $S[\phi] = S[\phi_c] + \emptyset + \frac{\hbar}{2} \tilde{\phi} \cdot S''[\phi_c] \cdot \tilde{\phi} + O(\hbar^{3/2})$

...ertex diagrams

$$\tilde{\phi}\cdot S''[\phi]\cdot\tilde{\phi} = \int d^d x_1 \int d^d x_2\, \tilde{\phi}(x_1)\,\tilde{\phi}(x_2)\, \underbrace{\frac{\delta^2 S[\phi]}{\delta\phi(x_1)\,\delta\phi(x_2)}}_{\text{Hessian of } S[\phi]}$$

1-loop calculation: 1^{st} order in $\hbar$

$\hbar \to 0$, dominates saddle point, $S[\phi]$ is minimal $\quad \phi_c : \dfrac{\delta S[\phi_c]}{\delta \phi} - j = 0$

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$$\phi = \phi_c + \sqrt{\hbar}\,\tilde{\phi}$$

...nd

expand $S[\phi] = S[\phi_c] + \emptyset + \dfrac{\hbar}{2}\,\tilde{\phi}\cdot S''[\phi_c]\cdot\tilde{\phi} + O\!\left(\hbar^{3/2}\right)$

...cable - vertex diagrams

$\tilde{\phi}\cdot S''[\phi]\cdot\tilde{\phi} = \int d^dx_1 \int d^dx_2$ ...

$\phi(x)$

1 - loop calculation: 1st order in $\hbar$

$\hbar \to 0$, dominates ϕ_c: $\frac{\delta S[\phi_c]}{\delta \phi} - j = 0$

saddle point

$S[\phi] - j\cdot\phi$ is minimal

ϕ_c is a "functional" of j

$\phi = \phi_c + \sqrt{\hbar}\,\tilde{\phi}$

Background Field

$$\text{expand}\left(\begin{array}{l} S[\phi] \\ -j\cdot\phi \end{array} \right) = \left\{ \begin{array}{l} S[\phi_c] + \quad + \frac{\hbar}{2}\tilde{\phi}\cdot S''[\phi_c]\cdot\tilde{\phi} + O(\hbar^{3/2}) \\ -j\cdot\phi_c \end{array} \right\}$$

...agrams

...p calculation: 1^{st} order in $\hbar$

..., dominates ϕ_c: $\frac{\delta S[\phi_c]}{\delta \phi} - j = 0$
saddle point

$S[\phi] - j\cdot\phi$ is minimal

...s a "functional" of j

$= \phi_c + \sqrt{\hbar}\,\tilde{\phi}$

$$\begin{pmatrix} S[\phi] \\ -j\cdot\phi \end{pmatrix} = \left\{ \begin{matrix} S[\phi_c] + \quad + \frac{\hbar}{2}\tilde{\phi}\cdot S''[\phi_c]\cdot\tilde{\phi} + o(\hbar^{3/2}) \\ -j\cdot\phi_c \end{matrix} \right\}$$

$$\tilde{\phi}\cdot S''[\phi]\cdot\tilde{\phi} = \int d^d x_1 \int d^d x_2\, \tilde{\phi}(x_1)\,\tilde{\phi}(x_2)\, \underbrace{\frac{\delta^2 S[\phi]}{\delta\phi(x_1)\,\delta\phi(x_2)}}_{\text{Hessian of } S[\phi]}$$

quadratic Form

$$Z[j] = \exp\left(-\frac{1}{\hbar}S[\phi_c]\right)\cdot\int D[\tilde{\phi}]\exp\left(-\frac{1}{2}\tilde{\phi}\cdot S''[\phi_c]\cdot\tilde{\phi}\right)\left(1 + o(\hbar^{1/2})\right)$$

$$W[j] = \begin{pmatrix} -S[\phi_c] \\ +j\,\phi_c \end{pmatrix} - \frac{1}{2}\hbar\, \mathrm{Tr}\left(\mathrm{Log}\left(S''[\phi_c]\right)\right)$$

"Log Det"

classical

quantum

Field

$$\varphi = \frac{\delta W[j]}{\delta j} = \phi_c + \frac{\delta \phi_c}{\delta j}\Big[\underbrace{\Big(-\frac{\delta S[\phi]}{\delta \phi} + j\Big)}_{=0 \text{ definition of } \phi_c} + O(\hbar)\Big]_{\phi=\phi_c}$$

ϕ_c depends on j

$$\varphi = \phi_c + \underline{O(\hbar)}$$

$$\Gamma[\varphi] = j \cdot \varphi - W[j] = \underbrace{j\,\varphi - j\,\phi_c + S[\phi_c]}\; + \frac{\hbar}{2} \mathrm{Tr}\big(\mathrm{Log}\,[S''[\phi_c]]\big) + \dots$$

$$\boxed{\Gamma[\varphi] = S[\varphi] + \hbar \frac{1}{2} \mathrm{Tr}\,\mathrm{Log}[S''[\varphi]] + O(\hbar^2)}$$

Valid for a QFT with scalar Fields (bosonic)
→ spin 1 & 2

$$\begin{vmatrix} S[\phi] \\ -j\cdot\phi \end{vmatrix} = \left\{ \begin{matrix} S[\phi_c] + \quad + \frac{\hbar}{2}\tilde{\phi}\cdot S''[\phi_c]\cdot\tilde{\phi} + O(\hbar^{3/2}) \\ -j\cdot\phi_c \end{matrix} \right\}$$

classical | quantum

ϕ^4 Theory $S[\phi] = \int d^d x \left(\frac{1}{2}\phi(-\Delta + m^2)\phi + \frac{g}{4!}\phi^4 \right)$

$$0 = \frac{\delta S}{\delta\phi(x)} - j(x) = (-\Delta_x + m^2)\phi_c(x) + \frac{g}{6}\phi_c^3(x) - j(x) = 0$$

