Title: AMATH 875/PHYS 786 - Fall 2015 - Lecture 17

Date: Nov 06, 2015 01:30 PM

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Abstract: <p>Course Description coming soon.</p>

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Note Title

GR for cosmology, Achim Kempf, Fall 2015, Lecture 17

Evolution of Friedmann-Lemaître spacetimes

- Depending on what is the major contributor to $T_{\mu\nu}$, there is an effective "Equation of State": $\boxed{p = p(\rho)}$

- Periods of time in which the eqn. of state can be approximated as:

$$p(\rho) = w\rho \qquad \text{with} \qquad w = \text{const.}$$

are called Cosmic Epochs. For example:

Radiation-dominated epoch: $\quad w = 1/3$

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Radiation-dominated epoch:	$w = 1/3$
Matter ("dust")-dominated epoch:	$w = 0$
Dark energy-dominated epoch:	$w = -1$

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- For any given epoch, use its $p(\rho)$ to solve (from previous lecture)

Continuity equation $\rightarrow$ $\frac{d}{da}\left(\rho a^3\right) = -3p(\rho)\, a^2$, i.e.: $\frac{d}{da}\left(\rho(a)\, a^3\right) = -3a^2 w\, \rho(a)$

to obtain $\rho(a)$, which shows how energy is diluting:

- **Solution:** $$\boxed{\rho(a) = \rho_o\, a^{-3(w+1)}}$$ Exercise: verify

- Key special cases:

dilution of matter, i.e., energy proportional to $\frac{1}{\text{Volume}} \sim \frac{1}{a^3}$

dilution of energy $\sim \frac{1}{\text{Volume}}$ and energy loss due to wavelength stretching $\sim \frac{1}{a}$

vacuum energy due to cosmological constant is of course constant.

$$\rho(a) = \begin{cases} \rho_m\, a^{-3} & \text{in matter-dominated epoch } (w=0) \\ \rho_r\, a^{-4} & \text{in radiation-dominated epoch } (w=1/3) \\ \rho_v\, a^{0} & \text{in dark energy-dominated epoch } (w=-1) \end{cases}$$

We know of no physical mechanism that could cause $w<-1$. Yet,

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We know of no physical mechanism that could cause $w<-1$. Yet, some evidence suggests it might be the case today $\rightarrow$ **Note:** $w<-1$ would mean $\rho(a) = \rho_s\, a^{\varepsilon}$ ($\varepsilon>0$) i.e. ρ increases with a.

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- Now use $\rho(a)$ to turn the Friedmann eqn. into an ordinary differential equation for $a(t)$:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho(a)$$

(we omit the Λ term by agreeing to incorporate Λ in the definition of ρ, p.)

- Observational evidence: the universe is spatially flat, i.e., $K=0$, in a good approximation.

- Solution for $K=0$ and $w \neq -1$:

$$\boxed{a(t) = \left(\frac{\pm t}{t_0}\right)^{\frac{2}{3(1+w)}}}$$

Note that, because $\dot{a}$ is squared in the Friedmann equation,

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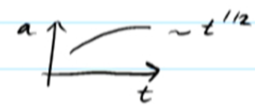
$$\boxed{a(t) = \left(\frac{\pm t}{t_0}\right)^{\frac{2}{3(1+w)}}}$$

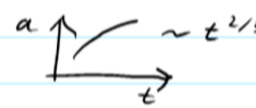
Note that, because $\dot{a}$ is squared in the Friedmann equation, there is always an expanding along with a contracting solution.

Key epochs: (Exercise/project: what if $K>0$ or $K<0$?)

