Title: Non-standard thermalization in critical quench in 2D

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Abstract: <p>We consider quantum quench from a gapped to a gapless system in 1+1 dimensions. We </p>

<p>provide a rigorous proof of the thermalization of the reduced density matrix, hence that of</p>

<p>an arbitrary string of local operators in an interval. In case the system is integrable, the "thermalization" leads to a generalized Gibbs ensemble (GGE). We model the critical quench in terms of an initial state in terms of a conformal boundary state deformed by exponential cutoffs involving hamiltonian and other charges. We justify this choice of the initial state by explicitly</p>

<p>deriving it in free boson and free fermion systems with time-dependent mass. A surprising result we find is that for generic quenches and observables the higher charges remain </p>

<p>important even if the initial gap is arbitrarily high, contrary to standard RG expectations.</p>

<p>(based on hep-th/1501.04580 and a couple of upcoming papers)</p>

Non-standard thermalization in critical quench in 2D

Gautam Mandal
TIFR, Mumbai

PI seminar, 27 October 2015

Based on: GM, R. Sinha, N. Sorokhaibam (1405.6695, 1501.04580), GM, T. Morita (1302.0859), P. Caputa, GM, R. Sinha (1306.4974), ongoing work with S. Paranjape, R. Sinha, N. Sorokhaibam and T. Ugajin

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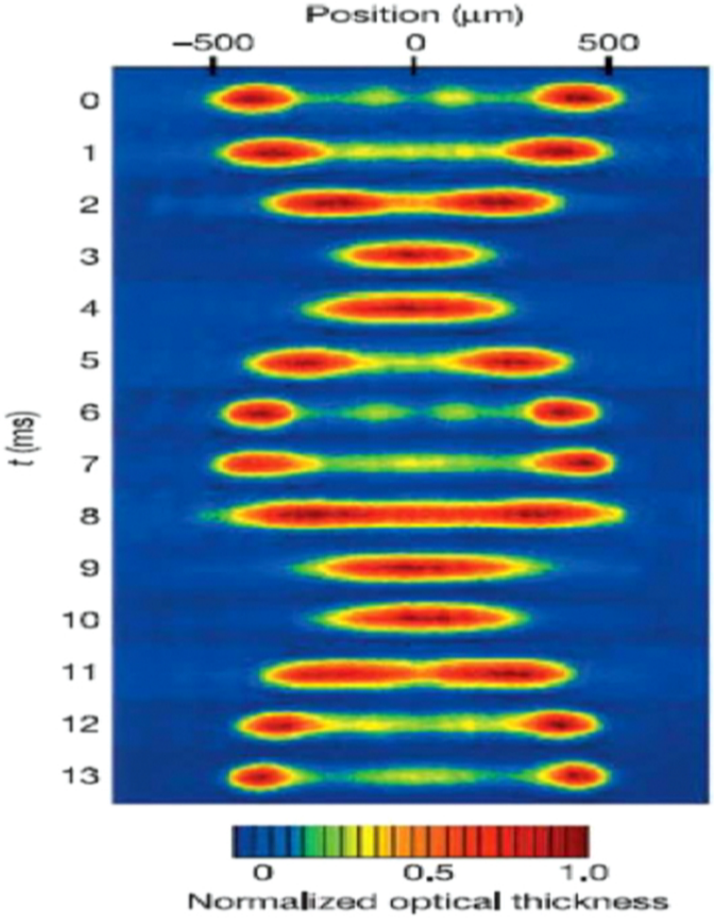
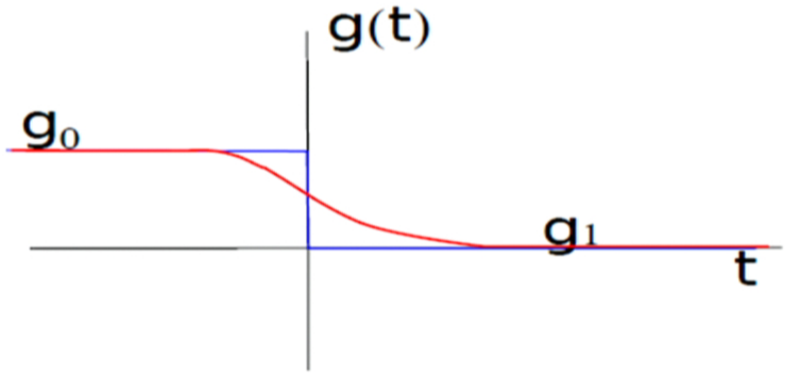
To thermalize or not to thermalize?



Quantum quench

Consider a quantum system in its ground state. Turn on a time-dependent coupling $g(t)$ for some time up to $t = t_1$.

$$\text{e.g.} \quad H(t) = -J \sum_{i=1}^{L} [\sigma_i^x \sigma_{i+1}^x + g(t)\sigma_i^z]$$



The post-quench dynamics is described by a final Hamiltonian H and an 'initial state' $|\psi_1\rangle$, which depends on $g(t)$.

Late time dynamics: thermalization

Post-quench:

$$|\psi(t)\rangle = \exp[-iH(t - t_1)]|\psi_1\rangle$$

Does the system reach a steady state at 'late times'?

Does the final state 'forget' most features of the initial state? In particular, is the state 'thermal'?

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Of course, we cannot have pure state $\rightarrow$ mixed state.

A more accurate statement of thermalization is ...

