Title: Fault-tolerant error correction with the gauge color code

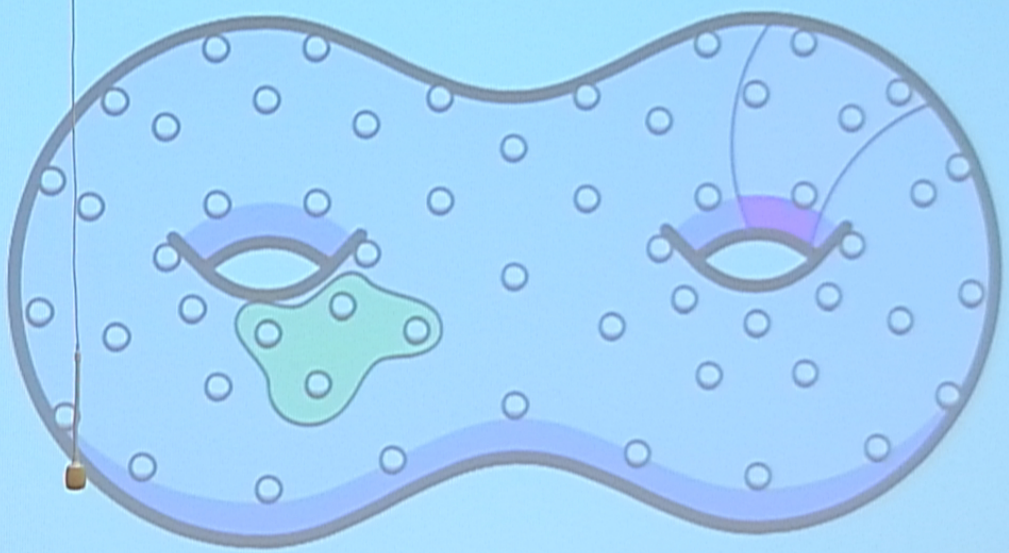
Date: Oct 29, 2015 04:30 PM

URL: http://pirsa.org/15100120

Abstract: The gauge color code is a quantum error-correcting code with local syndrome measurements that, remarkably, admits a universal transversal gate set without the need for resource-intensive magic state distillation. A result of recent interest, proposed by Bombin, shows that the subsystem structure of the gauge color code admits an error-correction protocol that achieves tolerance to noisy measurements without the need for repeated measurements, so called single-shot error correction. Here, we demonstrate the promise of single-shot error correction by designing a two-part decoder and investigate its performance. We simulate fault-tolerant error correction with the gauge color code by repeatedly applying our proposed error-correction protocol to deal with errors that occur continuously to the underlying physical qubits of the code over the duration that quantum information is stored. We estimate a sustainable error rate, i.e. the threshold for the long time limit, of ~0.31% for a phenomenological noise model using a simple decoding algorithm.

Topological Error-Correcting Codes

Quantum information can be encoded over topological degrees of freedom of a lattice



Stabilizer Codes

The Stabilizer Formalism

- Many quantum error-correcting codes can be described by a stabilizer group $\mathcal{S}$
- The Abelian group satisfies the property

$$S|\psi\rangle = (+1)|\psi\rangle$$

  for all elements $S \in \mathcal{S}$ and encoded state $|\psi\rangle$.
- In other words, all encoded states are in the common $+1$ eigenspace of elements of $\mathcal{S}$

D. Gottesman, Ph.D. Thesis (1997)

Stabilizer Codes

Stabilizers Identify Errors

- Stabilizer measurements identify errors E on encoded states
- We use measurement information

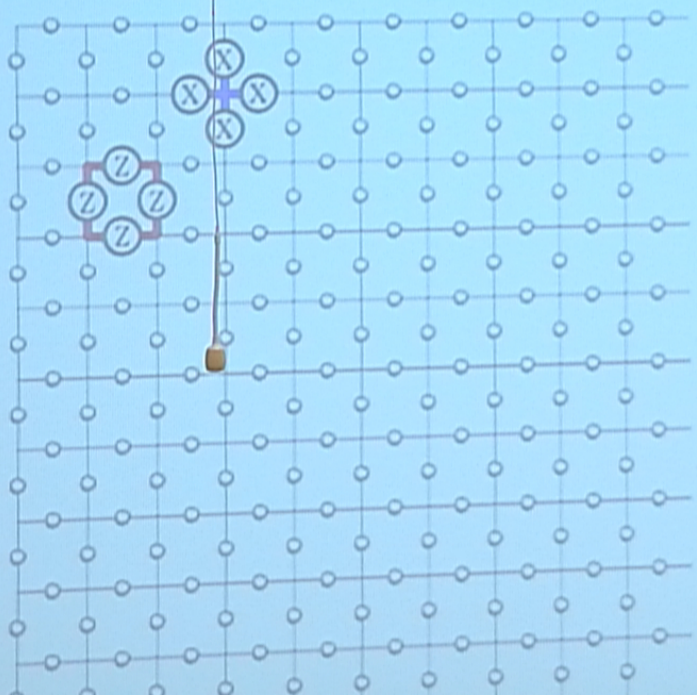
$$SE|\psi\rangle = (-1)E|\psi\rangle$$

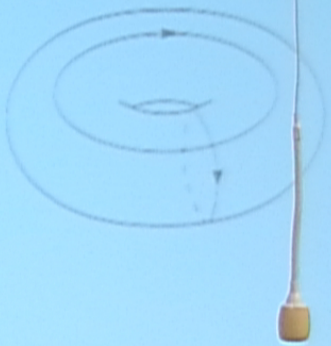
  to identify errors
- We call a -1 measurement outcome a *stabilizer defect*
- A collection of defects is known as a *stabilizer syndrome*
- We use a syndrome to estimate error E incident to the code

