

Title: Hydrodynamics and the eightfold way to dissipation

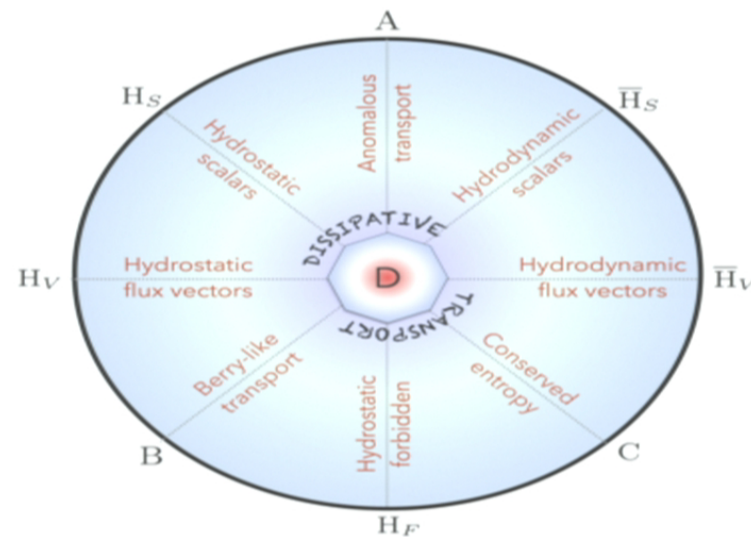
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Abstract: <p>This is the first of two talks on recent advances in our understanding of hydrodynamics as a generic theory of near-thermal dynamics of density matrices. In this talk I will focus on the structure of the hydrodynamic gradient expansion subject to the Second Law of thermodynamics. I will present an eightfold classification scheme and an explicit solution at all orders in derivatives of hydrodynamic transport consistent with the Second Law. I will also mention some connections with gravity and argue that the classification hints towards the existence of a gauge theory whose symmetry current is the entropy current. (The topic of my second, independently accessible, talk on Nov 20 will be a detailed field theoretic proposal for how these and other interesting features emerge from microscopic Schwinger-Keldysh formalism.)</p>

Adiabatic Hydrodynamics and the Eightfold Way to Dissipation

Felix Haehl (Durham University & PI)



Perimeter Institute — 12 November 2015

FH, R. Loganayagam, M. Rangamani

[1412.1090], [1502.00636], (see also [1312.0610], [1510.02494], [1511.xxxxx], ...)

Outline

- **The hydrodynamic gradient expansion**
- Classification of transport
- Example: conformal fluid
- Schwinger-Keldysh and emergent gauge symmetry
- Outlook and conclusion

The hydrodynamic gradient expansion

- Hydrodynamics: generic near-equilibrium eff. field theory for long wavelength fluctuations

microscopic theory

↓ $L \gg \ell_{\text{mfp}}$

macroscopic fluid variables: $u^\mu(x), T(x), \mu(x)$ ($u^2 = -1$)
 background sources: $g_{\mu\nu}(x), A_\mu(x)$

↓

Constitutive relations:

$$T^{\mu\nu} = T_{(0)}^{\mu\nu} + T_{(1)}^{\mu\nu} + \dots$$

$$J^\alpha = J_{(0)}^\alpha + J_{(1)}^\alpha + \dots$$

Dynamics:

$$\nabla_\nu T^{\mu\nu} \simeq F^{\mu\nu} J_\nu + \mathbb{T}_H^{\perp\nu}$$

$$\nabla_\alpha J^\alpha \simeq \mathbb{J}_H^\perp \quad (\text{cov. anomalies})$$

- E.g.: $T_{(0)}^{\mu\nu} = \varepsilon(T, \mu) u^\mu u^\nu + p(T, \mu) (g^{\mu\nu} + u^\mu u^\nu)$, $J_{(0)}^\alpha = q(T, \mu) u^\alpha$

Plan

This talk: the **structure** of hydrodynamics

- Understand constitutive relations allowed by Second Law:
 - ▷ **Classify** hydrodynamic transport in a physically useful way
 - ▷ Construct **most general solution** at all orders
- Suggest a **unifying framework** for adiabatic transport:
 - ▷ **New gauge symmetry** that explains the 2nd law constraint

Talk on Nov 20: the **field theory** behind hydrodynamics

- Use hydrodynamics as a tractable starting point to learn basic lessons about some important problems across physics:
 - ▷ How to understand hydrodynamics as a **Wilsonian field theorist?**
 - ▷ Ingredients: Schwinger-Keldysh, SUSY, topological σ -models, ...
 - ▷ **Black holes via AdS/CFT**: dissipation, entropy, unitarity, ...

