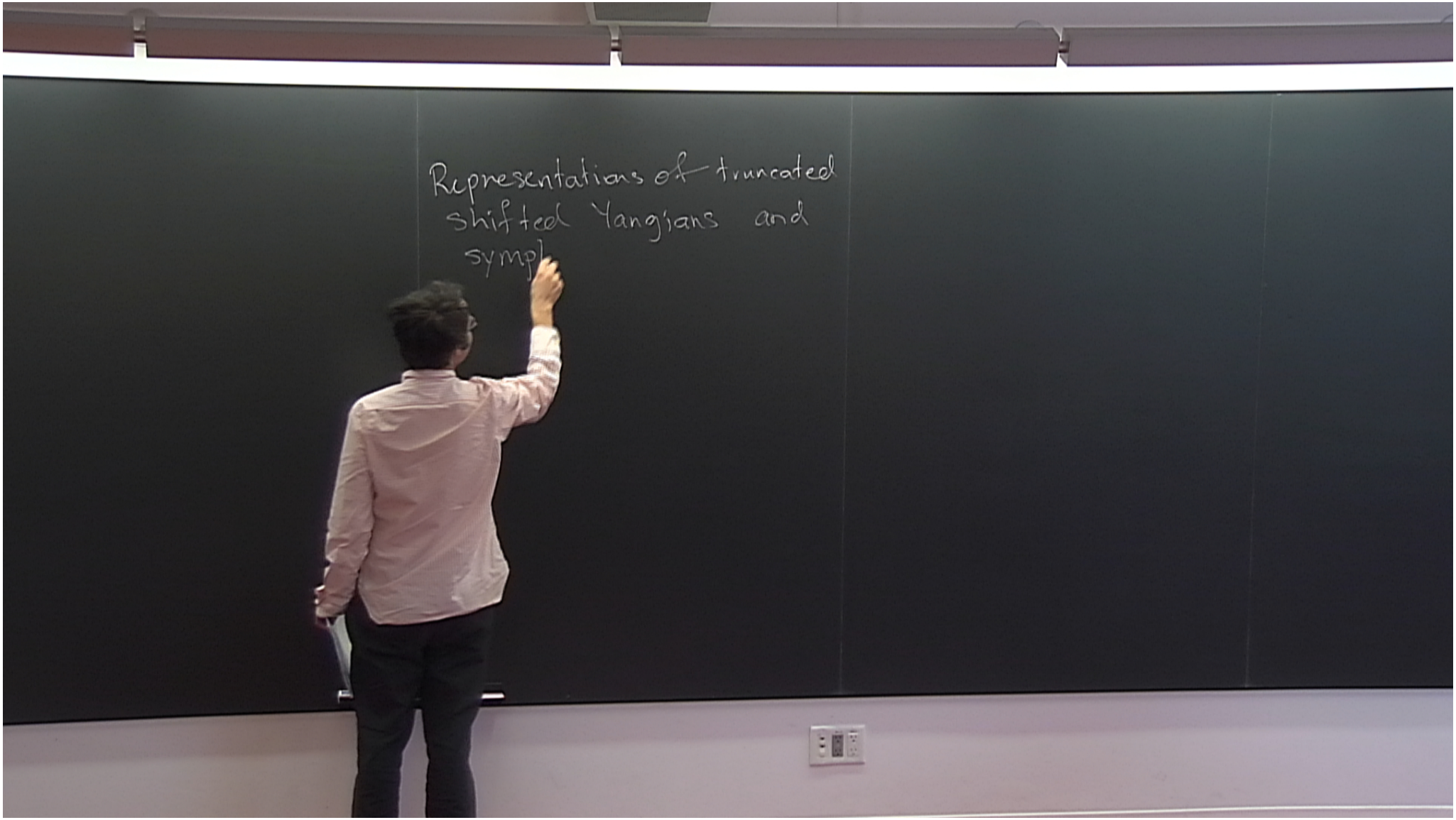
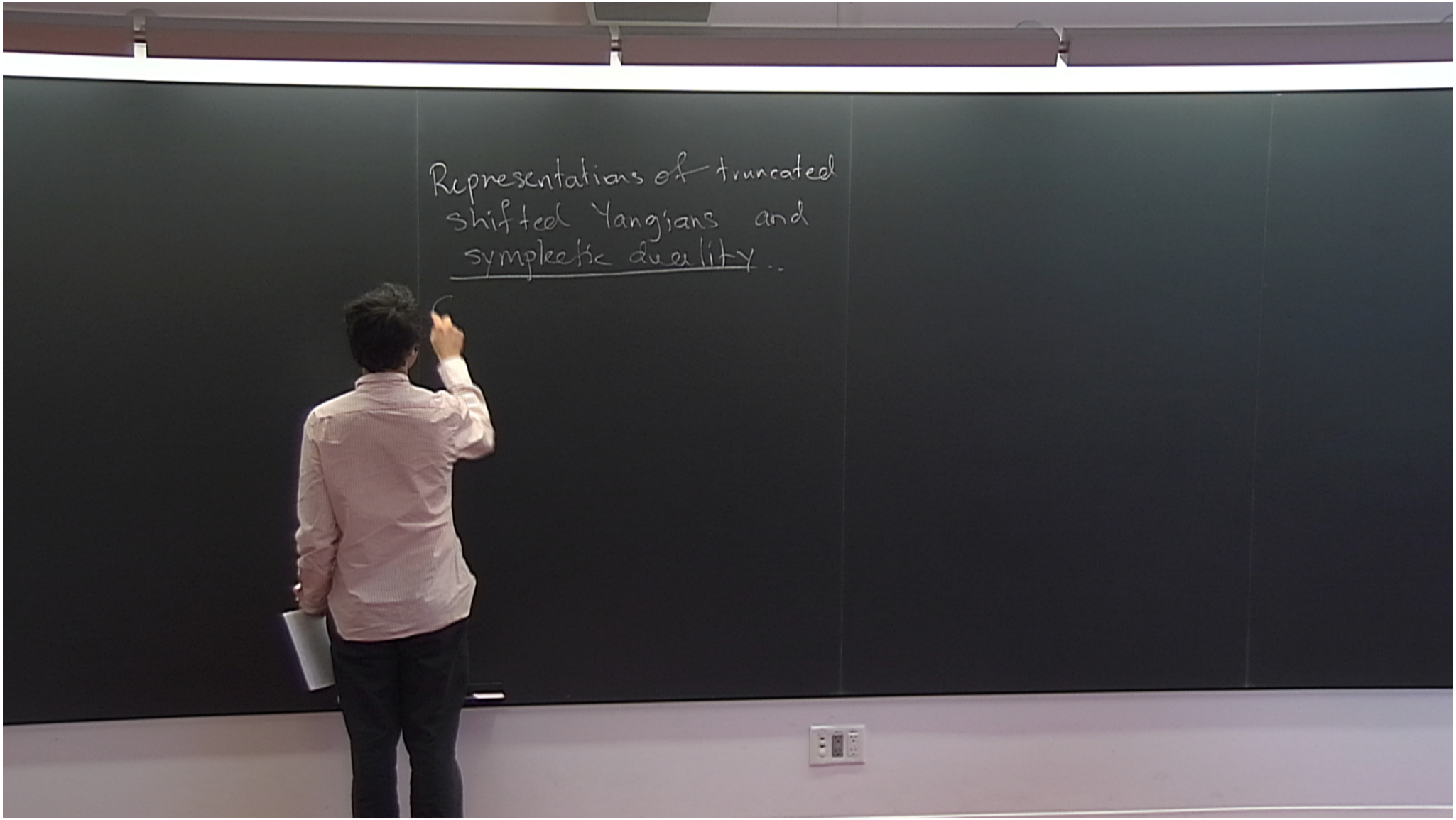
Title: Representations of truncated shifted Yangians and symplectic duality

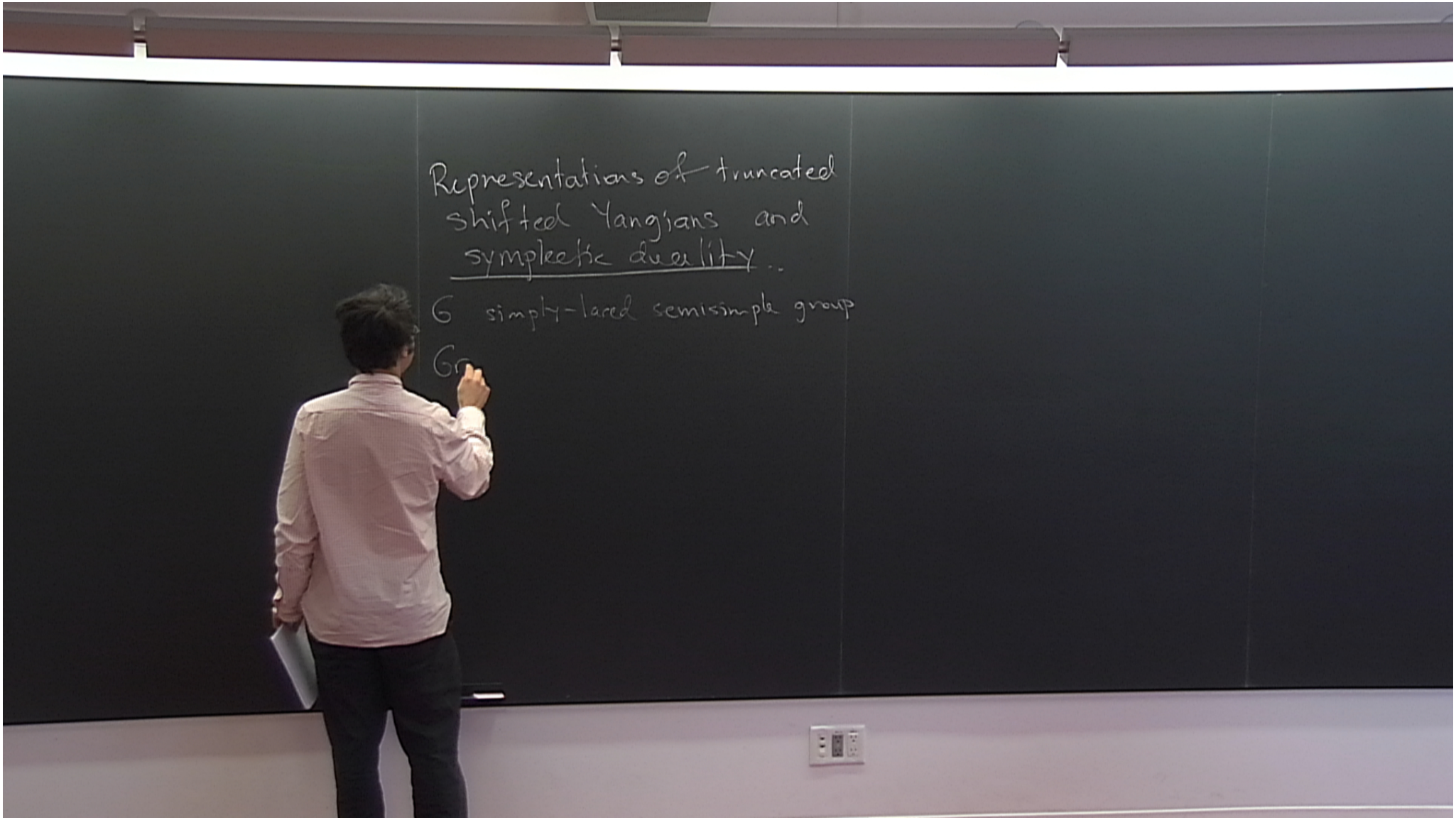
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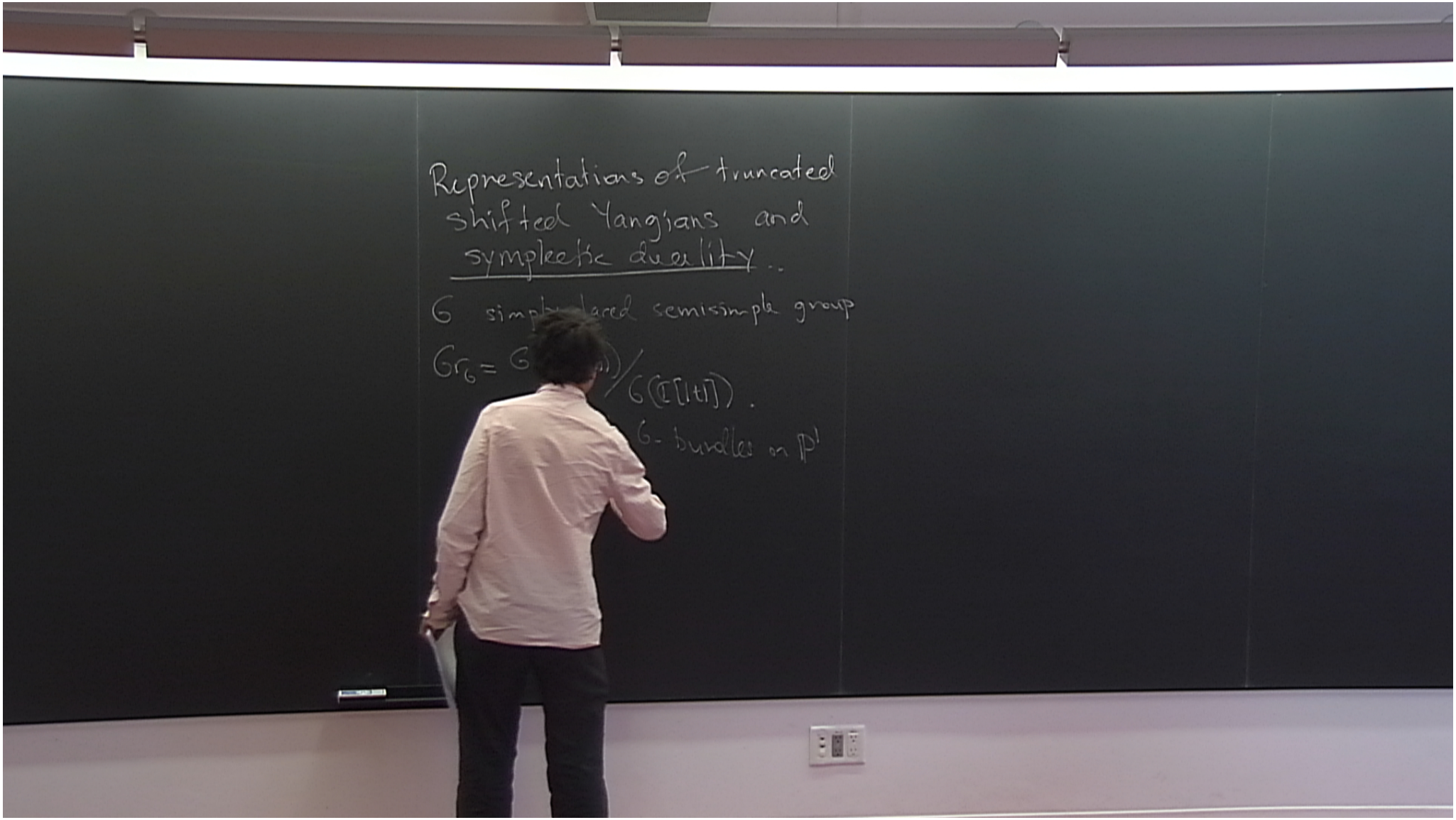
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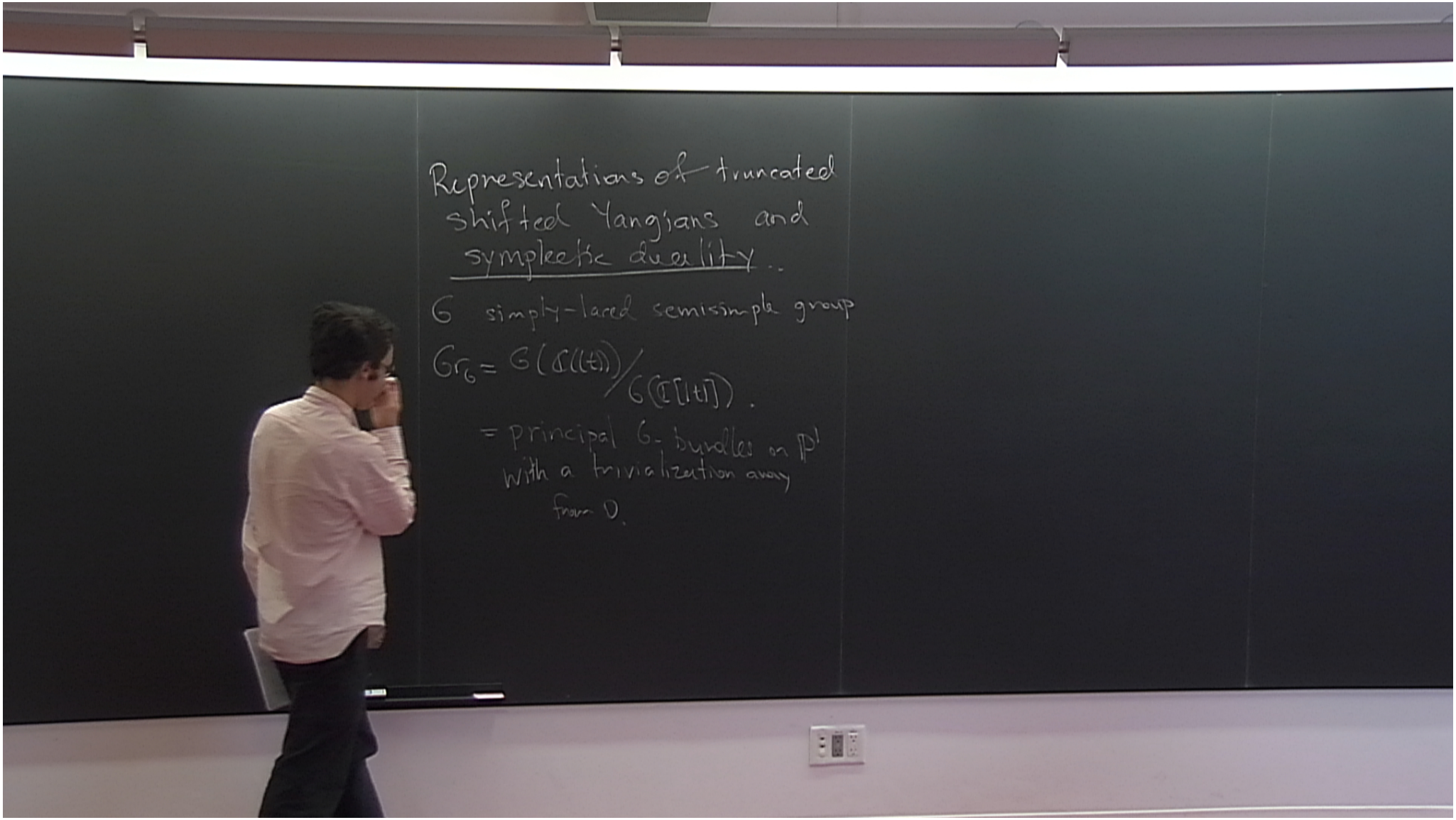
Abstract: We study the representation theory of truncated shifted Yangians. These algebras arise as quantizations of slices to Schubert varieties in the affine Grassmannian. We will describe the combinatorics of their highest weights, which is encoded in Nakajima's monomial crystal. We also prove Hikita's conjecture in this context.

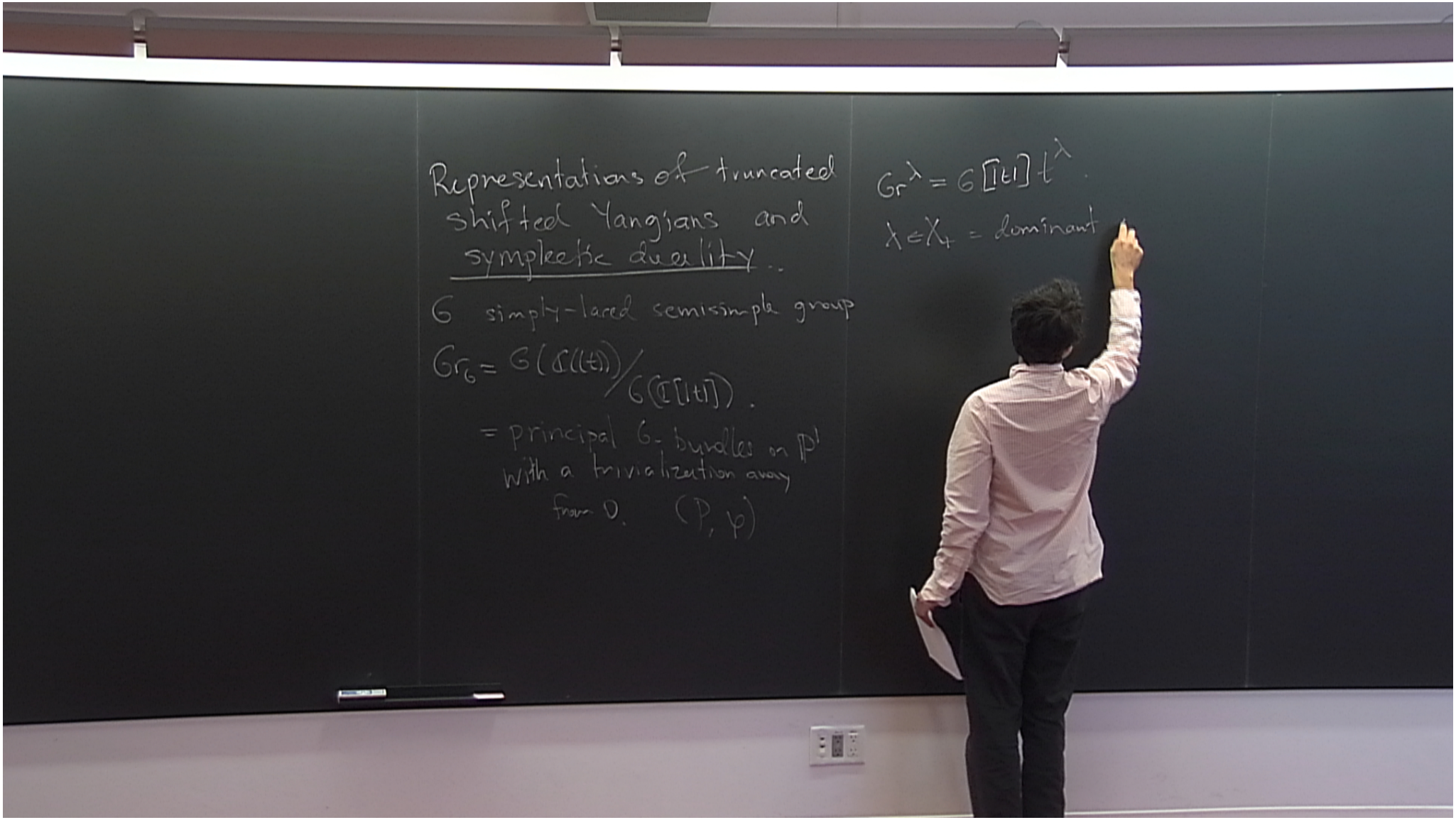

Representations of truncated
shifted Yangians and
sympl

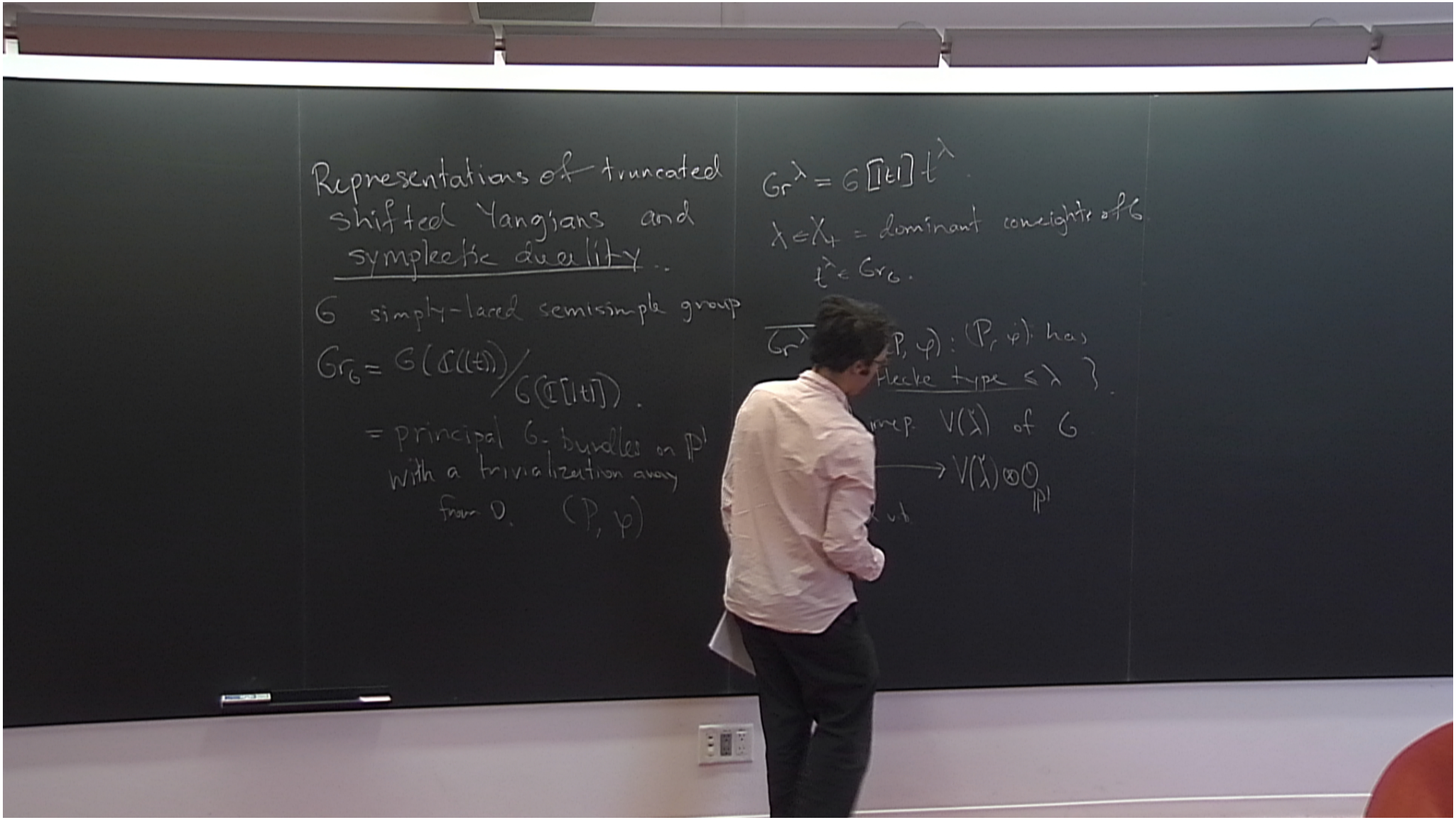

Representations of truncated
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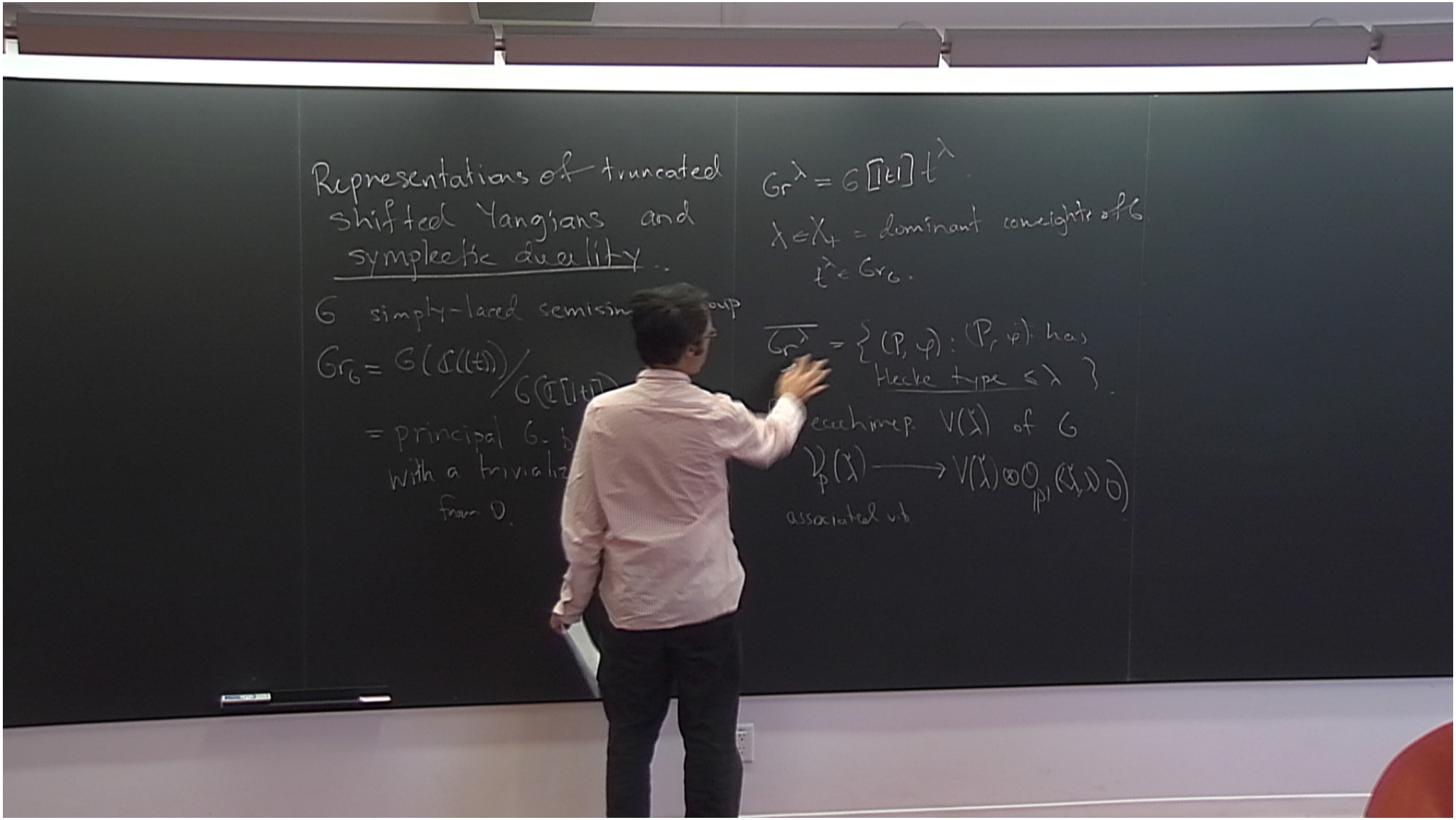

Representations of truncated shifted Yangians and symplectic duality..
G simply-laced semisimple group

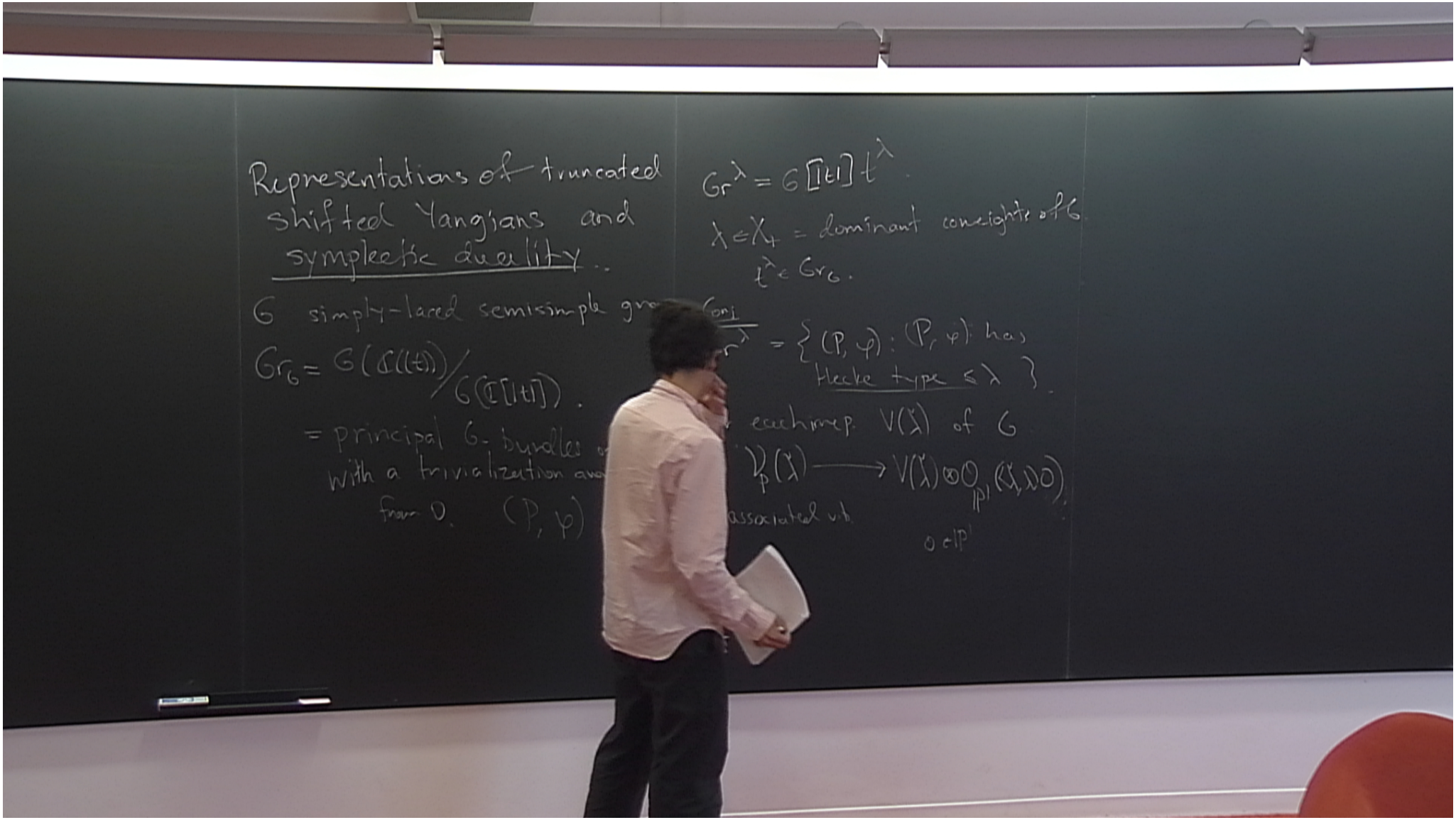

Representations of truncated shifted Yangians and symplectic duality..
G simply-laced semisimple group
G(C[[t]]).
G-bundles on P¹


Representations of truncated shifted Yangians and symplectic duality..
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$Gr_G = G(\mathcal{K}((t)))/G(\mathbb{C}[[t]])$.
= principal G-bundles on $\mathbb{P}^1$ with a trivialization away from 0.