The Toric Code

The stabilizers of the toric code

A. Kitaev, Ann. Phys. **303**, 2 (2003)

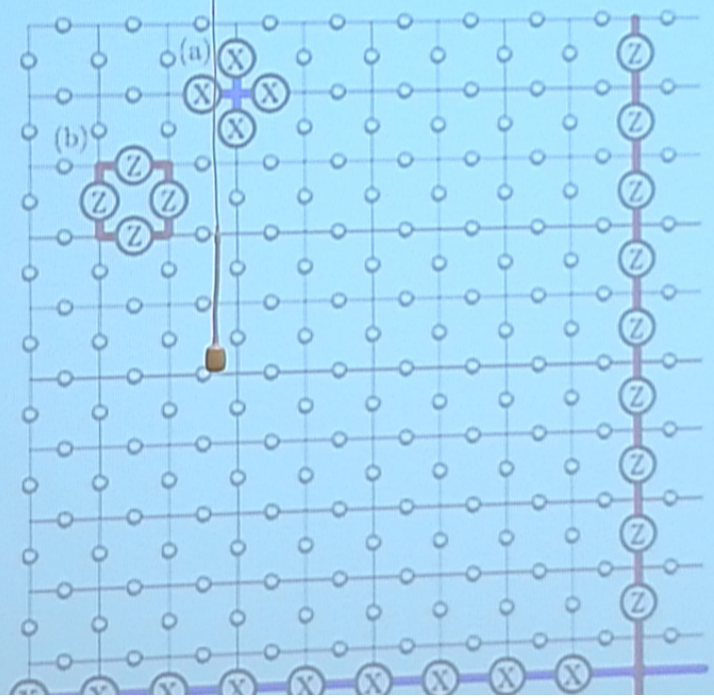


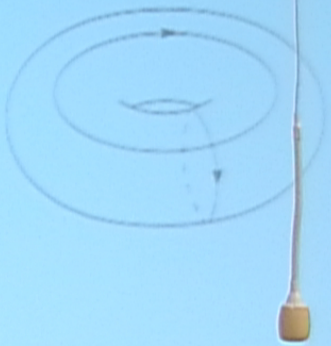


The Toric Code

Logical operators of the toric code

A. Kitaev, Ann. Phys. **303**, 2 (2003)

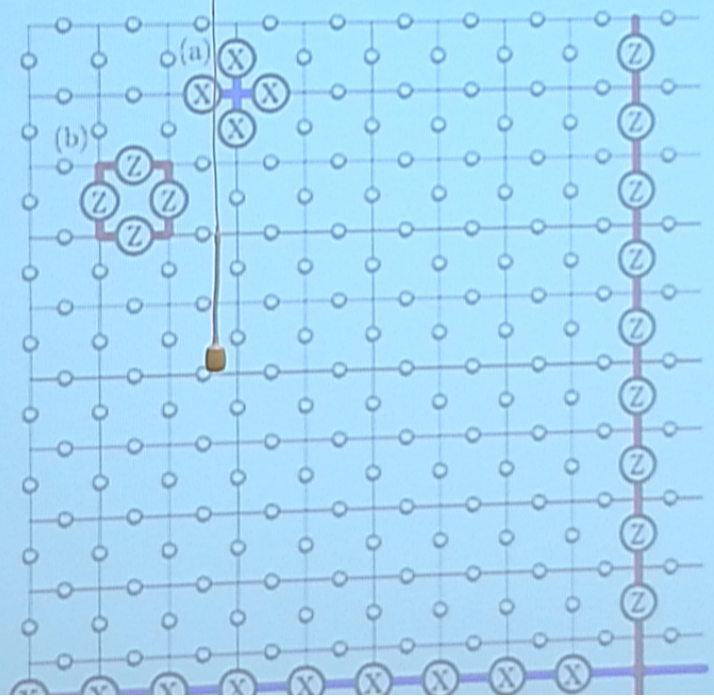


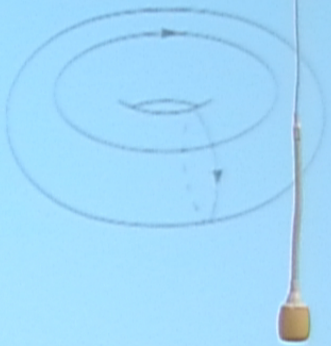


The Toric Code

Logical operators of the toric code

A. Kitaev, Ann. Phys. **303**, 2 (2003)

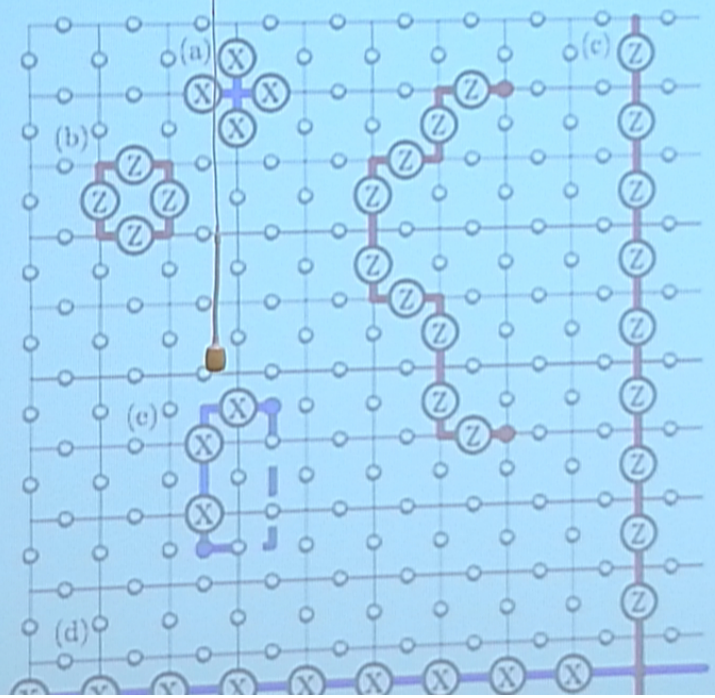


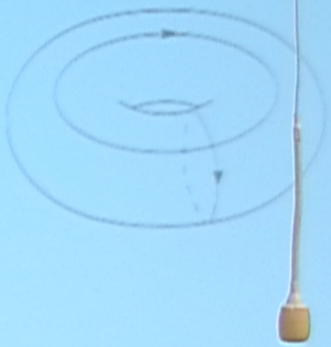


The Toric Code

Corrections are more difficult to find once errors become large

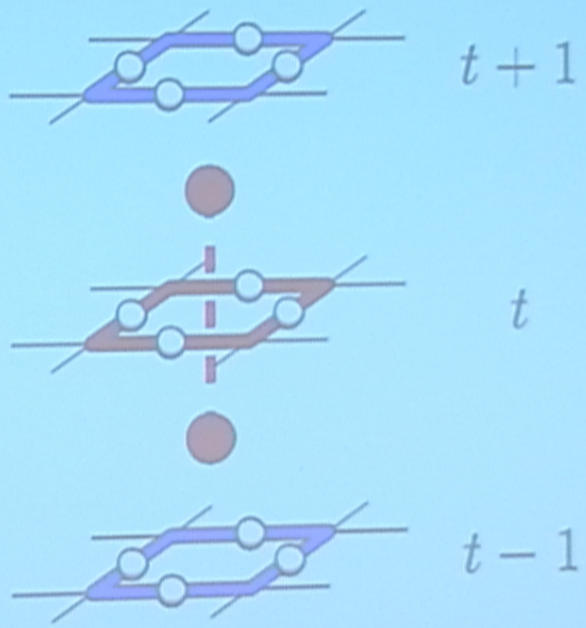
A. Kitaev, Ann. Phys. **303**, 2 (2003)





The Toric Code with Measurement Errors

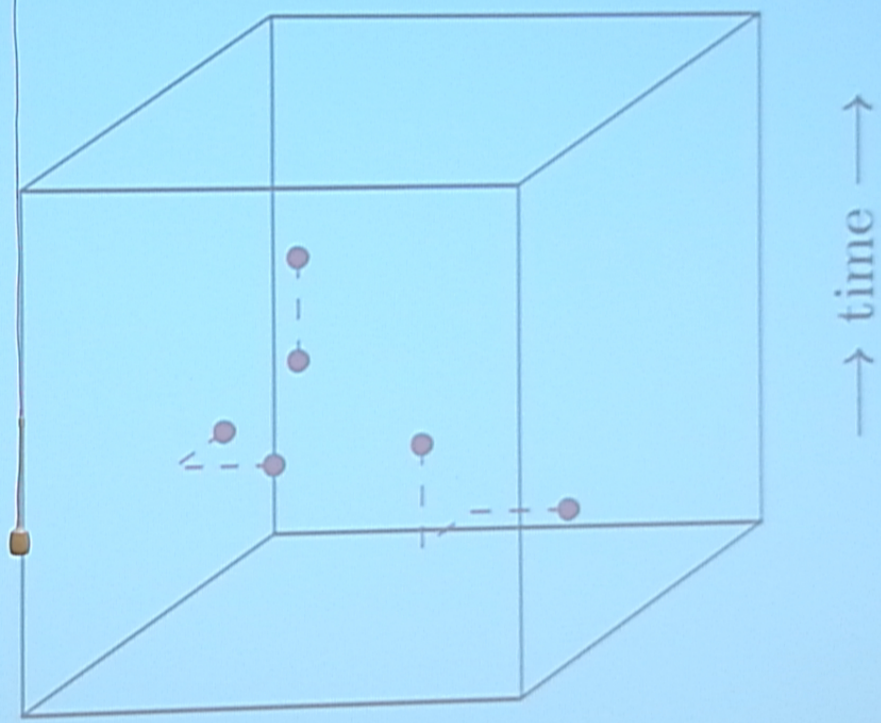
We check the parity of measurements over time



