Title: Orientation matters: interaction effects in topological insulators and superconductors

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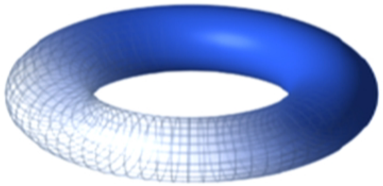
Abstract: <p>Topological phases of matter are phases of matter which are not characterized
by classical local order parameters of some sort. Instead, it is the global properties
of quantum many-body ground states which distinguish one topological phase from
another. One way to detect such global properties is to put the system on a topologically
non-trivial space (spacetime). For example, topologically ordered phases in (2+1)
dimensions exhibit ground state degeneracy which depends on the topology of the spatial manifold.
In this talk, I will discuss how one can use a {\it unoriented} space (spacetime)
to detect non-trivial properties of topological phases of matter in the presence
of discrete spacetime symmetry, such as time-reversal or reflection symmetry.
In particular, I will show how interaction effects on topological insulators and
superconductors can be understood using quantum anomalies on unoriented spacetime.</p>

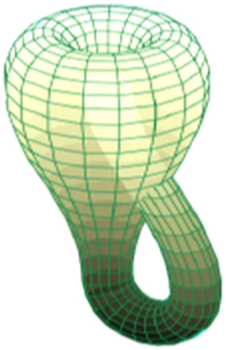
Orientation matters: Interaction effects in topological insulators and superconductors

Shinsei Ryu
Univ. of Illinois, Urbana-Champaign

There is some truth in it. Torus is everywhere when discussing topological phases of condensed matter ...



However, to discuss novel phases in condensed matter, like topological insulators, torus may not be enough ...



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- Introduction
  - Fractional quantum Hall effect (topological order)
  - Integer quantum Hall effect (invertible topological order)
  - Topological insulators (Sym. Protected Topological phases)
  - Question

- General scheme for SPT phases

  E.g. 1: (2+1)D topological insulator
  E.g. 2: (2+1)D topological superconductor
  E.g. 3: (3+1)D topological superconductor

- Closing

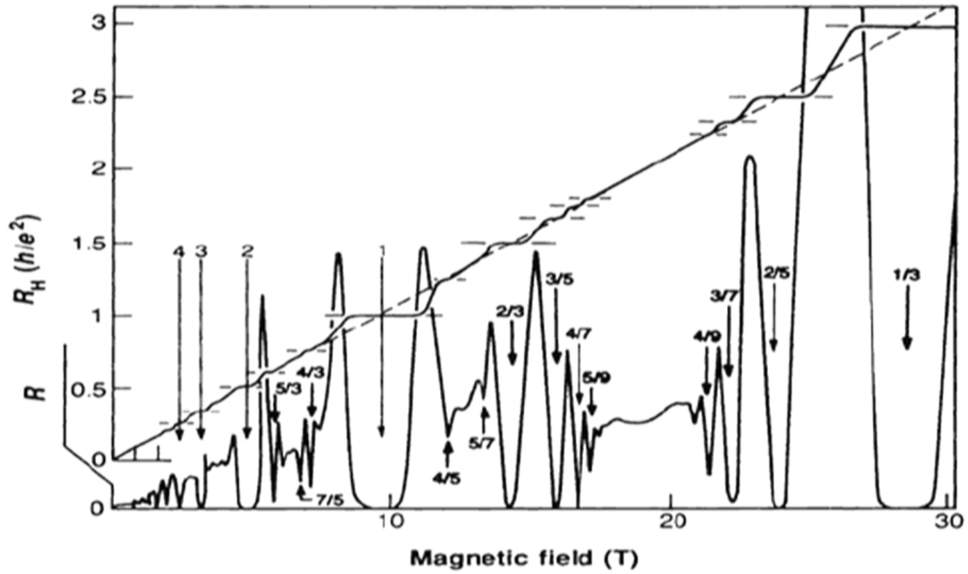
Topological phases (broad-brush intro)

There are, however, phases of matter which cannot be described by any local order parameter

Example: quantum Hall effect

different plateaus:

different gapped quantum phases

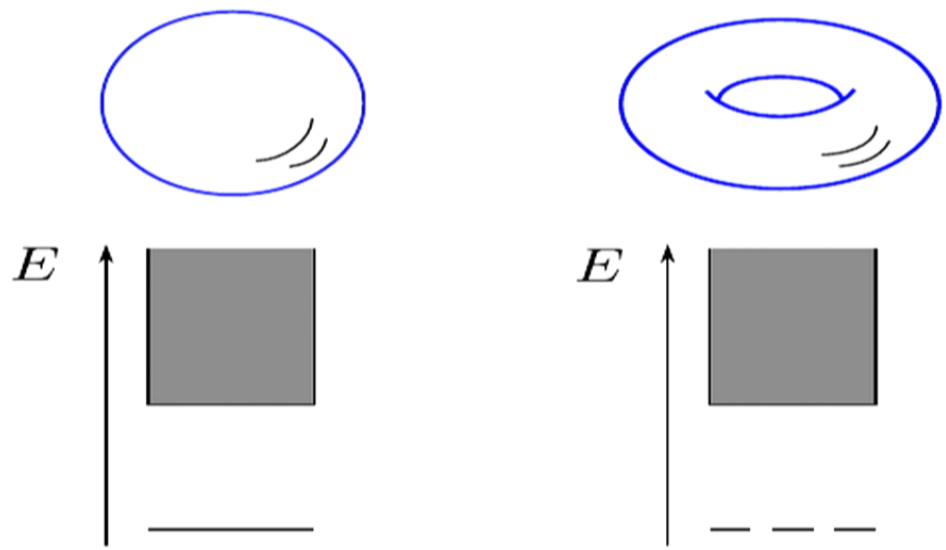


How to detect topological phases ?