Representations of truncated shifted Yangians and symplectic duality.
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$Gr^\lambda = G[[t]] \cdot t^\lambda$.
$\lambda \in X_+$ = dominant


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$\lambda \in X_+$ = dominant coweights of G
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$(P, \varphi) : (P, \varphi)$ has Hecke type $\leq \lambda$ }
rep. $V(\check{\lambda})$ of G
$V(\check{\lambda}) \otimes \mathcal{O}_{\mathbb{P}^1}$


Representations of truncated shifted Yangians and symplectic duality.
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$\lambda \in \Lambda_+$ = dominant coweights of G
$t^\lambda \in Gr_G$.
$\overline{Gr^\lambda}$ = { (P, φ) : (P, φ) has Hecke type ≤ λ }
eachimp. $V(\lambda)$ of G
$\mathcal{W}_p(\lambda) \longrightarrow V(\lambda) \otimes \mathcal{O}_{\mathbb{P}^1}(\langle \lambda, \lambda \rangle \mathcal{O})$
associated v.b.


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each irrep. $V(\lambda)$ of G
$V_p(\lambda) \longrightarrow V(\lambda) \otimes \mathcal{O}_{\mathbb{P}^1}(\ldots)$
associated v.b.
$0 \in \mathbb{P}^1$

Representations of truncated shifted Yangians and symplectic duality.

G simply-laced semisimple group

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$Gr_G^\lambda = G[[t]] t^\lambda$.

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$t^\lambda \in Gr_G$.

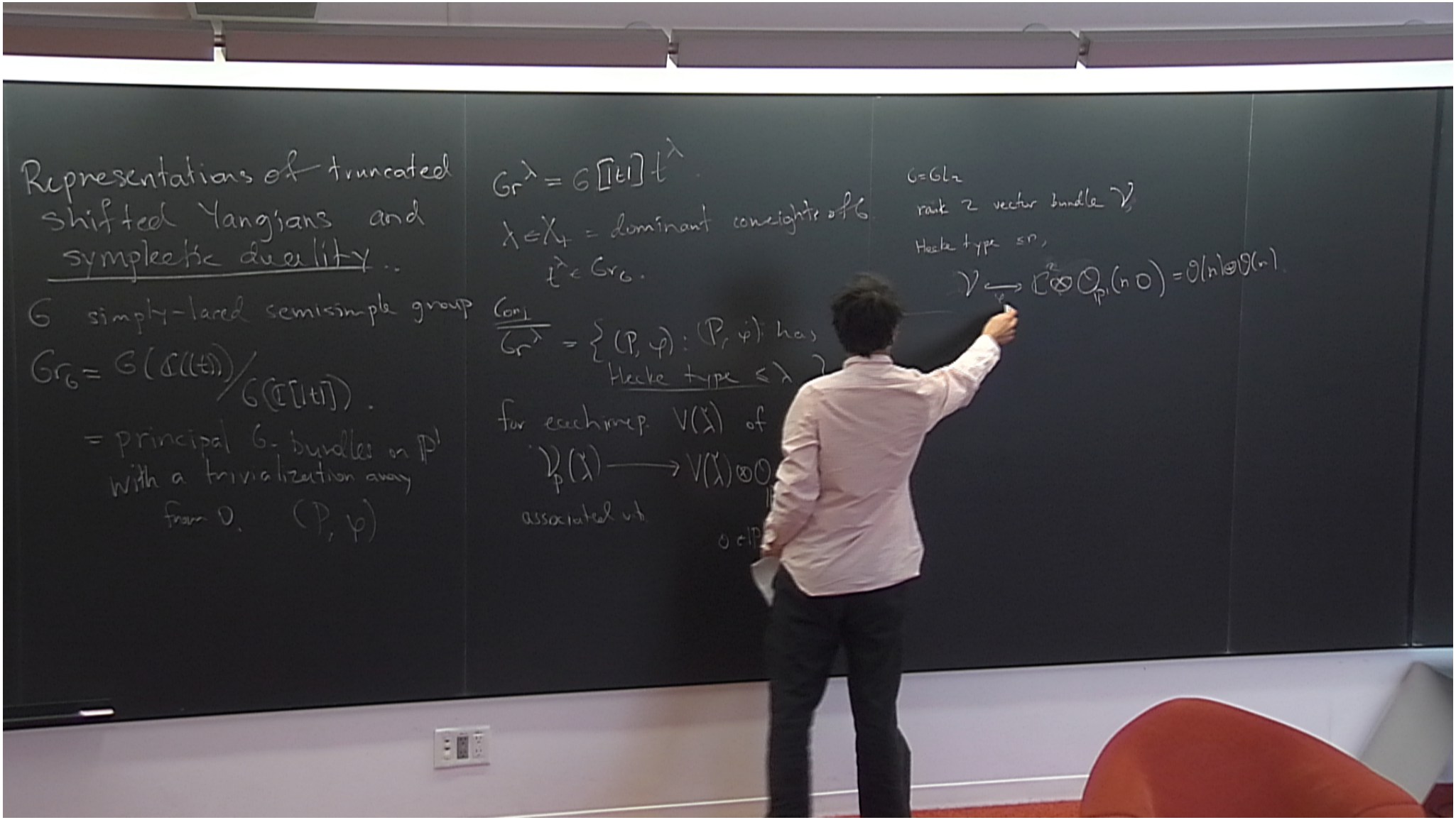
$\overline{Gr_G^\lambda} = \{(\mathcal{P}, \varphi) : (\mathcal{P}, \varphi)$ has Hecke type $\leq \lambda\}$.

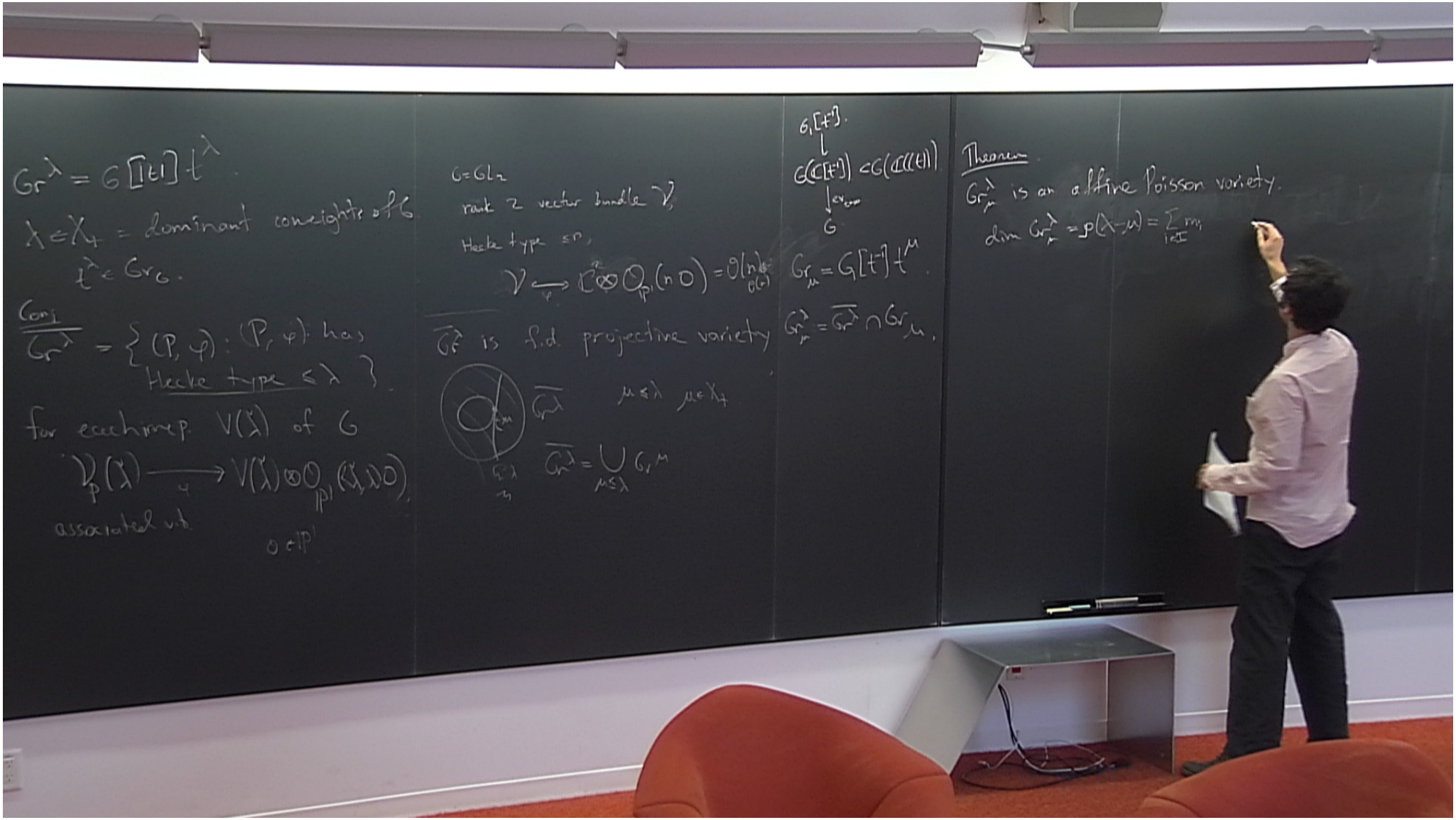
for each irrep. $V(\check{\lambda})$ of G

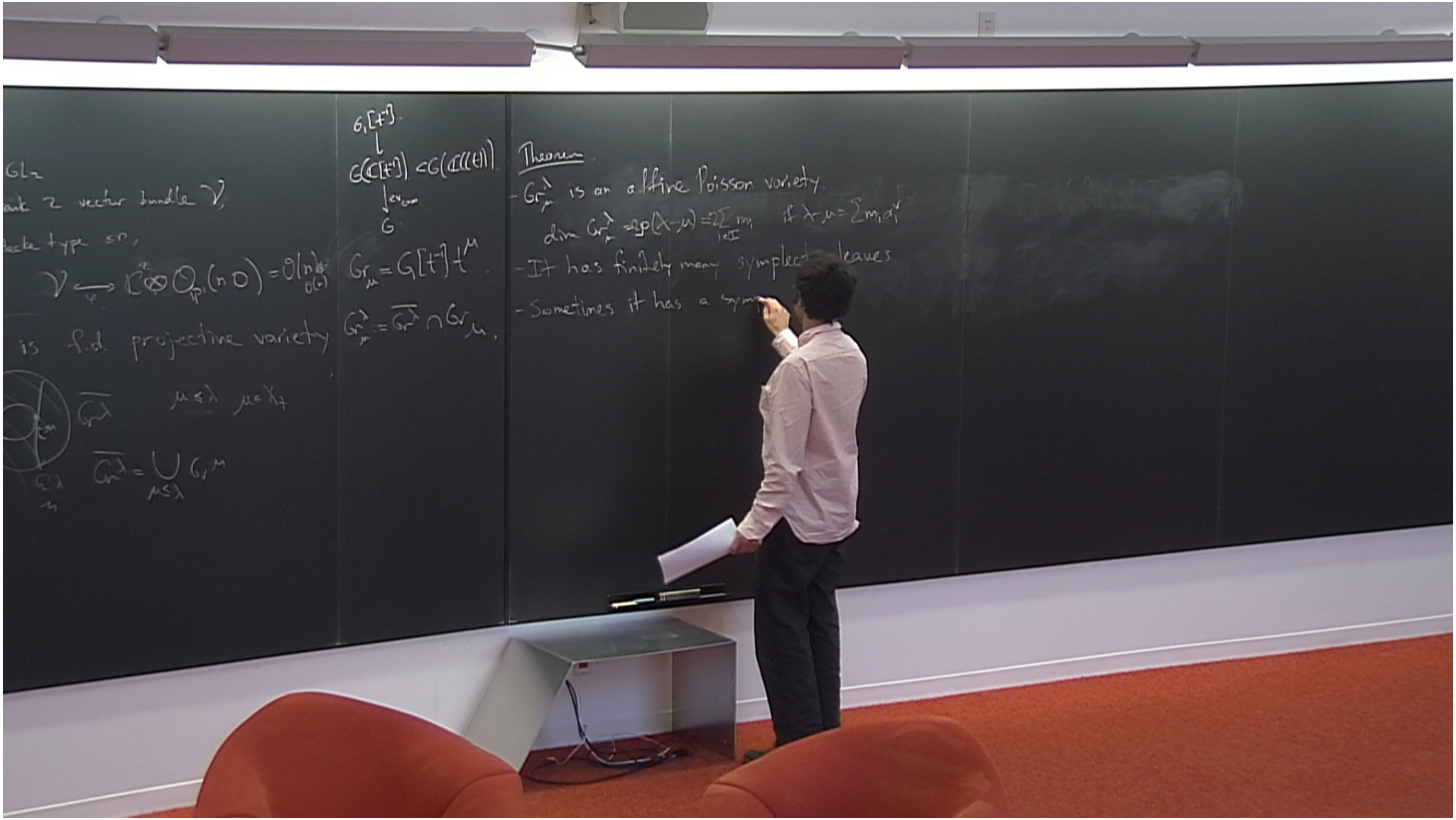
$\mathcal{V}_{\mathcal{P}}(\check{\lambda}) \longrightarrow V(\check{\lambda}) \otimes \mathcal{O}_{\mathbb{P}^1}(\check{\lambda}, 1 \cdot 0)$

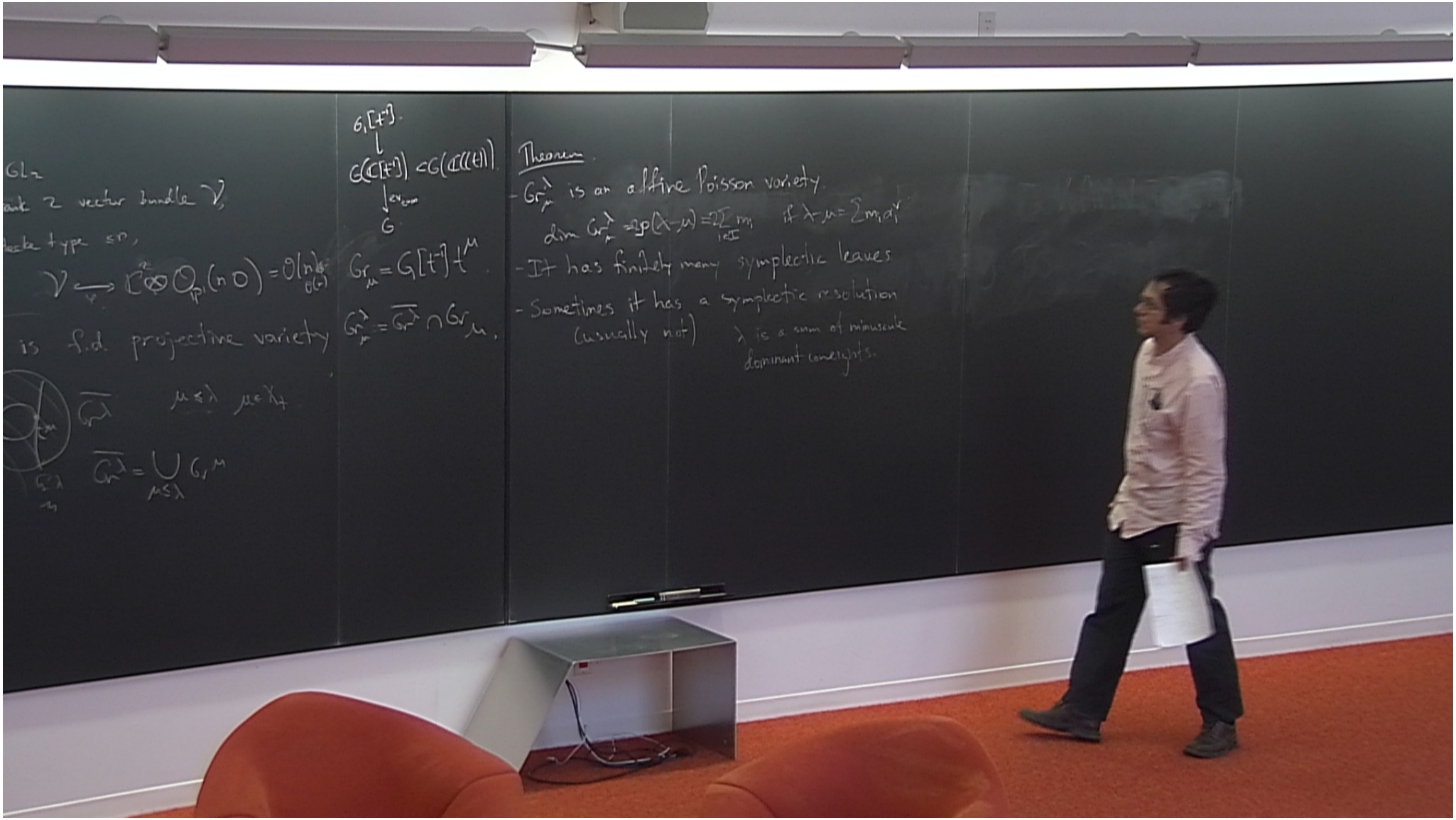
associated v.b.

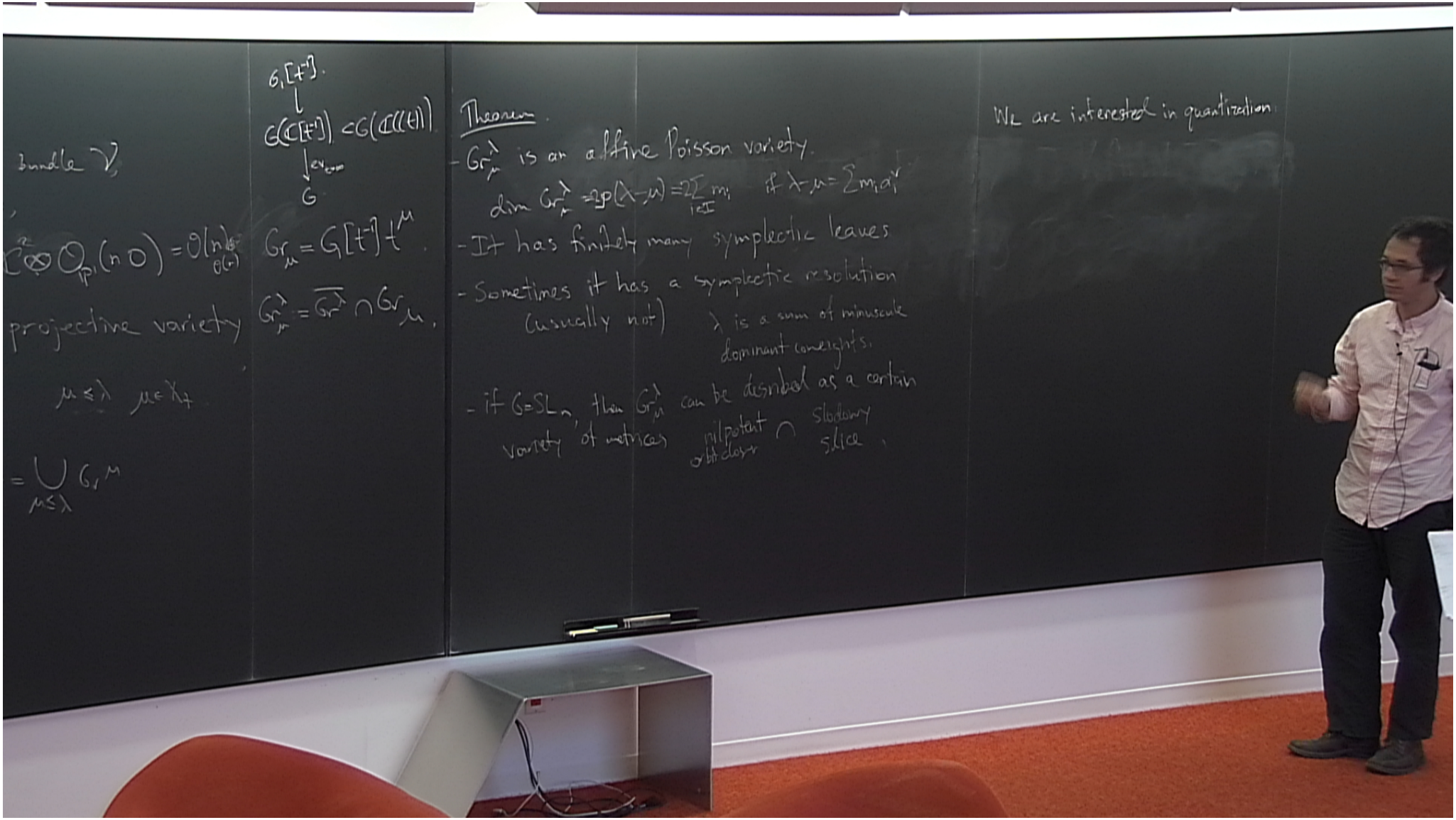
$0 \in \mathbb{P}^1$

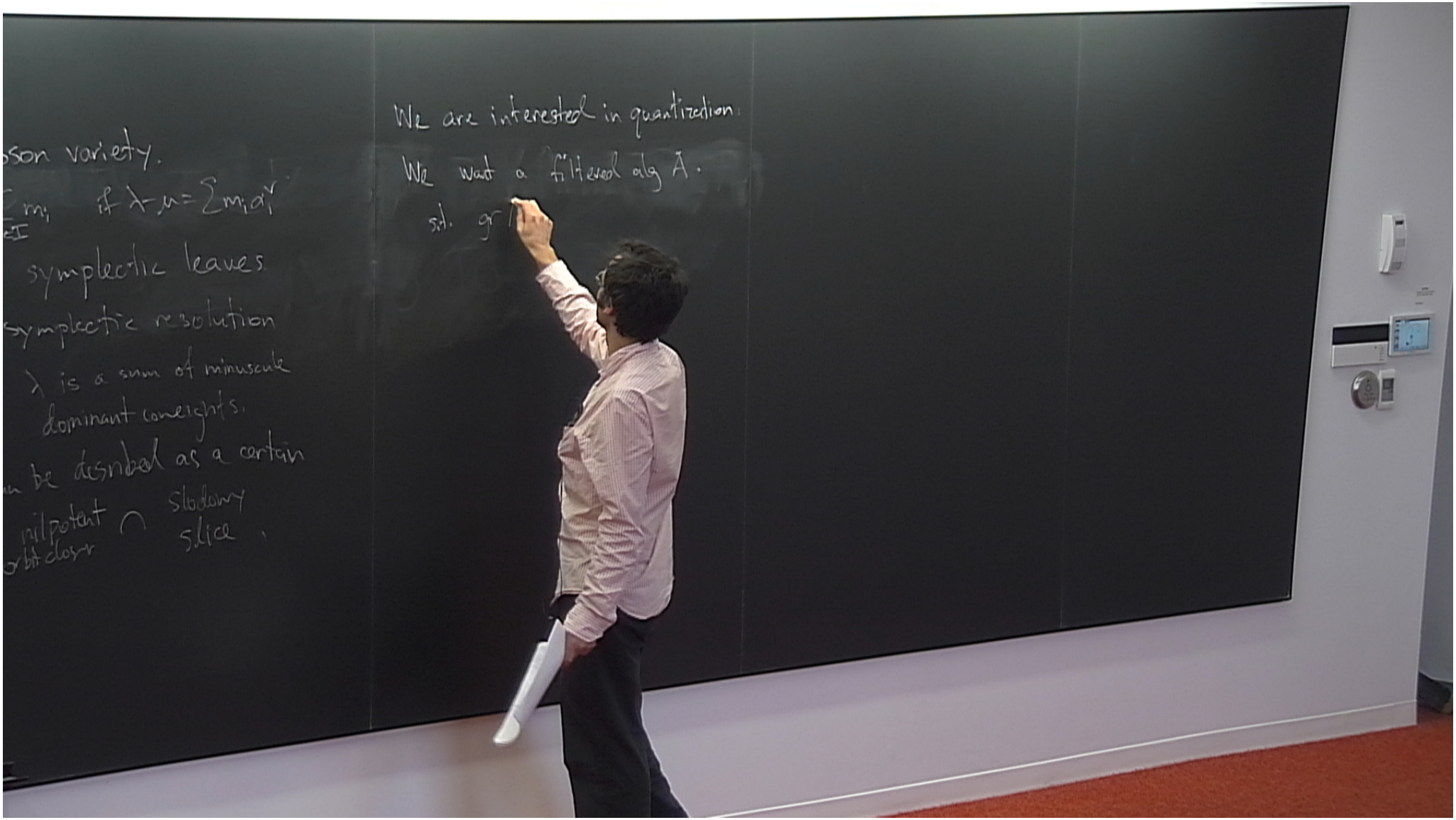

Representations of truncated shifted Yangians and symplectic duality.
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$Gr_G = G(\mathcal{K}((t)))/G(\mathbb{C}[[t]])$.
= principal G-bundles on $\mathbb{P}^1$ with a trivialization away from 0. $(\mathcal{P}, \varphi)$
$Gr^\lambda = G[[t]]t^\lambda$
$\lambda \in X_+$ = dominant coweights of G
$t^\lambda \in Gr_G$.
$\overline{Gr^\lambda} = \{(\mathcal{P}, \varphi) : (\mathcal{P}, \varphi)$ has Hecke type $\leq \lambda\}$
for each irrep $V(\lambda)$ of
$\mathcal{V}_p(\lambda) \longrightarrow V(\lambda) \otimes \mathcal{O}$
associated v.b.
$G = GL_2$
rank 2 vector bundle $\mathcal{V}$
Hecke type $\leq n$,
$\mathcal{V} \hookrightarrow \mathbb{C}^2 \otimes \mathcal{O}_{\mathbb{P}^1}(n \cdot 0) = \mathcal{O}(n) \oplus \mathcal{O}(n)$

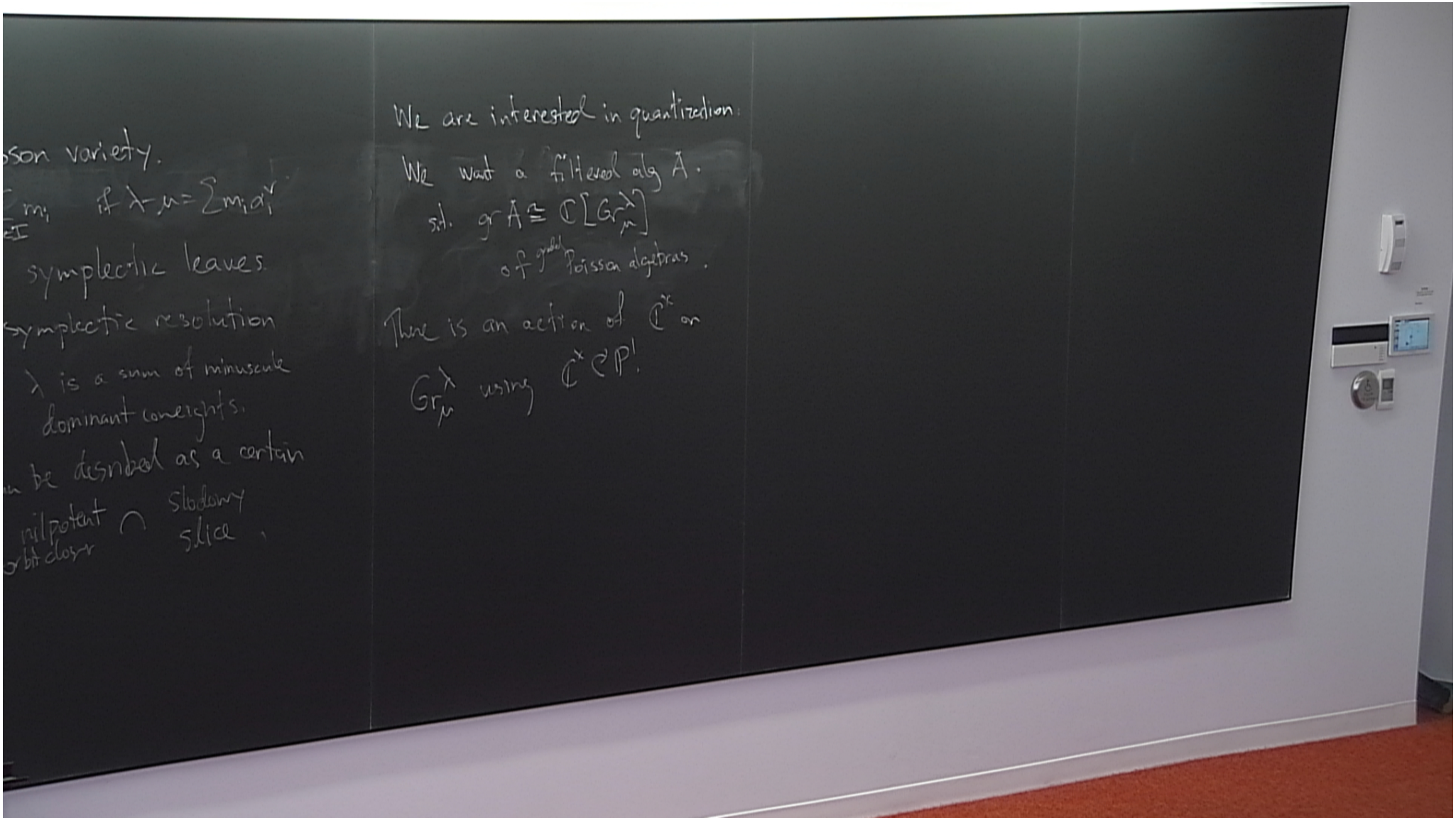

$Gr^\lambda = G[[t]]t^\lambda$
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Conj
$\overline{Gr^\lambda}$ = { $(\mathcal{P}, \varphi)$: $(\mathcal{P}, \varphi)$ has Hecke type $\leq \lambda$ }
for each irrep $V(\lambda)$ of G
$\mathcal{V}_{\mathcal{P}}(\lambda) \xrightarrow{\varphi} V(\lambda) \otimes \mathcal{O}_{\mathbb{P}^1}(\langle \lambda, 1 \rangle 0)$
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$0 \in \mathbb{P}^1$
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$\mathcal{V} \xrightarrow{\varphi} \mathbb{C}^2 \otimes \mathcal{O}_{\mathbb{P}^1}(n \cdot 0) = \mathcal{O}(n) \oplus \mathcal{O}$
$\overline{Gr^\lambda}$ is f.d. projective variety
$\mu \leq \lambda$ $\mu \in X_+$
$\overline{Gr^\lambda} = \bigcup_{\mu \leq \lambda} Gr^\mu$
$G_1[t^{-1}]$.
$G(\mathbb{C}[t^{-1}]) \subset G(\mathbb{C}((t)))$
$\downarrow ev_\infty$
G
$Gr_\mu = G_1[t^{-1}]t^\mu$.
$Gr_\mu^\lambda = \overline{Gr^\lambda} \cap Gr_\mu$.
Theorem
Gr_μ^λ is an affine Poisson variety.
$\dim Gr_\mu^\lambda = p(\lambda - \mu) = \sum_{i \in I} m_i$

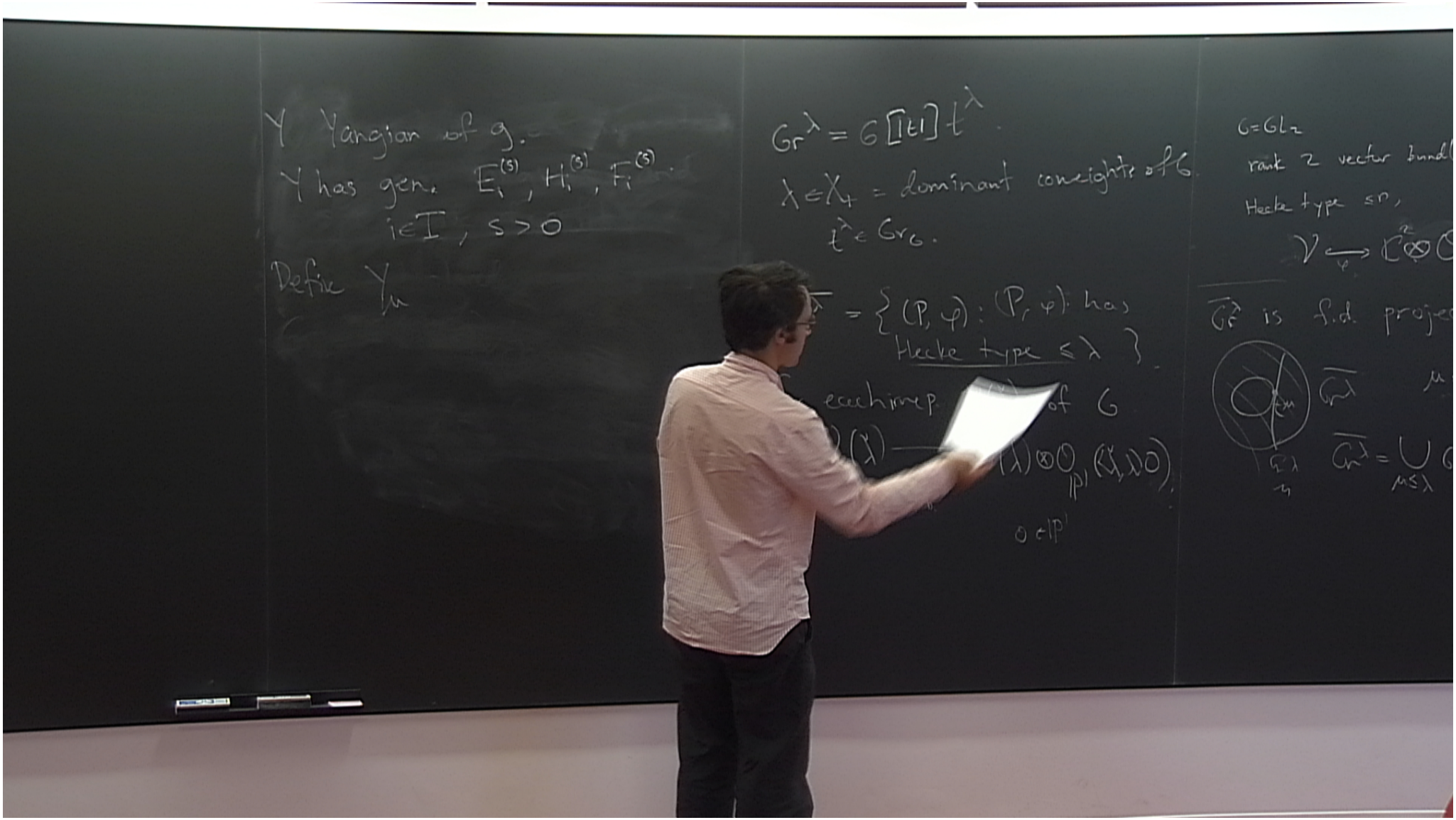

Theorem
- Gr_μ^λ is an affine Poisson variety
- It has finitely many symplectic leaves
$G_1[t^{-1}]$
$G(\mathbb{C}[t^{-1}]) < G(\mathbb{C}((t)))$
$ev_{t=\infty}$
$Gr_\mu = G[t^{-1}]t^\mu$
$Gr_\mu^\lambda = \overline{Gr^\lambda} \cap Gr_\mu$
$\overline{Gr^\lambda} = \bigcup_{\mu \le \lambda} G_1 \mu$

