Title: Kinetic Terms in Massive gravity

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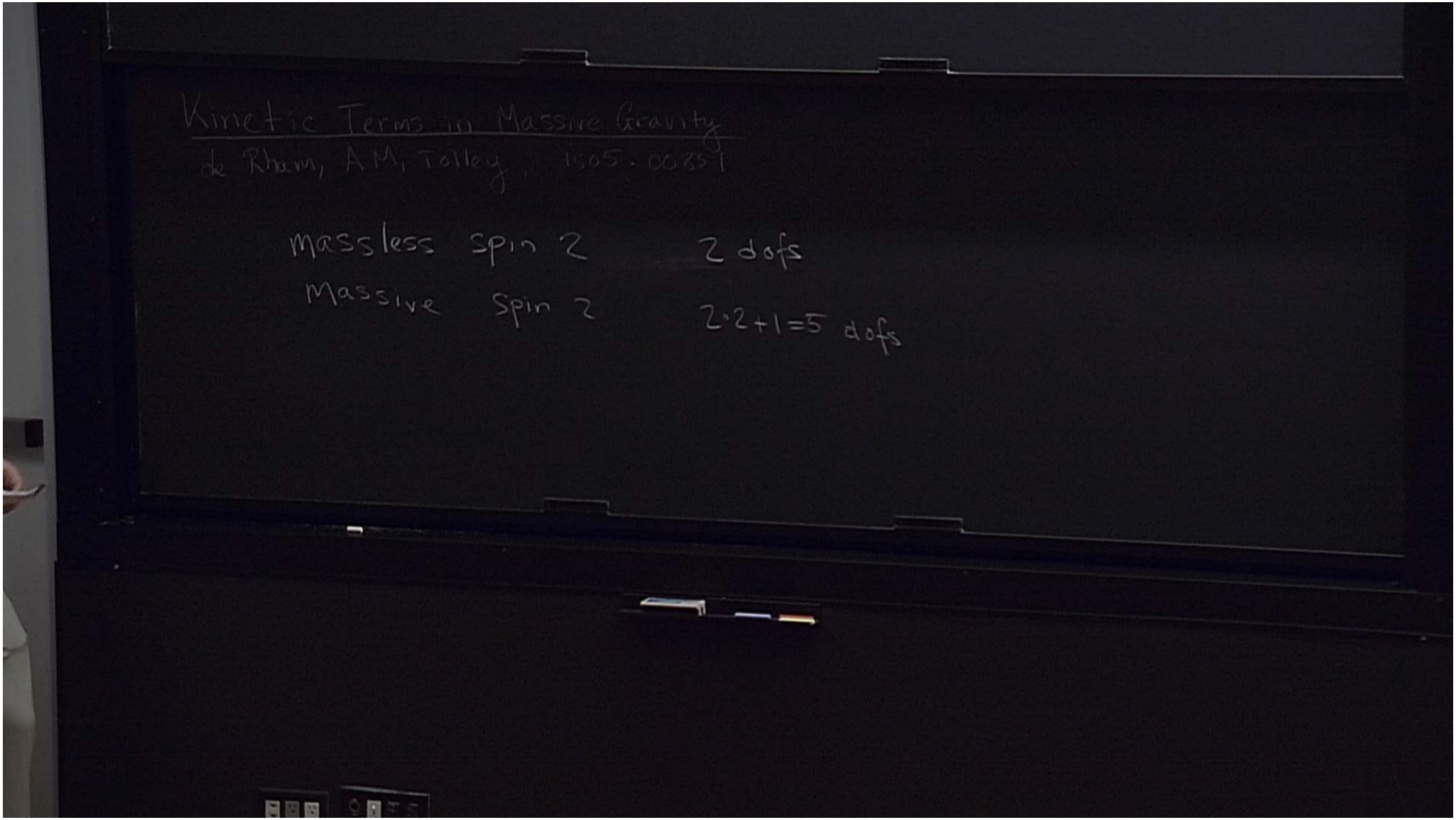
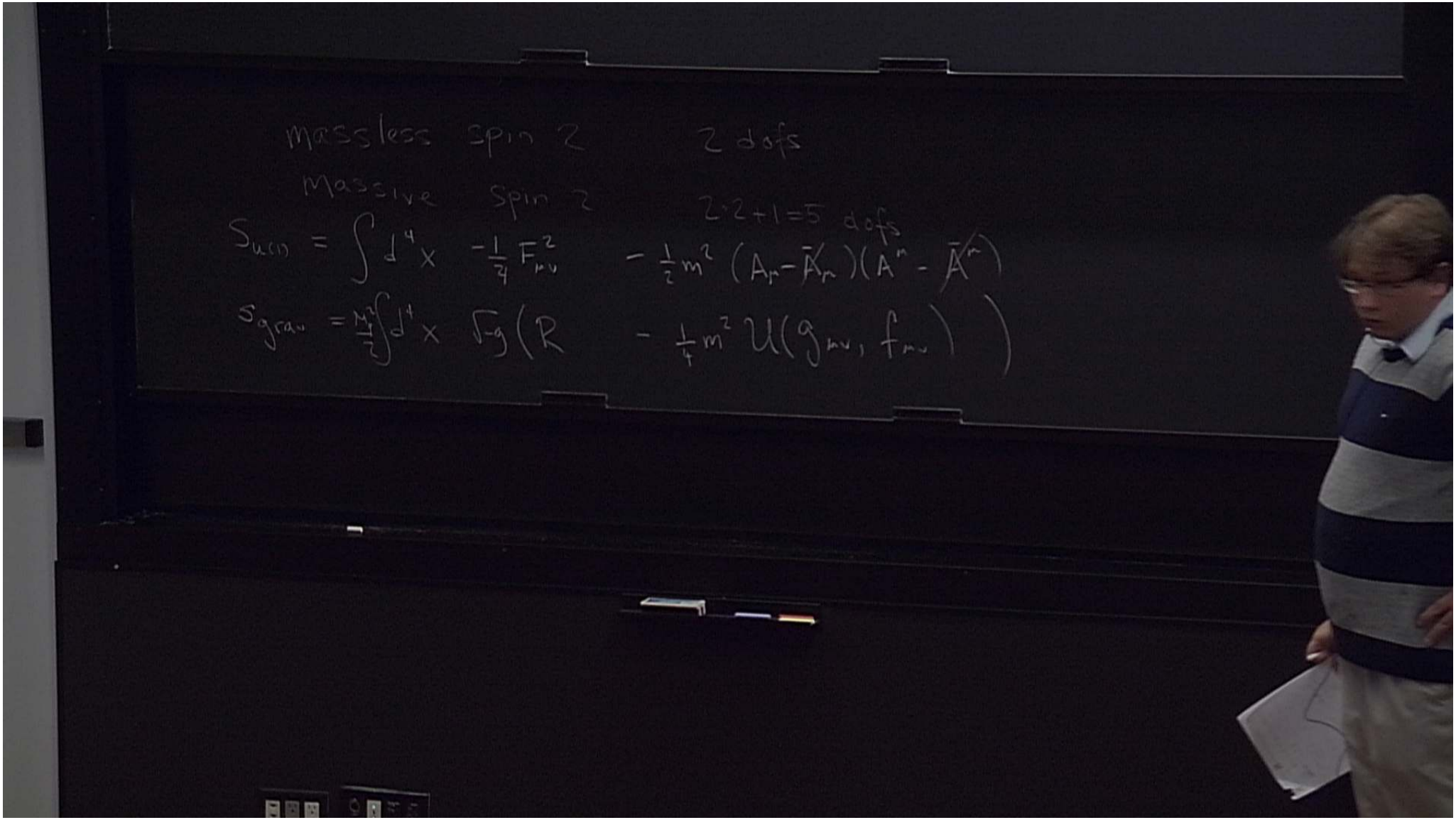
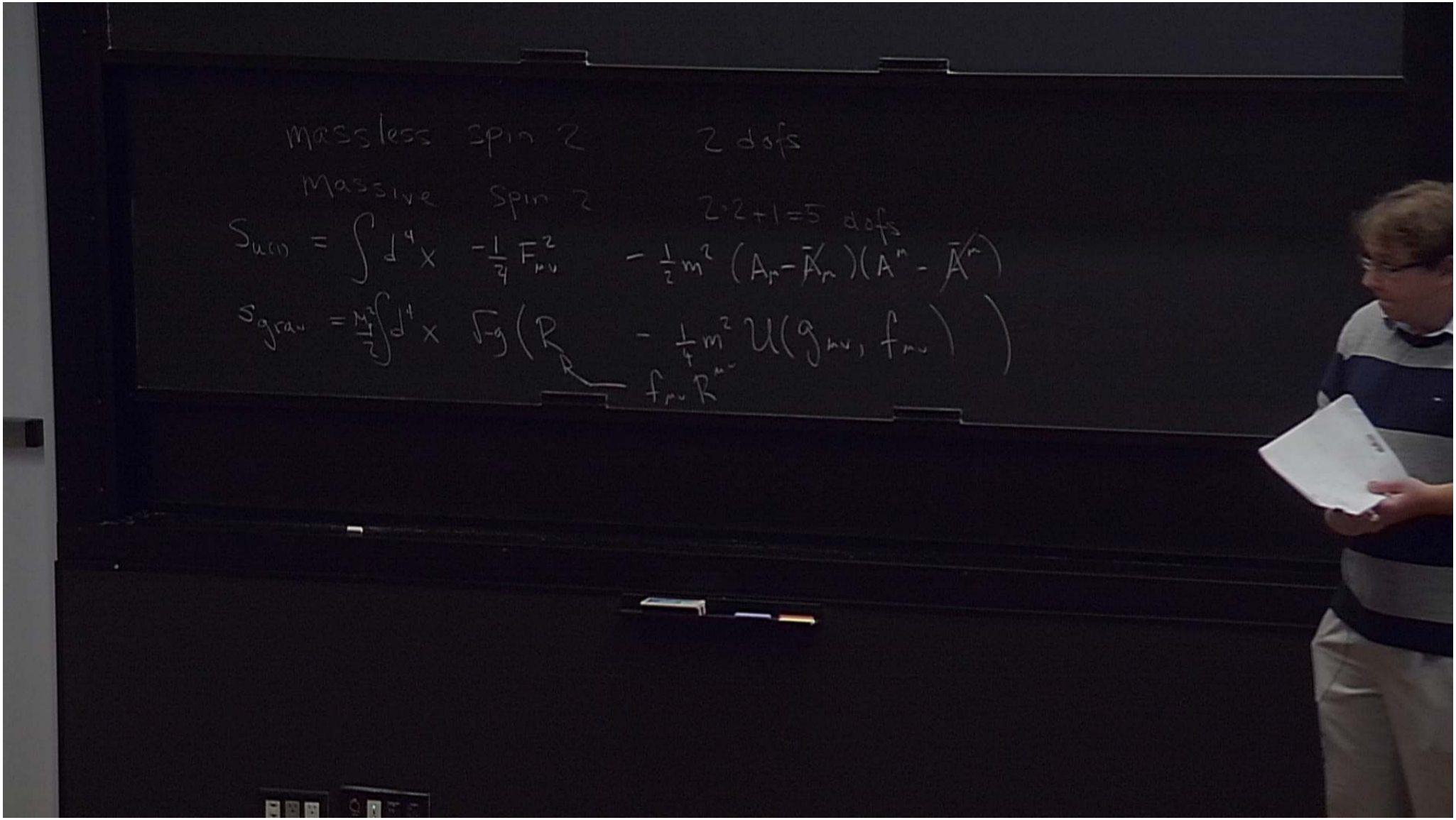
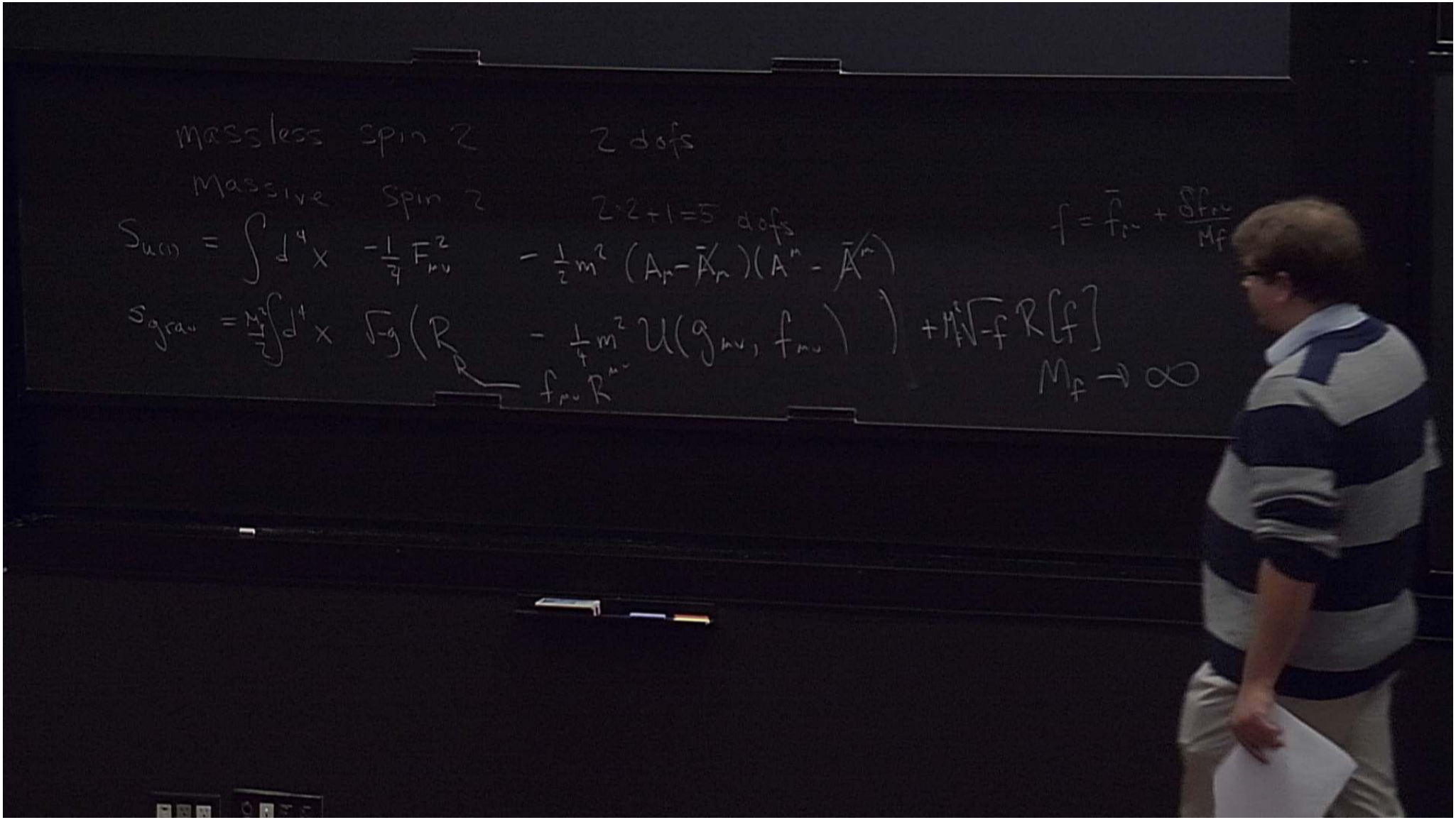
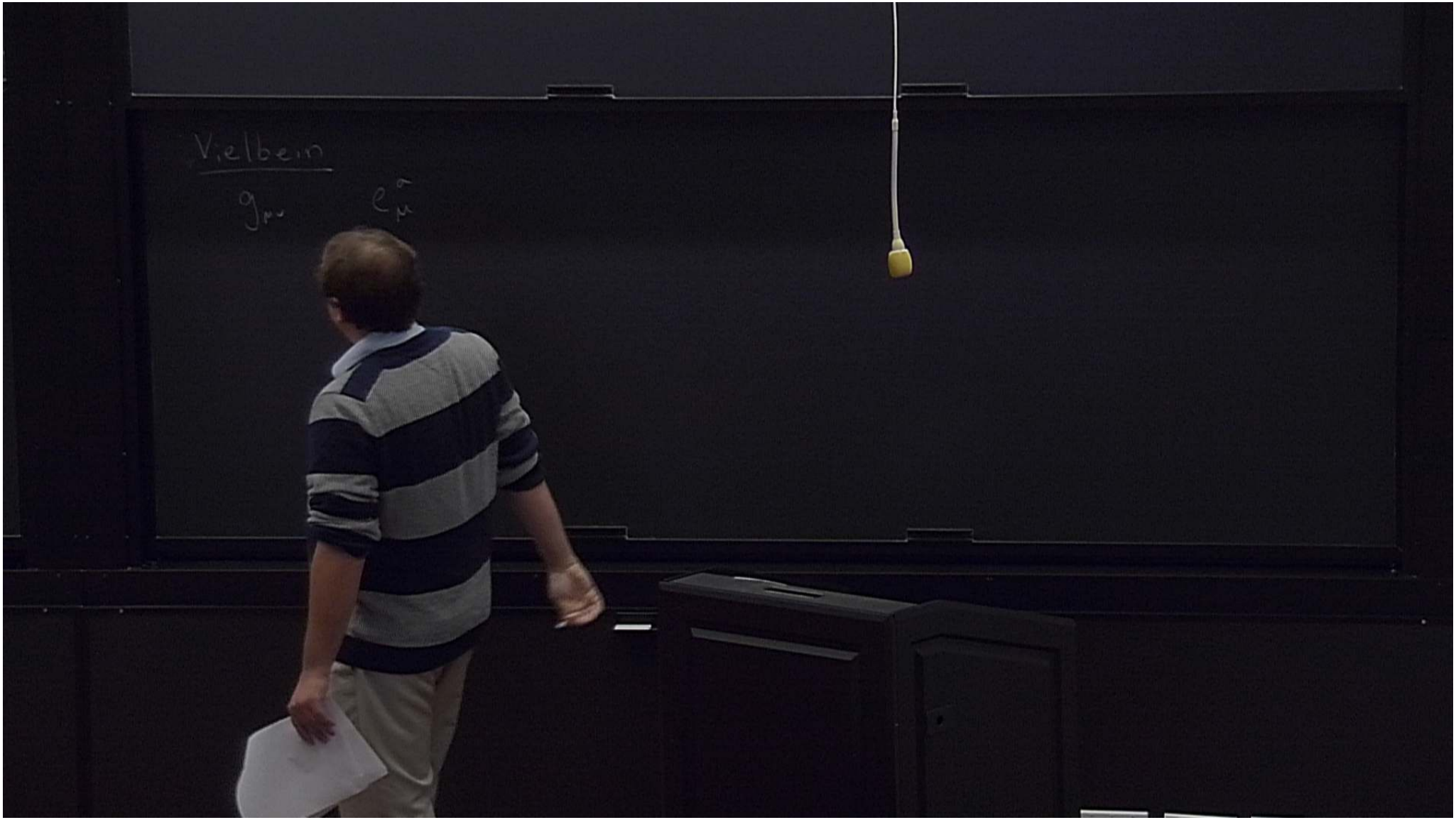
Abstract: Is the graviton a truly massless spin-2 particle, or can the graviton have a small mass? If the mass of the graviton is of order the Hubble scale today, it can potentially help to explain the observed cosmic acceleration. Previous attempts to study massive gravity have been spoiled by the fact that a generic potential for the graviton leads to an instability called the Boulware-Deser ghost. Recently, a special potential has been constructed which avoids this problem while maintaining Lorentz invariance. In this talk I will present recent arguments that suggest that the requirement of avoiding the Boulware-Deser ghost (or other degrees of freedom) is so powerful that the kinetic term for a massive graviton is fixed as well. In fact it must be exactly the same as in General Relativity. This is remarkable as we derive the structure of General Relativity on the basis of stability requirements, not on a symmetry principle. 

