Title: Kinetic Terms in Massive gravity

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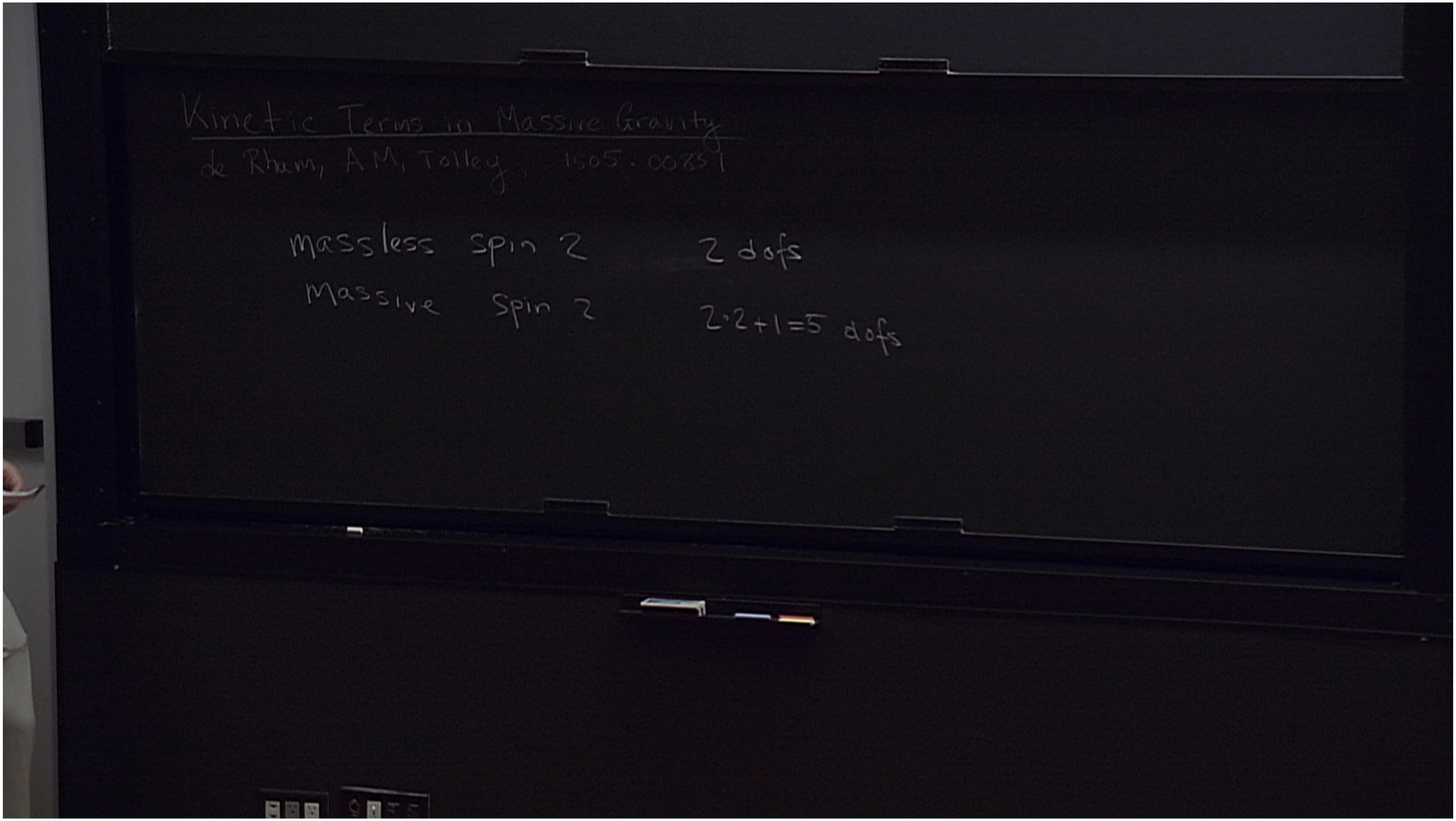
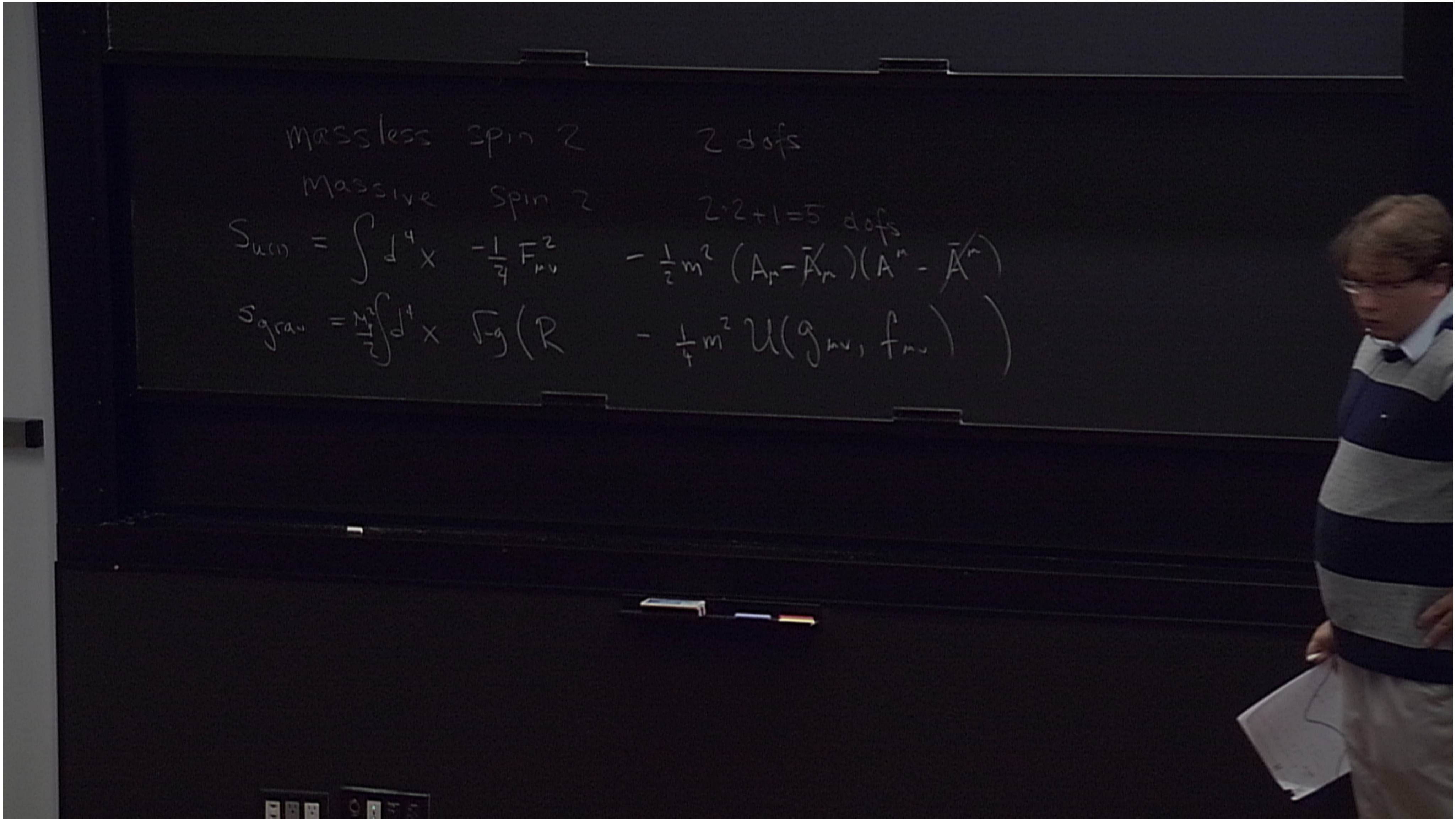
Abstract: <p>Is the graviton a truly massless spin-2 particle, or can the graviton have a small mass? If the mass of the graviton is of order the Hubble scale today, it can potentially help to explain the observed cosmic acceleration. Previous attempts to study massive gravity have been spoiled by the fact that a generic potential for the graviton leads to an instability called the Boulware-Deser ghost. Recently, a special potential has been constructed which avoids this problem while maintaining Lorentz invariance. In this talk I will present recent arguments that suggest that the requirement of avoiding the Boulware-Deser ghost (or other degrees of freedom) is so powerful that the kinetic term for a massive graviton is fixed as well. In fact it must be exactly the same as in General Relativity. This is remarkable as we derive the structure of General Relativity on the basis of stability requirements, not on a symmetry principle. </p>