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Disclaimer

From now on I will only discuss **neutral fluids**.

Adding an arbitrary number of abelian or non-abelian flavours is just a technical task without new conceptual ideas, see [1502.00636].

Off-shell entropy production and adiabaticity

- Inequality constraint $\nabla_\mu J_S^\mu \gtrsim 0$ is much more conveniently incorporated if we don't have to simplify it using equations of motion.
- Use Lagrange multiplier β^μ and consider **off-shell statement**:

$$\nabla_\mu J_S^\mu + \beta_\mu \left(\nabla_\nu T^{\mu\nu} - T_H^{\mu\perp} \right) \equiv \Delta \geq 0$$

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- Natural Lagrange multiplier:

▶ $\beta^\mu = \frac{1}{T} u^\mu$ (local thermal vector)

Task: solve for $\{J_S^\mu, T^{\mu\nu}\}$ as functionals of $\{\beta^\mu, g_{\mu\nu}\}$

▶ Ideally: find effective action which defines all solutions off-shell

- Marginal case $\Delta = 0$: **'adiabaticity equation'**
 - ▶ Particularly rich structure! \Rightarrow focus on this first

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Aside: adiabaticity equation for free energy current

$$\nabla_{\mu} J_S^{\mu} + \beta_{\mu} \left(\nabla_{\nu} T^{\mu\nu} - T_H^{\mu\perp} \right) = 0$$

- Can trade entropy current J_S^{μ} for **free energy current** \mathcal{G}^{μ} :

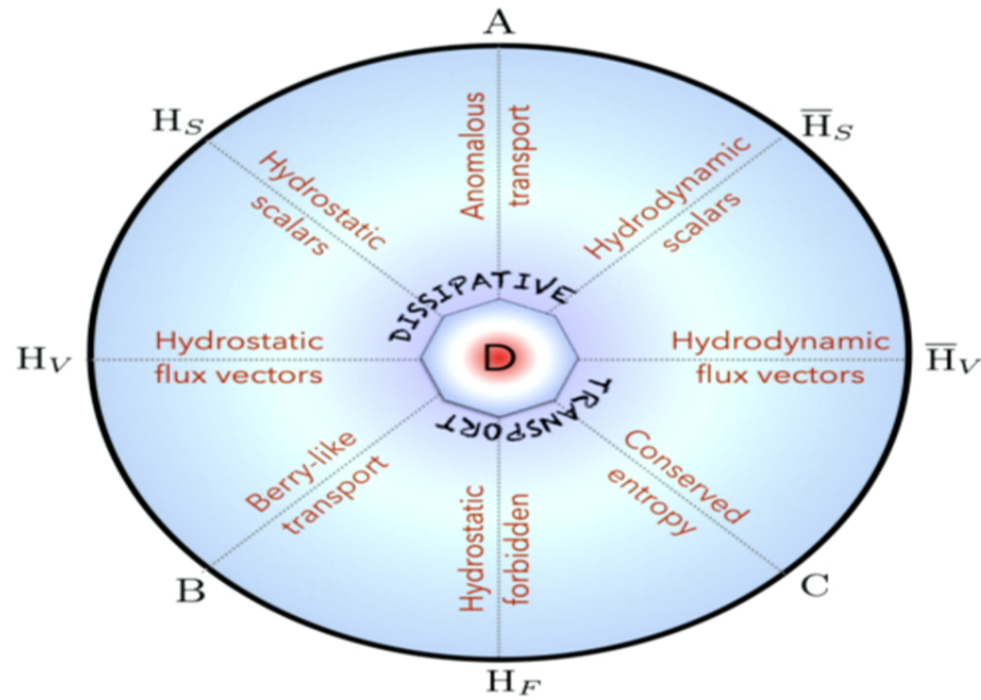
$$-\frac{\mathcal{G}^{\mu}}{T} \equiv J_S^{\mu} - (J_S^{\mu})_{\text{canonical}} \quad \text{with} \quad (J_S^{\mu})_{\text{canonical}} = -\beta_{\nu} T^{\mu\nu}$$

- Grand-canonical version of adiabaticity equation:

$$-\left[\nabla_{\mu} \left(\frac{\mathcal{G}^{\mu}}{T} \right) - \frac{\mathcal{G}_H^{\perp}}{T} \right] = \frac{1}{2} T^{\mu\nu} \mathcal{L}_{\beta} g_{\mu\nu}$$