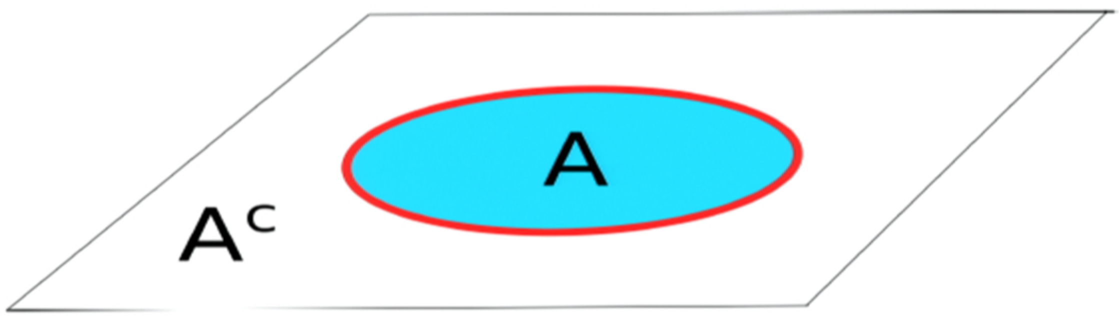
Definition of thermalization

$$\langle\psi_1|O_1(x_1,t)\ldots O_n(x_n,t)|\psi_1\rangle \xrightarrow{t>t_{eqm}} (O_1(x_1)\ldots O_n(x_n)\rho_{eqm})$$

Equivalent statement in terms of density matrix of subsystem A

$$\rho_A(t) \xrightarrow{t>t_{eqm}} \rho_{A,eqm}$$

where $\rho_A(t) = \mathrm{Tr}_{A^c}|\psi(t)\rangle\langle\psi(t)|,\ \rho_{A,eqm} = \mathrm{Tr}_{A^c}\rho_{eqm}$



This formalizes the concept of the rest of the system as a ‘bath’.

Quantum Ergodic Hypothesis

QEH: (i) An equilibrium state ρ_{eqm} exists, and it is given by the microcanonical ensemble

$$\rho_{eqm} = \rho_{micro}$$

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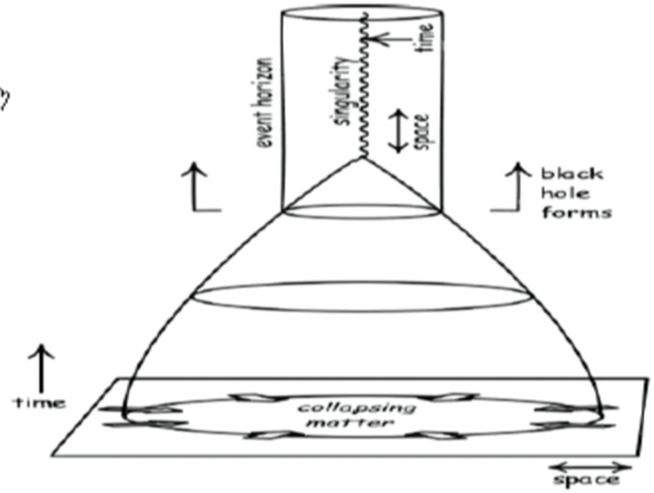
Is the QEH true?

Thermalization in gravity: gravitational collapse



No hair theorem: different forms and descriptions of matter, collapse into a black hole characterized by only the total mass (and angular momentum and charge) of the collapsing matter.

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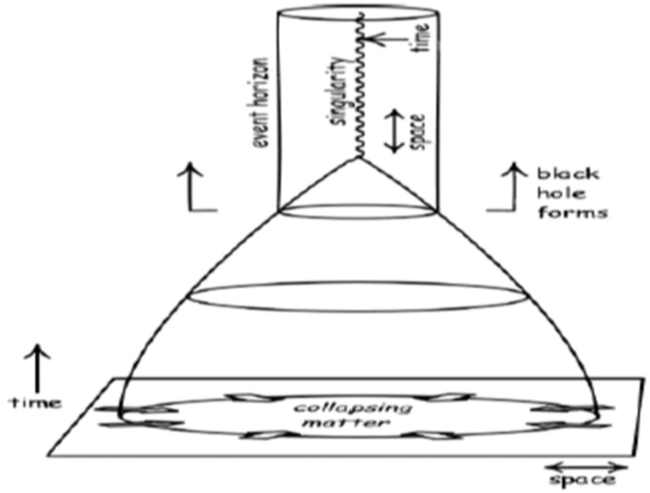

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$$\rho_{pure} = |\psi_1\rangle\langle\psi_1| \xrightarrow{?} \rho_{M,J,Q}$$ information loss

As mentioned above, the correct way to understand this is in terms of the reduced density matrix: $\rho_A(t) \to \rho_{A;M,J,Q}$

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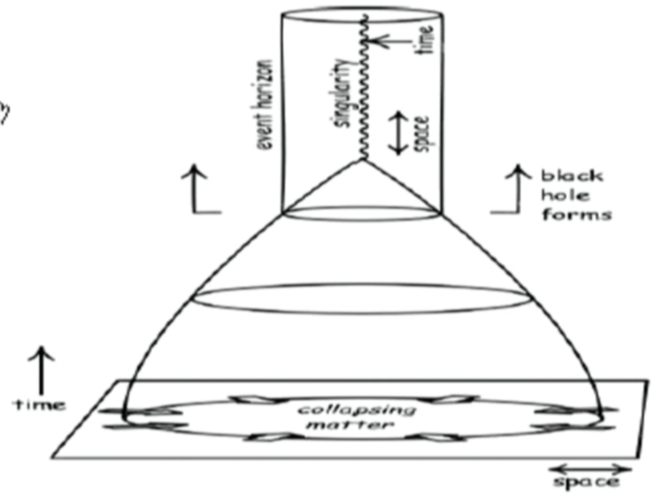