E. Dennis *et al.* J. Math. Phys. **43**, 4452 (2002)

The Toric Code Syndrome with Measurement Errors

We construct a 2+1 syndrome that we can decode successfully



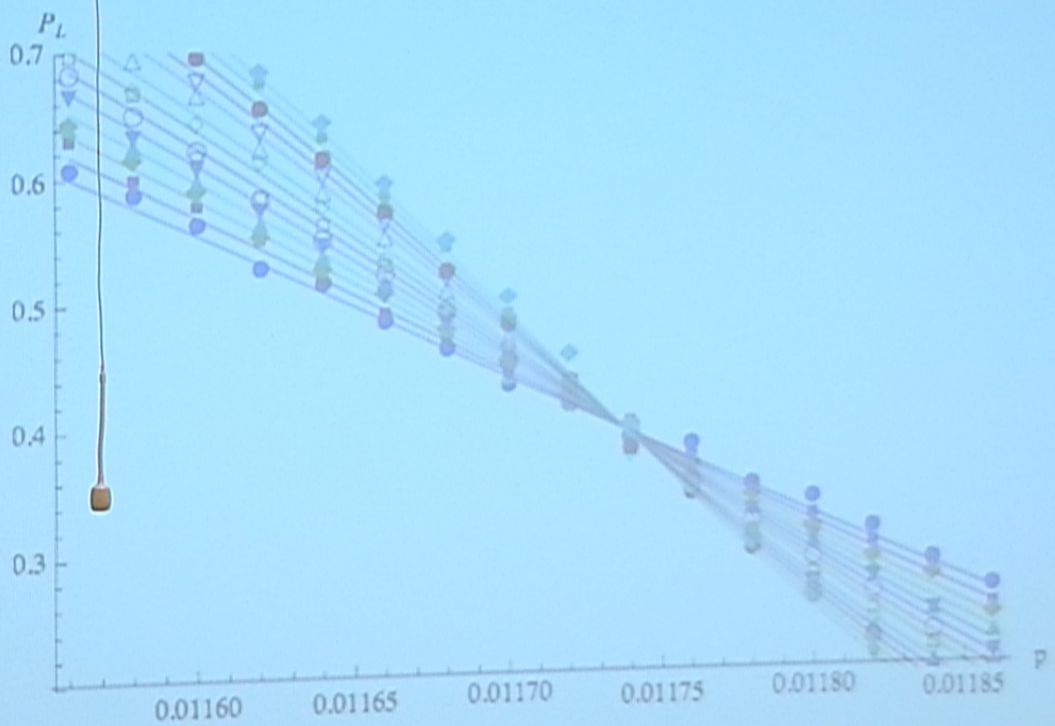
Noise Models

- We consider an **independent and identically distributed** noise model (I.I.D. noise)
- Qubits suffer a **bit-flip error** with probability p
- Incorrect measurement outcomes are returned with probability q (where typically $q = p$)
- This is known as a **phenomenological noise model** for fault-tolerant error correction
- The *gate error model* considers the underlying circuit for stabilizer measurements

Error Thresholds

We identify thresholds to analyze a code

A threshold is a critical noise rate below which we can reduce the logical failure rate arbitrarily by increasing *code distance*



B. Terhal, Rev. Mod. Phys. **87**, 307 (2015)

Error Thresholds for the Toric Code

Toric code, I.I.D. noise, perfect measurements - $q = 0$

- Threshold $p_{\text{th}} \approx 10.9\%$ [E. Dennis *et al.* (2002)]
- Decoding - $2D$-matching problem

Toric code, phenomenological noise model, $q = p$

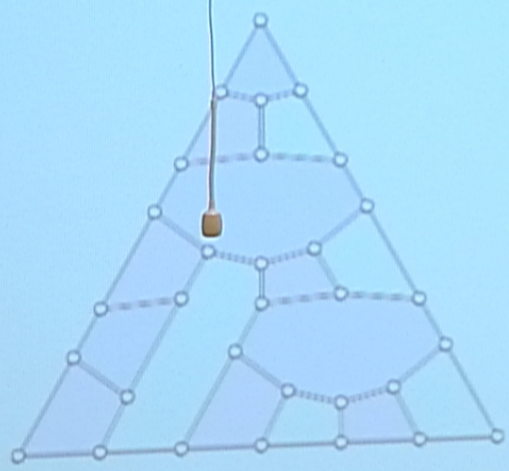
- Threshold $p_{\text{th}} \approx 2.9\%$ [C. Wang *et al.* (2003)]
- Decoding - $2 + 1D$-matching problem

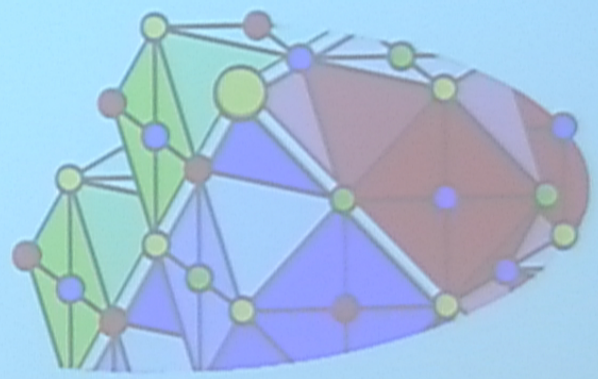
Color Codes

Color codes are error-correcting codes with interesting computational capabailities

H. Bombín - some milestones for color codes...

- 2D color code, Phys. Rev. Lett. **97**, 180501 (2006).
- 3D color code, Phys. Rev. Lett. **98**, 160502 (2007)
- Gauge color code (2013-2014).



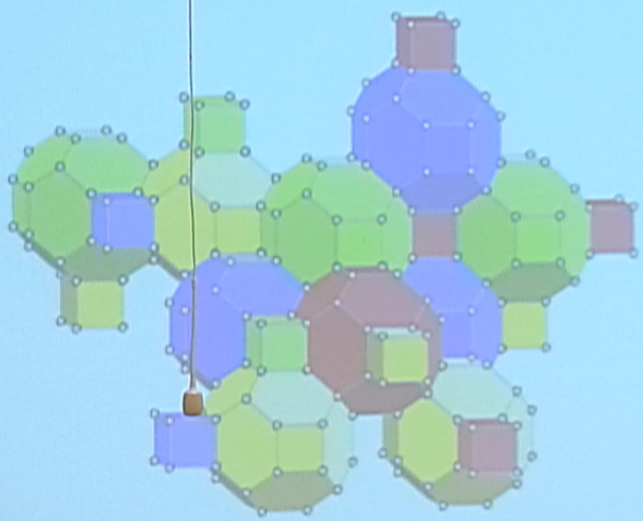


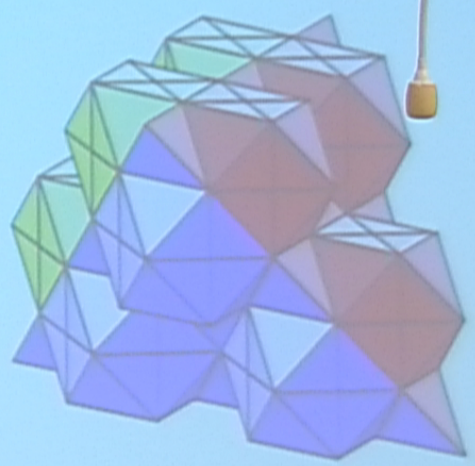
The Gauge Color Code

Describing the gauge color code lattice

The gauge color code is defined on a four-valent, *four colorable* lattice with a qubit on each vertex