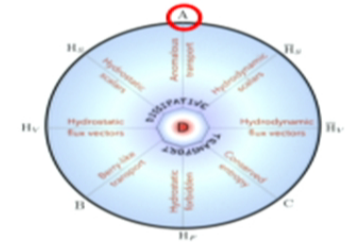
- ▶ Solve for $\{\mathcal{G}^{\mu}, T^{\mu\nu}\}$ as functionals of $\{\beta^{\mu}, g_{\mu\nu}\}$

Classification of hydrodynamic transport



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Anomaly induced transport (Class A)



$$-\left[\nabla_{\sigma}\left(\frac{\mathcal{G}^{\sigma}}{T}\right)-\frac{\mathcal{G}_{H}^{\perp}}{T}\right]=\frac{1}{2}T^{\mu\nu}\mathcal{L}_{\beta}g_{\mu\nu}+\Delta$$

- First of all: let's get rid of anomalies $\mathcal{G}_{H}^{\perp}=-u_{\nu}T_{H}^{\nu\perp}$
 - ▶ Can always split off from a solution $\{\mathcal{G}^{\sigma},T^{\mu\nu}\}$ a **particular solution** $\{(\mathcal{G}^{\sigma})_{A},(T^{\mu\nu})_{A}\}$ that takes care of anomalies with $(\Delta)_{A}=0$:

$$-\left[\nabla_{\sigma}\left(\frac{(\mathcal{G}^{\sigma})_{A}}{T}\right)-\frac{\mathcal{G}_{H}^{\perp}}{T}\right]=\frac{1}{2}(T^{\mu\nu})_{A}\mathcal{L}_{\beta}g_{\mu\nu}$$

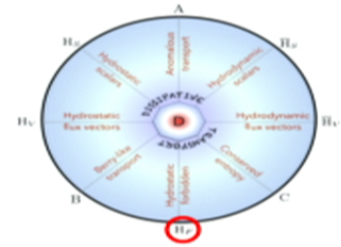
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Jensen-Loganayagam-Yarom '13

- ▶ Anomalous transport coefficients fixed in terms of anomaly polynomial
 \Rightarrow finite class

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Hydrostatically forbidden terms (Class H_F)



$$\Delta \equiv -\nabla_\sigma \left(\frac{g^\sigma}{T} \right) - \frac{1}{2} T^{\mu\nu} \mathcal{L}_\beta g_{\mu\nu} \geq 0$$

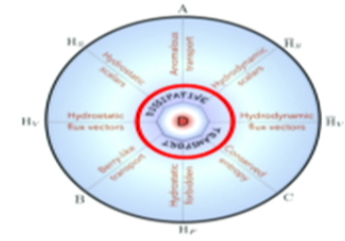
- Type ③: sign-indefinite terms at dominant order in ∂
 - ▶ Need to be zero for consistency with 2nd law!
 - ▶ **Example:** Ideal fluid

$$T_{(0)}^{\mu\nu} = \varepsilon u^\mu u^\nu + p (g^{\mu\nu} + u^\mu u^\nu), \quad J_{S,(0)}^\mu = s u^\mu$$

$$\Rightarrow \Delta \simeq \underbrace{(Ts - \varepsilon - p)}_{=0 (!)} \frac{\Theta}{T} + \underbrace{\left(T \frac{ds}{dT} - \frac{d\varepsilon}{dT} \right)}_{=0 (!)} \frac{(u\nabla)T}{T}$$

- ▶ A-priori: 3 parameters
 - ▶ But second law enforces: 2 relations
- This is **Class H_F** : combinations forbidden by 2nd law

Dissipative transport (Class D)



$$\Delta \equiv -\nabla_\sigma \left(\frac{\mathcal{G}^\sigma}{T} \right) - \frac{1}{2} T^{\mu\nu} \mathcal{L}_\beta g_{\mu\nu} \geq 0$$

- There's a simple way to construct Class D explicitly:

$$(T^{\mu\nu})_D = - \left(\mathcal{N}^{(\mu\nu)(\alpha\beta)} + \mathcal{N}^{(\alpha\beta)(\mu\nu)} \right) \mathcal{L}_\beta g_{\alpha\beta}$$

$$(\mathcal{G}^\sigma)_D = 0$$

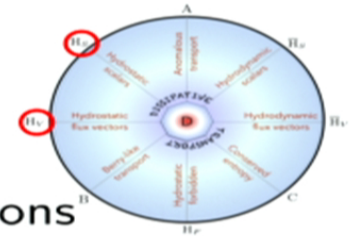
- ▶ Every term in Δ involves a total square of the form

$$\Delta = \mathcal{N}^{((\mu\nu)(\alpha\beta))} (\mathcal{L}_\beta g_{\mu\nu}) (\mathcal{L}_\beta g_{\alpha\beta})$$