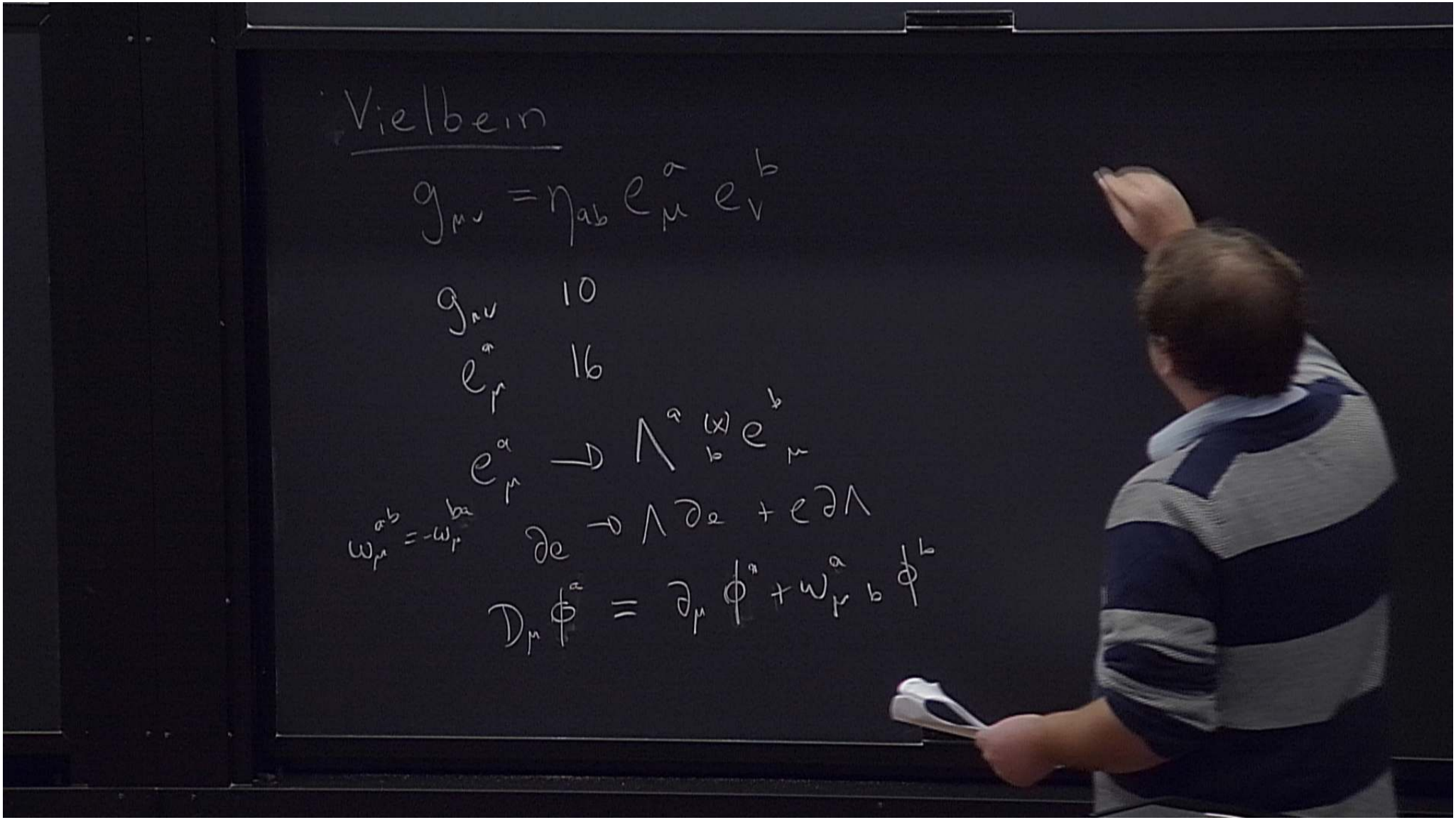

Kinetic Terms in Massive Gravity
de Rham, AM, Tolley, 1505.00351
massless spin 2 2 dofs
Massive Spin 2 2·2+1=5 dofs