Non-Lineal PDE

(time indept. N·L Schroedinger Equation)
with a source

$$\frac{\delta^2 S[\phi]}{\delta\phi(x_1)\delta\phi(x_2)} = S''[\phi]_{x_1,x_2} = \left(-\Delta + m^2 + \frac{g}{2}\phi^2\right)_{x_1,x_2}$$

$$\left(-\Delta + m^2 + \frac{g}{2}\phi^2\right) = S''[\phi]$$

integral Kernel of the operator

$S''[\phi]$ is a linear diff. operator

ψ on $\mathbb{R}^d$ $\quad \left(S''[\phi]\cdot\psi\right)(x) = \left(-\Delta_x + m^2 + \frac{g}{2}\phi^2(x)\right)\cdot\psi(x)$

linear on ψ $\leftarrow$ test function

depends N·L on ϕ

$$S''[\phi]_{x_1,x_2} = \left(-\Delta_{x_1} + m^2 + \frac{g}{2}\phi^2(x_1)\right)\delta(x_1 - x_2)$$

A distribution

$\rightarrow$ spin 1 & 2

in perturbation theory

$$\mathrm{Tr}\left[\mathrm{Log}\left(-\Delta + m^2 + \frac{g}{2}\phi^2\right)\right] = \mathrm{Tr}\left(\mathrm{Log}\left[(-\Delta + m^2)\times\left(\mathbb{1} + \frac{g}{2}(-\Delta+m^2)^{-1}\cdot\phi^2\right)\right]\right)$$

$$= \underbrace{\mathrm{Tr}\left[\mathrm{Log}\left(-\Delta + m^2\right)\right]}_{\text{Free Field}} + \boxed{\mathrm{Tr}\left(\mathrm{Log}\left[\mathbb{1} + \frac{g}{2}(-\Delta+m^2)^{-1}\phi^2\right]\right)} \qquad \square =$$

$$\square = \sum_{K=1}^{\infty}\frac{(-1)^{K-1}}{K}\frac{g^K}{2^K}\boxed{\mathrm{Tr}\left[\left[(-\Delta+m^2)^{-1}\circ\phi^2(x)\right]^K\right]}$$

$\rightarrow$ spin 1 & 2

in perturbation theory

$$\mathrm{Tr}\left[\mathrm{Log}\left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right)\right] = \mathrm{Tr}\left(\mathrm{Log}\left[(-\Delta + m^2)\times\left(\mathbb{1} + \frac{g}{2}(-\Delta + m^2)^{-1}\varphi^2\right)\right]\right)$$

$$= \mathrm{Tr}\left[\mathrm{Log}\left(-\Delta + m^2\right)\right] + \boxed{\mathrm{Tr}\left(\mathrm{Log}\left[\mathbb{1} + \frac{g}{2}(-\Delta + m^2)^{-1}\varphi^2\right]\right)} \qquad \Box =$$

Free Field

$$\Box = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} \frac{g^k}{2^k} \boxed{\mathrm{Tr}\left[\left[(-\Delta + m^2)^{-1} \cdot \varphi^2(x)\right]^k\right]}$$

$$\Gamma[\varphi] = S[\varphi] + \hbar\,\frac{1}{2}\,\mathrm{Tr}\,\mathrm{Log}\left[S''[\varphi]\right] + O(\hbar^2)$$

Valid for a QFT with scalar Fields (bosonic) → spin 1 & 2

linear on φ ← ... depends N.L. on ϕ

in perturbation theory, compute the $\hbar$ correction term as a functional of φ : bckgd :

$$(-\Delta+m^2)^{-1}_{x,y} =$$

$$(\varphi^2)_{xy} =$$

$$\mathrm{Tr}\left[\mathrm{Log}\left(-\Delta+m^2+\frac{g}{2}\varphi^2\right)\right] = \mathrm{Tr}\left(\mathrm{Log}\left[(-\Delta+m^2)\cdot\left(\mathbb{1}+\frac{g}{2}(-\Delta+m^2)^{-1}\varphi^2\right)\right]\right)$$

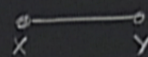
$$= \mathrm{Tr}\left[\mathrm{Log}\left(-\Delta+m^2\right)\right] + \boxed{\mathrm{Tr}\left(\mathrm{Log}\left[\mathbb{1}+\frac{g}{2}(-\Delta+m^2)^{-1}\varphi^2\right]\right)}$$

Free Field

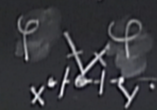
$$\square = \iint \cdot\, G_0(x_1,y_1)\,(\varphi^2)_{y_1x_2}\,G_0(x_2,y_2)\,(\varphi^2)_{y_2x_3}$$

$$\square = \sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\,\frac{g^k}{2^k}\,\boxed{\mathrm{Tr}\left[\left[(-\Delta+m^2)^{-1}\cdot\varphi^2(x)\right]^k\right]}$$

φ : bckgd.

$$(-\Delta + m^2)^{-1}_{x,y} = G_0(x,y) =$$  Propagator

$\varphi(x)$ = value of the background field at x

$$(\varphi^2)_{xy} = \varphi^2(x)\,\delta(x-y) =$$  vertex with 2 φ attached

$$\ldots)\,(\varphi^2)_{y_1 x_2}\, G_0(x_2,y_2)\,(\varphi^2)_{y_2 x_3} \cdots \quad G_0(x_k,y_k)\,(\varphi^2)_{y_k}$$

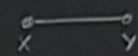
valid for a QFT
with scalar Fields (bosonic)
→ spin 1 & 2

linear on $\varphi \longleftarrow$ test function
depends N L on Φ

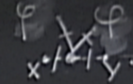
A distribution

...erm as a functional of φ : bckgd.