- ▶ Hence $\Delta \geq 0$ automatic as long as the leading order is sign-definite

- Easy task at any order in ∇ : find all tensor structures $\mathcal{N}^{(\alpha\beta)(\mu\nu)} [\beta^\mu, g_{\mu\nu}]$

Hydrostatics (Class H)

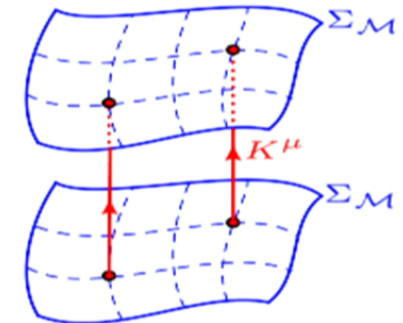


- Hydrostatic transport: time-independent equilibrium configurations

- ▶ \exists timelike Killing vector $K^\mu = \beta^\mu|_{equil.}$:

$$\mathcal{L}_K g_{\mu\nu} = 0$$

- ▶ Spacetime manifold \mathcal{M} :
Euclidean structure $\Sigma_{\mathcal{M}} \times S^1$

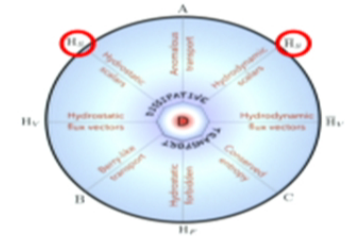


- Transport captured by **Euclidean path integral/partition function**:

$$W_{\text{Hydrostatic}} = -[\text{total free energy}] = - \left[\int_{\Sigma_{\mathcal{M}}} \left(\frac{\mathcal{G}^\sigma}{T} \right) d^{d-1} S_\sigma \right]_{\text{Hydrostatic}}$$

- ▶ Decompose: $\mathcal{G}^\sigma = \mathcal{S} \beta^\sigma + \mathcal{V}^\sigma$
- ▶ This splits Class H into two subclasses: $H = H_S \cup H_V$
- ▶ Variation w.r.t. $g_{\mu\nu}$ gives all hydrostatic $T^{\mu\nu}$

Lagrangian solutions (Class L)



- Consider Landau-Ginzburg action:

- Fields: fluid vector & background geometry = $\{\beta^\mu, g_{\mu\nu}\}$
- Symmetries: diffeomorphism invariance

$$S_{\text{eff}} = \int \sqrt{-g} \mathcal{L}[\beta^\mu, g_{\mu\nu}]$$

- Basic variation defines hydrodynamic currents:

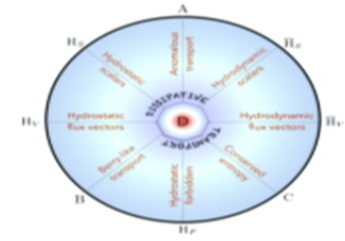
$$\delta S_{\text{eff}} = \int \sqrt{-g} \left[\frac{1}{2} T^{\mu\nu} \delta g_{\mu\nu} + T \mathfrak{h}_\sigma \delta \beta^\sigma + \underbrace{\nabla_\mu (\dots)^\mu}_{\text{surface term}} \right]$$

- Further, define entropy current:

$$J_S^\mu = s u^\mu \quad \text{with} \quad s \equiv \left[\frac{1}{\sqrt{-g}} \frac{\delta S_{\text{eff}}}{\delta T} \right]_{\{u^\mu, g_{\mu\nu}\} \text{ fixed}} = -\mathfrak{h}_\sigma \beta^\sigma$$

- Can show: $\{T^{\mu\nu}, J_S^\mu\}$ solve **adiabaticity equation** (= off-shell version of 2nd law constraint)

What do we have so far?