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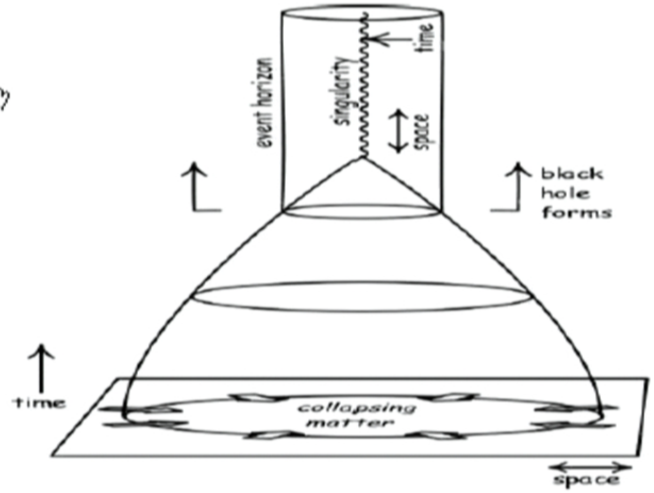

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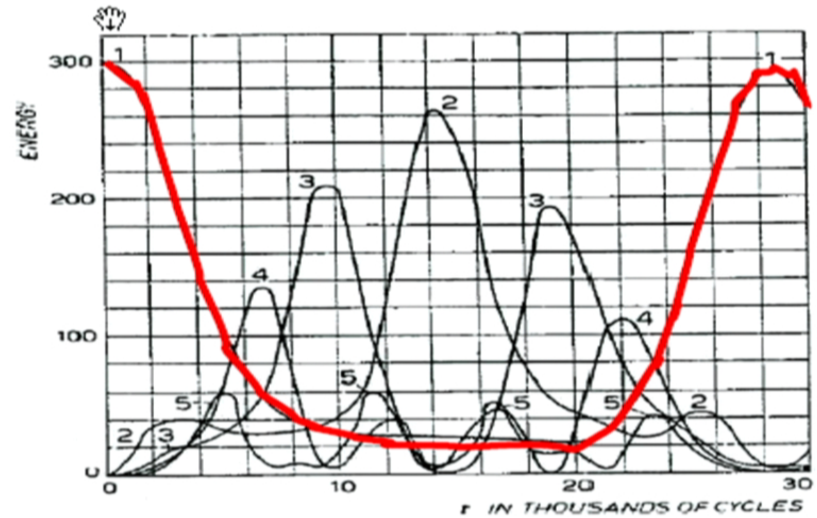
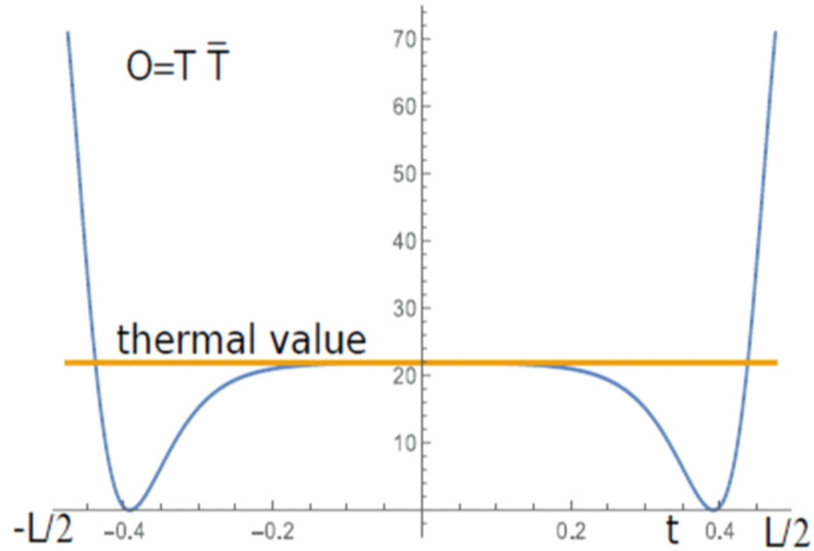
AdS/CFT: gravitational collapse= thermalization in field theory.

Non-thermalization



Weakly anhamonically coupled chain of oscillators show ‘revival’

Fermi, Pasta, Ulam 1953



1+1 dimensional critical system with spatial boundaries; shows periodicity $\Delta T = L/2$.

Mandal, Sinha, Ugajin 2015; see also Cardy 2014, Kuns, Marolf 2014

AdS/CFT: For gravitational duals of non-ergodic systems, see Balasubramanian, Buchel, Green, Lehner, Leibling 2014

Integrable systems: recent insights

Expect: integrability $\Rightarrow$ non-ergodicity.

But, examples of 2D integrable models have been discovered in the last 8 years, where QEH holds.

Transverse field Ising (Calabrese et al 2005)

$H = -J \sum_{i=1}^{L} \left[\sigma_i^x \sigma_{i+1}^x + h(t)\sigma_i^z\right]$

Hard core boson chain (Rigol et al 2007)

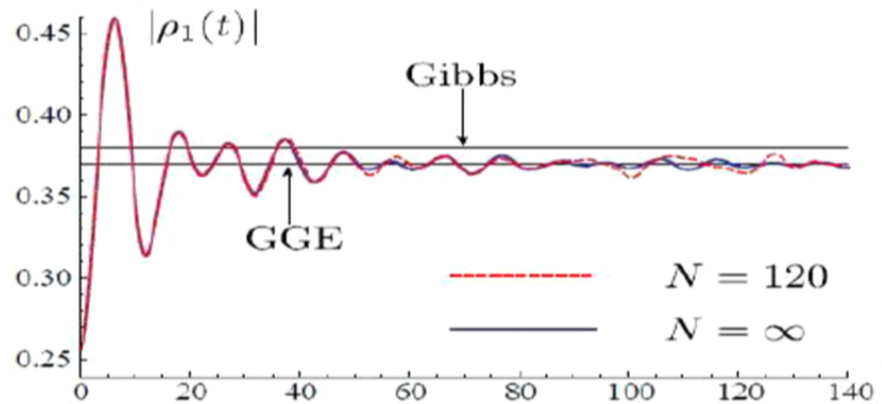
$H = -J \sum_{i=1}^{L(t)} b_i^\dagger b_{i+1} + \mathrm{h.c.}$