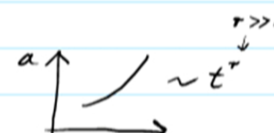
$(t/t_r)^{1/2}$ in a radiation-dominated epoch $w = 1/3$



$(t/t_d)^{2/3}$ in a matter-dominated epoch: $w = 0$

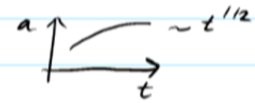


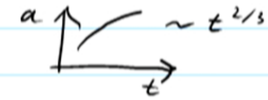
$(t/t_p)^{r}$ with $r \gg 1$ in a so-called "power

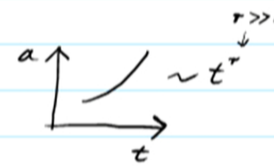


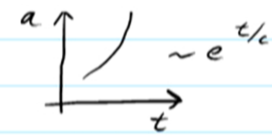
Key epochs: (Exercise/project: what if $K>0$ or $K<0$?)

$$a(t) = \begin{cases} (t/t_r)^{1/2} & \text{in a radiation-dominated epoch } w = 1/3 \\ (t/t_d)^{2/3} & \text{in a matter-dominated epoch: } w = 0 \\ (t/t_p)^{r} & \text{with } r \gg 1 \text{ in a so-called ``power law epoch'': } w = -1 + \frac{2}{3r} \text{ (Exercise: verify)} \\ e^{t/t_v} & \text{in a totally dark energy dominated epoch: } w = -1. \text{ Exercise: Show this.} \end{cases}$$









Definition: Any epoch in which $\ddot{a} > 0$, i.e., in which

$w < -1/3$ (exercise: verify), is called an "*inflationary epoch*".

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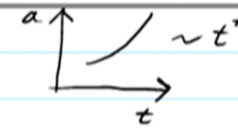

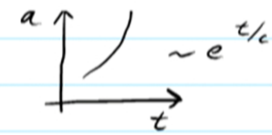

Definition: Any epoch in which $\ddot{a} > 0$, i.e., in which $w < -1/3$ (exercise: verify), is called an "inflationary epoch".

Most likely timeline:

see below for precise definition of $\rho_{critical}$

Best fit today: $K = 0$

$\Lambda \approx 0.7\, \rho_{critical}$ ("dark energy")

$\rho_{matter} \approx 0.3\, \rho_{critical}$

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Best fit today: $K = 0$

$\Lambda \approx 0.7\, \rho_{critical}$ ("dark energy")

$\rho_{matter} \approx 0.3\, \rho_{critical}$

$\rho_{dark\ matter} \approx 0.9\, \rho_{matter}$

$\rho_{visible\ matter} \approx 0.1\, \rho_{matter}$

Most likely timeline:

short period of matter domination by "inflaton" particles which then decay leaving a hot soup of all sorts of particles

A field, ϕ, inducing "dark energy" dominates ; radiation dominates ; matter dominates ; A Λ, inducing a "dark energy" dominates

.... ? 10^{-43}s — 10^{-32}s — 10^{-11}s — 10^{0}s — 10^{13}s — 10^{17}s $\approx \log(t)$

? Planck era

Inflation $\;w = -1 + \varepsilon$ ← very small

10^{0}s nuclei form

10^{13}s atoms form & CMB released (10^5 yrs)

10^{17}s today

Why inflation?

at this time, the temperature was so high that particle collisions occurred at a typical energy of $1\,TeV = 1.6 \cdot 10^{-19} \cdot 10^{12}\, J$ which is about

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particle collisions occurred at a typical energy of $1\,\text{TeV} = 1.6 \cdot 10^{-19} \cdot 10^{12}\,J$ which is about the maximal energy that accelerator experiments e.g. at CERN can currently impart on particles.

The flatness problem:

Reconsider the experimental finding of spatial flatness, $K = 0$:

- Rewrite the Friedmann equation

$$3\left(\frac{\dot{a}}{a}\right)^2 + 3\,\frac{K}{a^2} = 8\pi G \rho + \Lambda$$

Hubble param

/ (const. in sp...

