Title: S-duality of u(1) gauge theory with Î¸ = Ï€ on non-orientable manifolds: Applications to topological insulators and superconductors

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URL: http://pirsa.org/15100114

Abstract:

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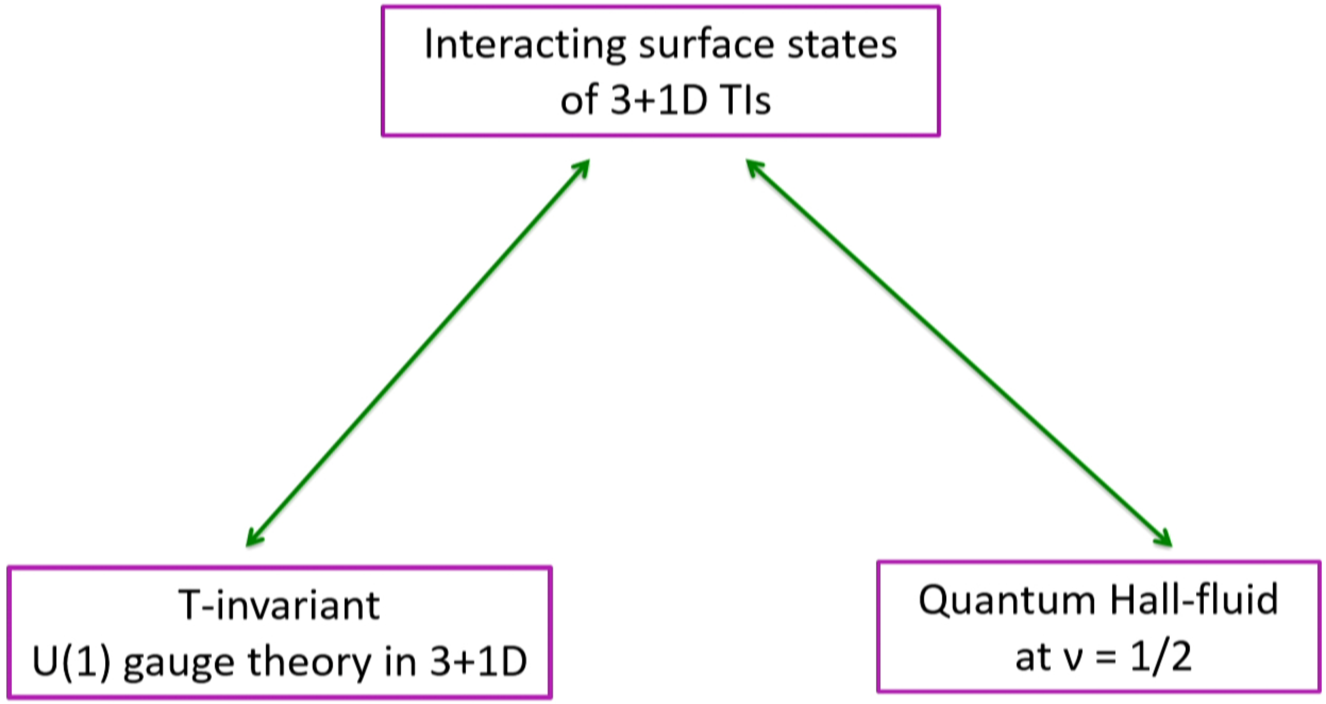
MM and A. Vishwanath, arXiv:1505.05142

MM, arXiv:1510.05663

Related work:

Motivation: D. Son, Phys. Rev. X 5, 031027 (2015)

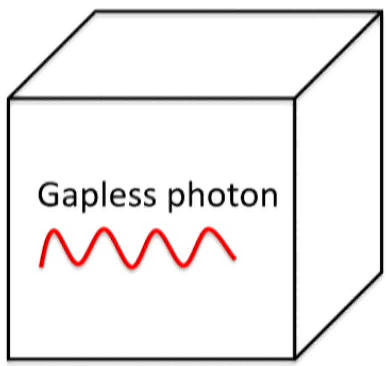
C. Wang and T. Senthil, arXiv:1505.03520, arXiv:1505.05141