Massive Scalar(Sotiriadis, Cardy 2010)

$S = \int d^2x \left[(\partial\phi)^2 - m^2(t)\phi^2\right]$

Matrix QM model (Morita, GM 2013)

$S = \int dt \left[\mathrm{Tr}(U^\dagger \partial_t U + a(t)(U + U^\dagger)\right]$



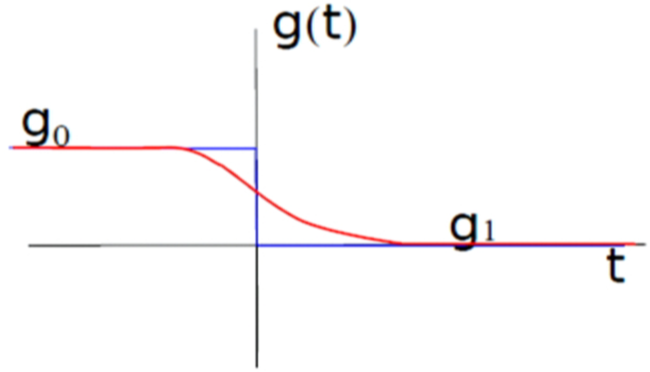
(a) $\rho_1(t)$ and GGE vs. Gibbs ensemble

Morita, GM 2013

Critical Quench

RG flow to a critical point leads to universality classes of systems.

Suppose in course of a quench, $g(t)$ stops at a value g_1 for which the system is critical.



Does the 'dynamics' of various systems show some universality in this case?

Rest of the talk

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Critical quench: first pass (Calabrese-Cardy ansatz)

For a sudden quench from a coupling g_0 to a quantum critical coupling g_1 ($m_0^{2-\Delta} = g_0 - g_1$)

$$S = S_{CFT}(g_1) + (g_0 - g_1) \int d^2x \theta(-t) O_\Delta(x,t)$$

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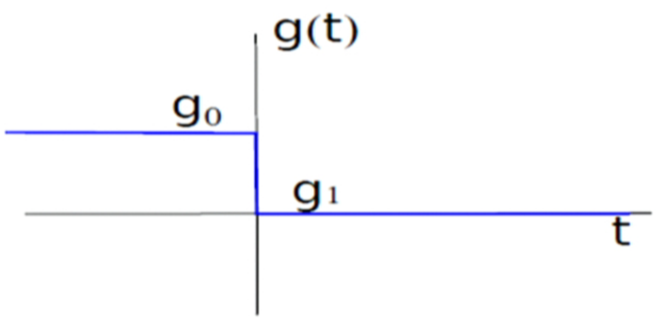

Calabrese and Cardy (2005) modelled the state at $t = 0$ as $|\psi_{CC}\rangle = e^{-\kappa H}|Bd\rangle$

Here $\kappa \sim 1/m_0$. $|Bd\rangle$ =conformal boundary state.

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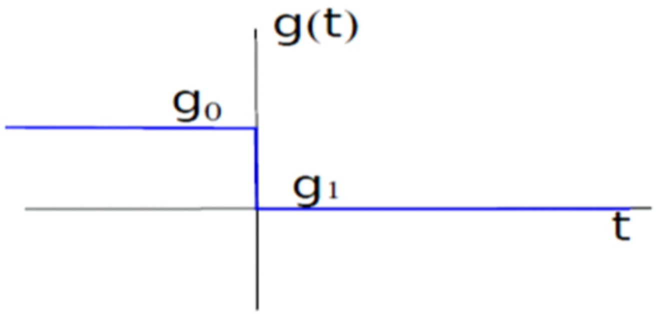

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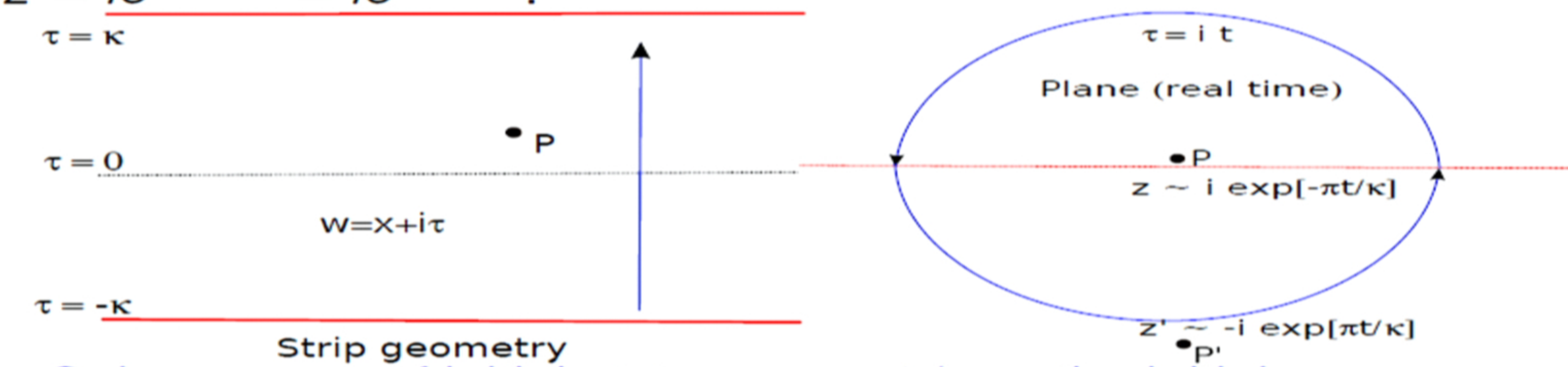
Logic: Single scale, not visible to $E \ll m_0$; the state is conformal ($\Rightarrow |Bd\rangle$). Modes $E \gg m_0$ are not excited by the quench ($\Rightarrow e^{-\kappa H} \sim e^{-H/m_0}$.)