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Hubble parameter (const. in space but not in time)

by incorporating Λ in $\rho_{tot} = \rho + \frac{\Lambda}{8\pi G}$ and setting $H := \frac{\dot a}{a}$:

$$H(t)^2 + \frac{K}{a(t)^2} = \frac{8\pi G}{3}\,\rho_{tot}(t)$$

$\Rightarrow$ At any given time, the critical energy density for $K=0$, i.e., for space to be flat, is:

$$\rho_{crit}(t) = \frac{3}{8\pi G}\,H(t)^2$$

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- How close to critical are we now, and at other times?

Definition:

how close to one is/was it?

$$\Omega(t) := \frac{\rho_{tot}(t)}{\rho_{crit}(t)} \quad \text{, i.e.: } \quad \rho_{tot}(t) = \Omega(t)\,\frac{3}{8\pi G}\,H(t)^2$$

- Thus, the Friedmann equation becomes:

$$H(t)^2 + \frac{K}{a(t)^2} = \Omega(t)\,H(t)^2$$

i.e.

$$\boxed{\Omega(t) - 1 = \frac{K}{\dot{a}(t)^2}}$$

Exercise: check

- Calculate backwards through the matter-dominated epoch, $a \sim t^{2/3}$, $\dot{a} \sim t^{-1/3}$:

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Calculate backwards through the matter-dominated epoch, $a \sim t^{2/3}$ $\dot{a} \sim t^{-1/3}$:

Thus: $\Omega(t) - 1 = K\, t^{2/3}$

(radiation-dominated epoch before: $a \sim t^{1/2} \Rightarrow \dot{a} \sim t^{-1/2}$
$\Rightarrow \Omega(t) - 1 = K\, t$)

$$\Rightarrow \quad \boxed{\frac{\Omega(t_1) - 1}{\Omega(t_0) - 1} = \left(\frac{t_1}{t_0}\right)^{2/3}}$$

Notice:
The unit-dependent K dropped out.

Given that $\Omega(t_1) - 1 = \mathcal{O}(1)$ today, at time t_1, much earlier, say at $t_0 = 10^{-6} t_1$, we had

$$\Omega(t_0) - 1 = \mathcal{O}(10^{-4})$$

accelerator physics goes so far

At $t = 10^{-30} t_1$ (i.e. at $t = 10^{-11}s$) we had

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accelerator physics goes so far

At $t_n = 10^{-30} t_1$ (i.e. at $t = 10^{-11}s$) we had

$$\Omega(t_n)-1 \approx \mathcal{O}(10^{-24})$$

in radiation-dominated epoch the effect is even greater since $\Omega - 1 \sim t$

$\Rightarrow$ Flatness is not stable! The universe must have started out flat with tremendous accuracy to be

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- Yes! (Brout, Englert, Starobinsky, Linde et al ≈ 1980)

To this end, conjecture an early cosm. epoch in which

$$\Omega(t) - 1 = \frac{k}{\dot{a}(t)^2}$$ with $\dot{a}(t)$ increasing with t.

- The current standard model of cosmology therefore postulates an early epoch with:

$$\ddot{a}(t) > 0$$

Recall: We call such an epoch inflationary and it arises whenever $w < -\frac{1}{3}$. ("Inflationary attractor")

- Experimental constraints?

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Experimental constraints?

In order to account for the degree of flatness observed today (and cross-checked in the CMB), a period of near-exponential inflation should have expanded the universe by a factor of at least

$$\frac{a(t_{\text{before}})}{a(t_{\text{after}})} \approx e^{60}$$

The conjecture of an early inflationary epoch also explains

- The absence of exotic high mass particles that would likely have been produced close to Planck time (and only then).

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- The absence of exotic high mass particles that would likely have been produced close to Planck time (and only then).

Namely: The inflationary expansion extremely dilutes all particles.

But also: At the end of the inflationary epoch, how did it happen that the universe was filled with a high density of matter?