Interacting surface states of 3+1D TIs
T-invariant U(1) gauge theory in 3+1D
Quantum Hall-fluid at ν = 1/2

Bulk duality in 3+1D

- EM duality of U(1) gauge theory with fermionic matter and $\theta = \pi$.

$$L = \frac{1}{4e^2} f_{\mu\nu}^2 + \frac{i\theta}{32\pi^2} \epsilon^{\mu\nu\lambda\sigma} f_{\mu\nu} f_{\lambda\sigma}$$



$\psi \longleftrightarrow$ Double monopole

$e \longleftrightarrow \frac{4\pi}{e}$

$T \longleftrightarrow CT$

Gauged topological insulator ($u(1) \rtimes T$) $\longleftrightarrow$ Gauged topological insulator ($u(1) \times CT$)

Surface “duality”

$$L_{free} = \bar{\psi}\gamma^{\mu}(\partial_{\mu} - iA_{\mu}^{ext})\psi$$

$$L_{QED_3} = \bar{\psi}_d\gamma^{\mu}(\partial_{\mu} - ia_{\mu})\psi_d + \frac{1}{4g^2}f_{\mu\nu}^2 + \frac{i}{4\pi}A_{\mu}^{ext}\epsilon^{\mu\nu\lambda}\partial_{\nu}a_{\lambda}$$

Same anomaly under $U(1)_{ext}$ and T symmetries - yes!

Same at the IR fixed point - ?

Surface "duality"

$$L_{free} = \bar{\psi}\gamma^{\mu}(\partial_{\mu} - iA_{\mu}^{ext})\psi$$

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Same anomaly under $U(1)_{ext}$ and T symmetries - yes!

Same at the IR fixed point - ?

Gapped, symmetric, topologically ordered surface: T-Pfaffian$_{+}$

U(1) gauge theory in 3+1D

$$L = \frac{1}{4e^2} f_{\mu\nu}^2 + \frac{i\theta}{32\pi^2} \epsilon^{\mu\nu\lambda\sigma} f_{\mu\nu} f_{\lambda\sigma} \qquad \tau = \frac{\theta}{2\pi} - \frac{2\pi i}{e^2}$$

- Assume microscopic Hilbert space has only bosons
- Bosonic charge matter or fermionic charge matter

S and T dualities

$$L = \frac{1}{4e^2} f_{\mu\nu}^2 + \frac{i\theta}{32\pi^2} \epsilon^{\mu\nu\lambda\sigma} f_{\mu\nu} f_{\lambda\sigma} \qquad \tau = \frac{\theta}{2\pi} - \frac{2\pi i}{e^2}$$

Bosonic matter:

$$S : \tau \to -\frac{1}{\tau}$$

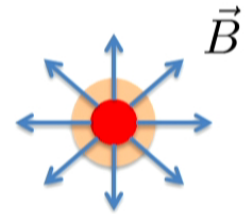
$$\mathcal{T} : \tau \to \tau + 2, \qquad (\theta \to \theta + 4\pi)$$

Generate a subgroup of SL(2, Z)

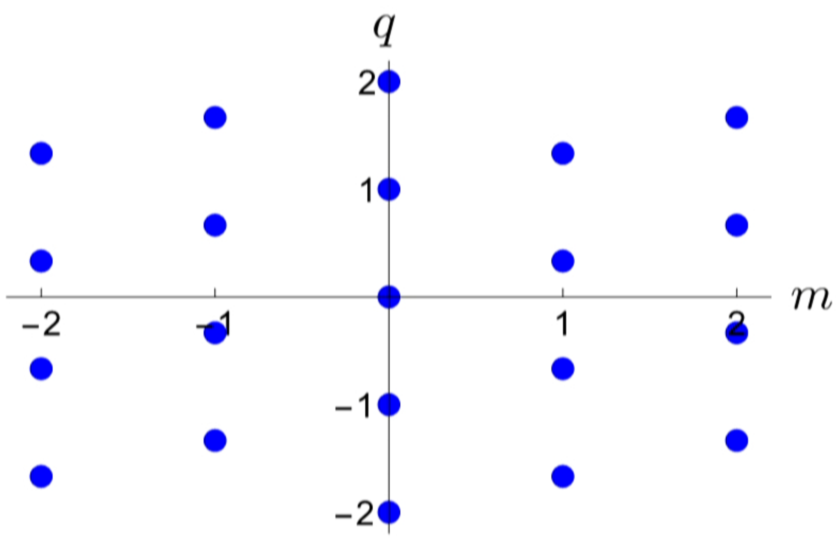
- Lattice of dyon excitations - Cardy, Rabinovici, 82
- Partition function on non-trivial manifolds – Witten, 95