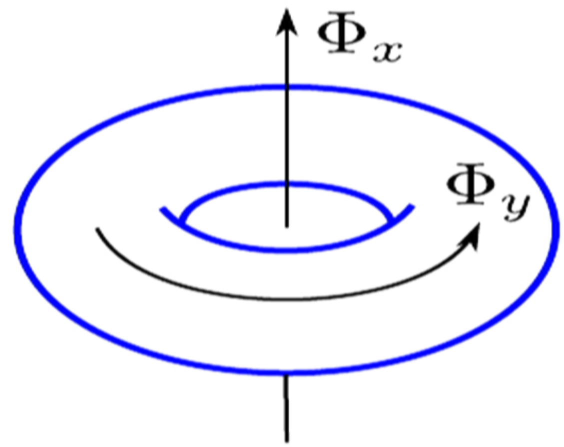
Fractional quantum Hall effect (Topologically ordered phases)

Idea: response of the system to topology of spacetime

Topological orders: ground-state degeneracy depending on the topology of the space (Topological degeneracy) [Wen '90]



Integer quantum Hall effect



- No topological order
  (Unique ground state on any manifold)

- Characterization still requires manifolds
  with non-trivial topology (e.g. torus)

- Ground state on a torus with flux $|\Psi(\Phi_x, \Phi_y)\rangle$

- Berry connection in parameter space

$$A_i = \langle \Psi(\Phi_x, \Phi_y)| \frac{\partial}{\partial \Phi_i} |\Psi(\Phi_x, \Phi_y)\rangle$$

- Topological invariant:

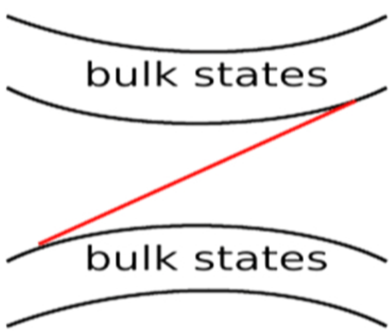
$$Ch := \frac{i}{2\pi} \int d\Phi_x d\Phi_y \left(\partial_{\Phi_x} A_y - \partial_{\Phi_y} A_x\right)$$

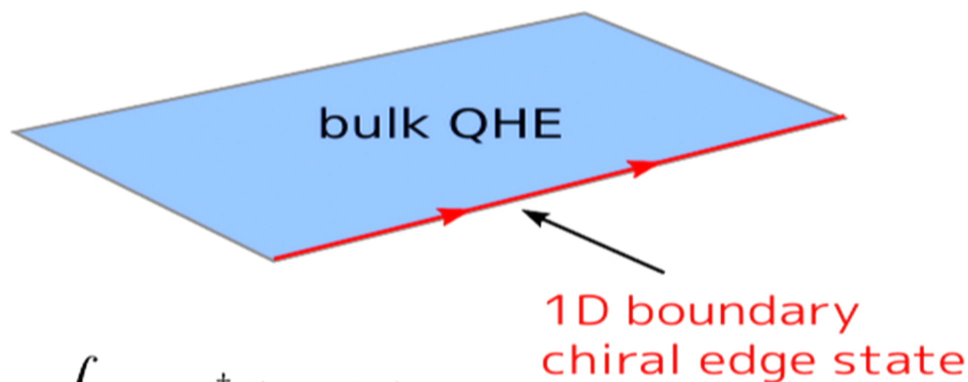
- Topological invariant (Chern number)

$$Ch := \frac{i}{2\pi} \int d\Phi_x d\Phi_y \left(\partial_{\Phi_x} A_y - \partial_{\Phi_y} A_x \right)$$

- Related to transport (Hall conductance)

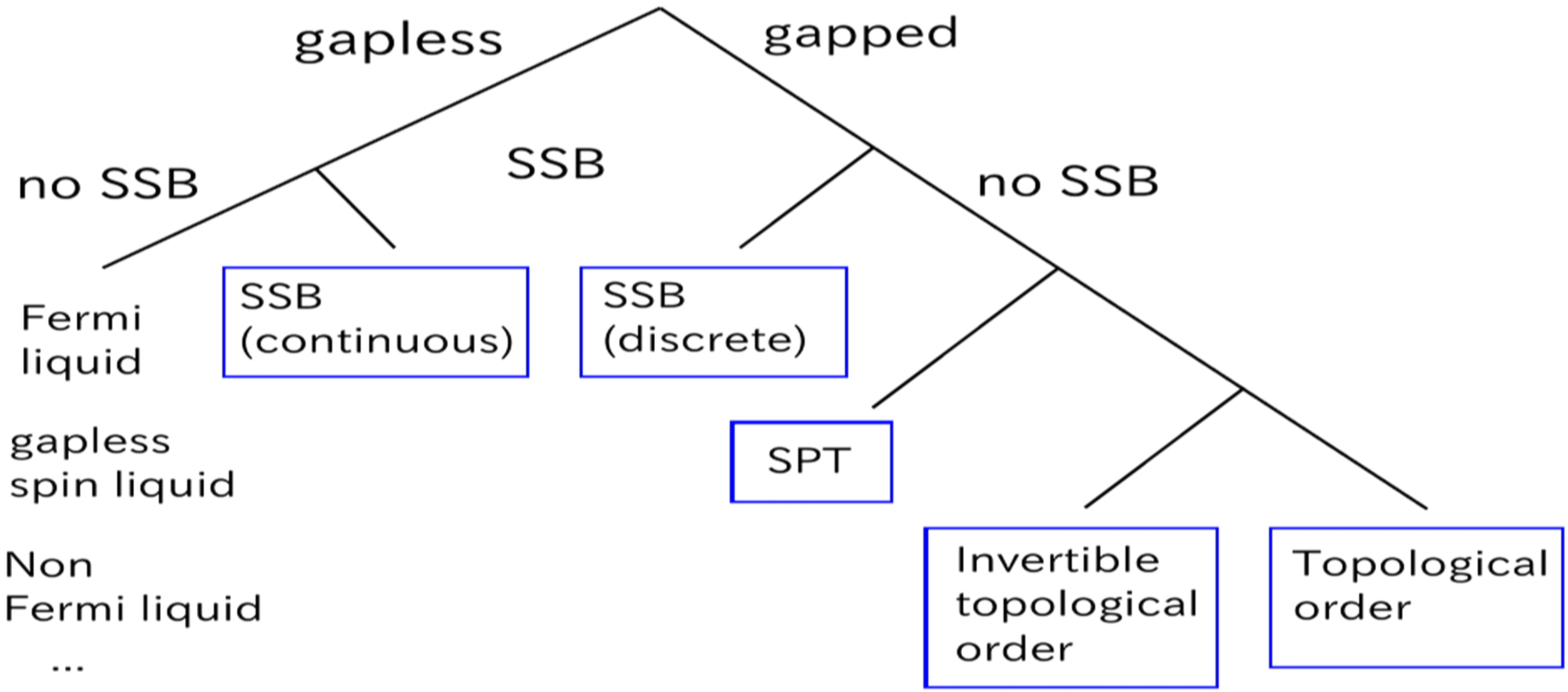
- Non-zero Chern number < --> Chiral edge theory
  "Bulk-boundary correspondence"