Thermalization in a CFT with initial state $|\psi_{CC}\rangle = e^{-\kappa H}|Bd\rangle$

Time-dependence of one- and two-point functions (Calabrese-Cardy 2005,...)

$$\langle\psi_{CC}|O(x,t)|\psi_{CC}\rangle \xrightarrow{t\gg\kappa} \mathrm{Tr}(\rho_\beta O(x,t)) + a\, e^{-\gamma t}, \quad \rho_\beta = e^{-\beta H}/Z$$

where $\beta = 4\kappa$, $\gamma = 2\pi\Delta/\beta$. Δ= scaling dimension of $O(x,t)$. Proof: $z = ie^{\pi w/2\kappa} = ie^{2\pi w/\beta}$.



- Only memory of initial state= $m_0 \sim 1/\kappa$ or the initial energy,
- Universality of late dynamics: $\gamma_i/\gamma_j = \Delta_i/\Delta_j$ irrespective of initial state.

Rigorous result on thermalization

Define reduced density matrices

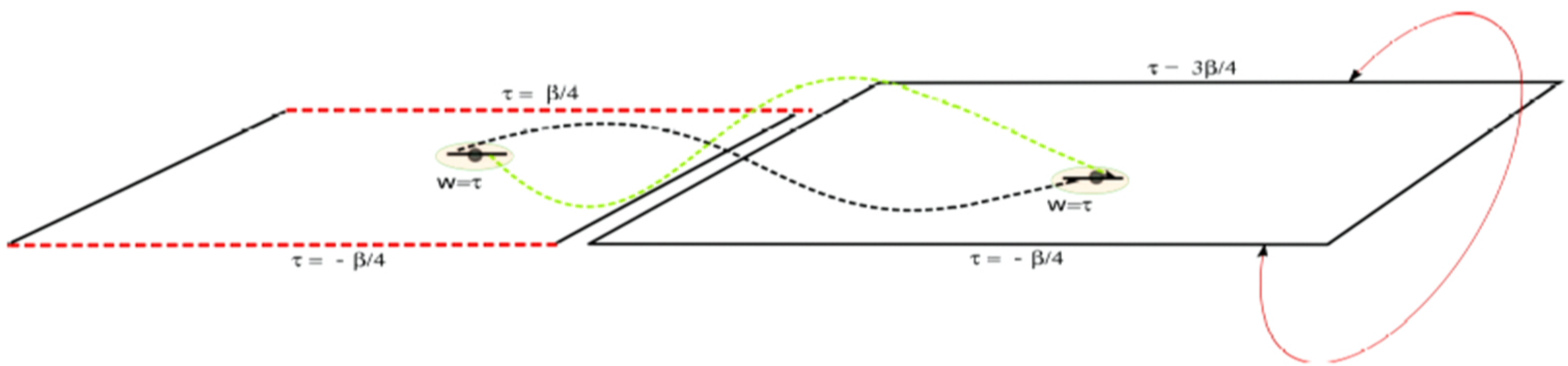
$$\rho_A(t) = \mathrm{Tr}_{A^c} |\psi_{CC}(t)\rangle\langle\psi_{CC}(t)|, \quad \rho_A(\beta) = \mathrm{Tr}_{A^c} \rho_\beta$$

and their square normalized variety: $\hat{\rho} = \rho/\sqrt{\mathrm{Tr}\rho^2}$.

We prove that (GM-Sinha-Sorokhaibam 1501.04580, Cardy 1507.07266)

$$\mathrm{Tr}\left(\hat{\rho}_A(t)\,\hat{\rho}_A(\beta)\right) = 1 - a\, e^{-2\gamma_m t} + \textit{faster transients}$$

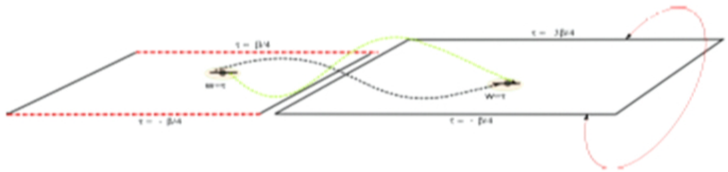
$$\gamma_m = 2\pi\Delta_m/\beta \quad \Leftarrow \text{the most relevant primary operator}$$



Details

The overlap function

$$I_A(t) = \mathrm{Tr}(\hat{\rho}_A(t)\hat{\rho}_A(\beta)) = \frac{\hat{Z}_{sc}}{\sqrt{\hat{Z}_{ss}\hat{Z}_{cc}}}$$



involves gluing a strip and a cylinder along an interval. We compute this by using the short interval expansion (Headrick 2010, Calabrese-Cardy-Tonni 2010) in which each interval is replaced by a direct sum of conformal fields.