Dyon excitations

Excitations: electric charges and magnetic monopoles

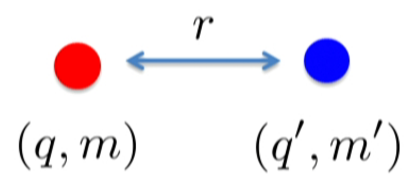


Witten effect: $q = n + \dfrac{\theta m}{2\pi}$

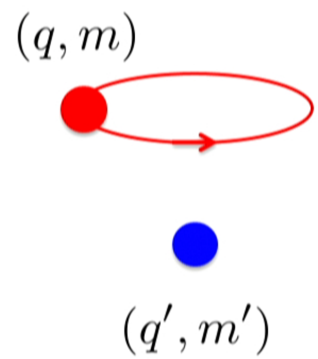


Dyon interactions

Coulomb: $$E = \frac{1}{4\pi r}\left(e^2 qq' + \frac{4\pi^2}{e^2} mm'\right)$$



Statistical: $$e^{i(qm' - mq')\Omega/2}$$



Dyon self-statistics

$\theta = 0:$

Charge: boson — $(q = 1, m = 0)$

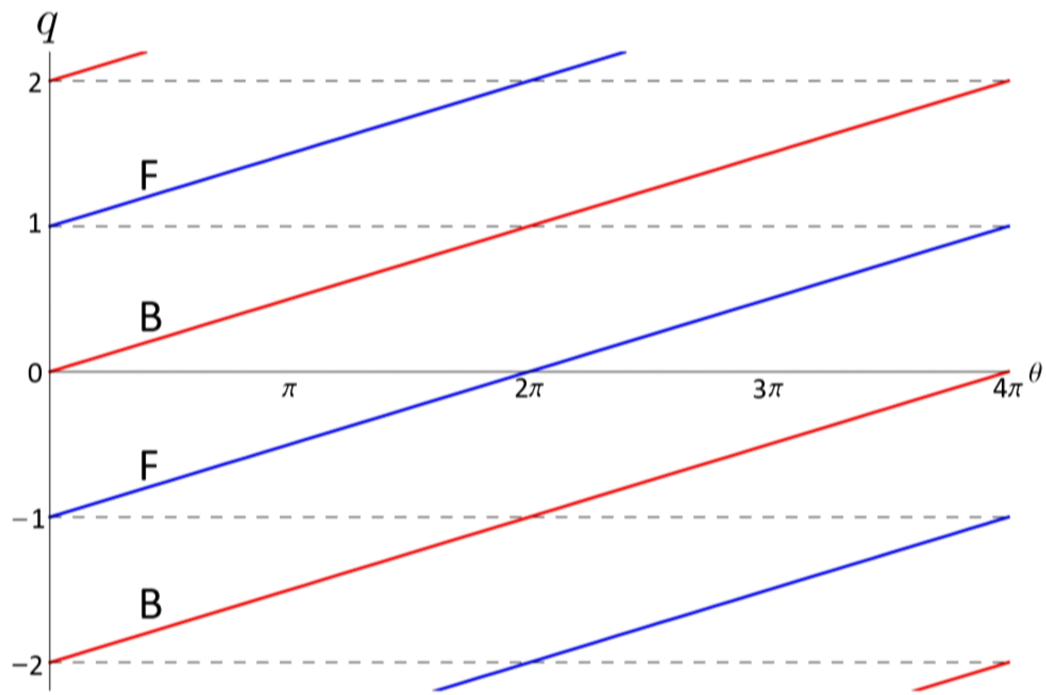
Monopole: boson — $(q = 0, m = 1)$

Bound state: fermion — $(q = 1, m = 1)$

More generally: $(q, m) \longrightarrow (-1)^{qm}$

A. Goldhaber, (1976)

Monopole (dyon) statistics at finite θ



$$\left(q = n + \frac{\theta m}{2\pi}, m\right)$$

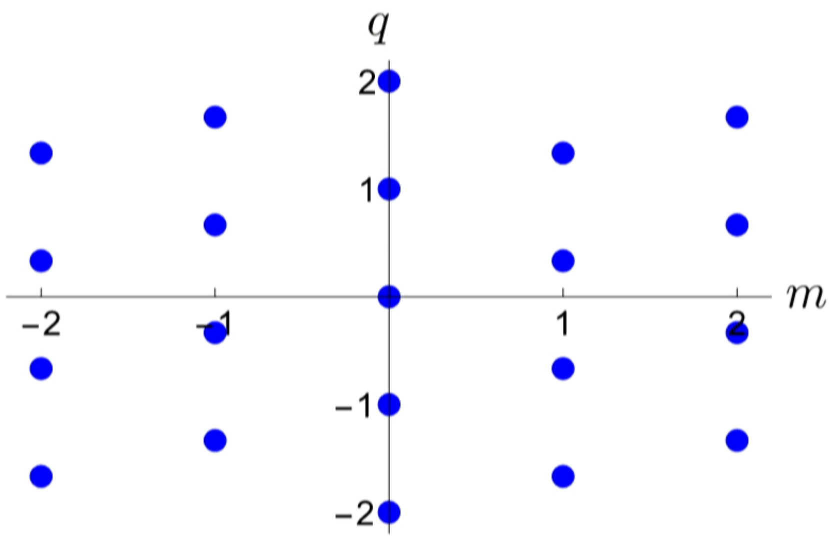
$$m = 1$$

Neutral monopole: $\theta = 0$ - boson

$\theta = 2\pi$ - fermion

$$\mathcal{T} : \theta \to \theta + 4\pi$$

S-duality



$$q = n + \frac{\theta m}{2\pi}$$

$$S: \; \tilde{n} = m, \quad \tilde{m} = -n, \qquad \tau \to -\frac{1}{\tau}$$

At $\theta = 0$ exchange electric and magnetic charges, $e \to \frac{2\pi}{e}$

Fermionic charge matter

$\theta = 0$:

Charge: fermion

$(q = 1, m = 0)$

Monopole: boson

$(q = 0, m = 1)$

Bound state: boson

$(q = 1, m = 1)$

More generally: $(q, m) \longrightarrow (-1)^{qm} \times (-1)^{q}$

S-duality, fermions

$\theta = 0:$

Charge: fermion
Monopole: boson

→

Charge: boson
Monopole: fermion

fermion matter, $\theta = 0$ $\xrightarrow{S_{bf}}$ boson matter, $\theta = 2\pi$