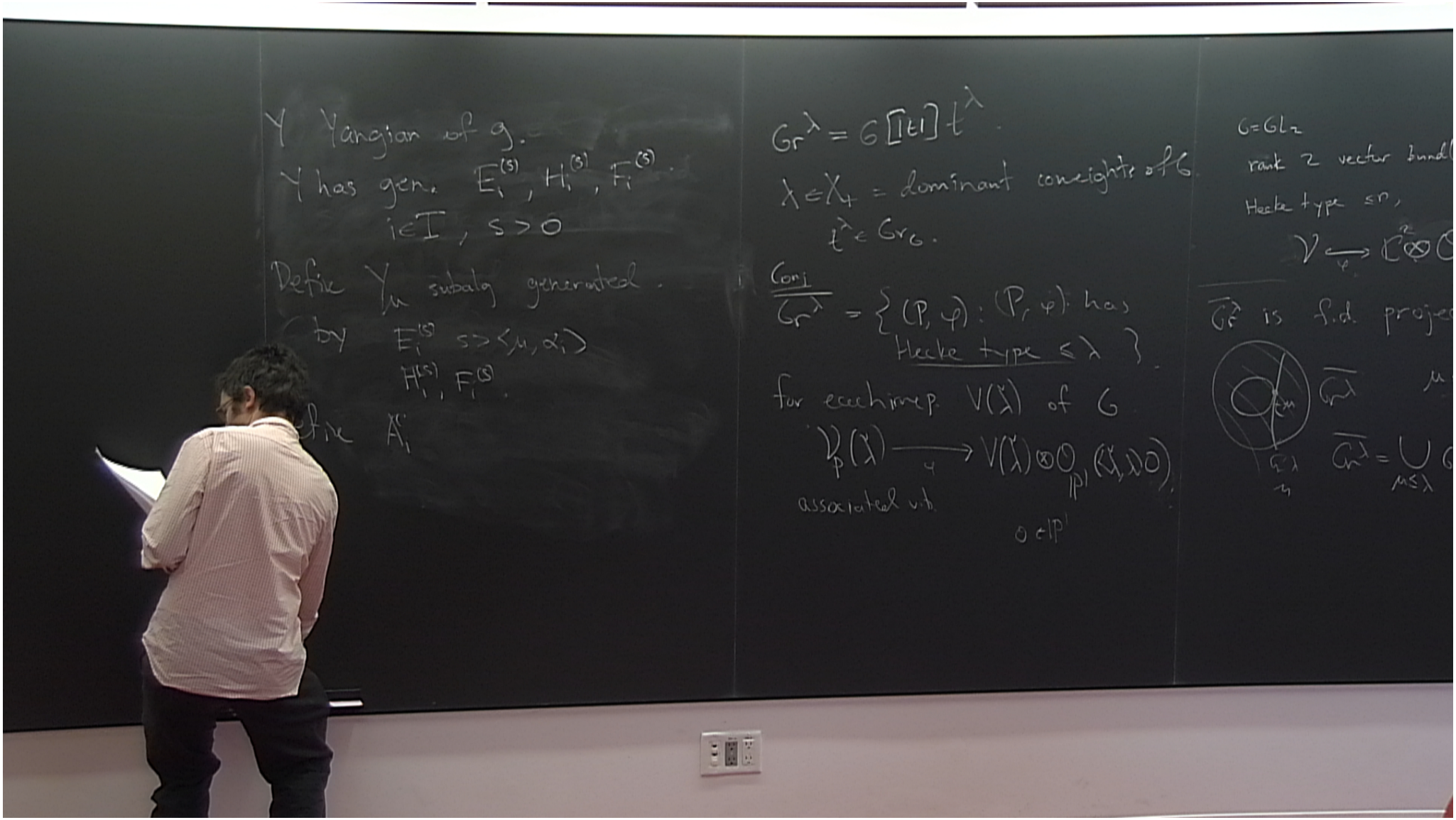

Theorem
- Gr^{λ}_{μ} is an affine Poisson variety.
$\dim Gr^{\lambda}_{\mu} = 2\rho(\lambda-\mu) = 2\sum_{i\in I} m_i$ if $\lambda-\mu = \sum m_i \alpha_i^\vee$
- It has finitely many symplectic leaves
- Sometimes it has a symplectic resolution (usually not)
λ is a sum of minuscule dominant coweights.
$Gr_\mu = G[t^{-1}]t^\mu$
$Gr^{\lambda}_{\mu} = \overline{Gr^{\lambda}} \cap Gr_\mu$.
$\overline{Gr^{\lambda}} = \bigcup_{\mu\le\lambda} Gr_\mu$

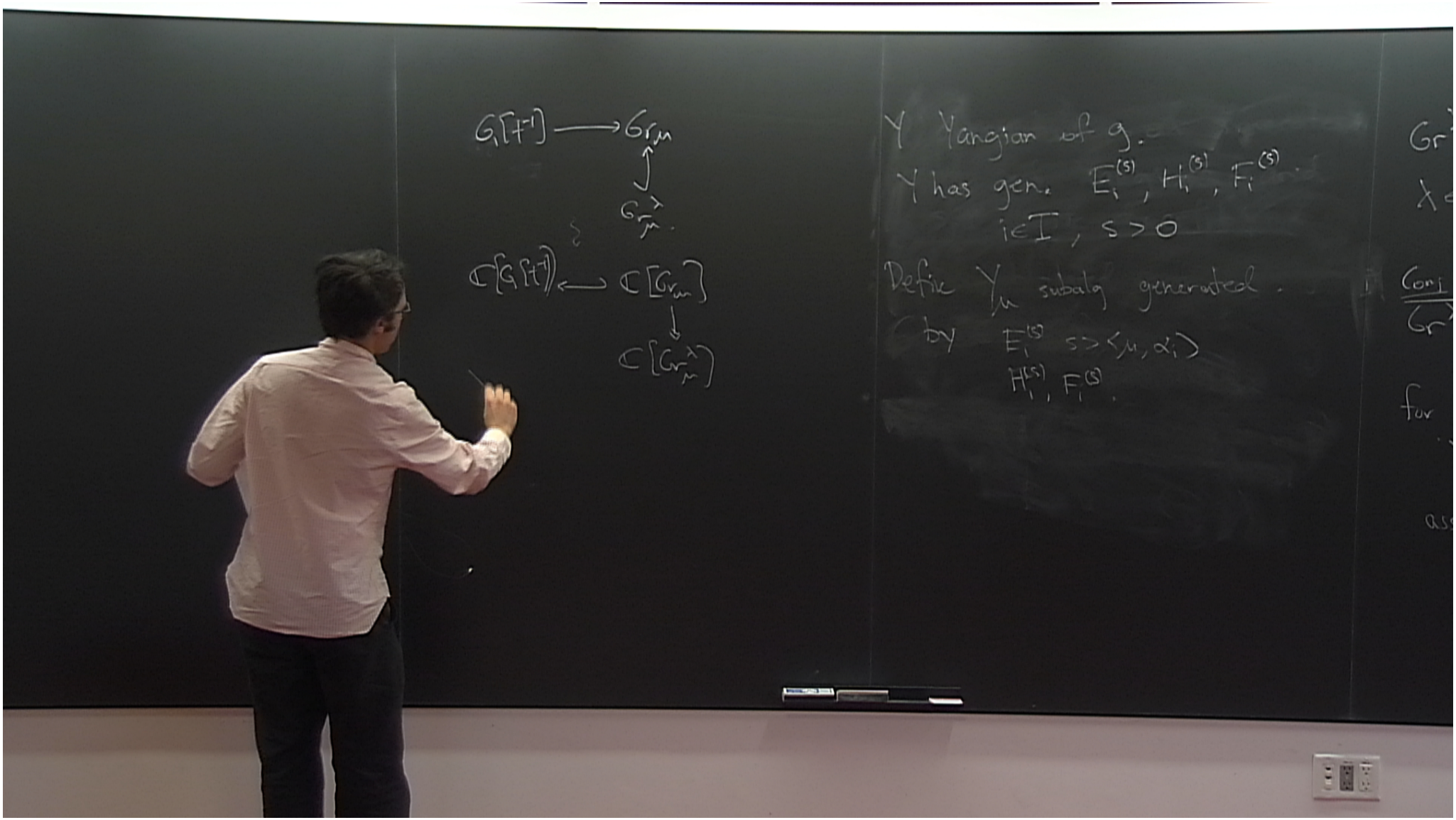

bundle V
projective variety
$G_1[t^{-1}]$
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Theorem
- Gr_μ^λ is an affine Poisson variety
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λ is a sum of minuscule dominant coweights.
- if $G=SL_n$, then Gr_μ^λ can be described as a certain variety of matrices
nilpotent orbit closure ∩ Slodowy slice
We are interested in quantization

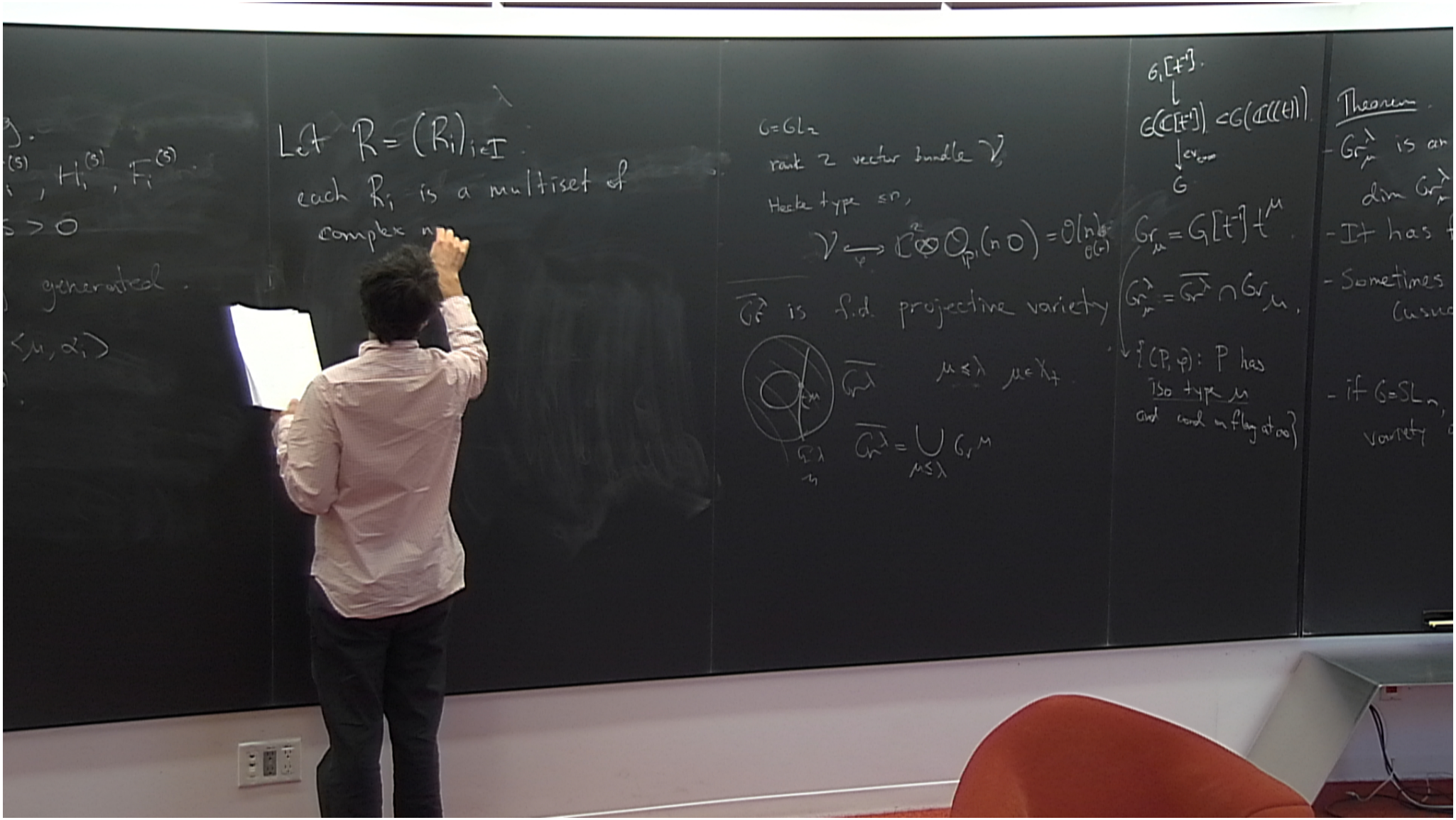

sson variety.
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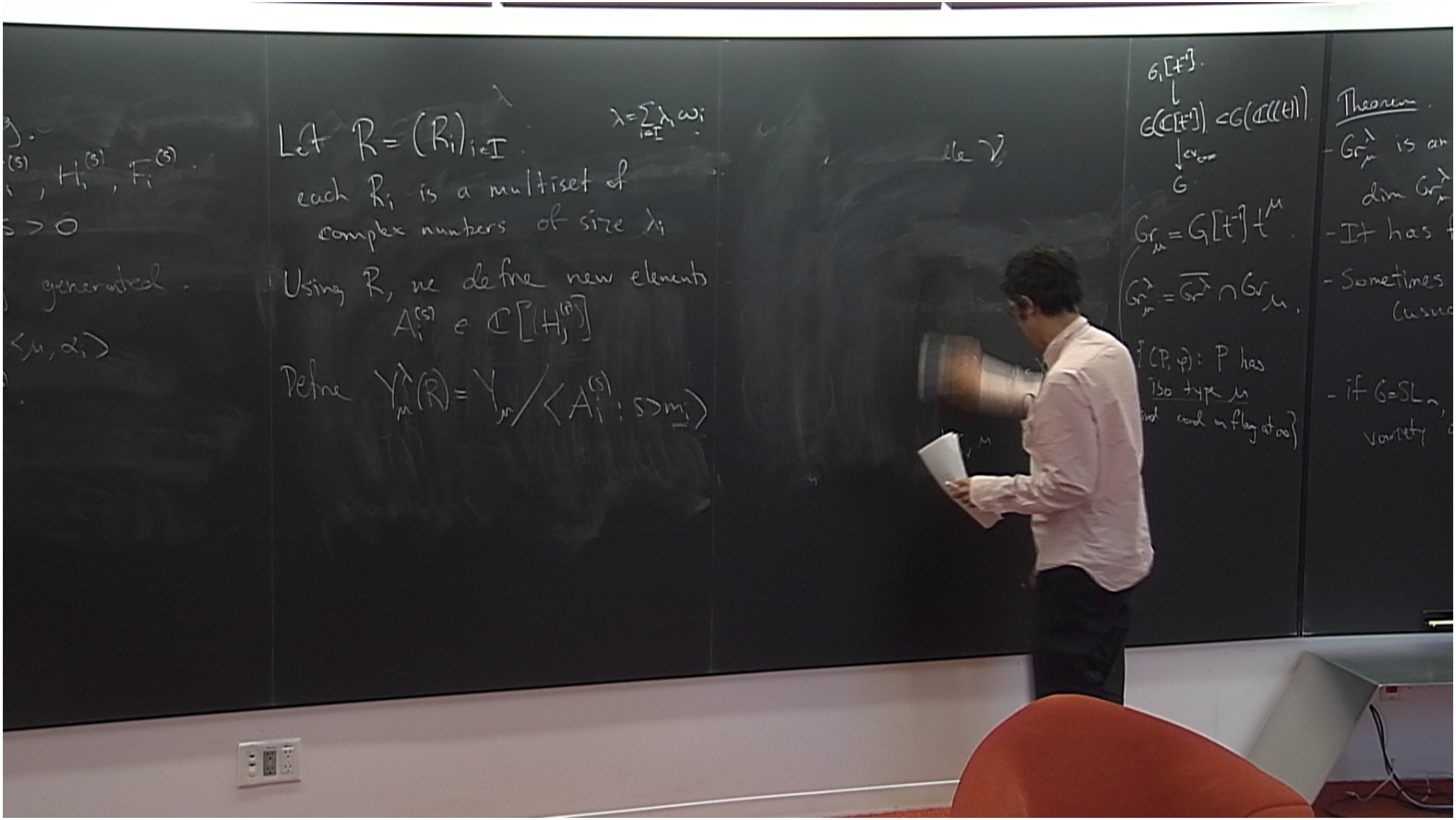

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s.t. gr A ≅ ℂ[Gr_μ^λ]
of graded Poisson algebras.
There is an action of ℂ^× on
Gr_μ^λ using ℂ^× ⊂ ℙ¹.

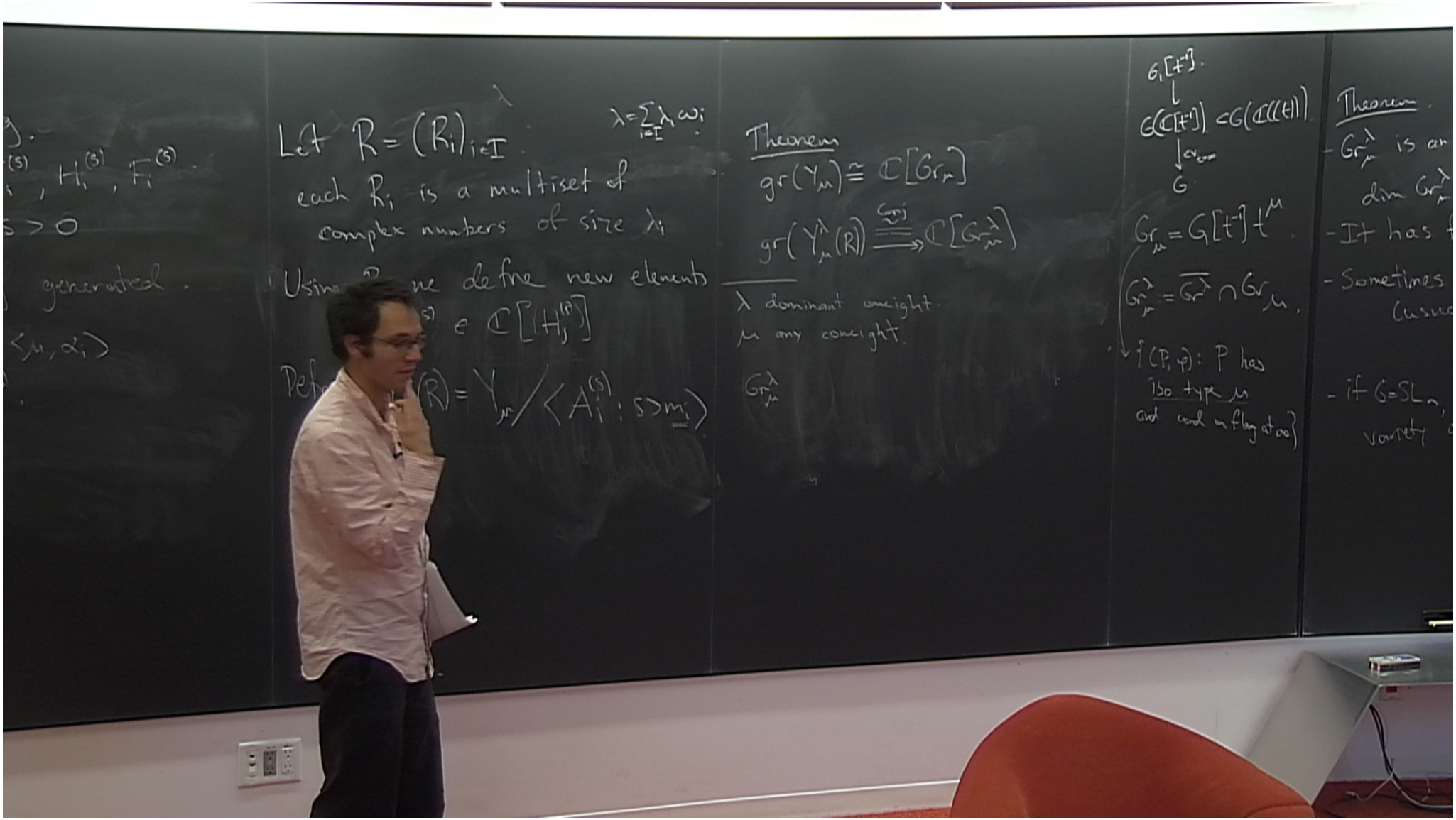

Y Yangian of g.
Y has gen. $E_i^{(s)}, H_i^{(s)}, F_i^{(s)}$
$i \in I$, $s > 0$
Define Y_μ
$Gr^\lambda = G[[t]] \cdot t^\lambda$
$\lambda \in X_+$ = dominant coweights of G
$t^\lambda \in Gr_G$.
Hecke type $\leq \lambda$
$G = GL_2$
rank 2 vector bund
Hecke type $\leq n$,
is f.d. proje

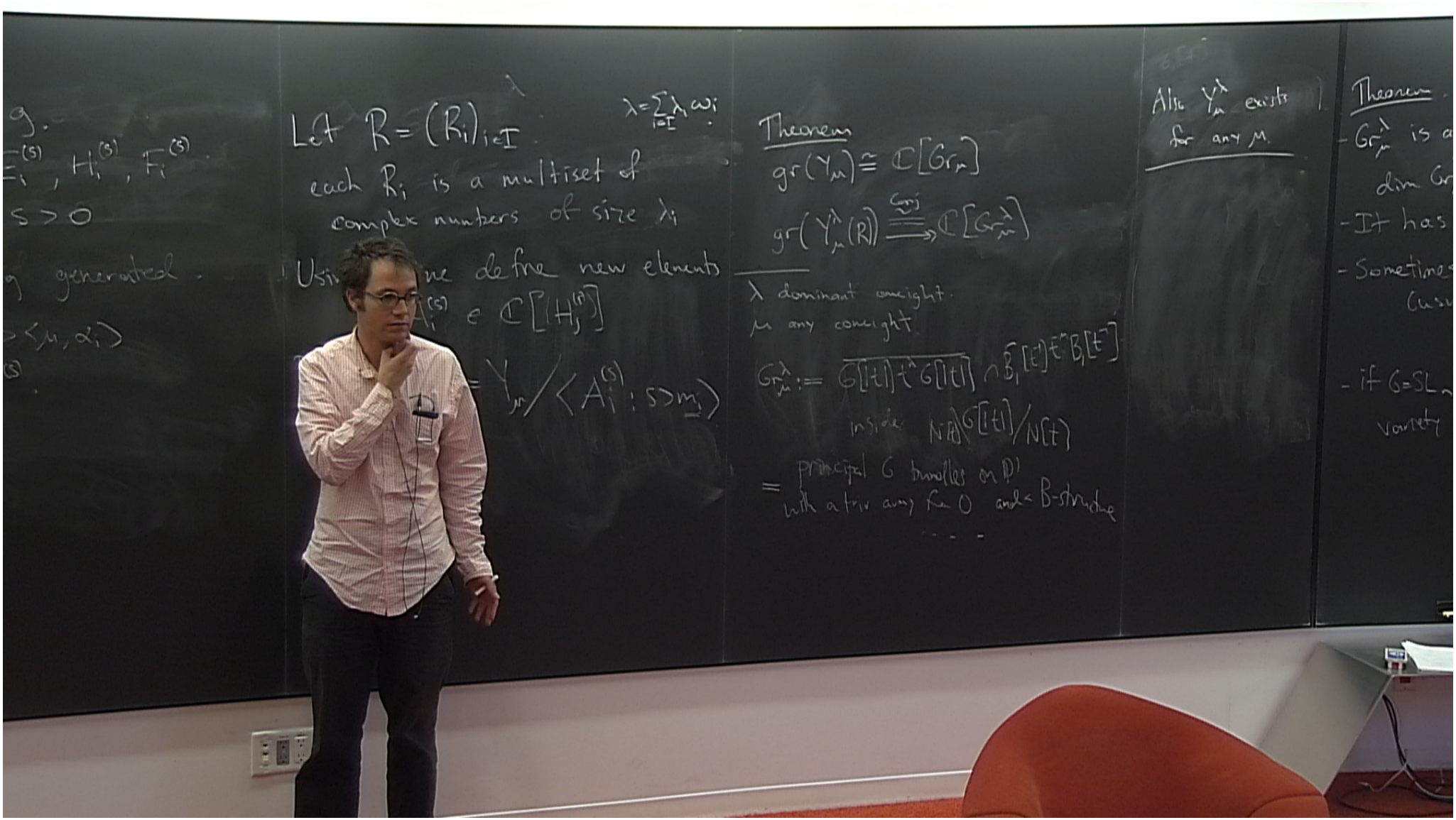

Y Yangian of g.
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$i \in I$, $s > 0$
Define Y_μ subalg generated
by $E_i^{(s)}$ $s > \langle \mu, \alpha_i \rangle$
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Fix $A_i^\times$
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for each irrep. $V(\lambda)$ of G
$V_p(\lambda) \longrightarrow V(\lambda) \otimes \mathcal{O}_{\mathbb{P}^1}(\langle \lambda, 1 \rangle 0)$
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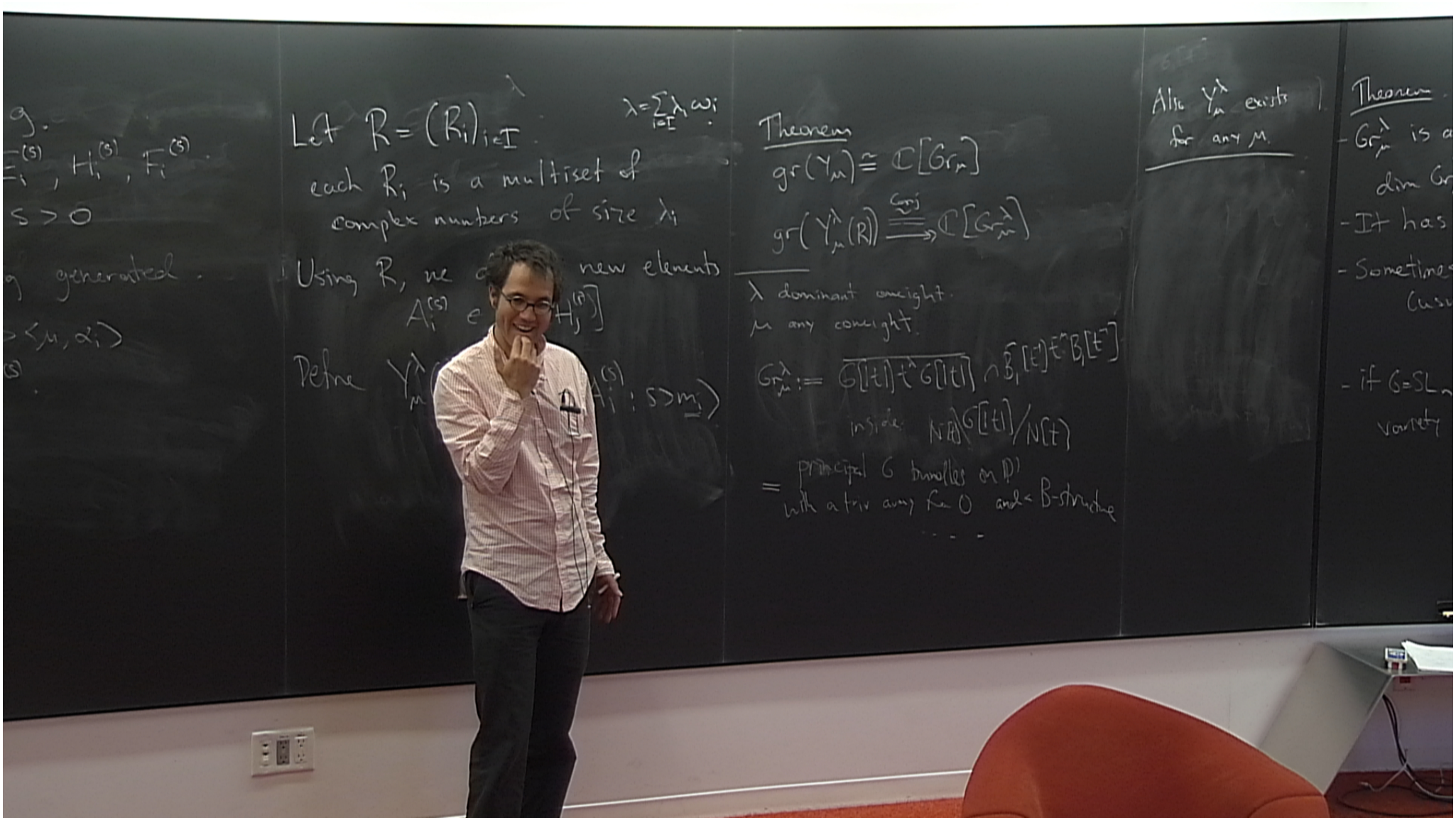

$G_1[t^{-1}] \longrightarrow G_{r_\mu}$
$G_{r_\mu^{-1}}$
$\mathbb{C}[G_1[t^{-1}]) \longleftarrow \mathbb{C}[G_{r_\mu}]$
$\mathbb{C}[G_{r_\mu^{-1}}]$
Y Yangian of g.
Y has gen. $E_i^{(s)}$, $H_i^{(s)}$, $F_i^{(s)}$
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Define Y_μ subalg generated
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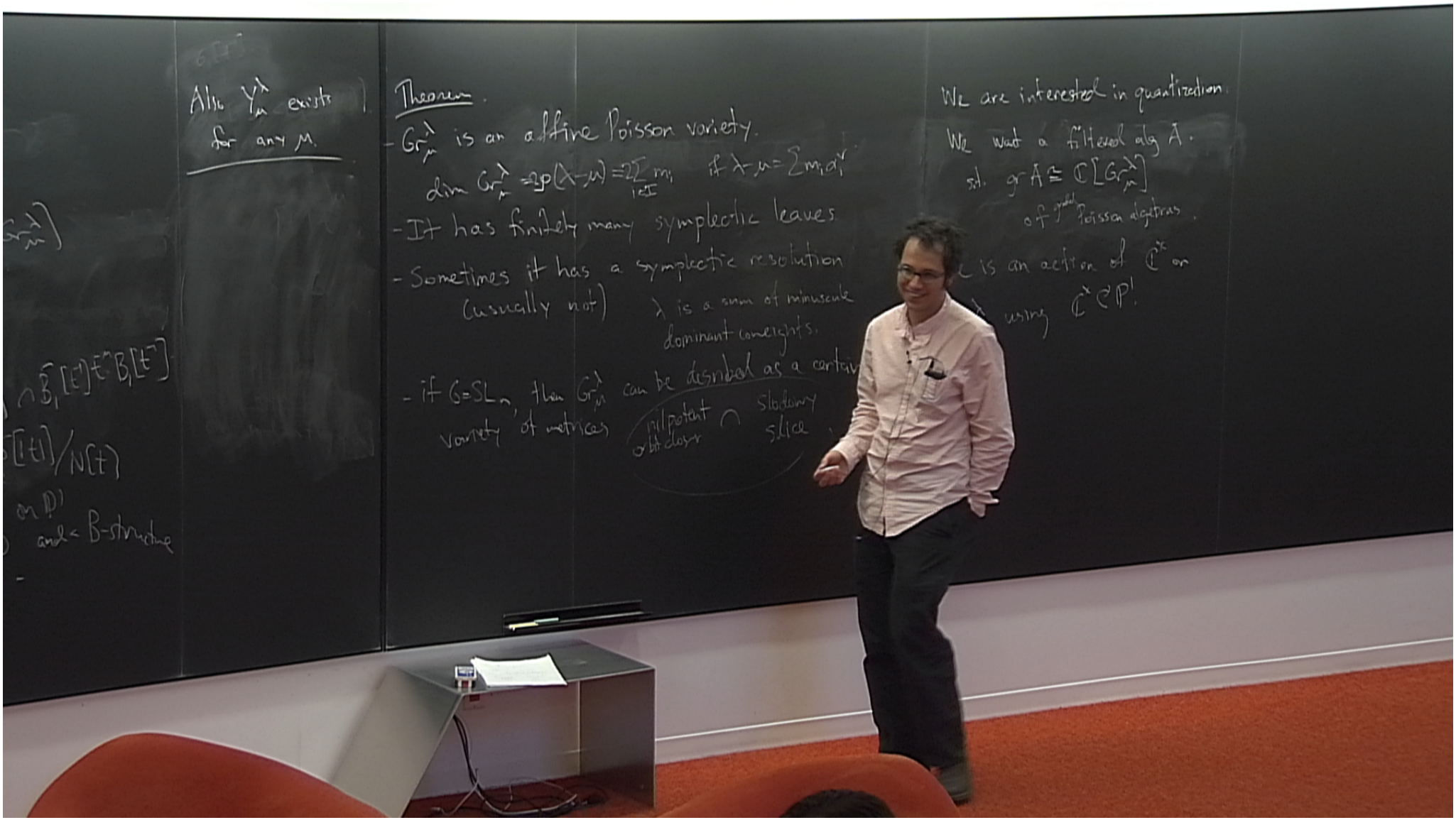

Let $R = (R_i)_{i \in I}$
each R_i is a multiset of
$G = GL_2$
rank 2 vector bundle $\mathcal{V}$
Hecke type $\leq n$,
$\mathcal{V} \hookrightarrow \mathbb{C}^2 \otimes \mathcal{O}_{\mathbb{P}^1}(n \cdot 0) = \mathcal{O}(n)^{\oplus 2}$
$\overline{Gr}^{\lambda}$ is f.d. projective variety
$\mu \leq \lambda$ $\mu \in X_+$
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$G(\mathbb{C}[t^{-1}]) \subset G(\mathbb{C}((t)))$
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G
$Gr_\mu = G_1[t^{-1}]\, t^\mu$
$Gr_\mu^{\lambda} = \overline{Gr}^{\lambda} \cap Gr_\mu$
{(P, φ): P has iso type μ and cond on flag at ∞}
Theorem
- Gr_μ^{λ} is an
dim Gr_μ^{λ}
- It has
- Sometimes
Cusu
- if $G = SL_n$,
variety