$$H = \int dx \psi_L^\dagger (-i\partial_x) \psi_L$$

Phases in condensed matter



T symmetric topological insulator in (2+1) dimensions

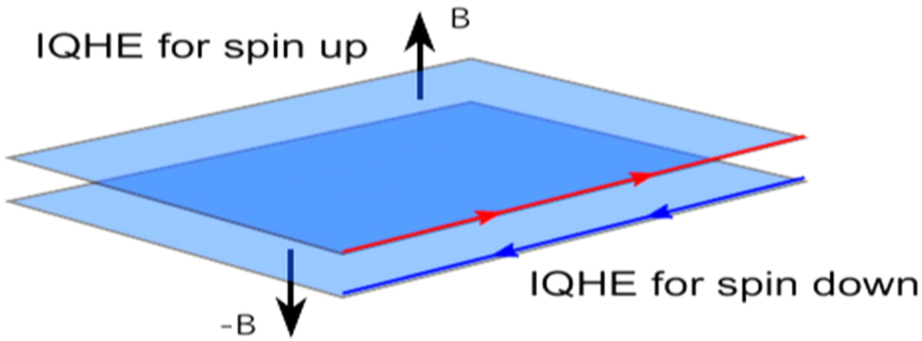
(A.k.a. quantum spin Hall effect)

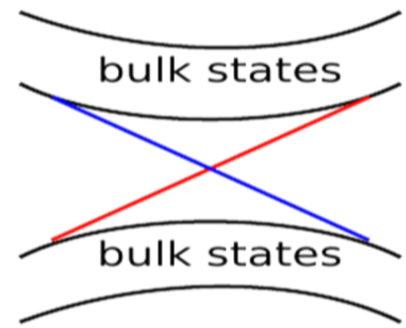
- Time-reversal invariant band insulator with strong spin-orbit interaction

TRS

$$(i\sigma_y)\mathcal{H}^*(-i\sigma_y) = \mathcal{H}$$

- gapless Kramers pair of edge modes





- Characterized by a binary (Z_2) topological quantity

$$W = \prod_{K} \frac{\mathrm{Pf}\,[w(K)]}{\sqrt{\det\,[w(K)]}}$$

T symmetric topological insulator in (2+1) dimensions

- Edge Hamiltonian ("helical" edge):

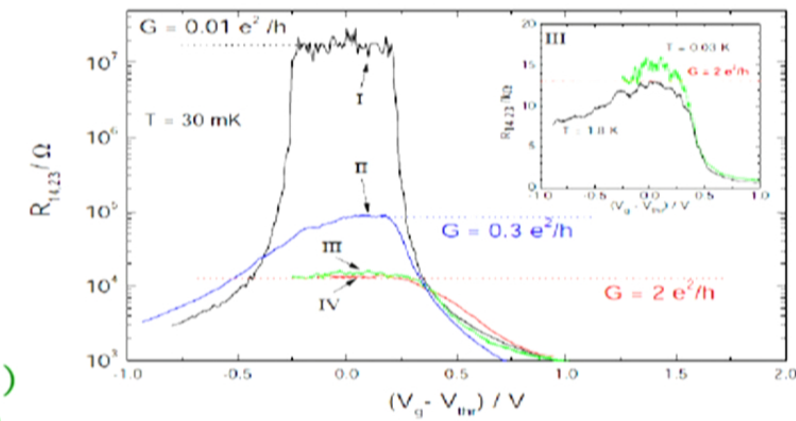
$$H = \int dx\ \left[\psi_L^\dagger i\partial_x \psi_L - \psi_R^\dagger i\partial_x \psi_R\right]$$

- T symmetry

$$\mathcal{T}\psi_L(x)\mathcal{T}^{-1} = \psi_R(x)$$
$$\mathcal{T}\psi_R(x)\mathcal{T}^{-1} = -\psi_L(x)$$

- Can check no mass terms are allowed

Z2 classification



Bernevig-Hughes-Zhang (2006)
M. Koenig et al. Science (2007)

Issues

- Single-particle topological invariants; are they clearly enough?

$$W = \prod_{\mathsf{K}} \frac{\mathrm{Pf}\left[w(\mathsf{K})\right]}{\sqrt{\det\left[w(\mathsf{K})\right]}}$$

- What is the framework to discuss interacting SPT phases?
  What is a proper definition of topological invariant?