More generally,

$S_{bf}: \ \tau \to -\frac{1}{\tau} + 1,$ fermion matter → boson matter

S-duality, fermions to fermions

$$S_{bf}\,:\;\tau \to -\frac{1}{\tau}+1,$$ F → B

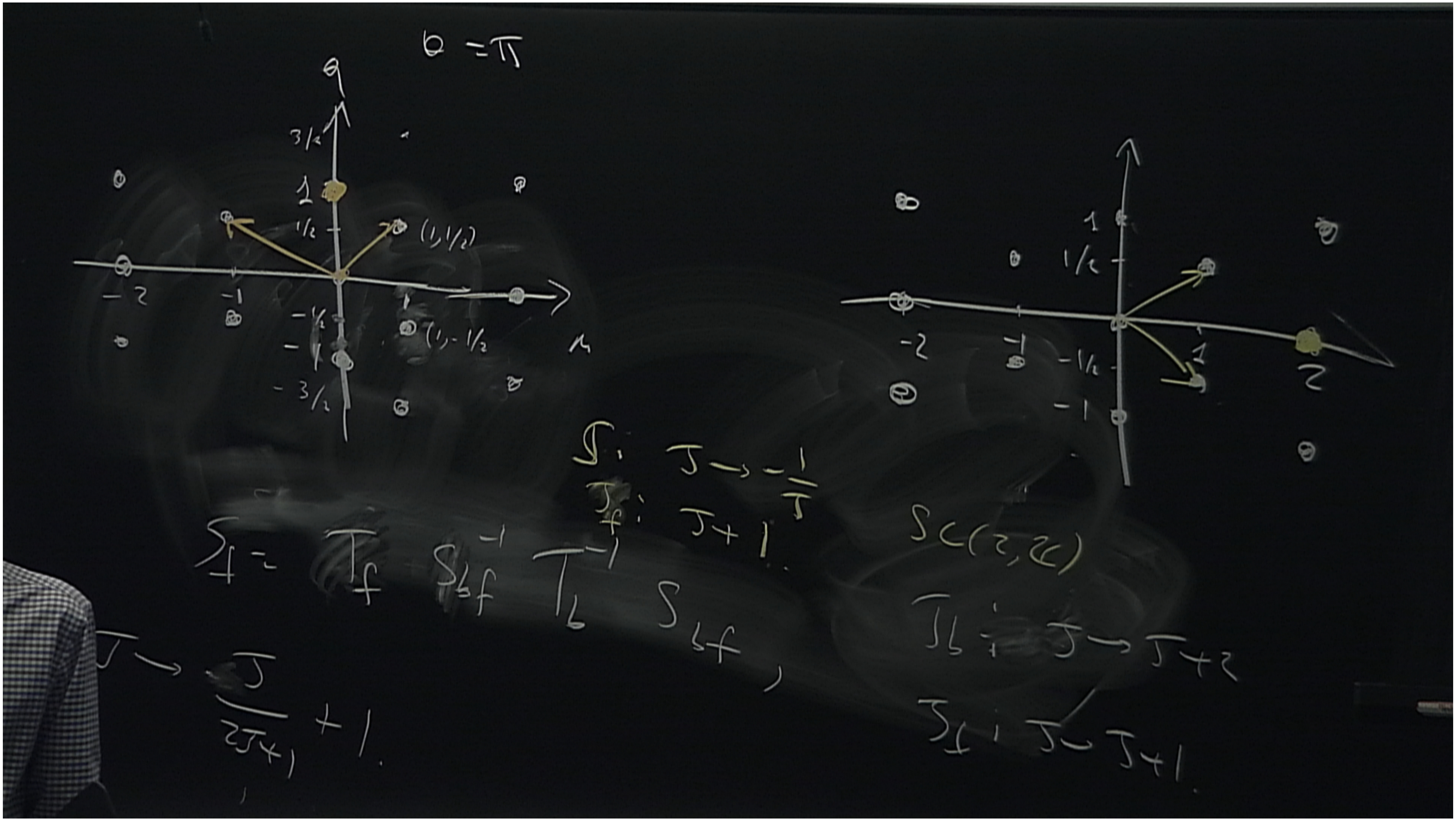
$$\mathcal{T}_b^{-1}\,:\;\tau \to \tau - 2,$$ B → B

$$S_{bf}^{-1}\,:\;\tau \to \frac{1}{1-\tau},$$ B → F

$$S_f\,:\;\tau \to \frac{\tau}{2\tau+1},$$ F → F

$$(e,\theta=-\pi) \xrightarrow{\;S_f\;} \left(\frac{4\pi}{e},\theta=\pi\right)$$

Fixes $\theta = \pi$!


θ = π
(1, 1/2)
(1, -1/2)
S: J → -1/J
T_f: J+1
S_f = T_f S_bf^{-1} T_b^{-1} S_bf
J → J/(2J+1) + 1
SL(2,Z)
T_b: J → J+2
T_f: J → J+1

S-duality from partition function

- How does pure gauge theory know about matter?

$$Z = \int Da_\mu \, e^{-S} \qquad\qquad L = \frac{1}{4e^2} f_{\mu\nu}^2 + \frac{i\theta}{32\pi^2} \epsilon^{\mu\nu\lambda\sigma} f_{\mu\nu} f_{\lambda\sigma}$$

Bosonic matter: sum over complex line bundles

Fermionic matter: sum over Spin_c bundles

View gauge-theory as emergent from Hilbert space of bosons,

Do not assume a Spin manifold!

