Title: What if Gravity is like QCD? -- Revisiting Quadratic Gravity in Analogy with QCD

Date: Nov 10, 2015 01:00 PM

URL: http://pirsa.org/15110010

Abstract: <p>It has been known for a long time that quadratic gravity, which generalizes Einstein gravity with quadratic curvature terms, is renormalizable and asymptotically free in the UV. However the theory is afflicted with a ghost problem if the perturbative spectrum is taken seriously. We explore the possibility that the dimensional scale of Einstein-Hilbert term is far smaller than the scale where the dimensionless gravitational couplings become strong. The propagation of the gravitational degrees of freedom can change character at this strong interaction scale. Lattice QCD studies show a particular suppression of gluon propagator in the IR, which removes the perturbative gluon from the physical spectrum. We propose that the same fate can apply to the spin-2 ghost. The Planck mass is associated with the strong dynamics scale below which the normal Einstein description can emerge. In this picture both the UV and IR limits have weakly coupled descriptions, similar to perturbative QCD and the chiral Lagrangian. Some implications of a small mass ratio in the theory are considered.</p>

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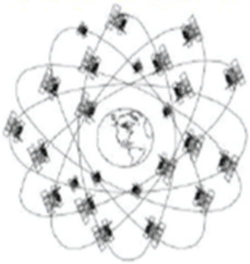
Einstein Gravity

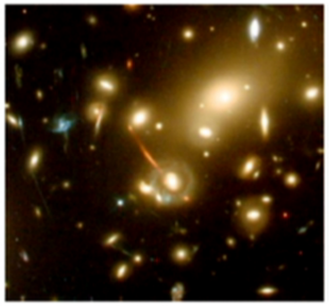
- 100 years of General relativity: huge success in large scale!

Daily life

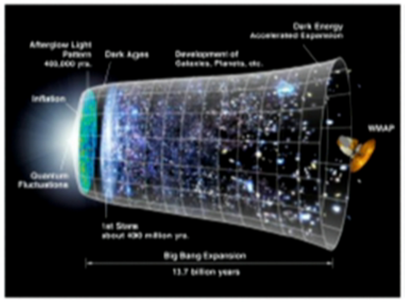
GPS CONSTELLATION



Astrophysics



Cosmology



- Quantum mechanics + General relativity??
  - Non-renormalizable, treat as an effective field theory

$$S_{\rm GR} = \int d^4x\sqrt{-g}\left[M_{\rm Pl}^2\left(-\Lambda + \frac{1}{2}R\right) + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + \ldots\right]$$

Gravitational interactions get strong at M_{Pl}, need UV completion.

  - Decades of efforts for quantum gravity: string, loop, asymptotic safety…

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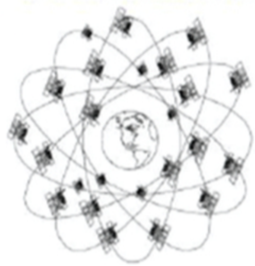

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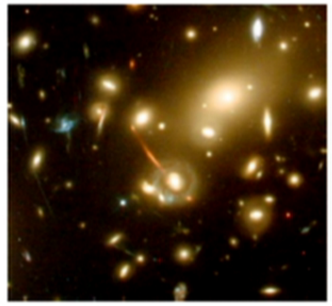

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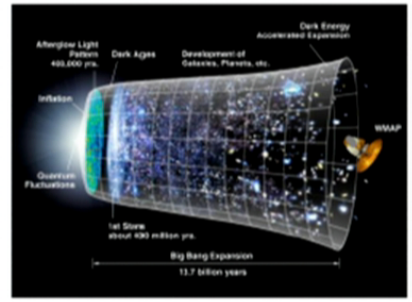