↓ Primal picture ↓





↑ dual picture ↑

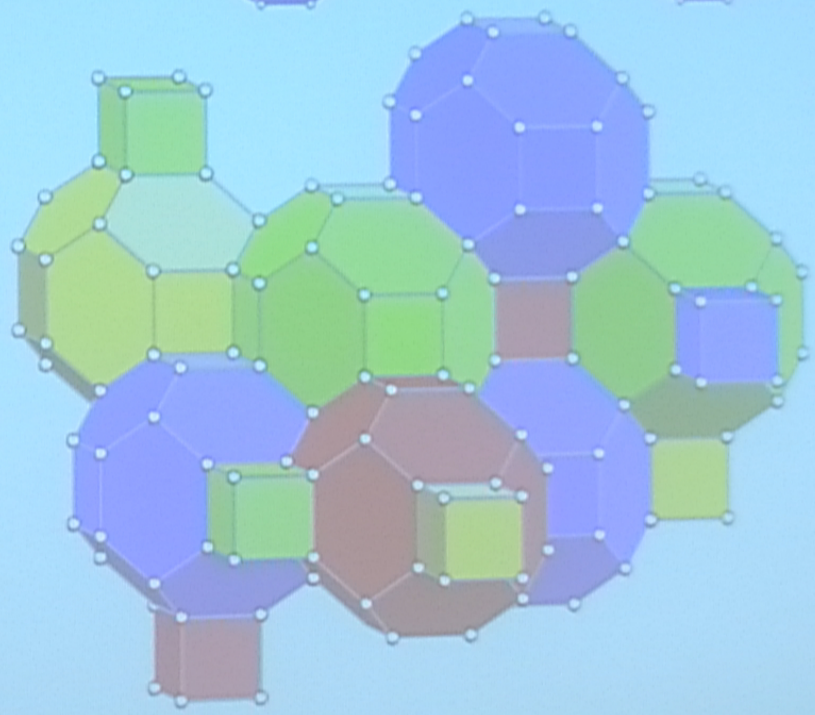
We require the dual picture to understand the boundaries

Stabilizers of the Gauge Color Code

Stabilizers are associated to the cells of the lattice

$S_c^X =$ 

$S_c^Z =$ 

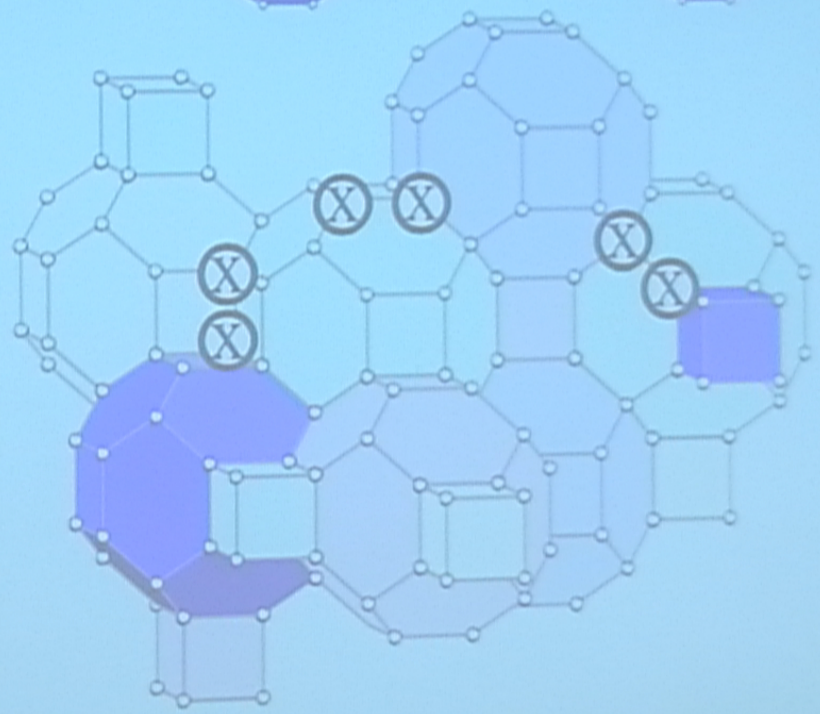


Errors of the Gauge Color Code

Two stabilizers are lit by the present error

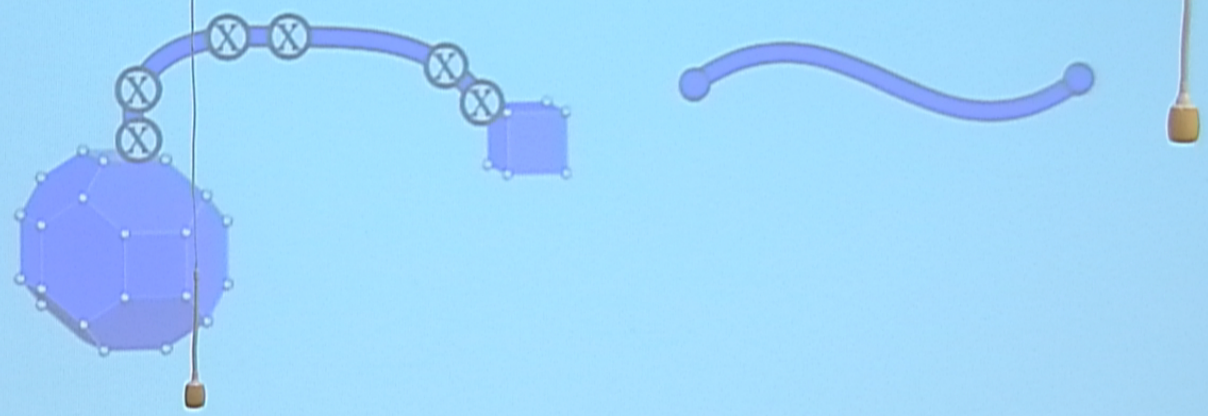
$S_c^X =$ 

$S_c^Z =$ 



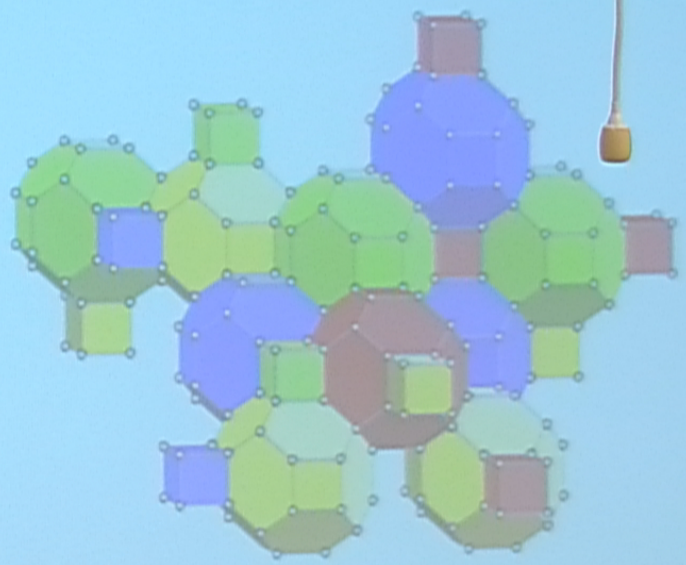
Errors of the Gauge Color Code

We can visualise errors independent of the lattice with a string picture



Stabilizers of the Gauge Color Code

The gauge color code is a topological quantum error-correcting code



. . . very much like the toric code -

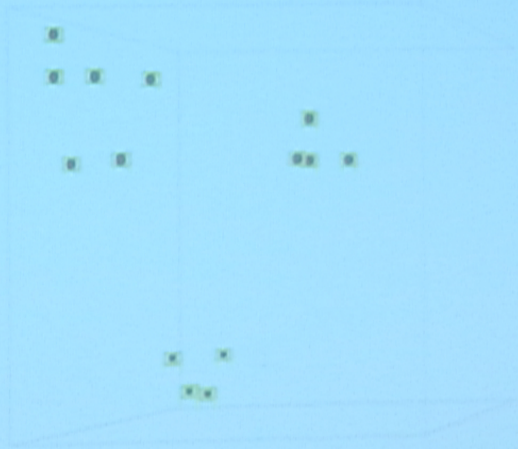
A. Kubica *et al.* arXiv:1503:02065 (2015)