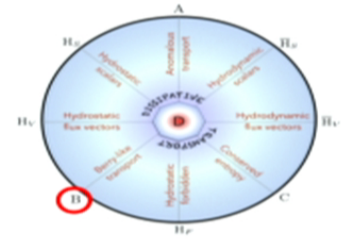


- **Class A:** anomalies can be dealt with once and forever
- **Class D:** can genuinely produce entropy ($\Delta \geq 0$)
- **Class H_F :** Constitutive relations forbidden by 2nd law

$$\text{_____ } \mathcal{G}^\sigma = \mathcal{S}\beta^\sigma + \mathcal{V}^\sigma \text{ _____}$$

- **Class H** = $H_S \cup H_V$: Hydrostatic response to free energy **density** and **flux**
- **Class L:** Wilsonian action giving currents consistent with 2nd law
 - ▶ But Lagrangians being scalars, we only get: Class L = $H_S \cup \bar{H}_S$
- Some more situations that we're missing so far:
 - ▶ Free energy current \mathcal{G}^σ could be **zero** or **topological**
 - ▶ **Non-hydrostatic** free energy **flux vectors** (\bar{H}_V)

Berry-curvature type solutions (Class B)



$$\Delta \equiv -\nabla_\sigma \left(\frac{\mathcal{G}^\sigma}{T} \right) - \frac{1}{2} T^{\mu\nu} \mathcal{L}_\beta g_{\mu\nu} = 0$$

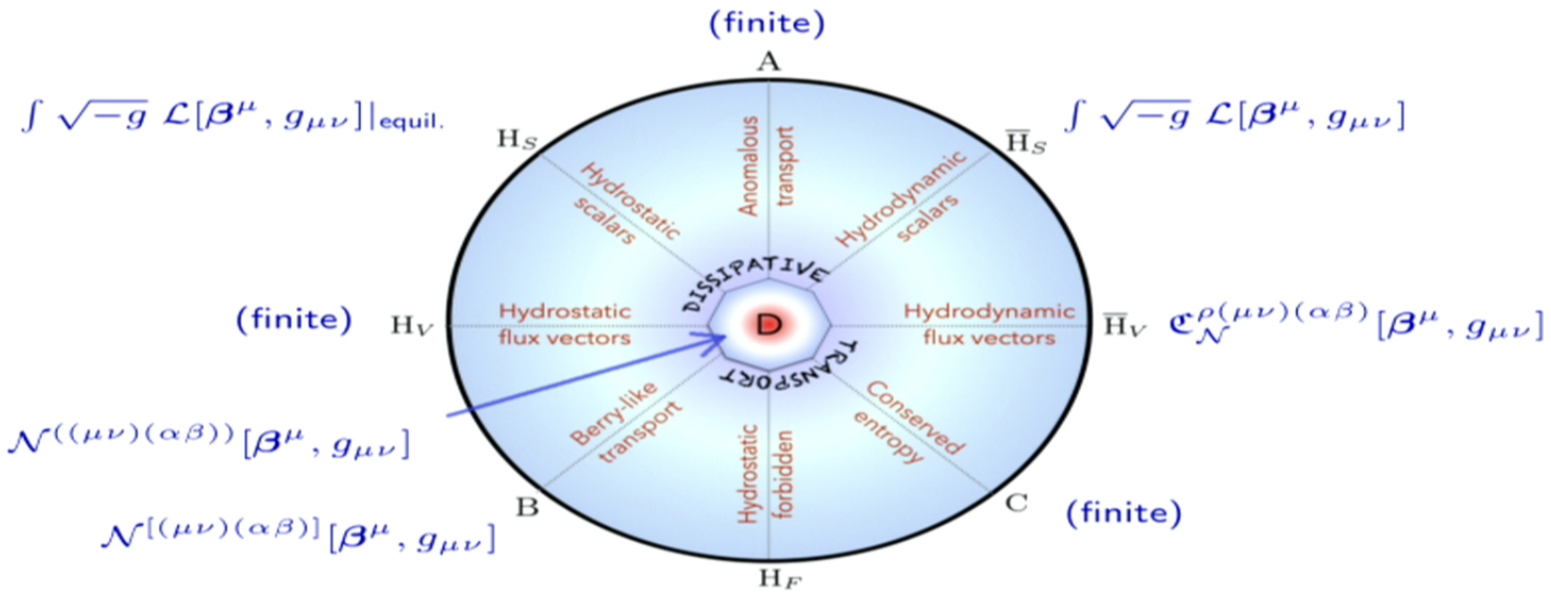
- Consider the following currents:

$$(T^{\mu\nu})_B \propto \left(\mathcal{N}^{(\mu\nu)(\alpha\beta)} - \mathcal{N}^{(\alpha\beta)(\mu\nu)} \right) \mathcal{L}_\beta g_{\alpha\beta}$$

$$(\mathcal{G}^\sigma)_B = 0$$

- ▶ Trivially solve adiabaticity equation
- ▶ Manifestly **non-hydrostatic** ($\mathcal{L}_\beta g_{\mu\nu} = 0$ in hydrostatics)
- ▶ Seemingly not captured by Lagrangians (Class L)
- Easy task at any order in ∇ :
find all tensor structures $\mathcal{N}^{(\alpha\beta)(\mu\nu)}$ built out of $\{\beta^\mu, g_{\mu\nu}\}$
- Examples in $d = 2 + 1$: Hall conductivity, Hall viscosity