Ex: 2d Topological crystalline superconductor

- Topological superconductor protected by parity (P) and time-reversal (T)

$$P : (x, y) \to (-x, y)$$

- BdG Hamiltonian on reflection symmetric edge:

$$H = \int dx \, [\psi_L(-i\partial_x)\psi_L + \psi_R(i\partial_x)\psi_R]$$

- TR symmetry

$$T\psi_L(x)T^{-1} = \psi_R(x)$$
$$T\psi_L(x)T^{-1} = -\psi_R(x)$$

Parity:

$$P\psi_R(x)P^{-1} = \psi_L(-x)$$
$$P\psi_L(x)P^{-1} = \psi_R(-x)$$

- Can check no mass terms are allowed. Classification: Z

- How about interactions?

Aside: Topological crystalline insulators

- "Topological crystalline insulators" protected by mirror symmetry

- Characterized by an integral bulk topological invariant "Mirror Chern number"

- Protected surface state on reflection symmetric surfaces

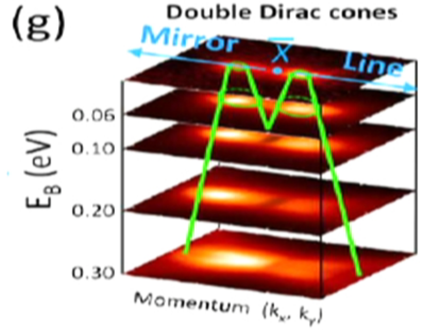
- SnTe, Pb1-xSnxTe, Pb1-xSnxSe

  L. Fu, PRL (2011)
  Y. Tanaka et al. Nature Phys (2012)
  T. H. Hsieh et al, Nature Commun. (2012)
  S.-Y. Xu et al, Nature Commun (2012)
  P. Dziawa et al, Nature Mat. (2012)

- Full classification:
  [Chui-Yao-Ryu (13), Furusaki-Morimoto (13), Shiozaki-Sato (13)]

ARPES on Pd0.6Sn0.4Te

(g)



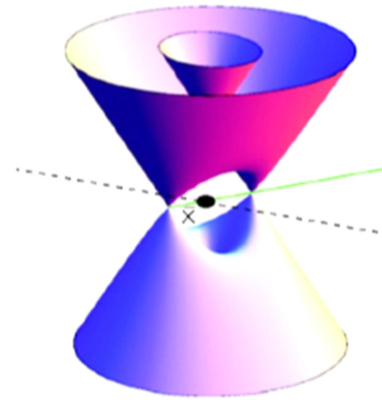
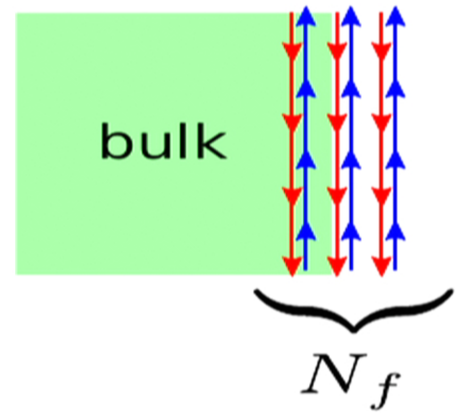


Image from Zejikovic et al. , Nat. Mat.

Collapse of non-interacting classification Z --> Z_8

H. Yao and SR (12)

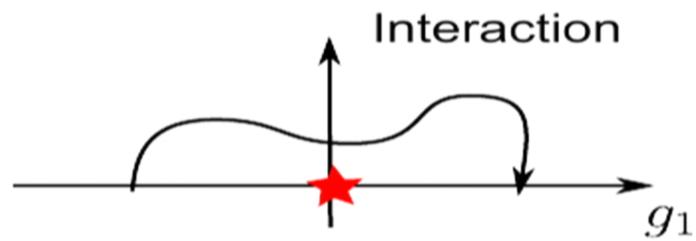
- 2d SCs with T-symmetry and reflection:
  Classified by an integer topological invariant.



- Edge states are stable at non-interacting level.

- Consider N_f copies of crystalline TSCs.
  By adding interactions,
  the edge state is unstable when N_f = 8.

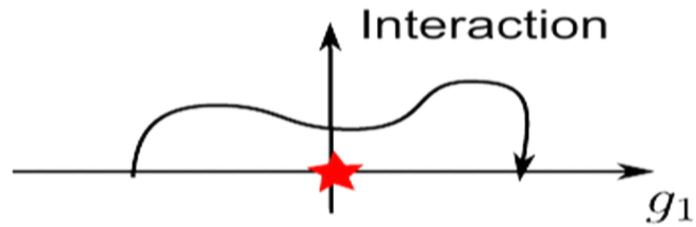
Two distinct phases ?



Single phase!

Other known "collapses" of non-interacting classification

Two distinct phases ?



Single phase!

- 1D TSC (Kitaev chain) with TRS (BDI) : Z --> Z_8 [Fidkowski and Kitaev (10)]

- 2D TSC w/ Z_2 symmetry: Z --> Z_8 [Qi (12), SR-Zhang (12)]
- 2D TSC w/ reflection symmetry: Z --> Z8 [Yao-SR(12)]

- 3D TSC (DIII): Z --> Z_{16}
  [Fidkowski et al (13), Metlitski et al (14), Wang-Senthil (14), ...]

- 3D crystalline TSC: Z --> Z_{16} [Hsieh-Cho-SR (15)]
- 3D crystalline TI: Z --> Z_8 [Isobe-Fu (15)]

- No symmetry breaking, General framework?

- Single-particle topological invariant is clearly not enough

- What is the framework to discuss interacting SPT phases?
  What is a proper definition of topological invariant?

- Key concept: **Quantum anomalies**
  -- failure of classical symmetry by quantum effects

- This talk focuses on calculations quantum anomalies in non-interacting SPT phases, and hence the stability of non-interacting SPTs against interactions.