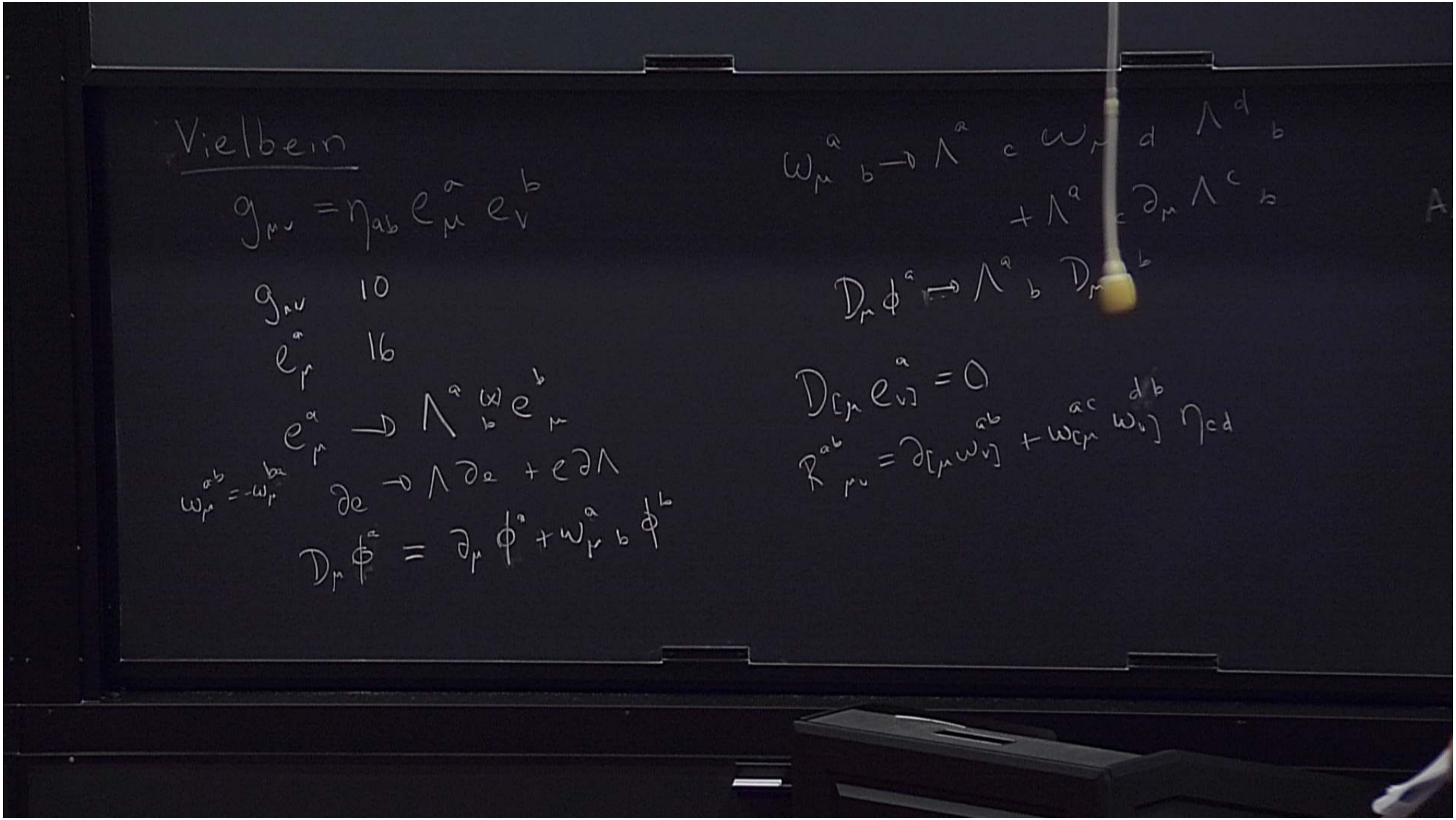

Massless Spin 2 2 dofs
Massive Spin 2 2·2+1=5 dofs
$S_{kin} = \int d^4x \; -\frac{1}{4} F_{\mu\nu}^2 \; - \frac{1}{2} m^2 (A_\mu - \bar{A}_\mu)(A^\mu - \bar{A}^\mu)$
$S_{grav} = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left( R \; - \frac{1}{4} m^2 \mathcal{U}(g_{\mu\nu}, f_{\mu\nu}) \right)$


Massless Spin 2 2 dofs
Massive Spin 2 2·2+1=5 dofs
$S_{mass} = \int d^4x \; -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} m^2 (A_\mu - \bar{A}_\mu)(A^\mu - \bar{A}^\mu)$
$S_{grav} = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left( R_R - \frac{1}{4} m^2 \mathcal{U}(g_{\mu\nu}, f_{\mu\nu}) \right)$
$f_{\mu\nu} R^{\mu\nu}$


Massless Spin 2 2 dofs
Massive Spin 2 2·2+1=5 dofs
$S_{kin} = \int d^4x \; -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} m^2 (A_\mu - \bar{A}_\mu)(A^\mu - \bar{A}^\mu)$
$f = \bar{f}_{\mu\nu} + \frac{\delta f_{\mu\nu}}{M_f}$
$S_{grav} = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left( R - \frac{1}{4} m^2 \mathcal{U}(g_{\mu\nu}, f_{\mu\nu}) \right) + M_f^2 \sqrt{-f} R[f]$
$M_f \to \infty$
$f_{\mu\nu} R^{\mu\nu}$