Kinetic Terms in Massive Gravity

de Rham, AM, Tolley, 1505.00851

massless spin 2	2 dofs
Massive spin 2	$2\cdot 2+1=5$ dofs

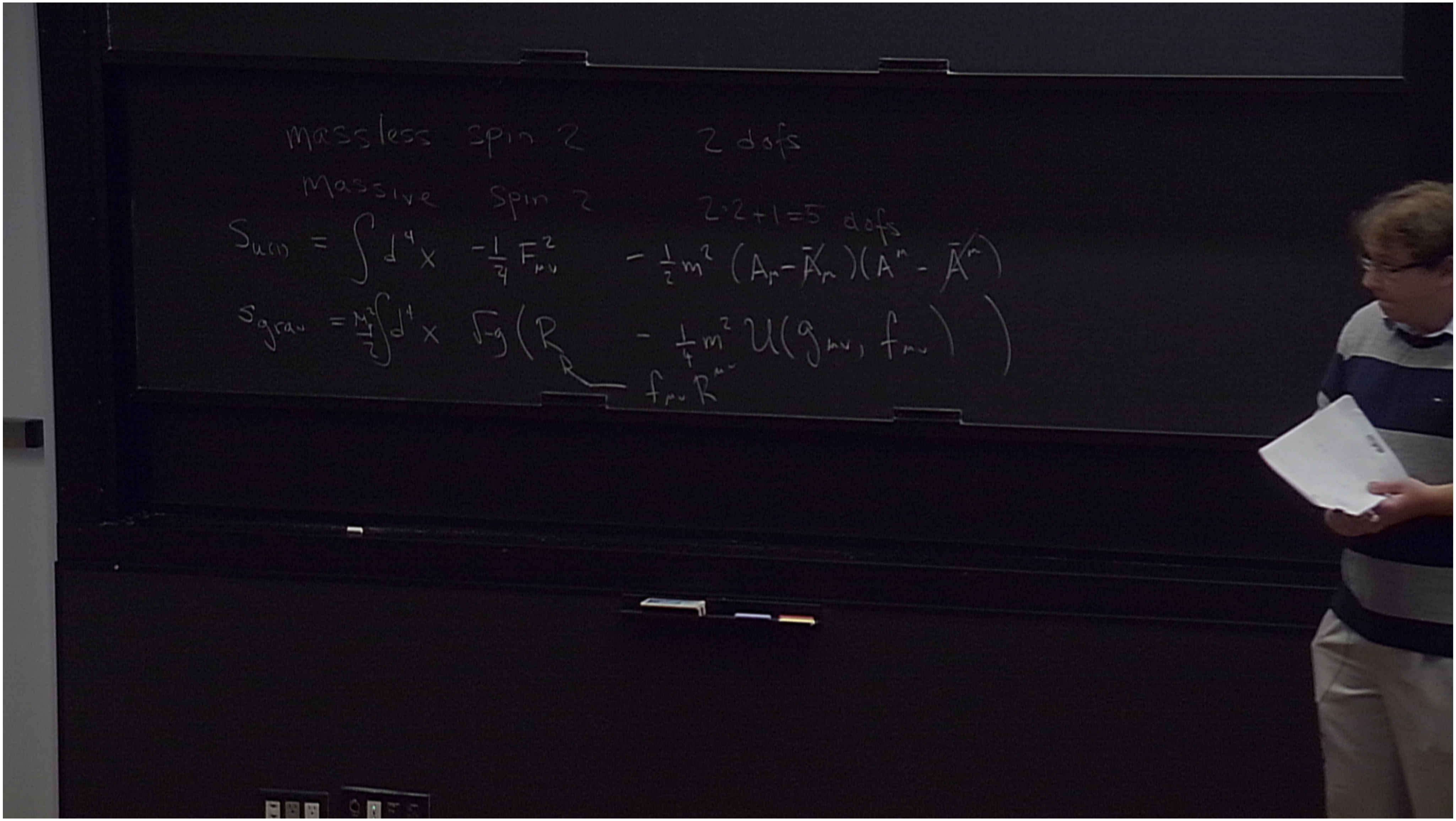


massless spin 2 2 dofs

Massive spin 2 $2 \cdot 2 + 1 = 5$ dofs

$$S_{U(1)} = \int d^4x \; -\frac{1}{4} F_{\mu\nu}^2 \quad - \frac{1}{2} m^2 (A_\mu - \cancel{\bar{A}_\mu})(A^\mu - \cancel{\bar{A}^\mu})$$

$$S_{grav} = \frac{M_4^2}{2} \int d^4x \; \sqrt{-g} \left(R \quad - \frac{1}{4} m^2 \, \mathcal{U}(g_{\mu\nu}, f_{\mu\nu}) \right)$$

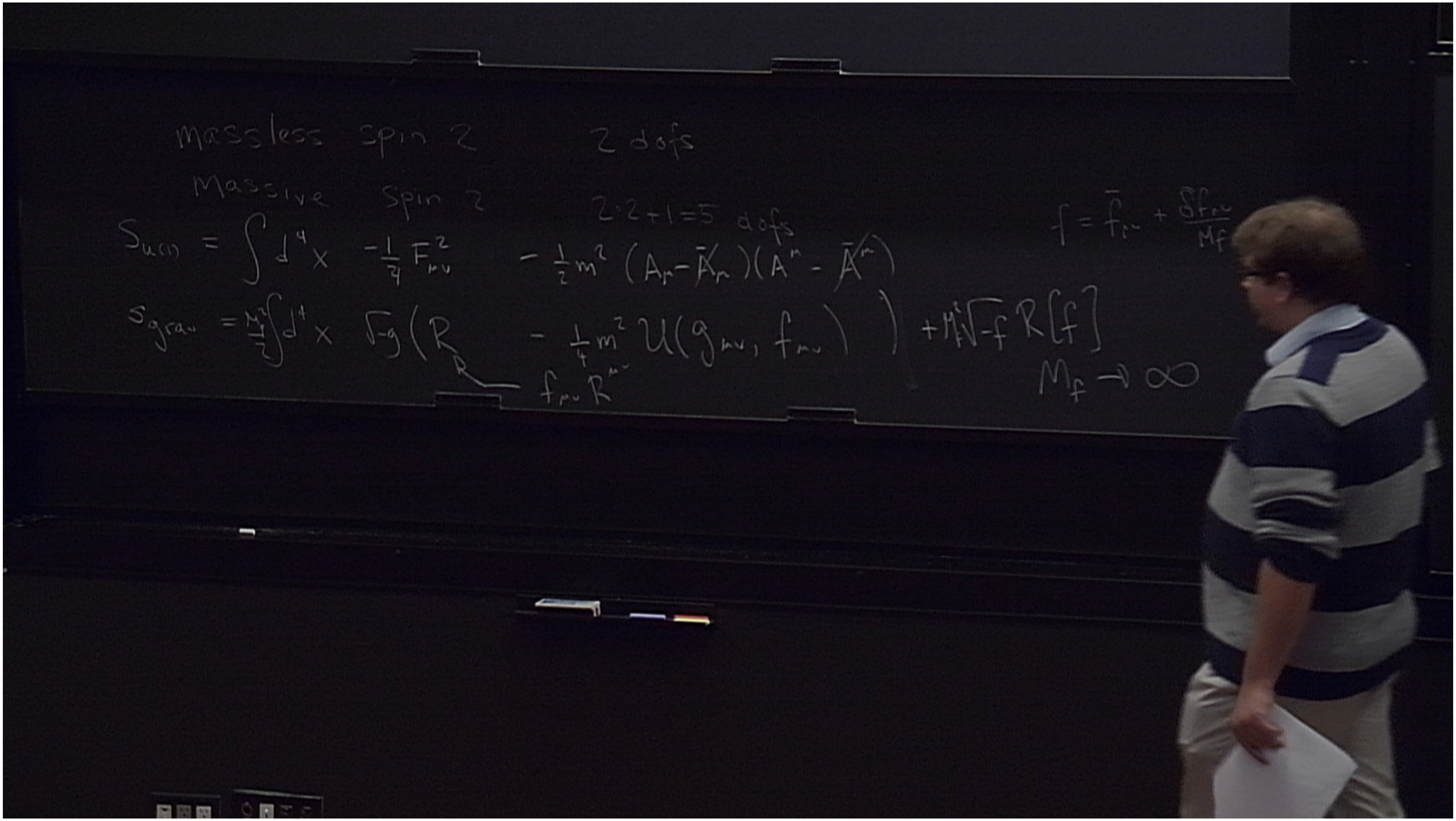


massless spin 2 $\quad$ 2 dofs

Massive spin 2 $\quad$ $2\cdot 2+1=5$ dofs

$$S_{U(1)} = \int d^4x \; -\frac{1}{4}F_{\mu\nu}^2 \quad -\frac{1}{2}m^2\left(A_\mu-\cancel{\bar{A}_\mu}\right)\left(A^\mu-\cancel{\bar{A}^\mu}\right)$$

$$S_{grav} = \frac{M_p^2}{2}\int d^4x \,\sqrt{-g}\Big(\underset{\nwarrow\, f_{\mu\nu}R^{\mu\nu}}{R} \quad -\frac{1}{4}m^2\,\mathcal{U}(g_{\mu\nu}, f_{\mu\nu})\Big)\Big)$$