$$\mathbb{1} + \frac{g}{2}\left(-\Delta+m^2\right)^{-1}\varphi^2 \,)\,]\,)$$

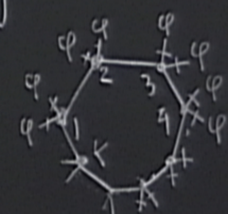
$$\left(-\Delta+m^2\right)^{-1}_{x,y} = G_0(x,y) =$$  Propagator

$\varphi(x)$ = value of the background field at x

$$(\varphi^2)_{xy} = \varphi^2(x)\,\delta(x-y) =$$  vertex with 2 φ attached

$$\square = \iiint \left[G_0(x_1,y_1)\,(\varphi^2)_{y_1x_2}\, G_0(x_2,y_2)\,(\varphi^2)_{y_2x_3} \cdots G_0(x_k,y_k)\,(\varphi^2)_{y_kx_1} \right] dx_1\,dy_1 \cdots dx_k\,dy_k$$

$$= \iiint dx_1 \cdots dx_k \left[G_0(x_1,x_2)\,\varphi^2(x_2)\, G_0(x_2,x_3)\,\varphi^2(x_3) \cdots G_0(x_k,x_1)\,\varphi^2(x_1) \right]$$



Integral of a Feynman diagram

A distribution

...pends NL on ϕ

$$(-\Delta + m^2)^{-1}_{xy} = G_0(x,y) = \overset{\circ}{x}\text{———}\overset{\circ}{y}$$

Propagator

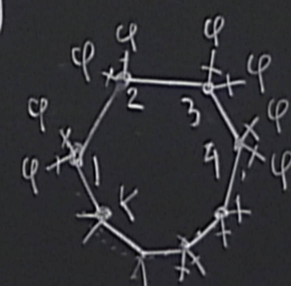
$\varphi(x)$ = value of the background field at x

$$(\varphi^2)_{xy} = \varphi^2(x)\,\delta(x-y) =$$

vertex with 2 φ attached

$$\ldots(x_2,y_2)(\varphi^2)_{y_2 x_3} \cdots \quad G_0(x_k,y_k)(\varphi^2)_{y_k x_1}\Big]\, dx_1 dy_1 \cdots dx_k dy_k$$

$$\ldots(x_2)\, G_0(x_2,x_3)\,\varphi^2(x_3) \cdots G_0(x_k,x_1)\,\varphi^2(x_1)\Big]$$



...agram

1 loop diagram with $2K$ truncated legs

$\varphi(y)$

$= \varphi(x) = \int dy\, \varphi(y)\, \delta(y-x)$

$x \quad y$

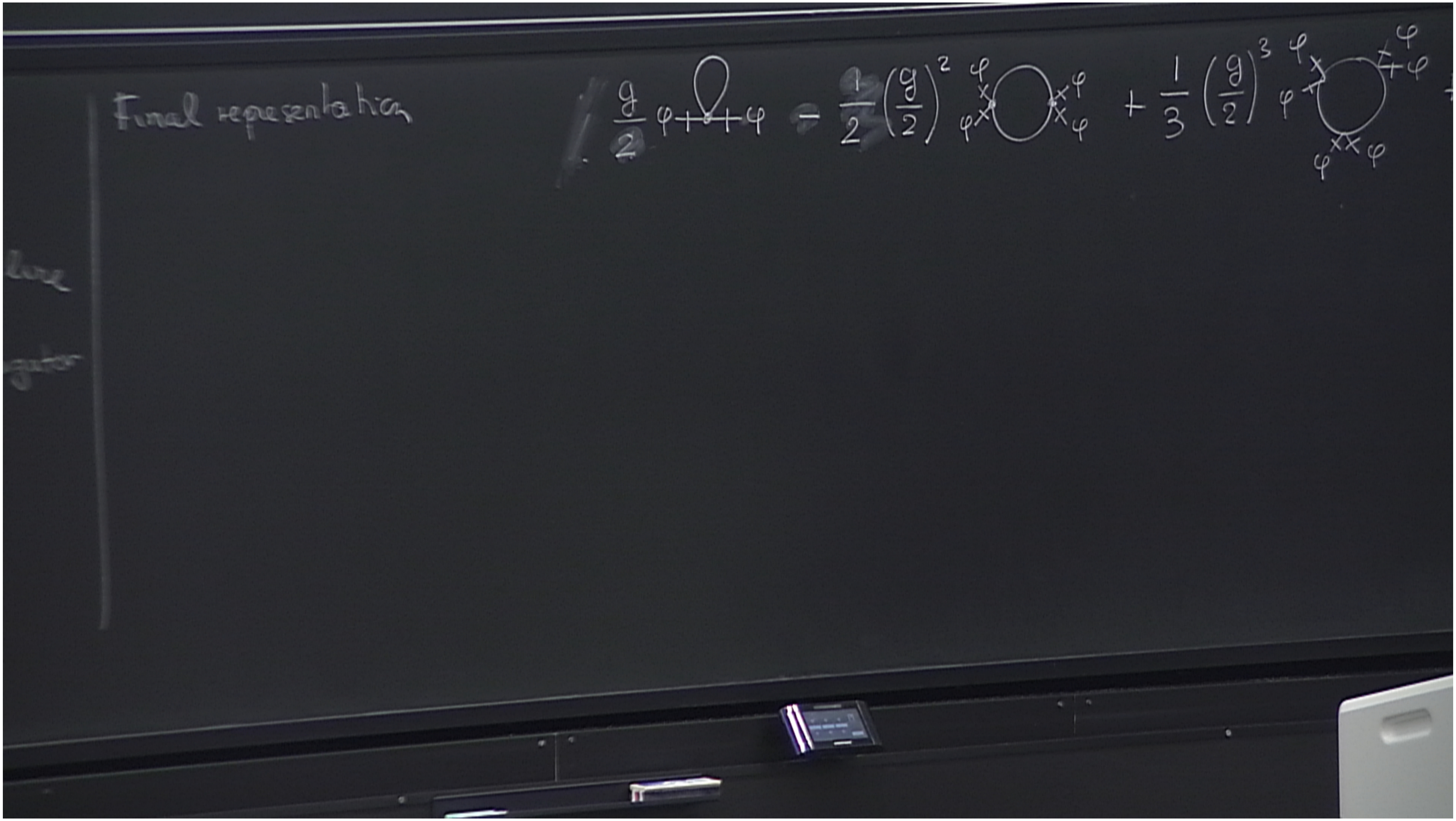
$= \circ \cdots \circ = \delta(x-y)$ Truncated line