SL(2,Z) duality from partition function

- Bosons:

$$Z^b(-1/\tau) = (i\tau)^{(\chi+\sigma)/8}(-i\bar{\tau})^{(\chi-\sigma)/8}Z^b(\tau), \qquad Z^b(\tau+2) = Z^b(\tau)$$

Witten (95)

Arbitrary oriented manifold

χ - Euler number

σ - Signature

SL(2,Z) duality from partition function

- Fermions (oriented manifold):

$$Z^f(-1/\rho) = e^{-\frac{\pi i \sigma}{16}(\rho+\bar{\rho}+\rho^{-1}+\bar{\rho}^{-1})}(i\rho)^{(\chi+\sigma)/4}(-i\bar{\rho})^{(\chi-\sigma)/4}Z^f(\rho)$$

$$Z^f(\rho+2) = Z^f(\rho)$$

$$\rho = 2\tau + 1 = \left(1 + \frac{\theta}{\pi}\right) - \frac{4\pi i}{e^2}$$

MM (15)

Time-reversal symmetry

- Is S-duality compatible with time-reversal?
- Focus on fermion matter,

$$L = \bar{\psi} i \gamma^\mu (\partial_\mu - i a_\mu) \psi - |m| cos\theta \bar{\psi}\psi - i|m| sin\theta \bar{\psi}\gamma^5 \psi$$

$$\frac{Z_\psi(\theta, |m|)}{Z_\psi(\theta = 0, |m|)} = \exp\left[i\theta \left(\frac{1}{32\pi^2} \int_M d^4x\, \epsilon^{\mu\nu\lambda\sigma} f_{\mu\nu} f_{\lambda\sigma} - \frac{1}{8}\sigma(M) \right) \right]$$

Time-reversal symmetry

$$L = \bar{\psi} i \gamma^{\mu}(\partial_{\mu} - i a_{\mu})\psi - |m| cos\theta \bar{\psi}\psi - i|m| sin\theta \bar{\psi}\gamma^5 \psi$$

$$T: \quad \psi(\vec{x}, t) \to C^{\dagger}\gamma^5 \psi(\vec{x}, -t), \quad i \to -i, \quad T^2 = (-1)^F, \; u(1) \rtimes T$$

$$CT: \quad \psi(\vec{x}, t) \to (\bar{\psi}(\vec{x}, -t)\gamma^5)^T, \quad i \to -i, \quad u(1) \times CT$$

Time-reversal symmetry

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$$\theta = 0, \, \pi$$

Time-reversal symmetry

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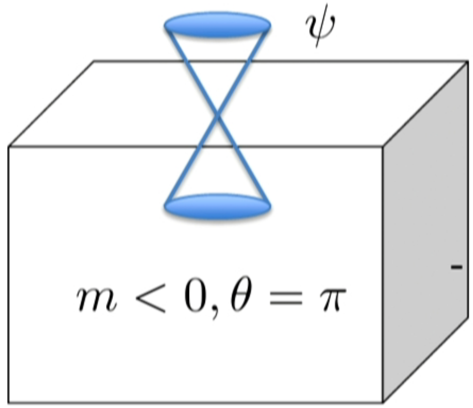
$$\theta = 0, \boxed{\pi}$$