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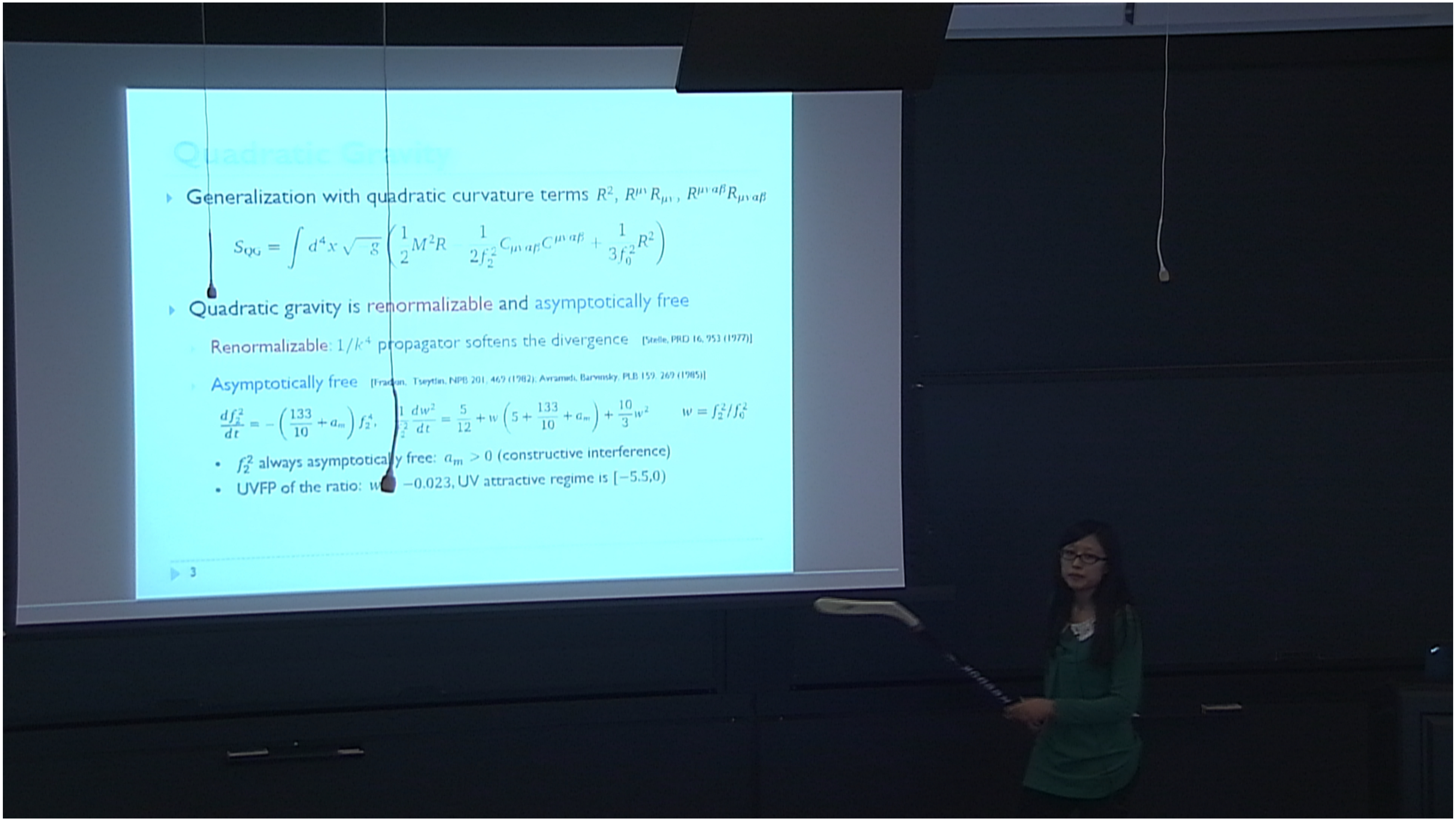
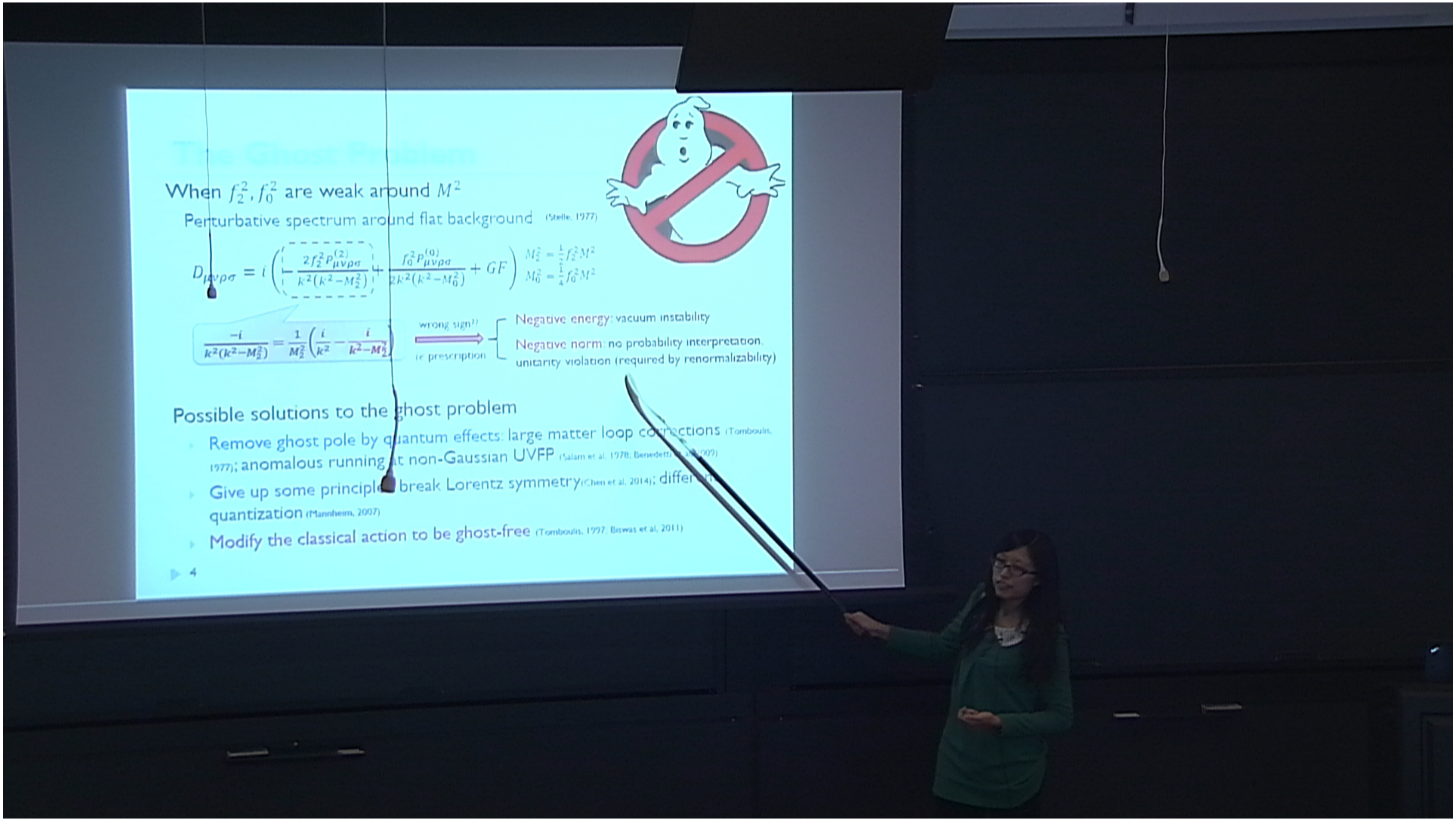

Quadratic Gravity
Generalization with quadratic curvature terms R^2, $R^{\mu\nu}R_{\mu\nu}$, $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$
$S_{QG} = \int d^4x \sqrt{-g}\left(\frac{1}{2}M^2R - \frac{1}{2f_2^2}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} + \frac{1}{3f_0^2}R^2\right)$
Quadratic gravity is renormalizable and asymptotically free
Renormalizable: $1/k^4$ propagator softens the divergence [Stelle, PRD 16, 953 (1977)]
Asymptotically free [Fradkin, Tseytlin, NPB 201, 469 (1982); Avramidi, Barvinsky, PLB 159, 269 (1985)]
$\frac{df_2^2}{dt} = -\left(\frac{133}{10} + a_m\right)f_2^4, \quad \frac{1}{f_2^2}\frac{dw^2}{dt} = \frac{5}{12} + w\left(5 + \frac{133}{10} + a_m\right) + \frac{10}{3}w^2 \qquad w = f_2^2/f_0^2$
f_2^2 always asymptotically free: $a_m > 0$ (constructive interference)
UVFP of the ratio: w -0.023, UV attractive regime is $[-5.5, 0)$
3

The Ghost Problem

When f_2^2, f_0^2 are weak around M^2

Perturbative spectrum around flat background (Stelle, 1977)