massless spin 2 2 dofs

Massive spin 2 $2\cdot 2+1=5$ dofs

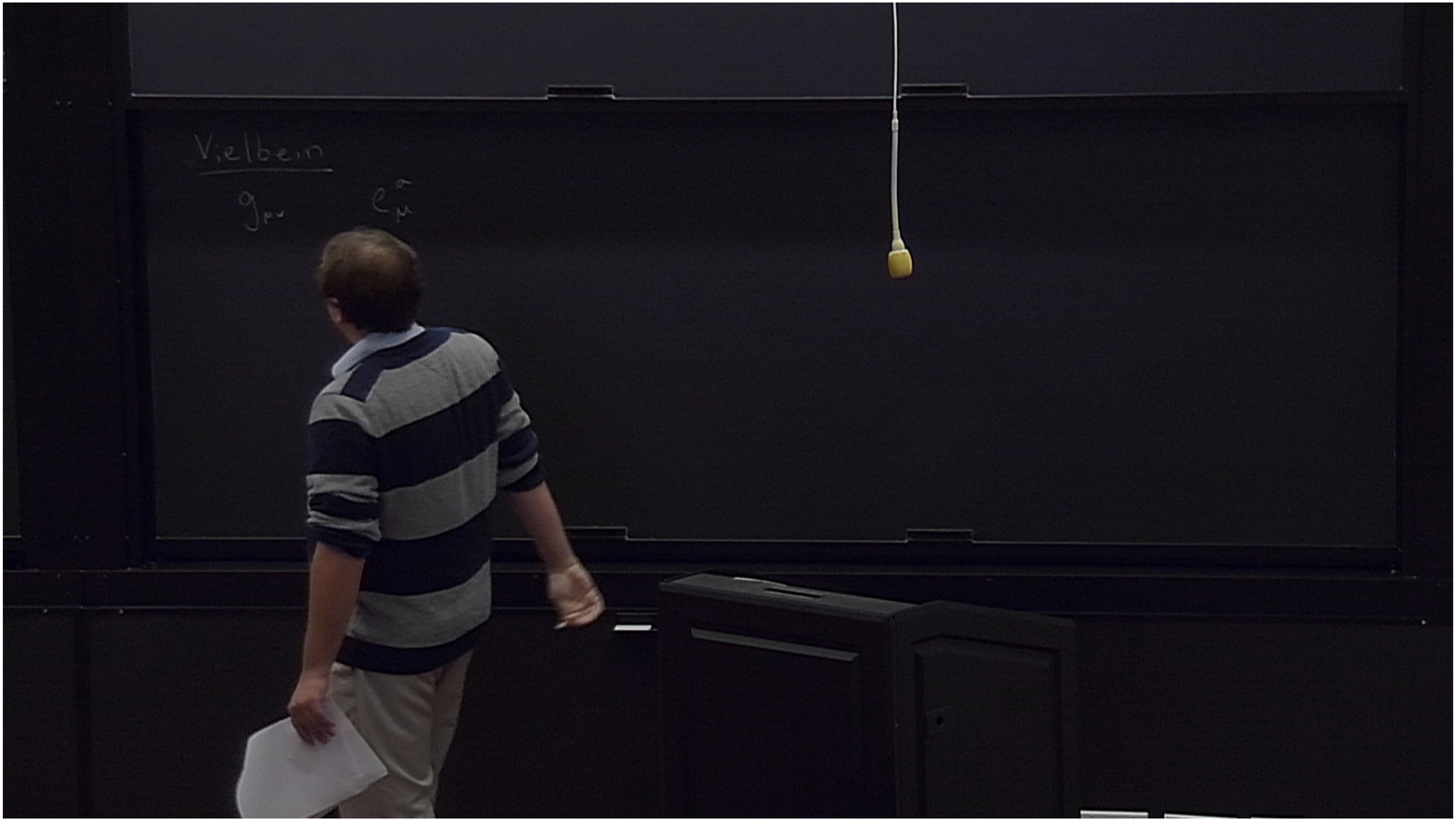
$$S_{U(1)} = \int d^4x \ -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2\left(A_\mu - \cancel{\bar{A}_\mu}\right)\left(A^\mu - \cancel{\bar{A}^\mu}\right)$$

$$f = \bar{f}_{\mu\nu} + \frac{\delta f_{\mu\nu}}{M_f}$$

$$S_{grav} = \frac{M_p^2}{2}\int d^4x \ \sqrt{-g}\left(R - \frac{1}{4}m^2\,\mathcal{U}(g_{\mu\nu}, f_{\mu\nu})\right) + M_f^2\sqrt{-f}\,R[f]$$

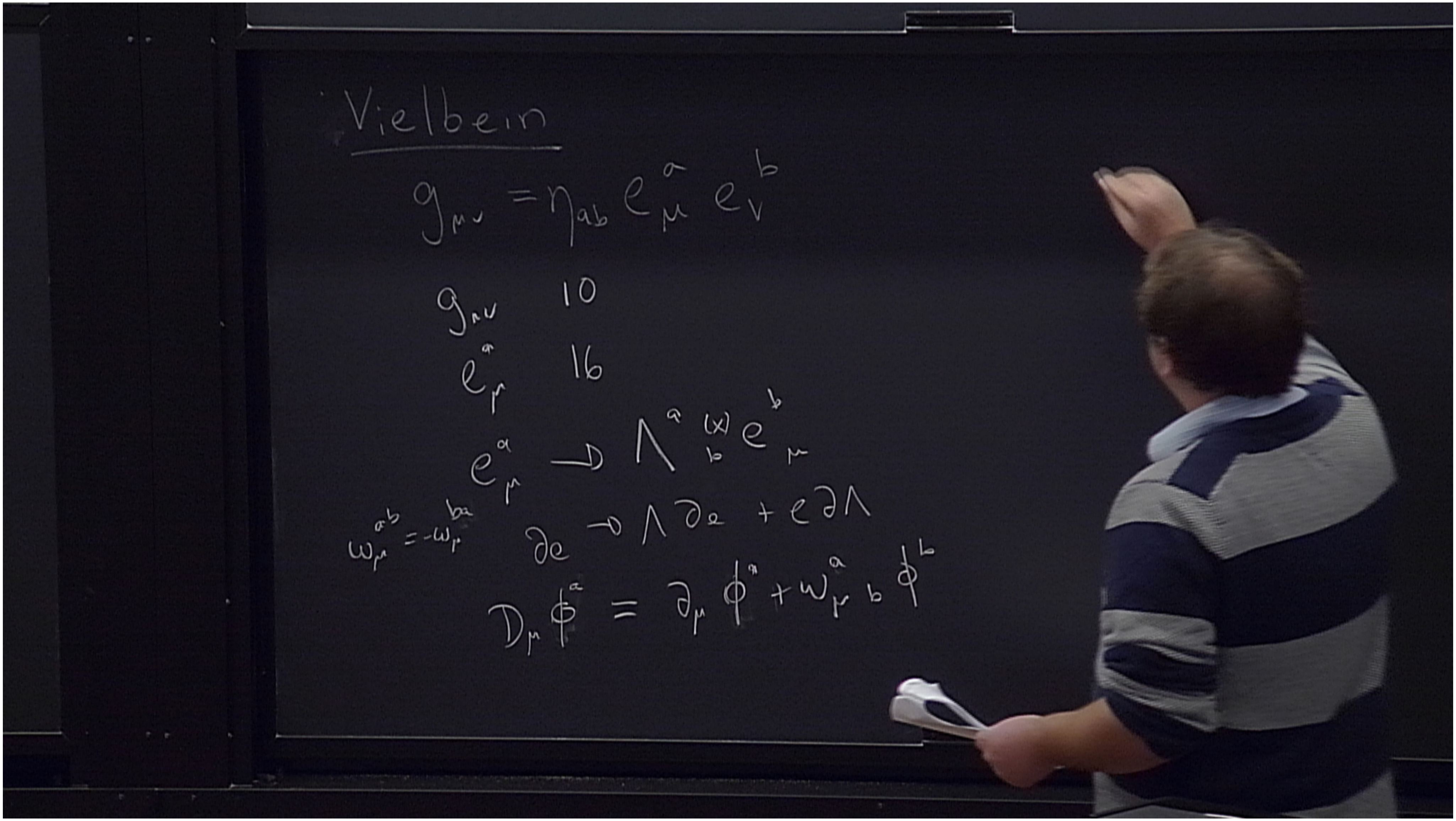
$f_{\mu\nu}R^{\mu\nu}$

$M_f \to \infty$



Vielbein

$g_{\mu\nu}$ $\quad e^{a}_{\mu}$



Vielbein

$$g_{\mu\nu} = \eta_{ab}\, e^a_\mu\, e^b_\nu$$

$g_{\mu\nu}$ 10

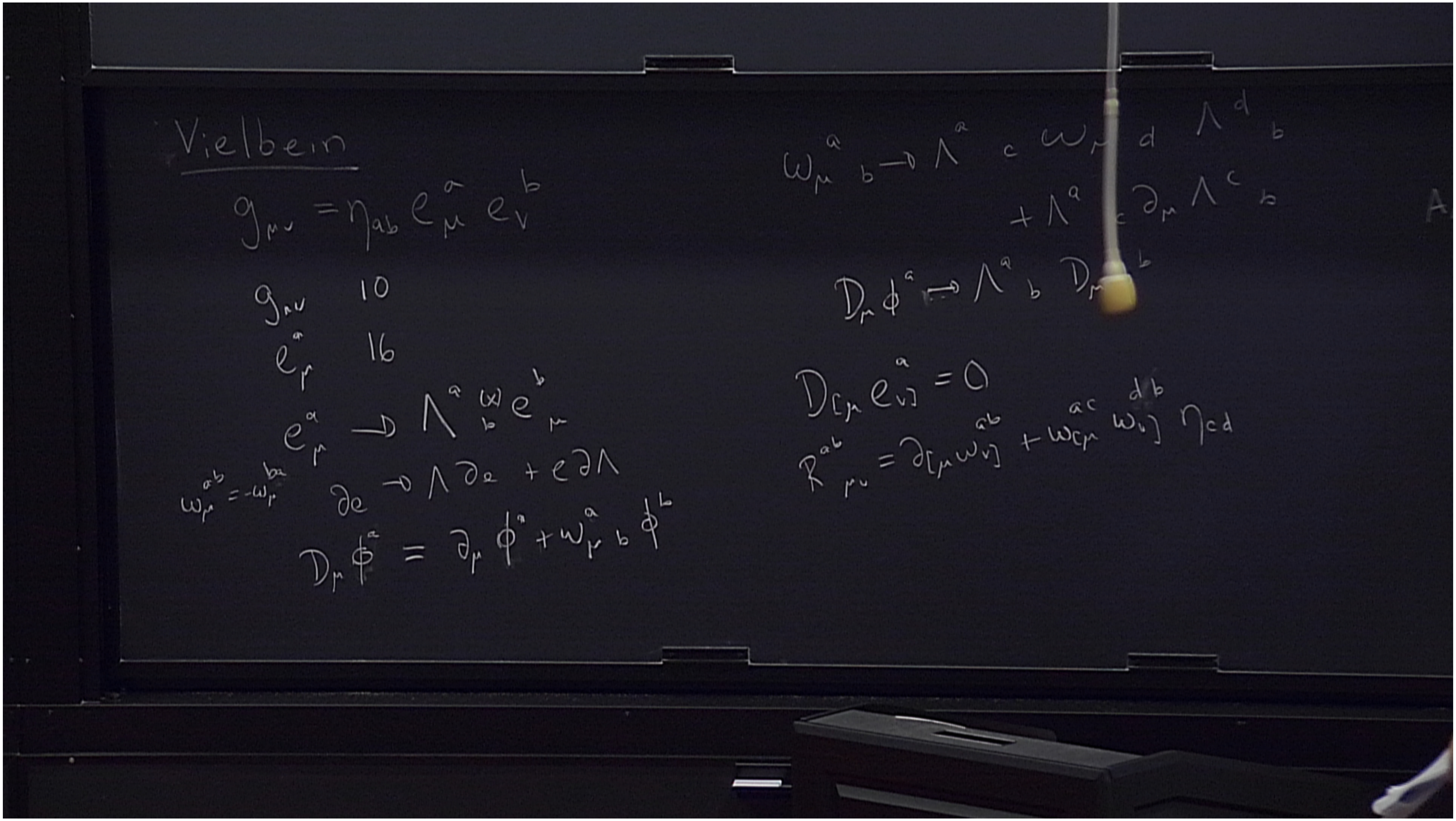
e^a_μ 16

$$e^a_\mu \to \Lambda^a{}_b(x)\, e^b_\mu$$

$\omega_\mu^{ab} = -\omega_\mu^{ba}$

$$\partial e \to \Lambda \partial e + e \partial \Lambda$$

$$D_\mu \phi^a = \partial_\mu \phi^a + \omega_\mu{}^a{}_b\, \phi^b$$



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$$e^a_\mu \longrightarrow \Lambda^a{}_b(x) e^b{}_\mu$$