$x \quad y$

$= G_0(x-y)$ ordinary propagator

diagrammatic representation

Final represen


Final representation

→ spin 1 & 2

in perturbation theory: compute the Tr correction term as a functional of φ

$$\mathrm{Tr}\left[\mathrm{Log}\left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right)\right] = \mathrm{Tr}\left(\mathrm{Log}\left[(-\Delta+m^2)\times\left(\mathbb{1} + \frac{g}{2}(-\Delta+m^2)^{-1}\cdot\varphi^2\right)\right]\right)$$

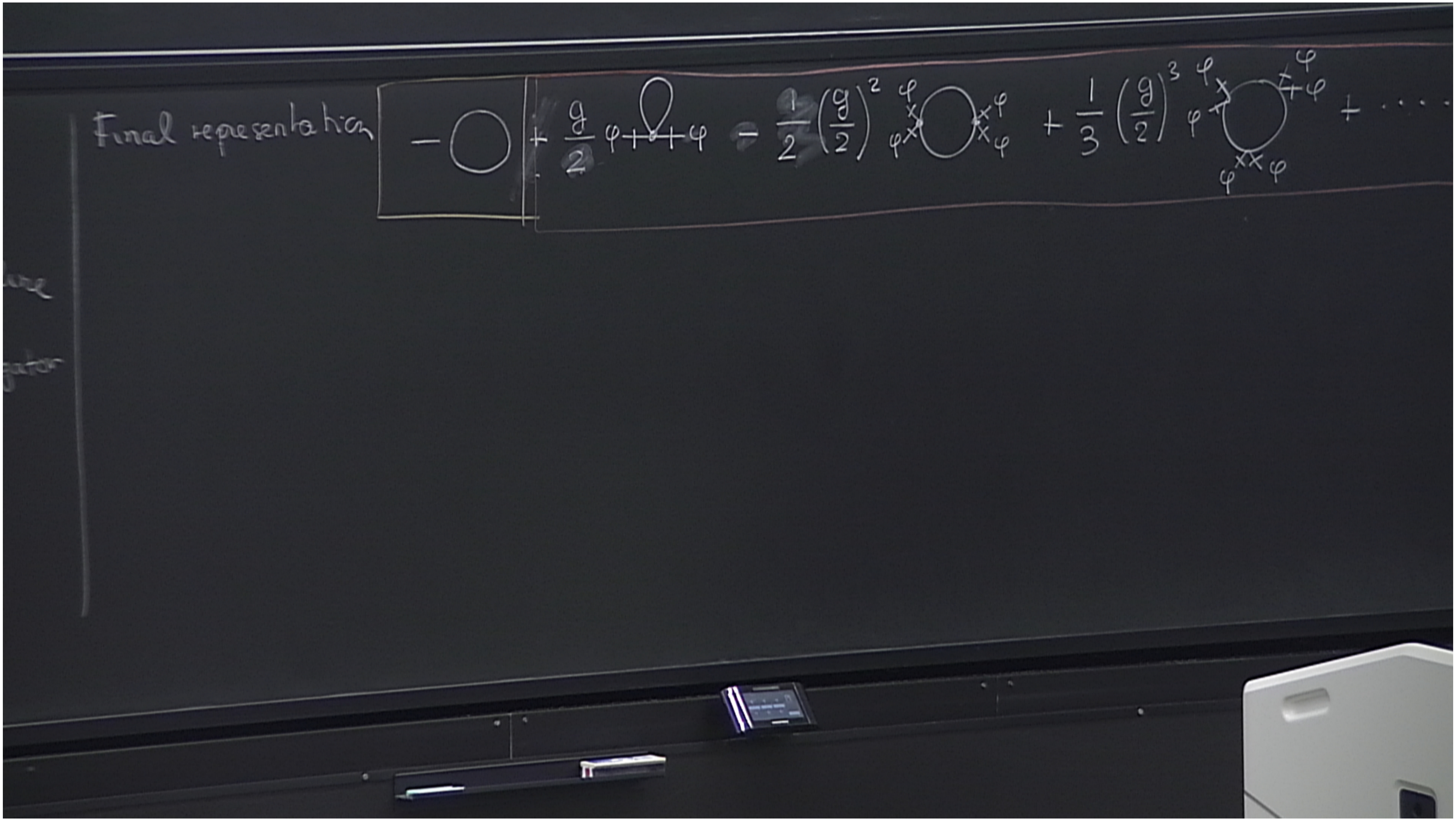
$$= \boxed{\underbrace{\mathrm{Tr}\left[\mathrm{Log}(-\Delta+m^2)\right]}_{\text{Free Field}}} + \boxed{\mathrm{Tr}\left(\mathrm{Log}\left[\mathbb{1} + \frac{g}{2}(-\Delta+m^2)^{-1}\varphi^2\right]\right)}$$