Decoding Topological Codes

Topological stabilizer codes can be decoded using a clustering algorithm

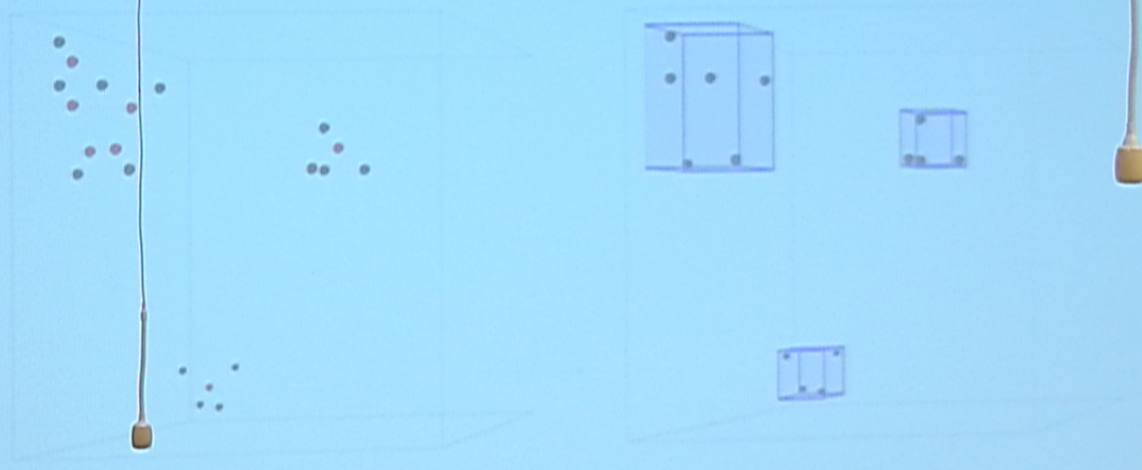
Decoding Topological Codes

Syndromes are placed in small boxes



Decoding Topological Codes

Small clusters can never contain a logical operator, and so correction will be successful



The Gauge Group of the Gauge Color Code

Subsystem Codes

- The gauge color code is specified by a gauge group $\mathcal{G}$
- Stabilizers are members of $\mathcal{G}$ that commute with $\mathcal{G}$

$$\mathcal{S} = Z(\mathcal{G}) \cap \mathcal{G}$$

- The logical operators of the code are not members of $\mathcal{G}$, but commute with $\mathcal{G}$

$$\mathcal{S} = Z(\mathcal{G}) \backslash \mathcal{G}$$

The Gauge Group of the Gauge Color Code

Subsystem Codes

- The gauge color code is specified by a gauge group $\mathcal{G}$
- Stabilizers are members of $\mathcal{G}$ that commute with $\mathcal{G}$

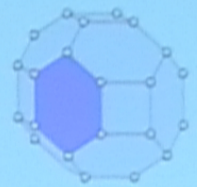
$$\mathcal{S} = Z(\mathcal{G}) \cap \mathcal{G}$$

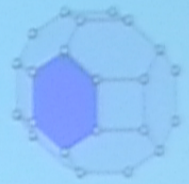
- The logical operators of the code are not members of $\mathcal{G}$, but commute with $\mathcal{G}$

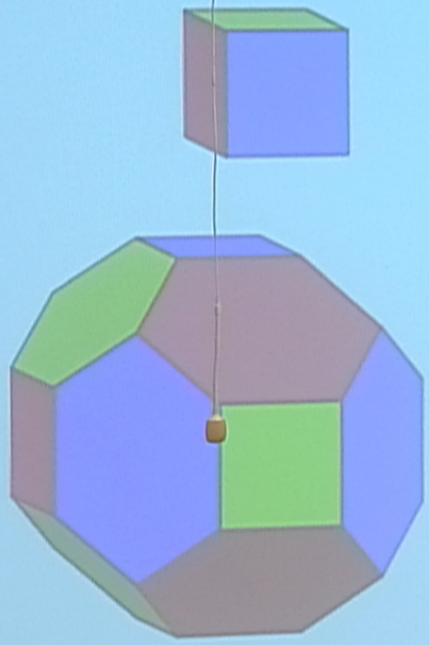
$$\mathcal{S} = Z(\mathcal{G}) \backslash \mathcal{G}$$