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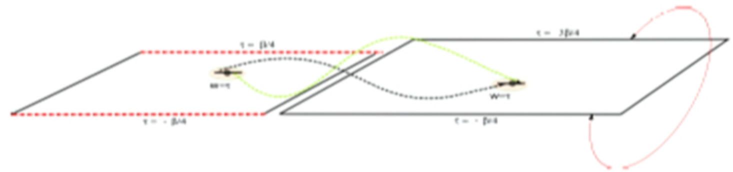

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$$\hat{Z}_{sc} = C_{0,0}(1 + S_1^{sc}),\ \ S_1^{sc} = \sum_a \hat{C}_{a,0}(\langle O_a\rangle^{\mu}_{str} + \langle O_a\rangle^{\mu}_{cyl}) + \sum_{ab} \hat{C}_{a,b}\langle O_a\rangle^{\mu}_{str}\langle O_b\rangle^{\mu}_{cyl}$$

$$\hat{Z}_{ss} = C_{0,0}(1 + S_1^{ss} + S_2^{ss}),\ \ S_1^{ss} = 2\sum_a \hat{C}_{a,0}\langle O_a\rangle^{\mu}_{str} + \sum_{ab} \hat{C}_{a,b}\langle O_a\rangle^{\mu}_{str}\langle O_b\rangle^{\mu}_{str},\ \ S_2^{ss} = \sum_k \hat{C}_{k,k}(\langle O_k\rangle^{\mu}_{str})^2$$

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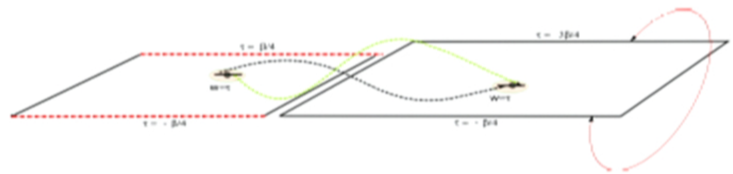

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At $t \to \infty$ all one-point functions reduce to thermal one-point function. Thus, $\hat{Z}_{sc} = \hat{Z}_{ss} = \hat{Z}_{ss}$. Hence $I_A(t = \infty) = 1$. The slowest transient comes from S_2^{ss} which contains $\langle O_m\rangle^{\mu}_{str})^2 \sim\sim \exp[-2\gamma_m t]$.

Consequence of the rigorous result

We find the following long time behaviour of the reduced density matrix itself

$$\rho_A(t) \xrightarrow{t \gg l/2} \rho_A(\beta) + a\, e^{-\gamma_m t}$$

where l = length of the interval A.

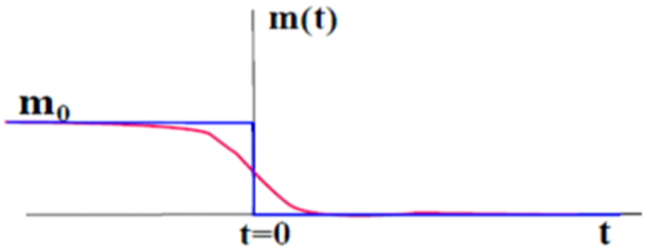
Thus, for any finite string of local operators, or even an infinite string of local operators all contained in the interval A, we conclude

$$\langle \psi_{cc} | O_1(x_1, t_1) O_2(x_2, t_2) \ldots O_n(x_n, t_n) | \psi_{cc} \rangle \xrightarrow{t \gg l/2, t_i} \mathrm{Tr}\left(\rho_\beta O_1(x_1, t_1) O_2(x_2, t_2) \ldots O_n(x_n, t_n)\right) + a\, e^{-\gamma_m t}$$

The only ingredient which goes into the proof is that the spectrum of conformal dimensions has a gap (note that this is different from a mass gap). (Finite size of space: near-thermalization followed by recurrence).

Actual quench, e.g. scalar field: generalized CC

$$S = \int d^2x \, \frac{1}{2} \left[(\partial\phi)^2 - m(t)^2\phi^2 \right]$$



Time-dependent coupling induces a Bogoliubov transformation:
$a_{in}(k) \propto a_{out}(k) - \gamma(k) a^\dagger_{out}(k)$ Hence
$|0, in\rangle = e^{\sum_k \gamma(k) a^\dagger_{out}(k) a^\dagger_{out}(-k)} |0, out\rangle$

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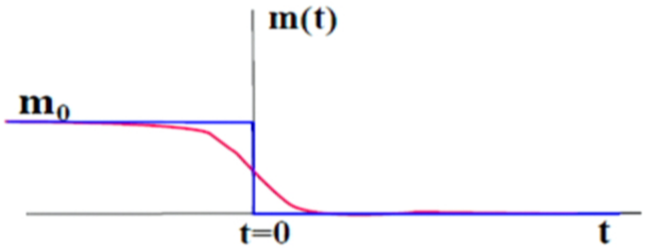

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For a simple form $m^2(t) = m_0^2(1 - \tanh(t/\delta t))/2$, explicit value of $\gamma(k)$: Birrell, Davies 1994
$|0, in\rangle = |\psi_{gCC}\rangle = e^{-(\kappa_2 H + \sum_n \kappa_n W_n + \dots)} |\text{Dirichlet}\rangle$, where κ_2, κ_4 depend on $m_0, \delta t$. For $\delta t \to 0$, we have $\kappa_2 = 1/m_0, \kappa_4 = -5/(160 m_0^3), \dots$

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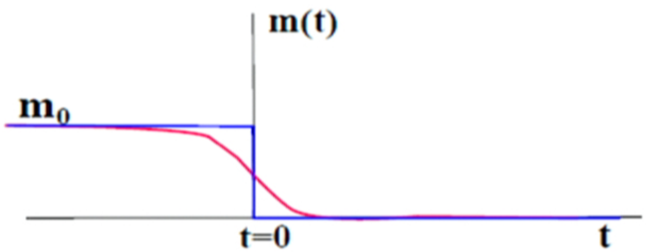