$$D_{\mu\nu\rho\sigma} = i\left(-\frac{2f_2^2 P^{(2)}_{\mu\nu\rho\sigma}}{k^2(k^2-M_2^2)} + \frac{f_0^2 P^{(0)}_{\mu\nu\rho\sigma}}{2k^2(k^2-M_0^2)} + GF\right) \quad \begin{matrix} M_2^2 = \frac{1}{2} f_2^2 M^2 \\ M_0^2 = \frac{1}{4} f_0^2 M^2 \end{matrix}$$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$\frac{-i}{k^2(k^2-M_2^2)} = \frac{1}{M_2^2}\left(\frac{i}{k^2} - \frac{i}{k^2-M_2^2}\right)$
wrong sign!!
$i\epsilon$ prescription
Negative energy: vacuum instability
Negative norm: no probability interpretation, unitarity violation (required by renormalizability)
Possible solutions to the ghost problem
Remove ghost pole by quantum effects: large matter loop corrections (Tomboulis, 1977); anomalous running at non-Gaussian UVFP (Salam et al. 1978; Benedetti
Give up some principle break Lorentz symmetry (Chen et al. 2014);
quantization (Mannheim, 2007)
Modify the classical action to be ghost-free (Tomboulis, 1997; Biswas et al. 2011)
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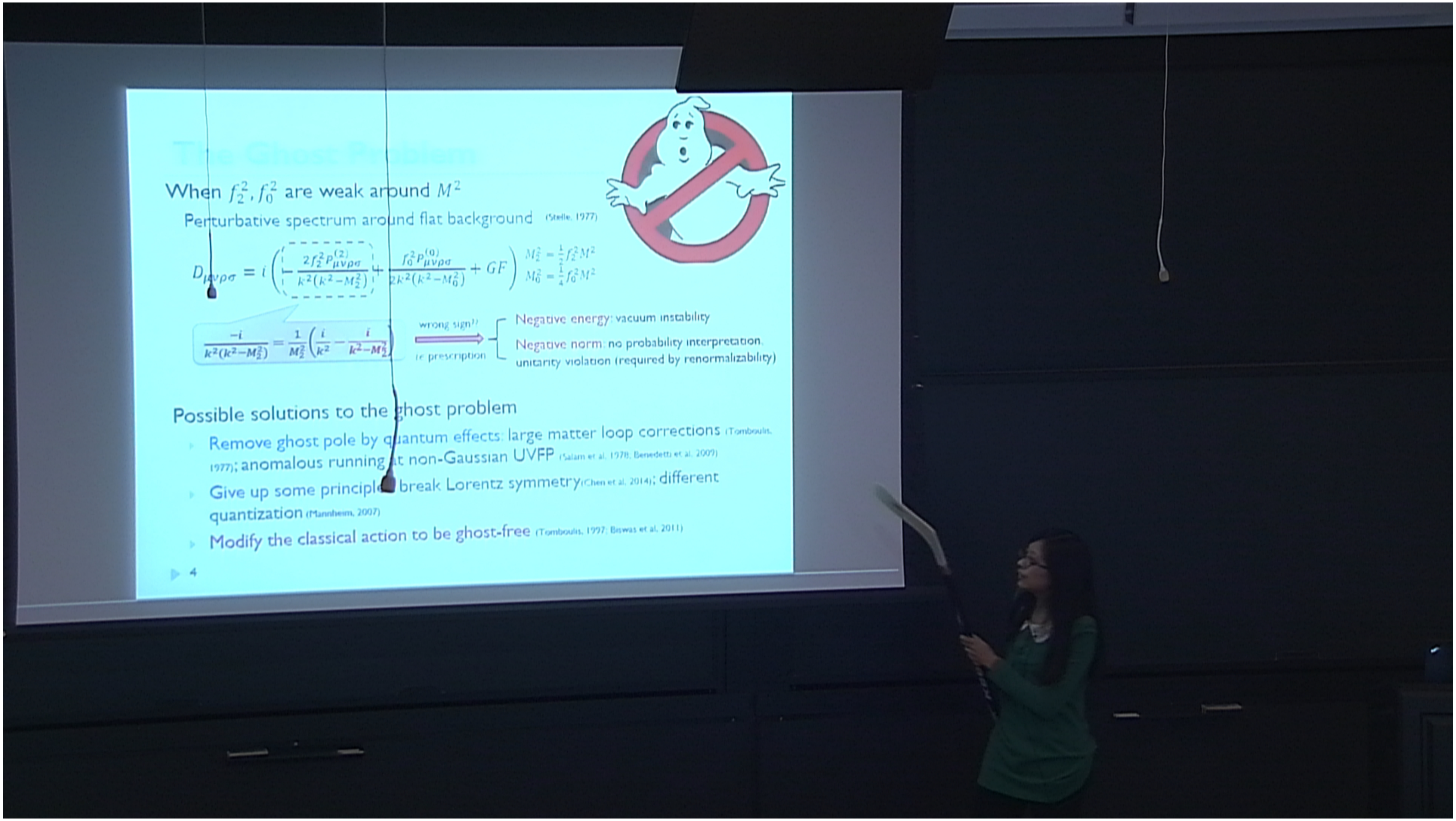
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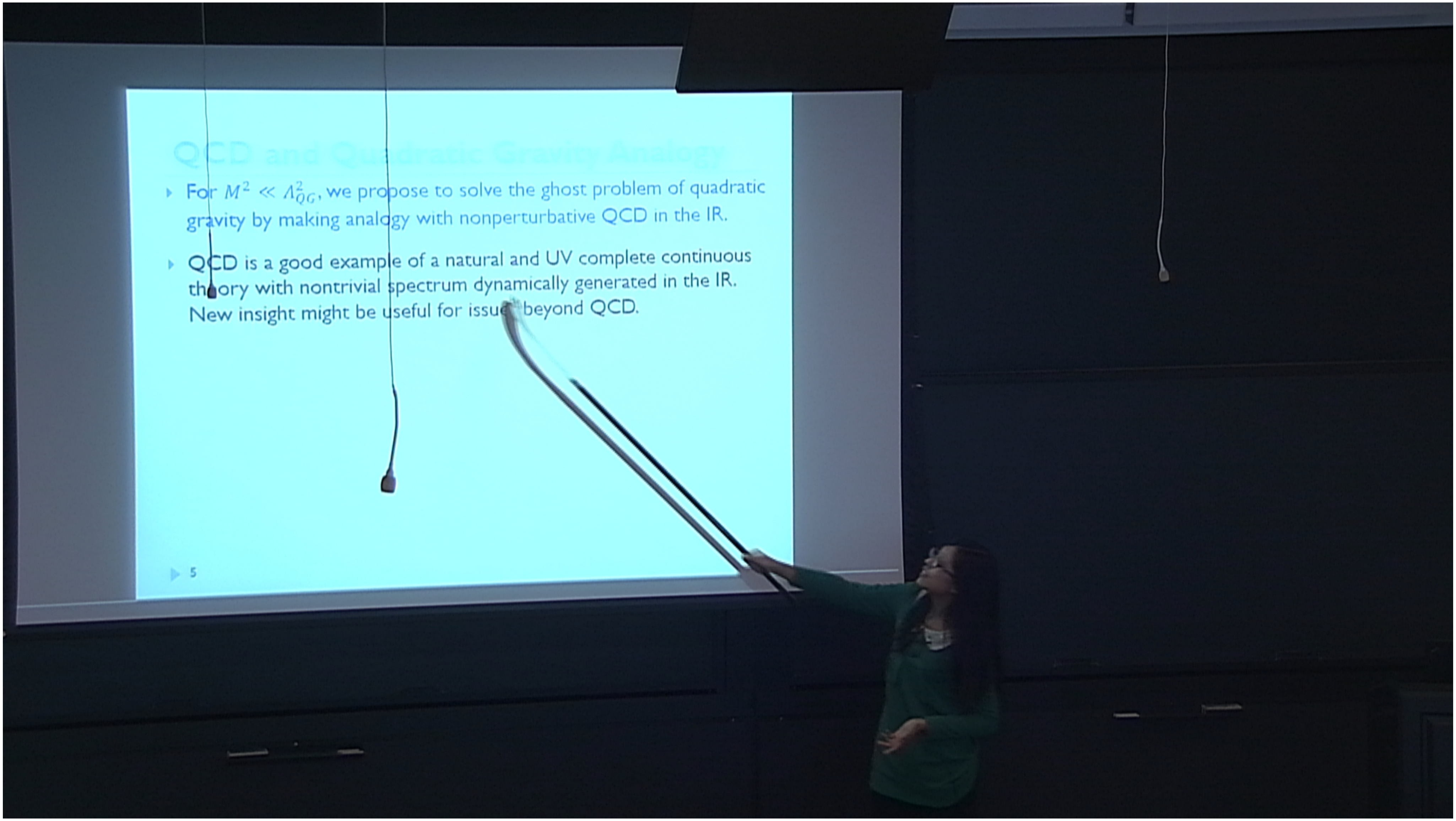
QCD and Quadratic Gravity Analogy
For $M^2 \ll \Lambda_{QG}^2$, we propose to solve the ghost problem of quadratic gravity by making analogy with nonperturbative QCD in the IR.
QCD is a good example of a natural and UV complete continuous theory with nontrivial spectrum dynamically generated in the IR. New insight might be useful for issues beyond QCD.
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- QCD is a good example of a natural and UV complete continuous theory with nontrivial spectrum dynamically generated in the IR. New insight might be useful for issues beyond QCD.
- Why QCD and quadratic gravity analogy?