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$$\partial e \to \Lambda \partial e + e \partial \Lambda$$

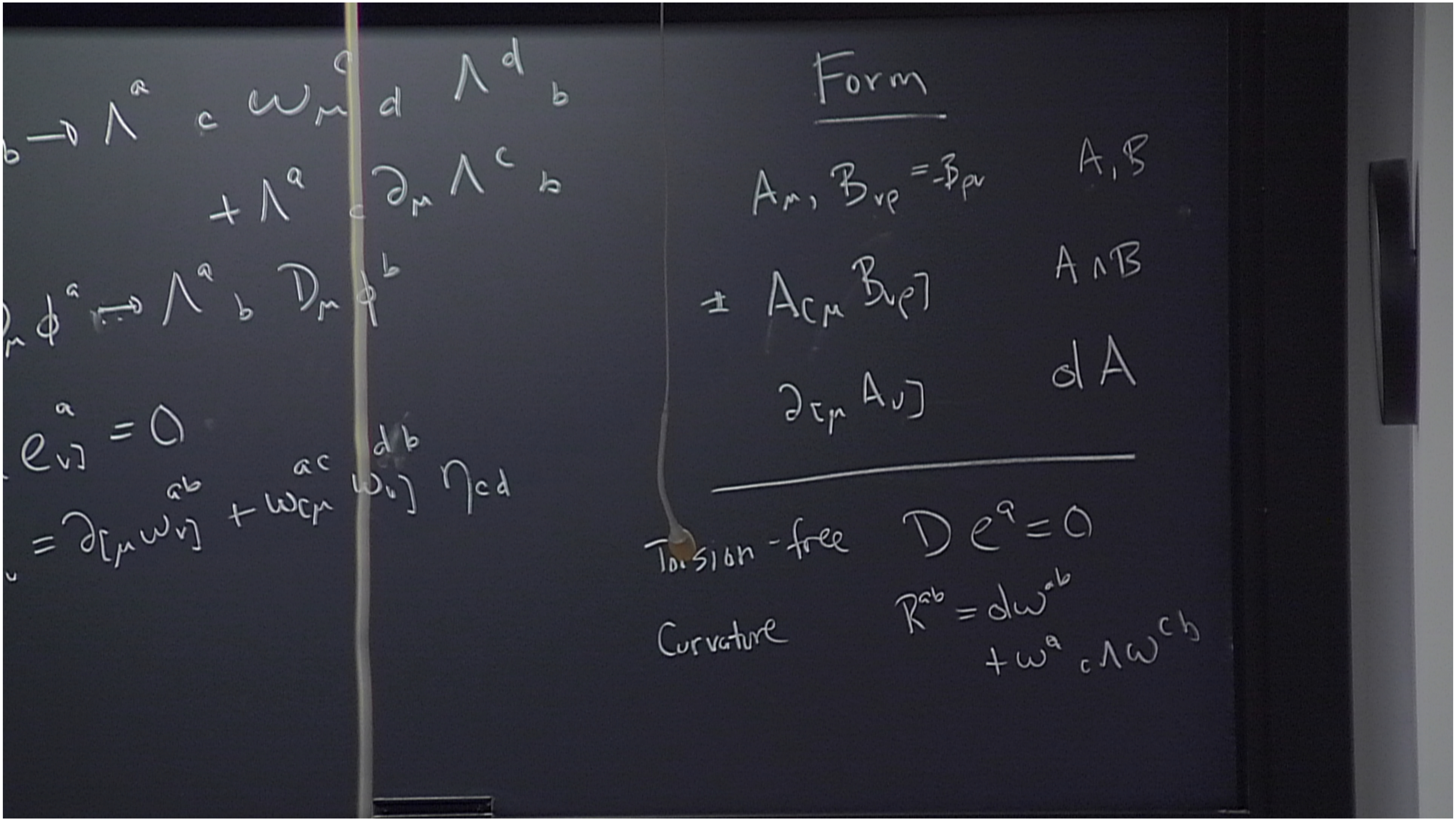
$$D_\mu \phi^a = \partial_\mu \phi^a + \omega_\mu{}^a{}_b \phi^b$$

$$\omega_\mu{}^a{}_b \longrightarrow \Lambda^a{}_c \omega_\mu{}^c{}_d \Lambda^d{}_b + \Lambda^a{}_c \partial_\mu \Lambda^c{}_b$$

$$D_\mu \phi^a \longrightarrow \Lambda^a{}_b D_\mu \phi^b$$

$$D_{[\mu} e^a_{\nu]} = 0$$

$$R^{ab}_{\mu\nu} = \partial_{[\mu} \omega^{ab}_{\nu]} + \omega^{ac}_{[\mu} \omega^{db}_{\nu]} \eta_{cd}$$



$$b \longrightarrow \Lambda^{a}{}_{c}\, \omega_{\mu}{}^{c}{}_{d}\, \Lambda^{d}{}_{b} + \Lambda^{a}{}_{c}\, \partial_{\mu} \Lambda^{c}{}_{b}$$

$$\partial_{\mu} \phi^{a} \longmapsto \Lambda^{a}{}_{b}\, D_{\mu} \phi^{b}$$

$$e^{a}_{\nu]} = 0$$

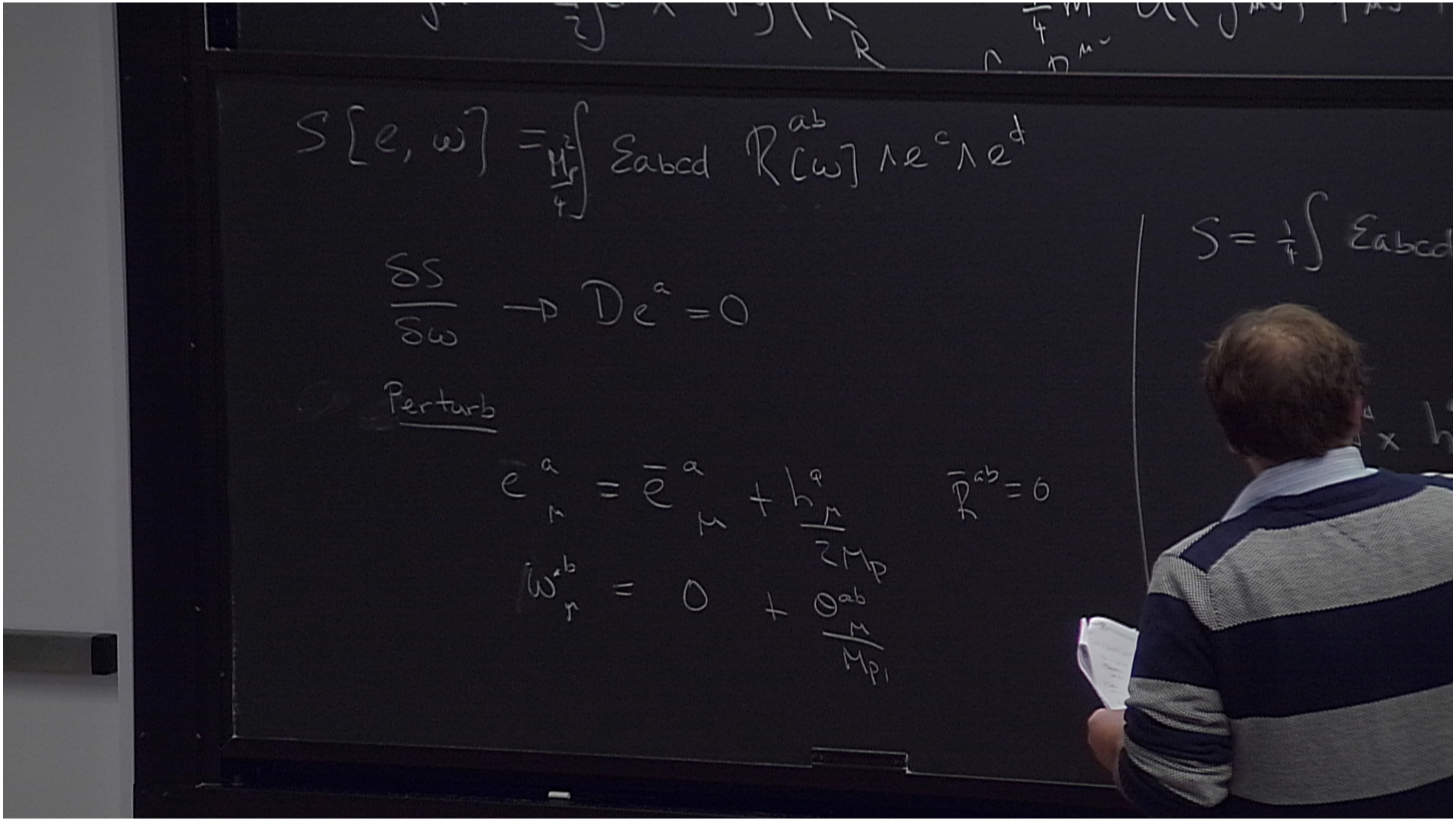
$$= \partial_{[\mu} \omega^{ab}_{\nu]} + \omega^{ac}_{[\mu}\, \omega^{db}_{\nu]}\, \eta_{cd}$$