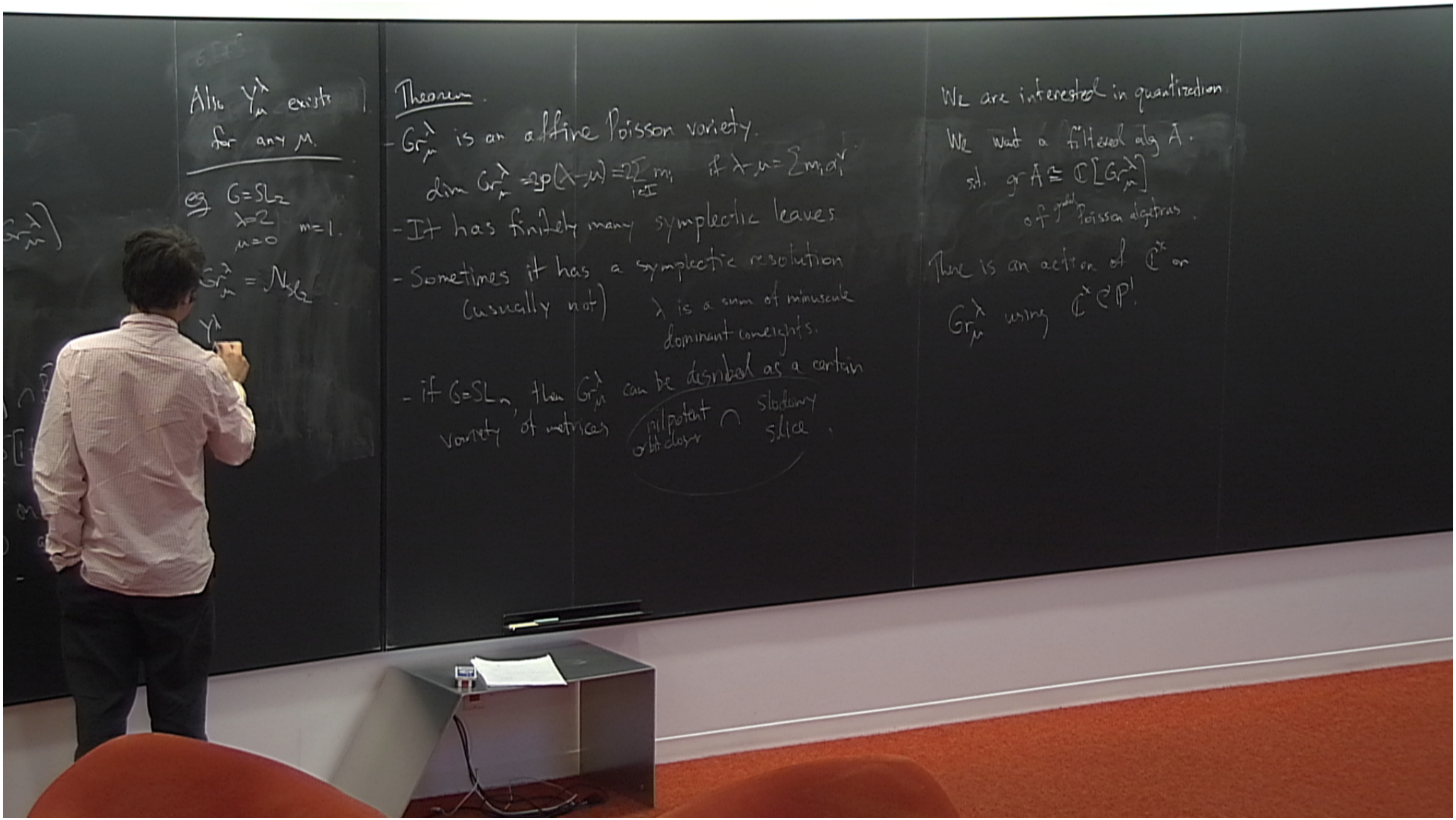

Let $R = (R_i)_{i \in I}$
$\lambda = \sum_{i \in I} \lambda_i \omega_i$
each R_i is a multiset of complex numbers of size λ_i
Using R, we define new elements
$A_i^{(s)} \in \mathbb{C}[(H_j^{(s)})]$
Define $Y_\mu^\lambda(R) = Y_\mu / \langle A_i^{(s)} : s > m_i \rangle$
$G_1[t^{-1}]$
$G(\mathbb{C}[t^{-1}]) \subset G(\mathbb{C}((t)))$
ev_∞
G
$Gr_\mu = G[t^{-1}]t^\mu$
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- Gr_μ^λ is an
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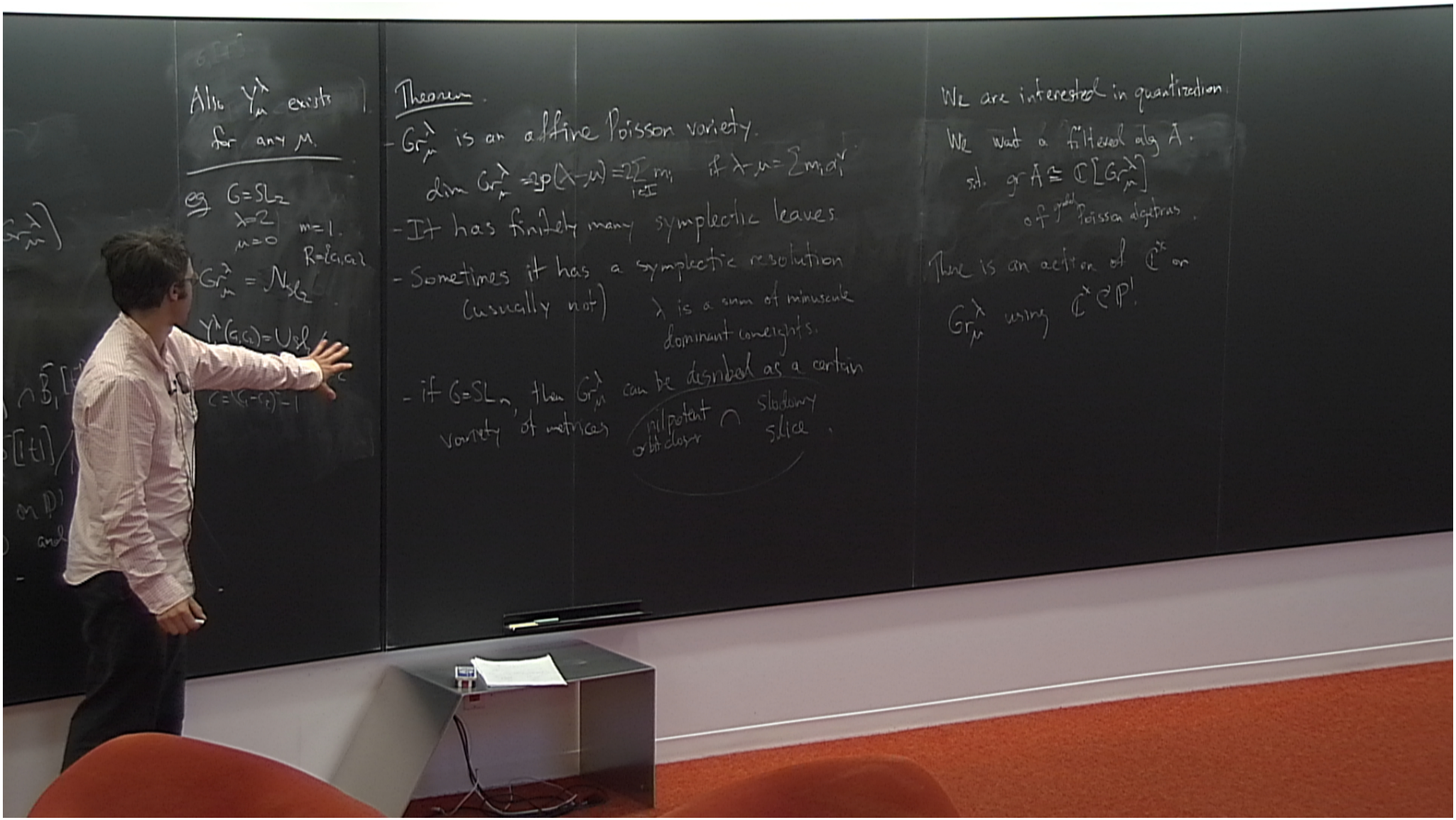

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Theorem
$gr(Y_\mu) \cong \mathbb{C}[Gr_\mu]$
$gr(Y_\mu^\lambda(R)) \xrightarrow{\text{Conj}} \mathbb{C}[Gr_\mu^\lambda]$
λ dominant coweight.
μ any coweight.
$G_1[t^{-1}]$.
$G(\mathbb{C}[t^{-1}]) \subset G(\mathbb{C}((t)))$
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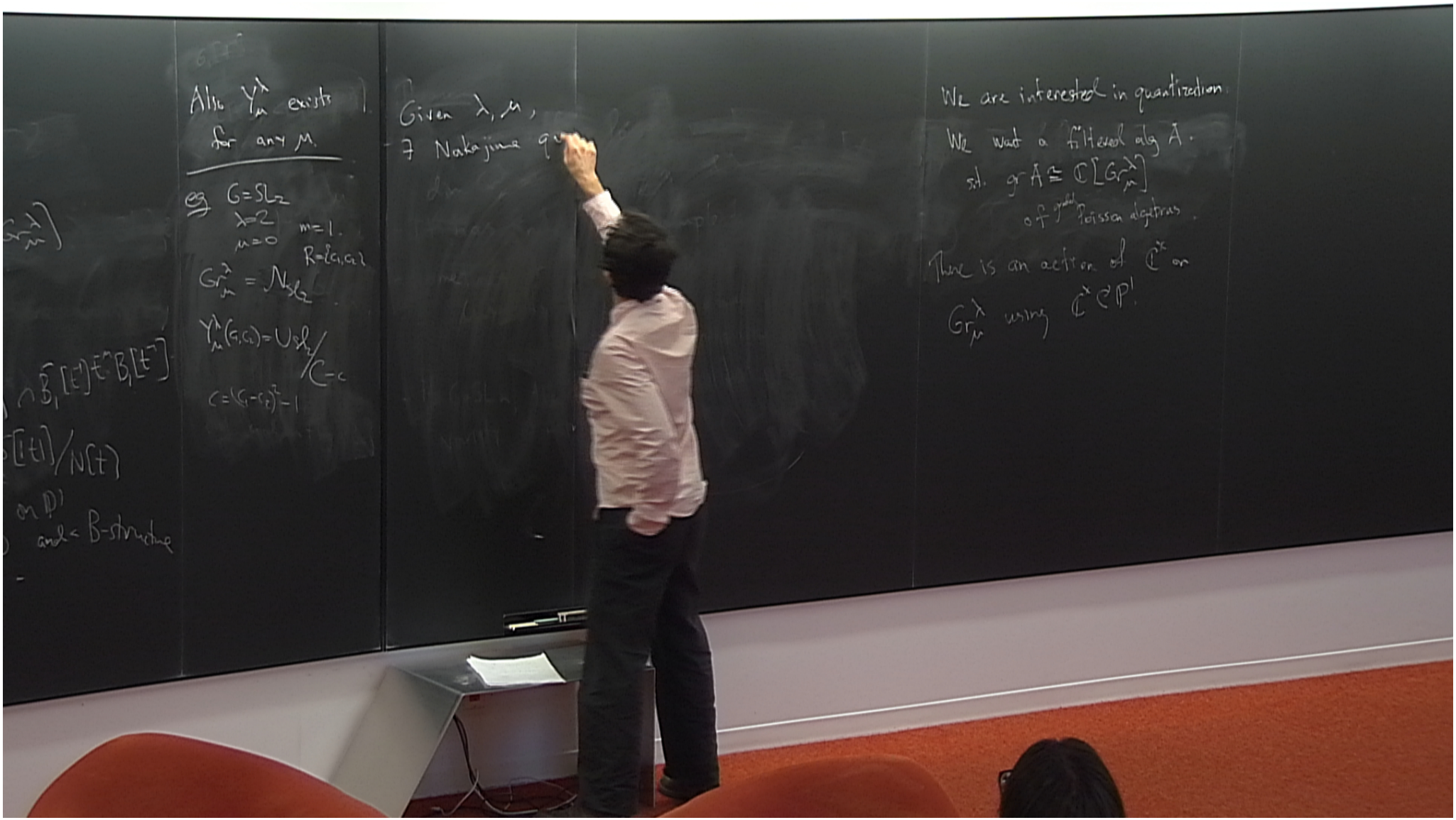

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λ dominant coweight.
μ any coweight
Also Y^λ_μ exists for any μ.
Theorem
principal G bundles on $\mathbb{P}^1$
with a triv away from 0 and $<$ B-structure

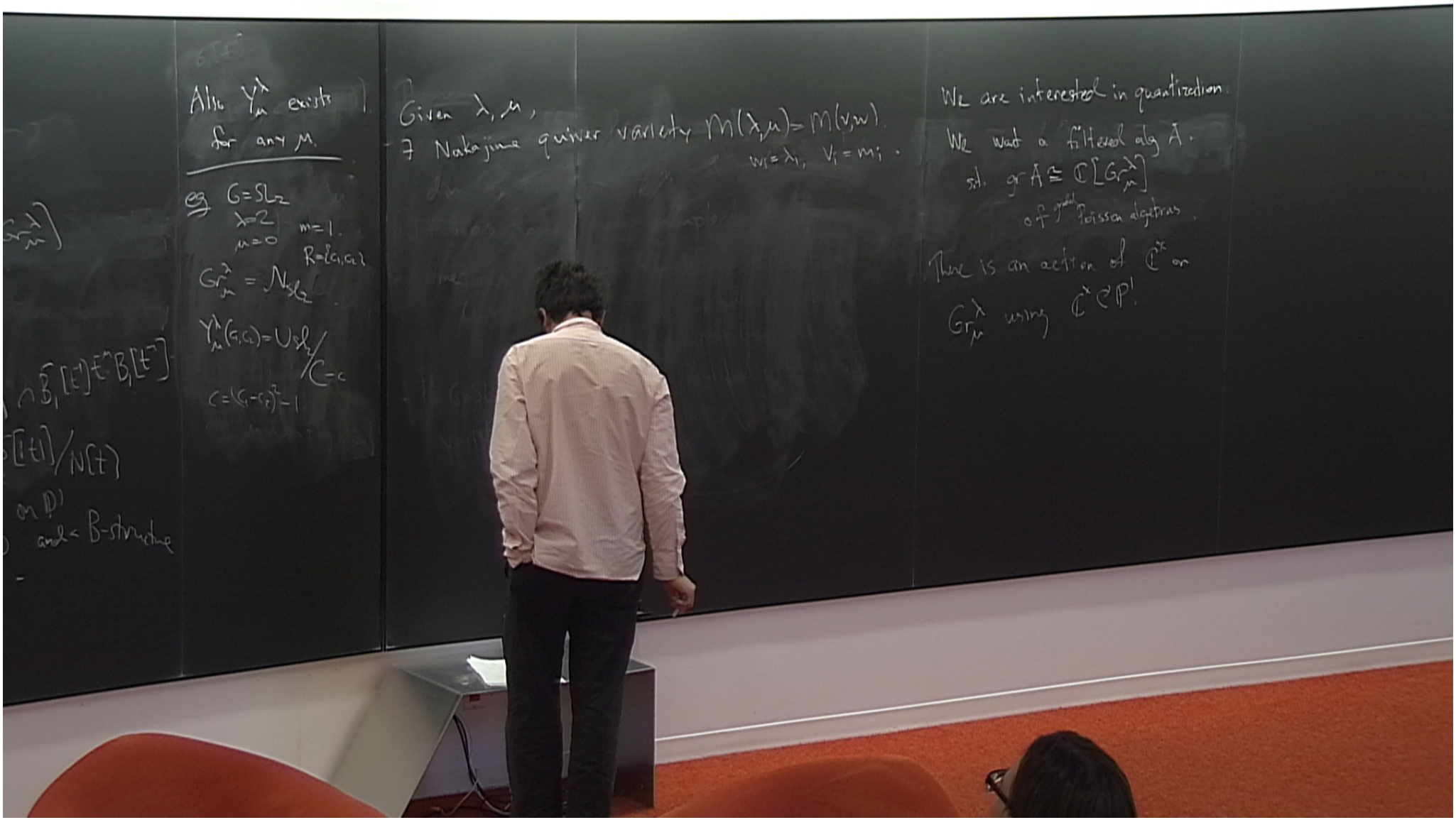

Let $R = (R_i)_{i \in I}$
each R_i is a multiset of complex numbers of size λ_i
$\lambda = \sum_{i \in I} \lambda_i \omega_i$
Theorem
$gr(Y_\mu) \cong \mathbb{C}[Gr_\mu]$
$gr(Y_\mu^\lambda(R)) \xrightarrow{\text{Conj}} \mathbb{C}[Gr_\mu^\lambda]$
λ dominant coweight.
μ any coweight.
Also Y_μ^λ exists for any μ.
Theorem
- Gr_μ^λ is a
- It has
- Sometimes
- if $G = SL_n$
variety

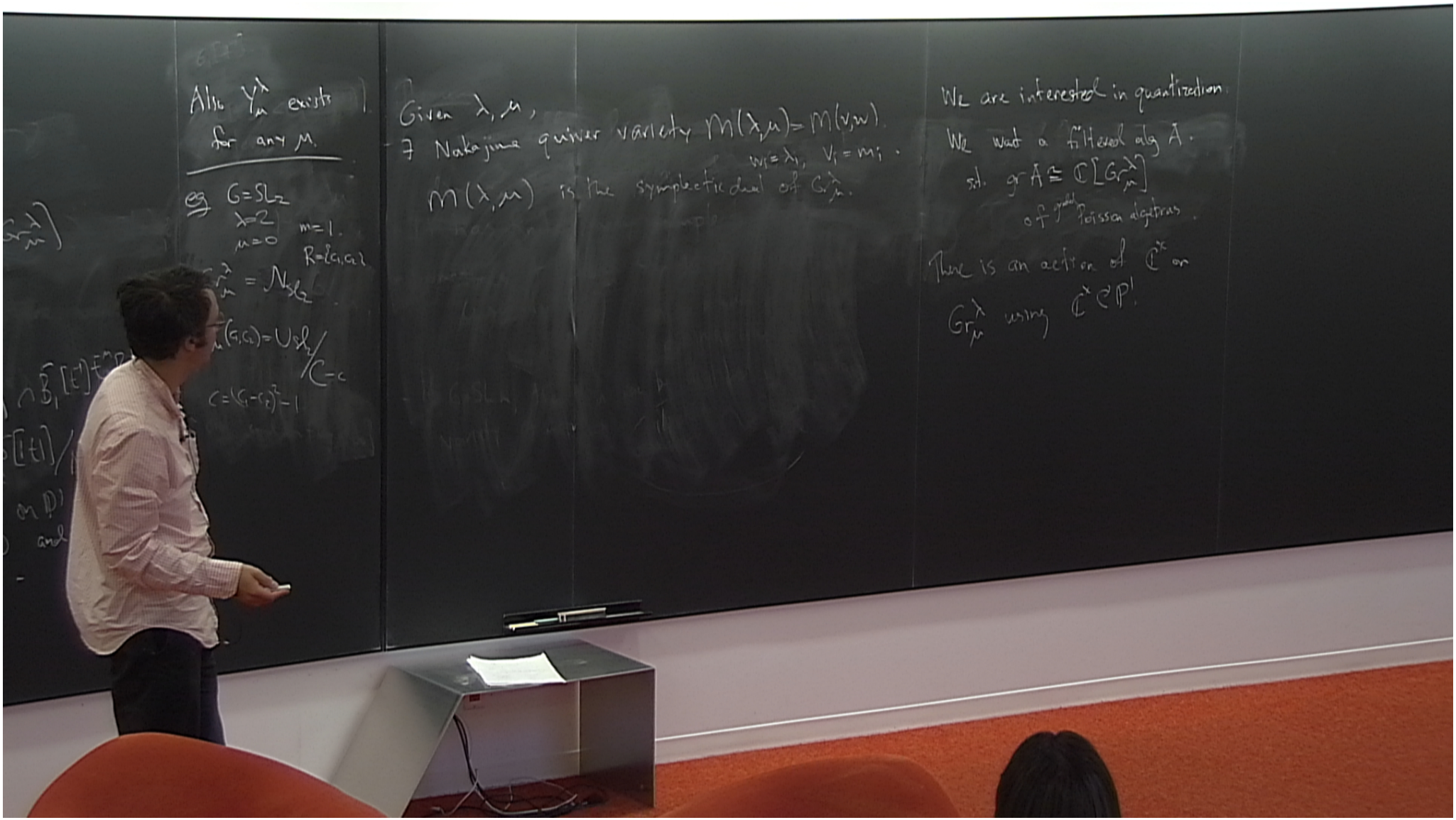

Alg Y^λ_μ exists for any μ.
Theorem.
- Gr^λ_μ is an affine Poisson variety.
$\dim Gr^\lambda_\mu = 2\rho(\lambda-\mu) = 2\sum_{i\in I} m_i$ if $\lambda-\mu=\sum m_i\alpha_i^\vee$
- It has finitely many symplectic leaves
- Sometimes it has a symplectic resolution (usually not) λ is a sum of minuscule dominant coweights.
- if $G=SL_n$, then Gr^λ_μ can be described as a certain
variety of matrices
nilpotent orbit closure ∩ Slodowy slice
We are interested in quantization.
We want a filtered alg A.
s.t. $\mathrm{gr}\,A \cong \mathbb{C}[Gr^\lambda_\mu]$
of graded Poisson algebras.
is an action of $\mathbb{C}^\times$ on
using $\mathbb{C}^\times \curvearrowright \mathbb{P}^1$.

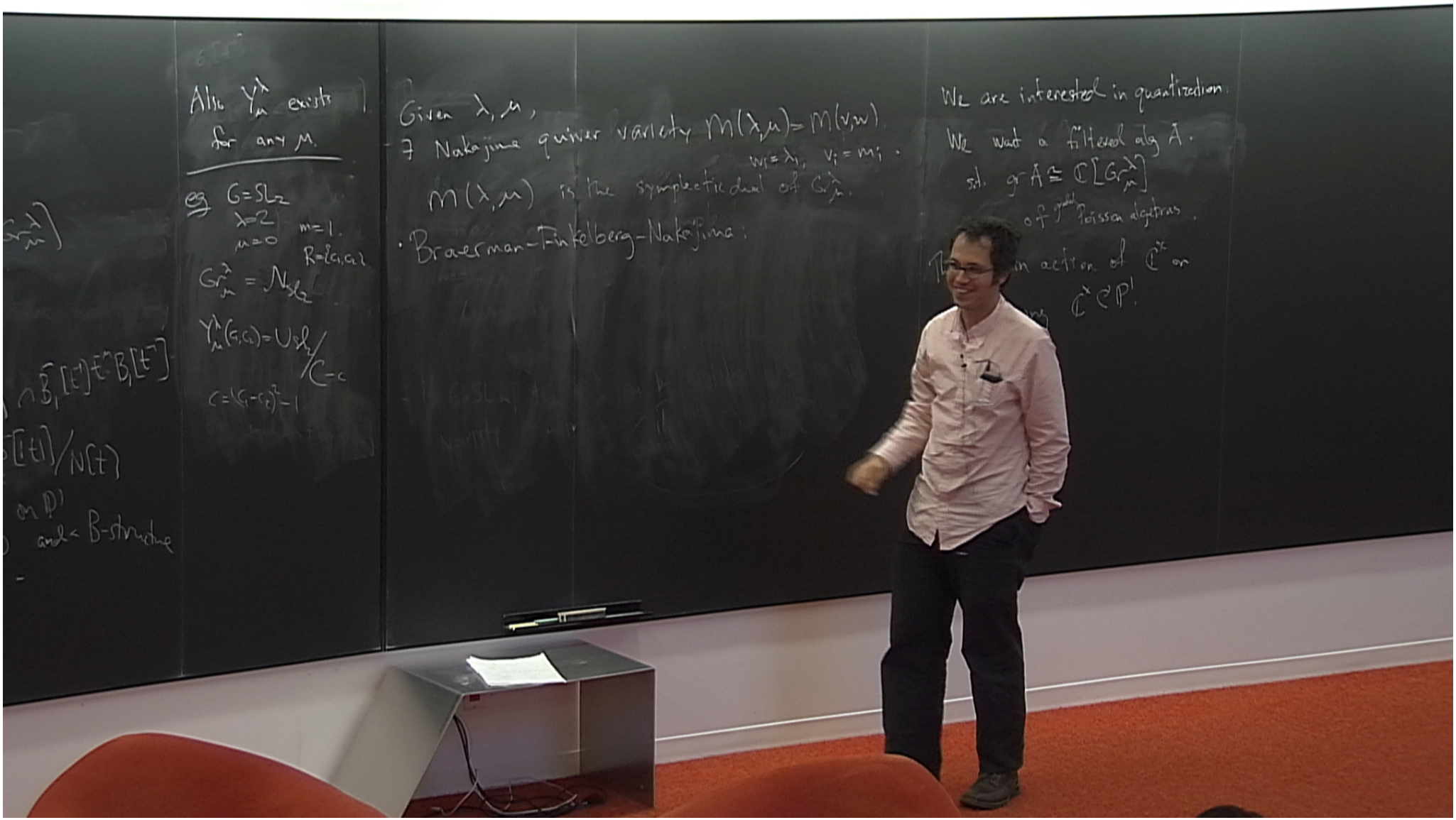

All Y^λ_μ exists for any μ.
eg $G=SL_2$
$\lambda=2$
$\mu=0$
$m=1$.
$Gr^\lambda_\mu = N_{SL_2}$
Y^λ
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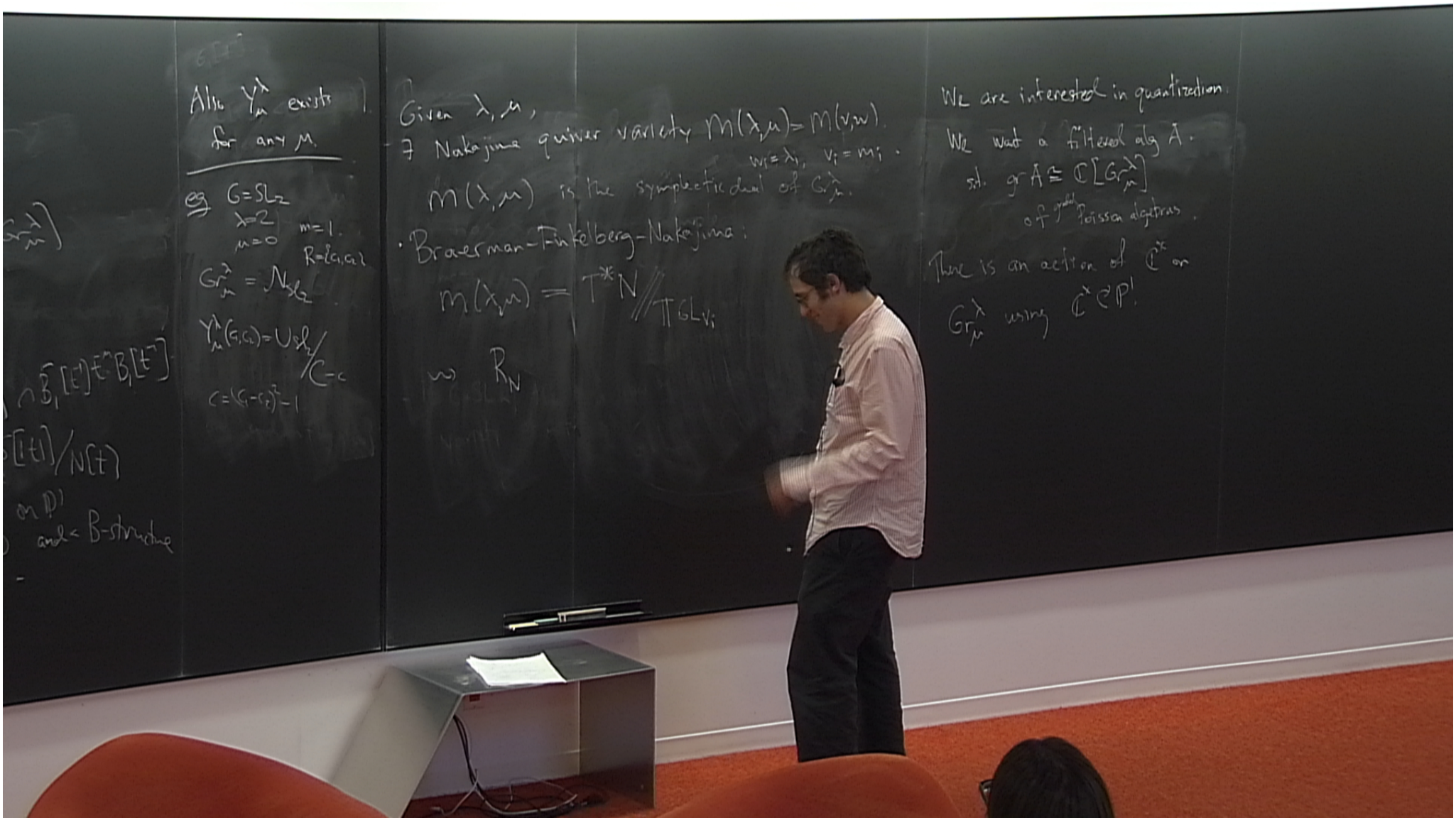

All Y^λ_μ exists for any μ.
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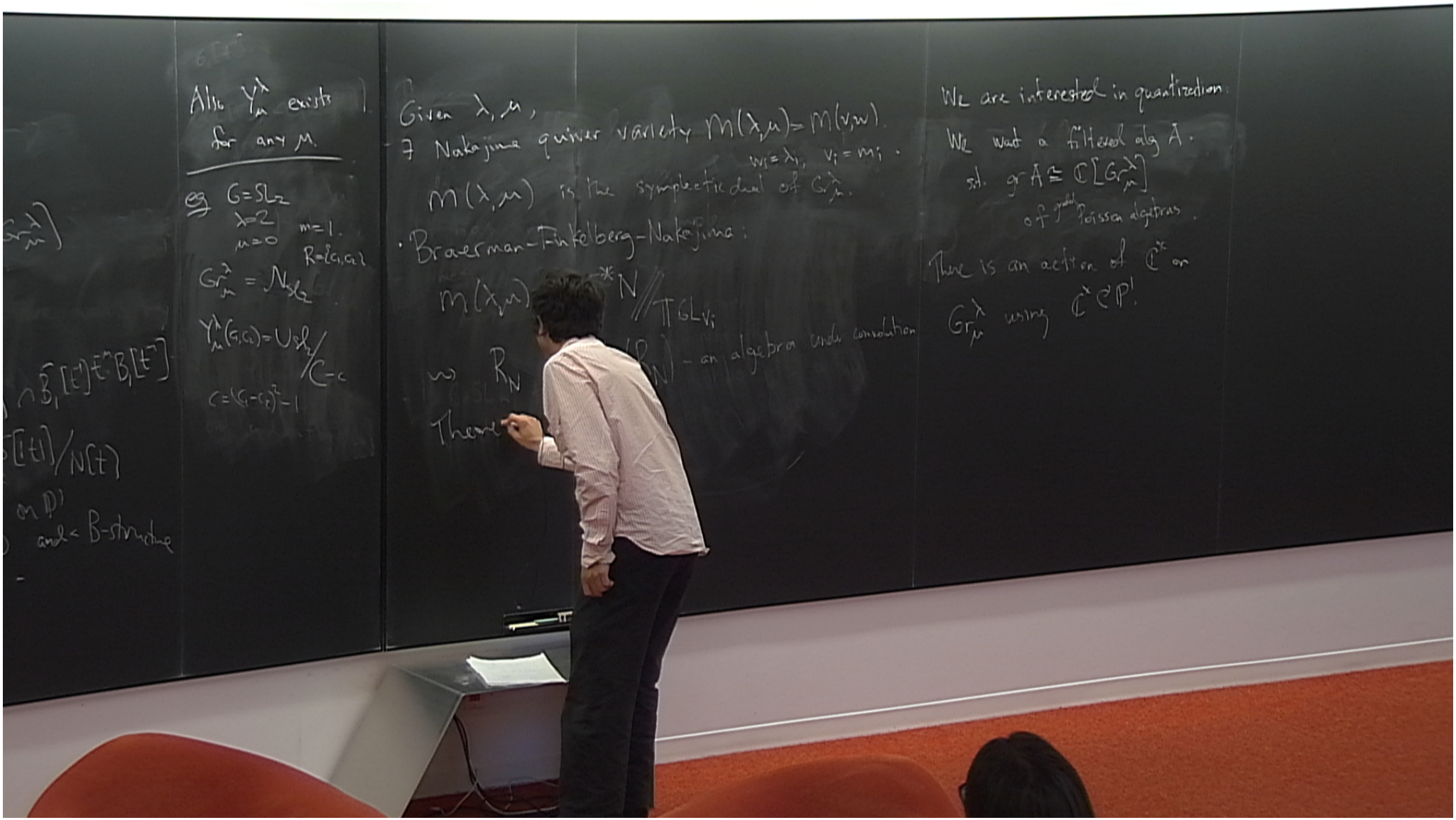

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$R = \mathbb{C}[c_1, c_2]$
$G^\lambda_\mu = N_{SL_2}$
$Y^\lambda_\mu(c_1, c_2) = U_{sh}/(C - c)$
$c = (c_1 - c_2)^2 - 1$
and < B-structure

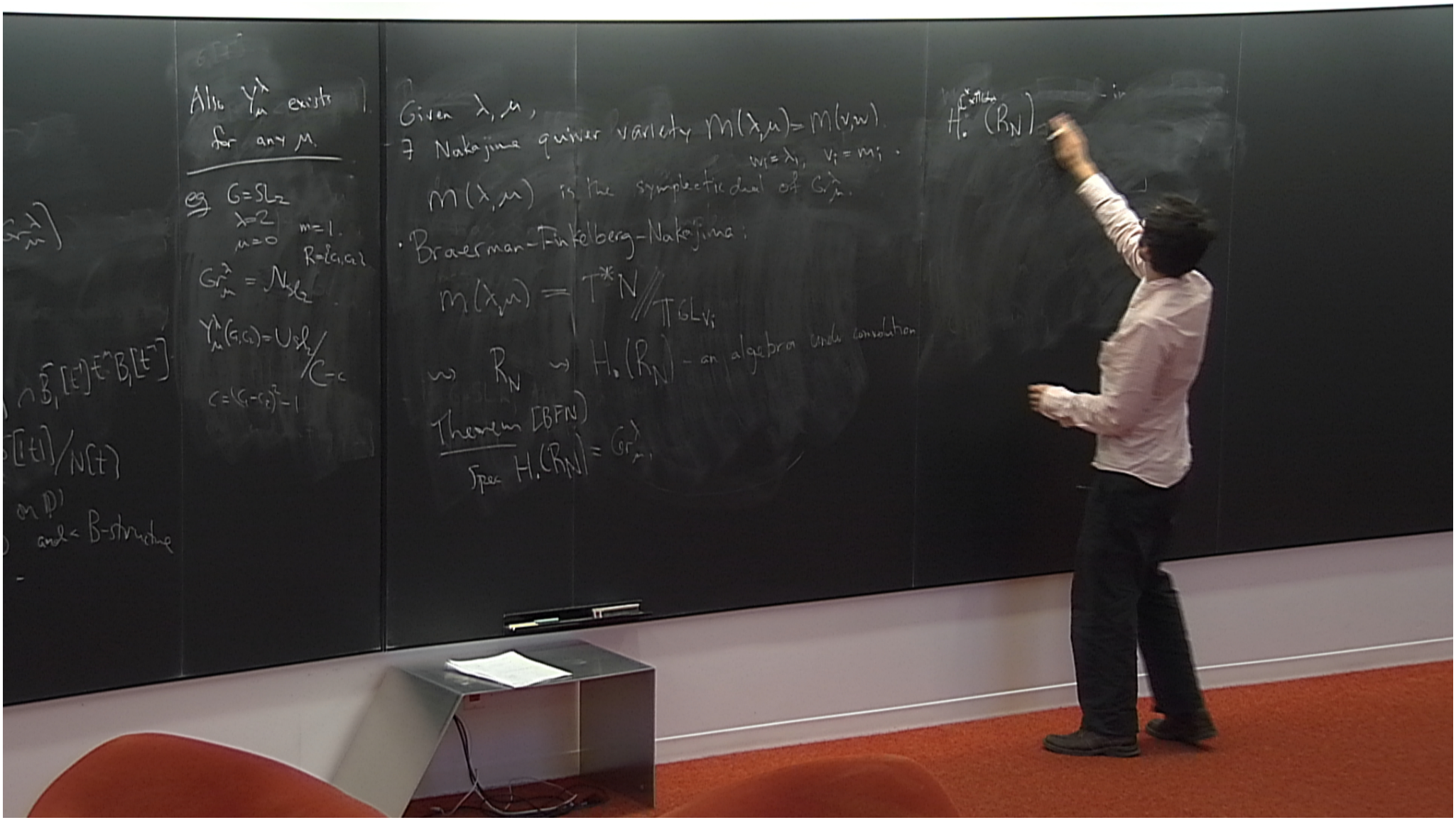

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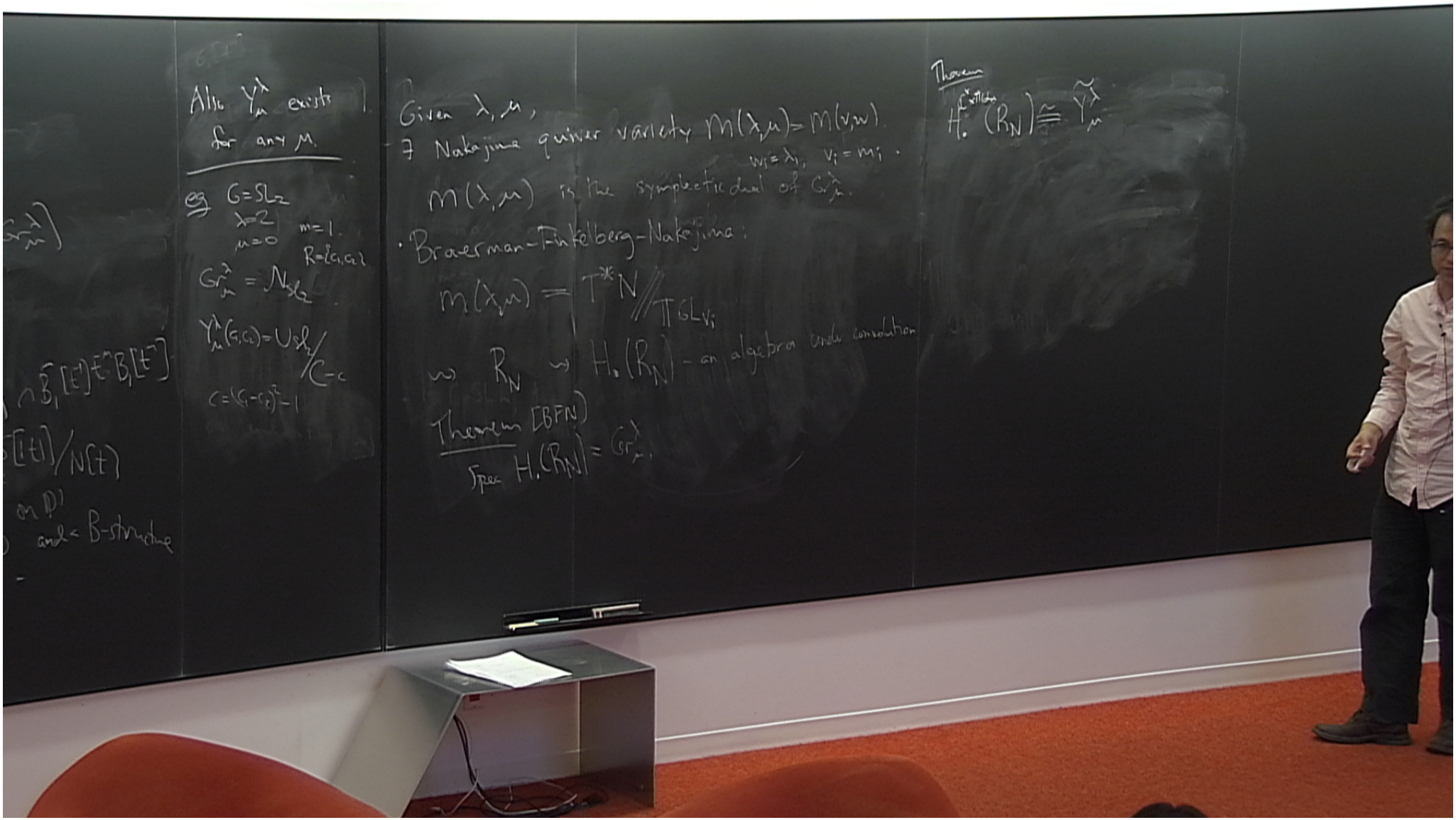