Summary: Classification of hydrodynamic transport



FH-Loganayagam-Rangamani '14 '15

Theorem: The eightfold way of hydrodynamic transport

- ▷ There are eight classes of $\{T^{\mu\nu}, J_S^\mu\}$ consistent with $\nabla_\mu J_S^\mu \gtrsim 0$.
- ▷ All of them can be constructed easily at all orders in ∇_μ .
- ▷ Constitutive relations not produced by this algorithm, are forbidden by second law (Class H_F).

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Example: neutral Weyl-invariant fluid at $\mathcal{O}(\partial^2)$

- Most general 2nd order (neutral, Weyl-invariant) stress tensor:

$$\begin{aligned} T_{(2)}^{\mu\nu} = & (\lambda_1 - \kappa) \sigma^{\langle\mu\alpha} \sigma_{\alpha}{}^{\nu\rangle} \\ & + (\lambda_2 + 2\tau - 2\kappa) \sigma^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle} \\ & + \tau (u^\alpha \mathcal{D}_\alpha^{\mathcal{W}} \sigma^{\mu\nu} - 2\sigma^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle}) \\ & + \lambda_3 \omega^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle} \\ & + \kappa \left(C^{\mu\alpha\nu\beta} u_\alpha u_\beta + \sigma^{\langle\mu\alpha} \sigma_{\alpha}{}^{\nu\rangle} + 2\sigma^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle} \right) \end{aligned}$$

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 T_{(2)}^{\mu\nu} = & (\lambda_1 - \kappa) \sigma^{\langle\mu\alpha} \sigma_{\alpha}{}^{\nu\rangle} \\
 & + (\lambda_2 + 2\tau - 2\kappa) \sigma^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle} \\
 & + \tau (u^\alpha \mathcal{D}_\alpha^{\mathcal{W}} \sigma^{\mu\nu} - 2\sigma^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle}) && \rightarrow \text{Class } \bar{\text{H}}_S \\
 & + \lambda_3 \omega^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle} && \rightarrow \text{Class } \text{H}_S \\
 & + \kappa \left(C^{\mu\alpha\nu\beta} u_\alpha u_\beta + \sigma^{\langle\mu\alpha} \sigma_{\alpha}{}^{\nu\rangle} + 2\sigma^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle} \right) && \rightarrow \text{Class } \text{H}_S
 \end{aligned}$$

τ, λ_3, κ

Are all derivable from a Lagrangian (Class L)

$$\mathcal{L}_2^{\mathcal{W}} = \frac{1}{4} \left[-\frac{2\kappa}{(d-2)} ({}^{\mathcal{W}}R) + 2(\kappa - \tau) \sigma^2 + (\lambda_3 - \kappa) \omega^2 \right]$$

Note: λ_3 and κ are hydrostatic, τ is genuinely hydrodynamic

Example: neutral Weyl-invariant fluid at $\mathcal{O}(\partial^2)$

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$$\begin{aligned}
 T_{(2)}^{\mu\nu} &= (\lambda_1 - \kappa) \sigma^{\langle\mu\alpha} \sigma_{\alpha}{}^{\nu\rangle} && \rightarrow \text{Class D} \\
 &+ (\lambda_2 + 2\tau - 2\kappa) \sigma^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle} \\
 &+ \tau (u^\alpha \mathcal{D}_\alpha^{\mathcal{W}} \sigma^{\mu\nu} - 2\sigma^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle}) && \rightarrow \text{Class } \bar{H}_S \\
 &+ \lambda_3 \omega^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle} && \rightarrow \text{Class } H_S \\
 &+ \kappa \left(C^{\mu\alpha\nu\beta} u_\alpha u_\beta + \sigma^{\langle\mu\alpha} \sigma_{\alpha}{}^{\nu\rangle} + 2\sigma^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle} \right) && \rightarrow \text{Class } H_S
 \end{aligned}$$

$$(\lambda_1 - \kappa)$$

Leads to entropy production $\Delta \simeq -(\lambda_1 - \kappa) \frac{1}{T} \sigma^\mu{}_\nu \sigma^\nu{}_\rho \sigma^\rho{}_\mu$
 \Rightarrow Dissipative (\Rightarrow not captured by L-G Lagrangian)
 (but unconstrained by second law, since $\sigma^3 \ll \sigma^2$)