Form

$A_{\mu}, B_{\nu\rho} = -B_{\rho\nu}$	A, B
$\pm A_{[\mu} B_{\nu\rho]}$	$A \wedge B$
$\partial_{[\mu} A_{\nu]}$	dA

Torsion-free $De^{a} = 0$

Curvature $R^{ab} = d\omega^{ab} + \omega^{a}{}_{c} \wedge \omega^{cb}$



$$S[e,\omega] = \frac{M_p^2}{4}\int \varepsilon_{abcd}\, R^{ab}[\omega]\wedge e^c\wedge e^d$$

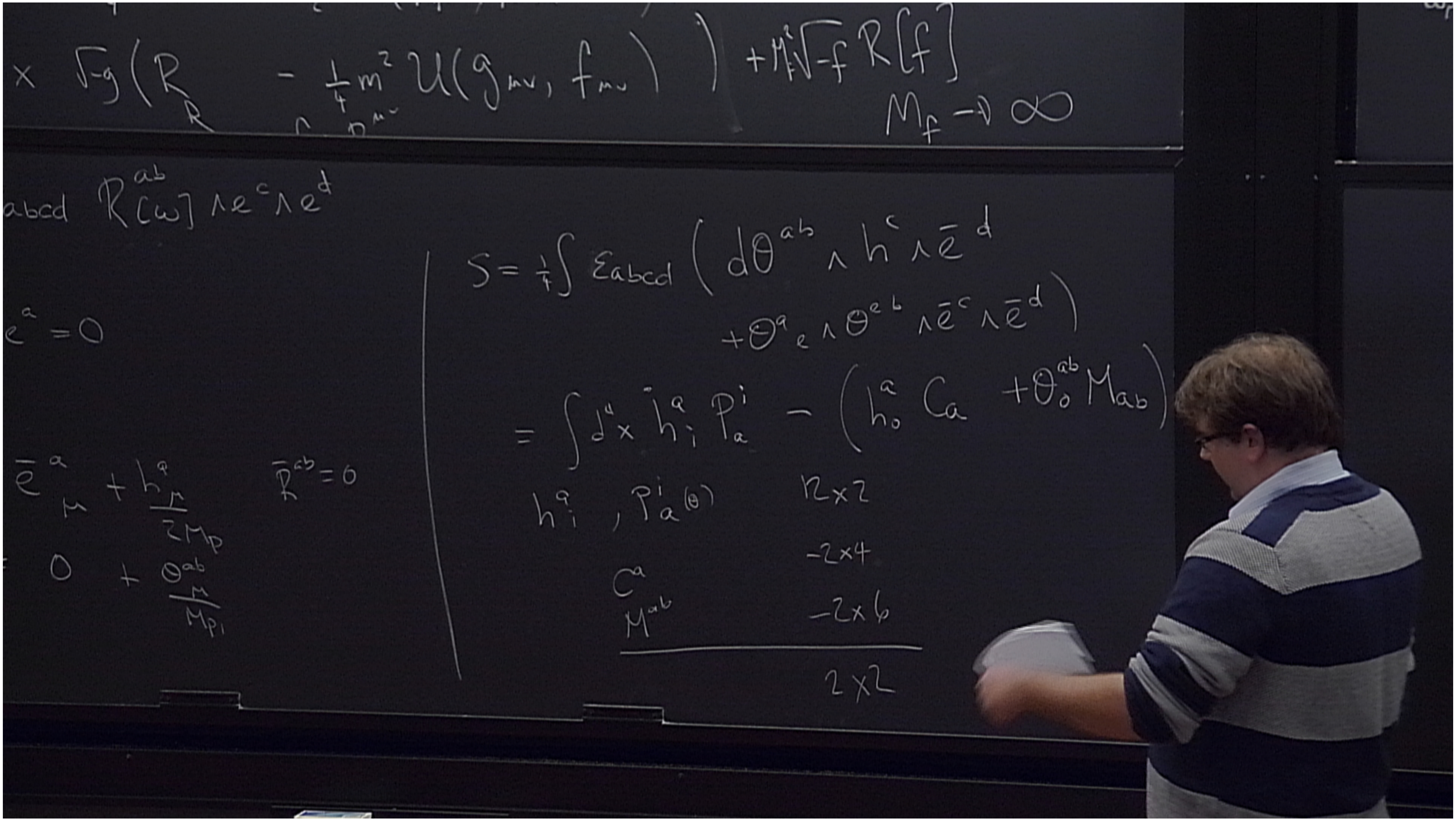
$$\frac{\delta S}{\delta \omega} \to De^a = 0$$

Perturb

$$e^a_\mu = \bar{e}^a_\mu + \frac{h^a_\mu}{2M_p} \qquad \bar{R}^{ab}=0$$

$$\omega^{ab}_\mu = 0 + \frac{\theta^{ab}_\mu}{M_{pl}}$$

$$S = \frac{1}{4}\int \varepsilon_{abcd}$$



$$x \ \sqrt{-g}\left(R - \frac{1}{4} m^2\, \mathcal{U}(g_{\mu\nu}, f_{\mu\nu})\right) + M_f^2 \sqrt{-f}\, R[f]$$

$$M_f \to \infty$$

$$abcd \ R^{ab}[\omega] \wedge e^c \wedge e^d$$

$$e^a = 0$$

$$\bar{e}^a{}_\mu + \frac{h^a{}_\mu}{2M_p} \qquad \bar{R}^{ab} = 0$$

$$0 + \frac{\theta^{ab}{}_\mu}{M_{pl}}$$

$$S = \frac{1}{4}\int \varepsilon_{abcd}\left(d\theta^{ab} \wedge h^c \wedge \bar{e}^d + \theta^a{}_e \wedge \theta^{eb} \wedge \bar{e}^c \wedge \bar{e}^d \right)$$

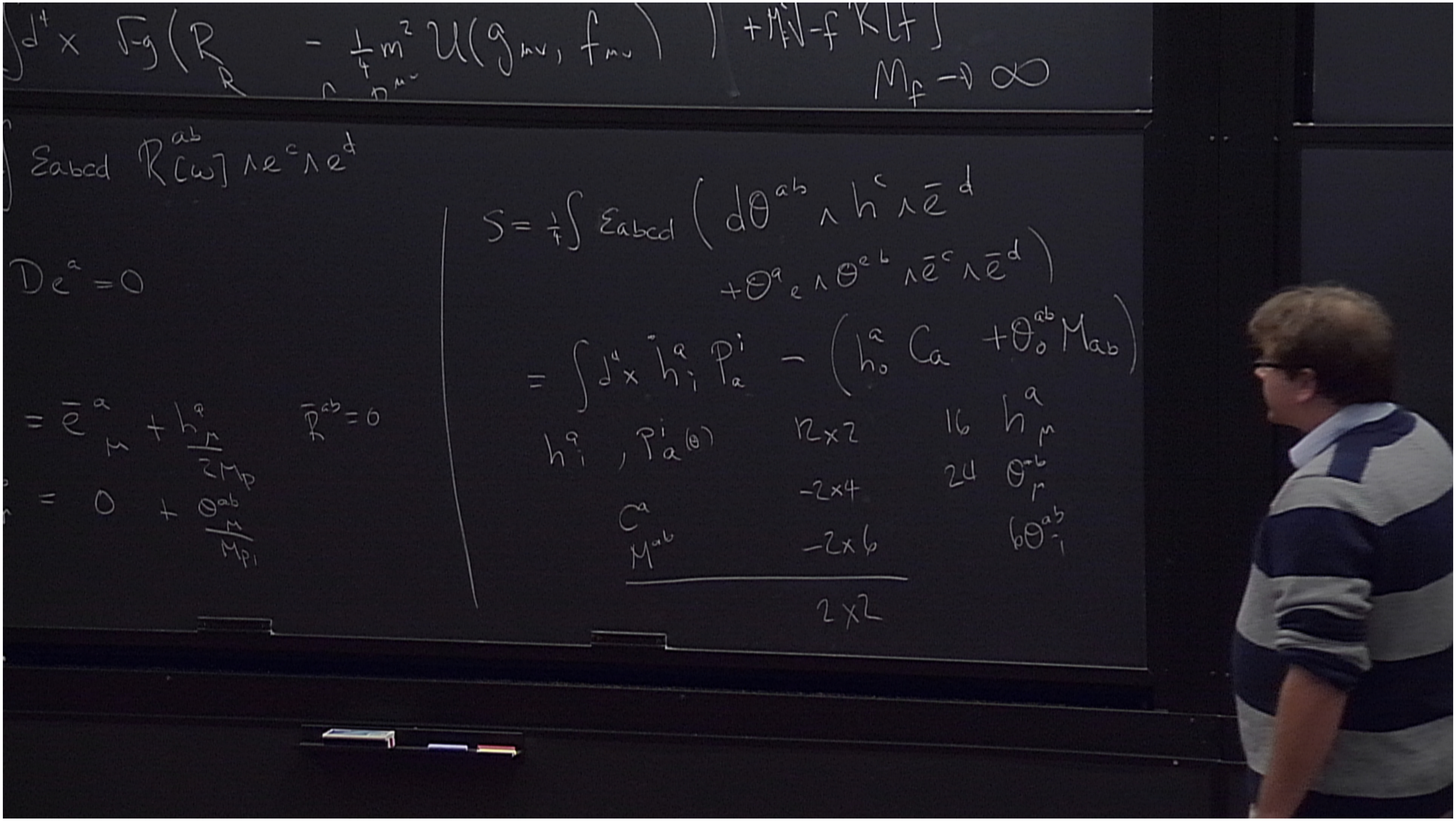
$$= \int d^4x \ \dot{h}^a_i P^i_a - \left(h^a_0 C_a + \theta^{ab}_0 M_{ab} \right)$$