QCD	Quadratic Gravity
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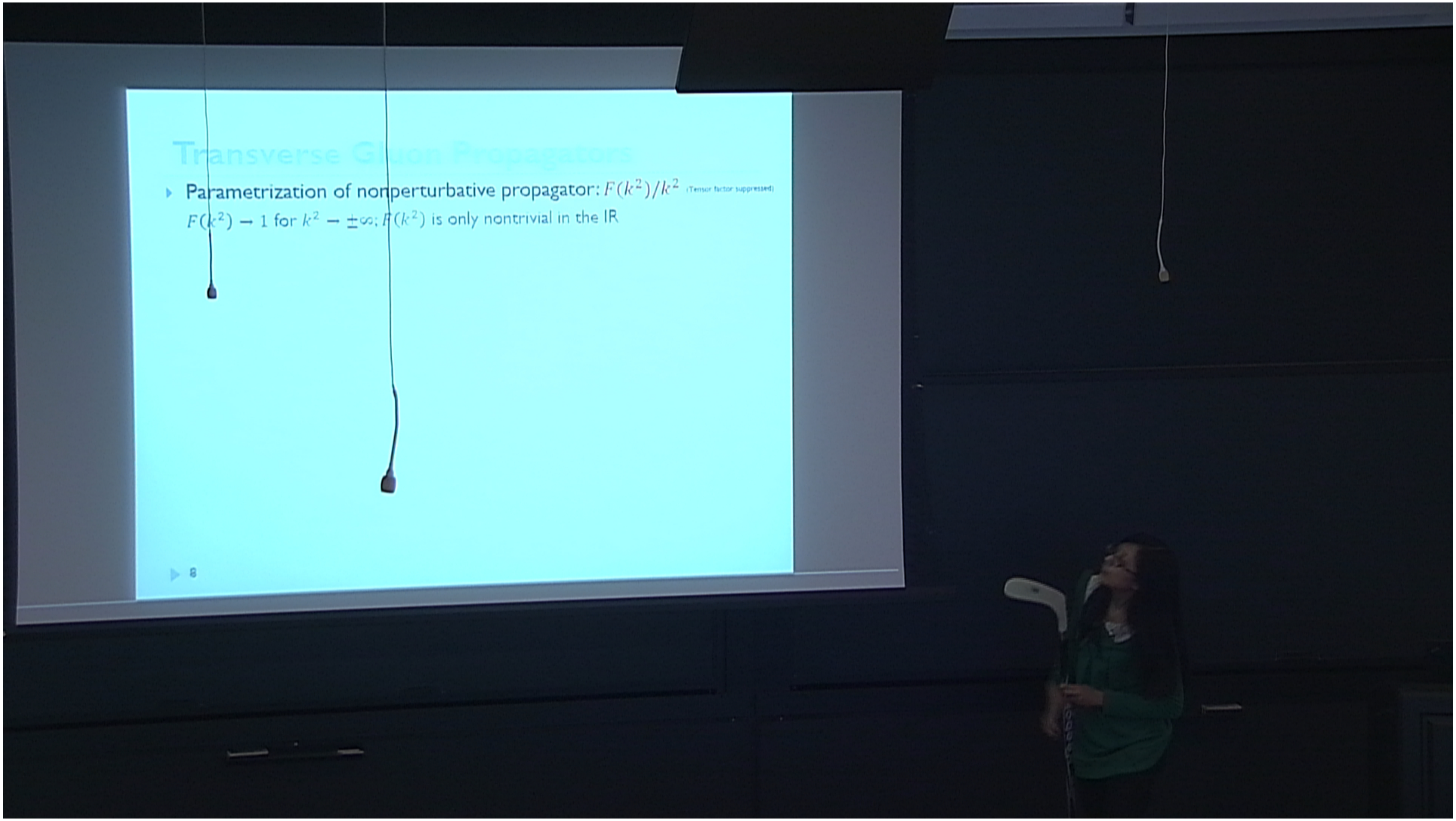
Transverse gluon removed from physical spectrum nonperturbatively	The ghost pole removed from physical spectrum nonperturbatively
Color singlet states described by Chiral Lagrangian in the IR	Massless graviton described by Einstein theory in the IR with $M_{pl} \sim \Lambda_{QG}$


Transverse Gluon Propagators
Parametrization of nonperturbative propagator: $F(k^2)/k^2$ (Tensor factor suppressed)
$F(k^2) \to 1$ for $k^2 \to \pm\infty$; $F(k^2)$ is only nontrivial in the IR
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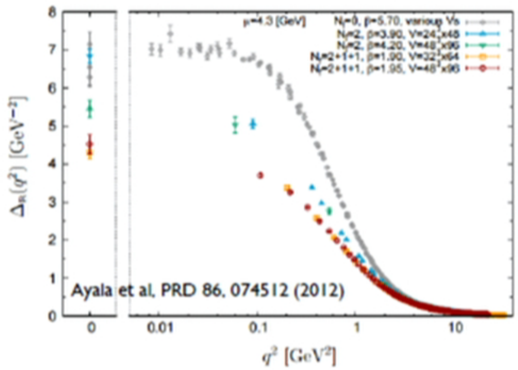

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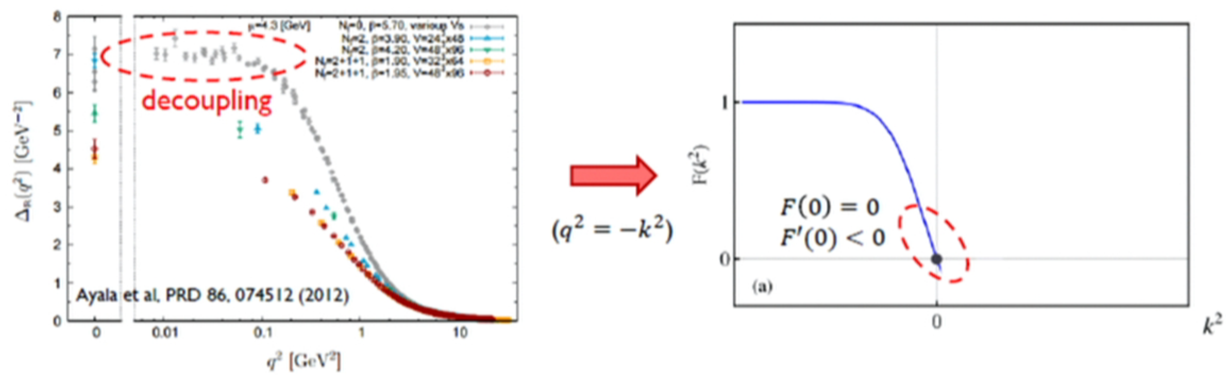

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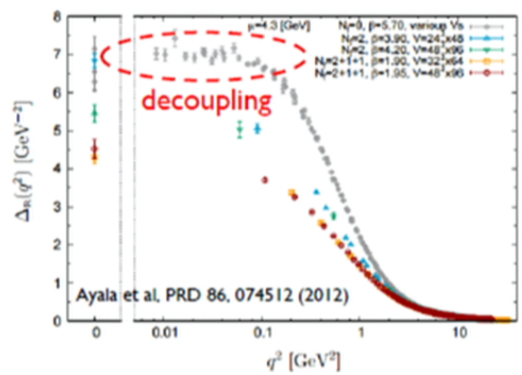