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Here $W_n \sim \sum_k k^{n-1} a^\dagger_{out}(k) a_{out}(k)$, $n = 2, 4, 6, \ldots$

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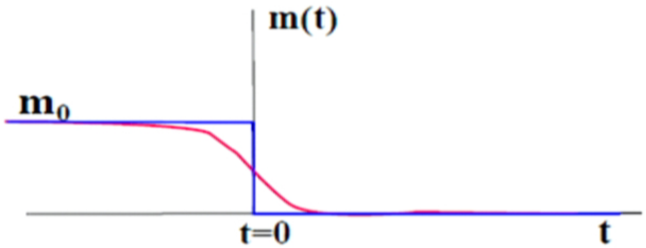
$$S = \int d^2x \, \frac{1}{2} \left[(\partial\phi)^2 - m(t)^2 \phi^2 \right]$$



Time-dependent coupling induces a Bogoliubov transformation: $a_{in}(k) \propto a_{out}(k) - \gamma(k) a^{\dagger}_{out}(k)$ Hence $|0, in\rangle = e^{\sum_k \gamma(k) a^{\dagger}_{out}(k) a^{\dagger}_{out}(-k)} |0, out\rangle$

For a simple form $m^2(t) = m_0^2(1 - \tanh(t/\delta t))/2$, explicit value of $\gamma(k)$: Birrell, Davies 1994 $|0, in\rangle = |\psi_{gCC}\rangle = e^{-(\kappa_2 H + \sum_n \kappa_n W_n + \dots)} |\text{Dirichlet}\rangle$, where κ_2, κ_4 depend on $m_0, \delta t$. For $\delta t \to 0$, we have $\kappa_2 = 1/m_0, \kappa_4 = -5/(160 m_0^3), \dots$

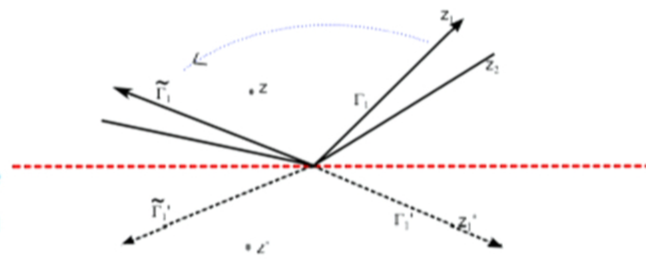
• With additional charges, whether finite or infinite in number, the initial state is not of the naive CC form. • It retains more memory in case of multi-scale quench. • These remarks also hold for non-critical quench (e.g. if the final mass $\neq 0$), or with the intial state $|0, in\rangle$ replaced by excited states, e.g. a squeezed state. • We find similar behaviour in a theory of fermions.

Rigorous result on thermalization with $|\psi_{gCC}\rangle$

Consider a generalized CC state, in a CFT, of the form

$$|\psi_{gCC}\rangle = e^{-(\kappa_2 H + \sum_n \kappa_n Q_n + \dots)} |Bd\rangle$$

where the number of additional charges Q_n can be finite or infinite in number (we take them to be derivable from local currents).



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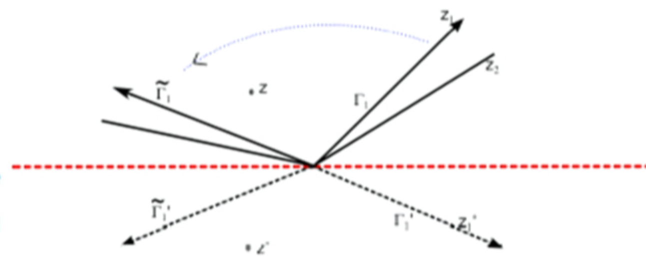

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We prove that (GM-Sinha-Sorokhaibam 1501.04580, Cardy 1507.07266)

$$\rho_A(t) \xrightarrow{t \gg l/2} \rho_A(\beta, \{\mu_n\}) + a\, e^{-\gamma t}$$

where the equilibrium state is given by

$$\rho_A(\beta, \{\mu_n\}) = e^{-\beta H - \sum_n \mu_n Q_n}/Z, \text{with} \quad \beta = 4\kappa_2, \mu_n = 4\kappa_n$$

and the relaxation time involves conformal properties (dimension Δ, charges q_n) of the most relevant operator

$$\gamma = \frac{2\pi}{\beta}\left[\Delta + \sum_n \tilde{\mu}_n q_n + O(\tilde{\mu}^2)\right], \quad \tilde{\mu}_n \equiv \frac{\mu_n}{\beta^{n-1}}$$

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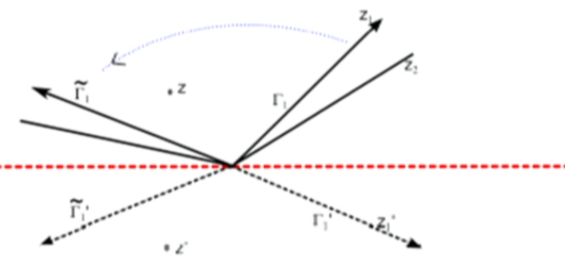

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For a sudden quench, $\mu_n \sim m_0^{-n+1} \sim \beta^{n-1}$, hence m_0 cancels from $\tilde{\mu}_n$'s!!

Non-Wilsonian behaviour?