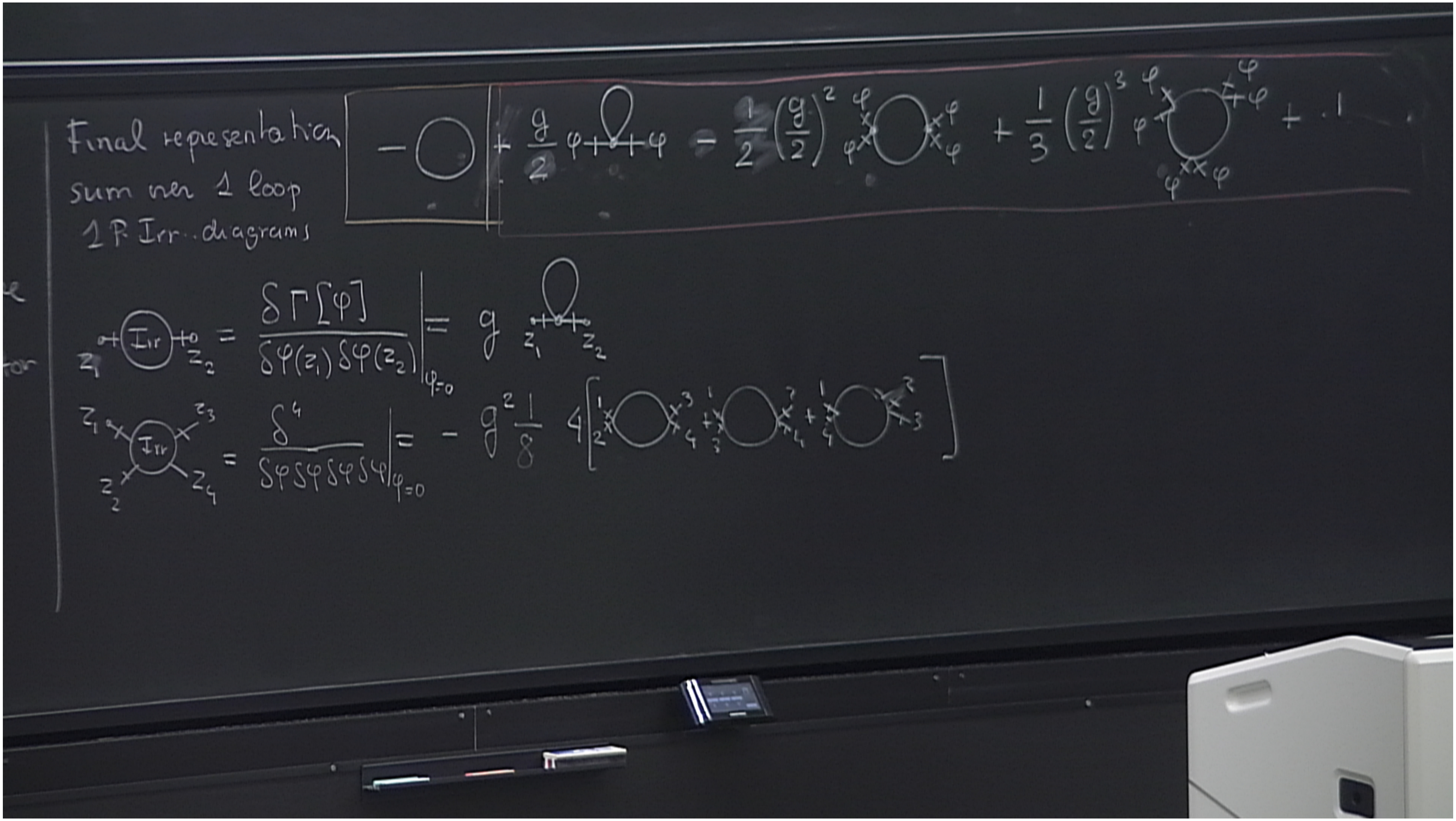
$$\square = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k}\frac{g^k}{2^k}\,\boxed{\mathrm{Tr}\left[\left[(-\Delta+m^2)^{-1}\cdot\varphi^2(x)\right]^k\right]}$$

$$\square = \iiint \left[G_0(x_1, y_1)\,(\varphi \ldots \right.$$

$$= \iiint dx_1 \ldots dx_k \left[G_0(x \ldots \right.$$

Integral of a Fe…


Final representation


Final representation
sum over 1 loop
1 P. Irr. diagrams
Irr
δΓ[φ] / δφ(z1)δφ(z2) |φ=0
δ^4 / δφδφδφδφ |φ=0
- g^2 1/8 4

the

ld at x

ncated

gs

$\varphi(y)$

y

x

$= \varphi(x) = \int dy\, \varphi(y)\, \delta(y-x)$

$x \quad y \quad = \circ \cdots \circ = \delta(x-y)$ Truncated line

$x \quad y \quad = G_0(x-y)$ ordinary propagator

diagrammatic representation

$$\Gamma[\varphi] = \sum_N \frac{1}{N!} \varphi^N \; \mathrm{Irr}$$

N legs

Final representation

sum over 1 loop

1 P. Irr.. diagrams

$z_1 \; \mathrm{Irr} \; z_2 = \dfrac{\delta \Gamma[}{\delta \varphi(z_1)}$

$z_1, z_2, z_3, z_4 \; \mathrm{Irr} = \dfrac{\delta^4}{\delta\varphi\,\delta\varphi\,\delta}$

$-x)$

ncated line

propagator

Final representation
sum over 1 loop
1P Irr. diagrams

$$-\bigcirc + \frac{g}{2}\,\varphi\ \varphi - \frac{1}{2}\left(\frac{g}{2}\right)^2 \ \varphi\ \varphi\ \varphi\ \varphi + \frac{1}{3}\left(\frac{g}{2}\right)^3 \ \varphi\ \varphi\ \varphi\ \varphi\ \varphi\ \varphi + \cdots$$

$$z_1 \ \mathrm{Irr}\ z_2 = \frac{\delta \Gamma[\varphi]_{1\,\mathrm{loop}}}{\delta\varphi(z_1)\,\delta\varphi(z_2)}\Bigg|_{\varphi=0} = g \quad z_1 \ z_2$$