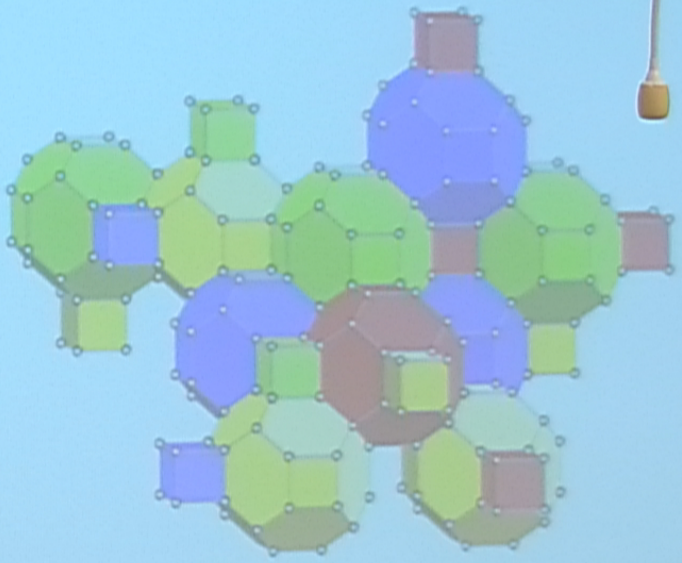
The Gauge Group of the Gauge Color Code

Gauge operators live on the faces of the lattice

$G_f^X =$ 

$G_f^Z =$ 

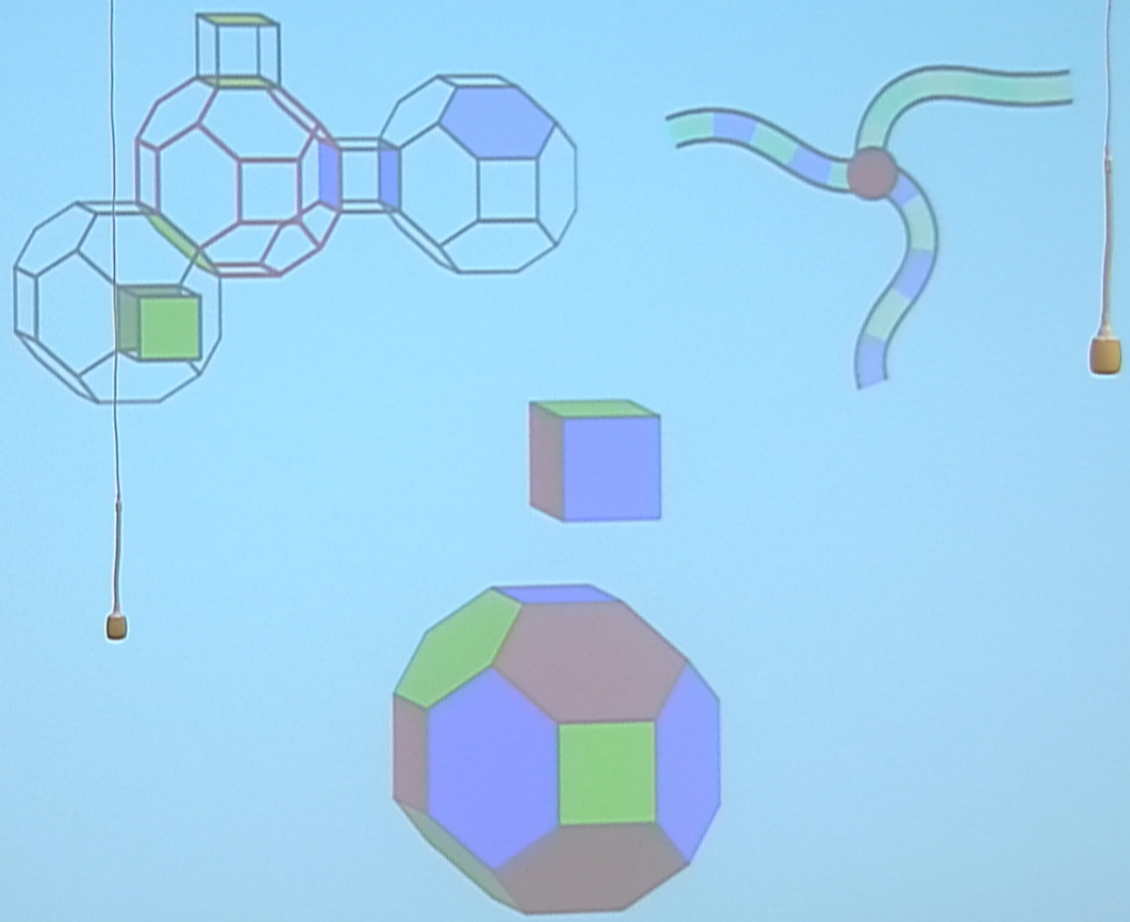




H. Bombín arXiv: 1404.5504 (2014)

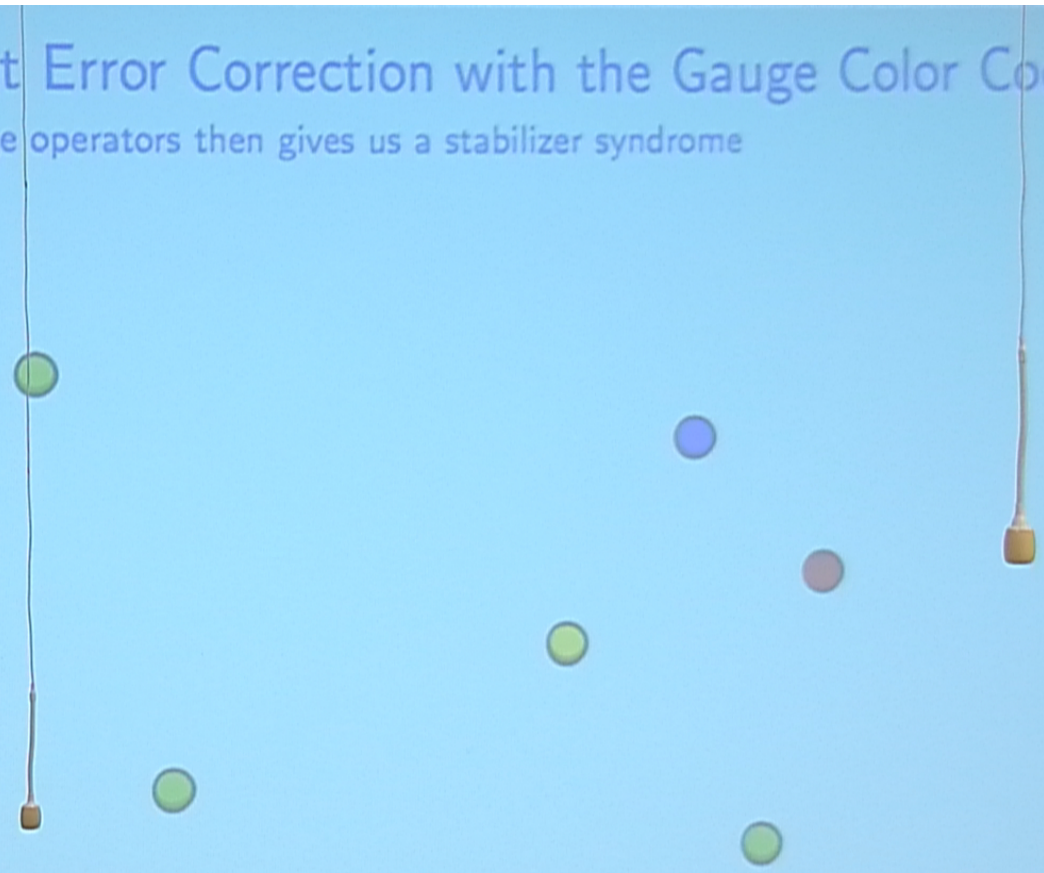
Single-Shot Error Correction with the Gauge Color Code

Measuring all the faces of the code return stabilizer information with some redundancy



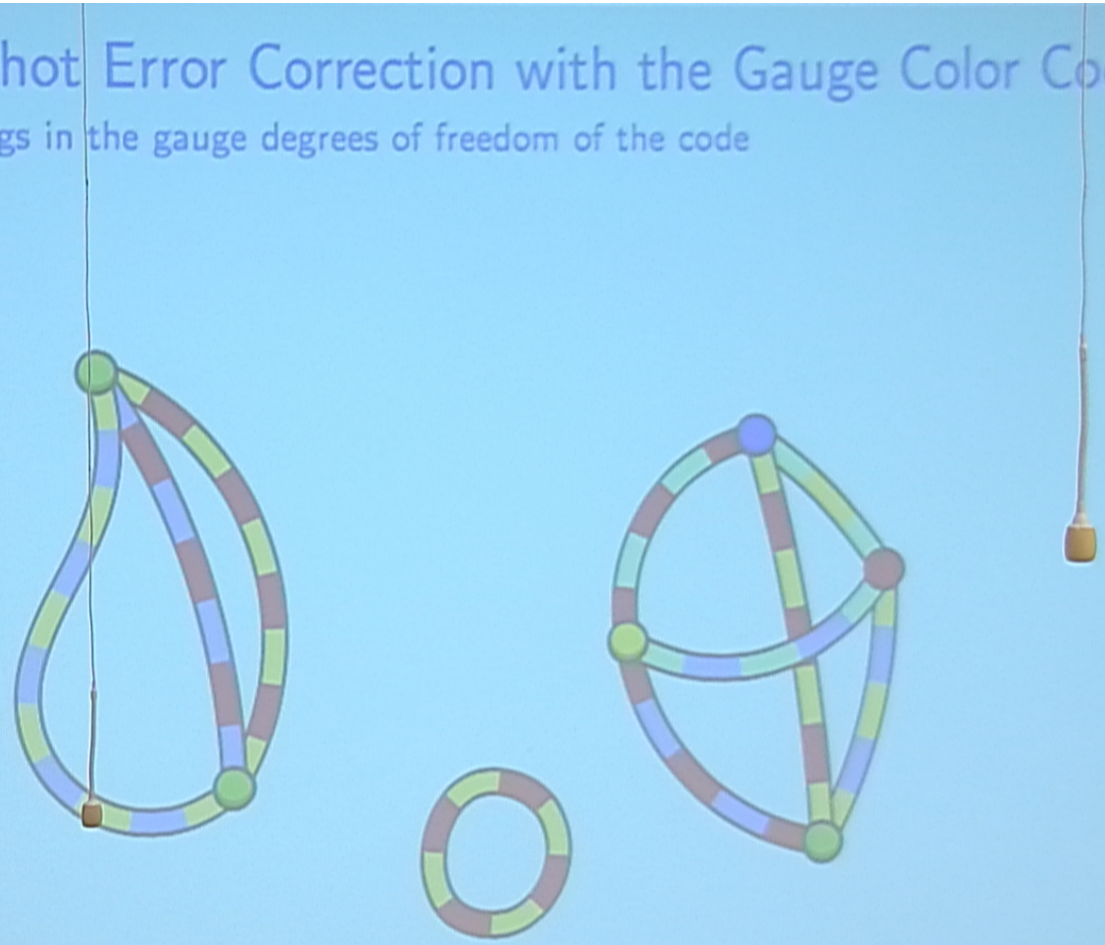
Single-Shot Error Correction with the Gauge Color Code

Measuring gauge operators then gives us a stabilizer syndrome



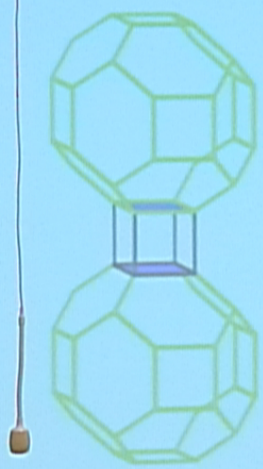
Single-Shot Error Correction with the Gauge Color Code