Vielbein


Vielbein
$g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu$
$g_{\mu\nu}$ 10
$e^a_\mu$ 16
$e^a_\mu \to \Lambda^a_b e^b_\mu$
$\omega_\mu^{ab} = -\omega_\mu^{ba}$
$\partial e \to \Lambda \partial e + e \partial \Lambda$
$D_\mu \phi^a = \partial_\mu \phi^a + \omega^a_{\mu b} \phi^b$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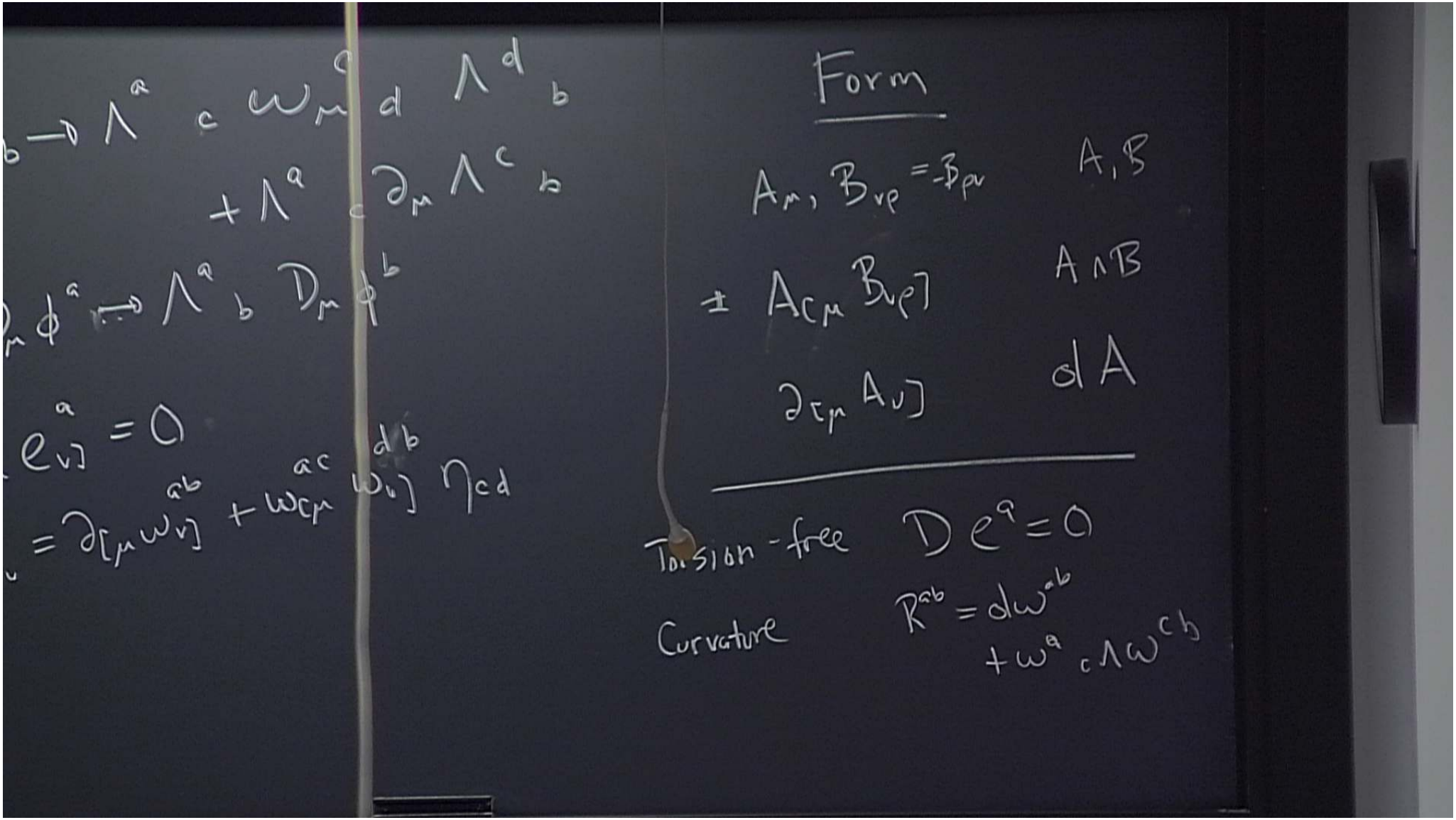
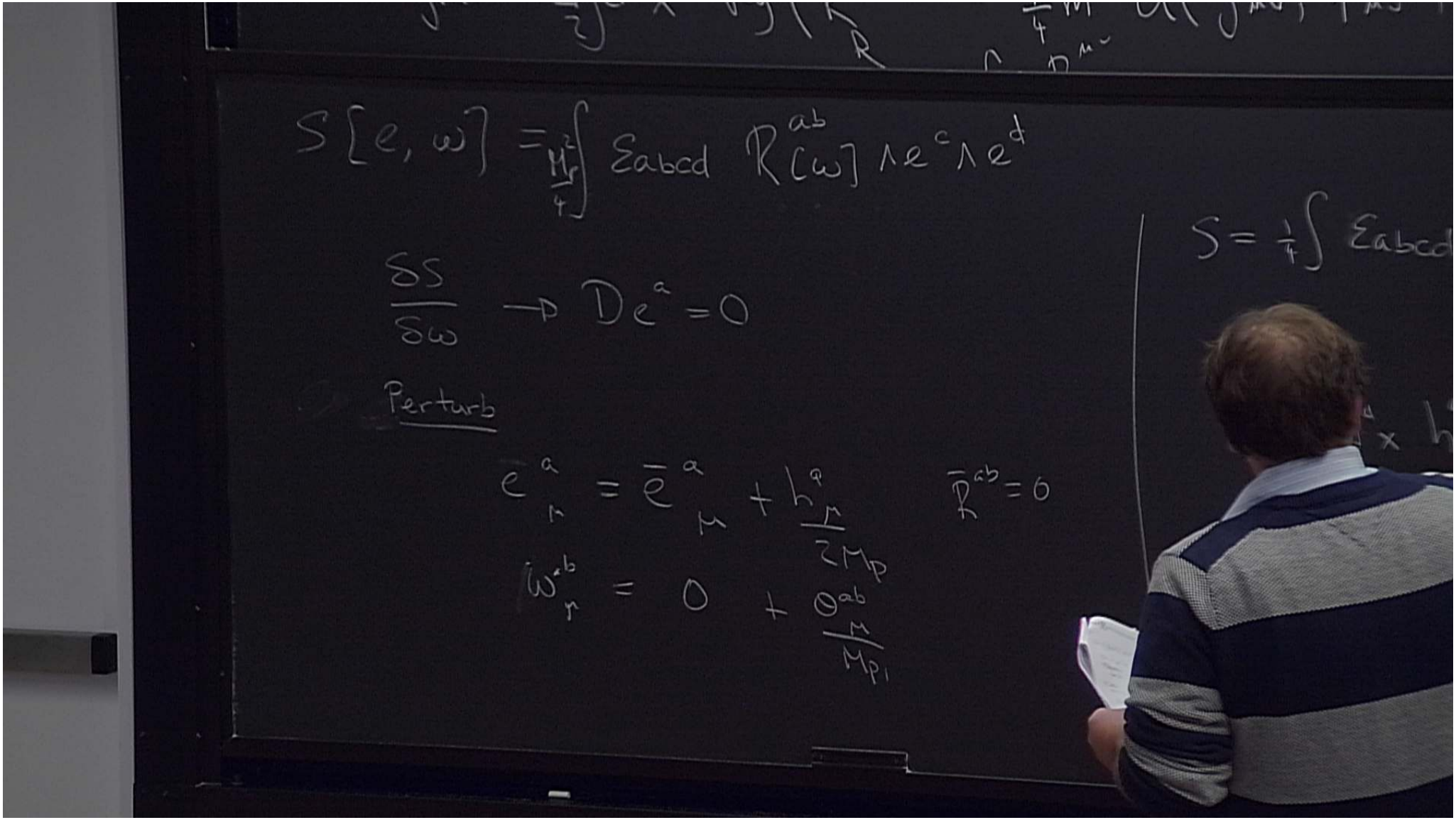
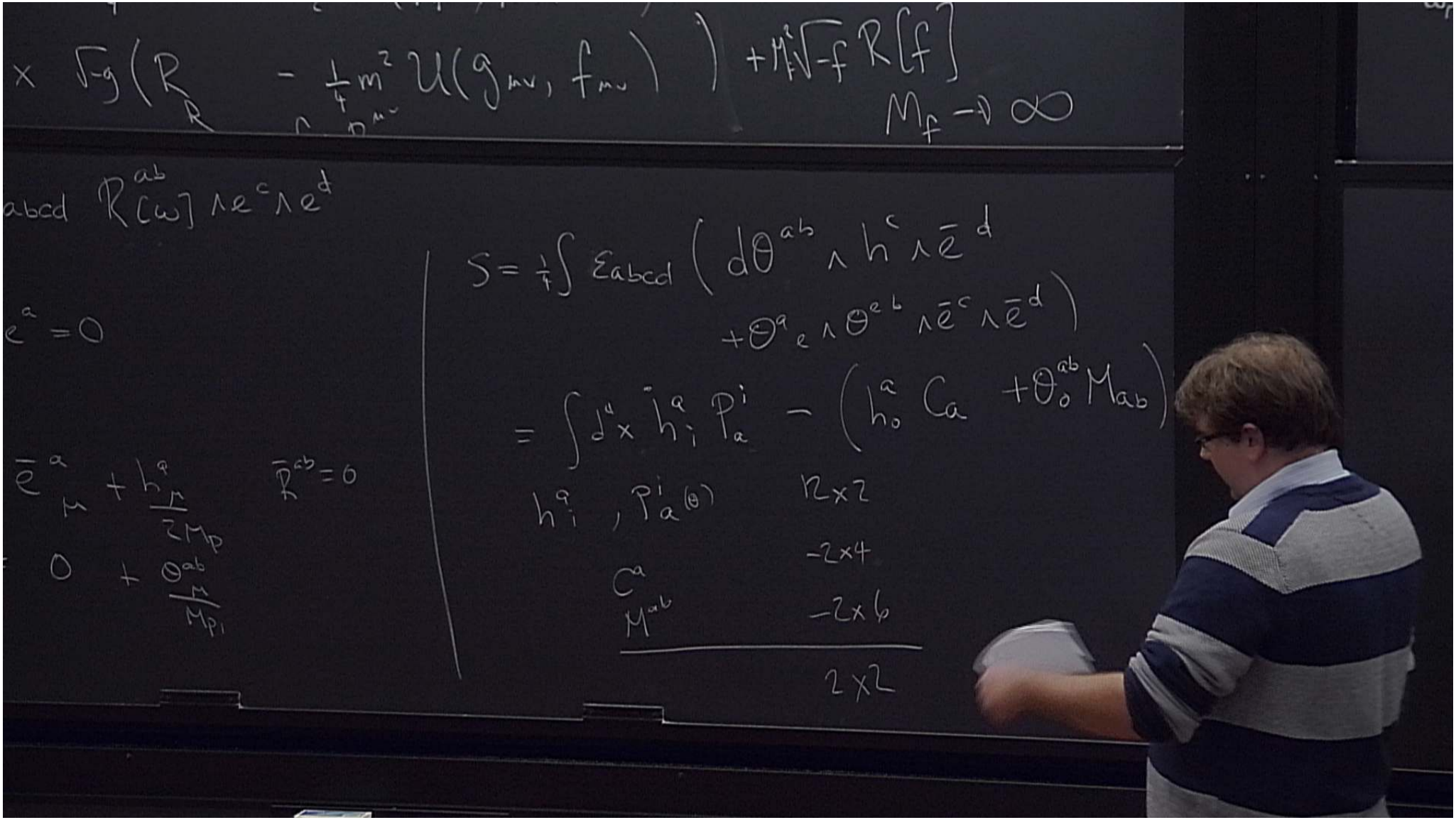