2pt function

$$z_1,\ z_2,\ z_3,\ z_4\ \mathrm{Irr} = \frac{\delta^4 \Gamma_{1\,\mathrm{loop}}}{\delta\varphi\,\delta\varphi\,\delta\varphi\,\delta\varphi}\Bigg|_{\varphi=0} = -\,g^2\,\frac{1}{8}\,4\left[\ {}^{1}_{2}\bigcirc^{3}_{4} + {}^{1}_{3}\bigcirc^{2}_{4} + {}^{1}_{4}\bigcirc^{2}_{3}\ \right]$$

4pt function

$$\hat{\Gamma}^{(2)}(p_1, p_2) = (2\pi)^d \delta(p_1 + p_2)\, \hat{\Gamma}^{(2)}(p)$$

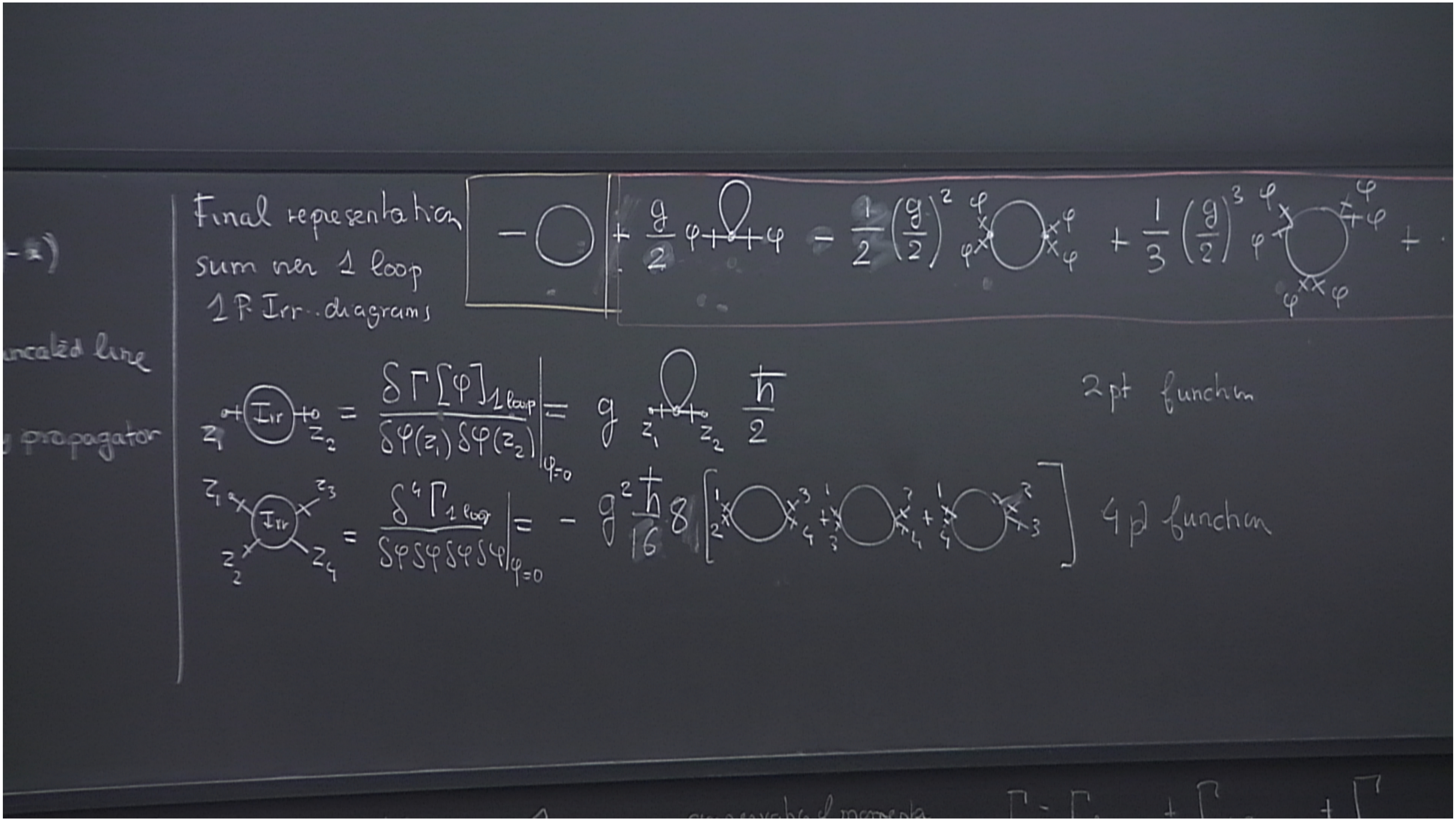
conservation of momenta

$$\Gamma = \Gamma_{\text{classical}} + \Gamma_{1\,\text{loop}} + \Gamma_{2\,\text{loop}}$$

$$S''$$

$$+ \hbar \frac{g}{2} + o(\hbar^2) = \; = \; = \frac{\delta \Gamma[\varphi]}{\delta\varphi(z_1)\,\delta\varphi(z_2)}\Bigg|_{\varphi=0}$$

$$\hbar G_0(0) + o(\hbar^2) \qquad G_0(0) = \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2 + m^2}$$


Final representation
sum over 1 loop
1P. Irr.. diagrams
truncated line
propagator
Irr
$\frac{g}{2}$
$-\frac{1}{2}\left(\frac{g}{2}\right)^2$
$+\frac{1}{3}\left(\frac{g}{2}\right)^3$
$= \frac{\delta \Gamma[\varphi]_{1\,loop}}{\delta\varphi(z_1)\,\delta\varphi(z_2)}\Big|_{\varphi=0} = g \; \frac{\hbar}{2}$
2pt function
$= \frac{\delta^4 \Gamma_{1\,loop}}{\delta\varphi\delta\varphi\delta\varphi\delta\varphi}\Big|_{\varphi=0} = -g^2\frac{\hbar}{16}\,8\,[\;+\;+\;]$
4pt function