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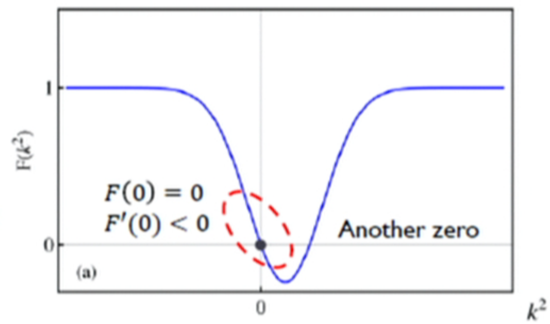
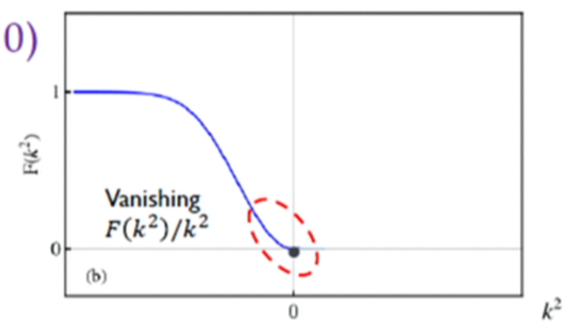


- Lattice data in Coulomb gauge ($\partial_i A^i = 0$)

  Evidence less clear, but Gribov picture and some lattice results suggest vanishing $F(k^2)/k^2$ in deep IR



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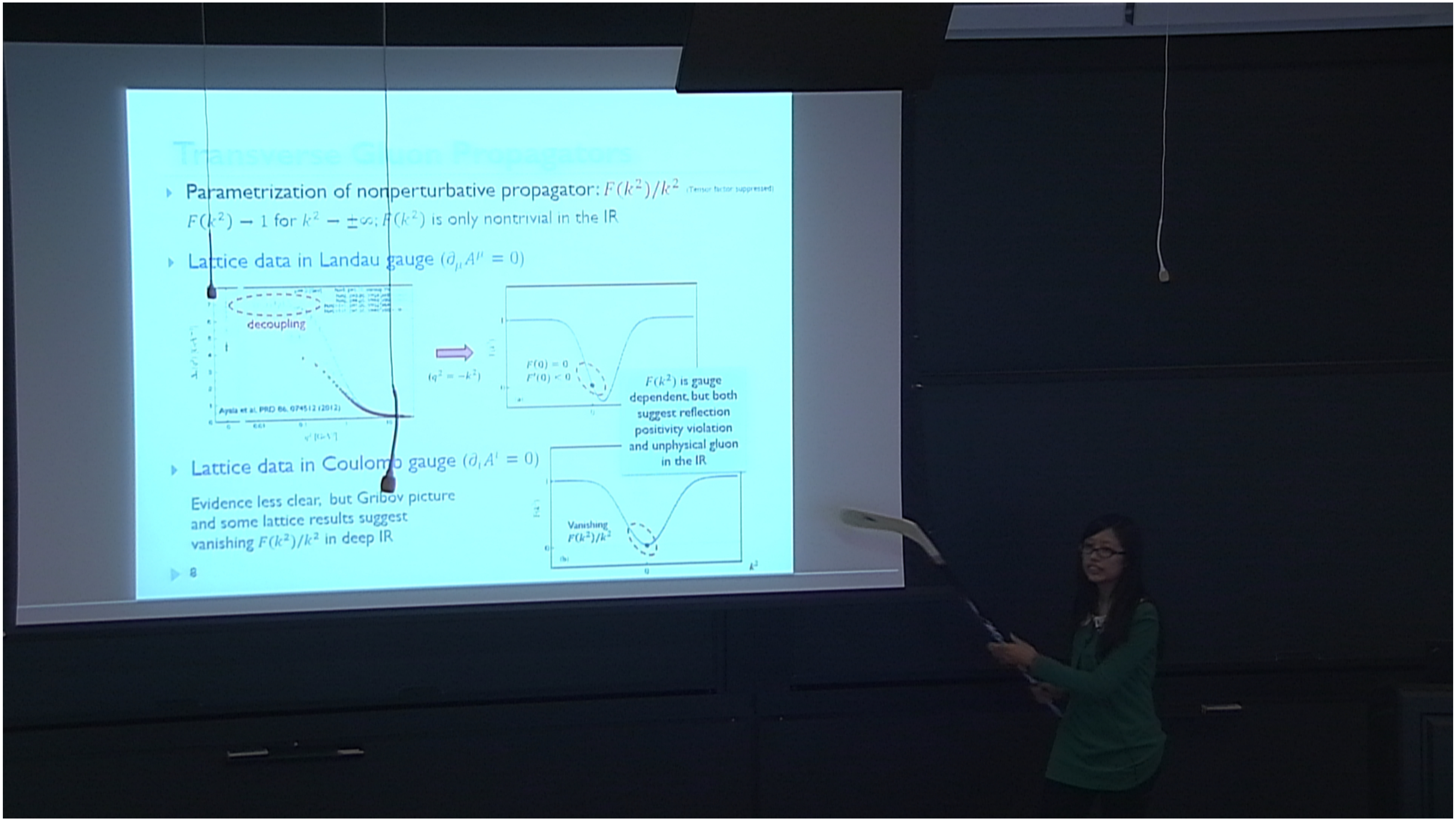

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Lattice data in Landau gauge $(\partial_\mu A^\mu = 0)$
decoupling
Ayala et al, PRD 86, 074512 (2012)
$(q^2 = -k^2)$
$F(0) = 0$
$F'(0) < 0$
$F(k^2)$ is gauge dependent, but both suggest reflection positivity violation and unphysical gluon in the IR
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Vanishing $F(k^2)/k^2$
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Graviton Propagators ($M^2 = 0$)

- First focus on pure quadratic gravity (study propagator on flat background)
- Parametrization of nonperturbative propagator: $-G(k^2)/k^4$ (Tensor factor suppressed)
- Assume that nonperturbative effects in quadratic gravity operate in a way similar to QCD. Consider the same two possibilities for $G(k^2)$ as found for $F(k^2)$. Plot $-G(k^2)/k^4$.

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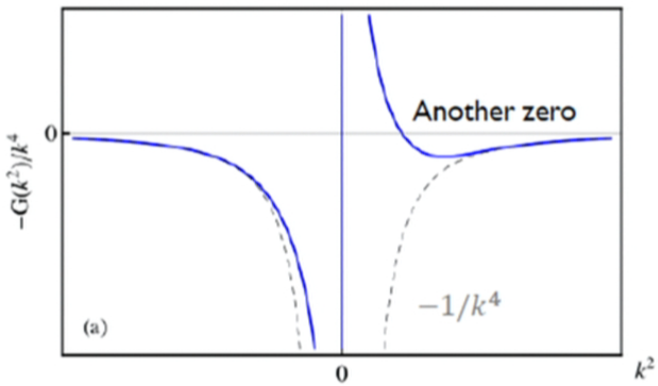
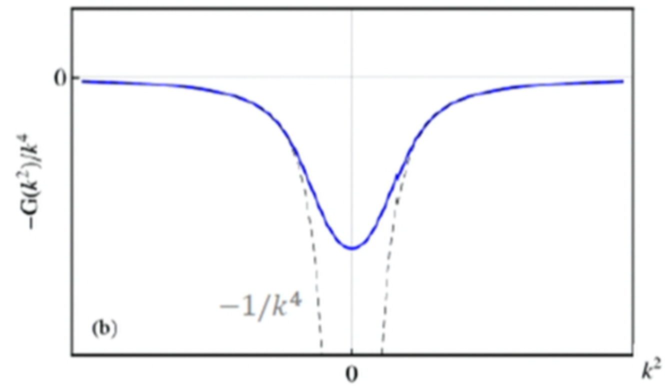


$-1/k^4$ pole softened to a $1/k^2$ pole with positive sign (massless state). **The ghost is removed by the strong gravity!**



The propagator develops a mass gap, no propagating graviton. More like QCD.