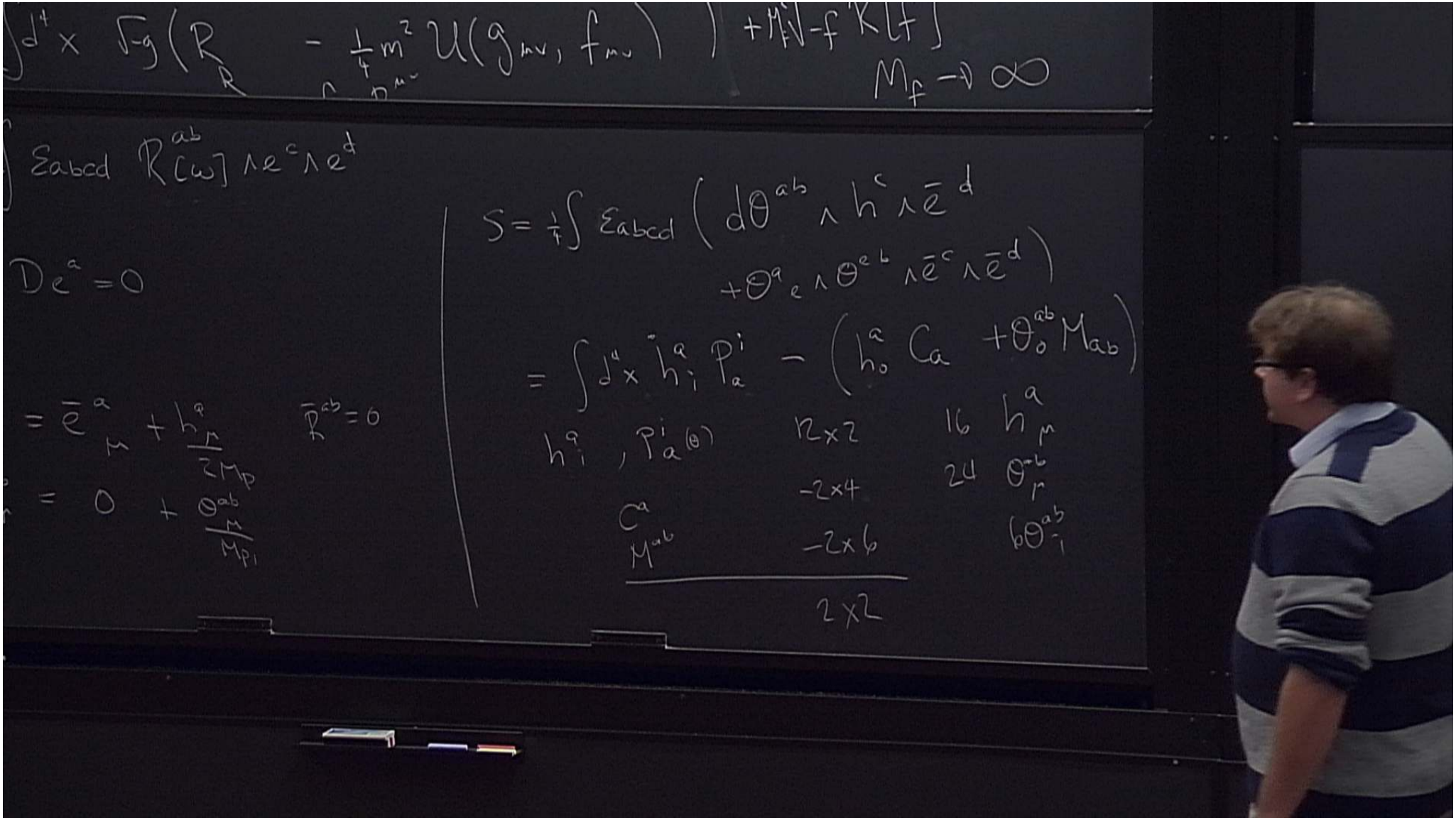
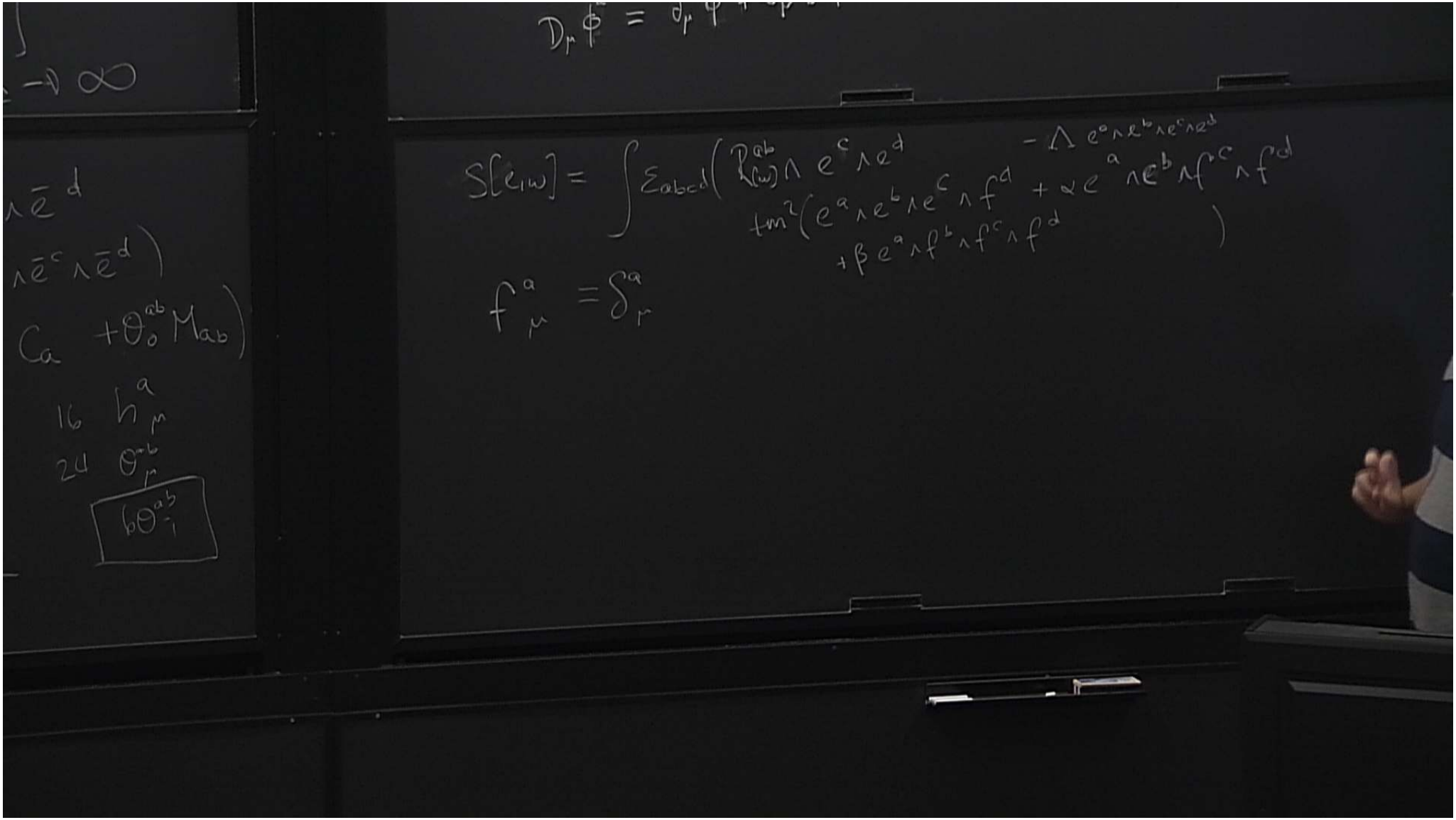
$\partial e \to \Lambda \partial e + e \partial \Lambda$
$D_\mu \phi^a = \partial_\mu \phi^a + \omega^a_{\mu b} \phi^b$
$\omega^a_{\mu\ b} \to \Lambda^a_{\ c} \omega_{\mu\ d}^{\ c} \Lambda^d_{\ b}$
$+ \Lambda^a_{\ c} \partial_\mu \Lambda^c_{\ b}$
$D_\mu \phi^a \to \Lambda^a_{\ b} D_\mu$
$D_{[\mu} e^a_{\nu]} = 0$
$R^{ab}_{\mu\nu} = \partial_{[\mu} \omega^{ab}_{\nu]} + \omega_{[\mu}^{ac} \omega_{\nu]}^{db} \eta_{cd}$