Example: neutral Weyl-invariant fluid at $\mathcal{O}(\partial^2)$

- Most general 2nd order (neutral, Weyl-invariant) stress tensor:

$$\begin{aligned}
 T_{(2)}^{\mu\nu} &= (\lambda_1 - \kappa) \sigma^{\langle\mu\alpha} \sigma_{\alpha}{}^{\nu\rangle} && \rightarrow \text{Class D} \\
 &+ (\lambda_2 + 2\tau - 2\kappa) \sigma^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle} && \rightarrow \text{Class B} \\
 &+ \tau (u^\alpha \mathcal{D}_\alpha^{\mathcal{W}} \sigma^{\mu\nu} - 2\sigma^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle}) && \rightarrow \text{Class } \bar{H}_S \\
 &+ \lambda_3 \omega^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle} && \rightarrow \text{Class } H_S \\
 &+ \kappa \left(C^{\mu\alpha\nu\beta} u_\alpha u_\beta + \sigma^{\langle\mu\alpha} \sigma_{\alpha}{}^{\nu\rangle} + 2\sigma^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle} \right) && \rightarrow \text{Class } H_S
 \end{aligned}$$

$$(\lambda_2 + 2\tau - 2\kappa)$$

Is of the form of a Class B constitutive relation

$$\begin{aligned}
 (T^{\mu\nu})_B &\equiv -\frac{1}{4} \left(\mathcal{N}^{(\mu\nu)(\alpha\beta)} - \mathcal{N}^{(\alpha\beta)(\mu\nu)} \right) \mathcal{L}_\beta g_{\alpha\beta} \\
 (\mathcal{G}^\sigma)_B &= 0
 \end{aligned}$$

because of orthogonality: $\sigma^{\langle\mu\alpha} \omega_{\alpha}{}^{\nu\rangle} \mathcal{L}_\beta g_{\mu\nu} = 0$

Example: neutral Weyl-invariant fluid at $\mathcal{O}(\partial^2)$

- Out of 5 transport coefficients, 3 come from a Lagrangian: τ , λ_3 and κ
- For fluids described by $\mathcal{L}[\beta^\mu, g_{\mu\nu}]$, the other 2 combinations are zero:

$$(\lambda_1 - \kappa) = 0 \quad \text{and} \quad (\lambda_2 + 2\tau - 2\kappa) = 0$$

- ▶ These relations have been **observed in Einstein gravity** *Haack-Yarom '08*
 - ★ Our simple Lagrangians seem to know about holography
 - ★ Derive $\mathcal{L}[\beta^\mu, g_{\mu\nu}]$ from gravity directly? *Nickel-Son '10*
de Boer et al. '15, Crossley et al. '15
- ▶ First relation ensures **no entropy production at subleading order** (this is not required by second law!)
→ **"Principle of minimum dissipation"** in holography?

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Outline

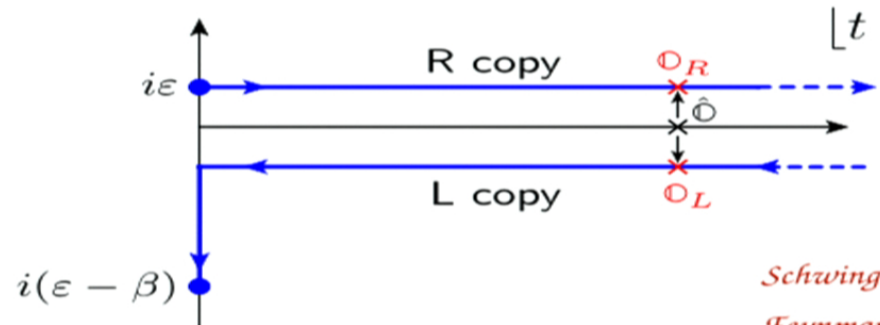
- The hydrodynamic gradient expansion
- Classification of transport
- Example: conformal fluid
- **Schwinger-Keldysh and emergent gauge symmetry**
- Outlook and conclusion

Schwinger-Keldysh doubling

- Non-equilibrium effective field theory in general described by **Schwinger-Keldysh** formalism
 - ▶ Systems in mixed state: path integral evolves both $|\cdot\rangle$ and $\langle\cdot|$