Topological insulators

$$L = \bar{\psi} i \gamma^\mu (\partial_\mu - i a_\mu)\psi - m\bar{\psi}\psi$$

- Topological insulator with $u(1) \rtimes T$ or $u(1) \times CT$

$m > 0, \theta = 0$ - trivial TI



- nontrivial ν = 1 TI

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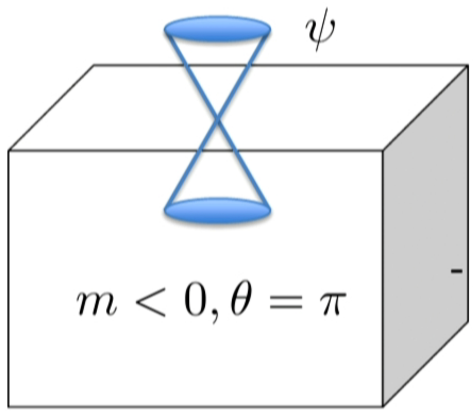
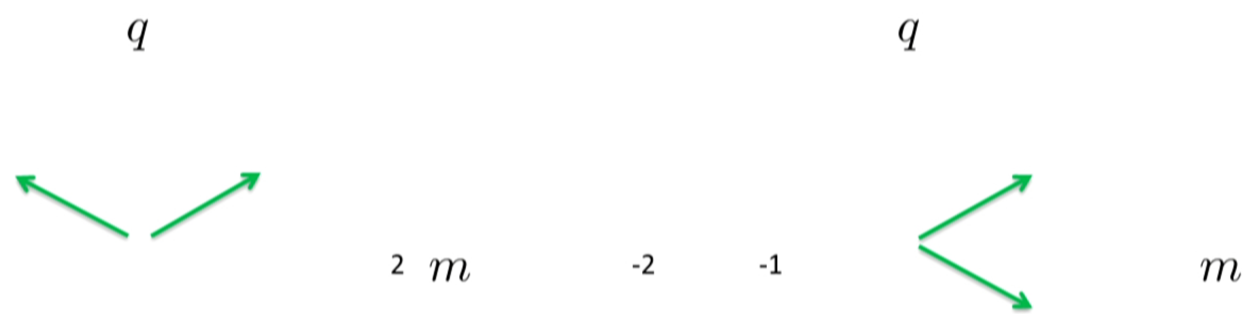


- nontrivial ν = 1 TI

- Non-interacting classification: $u(1) \rtimes T: \ \mathbb{Z}_2$

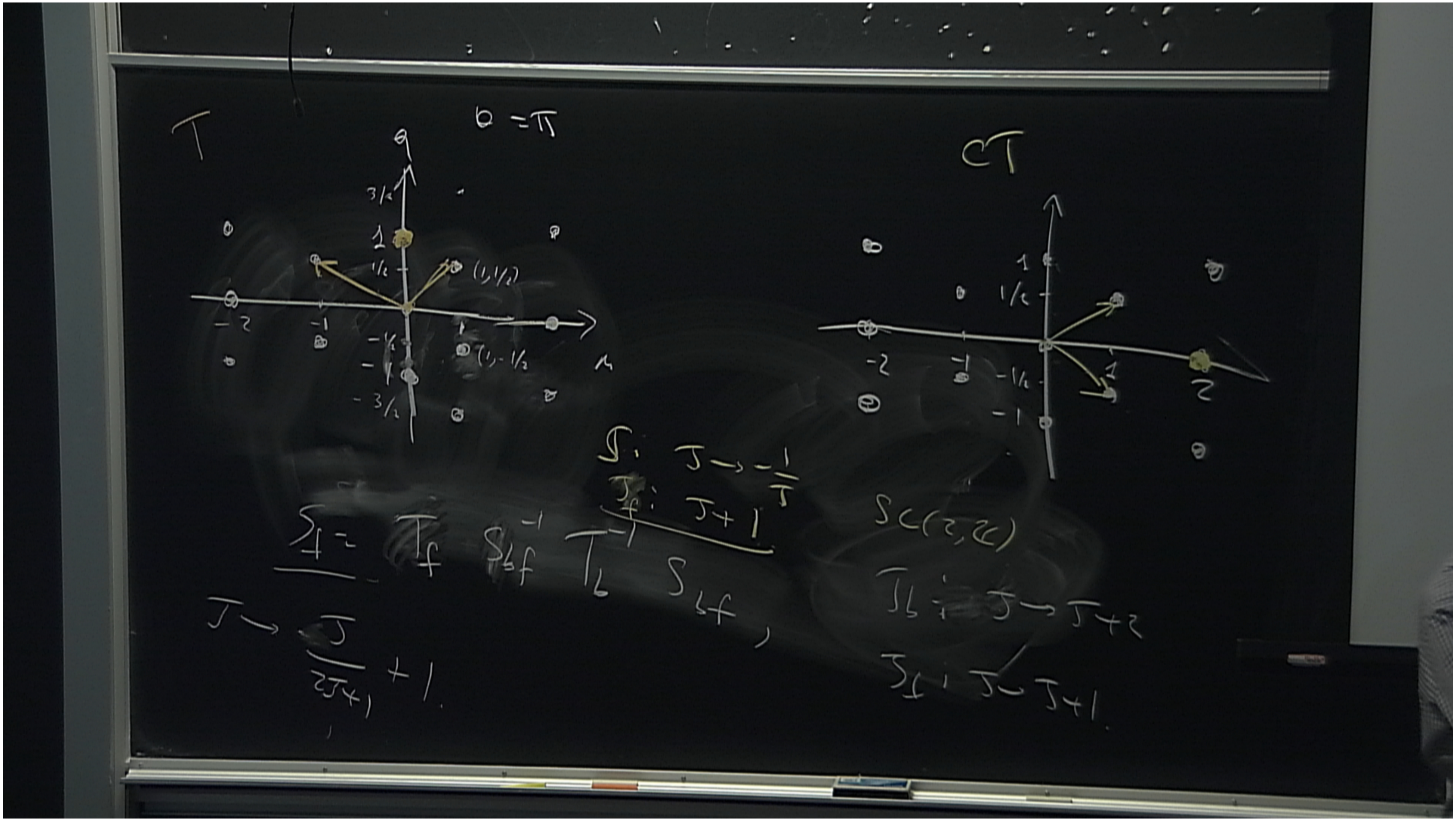
  $u(1) \times CT: \ \mathbb{Z} \to \mathbb{Z}_8$