Currently favored solution:

The inflationary epoch occured when a scalar field ϕ had a large potential: — temporarily large

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Currently favored solution:

The inflationary epoch occurred when a scalar field ϕ had a large potential:

temporarily large

Recall:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

so that, because of the large $V(\phi)$ we had:

$$w = \frac{p_\phi}{\rho_\phi} \simeq -1$$

i.e. power law inflation

After inflation, $V(\phi)$ becomes the kinetic and mass energy of all sorts of particles, thus making a hot primordial soup. After this "re-heating", followed ordinary big bang cosmology.

Speculation:

How did inflation start? In any spacetime a single quantum fluctuation of ϕ might elevate $V(\phi)$ locally so as to spawn a new universe!

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- The "horizon problem":

  Why does the CMB have the same properties even when checking in opposite directions in the sky?

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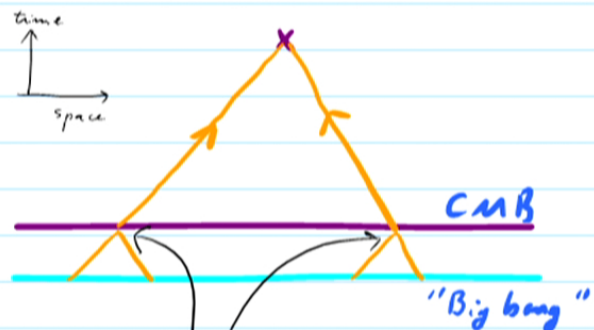

these two areas of the surface that emitted CMB photons do not have a common past. How come they are so similar?

Concretely: Only patches on the CMB sky of angular extent < 1 degree have a common past, if there was no inflation.

▫ Answer: If the

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The conjecture of an early inflationary epoch also explains

- the occurrence and precise statistics of inhomogeneities in the universe!

- How? The quantum fluctuations of scalar fields (unlike those of spinor fields of, e.g., electrons and vector fields of, e.g., photons) are being amplified in an inflationary epoch, along with those of g.

  ⇒ They are thought to have seeded the inhomogeneities in the CMB and therefore ultimately the condensation of hydrogen into galaxies and stars.

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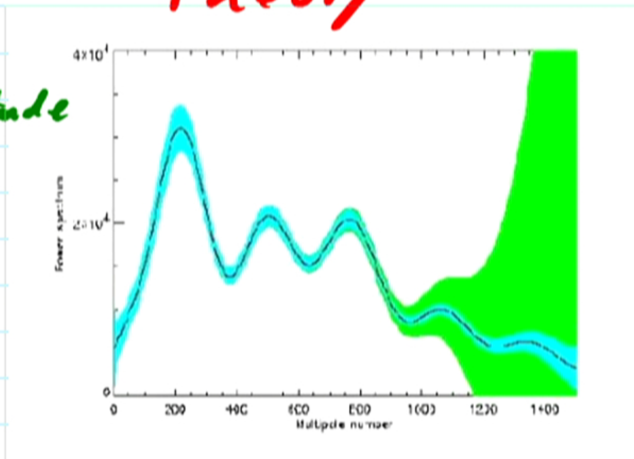
then the predicted statistics was (1980s):

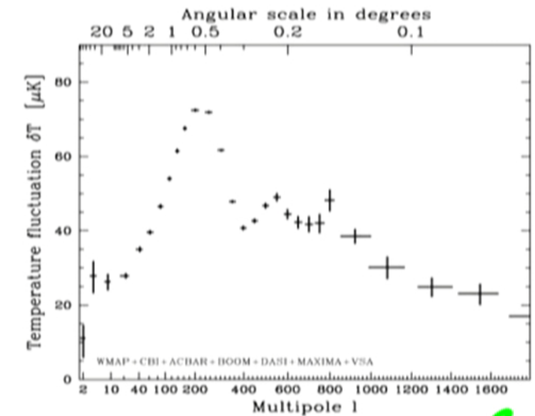
Theory

Amplitude



"angular wavelength^{-1}" ℓ

Experiment



ℓ

Remark: A competing theory held that phase transitions, as the universe cooled, left behind "topological defects" in the vacuum, much like crystal imperfections. Their statistics would be measurably differ