$$h^a_i \ , \ P^i_a(\theta) \qquad 12 \times 2$$

$$C^a \qquad -2 \times 4$$

$$M^{ab} \qquad -2 \times 6$$

$$\overline{\qquad\qquad 2 \times 2}$$



$$\int d^4x \sqrt{-g}\left(R - \frac{1}{4}m^2\,\mathcal{U}(g_{\mu\nu}, f_{\mu\nu})\right)\Big) + M_f\sqrt{-f}\,R[f]\Big]$$

$$M_f \to \infty$$

$$\int \varepsilon_{abcd}\, R^{ab}[\omega] \wedge e^c \wedge e^d$$

$$De^a = 0$$

$$= \bar{e}^a_\mu + \frac{h^a_\mu}{2M_p} \qquad \bar{R}^{ab} = 0$$

$$= 0 + \frac{\theta^{ab}_\mu}{M_{pl}}$$

$$S = \frac{1}{4}\int \varepsilon_{abcd}\left(d\theta^{ab} \wedge h^c \wedge \bar{e}^d + \theta^a{}_e \wedge \theta^{eb} \wedge \bar{e}^c \wedge \bar{e}^d\right)$$

$$= \int d^4x\, \dot{h}^a_i P^i_a - \left(h^a_0 C_a + \theta^{ab}_0 M_{ab}\right)$$

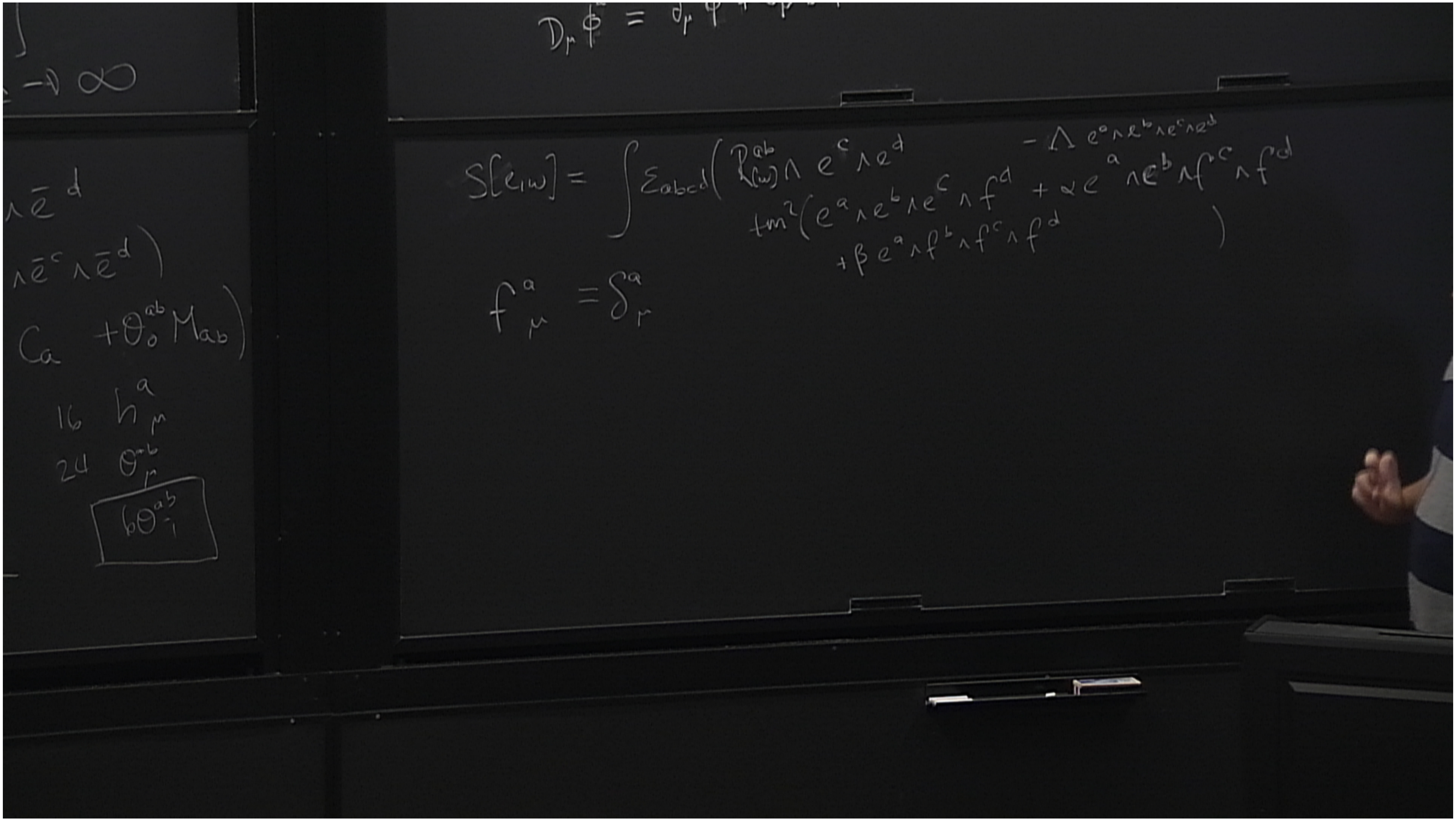
h^a_i, $P^i_a(\theta)$ 12×2 16 h^a_μ

−2×4 24 θ^{ab}_μ

C^a

M^{ab} −2×6 $6\,\theta^{ab}_{i}$

2×2



$D_\mu \phi = \partial_\mu \phi$

$\to \infty$

$\wedge \bar{e}^d$

$\wedge \bar{e}^c \wedge \bar{e}^d)$

$C_a \quad + \theta_0^{ab} M_{ab})$

$16 \quad h^a_\mu$

$24 \quad \theta^{ab}_\mu$

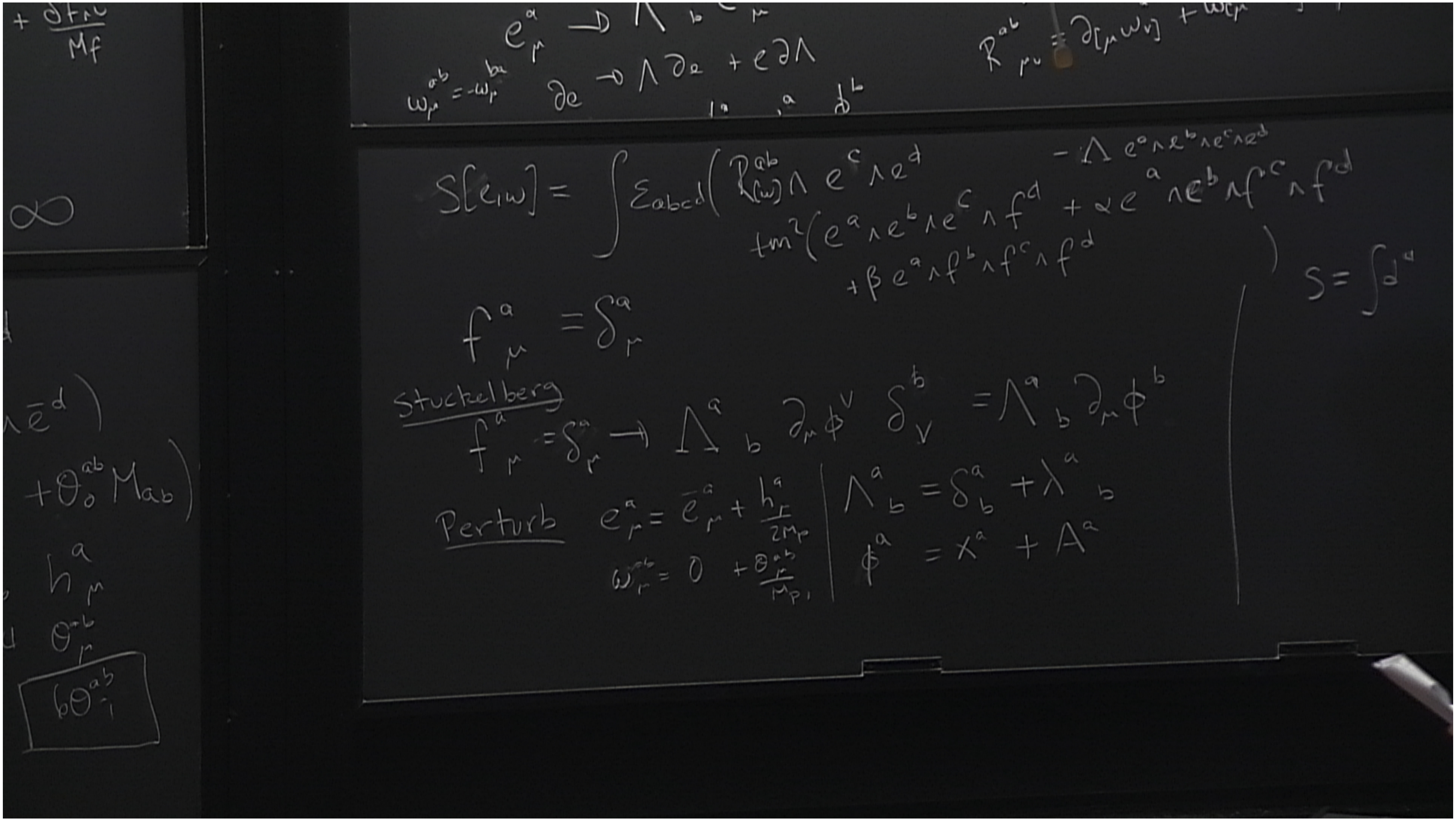
$\boxed{6\theta^{ab}_i}$

$$S[e,\omega] = \int \varepsilon_{abcd} \Big(R^{ab}_{(\omega)} \wedge e^c \wedge e^d \qquad - \Lambda\, e^a \wedge e^b \wedge e^c \wedge e^d$$