. . . and strings in the gauge degrees of freedom of the code



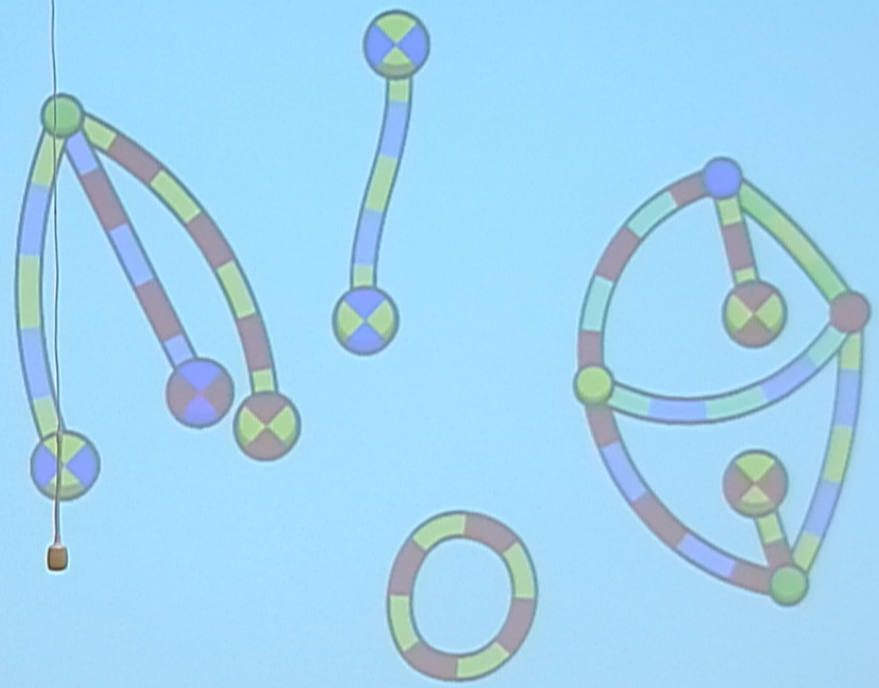
Single-Shot Error Correction with the Gauge Color Code

Studying the gauge measurements enables us to identify measurement errors



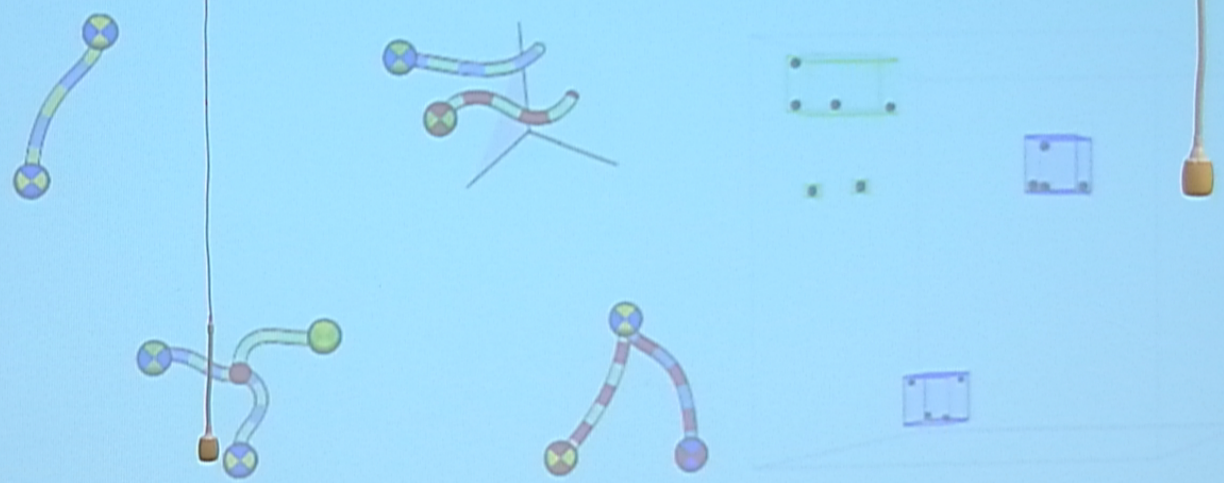
Single-Shot Error Correction with the Gauge Color Code

Breaks in strings then indicate that a measurement error has occurred



Gauge Defects

Gauge defects can be decoded by clustering



The Simulation

We simulate fault-tolerant error correction to demonstrate single-shot error correction

- Prepare code state ρ
- Add physical noise $\mathcal{E}_p$
- Attempt noisy recovery $\mathcal{R}_q$ with $q = p$
- Read out state $\mathcal{M}$ by collapsing onto product state
- Additional I.I.D. Noise is introduced at $\mathcal{M}$
- Effectively, $\mathcal{M} = \mathcal{R}_0 \circ \mathcal{E}_p$

The Simulation

We simulate fault-tolerant error correction to demonstrate single-shot error correction

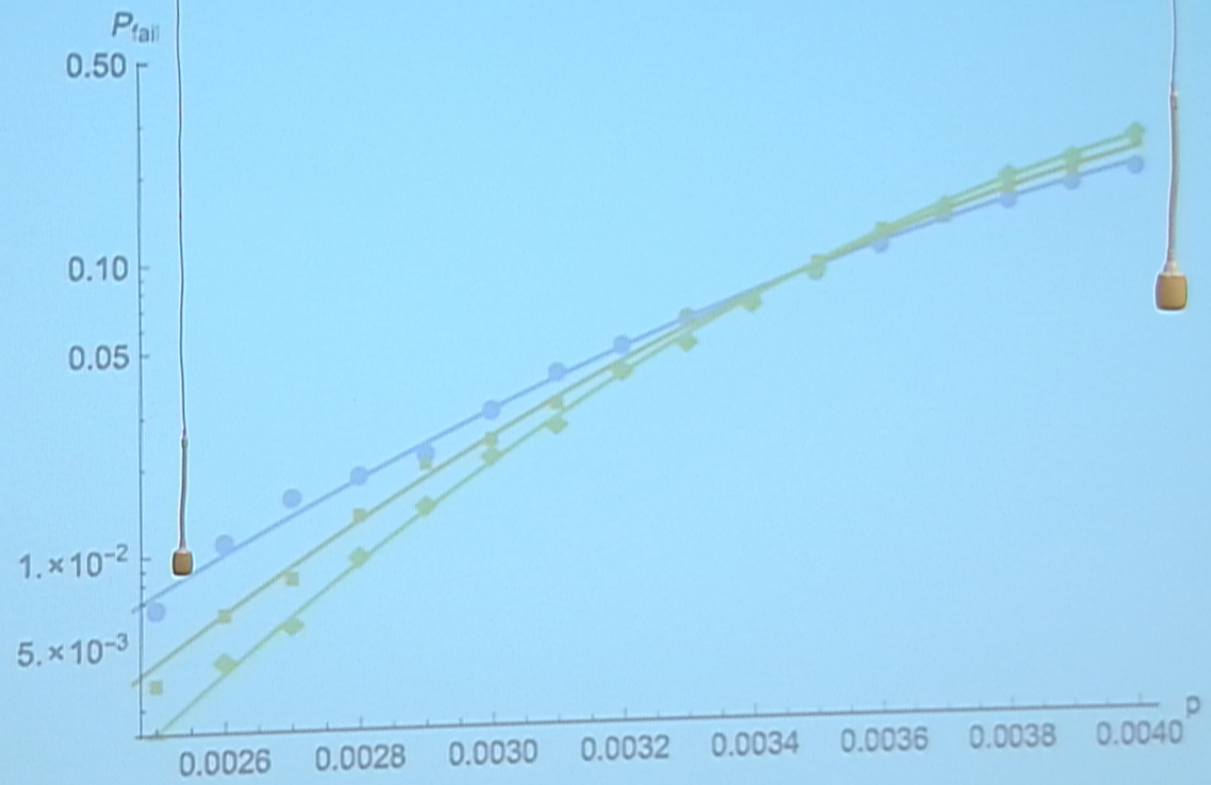
We seek to find

$$\text{prob}(\mathcal{M} \circ \mathcal{R}_q \circ \mathcal{E}_p[\rho] = \rho).$$