E.g. 1: (2+1)d Topological insulator Z_2
E.g. 2: (2+1)d Topological superconductor Z_8
E.g. 3: (3+1)d Topological superconductor Z_{16}

- Single-particle topological invariant is clearly not enough

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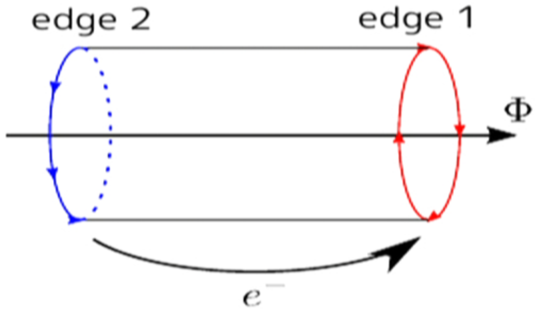
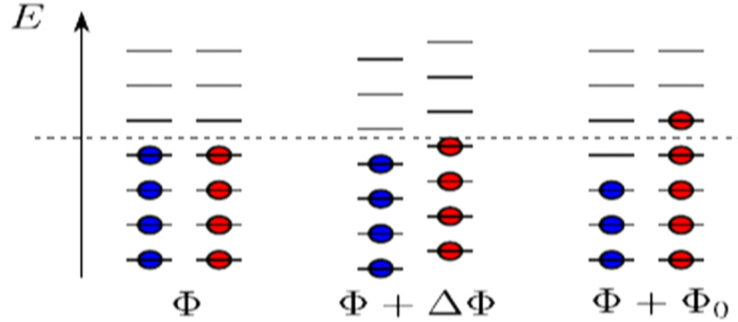
Laughlin's gauge argument





- Adiabatic process $\Phi \to \Phi + \Delta\Phi$

- When $\Delta\Phi = \text{integer} \times \Phi_0$ system goes back to itself ("large gauge equivalent") $H(\Phi) \equiv H(\Phi + n\Phi_0)$

- However, by this adiabatic process, an integer multiple of charge is transported from the left (right) to right (left) edge.

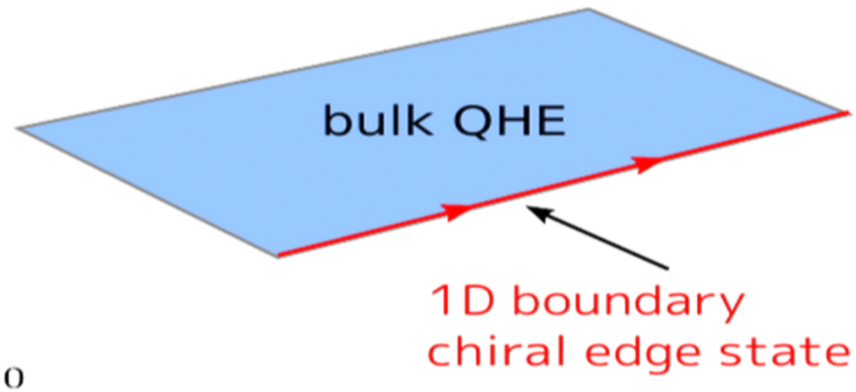
- Charge is not conserved for a given edge.

$$Z_{edge}(\Phi + \Phi_0) \neq Z_{edge}(\Phi)$$

Laughlin's argument: edge theory point of view

- Chiral edge theory

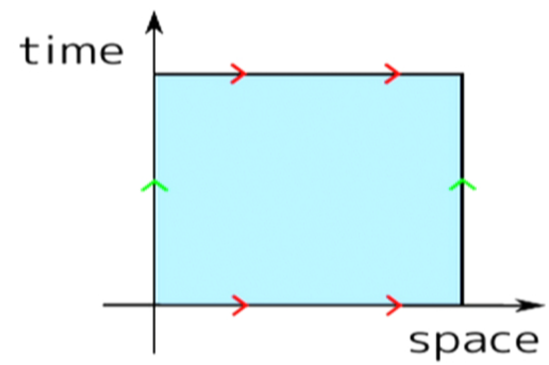
$$\mathcal{L} = \frac{1}{2\pi}\psi^\dagger i(\partial_t + v\partial_x)\psi$$



- Boundary conditions

$$\psi(t, x+L) = e^{2\pi i a}\psi(t,x), \quad a := \Phi/\Phi_0$$
$$\psi(t+\beta, x) = e^{2\pi i b}\psi(t,x). \quad b\text{: chemical potential}$$

- Classical system (Lagrangian + BCs) is invariant under a -> a+1 and b-> b+1



- Quantum mechanics --> Large gauge anomaly

$$Z_{[a,b]} := \int \mathcal{D}[\psi^\dagger, \psi] e^{-S}$$

$$Z_{[a,b]} \neq Z_{[a,b+1]} \qquad Z_{[a,b]} \neq Z_{[a+1,b]}$$

Anomalous symmetry breaking

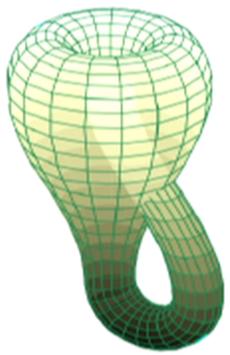
- *Starting point*: Edge/surface theory Z_{bdy}

- *Step1*: Twist boundary conditions using symmetries $Z_{bdy}^{twisted}$

Unitary on-site symmetry (orbifold)

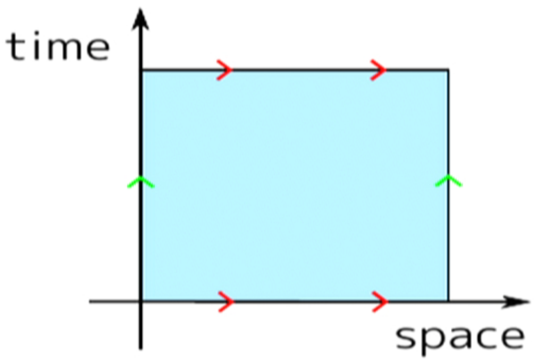
Reflection symmetry: putting the edge theory on a unoriented surface. e.g. Klein bottle.