N legs

2pt- Irreducible function

$$\hat{\Gamma}^{(2)}(p_1,p_2) = (2\pi)^d\,\delta(p_1+p_2)\,\hat{\Gamma}^{(2)}(p)$$ conservation of momenta

$\Gamma =$

$$\Gamma^{(2)}(z_1,z_2) = (-\Delta+m^2)_{z_1,z_2} + \hbar\,\frac{g}{2}\;(z_1 \quad z_2) + o(\hbar^2) = z_1\;\text{Irr}\;z_2 = p_1\;\text{Irr}\;p_2 = \frac{\delta\Gamma[\varphi]}{\delta\varphi(z_1)\,\delta\varphi(z_2)}\bigg|_{\varphi=0}$$

$\downarrow$ FT

$$\hat{\Gamma}^{(2)}(p) = p^2+m^2+\frac{g\hbar}{2}G_0(0)+o(\hbar^2) \qquad G_0(0)=\int\frac{d^dk}{(2\pi)^d}\,\frac{1}{k^2+m^2}$$

$$p\;\text{Irr}\;p = \Sigma(p)$$

Free theory

$$\hat{\Gamma}^{(2)}_0(p) = p^2+m^2 \qquad \hat{G}^{(2)}_0(p) = \frac{1}{p^2+m^2} \qquad \hat{G}^{(2)}(p) = \frac{1}{\hat{\Gamma}^{(2)}(p)}$$

General relation

Connected 2pt Function

Irreducible 2pt function

Tree decomp

$$\text{Conn.} = \sum_K \underbrace{\text{Ir.}\;\text{Irr}\;\text{Ir.}\;\text{Irr}}_{\text{chain of } K \text{ irreducible}} = \left(\frac{1}{1-\cdots}\right.$$

in p. space

$(2\pi)^d \delta(p_1+p_2)\, \hat{\Gamma}^{(2)}(p)$ conservation of momenta

$\Gamma = \Gamma_{classical} + \Gamma_{1\,loop} + \Gamma_{2\,loop}$

S''

$= \underset{z_1}{\circ}\!-\!(\mathrm{Irr})\!-\!\underset{z_2}{\circ} = \overset{p_1}{\rightarrow} \circ\!-\!(\mathrm{Irr})\!-\!\circ \overset{p_2}{\leftarrow} = \left.\frac{\delta \Gamma[\varphi]}{\delta\varphi(z_1)\,\delta\varphi(z_2)}\right|_{\varphi=0}$

$\Gamma_{1\,loop} = \frac{\hbar}{2}\, \mathrm{Tr}\cdot\log(\quad)$

$\int \frac{d^dk}{(2\pi)^d}\,\frac{1}{k^2+m^2}$

$\overset{p}{\rightarrow}(\mathrm{Irr})\overset{p}{\rightarrow} = \Sigma(p)$

Tree decomp

$\circ\!-\!(\mathrm{Conn.})\!-\!\circ = \sum_K \underbrace{-(\mathrm{Ir.})-(\mathrm{Irr})-(\mathrm{Ir.})-(\mathrm{Irr})-}_{\text{chain of K irreducible}} = \circ\!-\!\circ\left(\frac{1}{1-\circ\!-\!(\mathrm{Irr})}\right)$

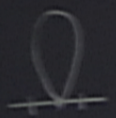
in p. space

$\hat{G}(p) = \frac{1}{p^2+m^2}\cdot\frac{1}{\left(1-\frac{1}{p^2+m^2}\Sigma(p)\right)} = \frac{1}{\underbrace{p^2+m^2-\Sigma(p)}_{\hat{\Gamma}^{(2)}(p)}}$

Connecting 2pt Function

Irreducible 2pt function

Chain of K irreducible

 → amplitude of the tadpole diagram

$$I_{\mathcal{O}} = T = \int_{\mathbb{R}^d} \frac{d^d k}{(2\pi)^k} \frac{1}{k^2+m^2}$$

$$= G_0(x=0)$$

UV divergent at $|k| \nearrow \infty$ if $d \geq 2$

$$G_0(x) \simeq |x|^{2-d}$$

Feature of QFT!

Regularization Procedure

- Lattice in $\mathbb{R}^d \longrightarrow \mathbb{Z}^d$
- Sharp momentum cut-off $|k| < \Lambda$

  $\Lambda \gg m$ ← physical scale
- Pauli-Villars
- $d \neq 4$ but complex

  ☺ ← Gauge theories
- The ultimate regular

  ℓ_{planck} or M_{planck}

$$T(m,\Lambda) = \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2+m^2} = \frac{1}{(4\pi)^2}\Lambda^2 - \frac{m^2}{(4\pi)^2}\log \Lambda^2 + \text{finite terms when } \Lambda \nearrow \infty$$

Ball

leading quadratic div

subleading log. divergence

of the irreducible in p. space $\hat{\Gamma}^{(2)}(p)$

$$T(m,\Lambda) = \int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2+m^2} = \frac{1}{(4\pi)^2}\Lambda^2 - \frac{m^2}{(4\pi)^2}\log\Lambda^2 + \text{finite terms when } \Lambda \nearrow \infty$$