We find this value using Monte Carlo trials

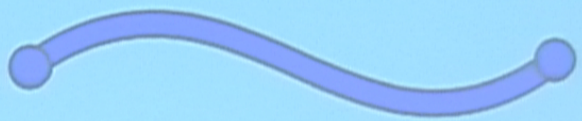
Results

Threshold Error Rate After One Round of Error Correction



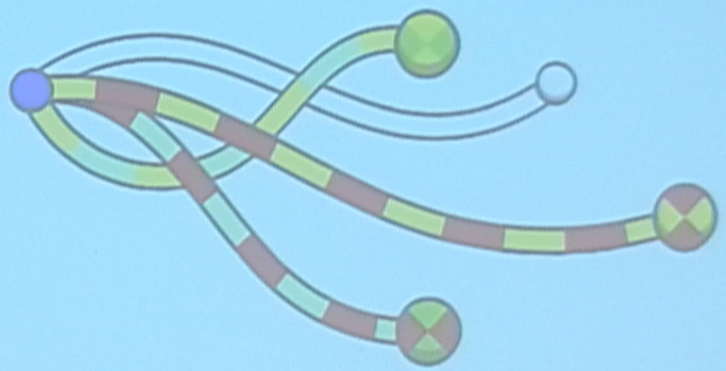
Error Correlations from Single Shot Error Correction

One might worry about the development of correlations



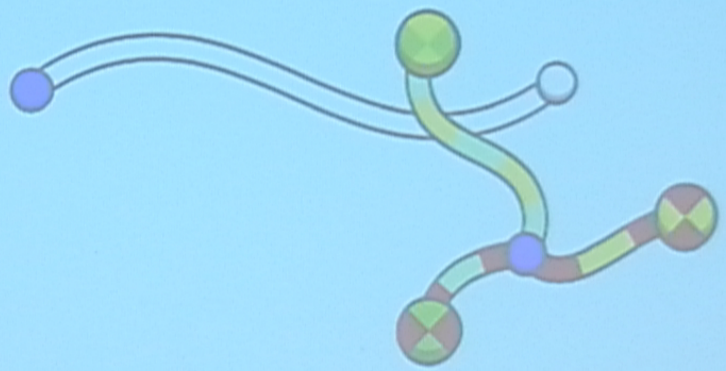
Error Correlations from Single Shot Error Correction

One might worry about the development of correlations



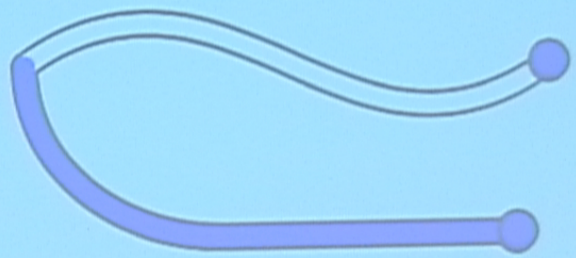
Error Correlations from Single Shot Error Correction

One might worry about the development of correlations



Error Correlations from Single Shot Error Correction

One might worry about the development of correlations



Error Correlations from Single Shot Error Correction

One might worry about the development of correlations

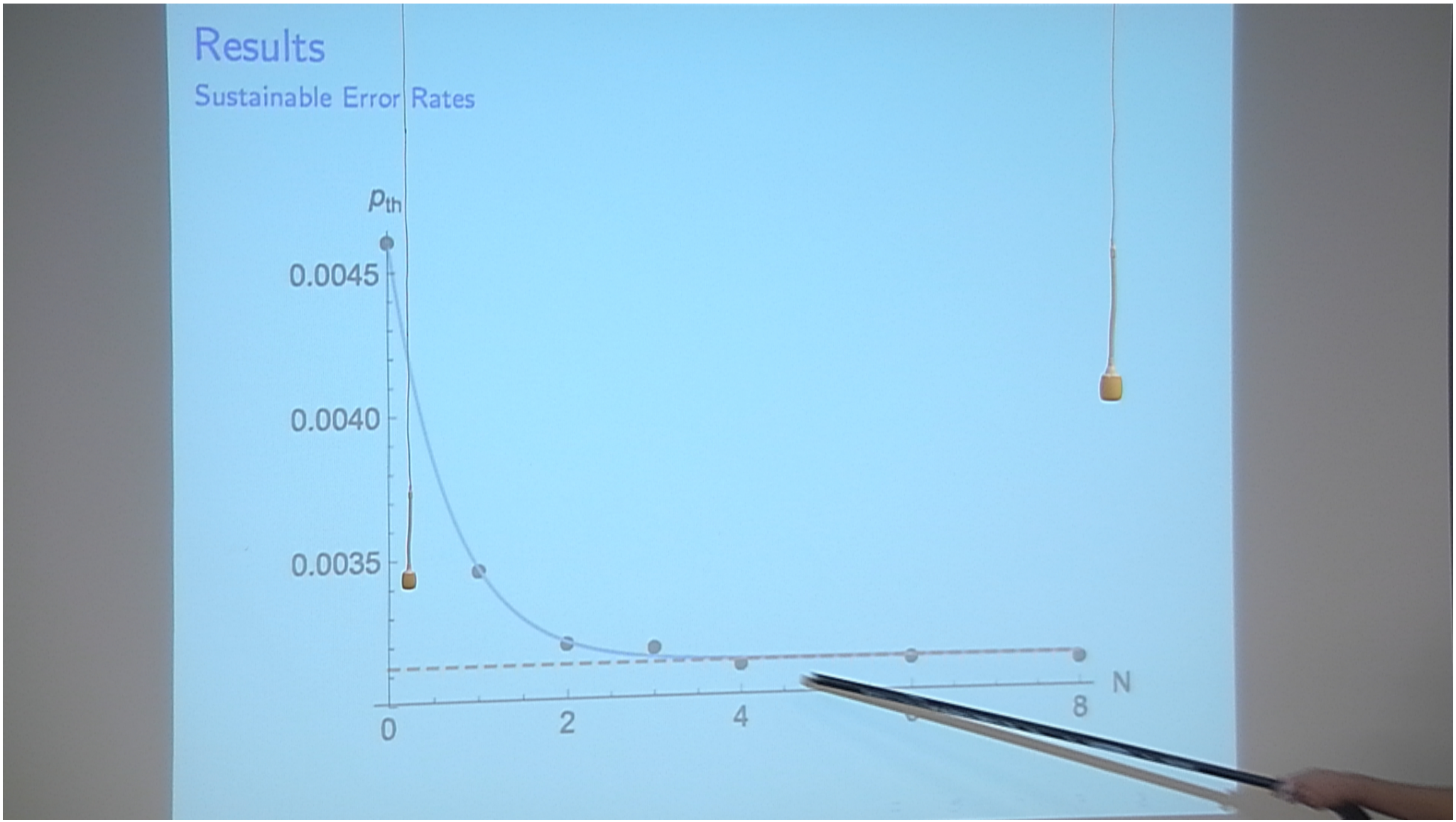
Simulation # 2

We perform simulations with repeated application of the noisy error correction procedure

We study correlations by evaluating the following quantity as a function of N

$$\text{prob}(\mathcal{M} \circ (\mathcal{R}_q \circ \mathcal{E}_p)^N[\rho] = \rho).$$

We argue that convergence in the $N \to \infty$ indicates correlations do not develop beyond the capabilities of the decoder.

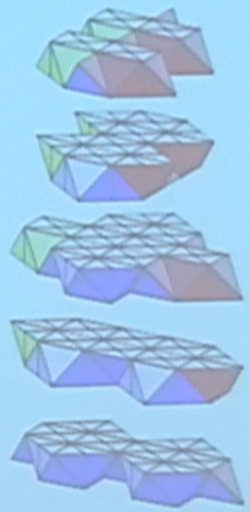

Results
Sustainable Error Rates
p_{th}
0.0045
0.0040
0.0035
0
2
4
8
N

Results

Sustainable Error Rates

Conclusions

- We have demonstrated single-shot error correction with a three-dimensional subsystem code
- We obtain respectable thresholds using a code that supports universal quantum gates
- We attribute this to single-shot error correction



Future Directions

- The model must be studied with more realistic noise
- Ultimately, we must compare the overhead requirements with other leading codes (the toric code)
- What are the fundamental features required of a single-shot code (self correction?)