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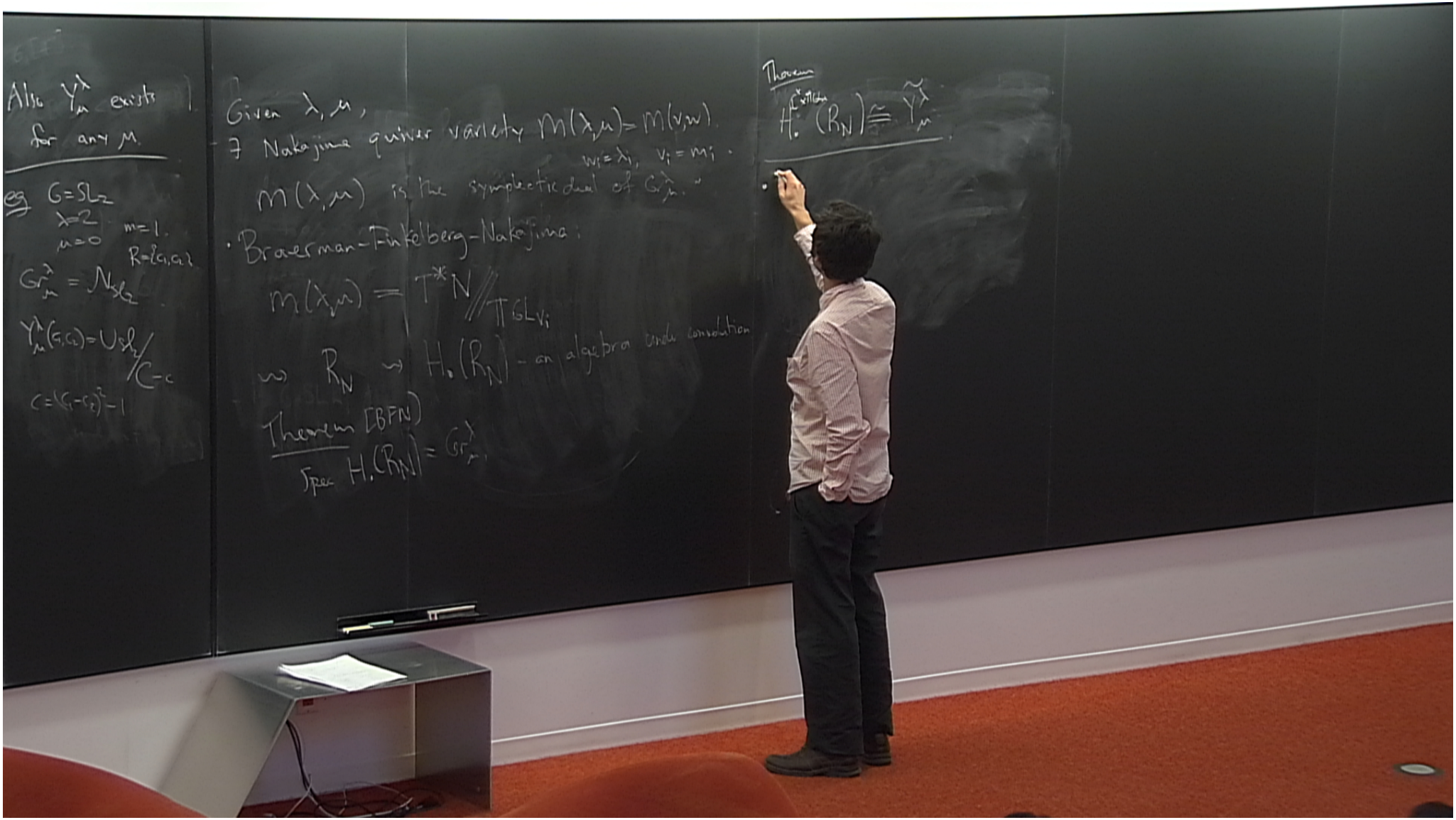

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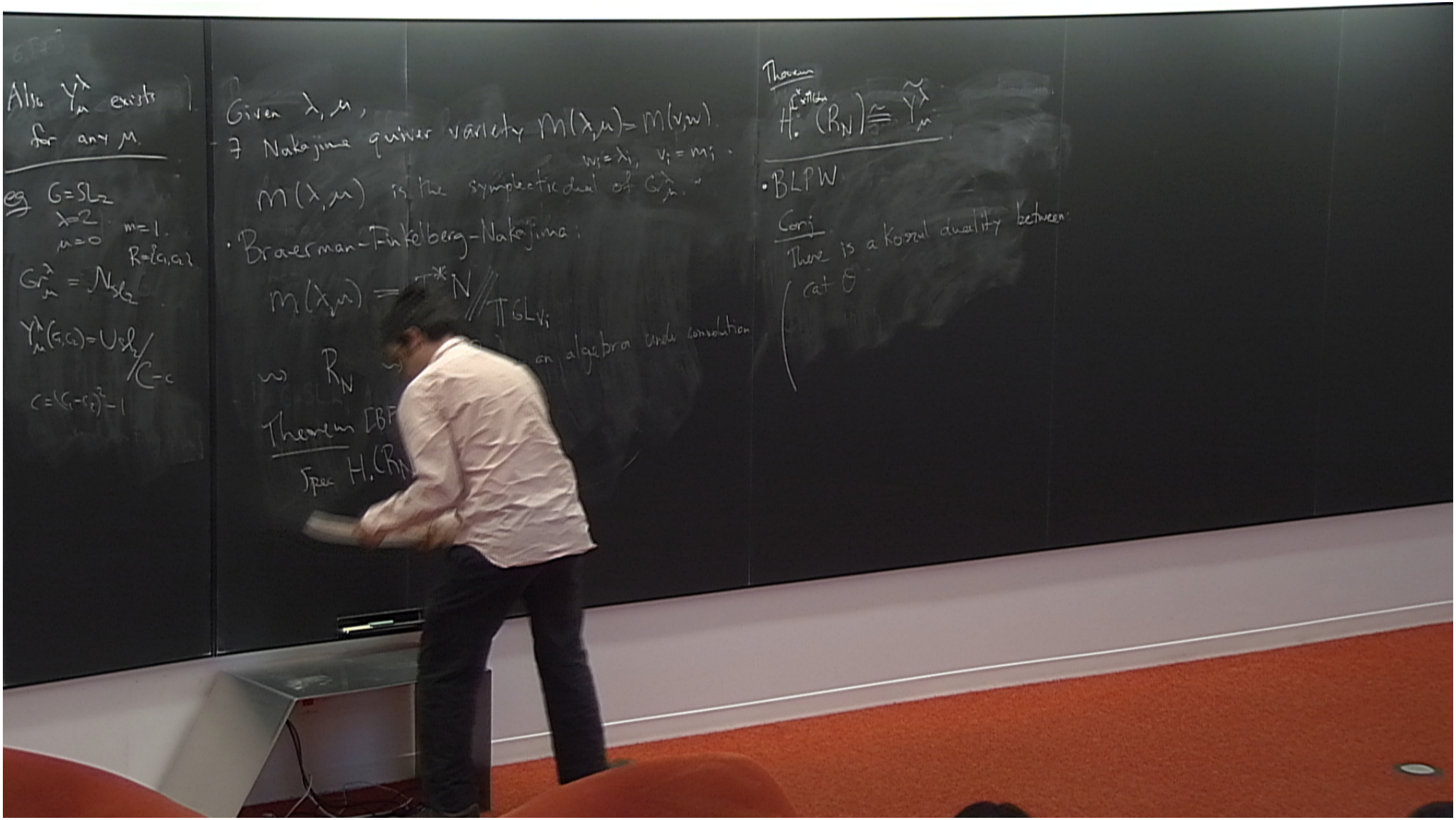

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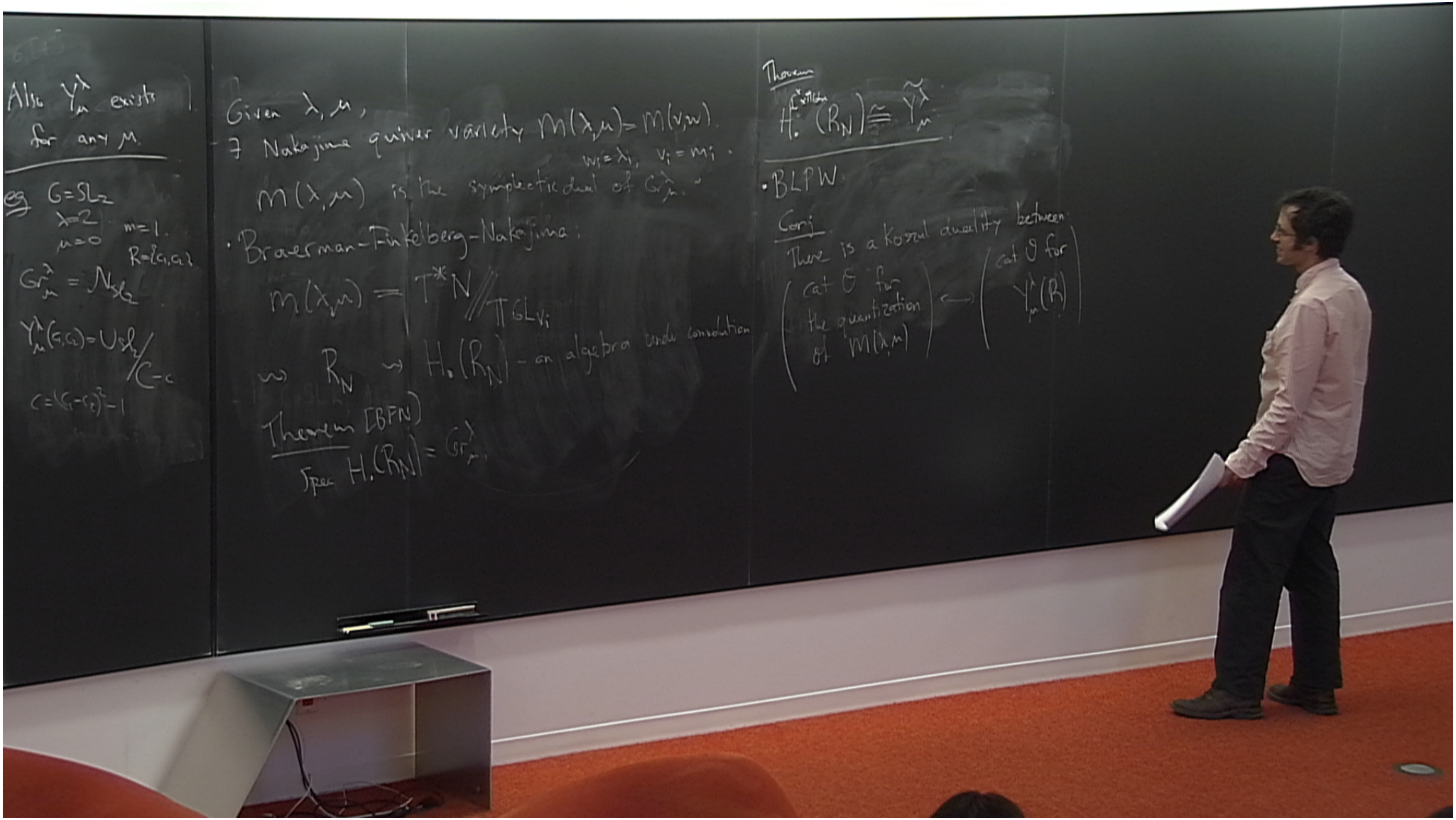

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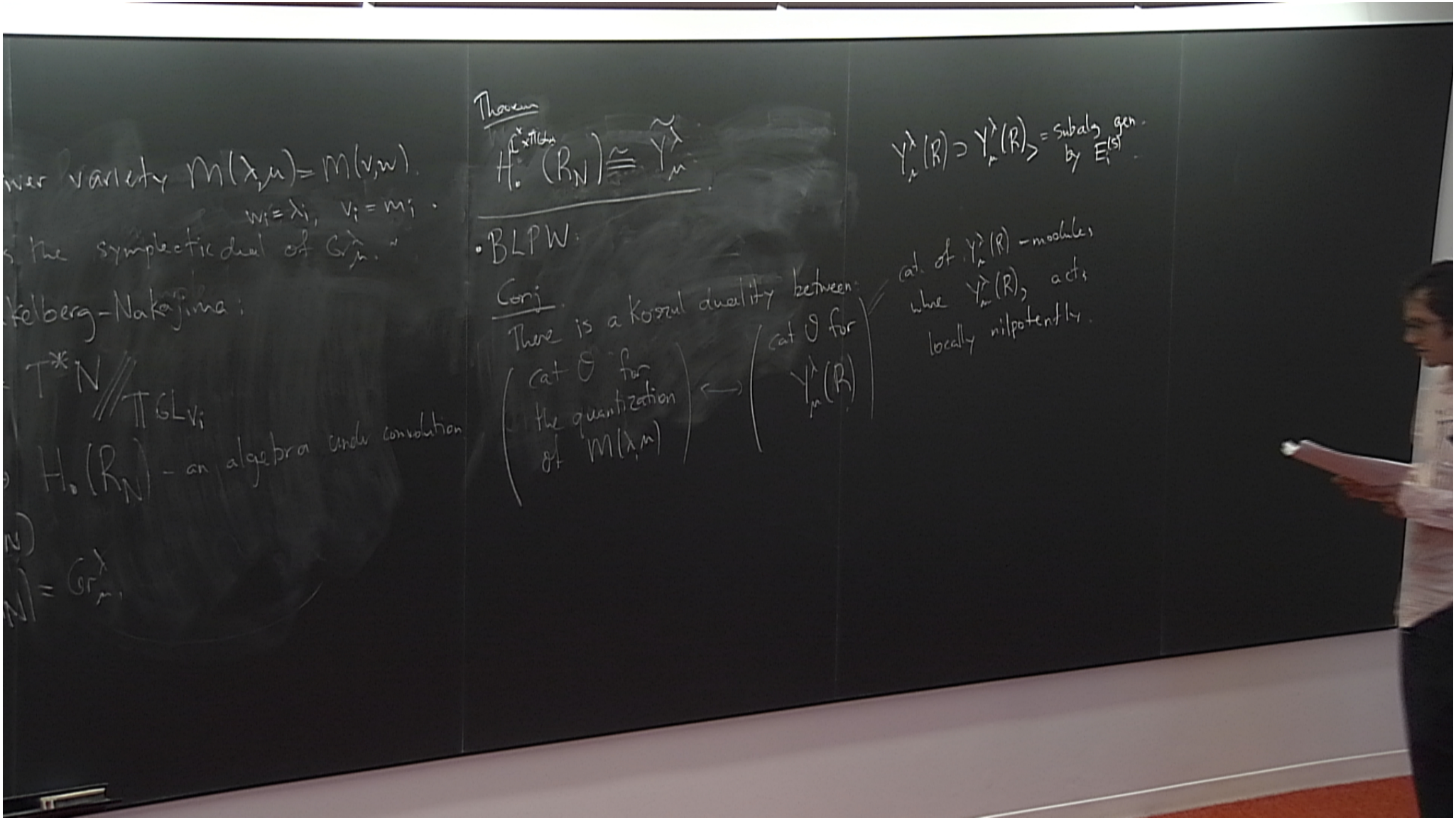

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Braverman-Finkelberg-Nakajima:
$\mathcal{M}_0(\lambda,\mu) = T^*N /\!\!/ \prod GLv_i$
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Theorem [BFN]
$\operatorname{Spec} H_*(R_N) = Gr^\lambda_\mu$.
eg $G = SL_2$
$\lambda = 2$ $m = 1$.
$\mu = 0$ $R = \mathbb{C}[z_1, z_2]$
$Gr^\lambda_\mu = \mathcal{N}_{SL_2}$
$Y^\lambda_\mu(z_1,z_2) = U\mathfrak{sl}_2 / C - c$
$c = (z_1 - z_2)^2 - 1$
$H^{fin}_*(R_N)$

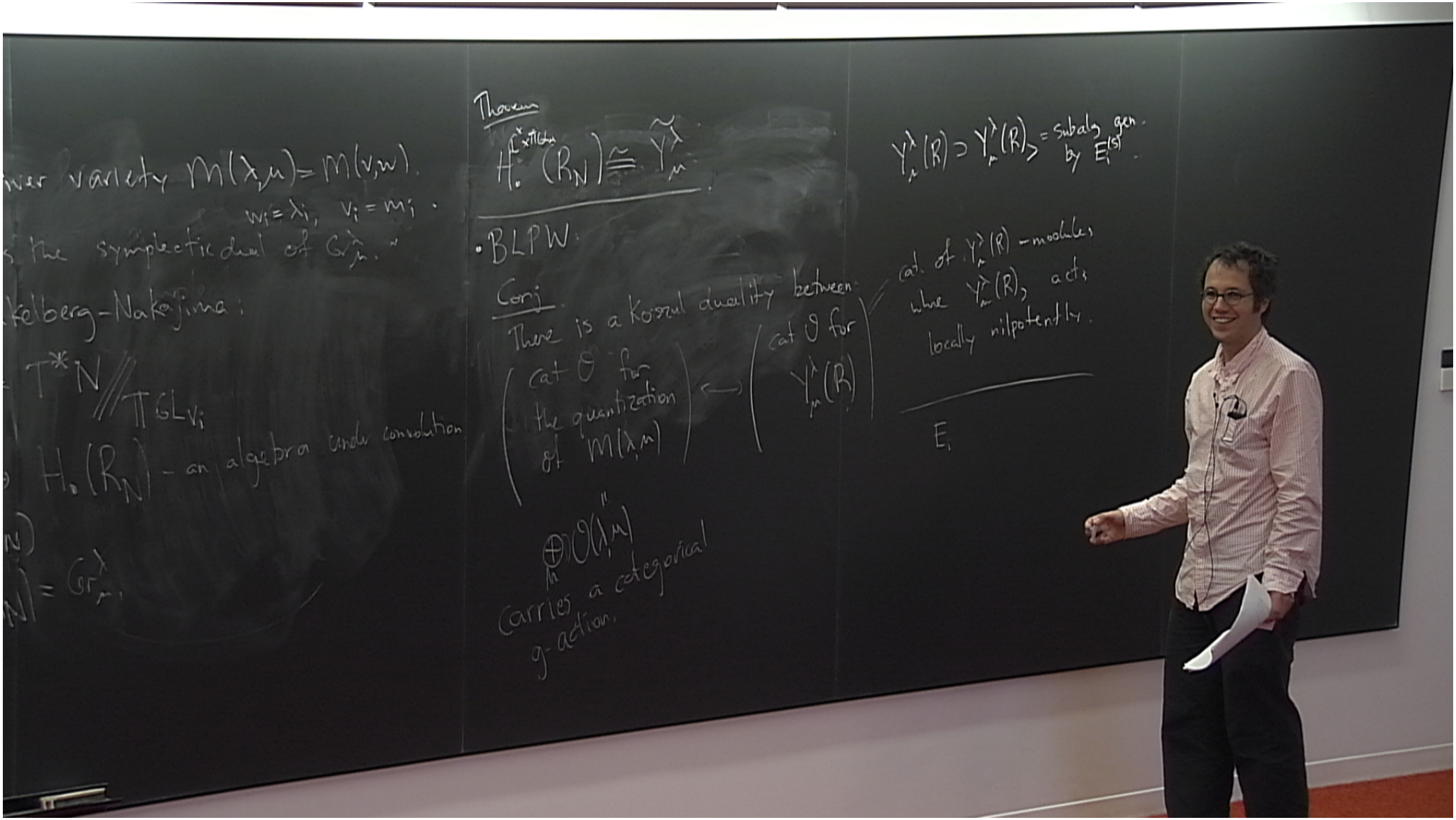

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Theorem
$H^{\text{Extension}}_\bullet(R_N) \cong \widetilde{Y}^\lambda_\mu$

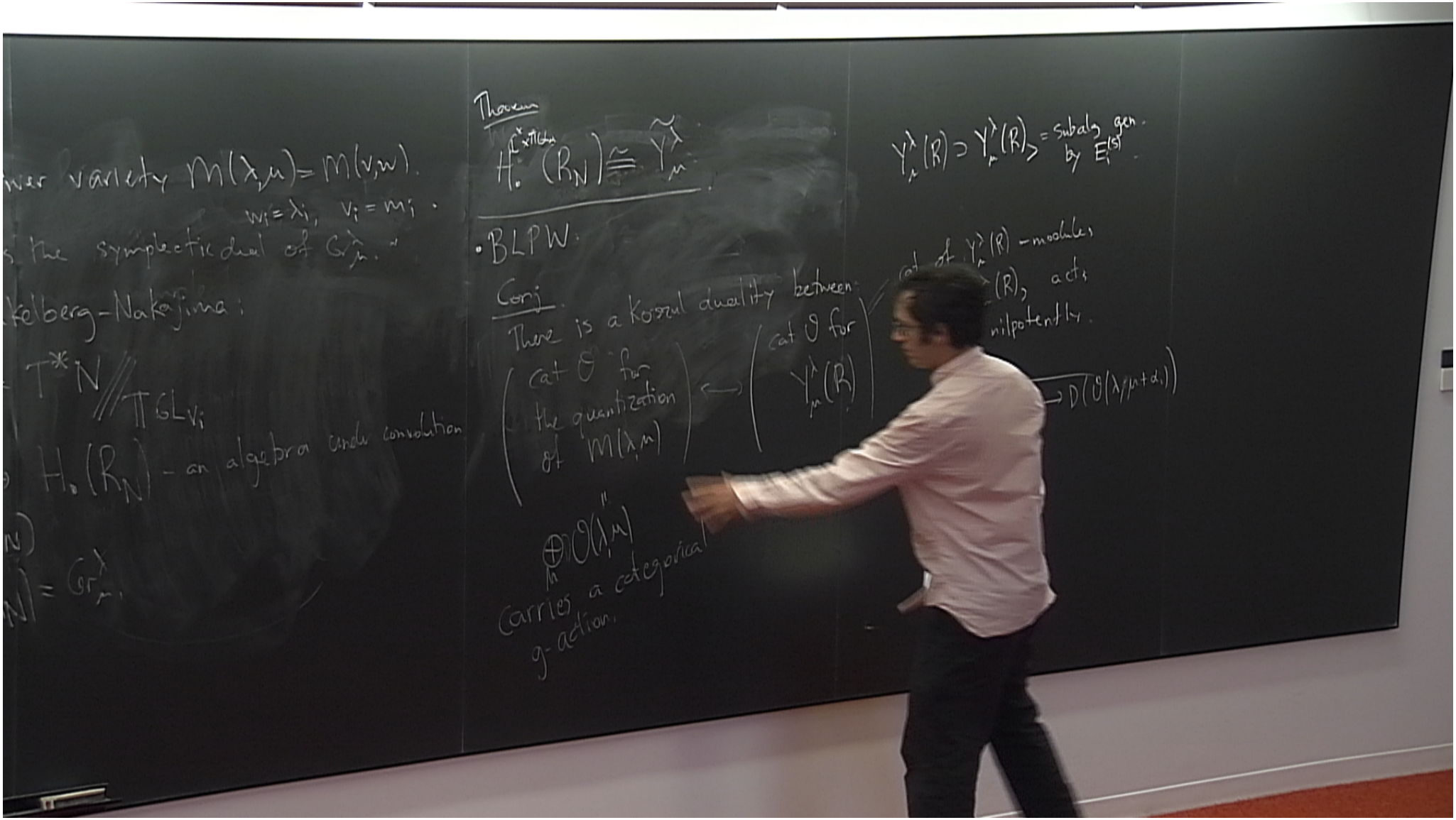

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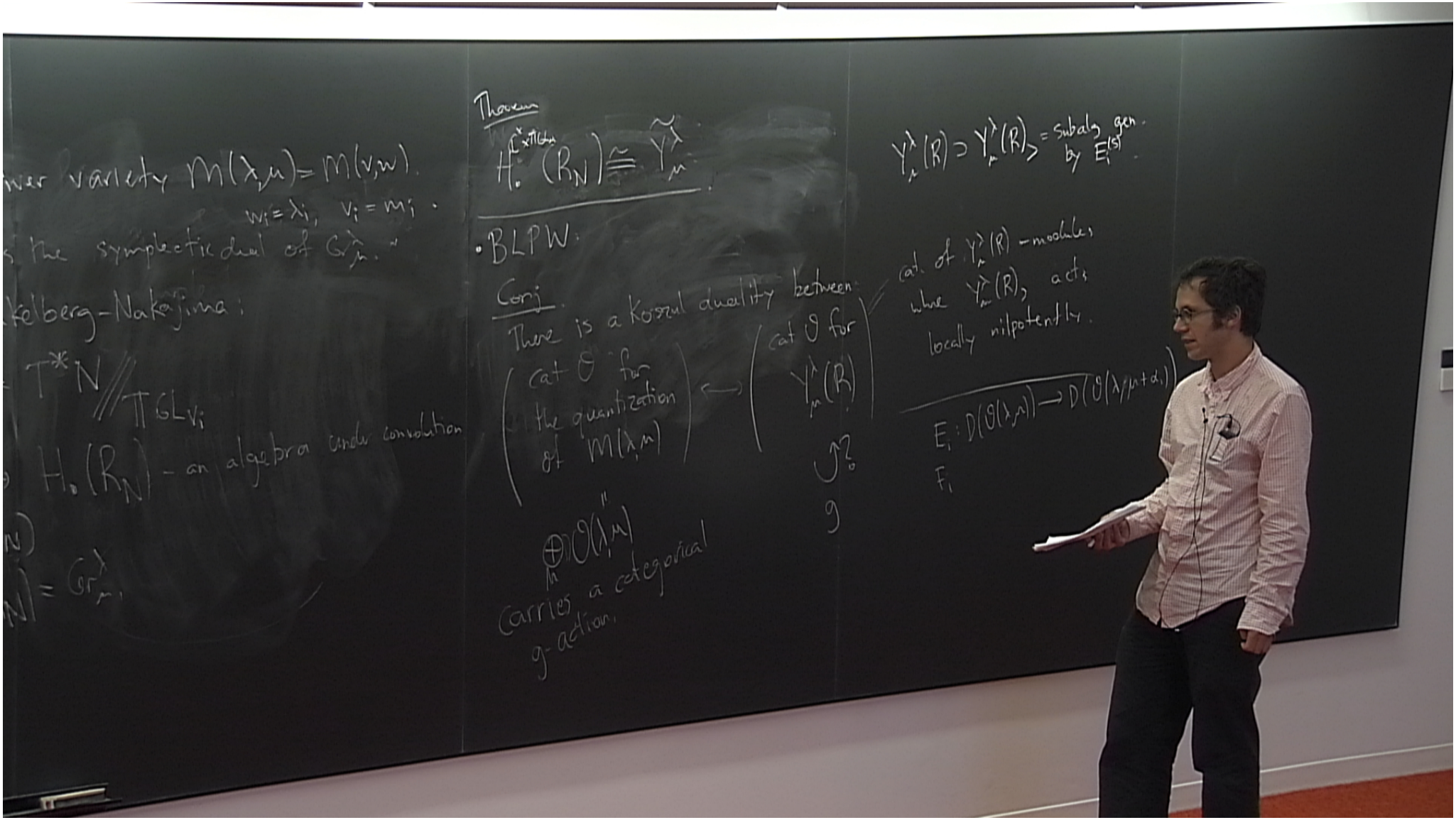

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Theorem [BF
Spec $H_*(R_N$
Theorem
$H^{Gr_{affine}}_*(R_N) \cong \widetilde{Y}^\lambda_\mu$
• BLPW
Conj
There is a Koszul duality between
cat $\mathcal{O}$
(

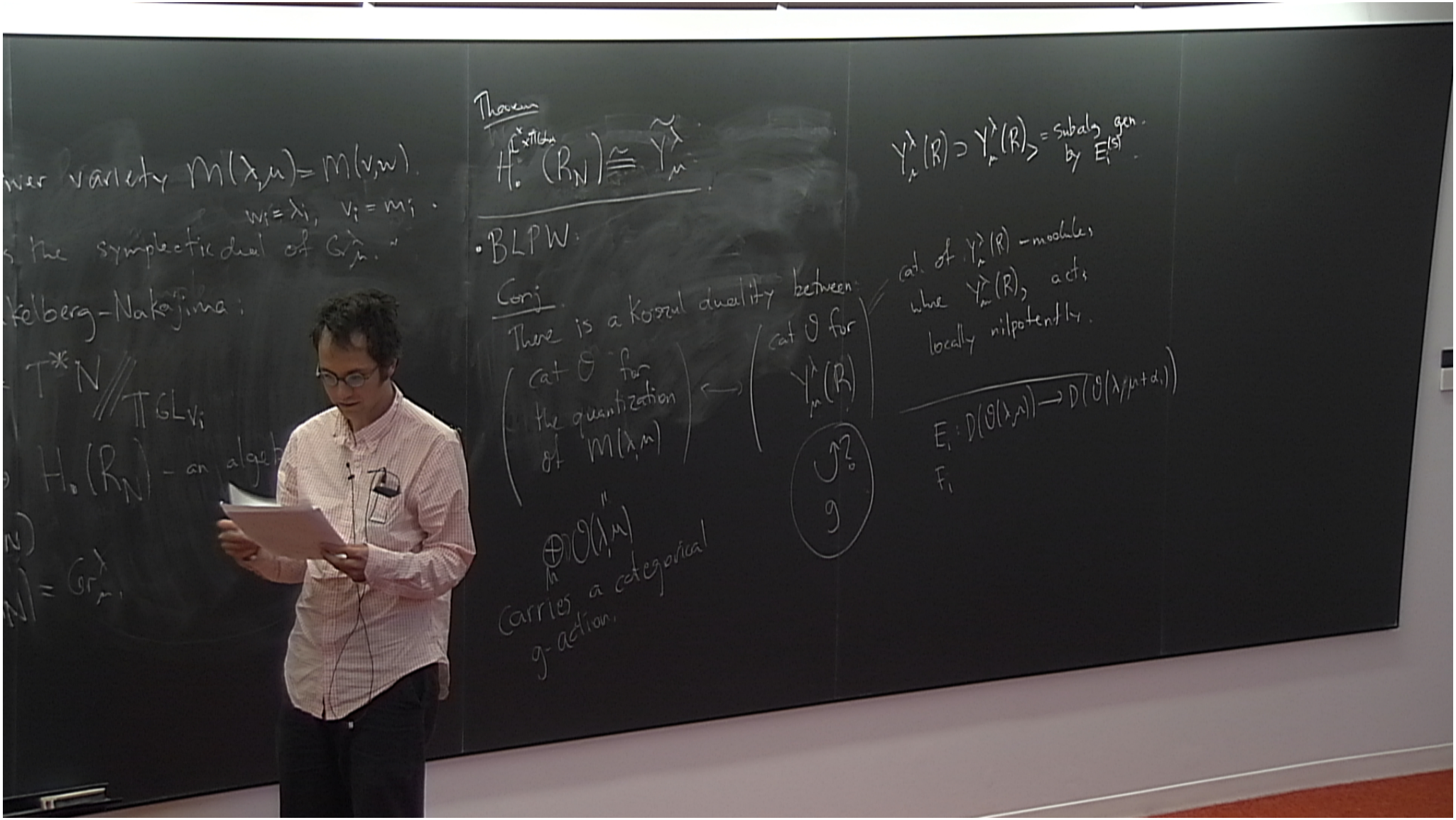

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There is a Koszul duality between
(cat $\mathcal{O}$ for the quantization of $\mathcal{M}(\lambda,\mu)$) $\leftrightarrow$ (cat $\mathcal{O}$ for $Y^\lambda_\mu(R)$)

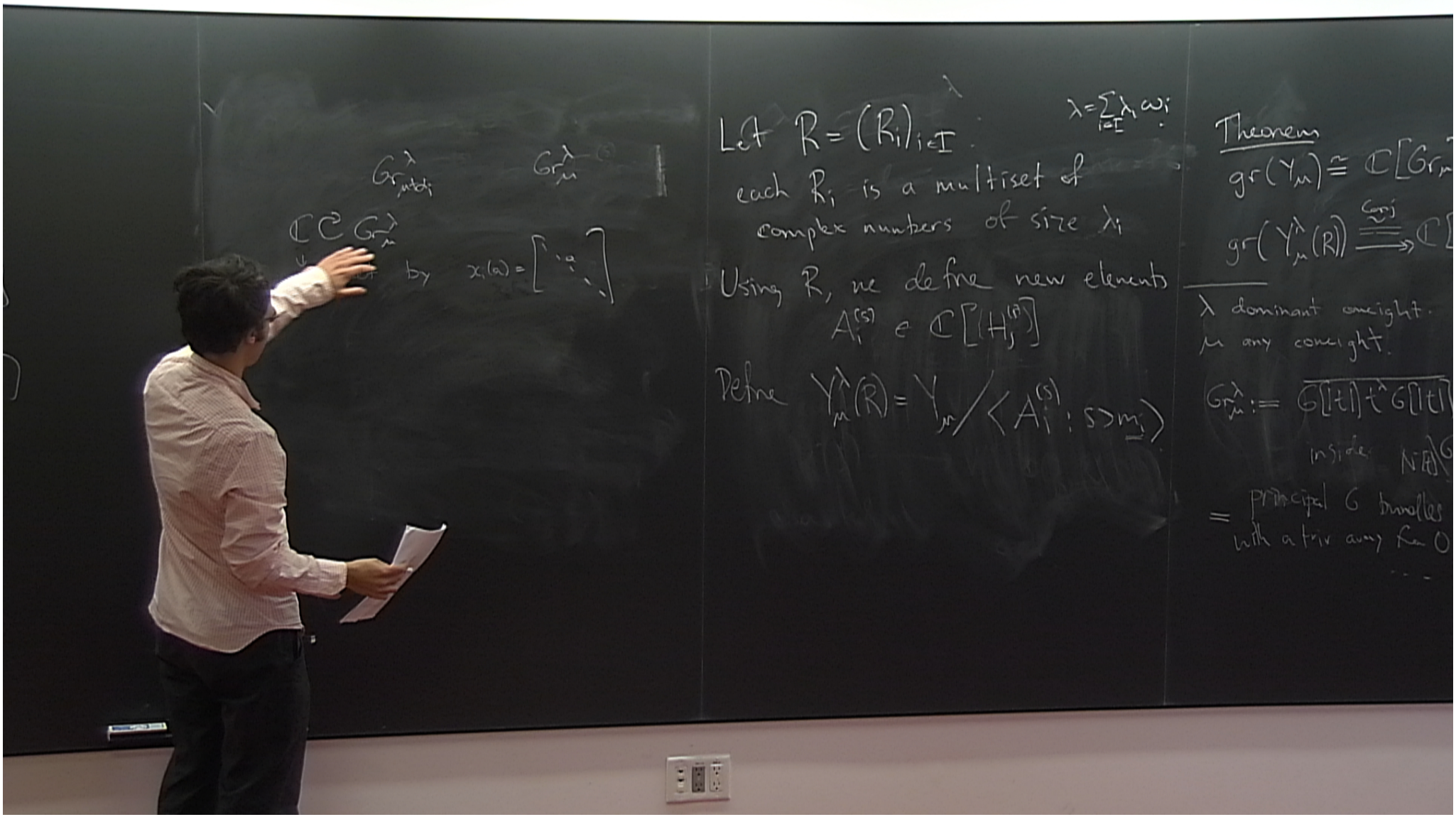

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Theorem
$H^{G_v \times T_{\mathbb{C}^N}}_\bullet(R_N) \cong \widetilde{Y}^{\lambda}_{\mu}$.
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There is a Koszul duality between:
(cat $\mathcal{O}$ for the quantization of $\mathcal{M}(\lambda,m)$) $\longleftrightarrow$ (cat $\mathcal{O}$ for $Y^{\lambda}_{m}(R)$)
$Y^{\lambda}_{\mu}(R) \supset Y^{\lambda}_{\mu}(R)_> =$ subalg gen. by $E_i^{(s)}$.
cat. of $Y^{\lambda}_{m}(R)$-modules where $Y^{\lambda}_{m}(R)_>$ acts locally nilpotently.