$$+ m^2 \big(e^a \wedge e^b \wedge e^c \wedge f^d + \alpha\, e^a \wedge e^b \wedge f^c \wedge f^d$$

$$+ \beta\, e^a \wedge f^b \wedge f^c \wedge f^d \big)$$

$$f^a_\mu = \delta^a_\mu$$



$+\frac{\partial f_{\mu\nu}}{Mf}$

∞

$\wedge \bar{e}^d)$

$+\theta^{ab}_0 M_{ab})$

h^a_μ

θ^{ab}_μ

$\boxed{b\theta^{ab}_1}$

$e^a_\mu \to \Lambda_b e_\mu$

$\omega^{ab}_\mu = -\omega^{ba}_\mu \qquad \partial e \to \Lambda \partial e + e \partial \Lambda$

$R^{ab}_{\mu\nu} = \partial_{[\mu}\omega_{\nu]} + \omega_{[\mu}$

$$S[e,\omega] = \int \varepsilon_{abcd}\Big(R^{ab}_{(\omega)} \wedge e^c \wedge e^d \quad - \Lambda\, e^a \wedge e^b \wedge e^c \wedge e^d$$
$$+ m^2\big(e^a \wedge e^b \wedge e^c \wedge f^d + \alpha\, e^a \wedge e^b \wedge f^c \wedge f^d$$
$$+ \beta\, e^a \wedge f^b \wedge f^c \wedge f^d \big)$$

$S = \int d^4$

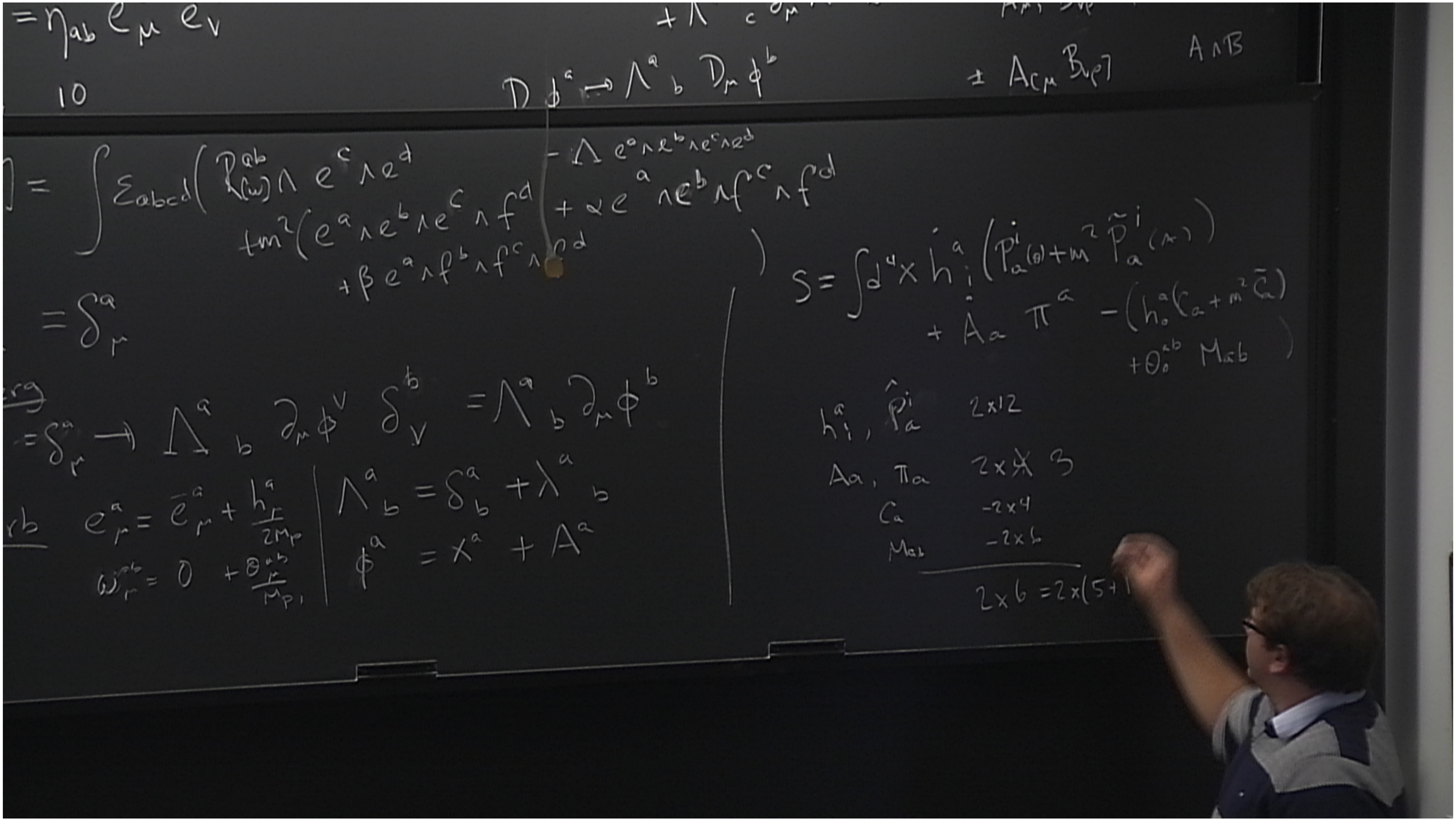
$f^a_\mu = \delta^a_\mu$

Stuckelberg

$$f^a_\mu = \delta^a_\mu \to \Lambda^a{}_b\, \partial_\mu \phi^\nu\, \delta^b_\nu = \Lambda^a{}_b\, \partial_\mu \phi^b$$

Perturb $e^a_\mu = \bar{e}^a_\mu + \frac{h^a_\mu}{2M_p}$ | $\Lambda^a{}_b = \delta^a_b + \lambda^a{}_b$

$\omega^{ab}_\mu = 0 + \frac{\theta^{ab}_\mu}{M_{p}}$ | $\phi^a = x^a + A^a$



$= \eta_{ab}\, e_\mu\, e_\nu$

10

$+\Lambda \quad c\, \partial_\mu$

$D\phi^a \mapsto \Lambda^a{}_b\, D_\mu \phi^b$

$\pm A_{[\mu} B_{\nu\rho]}$

$A \wedge B$

$$= \int \varepsilon_{abcd}\Big(R^{ab}_{(\omega)} \wedge e^c \wedge e^d - \Lambda\, e^a\wedge e^b\wedge e^c\wedge e^d$$
$$+ m^2\big(e^a\wedge e^b\wedge e^c\wedge f^d + \alpha\, e^a\wedge e^b\wedge f^c\wedge f^d$$
$$+ \beta\, e^a\wedge f^b\wedge f^c\wedge f^d\big)\Big)$$

$= \delta^a_\mu$

erg

$= \delta^a_\mu \longrightarrow \Lambda^a{}_b\, \partial_\mu \phi^\nu\, \delta^b_\nu = \Lambda^a{}_b\, \partial_\mu \phi^b$

rb

$e^a_\mu = \bar{e}^a_\mu + \frac{h^a_\mu}{2M_p}$

$\omega^{ab}_\mu = 0 + \frac{\theta^{ab}_\mu}{M_p}$

$\Lambda^a{}_b = \delta^a_b + \lambda^a{}_b$

$\phi^a = x^a + A^a$

$$S = \int d^4x\, \dot{h}^a_i \left(p^i_a(\theta) + m^2\, \tilde{p}^i_a(\kappa)\right) + \dot{A}_a\, \pi^a - \big(h^a_0 (C_a + m^2 \tilde{C}_a) + \theta^{ab}_0 M_{ab}\big)$$