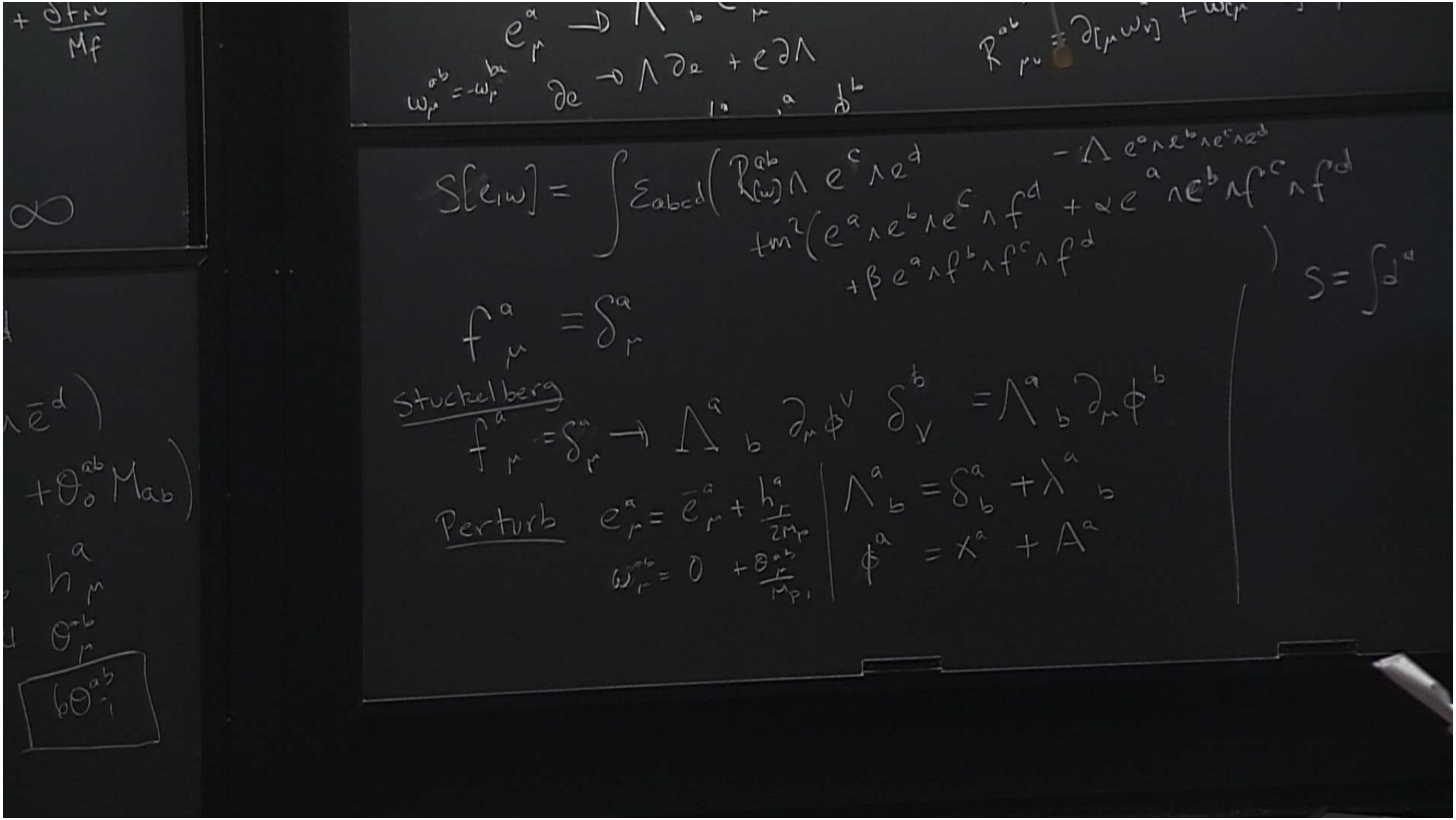

$\to \Lambda^a{}_c \omega_\mu{}^c{}_d \Lambda^d{}_b$
$+ \Lambda^a{}_c \partial_\mu \Lambda^c{}_b$
$\partial_\mu \phi^a \mapsto \Lambda^a{}_b D_\mu \phi^b$
$e^a_{\nu]} = 0$
$= \partial_{[\mu} \omega^{ab}_{\nu]} + \omega_{c[\mu}{}^{ac} \omega^{db}_{\nu]} \eta_{cd}$
Form
$A_\mu, B_{\nu\rho} = -B_{\rho\nu}$ $A, B$
$\pm A_{[\mu} B_{\nu\rho]}$ $A \wedge B$
$\partial_{[\mu} A_{\nu]}$ $dA$
Torsion-free $De^a = 0$
Curvature $R^{ab} = d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb}$