Klein bottle

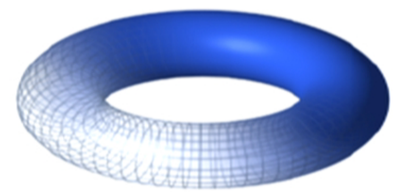


Twisting boundary conditions in edge theories

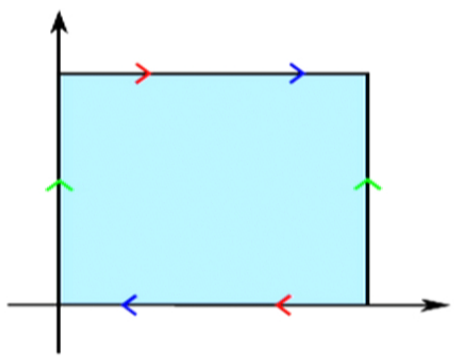
- Laughlin's scheme for the quantum Hall effect: twist by U(1)



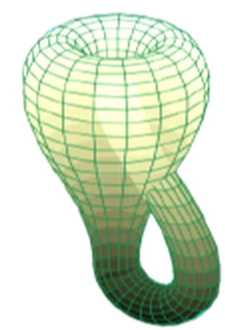
$$\Psi(t+T,x) = e^{i\Phi_t}\Psi(t,x)$$
$$\Psi(t,x+L) = e^{i\Phi_x}\Psi(t,x)$$



- Twist by reflection symmetry:



$$\Psi(t+T,x) \sim \Psi(t,L-x)$$
$$\Psi(t,x+L) \sim \Psi(t,x)$$



- *Step2*: Study the effects of symmetries of the twisted theory
(e.g., U(1), time-reversal, diffeomorphism inv. ...)

$$Z_{bdy}^{twisted} \longrightarrow \tilde{Z}_{bdy}^{twisted}$$

- Criterion:

Broken symmetry --> topological phase $\quad Z_{bdy}^{twisted} \neq \tilde{Z}_{bdy}^{twisted}$

Unbroken symmetry --> trivial phase $\quad Z_{bdy}^{twisted} = \tilde{Z}_{bdy}^{twisted}$

- Comment 1:
A proper generalization of Laughlin's charge pumping argument.

- Comment 2:
Subtle form of symmetry breaking by quantum effects: "quantum anomaly"

E.g. 1: CP symmetric topological insulator

- System with CP and charge U(1) symmetries

$$P : (x, y) \to (-x, y)$$

"CPT-dual" of the Quantum spin Hall effect

- Edge Hamiltonian:

$$H = \int dx \left[\psi_L^\dagger i\partial_x \psi_L - \psi_R^\dagger i\partial_x \psi_R\right]$$

- CP symmetry

$$\mathcal{U}\psi_L(x)\mathcal{U}^{-1} = \psi_R^\dagger(-x)$$
$$\mathcal{U}\psi_R(x)\mathcal{U}^{-1} = \psi_L^\dagger(-x)$$

- Can check no mass terms are allowed when topological. Z_2 classification

- How about interactions ?

Calculation of anomaly

- Edge Hamiltonian: $$H = \int dx\ \left[\psi_L^\dagger i\partial_x \psi_L - \psi_R^\dagger i\partial_x \psi_R\right]$$ $$\mathcal{U}\psi_L(x)\mathcal{U}^{-1} = \psi_R^\dagger(-x)$$ $$\mathcal{U}\psi_R(x)\mathcal{U}^{-1} = \psi_L^\dagger(-x)$$

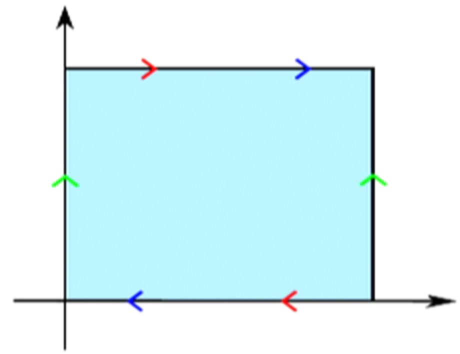
- Klein bottle partition function:

$$\psi_L(t+T,x) = \psi_R^\dagger(t,\ell - x)$$
$$\psi_R(t+T,x) = \psi_L^\dagger(t,\ell - x)$$

Twisting by CP

$$\psi_L(x+\ell) = e^{2\pi i a}\psi_L(x),$$
$$\psi_R(x+\ell) = e^{2\pi i a}\psi_R(x).$$

Twisting by U(1)



- Classically invariant under a-> a+1

- But not quantum mechanically:

$$Z^{\text{Klein}}_{[a+1]} = -Z^{\text{Klein}}_{[a]}$$

[Hsieh-Sule-Cho-SR-Leigh (2015)]

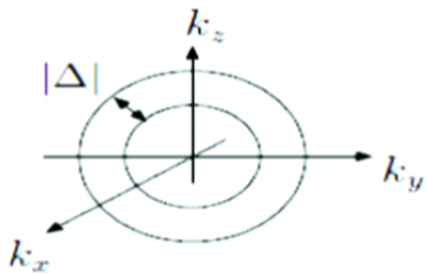
3He B is a topological "superconductor"