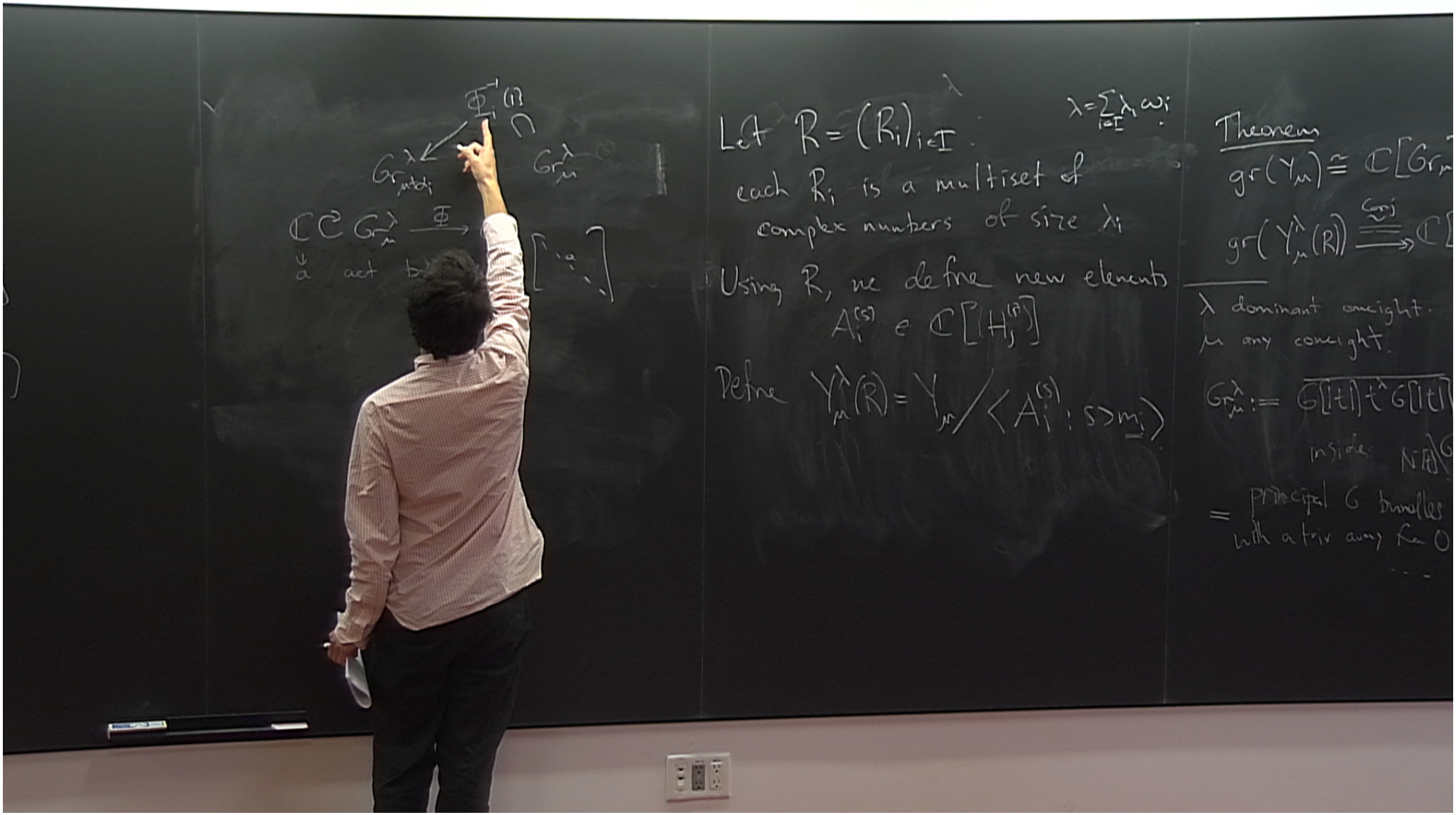

Theorem
$H_*^{G_c \times T_{\mathbb{C}}}(R_N) \cong \widetilde{Y}^\lambda_\mu$
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E_i
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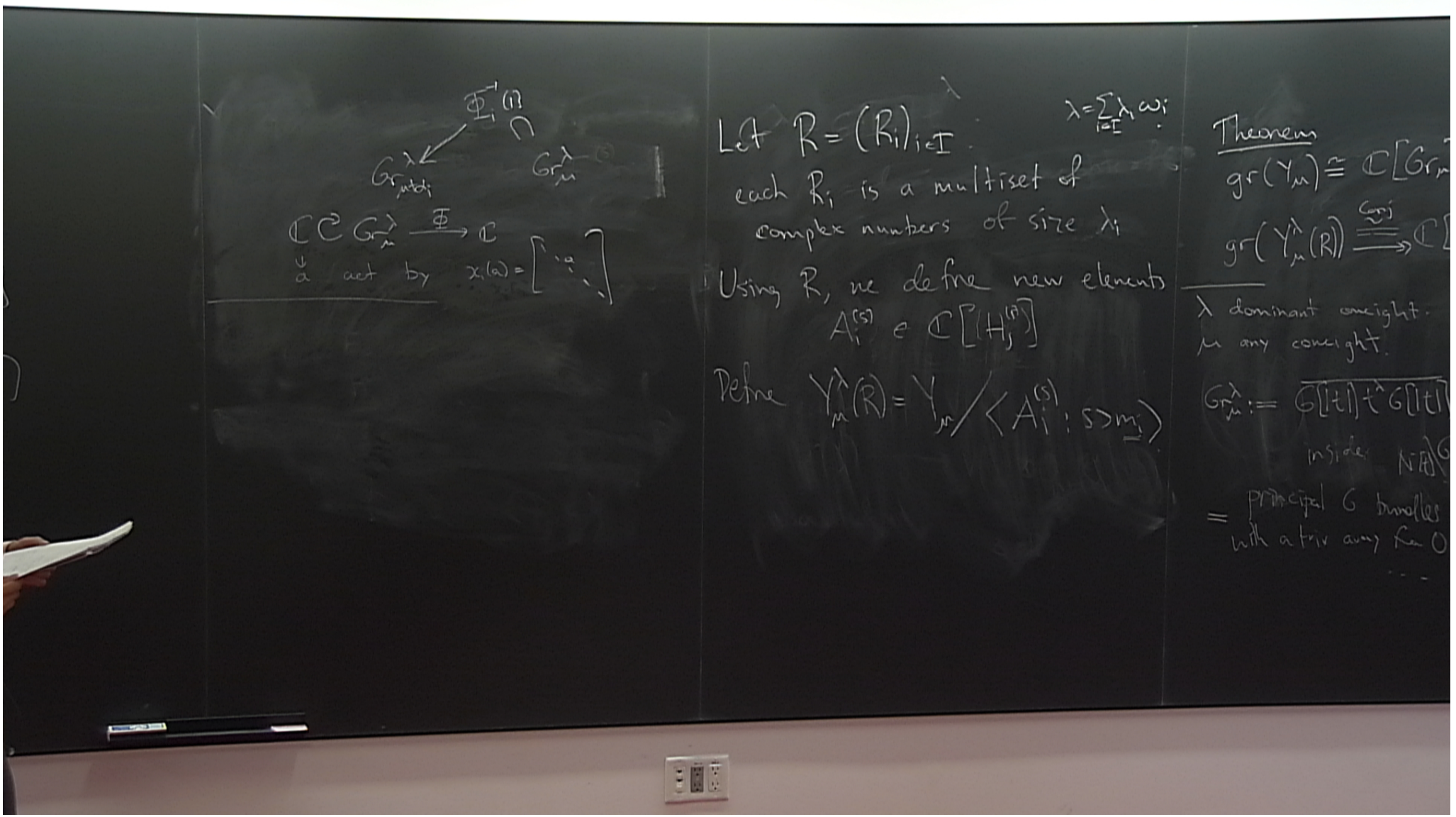

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$H_\bullet^{\mathbb{C}^\times \ltimes G_{\mathcal{O}}}(R_N) \cong \widetilde{Y}^\lambda_\mu$
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Kelberg-Nakajima:
$= Gr^\lambda_\mu$.
$\to D(\mathcal{O}(\lambda/\mu + \alpha_i))$

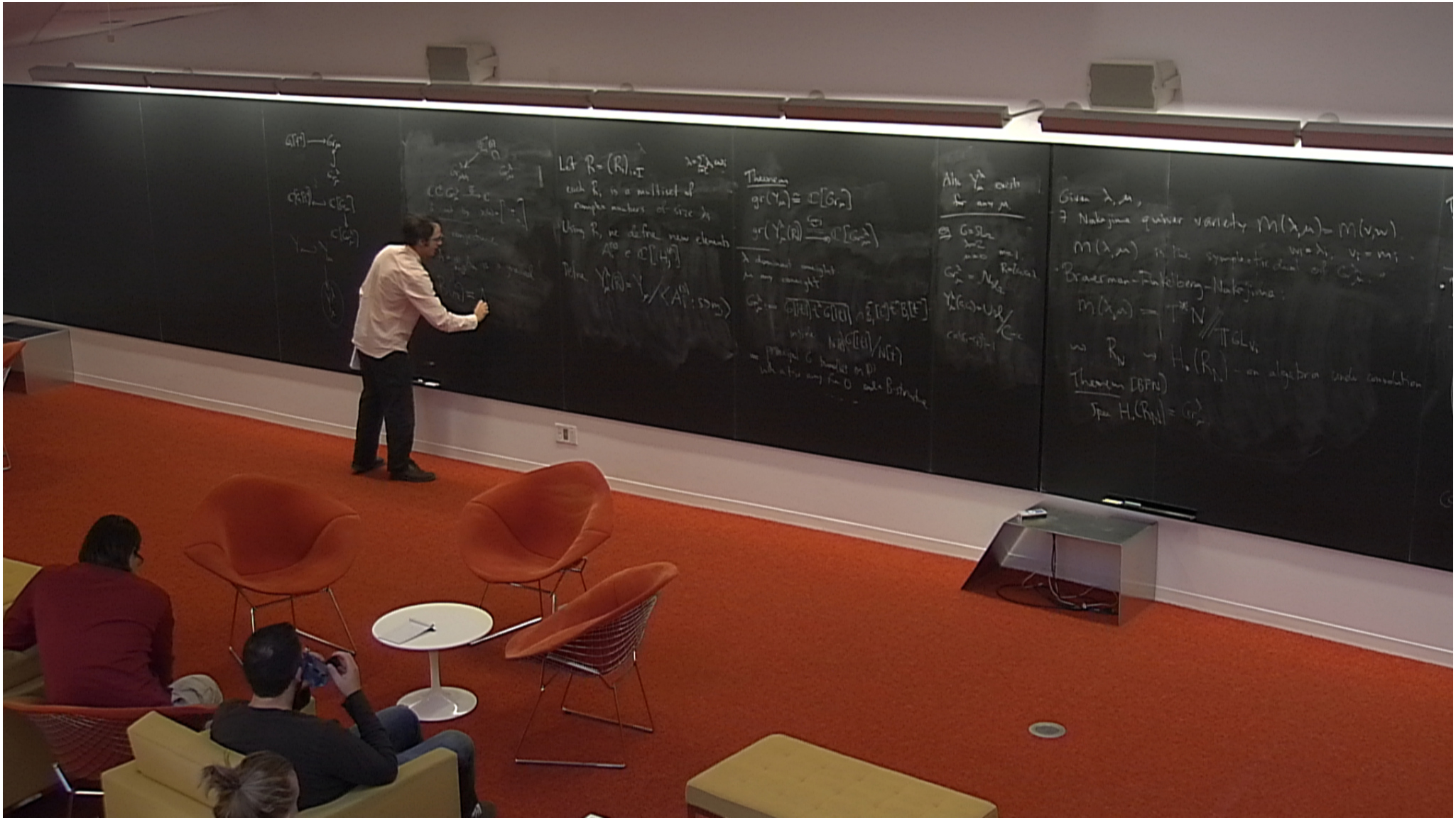

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$E_i : D(\mathcal{O}(\lambda,\mu)) \to D(\mathcal{O}(\lambda,\mu+\alpha_i))$
F_i
$\mathfrak{g}$
$\bigoplus_\mu \mathcal{O}(\lambda,\mu)$
Carries a categorical $\mathfrak{g}$-action.
$H_0(R_N)$ - an algebra under convolution
$T^*N /\!\!/ \prod GL_{V_i}$
$w_i = \lambda_i$, $v_i = m_i$.
$= Gr^\lambda_\mu$.

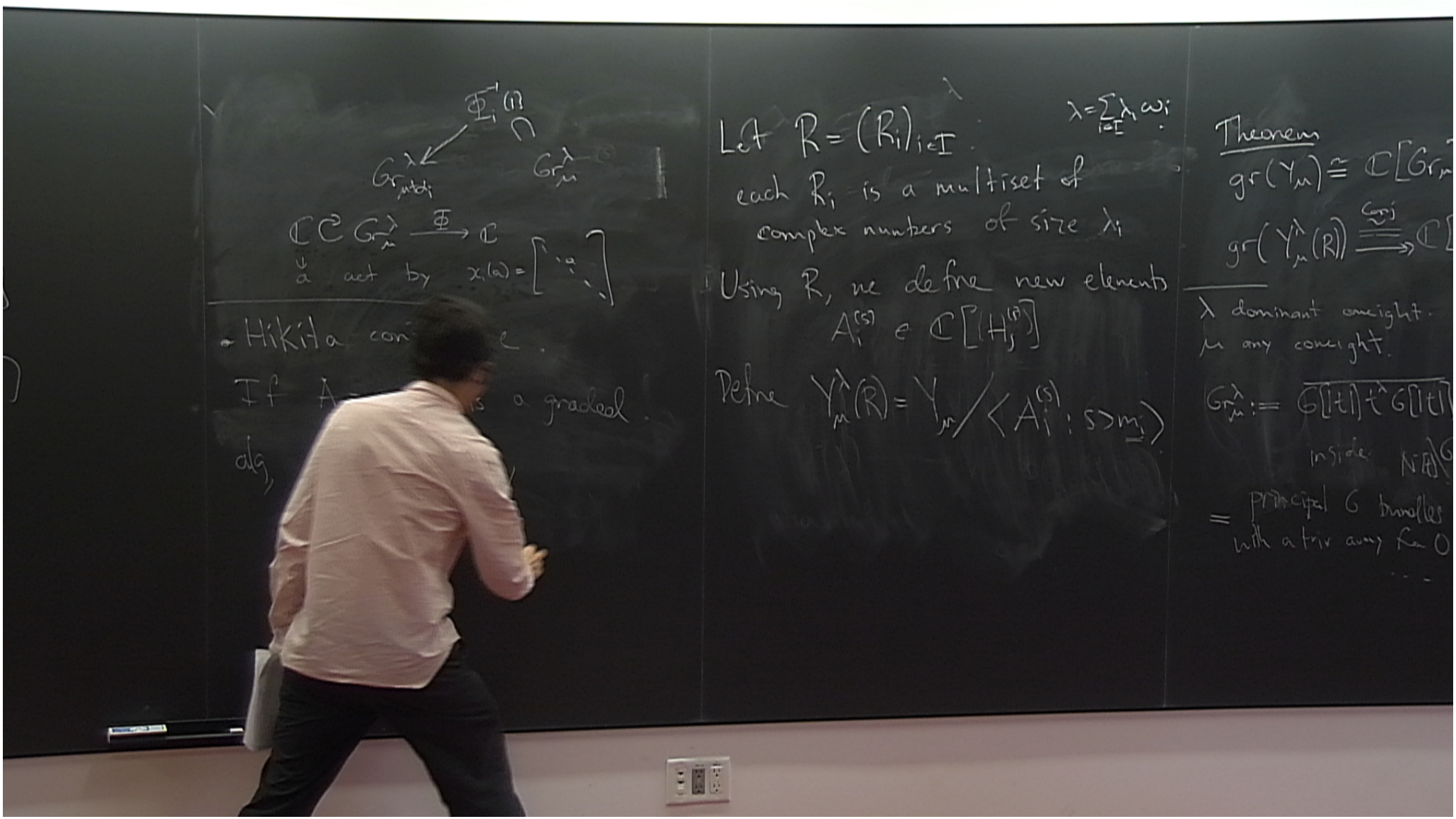

Theorem
$H_0^{\mathbb{C}^\times \rtimes G_N}(R_N) \cong \widetilde{Y}^\lambda_\mu$
$Y^\lambda_\mu(R) \supset Y^\lambda_\mu(R)_> =$ subalg gen. by $E_i^{(s)}$
• BLPW:
Conj
There is a Koszul duality between
(cat $\mathcal{O}$ for the quantization of $\mathcal{M}(\lambda,\mu)$) $\leftrightarrow$ (cat $\mathcal{O}$ for $Y^\lambda_\mu(R)$)
cat of $Y^\lambda_\mu(R)$-modules where $Y^\lambda_\mu(R)_>$ acts locally nilpotently.
$\mathfrak{g}$
$E_i : D(\mathcal{O}(\lambda,\mu)) \to D(\mathcal{O}(\lambda,\mu+\alpha_i))$
F_i
Carries a categorical $\mathfrak{g}$-action.

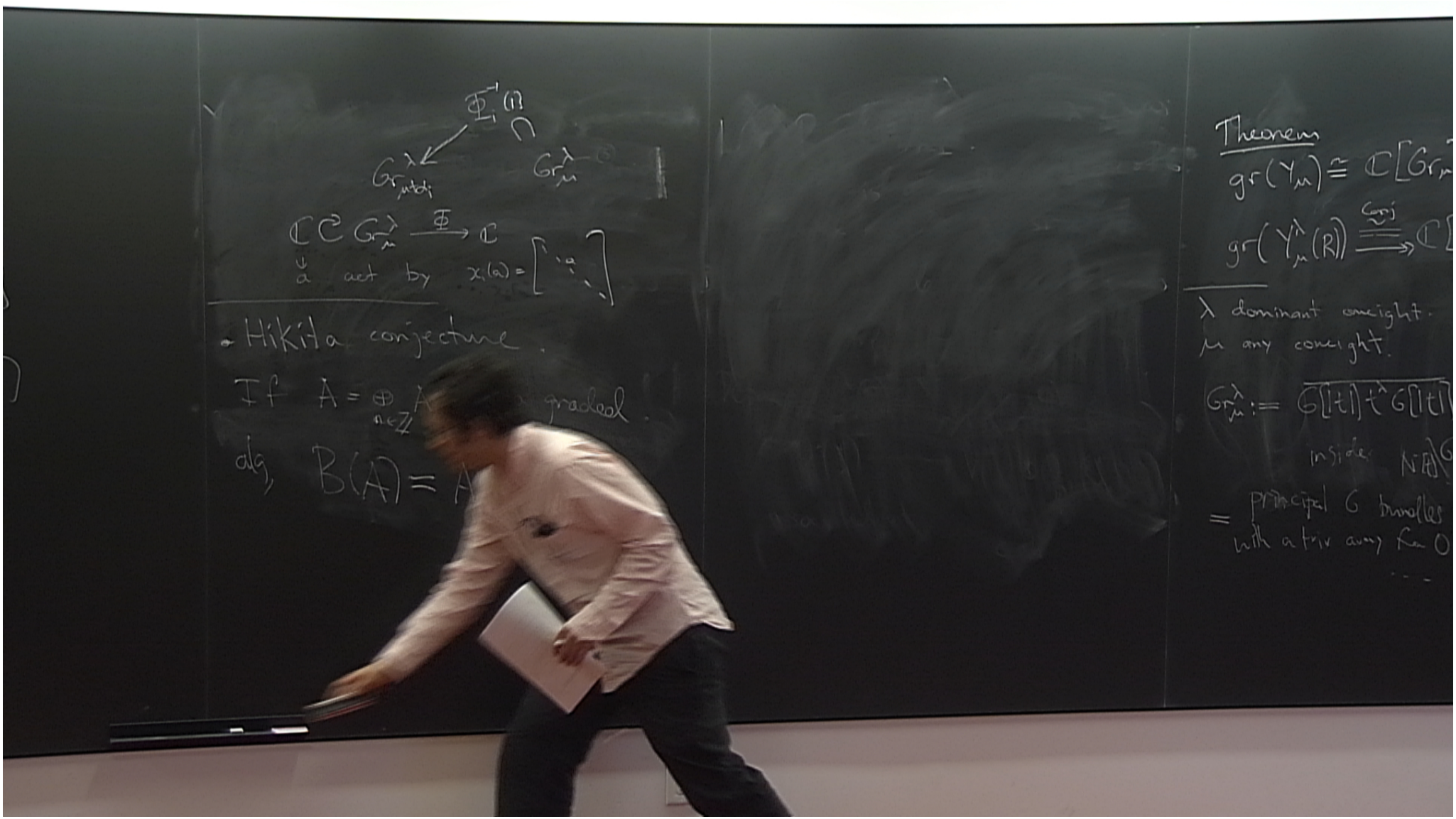

Let $R = (R_i)_{i \in I}$
each R_i is a multiset of complex numbers of size λ_i
Using R, we define new elements $A_i^{(s)} \in \mathbb{C}[H_j^{(s)}]$
Define $Y_\mu^\lambda(R) = Y_\mu / \langle A_i^{(s)} : s > m_i \rangle$
$\lambda = \sum_{i \in I} \lambda_i \omega_i$
Theorem
$gr(Y_\mu) \cong \mathbb{C}[Gr$
$gr(Y_\mu^\lambda(R)) \xrightarrow{\text{Conj}} \mathbb{C}[$
λ dominant coweight.
μ any coweight.
$Gr_\mu^\lambda :=$
inside
principal G
with a triv

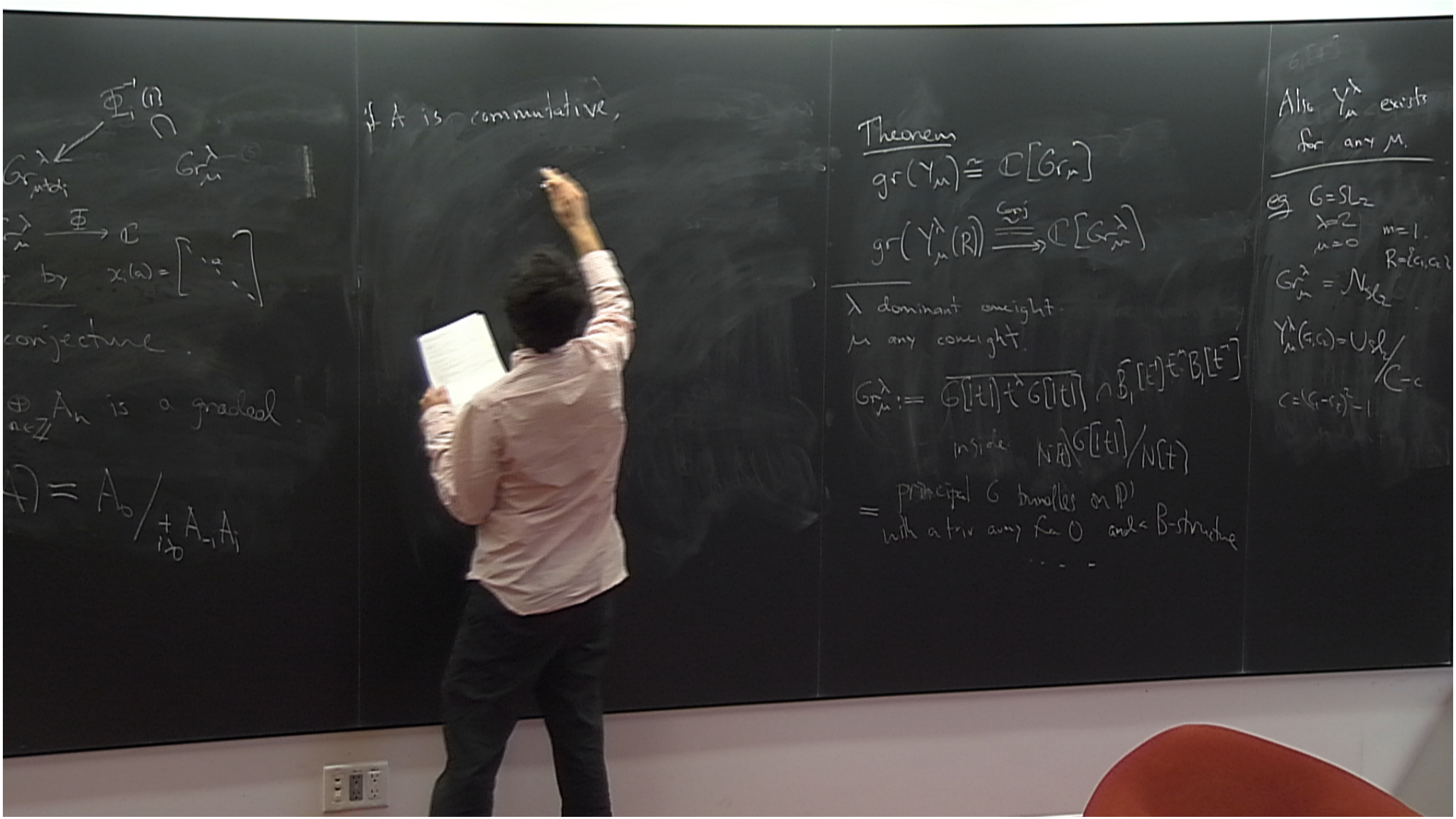

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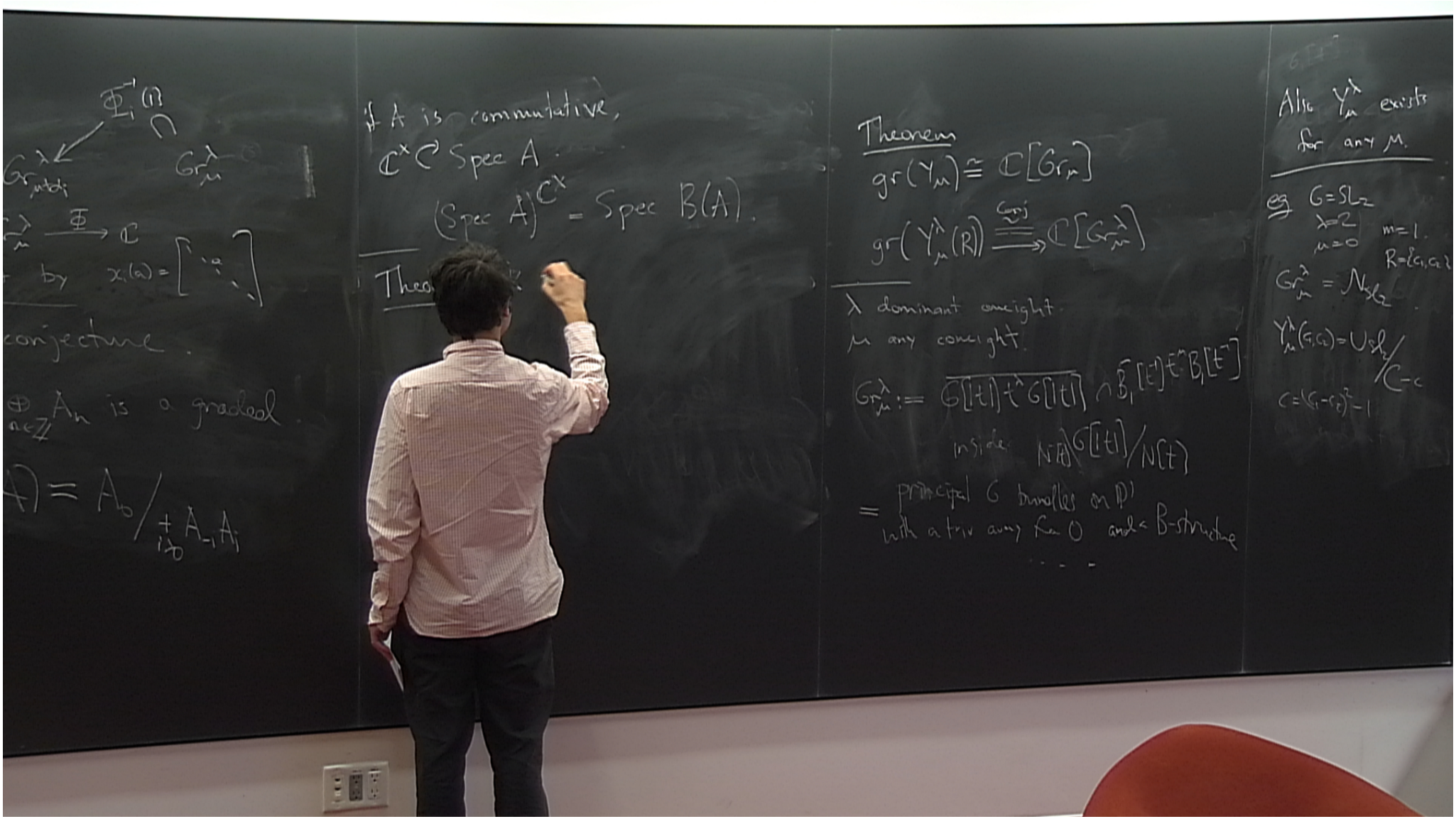

$\Phi_i^{-1}(1)$
$Gr^{\lambda}_{\mu,\mathrm{ntd}_i}$
Gr^{λ}_{μ}
$\mathbb{C}^{\times}\subset G^{\lambda}_{\mu} \xrightarrow{\Phi} \mathbb{C}$
a act by $x_i(a)=\begin{bmatrix} \ddots & & \\ & a & \\ & & \ddots \end{bmatrix}$
Let $R=(R_i)_{i\in I}$. $\lambda=\sum_{i\in I}\lambda_i\omega_i$.
each R_i is a multiset of complex numbers of size λ_i
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$Gr^{\lambda}_{\mu}:=\overline{G[[t]]t^{\lambda}G[[t]]}$
inside
$=$ principal G bundles
with a triv away fm 0