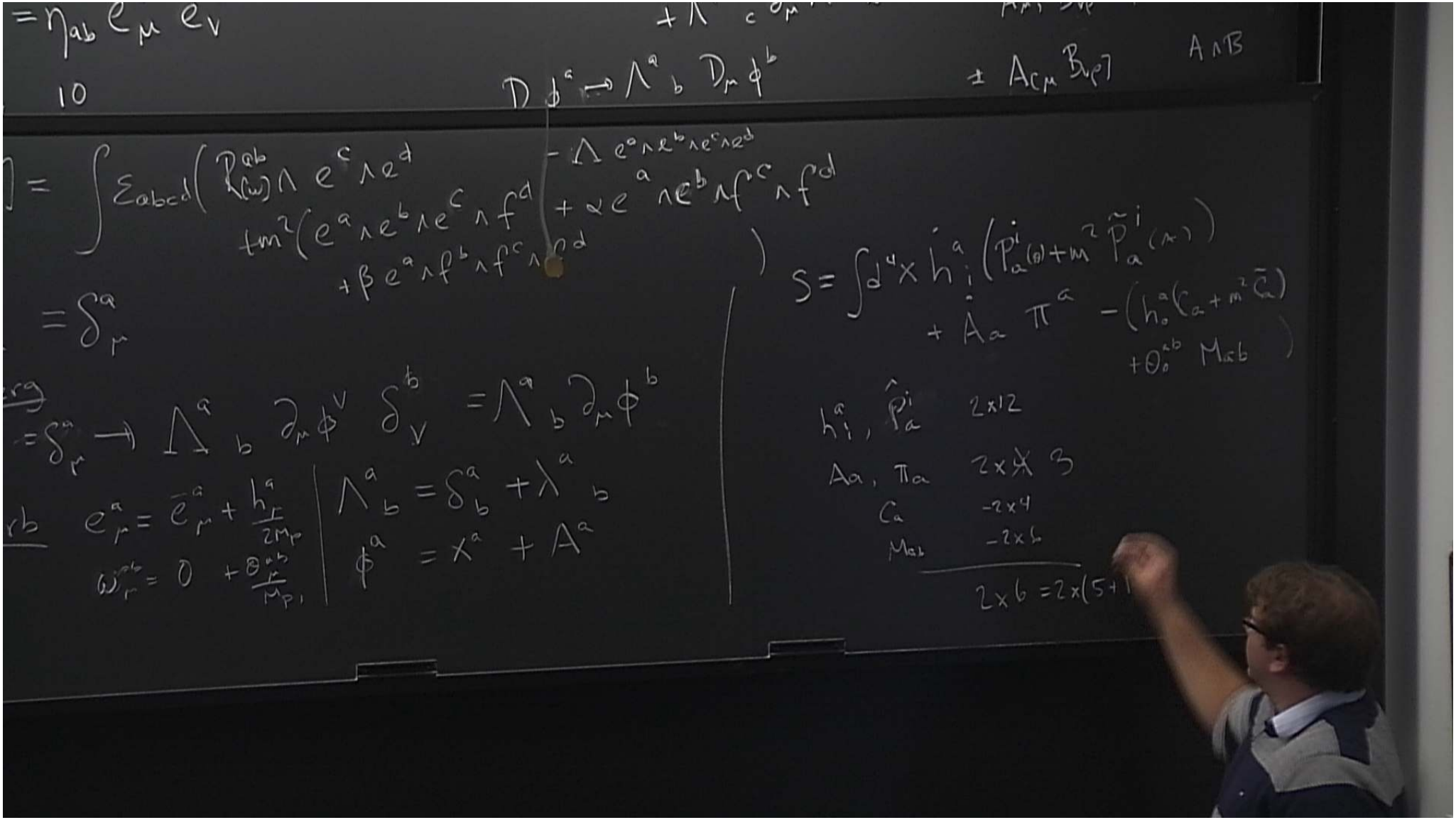

$S[e,\omega] = \frac{M_P^2}{4}\int \varepsilon_{abcd} R^{ab}[\omega] \wedge e^c \wedge e^d$
$\frac{\delta S}{\delta \omega} \to De^a = 0$
Perturb
$e^a_\mu = \bar{e}^a_\mu + \frac{h^a_\mu}{2M_P}$
$\omega^{ab}_\mu = 0 + \frac{\Theta^{ab}_\mu}{M_{Pl}}$
$\bar{R}^{ab} = 0$
$S = \frac{1}{4}\int \varepsilon_{abcd}$

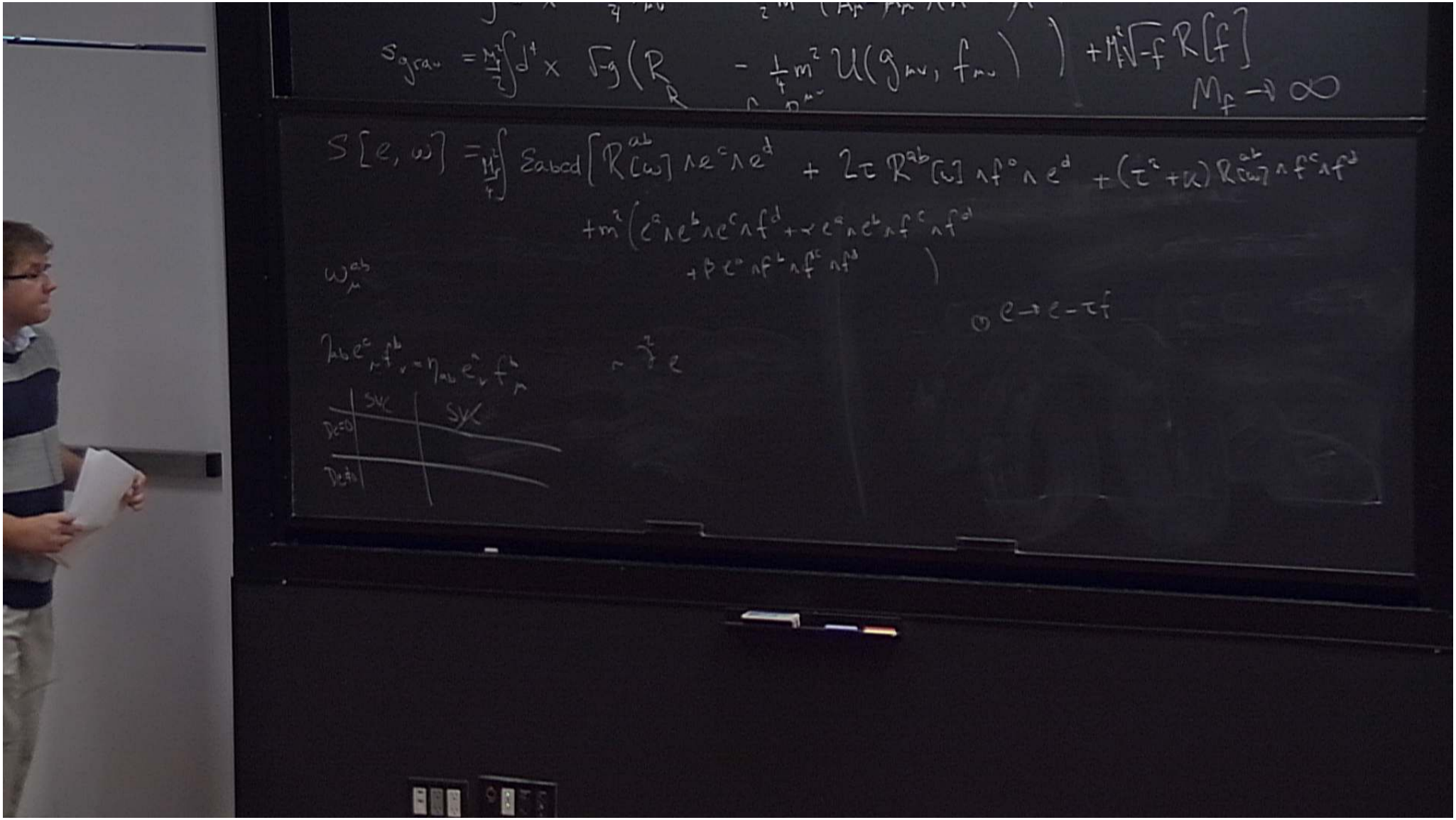

$\sqrt{-g}\left(R_R - \frac{1}{4}m^2\,\mathcal{U}(g_{\mu\nu}, f_{\mu\nu})\right) + M_f^2\sqrt{-f}\,R[f]$
$M_f \to \infty$
$\varepsilon_{abcd}\, R^{ab}[\omega] \wedge e^c \wedge e^d$
$e^a = 0$
$\bar{e}^a_\mu + h^a_\mu \quad \bar{R}^{ab} = 0$
$0 + \frac{\Theta^{ab}}{M_{Pl}}$
$S = \frac{1}{4}\int \varepsilon_{abcd}\left(d\theta^{ab} \wedge h^c \wedge \bar{e}^d + \theta^a{}_e \wedge \theta^{eb} \wedge \bar{e}^c \wedge \bar{e}^d\right)$
$= \int d^4x\, \dot{h}^a_i P^i_a - \left(h^a_0 C_a + \theta^{ab}_0 M_{ab}\right)$
$h^a_i, P^i_a(\theta)$ 12x2
$C_a$ -2x4
$M^{ab}$ -2x6
2x2