- Bulk BdG Hamiltonian

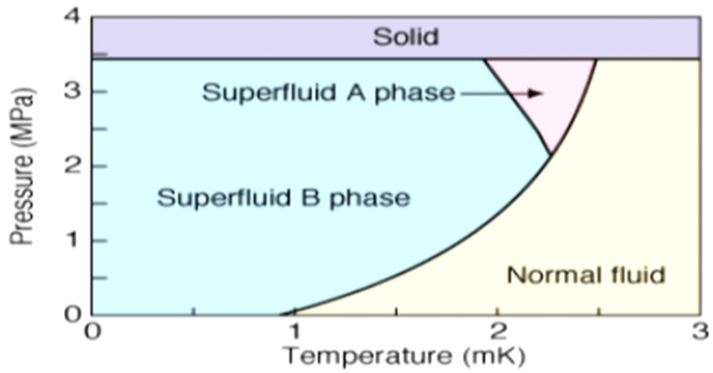
Schnyder, SR, Furusaki, Ludwig (08)
Saloama-Volovik (88)
Roy (08) Qi, Hughes, Raghu, Zhang (08)

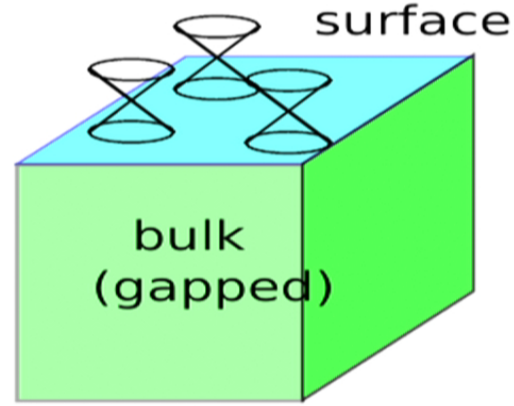
$$H = \frac{1}{2}\int d^3r \Psi^\dagger \mathcal{H} \Psi \quad \mathcal{H} = \begin{pmatrix} \xi & \Delta \\ \Delta^\dagger & -\xi \end{pmatrix}$$

$$\xi_{\boldsymbol{k}} = \frac{\boldsymbol{k}^2}{2m} - \mu \qquad \Delta_{\boldsymbol{k}} = |\Delta| i\sigma_y \boldsymbol{k}\cdot\boldsymbol{\sigma}$$



isotropic gap



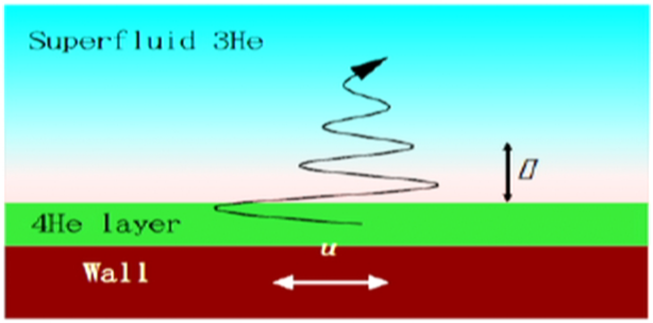


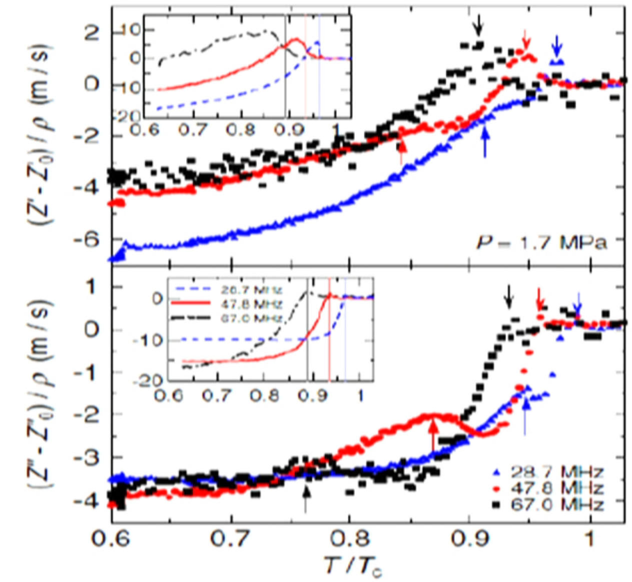
- Prediction:
Topologically protected surface
Majorana cone (stable when TR symmetric)

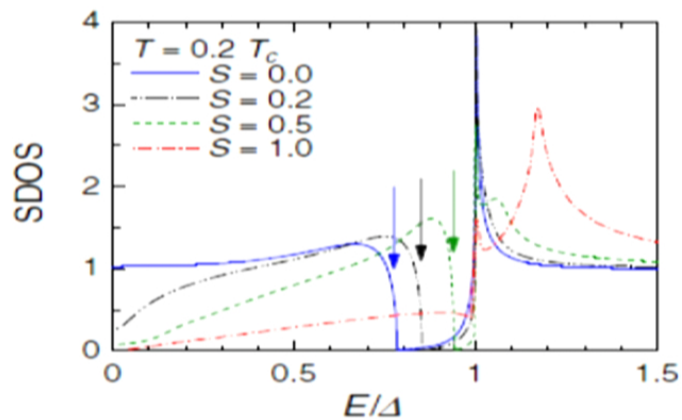
3 He B phase = 3d topological "superconductor" (superfluid)

- Detected by surface acoustic impedance measurement

S. Murakawa et al. PRL (2009)







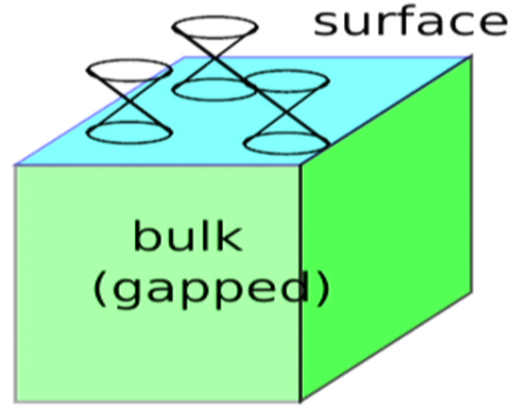
Y. Nagato et al. JLTP (2007)