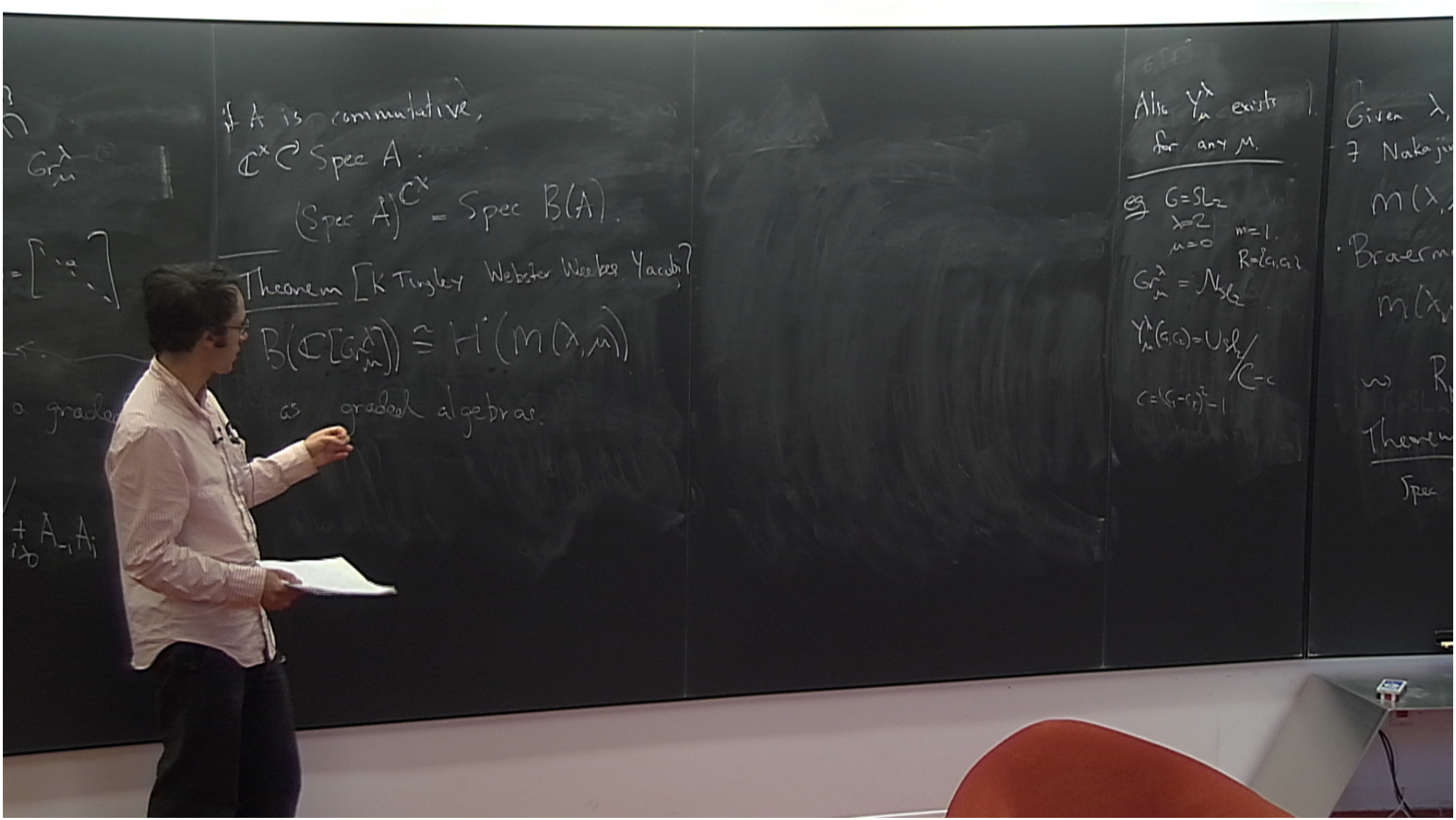

Let $R = (R_i)_{i \in I}$
Theorem
$gr(Y_\lambda) \cong \mathbb{C}[Gr_\lambda]$
$gr(Y_\lambda(R)) \cong \mathbb{C}[Gr_\lambda^R]$
Given λ, μ,
Nakajima quiver variety $\mathcal{M}(\lambda,\mu) = \mathcal{M}(v,w)$
$\mathcal{M}(\lambda,\mu)$ is the symplectic dual of $Gr_\lambda^{\mu^\vee}$
Braverman-Finkelberg-Nakajima:
$\mathcal{M}(\lambda,\mu) = T^*N /\!\!/ G(\mathcal{O})$
$H_*(R_N)$ – an algebra under convolution
Theorem [BFN]
$H_*(R_N) \cong$

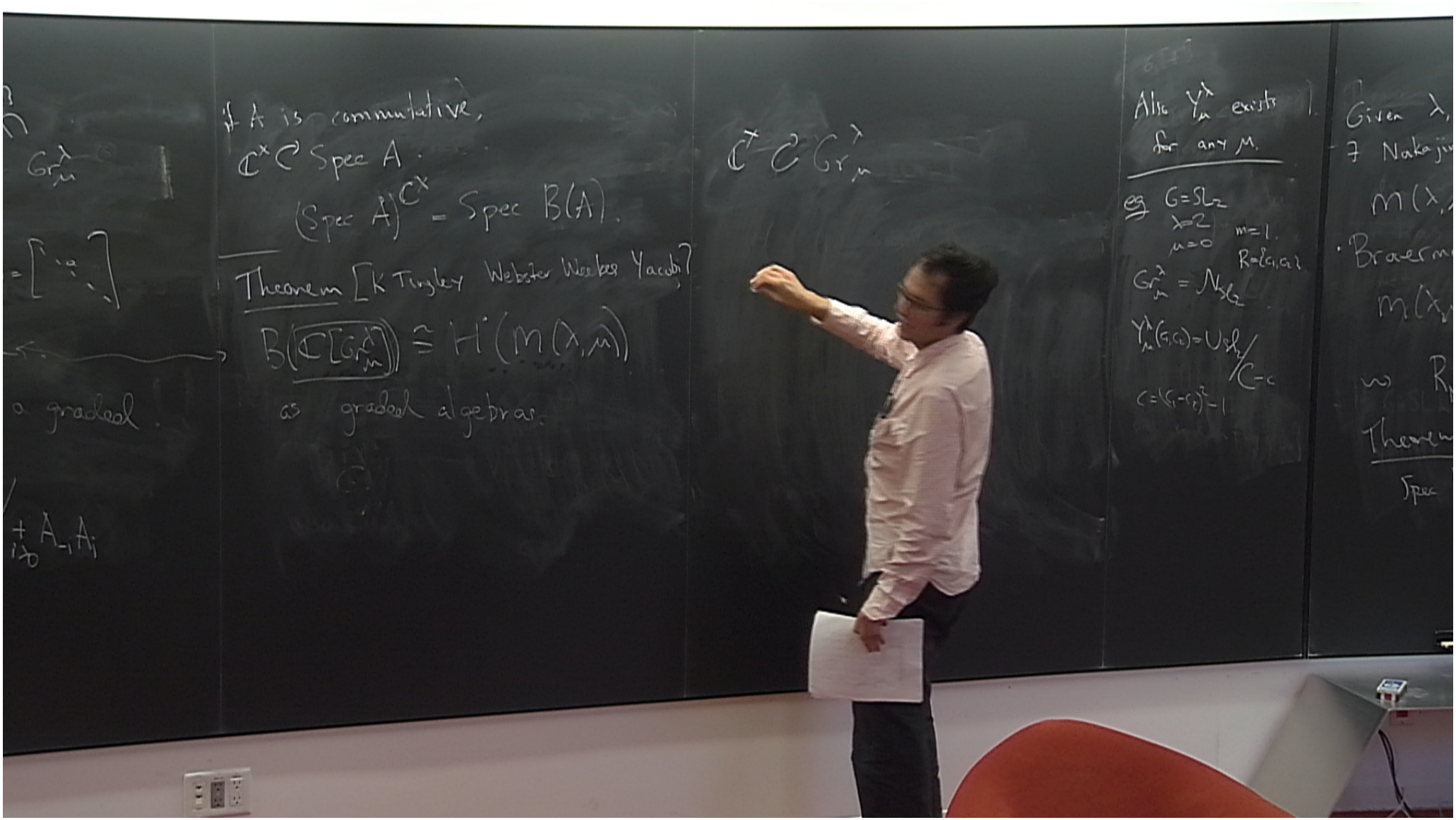

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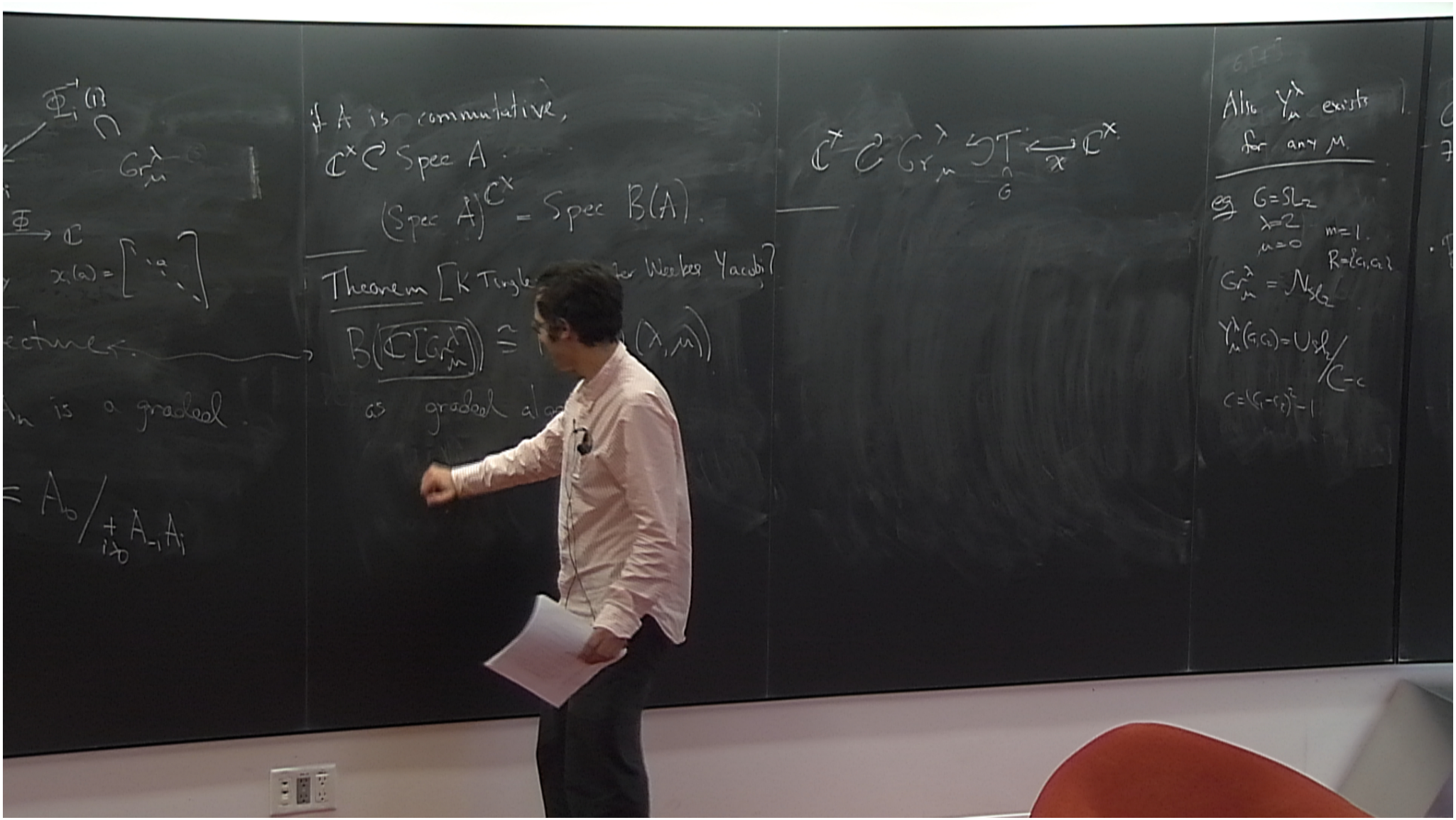

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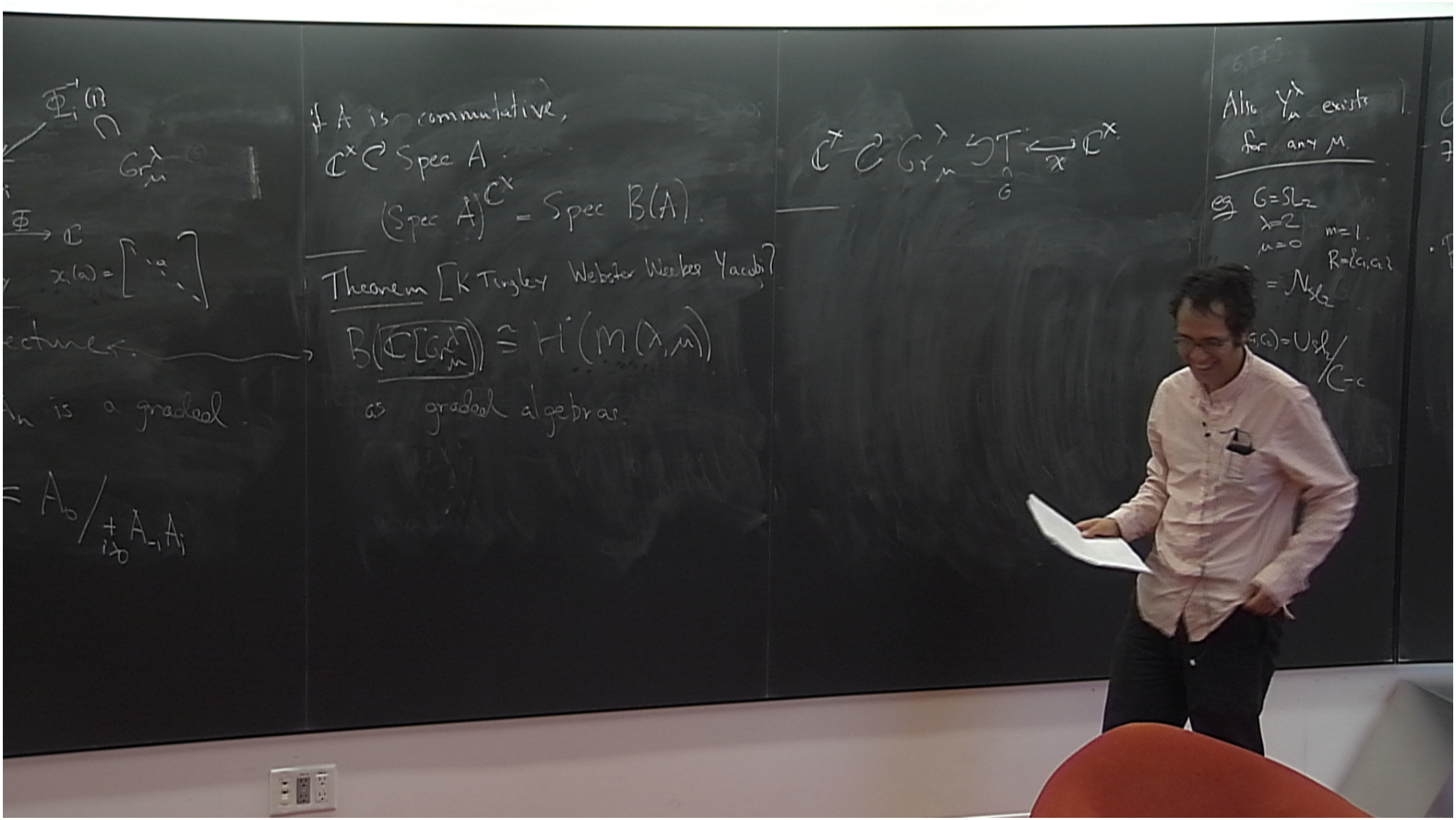

If A is commutative,
Theorem
gr(Y_μ^λ) ≅ ℂ[Gr_μ^λ]
λ dominant coweight
μ any coweight
Also Y_μ^λ exists for any μ.
eg G = SL_2
λ = 2
μ = 0
m = 1
R = {c_1, c_2}
Gr_μ^λ = N_{SL_2}
Y_μ^λ(c_1, c_2) = U_{SL_2}/C
C = (c_1 - c_2)^2 - 1
conjecture.
inside
principal G bundles on P^1
with a triv away from 0 and ∞ B-structure

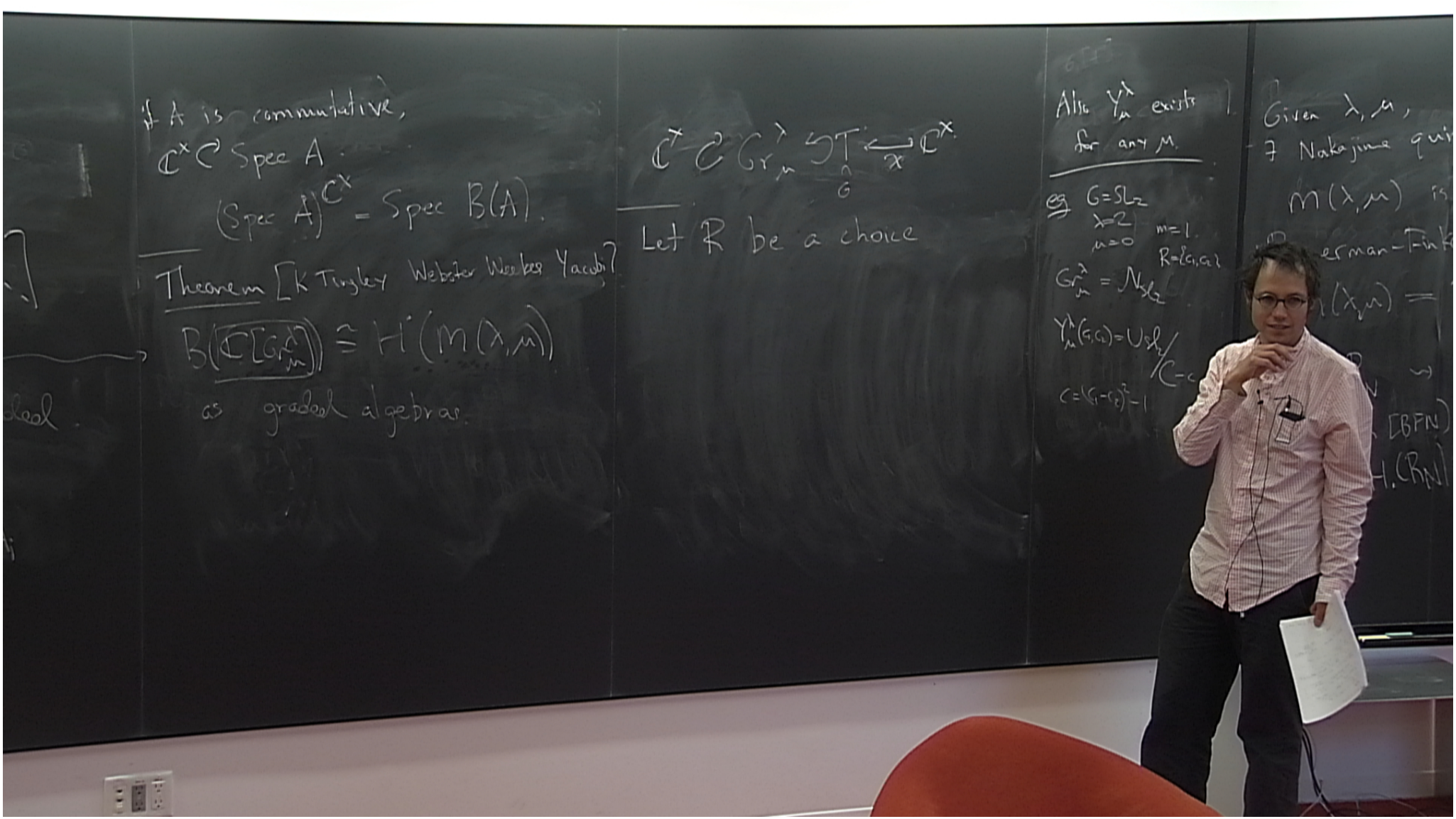

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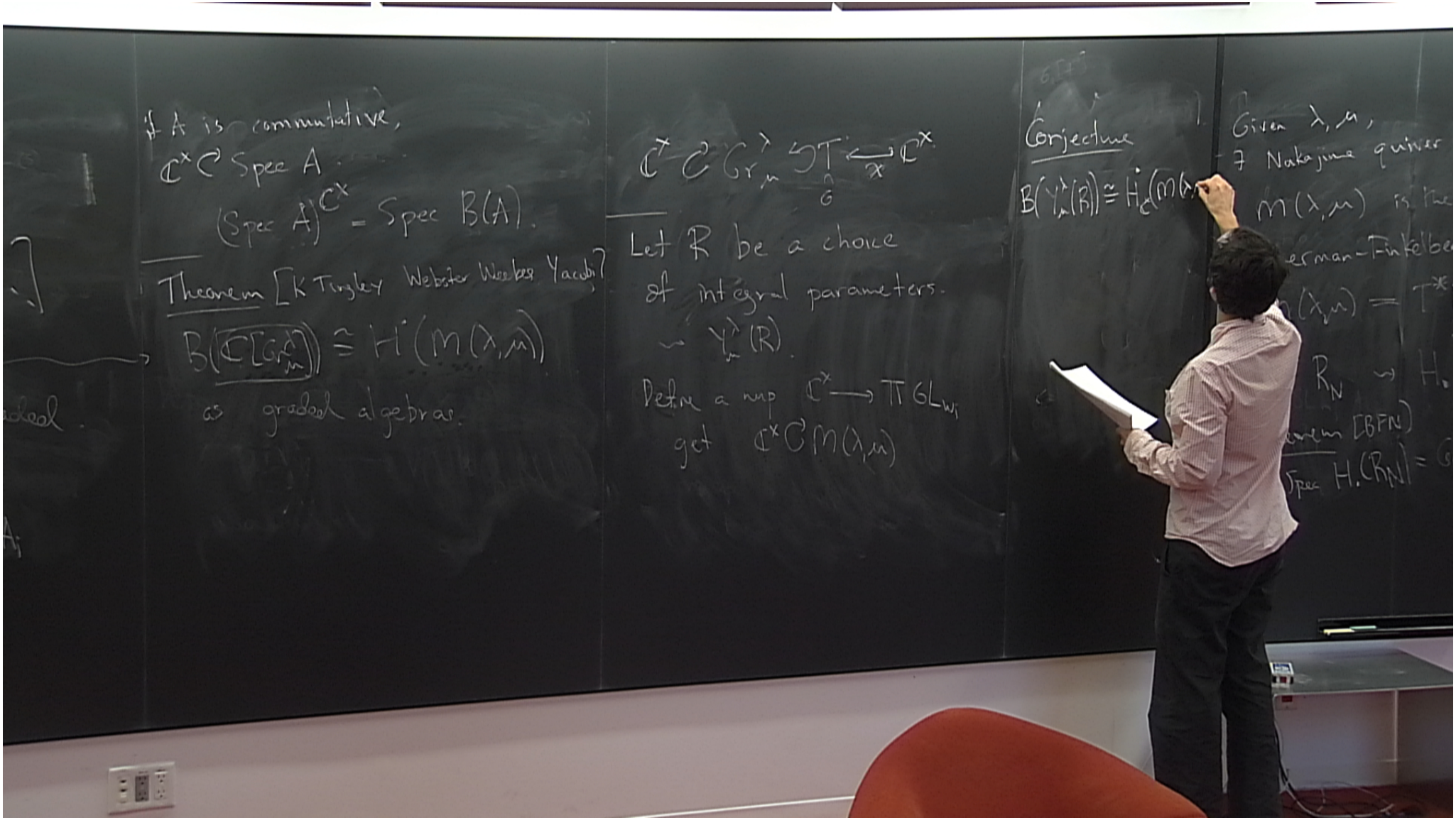

If A is commutative,
$\mathbb{C}^\times \subset \operatorname{Spec} A$.
$(\operatorname{Spec} A)^{\mathbb{C}^\times} = \operatorname{Spec} B(A)$.
Theorem [K Tingley Webster Weekes Yacobi]
$B(\mathbb{C}[G^\lambda_{r,\mu}]) \cong H^*(\mathcal{M}(\lambda,\mu))$
as graded algebras.
All Y^λ_μ exists for any μ.
eg $G = SL_2$
$\lambda = 2$ $m = 1$.
$\mu = 0$ $R = \mathbb{C}[c_1, c_2]$
$G^\lambda_\mu = N_{SL_2}$
$Y^\lambda_\mu(c_1,c_2) = U\mathfrak{sl}_2 / C - c$
$c = (c_1 - c_2)^2 - 1$
Given λ,
7 Nakaji
$\mathcal{M}(\lambda,$
Braverm
$\mathcal{M}(\lambda,$
Theorem
Spec

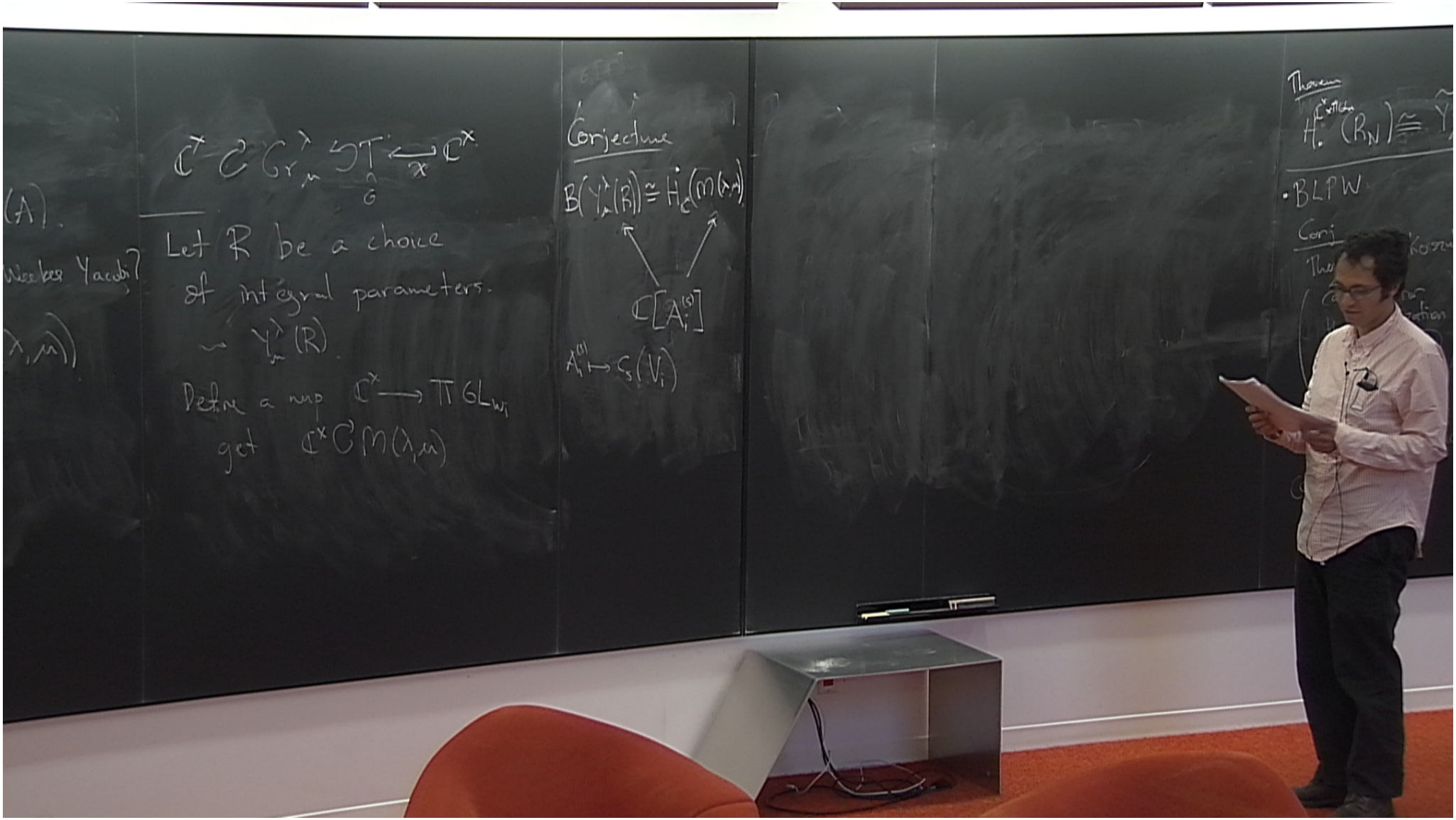

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Nakaj
$\mathcal{M}(\lambda,$
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Theorem
Spec
graded
$\sum_{i>0} A_{-i} A_i$

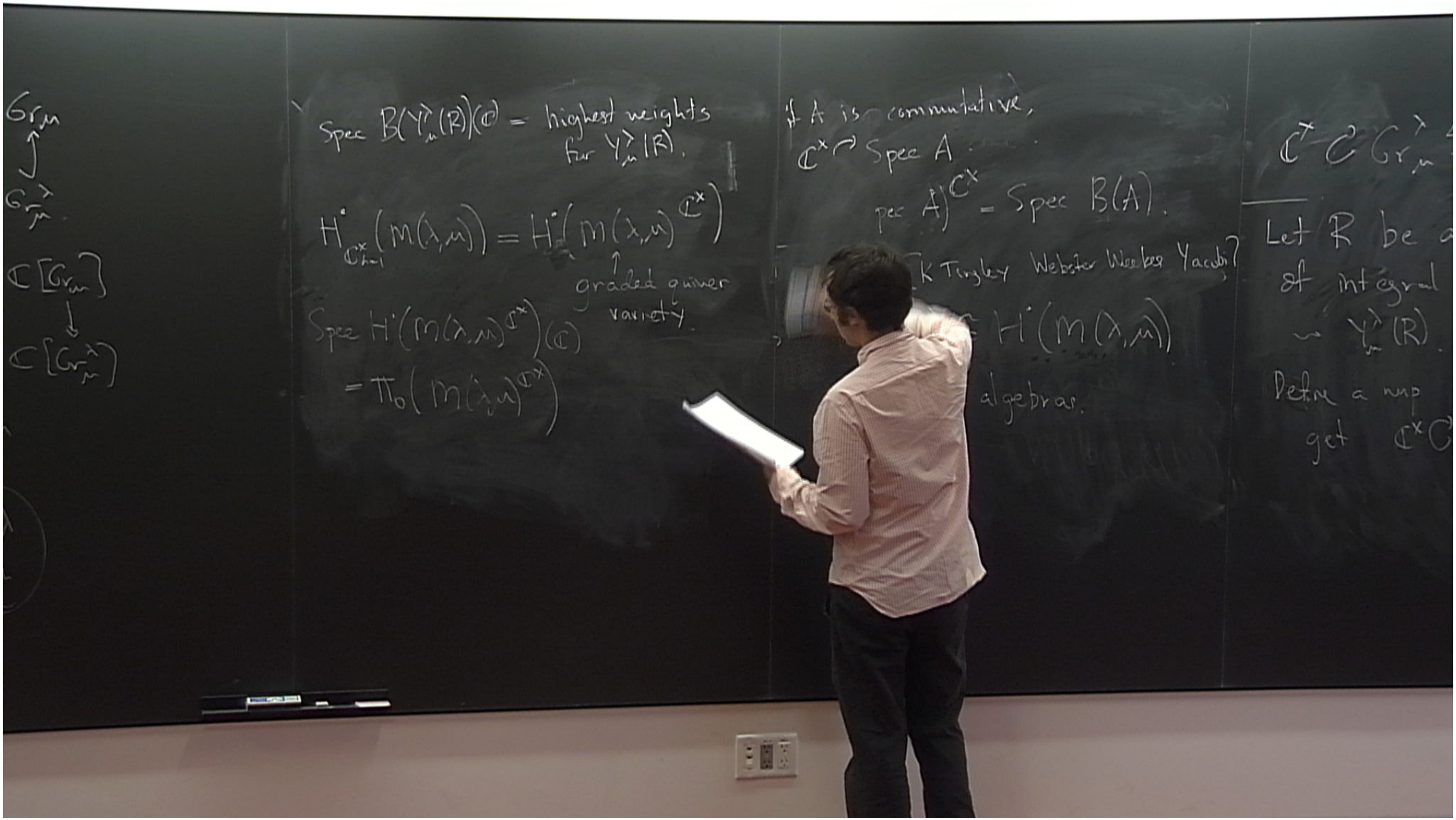

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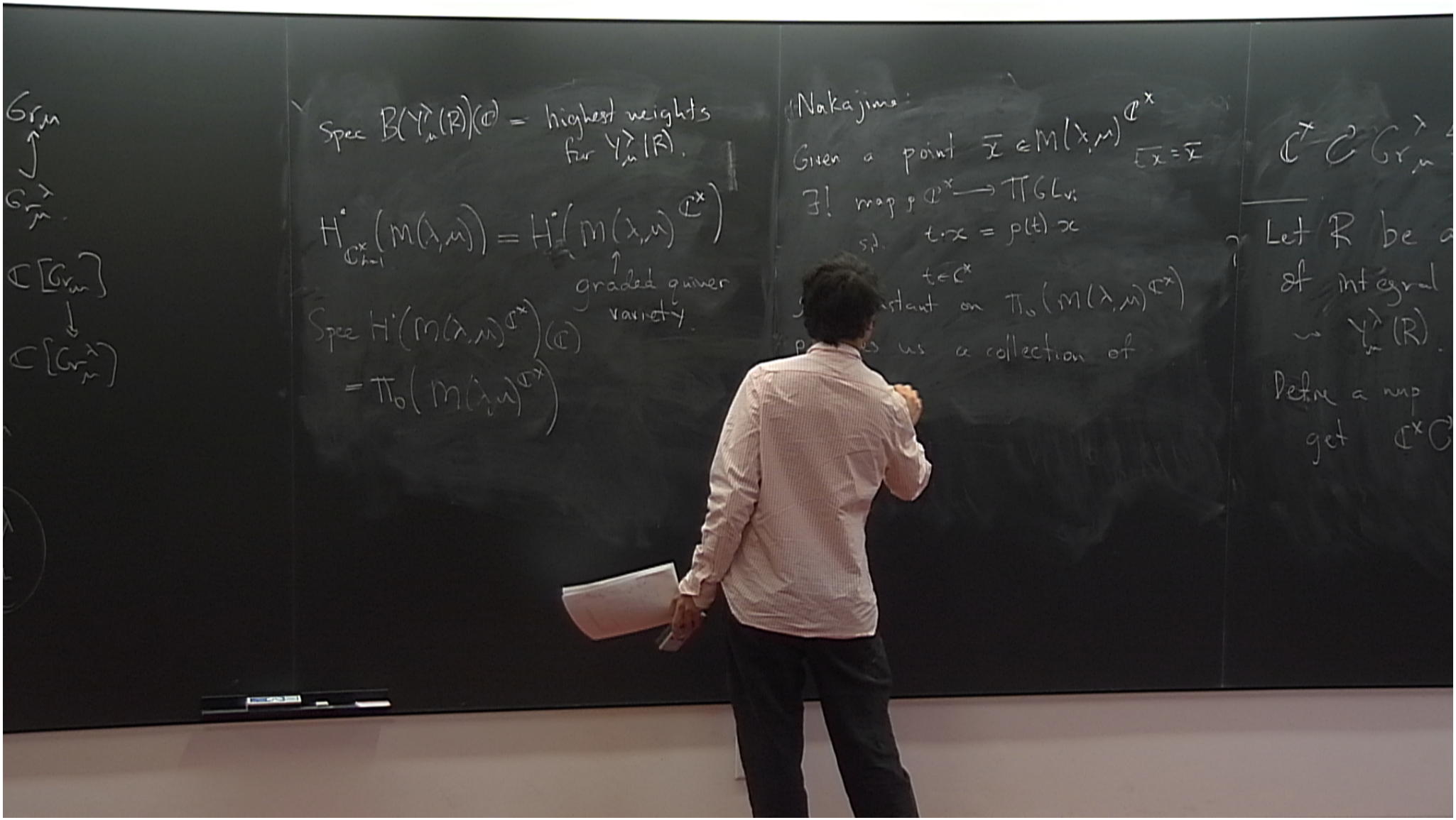