$\int d^4x \sqrt{-g}\left(R_R - \frac{1}{4}m^2 \mathcal{U}(g_{\mu\nu}, f_{\mu\nu})\right) + M_f^2 \sqrt{-f}\, K[f]$
$M_f \to \infty$
$\int \varepsilon_{abcd} R^{ab}[\omega] \wedge e^c \wedge e^d$
$De^a = 0$
$= \bar{e}^a_\mu + \frac{h^a_\mu}{2M_P}$
$= 0 + \frac{\theta^{ab}_\mu}{M_{Pl}}$
$\bar{R}^{ab} = 0$
$S = \frac{1}{4}\int \varepsilon_{abcd}\left(d\theta^{ab} \wedge h^c \wedge \bar{e}^d + \theta^a{}_e \wedge \theta^{eb} \wedge \bar{e}^c \wedge \bar{e}^d\right)$
$= \int d^4x\, \dot{h}^a_i P^i_a - \left(h^a_0 C_a + \theta^{ab}_0 M_{ab}\right)$
$h^a_i, P^i_a(\theta)$ 2×2 16 $h^a_\mu$
24 $\theta^{ab}_\mu$
$C_a$ −2×4
$M^{ab}$ −2×6 6$\theta^{ab}_i$
2×2


$D_\mu \phi = \partial_\mu \phi$
$S[e,\omega] = \int \varepsilon_{abcd} \left( R^{ab}(\omega) \wedge e^c \wedge e^d - \Lambda\, e^a \wedge e^b \wedge e^c \wedge e^d \right.$
$+ tm^2 (e^a \wedge e^b \wedge e^c \wedge f^d + \alpha e^a \wedge e^b \wedge f^c \wedge f^d$
$\left. + \beta e^a \wedge f^b \wedge f^c \wedge f^d \right)$
$f^a_\mu = \delta^a_\mu$
$\wedge \bar{e}^d$
$\wedge \bar{e}^c \wedge \bar{e}^d)$
$Ca + \theta_0^{ab} M_{ab})$
16 $h^a_\mu$
24 $\theta^{ab}_\mu$
$b\theta^{ab}_i$


$S[e,\omega] = \int \varepsilon_{abcd}\left( R^{ab}(\omega) \wedge e^c \wedge e^d - \Lambda\, e^a \wedge e^b \wedge e^c \wedge e^d \right.$
$+ m^2 (e^a \wedge e^b \wedge e^c \wedge f^d + \alpha e^a \wedge e^b \wedge f^c \wedge f^d$
$\left. + \beta e^a \wedge f^b \wedge f^c \wedge f^d \right)$
$f^a_\mu = \delta^a_\mu$
Stuckelberg
$f^a_\mu = \delta^a_\mu \to \Lambda^a{}_b\, \partial_\mu \phi^b\, \delta^b_\nu = \Lambda^a{}_b\, \partial_\mu \phi^b$
Perturb $e^a_\mu = \bar{e}^a_\mu + \frac{h^a_\mu}{2M_P}$
$\omega^{ab}_\mu = 0 + \frac{\theta^{ab}_\mu}{M_P}$
$\Lambda^a{}_b = \delta^a_b + \lambda^a{}_b$
$\phi^a = x^a + A^a$
$S = \int d^4$


$= \eta_{ab} e_\mu e_\nu$
10
$D\phi^a \to \Lambda^a{}_b D_\mu \phi^b$
$\pm A_{[\mu} B_{\nu\rho]}$
$A \wedge B$
$= \int \varepsilon_{abcd} ( R^{ab}_{(\omega)} \wedge e^c \wedge e^d - \Lambda e^a \wedge e^b \wedge e^c \wedge e^d$
$+ m^2 ( e^a \wedge e^b \wedge e^c \wedge f^d + \alpha e^a \wedge e^b \wedge f^c \wedge f^d$
$+ \beta e^a \wedge f^b \wedge f^c \wedge f^d )$
$= \delta^a_\mu$
$= \delta^a_\mu \to \Lambda^a{}_b \partial_\mu \phi^\nu \delta^b_\nu = \Lambda^a{}_b \partial_\mu \phi^b$
$e^a_\mu = \bar{e}^a_\mu + \frac{h^a_\mu}{2M_p}$
$\omega^{ab}_\mu = 0 + \frac{\theta^{ab}_\mu}{M_{pl}}$
$\Lambda^a{}_b = \delta^a_b + \lambda^a{}_b$
$\phi^a = x^a + A^a$
$S = \int d^4x \, \dot{h}^a_i ( P^i_a(\theta) + m^2 \tilde{P}^i_a(x) )$
$+ \dot{A}_a \pi^a - ( h^a_0 ( C_a + m^2 \tilde{C}_a )$
$+ \theta^{ab}_0 M_{ab} )$
$h^a_i, \hat{P}^i_a$ 2x12
$A_a, \pi_a$ 2x~~4~~ 3
$C_a$ -2x4
$M_{ab}$ -2x6
2x6 = 2x(5+


$S_{grav} = \frac{M_f^2}{2}\int d^4x \sqrt{-g}\left(R_R - \frac{1}{4}m^2 U(g_{\mu\nu}, f_{\mu\nu})\right) + M_f^2\sqrt{-f}\,R[f]$
$M_f \to \infty$
$S[e,\omega] = \frac{M_f^2}{4}\int \varepsilon_{abcd}\left[R^{ab}[\omega]\wedge e^c\wedge e^d + 2\tau R^{ab}[\omega]\wedge f^c\wedge e^d + (\tau^2+\alpha)R^{ab}[\omega]\wedge f^c\wedge f^d\right.$
$+ m^2\left(e^a\wedge e^b\wedge e^c\wedge f^d + \alpha e^a\wedge e^b\wedge f^c\wedge f^d + \beta e^a\wedge f^b\wedge f^c\wedge f^d\right)$
$\omega^{ab}_\mu$
$e \to e - \tau f$
$\eta_{ab}e^a_\mu f^b_\nu = \eta_{ab}e^a_\nu f^b_\mu$
SVC
SVC
De=0
De≠0

massless spin 2    2 dofs

Massive spin 2    $2\cdot 2+1=5$ dofs

$G(\bar{f})_{\mu\nu} + \cancel{m^2 X_{\mu\nu}} = 0$

$f = \bar{f}_{\mu\nu} + \frac{\delta f_{\mu\nu}}{M_f}$

$$S[e,\omega] = \frac{M_f^2}{4}\int \varepsilon_{abcd}\Big[R^{ab}[\omega]\wedge e^c\wedge e^d + 2\tau R^{ab}[\omega]\wedge f^c\wedge e^d + (\tau^2+\kappa) R^{ab}[\omega]\wedge f^c\wedge f^d$$

$$+ m^2\big(e^a\wedge e^b\wedge e^c\wedge f^d + \alpha\, e^a\wedge e^b\wedge f^c\wedge f^d + \beta\, e^a\wedge f^b\wedge f^c\wedge f^d\big)$$

$\boxed{\tau=0}$

$\omega^{ab}_\mu$

$$S^{(2)}_{\text{interesting}}(e,\omega) = \int \varepsilon_{abcd}\, d\theta^{ab}\wedge\big(2h^c\wedge \bar{e}^d + \kappa\,\lambda^c\, e\, 1^e\wedge 1^d\big)$$

$\eta_{ab} e^a_\mu f^b_\nu = \eta_{ab} e^a_\nu f^b_\mu$    $\sim \partial^2 e$

| | SVC | SVC |
|---|---|---|
| De=0 | | |
| De≠0 | | |

$$S = \int d^4x\, \dot{\theta}^{ab}_i \tilde{P}^i_{ab} + \dot{A}^a \pi_a - \big(h^a_0 C_a + \theta^{ab}_0 M_{ab}\big)$$

$\{18\,\theta^{ab}_i, (12 h^a_i, 6\lambda^a_i)\}$    $2\times 18$

$A^a$    $0$

$C_a$    $-2\times 4$

$M_{ab}$    $-2\times 6$    $2\times 8 = 2\times(5+3)$