Ball $|k|<\Lambda$

leading quadratic div

subleading log. divergence

$K_E = (\pm iE, \vec{k})$

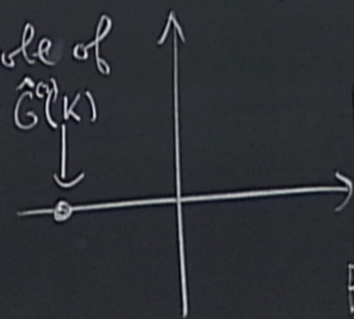
$$G^{(2)}(x) = \int \frac{d^4k}{(2\pi)^D}\, e^{ikx}\, \frac{1}{\Gamma^{(2)}(K)}$$

2pt Function

$$\Gamma^{(2)}(K) = \underbrace{K^2+m^2+\hbar\frac{g}{2}T}_{M^2} = 0$$

$$K^2 = -M^2$$

pole of $\hat{G}^{(2)}(K)$



Euclidean

$$|K|^2 = (\vec{k}^2 - E^2)$$

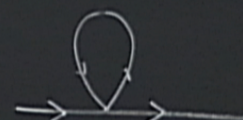
$$\boxed{\text{Theory has a 1 particle state with physical mass } M}$$

→ spin 1 & 2

Free theory : 1 particle state has mass m

Int Theory 1 " " has mass M $\quad M^2 = m^2 + g\hbar\frac{1}{2}T$

$M \neq m$ quantum corrections

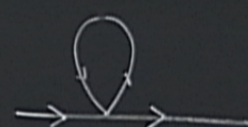


physical mass > naive mass

→ spin 1 & 2

Free theory : 1 particle state has mass m

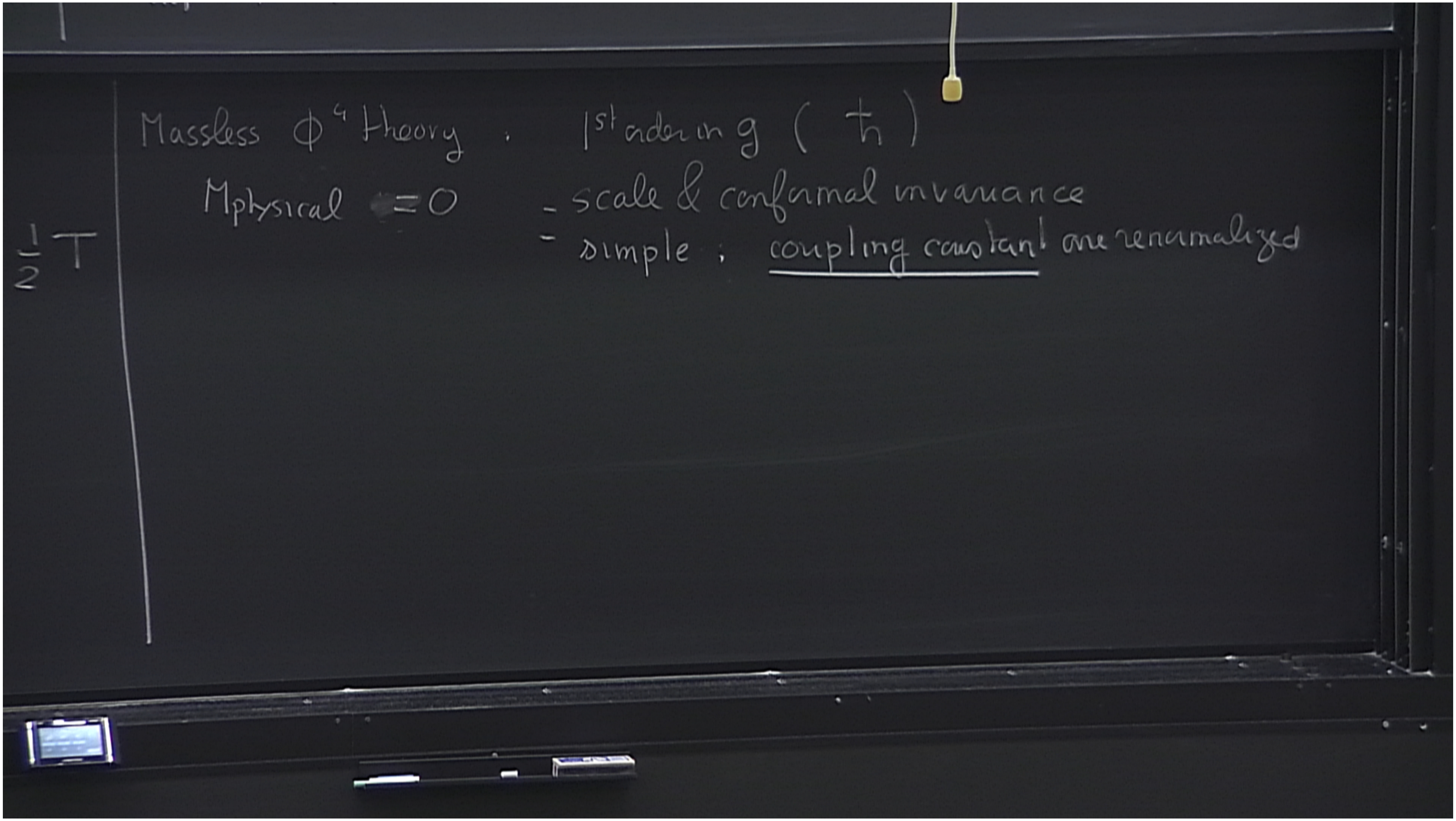
Int. Theory 1 " " has mass M $\qquad M^2 = m^2 + g\hbar \frac{1}{2} T$

$M \neq m$ quantum corrections  

physical mass > naive mass

m_0 is finite, 1st order quantum $\longrightarrow M_{phys} = \infty = O(\Lambda^2)$

? $\leftarrow M_{phys}$. finite


$\frac{1}{2}T$
Massless ϕ^4 theory : 1st order in g (ħ)
$M_{physical} = 0$
- scale & conformal invariance
- simple ; coupling constant are renormalized