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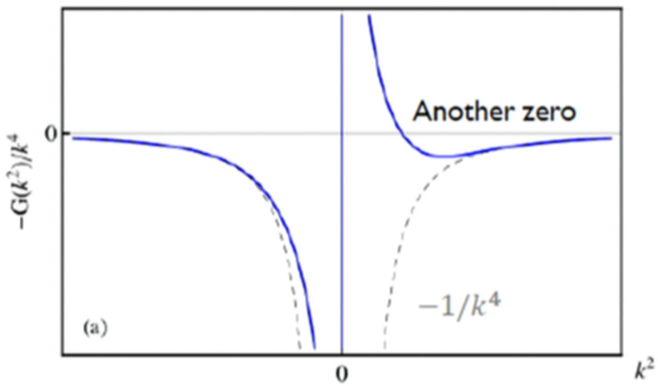

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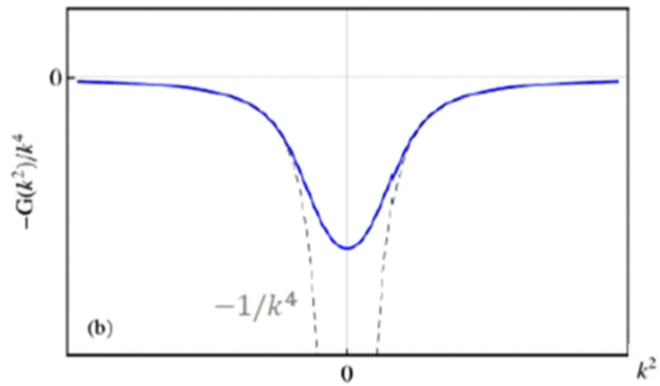

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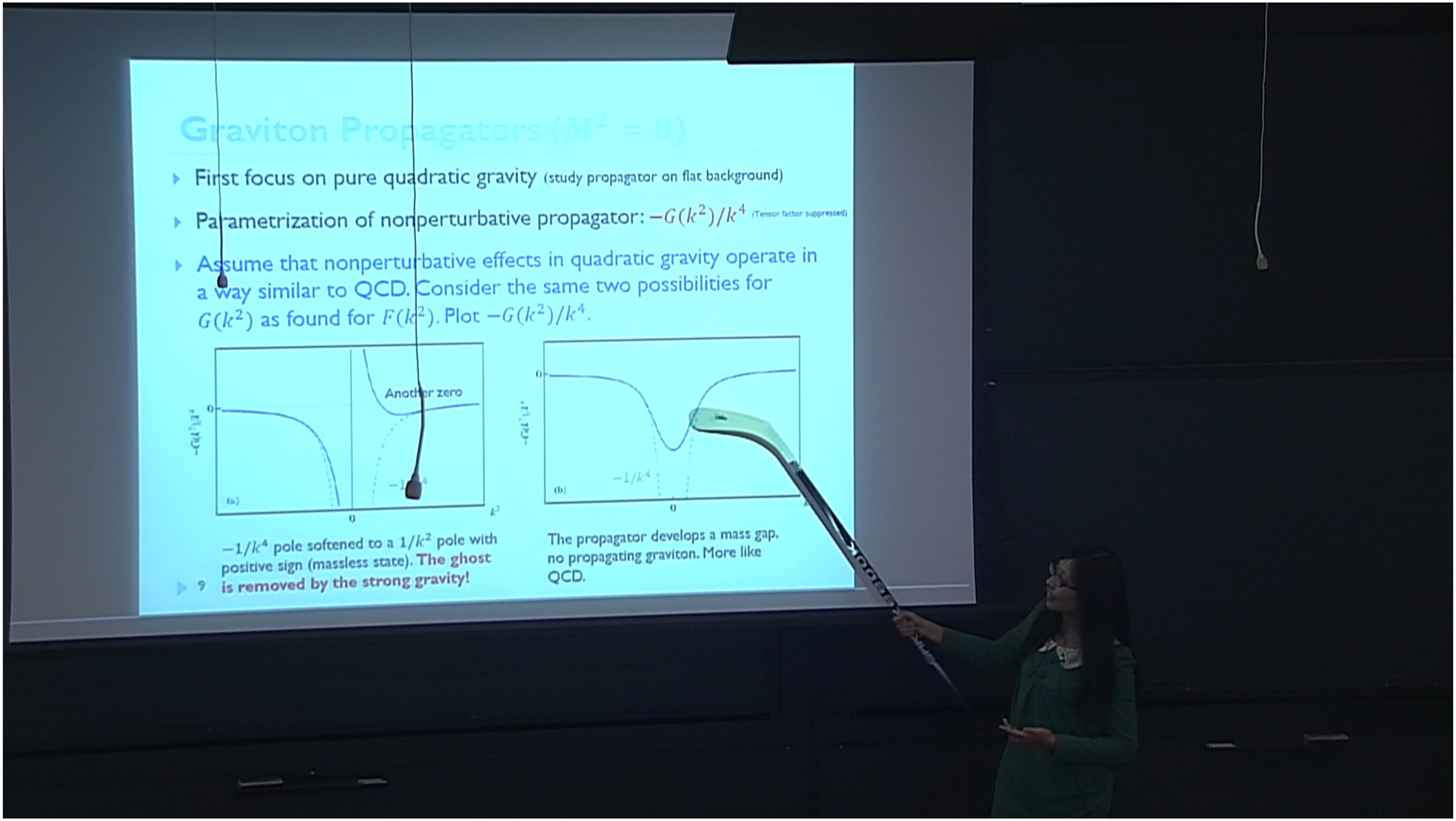