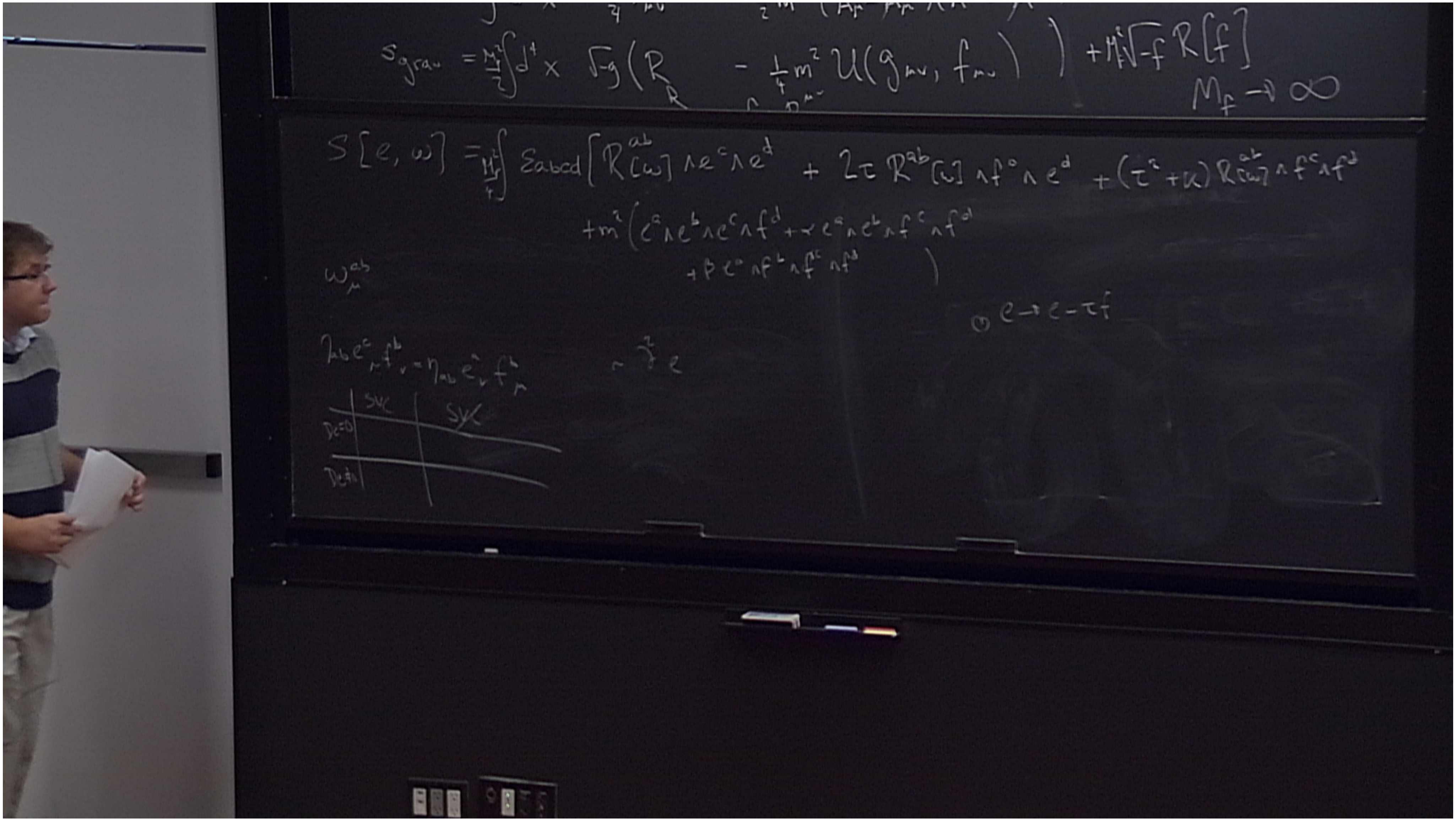
$h^a_i,\ \hat{p}^i_a$ $\quad 2\times 12$

$A_a,\ \pi_a$ $\quad 2\times \cancel{4}\ 3$

C_a $\quad -2\times 4$

M_{ab} $\quad -2\times 6$

$2\times 6 = 2\times(5+1$



$$S_{grav} = \frac{M_g^2}{2}\int d^4x \sqrt{-g}\left(R - \frac{1}{4}m^2\,\mathcal{U}(g_{\mu\nu}, f_{\mu\nu})\right) + M_f^2\sqrt{-f}\,R[f]$$

$$M_f \to \infty$$

$$S[e,\omega] = \frac{M_P^2}{4}\int \varepsilon_{abcd}\Big[R^{ab}[\omega]\wedge e^c\wedge e^d + 2\tau R^{ab}[\omega]\wedge f^c\wedge e^d + (\tau^2+\kappa)R^{ab}[\omega]\wedge f^c\wedge f^d$$
$$+ m^2\big(e^a\wedge e^b\wedge e^c\wedge f^d + \alpha\, e^a\wedge e^b\wedge f^c\wedge f^d + \beta\, e^a\wedge f^b\wedge f^c\wedge f^d\big)$$

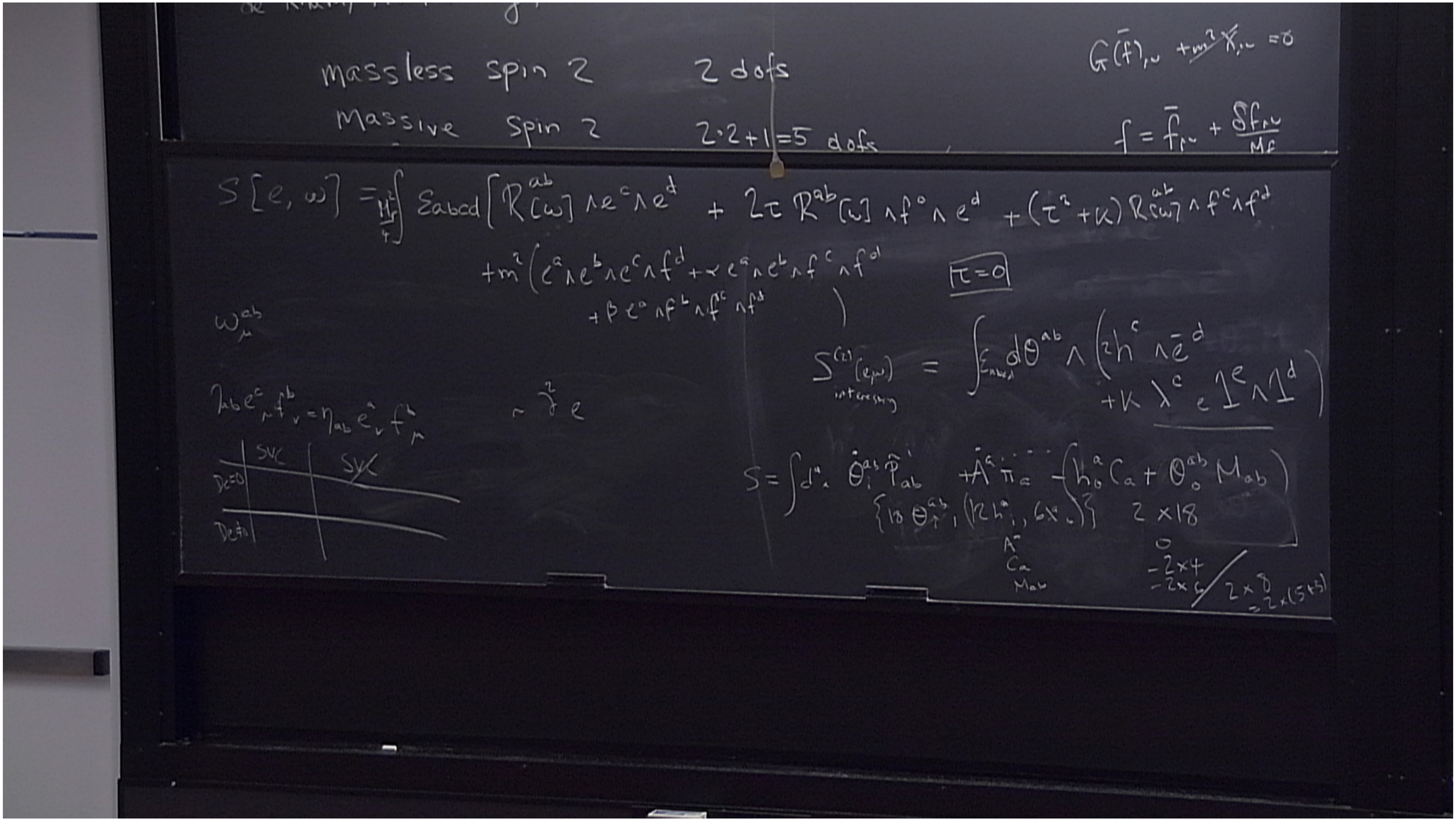
ω_μ^{ab}

① $e \to e - \tau f$

$\eta_{ab} e^a_\mu f^b_\nu = \eta_{ab} e^a_\nu f^b_\mu$

$\sim \gamma^2 e$

	SVC	$\cancel{\text{SVC}}$
$De=0$		
$De\neq 0$		



massless spin 2 2 dofs

Massive spin 2 $2\cdot 2+1=5$ dofs

$$G(\bar{f})_{\mu\nu} + \cancel{m^2 X_{\mu\nu}} = 0$$

$$f = \bar{f}_{\mu\nu} + \frac{\delta f_{\mu\nu}}{M_f}$$

$$S[e,\omega] = \frac{M_P^2}{4}\int \varepsilon_{abcd}\Big[R^{ab}[\omega]\wedge e^c\wedge e^d + 2\tau R^{ab}[\omega]\wedge f^c\wedge e^d + (\tau^2+\kappa)R^{ab}[\omega]\wedge f^c\wedge f^d$$
$$+m^2\big(e^a\wedge e^b\wedge e^c\wedge f^d + \alpha\, e^a\wedge e^b\wedge f^c\wedge f^d + \beta\, e^a\wedge f^b\wedge f^c\wedge f^d\big)$$

$$\boxed{\tau=0}$$

ω_μ^{ab}

$$\eta_{ab}e^a_\mu f^b_\nu = \eta_{ab}e^a_\nu f^b_\mu \qquad \sim \gamma^2 e$$

	SVC	S~~V~~C
$D_c=0$		
$D_c\neq 0$		

$$\underset{\text{interesting}}{S^{(2)}(e,\omega)} = \int \varepsilon_{abcd}\, d\theta^{ab}\wedge\big(2h^c\wedge\bar{e}^d + \kappa\,\lambda^c{}_e\,\underline{1^e\wedge 1^d}\big)$$

$$S = \int d^4x\ \dot{\theta}^{ab}_i\,\hat{P}^i_{ab} + \dot{A}^a_i\,\pi_a^i - \big(h^a_0 C_a + \theta^{ab}_0 M_{ab}\big)$$

$\{18\ \theta^{ab}_i,\ (12\ h^a_i,\ 6\ \chi^a_i)\}$ 2×18

A^a 0

C_a -2×4

M_{ab} -2×6

2×8
$= 2\times(5+3)$