M. Saitoh et al. PRB(R) (2006)

Ex: 3d Topological crystalline superconductors

[Hsieh-Cho-SR (2015)]

- Topological superconductor protected by parity (P)



- Surface theory: Majorana cone

$$H = \int d^2r\, \lambda^T(-i\sigma_z\partial_x - i\sigma_x\partial_y)\lambda$$

$\lambda(x,y)$: 2-component, real fermion

- P symmetry

$$P\lambda(x,y)P^{-1} = \sigma_z\lambda(-x,y)$$

- Can check no mass terms are allowed. Classification: Z

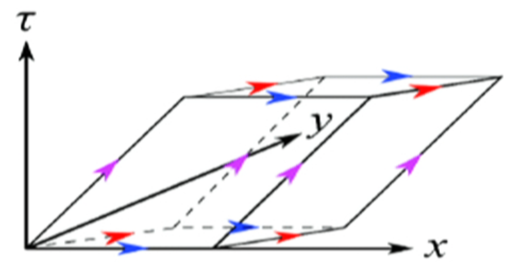
- CPT-related to topological superconductors with TR symmetry (in class DIII)

Calculation of anomaly

- Step 0: Surface theory put on T^3 with (flat) background metric g

5 modular parameters

$$R_1/R_0,\ R_2/R_0,\ \alpha,\ \beta,\ \gamma$$



Symmetry (coord. transfomation): SL(3,Z)

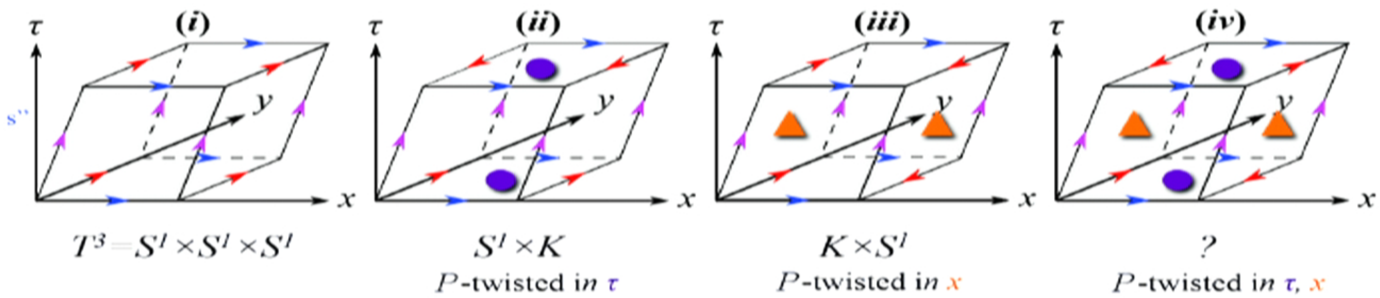
$$Z_{surf}(g)$$

$$Z_{surf}(g) \to Z_{surf}(g') = Z_{surf}(g)$$

Calculation of anomaly

[Hsieh-Cho-SR (2015)]

- Step 1: Twist b.c. by reflection $Z_{surf}^{unoriented}(g)$



$$Z_{surf}^{unoriented}(g) = \sum_{\vec{G} \in SG^3} \varepsilon(\vec{G}) \left[\chi_{\vec{G}}(g)\right]^{N_f}$$

$$\chi_{\vec{G}} = \int \mathcal{D}[\bar{\psi}\psi] e^{-S}$$

$$\psi(x_\mu + R_\mu) = G_\mu \cdot \psi(x_\mu)$$

ferm. # parity **reflection/parity**

$$SG = \{1, \mathscr{G}_f, \mathscr{P}, \mathscr{P}\mathscr{G}_f\}$$

Calculation of anomaly

[Hsieh-Cho-SR (2015)]

- Step 2: Look for anomalies

Gravitational anomaly unless Nf = 0 mod 8

$$Z_{surf}^{unoriented}(g') \neq Z_{surf}^{unoriented}(g)$$

- E. Witten [arxiv:1508.04715]:
N_f = 0 mod 16 is the sufficient and necessary condition to define a consistent parity-conserving path integral of massless Maj. ferm. in (2+1)d (on a possibly unorientable three-manifold)

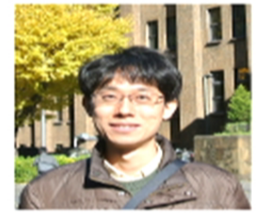
- Other approaches:
[Fidkowski et al: surface topological order
Metlitski et al, Wang-Senthil: vortex condensation
Kapustin et al: cobordism inv effective field theory
You-Xu: non-linear sigma model
Kitaev, Morimoto-Mudry-Furusaki: non-linear sigma model]

People and papers

Chang-Tse Hsieh (UIUC)

AtMa Pakon Chan (UIUC)

Bode Sule (UIUC)



Takahiro Morimoto (Berkeley)

Gil Young Cho (UIUC -> KAIST)

Hong Yao (Tsinghua)

Jeffrey Teo (Virginia)

Rob Leigh (UIUC)

- Topological crystalline superconductors: [Hong Yao and SR (2012)]
- Edge theory on Klein bottle: [Hsieh, Sule, Cho, SR and R. G. Leigh (2014)]
- Crosscap states: [Cho, Hsieh, Morimoto, SR (2015)]
- Surface theory on unoriented spacetime: [Hsieh, Cho, SR (2015)]
- Abelian top. phases on unoriented surfaces: [Chan, Teo, SR (2015)]