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Another zero
(a)
(b)
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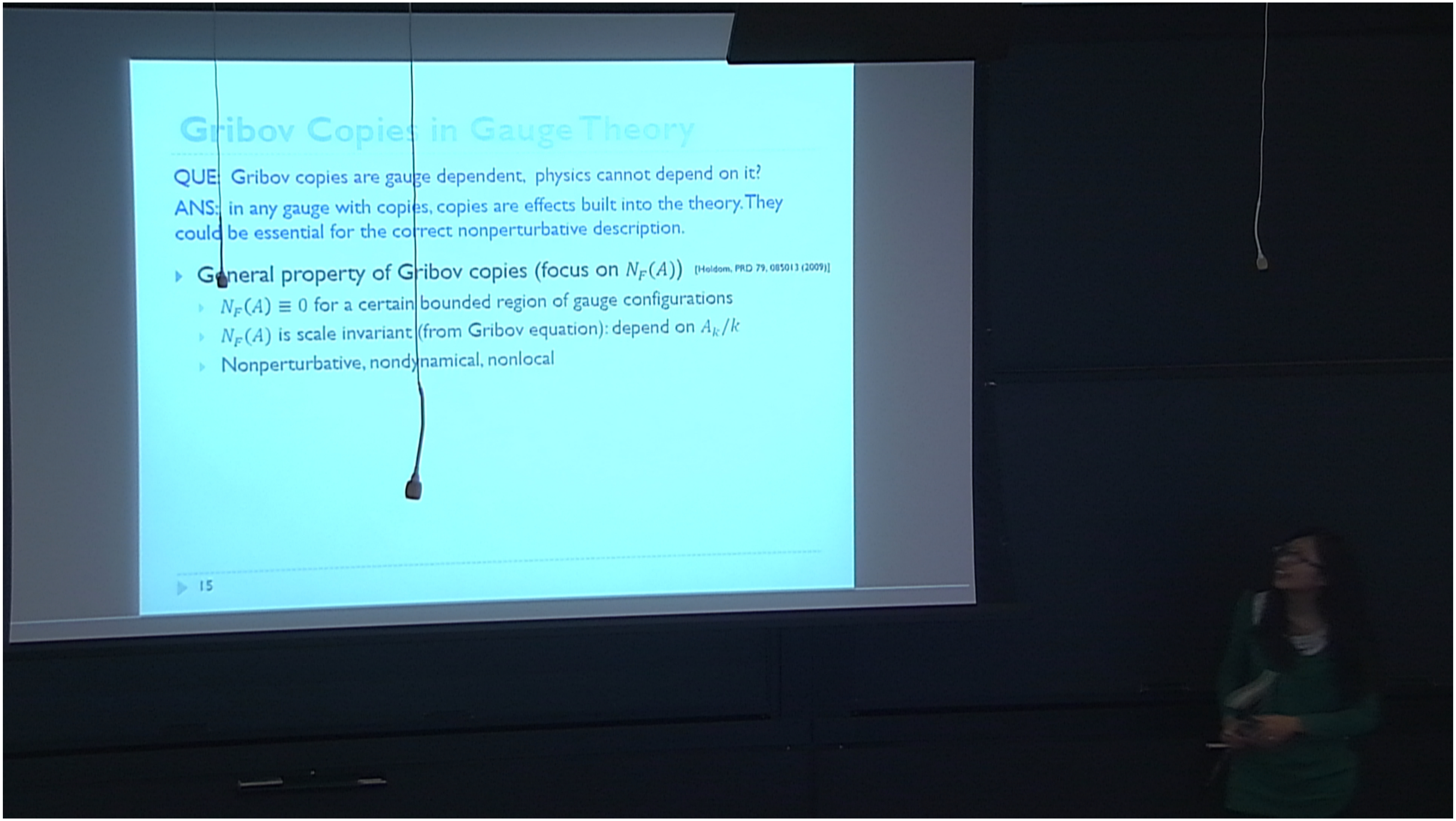
Gribov Copies in Gauge Theory
QUE: Gribov copies are gauge dependent, physics cannot depend on it?
ANS: in any gauge with copies, copies are effects built into the theory. They could be essential for the correct nonperturbative description.
General property of Gribov copies (focus on $N_F(A)$) [Holdom, PRD 79, 085013 (2009)]
$N_F(A) \equiv 0$ for a certain bounded region of gauge configurations
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Nonperturbative, nondynamical, nonlocal
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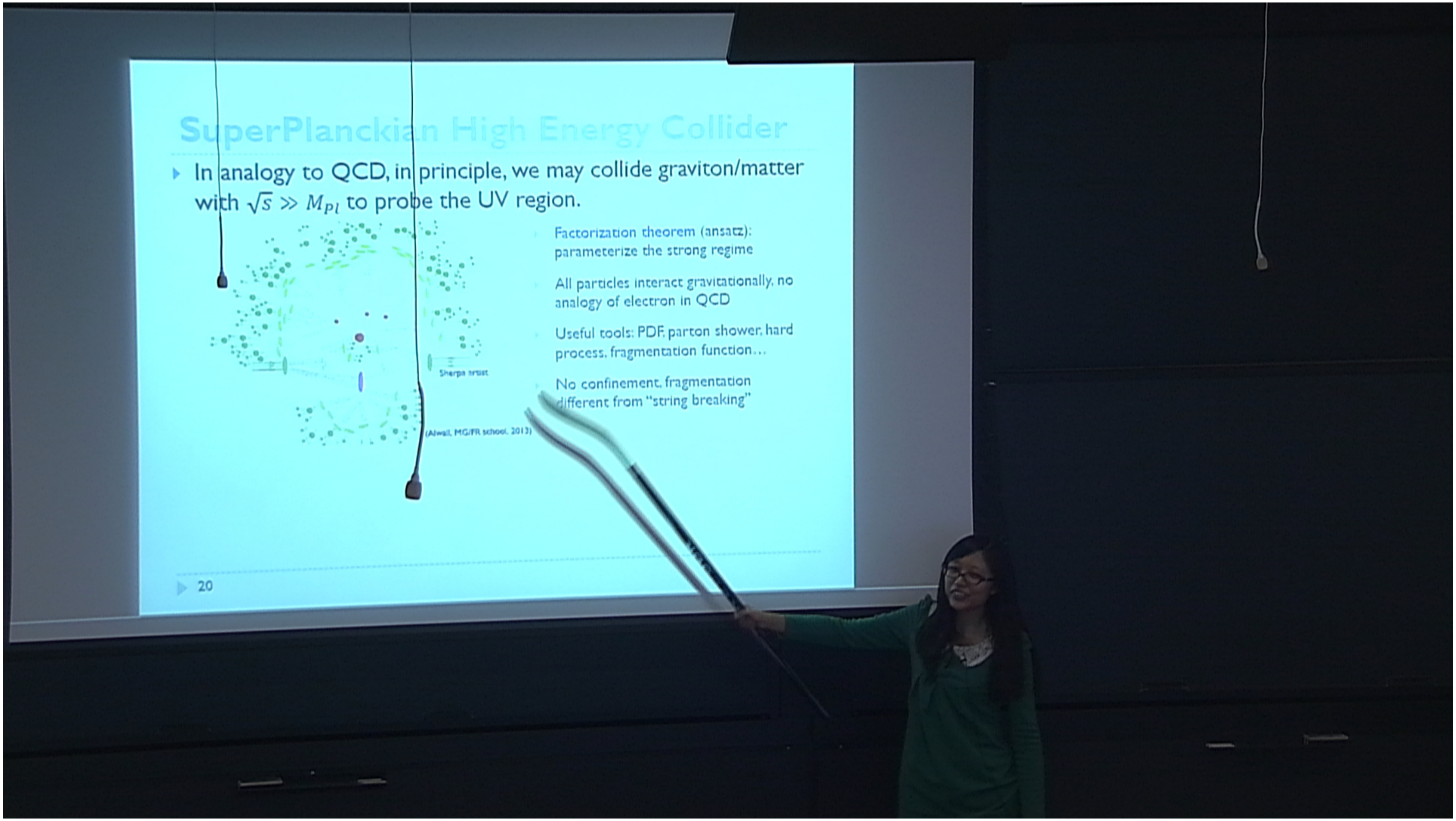
  - Nonperturbative, nondynamical, nonlocal
- Impact of $1/(1+N_F(A))$ on gluon propagator $F(k^2)$ factor
  - The effects of copies turn on at the strong scale Λ_{QCD}. Copies are important only if $A_k^2 \gtrsim k^2/\Lambda_{QCD}^4$ (typical size $A_k^2 \sim 1/k^2$).
  - At high k^2, exponentially small corrections, $F(k^2) \sim 1 + O(exp(-k^4/\Lambda^4))$
  - In deep IR, easily sample configurations above critical value, $N_F(A)$ grows fast, $F(k^2)$ suppressed (explicit form sensitive to how $N_F(A)$ grows).