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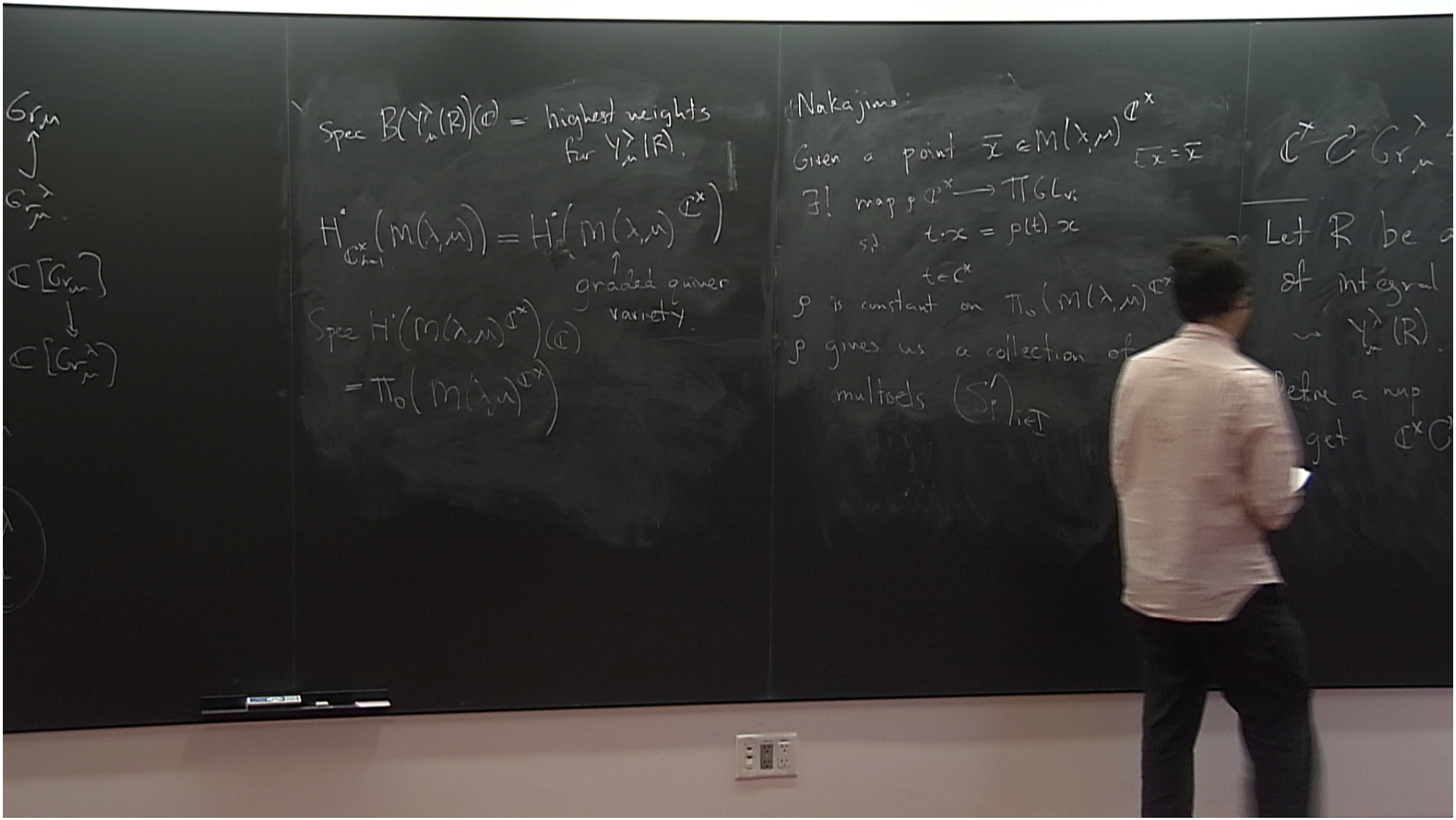

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[BFN]

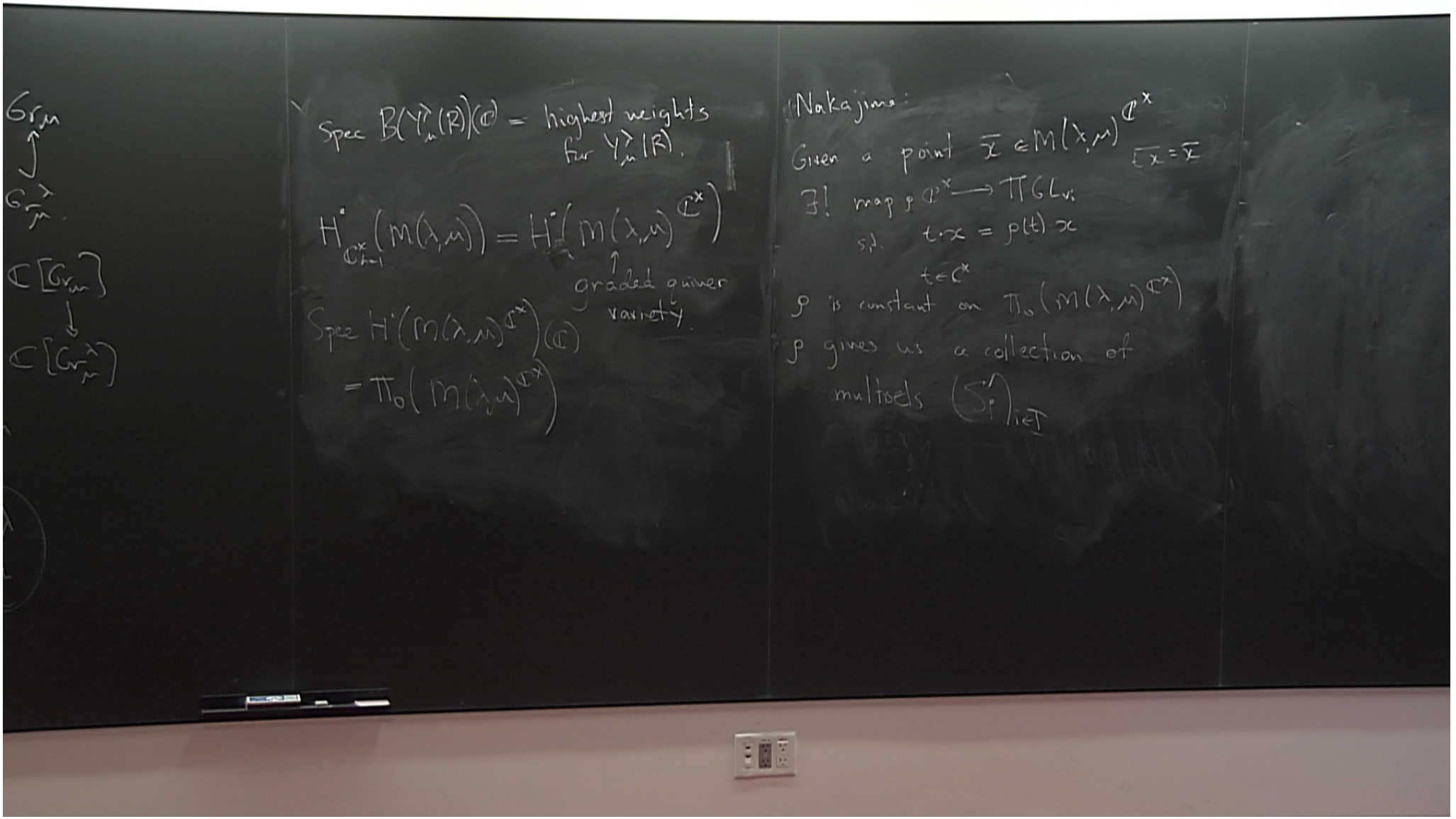

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Theorem [K Tingley Webster Weekes Yacobi]
$B(\mathbb{C}[G_{r,m}^{\lambda}]) \cong H^{\cdot}(\mathcal{M}(\lambda,\mu))$
as graded algebras.
$\mathbb{C}^\times \curvearrowright Gr_m^\lambda \supset T^* \rightleftarrows \mathbb{C}^\times$
Let $\mathcal{R}$ be a choice of integral parameters.
$\rightsquigarrow Y_m^\lambda(\mathcal{R})$
Define a map $\mathbb{C}^\times \longrightarrow T\,GL_{w_i}$
get $\mathbb{C}^\times \curvearrowright \mathcal{M}(\lambda,\mu)$
Conjecture
$B(Y_m^\lambda(\mathcal{R})) \cong H_{\mathbb{C}}^{\cdot}(\mathcal{M}(\lambda,\mu))$
Given λ, μ,
$\mathcal{M}(\lambda,\mu)$ is the
R_N
Spec $H.(R_N)$

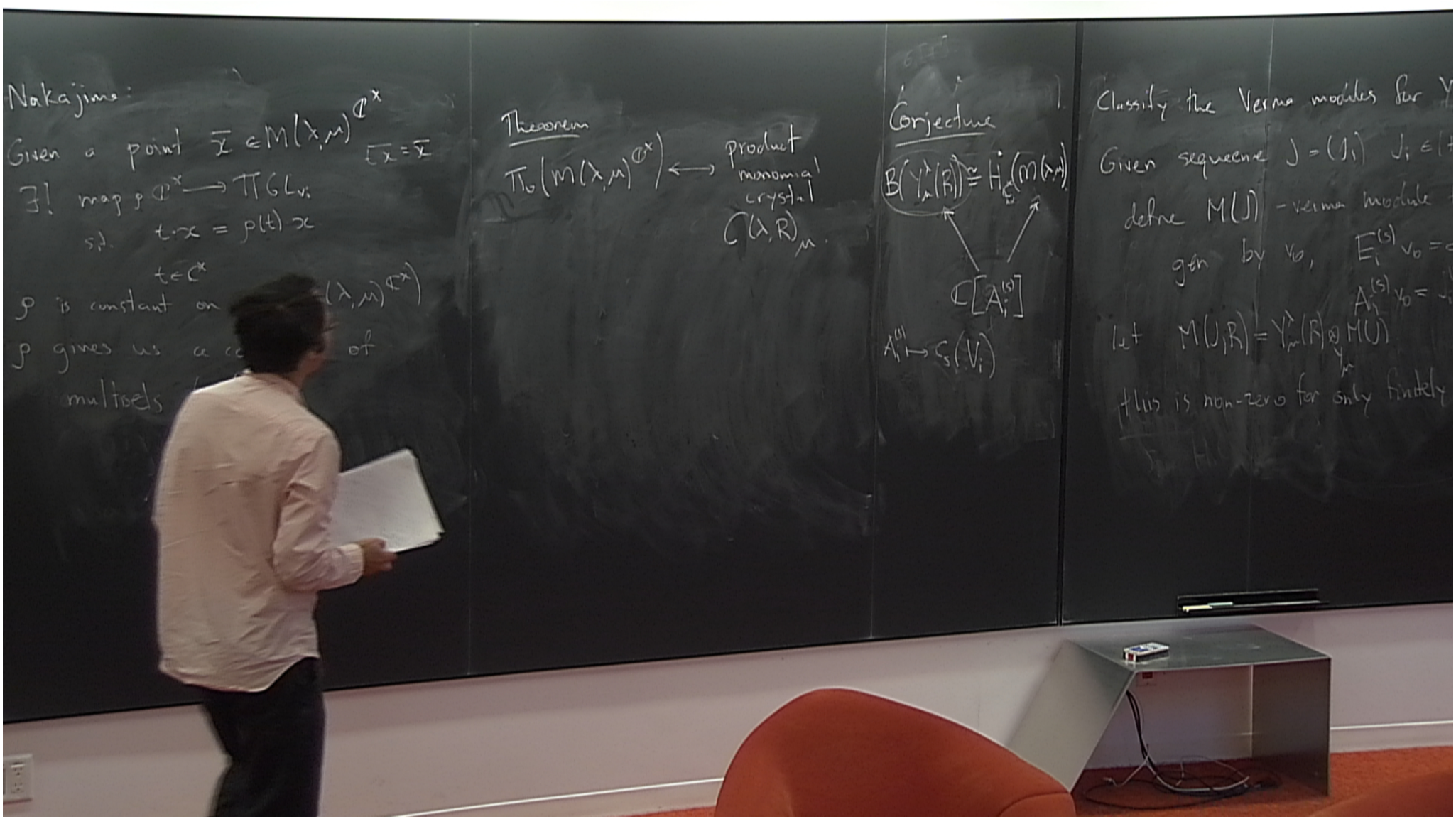

Let R be a choice
of integral parameters.
Conjecture
C[A_i^(s)]
Theorem
BLPW

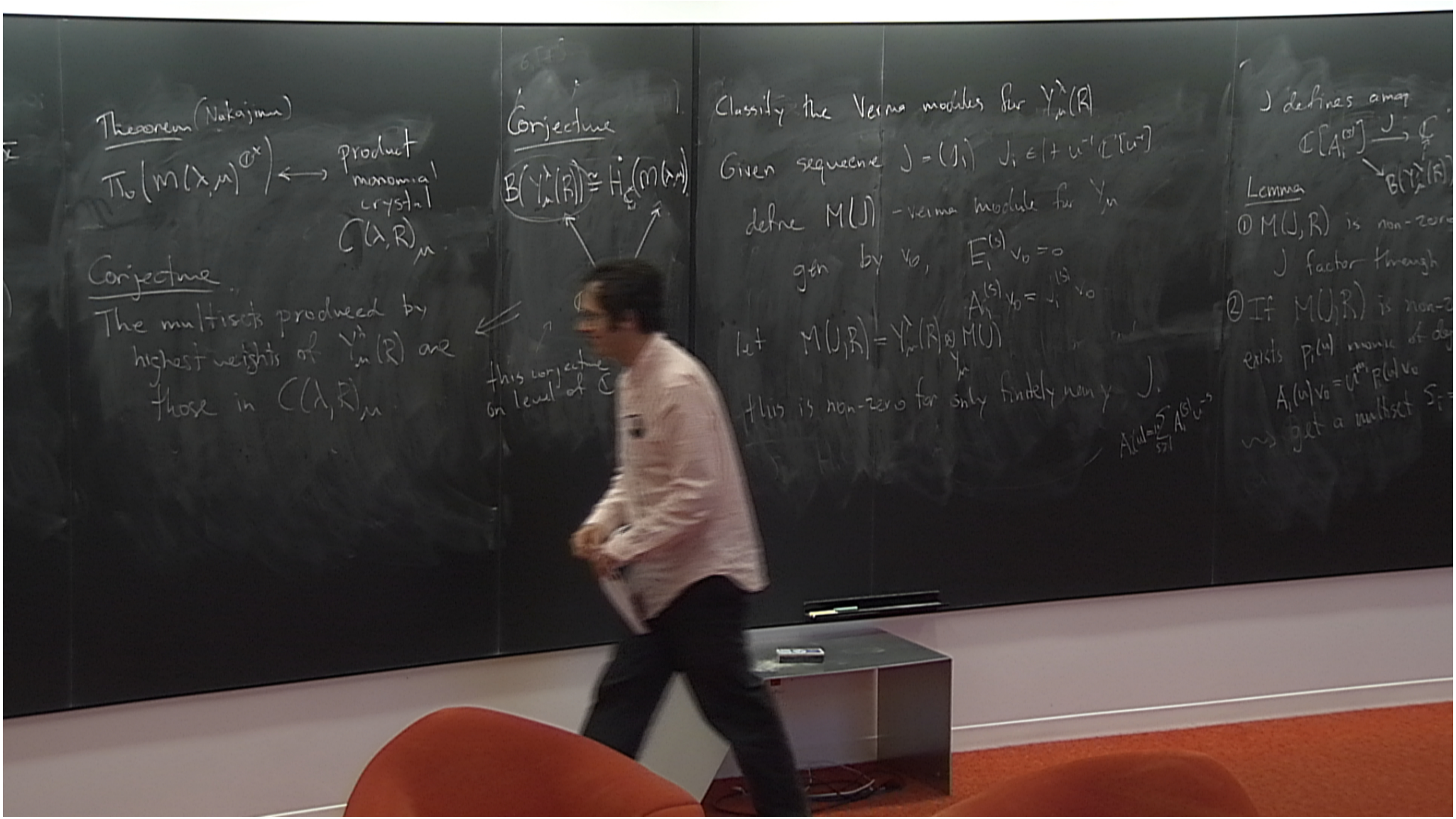

Spec $B(Y^{\lambda}_{\mu}(R))(\mathbb{C})$ = highest weights for $Y^{\lambda}_{\mu}(R)$.
$H^{\bullet}_{\mathbb{C}^{\times n-1}}(M(\lambda,\mu)) = H^{\bullet}(M(\lambda,\mu)^{\mathbb{C}^{\times}})$
graded quiver variety.
Spec $H^{\bullet}(M(\lambda,\mu)^{\mathbb{C}^{\times}})(\mathbb{C})$
$= \pi_0(M(\lambda,\mu)^{\mathbb{C}^{\times}})$
if A is commutative, $\mathbb{C}^{\times} \curvearrowright$ Spec A.
Spec $A)^{\mathbb{C}^{\times}}$ = Spec $B(A)$.
algebras.
Let R be
of integral
$\rightsquigarrow Y^{\lambda}_{\mu}(R)$.
Define a map
get

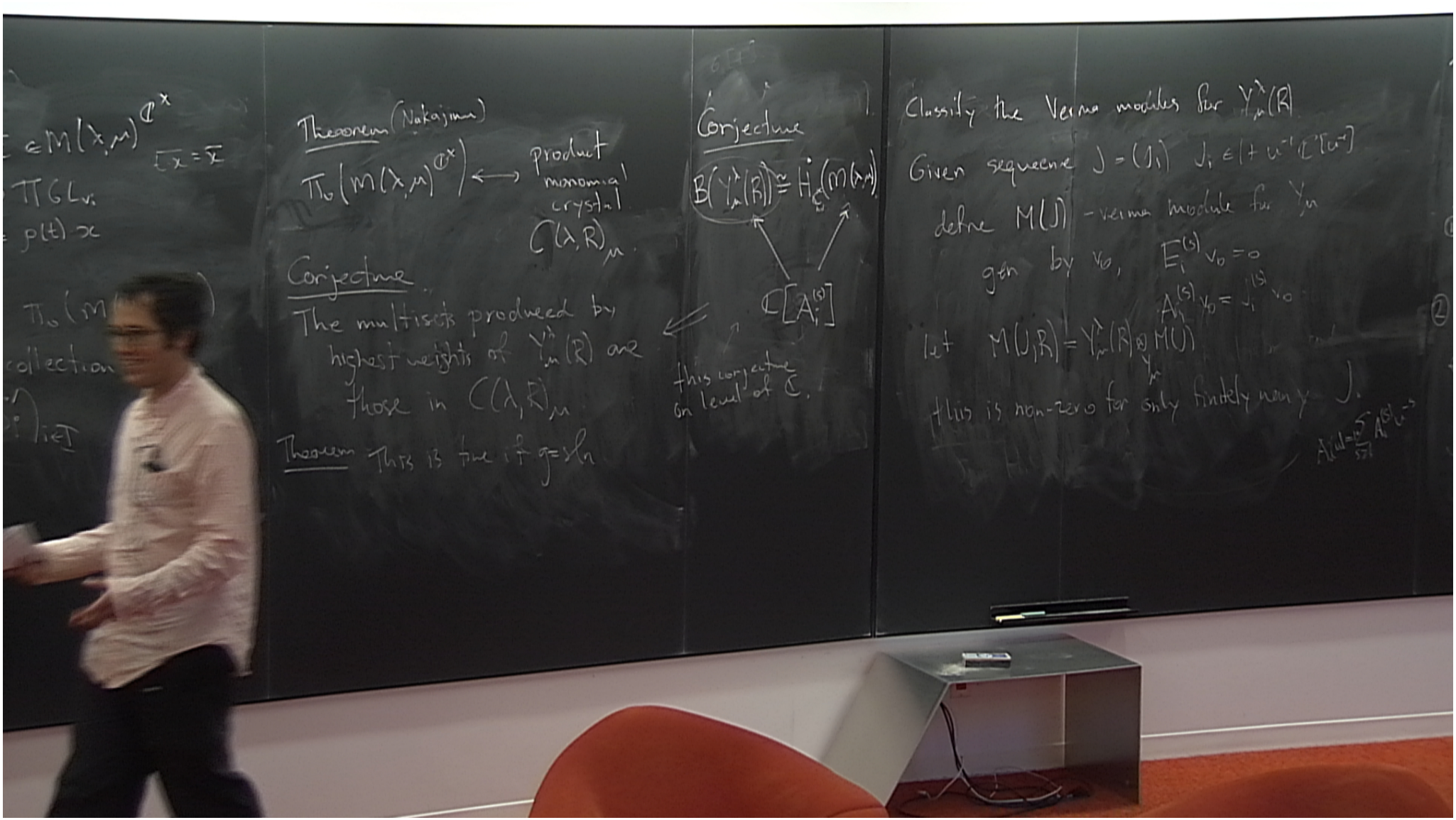

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graded quiver variety
Spec $H^{\bullet}(M(\lambda,\mu)^{\mathbb{C}^{\times}})(\mathbb{C})$
$= \pi_0(M(\lambda,\mu)^{\mathbb{C}^{\times}})$
Nakajima:
Given a point $\overline{x} \in M(\lambda,\mu)^{\mathbb{C}^{\times}}$
$\overline{tx} = \overline{x}$
$\exists!$ map $\rho: \mathbb{C}^{\times} \to \prod GL_{v_i}$
s.t. $t \cdot x = \rho(t)\, x$
$t \in \mathbb{C}^{\times}$
$\pi_0(M(\lambda,\mu)^{\mathbb{C}^{\times}})$
Let R be
of integral
$\rightsquigarrow Y^{\lambda}_{\mu}(R)$.
Define a map
get

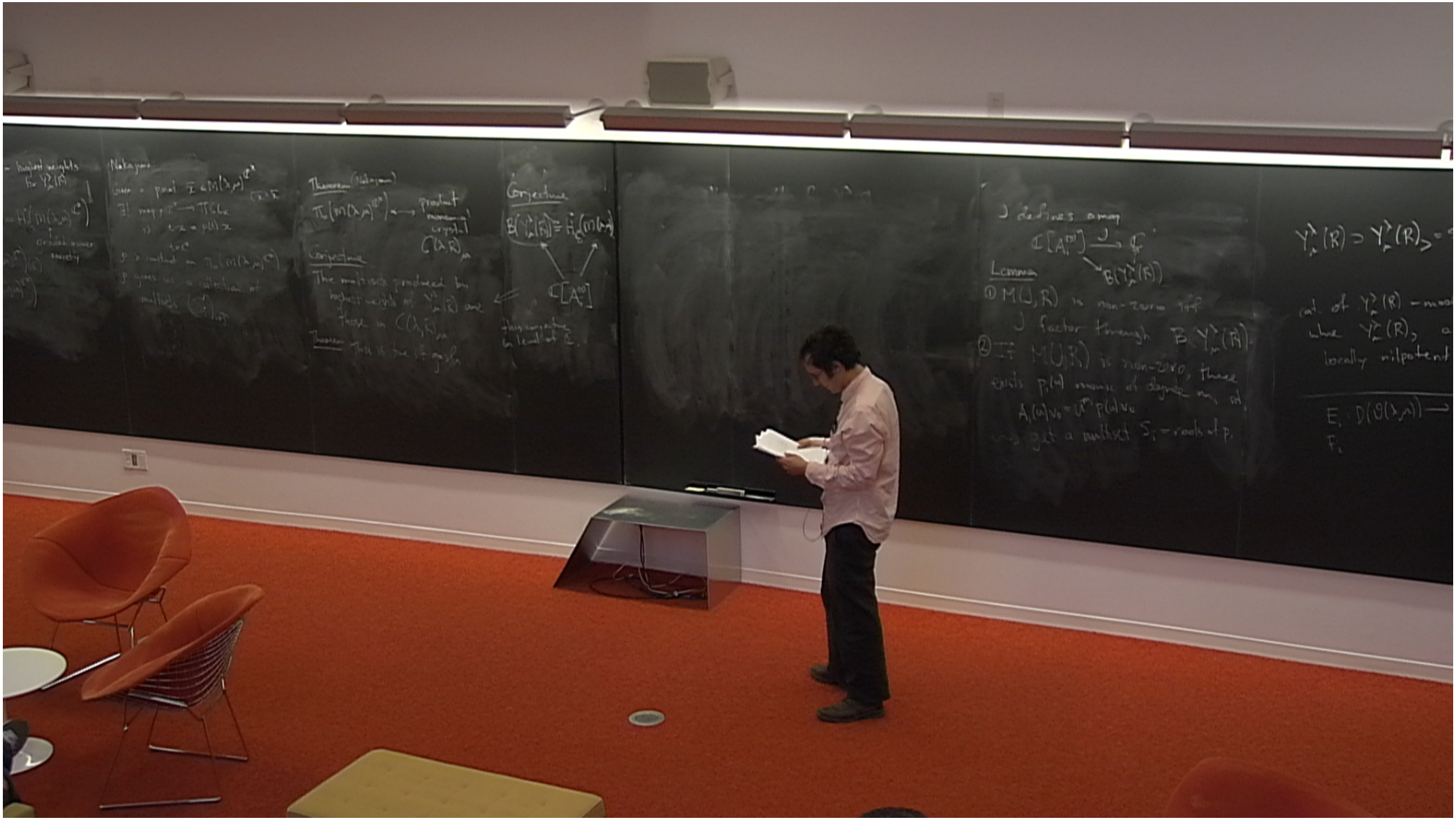

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$t \in \mathbb{C}^{\times}$
ρ is constant on $\pi_0(M(\lambda,\mu)^{\mathbb{C}^{\times}})$
ρ gives us a collection of multisets $(S^{\vee}_i)_{i \in I}$
Let R be a
of integral
$\rightsquigarrow Y^{\lambda}_{\mu}(R)$
Define a map
get $\mathbb{C}^{\times}$

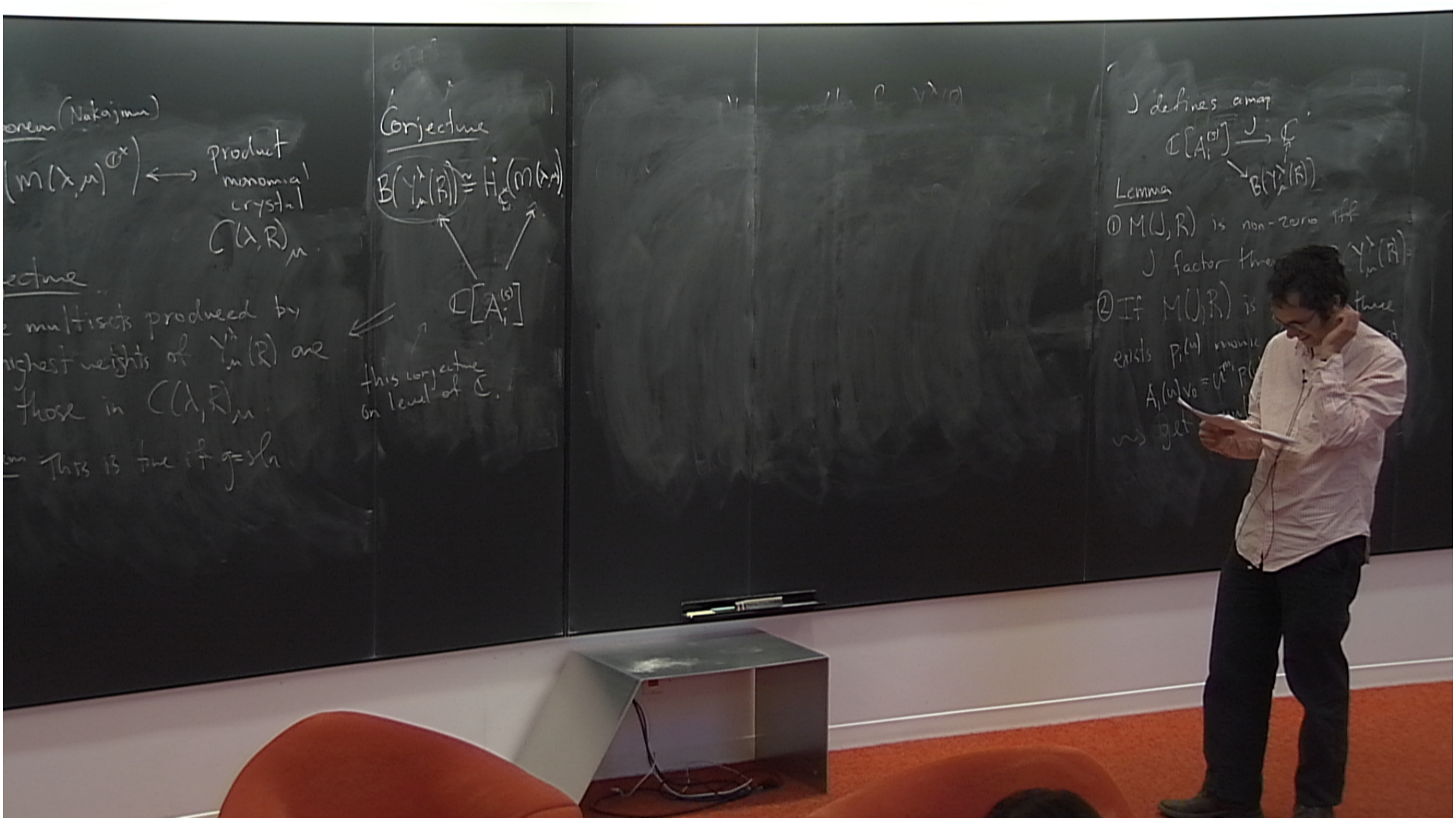

Gr_{λ,μ}
Gr^λ_μ
ℂ[Gr_{λ,μ}]
ℂ[Gr^λ_μ]
Spec B(Y^λ_μ(R))(ℂ) = highest weights for Y^λ_μ(R).
H^•_{ℂ^×_{h=1}}(M(λ,μ)) = H^•(M(λ,μ)^{ℂ^×})
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τx = x̄
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s.t. t·x = ρ(t)x
t∈ℂ^×
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ρ gives us a collection of
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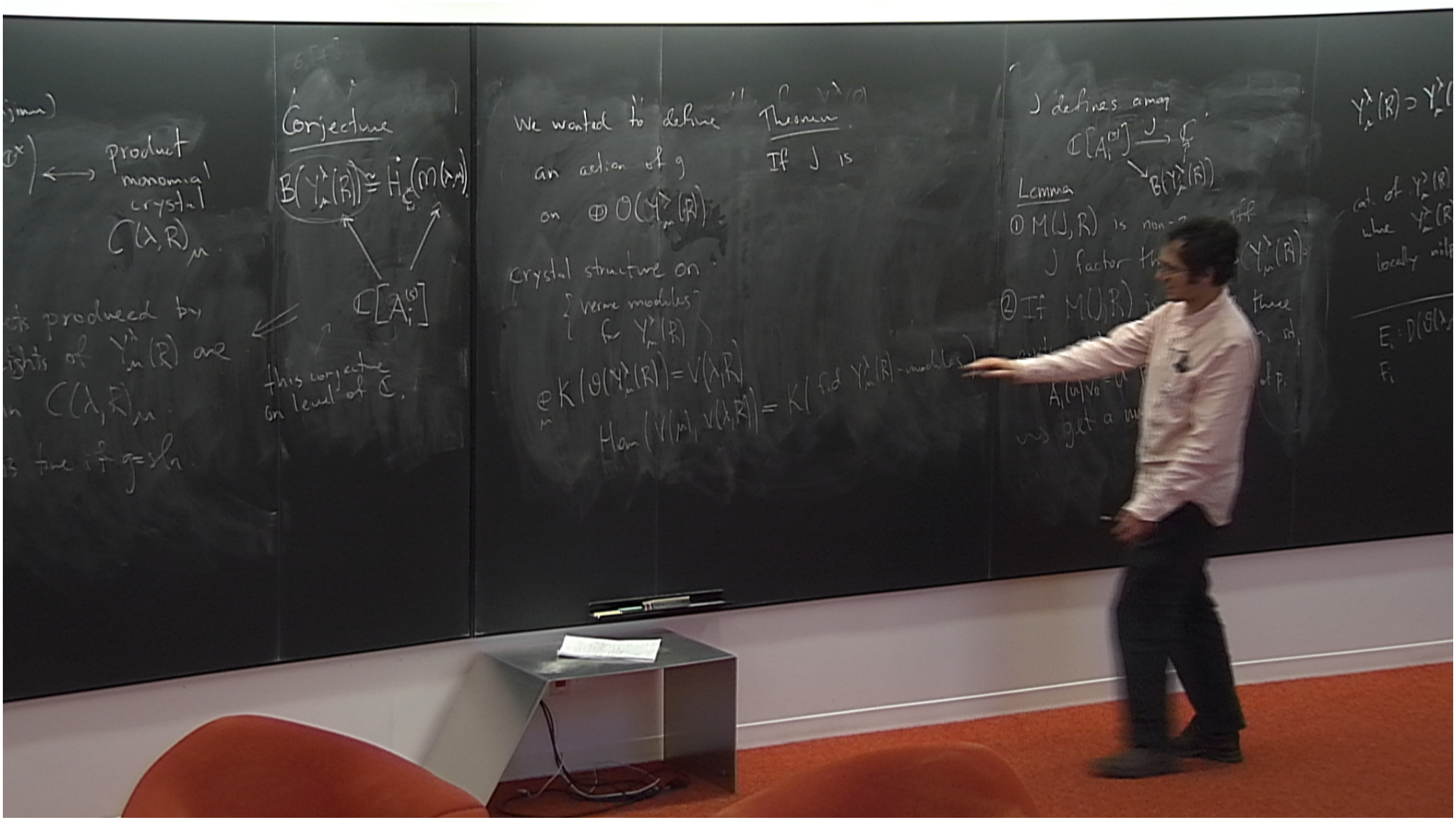

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Given a point $\overline{x} \in M(\lambda,\mu)^{\mathbb{C}^\times}$
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ρ is constant on
ρ gives us a
multisets
Theorem
$\pi_0\left(M(\lambda,\mu)^{\mathbb{C}^\times}\right) \longleftrightarrow$ product monomial crystal $C(\lambda,R)_\mu$
Conjecture
$B(Y_m^\lambda(R)) \cong H_*(M(\lambda,\mu))$
$\mathbb{C}[A_i^{(s)}]$
$A_i^{(s)} \mapsto S_s(V_i)$
Classify the Verma modules for
Given sequence $J = (J_i)$
define $M(J)$ - verma module
gen by v_0,
let $M(J,R) = Y_m^\lambda(R) \otimes_{Y_m} M(J)$
this is non-zero for only finitely


Theorem (Nakajima)
Product
monomial
crystal
Conjecture
Conjecture
Classify the Verma modules for
Given sequence
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Let
this is non-zero for only finitely many
J defines a map
Lemma
exists
get a multiset


Theorem (Nakajima)
Product monomial crystal
Conjecture
Conjecture
The multisets produced by highest weights of
those in
Theorem This is true if
this conjecture on level of
Classify the Verma modules for
Given sequence
define - verma module for
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Notation
Theorem (Nakajima)
Conjecture
Conjecture
Theorem
Lemma


Conjecture
Product
monomial
crystal
this conjecture
on level of
Lemma


Product
monomial
crystal
Conjecture
this conjecture
on level of C.
We wanted to define
an action of g
on
Theorem
If J is
Crystal structure on
Lemma
J defines a map
Hom