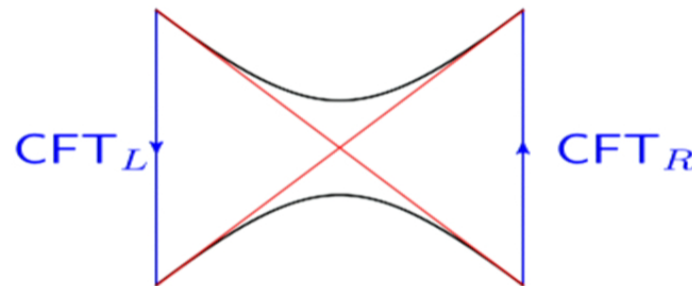
- **Double fields & symmetries:**

$$\mathcal{H}_{phys} \subset \mathcal{H}_R \otimes \mathcal{H}_L$$



*Schwinger, Keldysh,
Feynman-Vernon, '60s*

- In principle many applications: off-equilibrium physics, dissipation, gravity with horizons, complementarity, ...



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Schwinger-Keldysh hydrodynamics?

- Can we upgrade Class L Lagrangians to describe all 8 classes, using Schwinger-Keldysh?
- Would like to double the sources: $g_{\mu\nu} \rightarrow \{g_{\mu\nu}, \tilde{g}_{\mu\nu}\}$
 - ▶ ...and write something like

$$S_{\text{eff}} = \int \sqrt{-g} \frac{1}{2} T^{\mu\nu}[\beta^\mu, g_{\mu\nu}] \tilde{g}_{\mu\nu}$$
$$\Rightarrow \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{eff}}}{\delta \tilde{g}_{\mu\nu}} = T^{\mu\nu}$$

- This would allow us to get **any** $T^{\mu\nu}$ from an action. Including Class H_F !

The danger of doubling

Just blindly doubling gives too much freedom.
Can easily violate 2nd law.

- ▶ Doubling is very powerful. But how to control it?
- ▶ Important obstacle for systematic understanding of eff. field theory of mixed states (dissipation, entanglement, horizons, ...)

A new macroscopic symmetry

We found a consistent **Schwinger-Keldysh doubling for hydrodynamics**. It is constrained in exactly the right way to reproduce the eightfold way. “Danger of doubling” controlled by introducing an **emergent $U(1)_T$ symmetry**.

- Remember Class L free energy current: $\mathcal{G}^\mu = -\mathcal{L} u^\mu + T(\text{bdy. term}_\beta)^\mu$
 - ▶ $N^\mu \equiv -\frac{\mathcal{G}^\mu}{T}$ is **Noether current for diffeos along β^μ**
- Postulate a $U(1)_T$ gauge symmetry with gauge field $A^{(T)}_\mu$ coupling to N^μ

Proposed field content:

- | | |
|-------------------------|----------------------|
| ▷ Hydrodynamic field: | β^μ |
| ▷ Background source: | $g_{\mu\nu}$ |
| ▷ SK copy of source: | $\tilde{g}_{\mu\nu}$ |
| ▷ $U(1)_T$ gauge field: | $A^{(T)}_\mu$ |

Outlook: Talk next Friday (Nov 20, string group meeting)

- Try to derive this scenario from first principles
- Study Schwinger-Keldysh path integrals in detail
- Find very interesting and universal structures:
 - ▶ Hidden **topological (BRST) supersymmetry** behind every SK path integral (incl. relativistic fluids)
 - ▶ Associated **ghosts** are crucial to retain unitarity
 - ▶ **Gauge theory of entropy**: general field theory argument
- Formulate eff. action for **all 8 classes**
 - ▶ Dissipation comes from ghosts
 - ▶ Via AdS/CFT: all these features should play some role in black holes → lots of interesting conjectures to work on

Summary

- 8 classes of constitutive relations consistent with the Second Law
- I gave a recipe for constructing them at any order
- The classification is useful and physical:
 - ▶ Computations become simpler in this framework (classification often tells what is the “nicest” basis to work in)
 - ▶ Conjecture: long-wavelength **near-horizon AdS dynamics** can be usefully characterized using the Eightfold Way
 - ▶ **“Minimum dissipation conjecture”**:
Holographic fluids optimize entropy production.
- Schwinger-Keldysh doubling is powerful but dangerous
 - ▶ Proposal: **emergent $U(1)_T$ gauge symmetry** controls the doubling in hydrodynamics
 - ▶ Next week: systematic justification of $U(1)_T$