SuperPlanckian High Energy Collider
In analogy to QCD, in principle, we may collide graviton/matter with $\sqrt{s} \gg M_{Pl}$ to probe the UV region.
Factorization theorem (ansatz): parameterize the strong regime
All particles interact gravitationally, no analogy of electron in QCD
Useful tools: PDF, parton shower, hard process, fragmentation function…
No confinement, fragmentation different from "string breaking"
Sherpa artist
(Alwall, MG/FR school, 2013)
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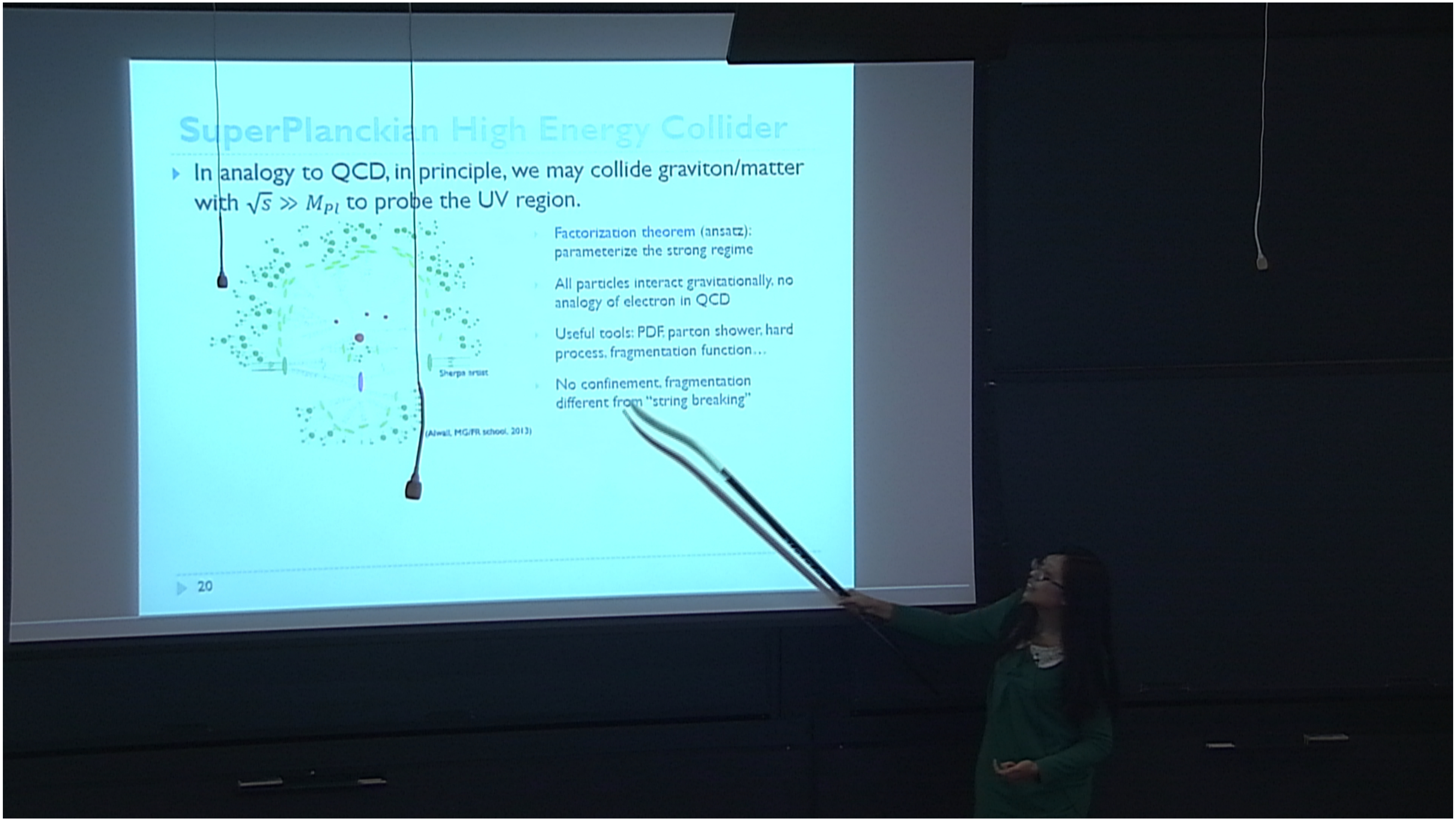

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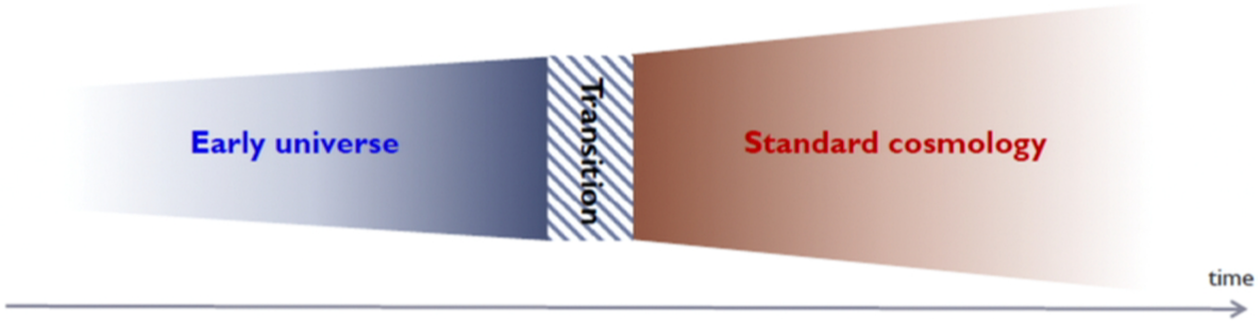

- Currently observations just probe effective theory region, not much to say even about inflation.
- For a circumstance characterized by high T or large curvature, it is described by perturbative quadratic gravity.
- Solution to initial singularity in quadratic gravity. For FRW metric, determined by R^2 (small M^2R negligible).

Implication for Matter Sector

Motivate asymptotically free extension of the SM

- For the SM structure, there are UV Landau pole problems for U(1) gauge coupling and scalar quartic couplings

- Solve problems within matter sector
  - Stable Asymptotically Free Extensions (SAFEs) of the SM   Holdom, JR, Zhang, JHEP 1503, 028 (2015)
  - Non-Abelian gauge couplings drive yukawa and scalar quartic couplings asymptotically free $(4\pi)^2\beta_y = a_y y^3 - a_g g^2 y$ $(4\pi)^2\beta_\lambda = a_{\lambda\lambda}\lambda^2 - a_{\lambda g}\lambda g^2 + a_{gg} g^4$ $a_y, a_g, a_{\lambda\lambda}, a_{\lambda g}, a_{gg} > 0)$
  - For example, study $SU(N_A) \times SU(N_B)$ with one scalar (N_A, N_B). The simplest possibility is Pati-Salam model with one (4,2,1), SAFEs require $2n_F + n_f = 21$

Fields	Number	$SU(4)$	$SU(2)_L$	$SU(2)_R$
F_L	n_F	4	2	1
F_R	n_F	4	1	2
$f_{L,R}$	n_f	4	1	1
Φ	1	4	2	1

- For weak couplings in matter sector, strong region effects on the matching between UV and IR running are mild

Summary

- We propose to solve the ghost problem in quadratic gravity by non-perturbative effects in analogy with QCD. Einstein theory emerges in deep IR, describing the weakly interacting massless graviton.
- Both QCD and quadratic gravity are based on path integrals over space of orbits — similar nontrivial infrared effects are built in.
- Although strong gravity around M_{Pl} still unknown, the existence of UV asymptotically free region has interesting implications.
- Open questions…
  - Implication of no fundamental holographic picture??
  - Gravity on the lattice??
  - …