In the scalar field example, the additional charges have higher dimensions than that of the hamiltonian, and are expected to be irrelevant in the IR. We found above, however, that the contribution of the additional charges to the relaxation time is of the same order as the initial value $2\pi\Delta/\beta$, at least to linear order in μ_n's.
In case of sudden quench of the ground state, the order $O(\tilde{\mu}^2)$ terms are not negligible. We have, however, explicitly computed the relaxation times and equilibrium correlation lengths in non-interacting scalar and fermion models.
We find that (through *ab initio* calculation in scalar model)

$$\text{limit}_{t\to\infty}\left[\langle\psi_{gCC}|O(x_1,t)O(x_2,t)|\psi_{gCC}\rangle\right] \sim \exp[-|x_1-x_2|/\xi]$$

where the correlation lengths for various operators are given by

Operator $O(x,t)$	ξ (quench)	ξ (expected in a thermal state with $\beta = 4/m_0$)
$\partial\phi$	$1/m_0$	$\beta/(2\pi) = 2/(\pi m_0)$ (does not match)
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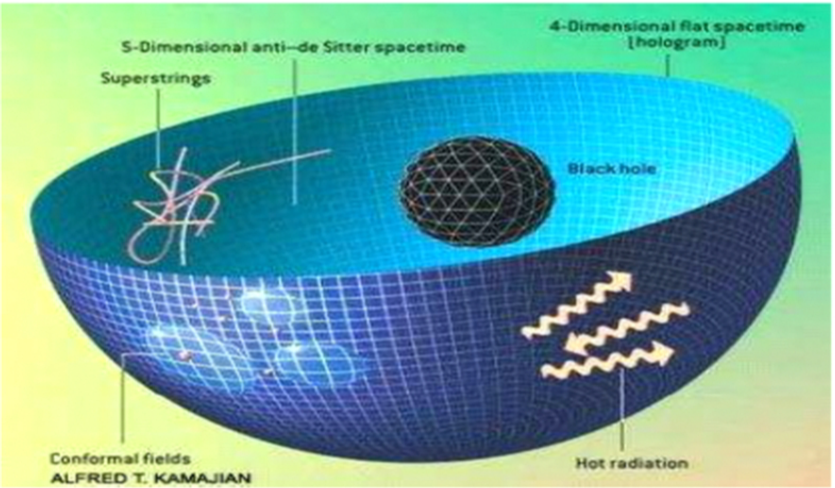
Higher dimensional ('irrelevant') operators affect large distance behaviour!

AdS/CFT Dictionary (holography)



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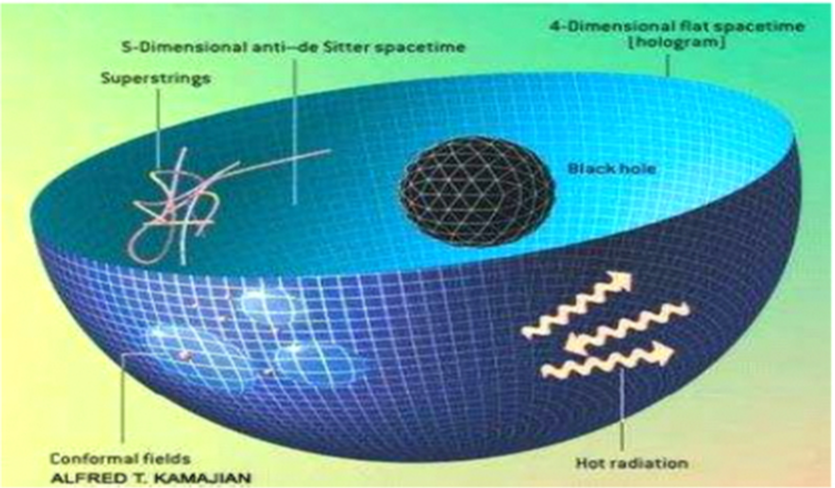

Thermalization in the boundary field theory= gravitational collapse into a black hole in the bulk!

Thermalization and holography

CFT	Gravity
Vacuum	Anti de Sitter space
Thermal state (β)	AdS-Schwarzschild ($M(\beta)$)
Thermal state with μ	Charged BH
Quantum Quench	Gravitational collapse
Ergodicity	No hair theorem
Thermal decay	Quasinormal mode
Thermalization/relaxation rate	Quasinormal frequency (QNF)
Integrable CFT (2D)	**?**
GGE (generalized Gibbs ensemble)	**?**
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Thermalization rate to GGE= QNF of HS BH

The holographic dual to 2D CFT = AdS gravity in 3D

Higher spin gravity theories are a mini version of string theory with a single Regge trajectory. In 3D, this is characterized by an infinite dimensional symmetry and infinite number of conserved charges.

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These theories possess BH solutions. The BH's carry an infinite number of conserved charges ! (infinite number of 'hairs')

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The holographic dual to integrable 2D CFT = Higher spin AdS gravity in 3D

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QNF has been calculated for bulk scalar Cabo-Bizet, Gava, Giraldo-Rivera, Narain 2014 for a HS BH with a single chemical potential μ_3

Match made!

Imaginary part of QNF

$$\mathrm{Im}\,\omega = \frac{2\pi}{\beta}\left(1 + \lambda + \frac{\tilde{\mu}_3}{3}(1+\lambda)(2+\lambda)\right)$$

Thermalization rate of the dual CFT operator is

$$\gamma_{CFT} = \frac{2\pi}{\beta}\left[\Delta + \sum_n \tilde{\mu}_n Q_n\right]?$$

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Summary

- Modelling critical quench: first pass (Calabrese-Cardy 2004)
- Proof of thermalization in general CFT with CC initial conditions
- Realistic quench: generalized CC state
- Proof of thermalization in general CFT with generic initial conditions
- Non-universality and 'non-Wilsonian' late time dynamics.
- Holographic interpretation. QEH can be interpreted as gravitational collapse to black holes with infinite number of extra charges! CFT Relaxation rate= Quasinormal frequency of BH.