S-duality, θ=π



$$T: \quad q \to q, \quad m \to -m$$

$$CT: \quad q \to -q, \quad m \to m$$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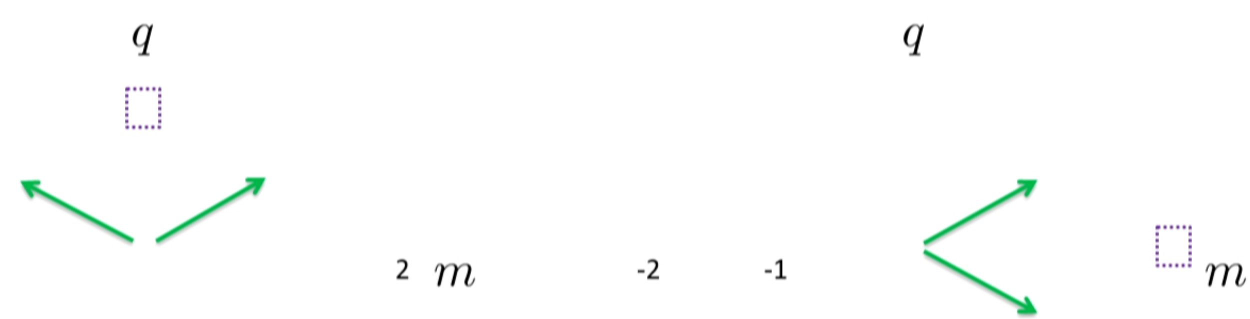
S-duality, θ=π



$$T: \quad q \to q, \quad m \to -m$$

$$CT: \quad q \to -q, \quad m \to m$$

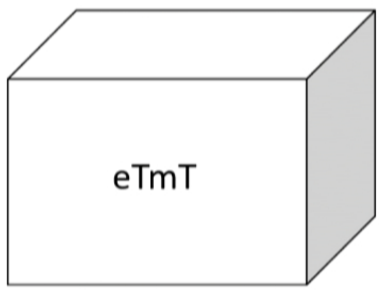
Is this sufficient to establish S_f duality?

T-SPT ambiguity

- SPTs of bosons with T-symmetry in 3+1D: $\mathbb{Z}_2^2$

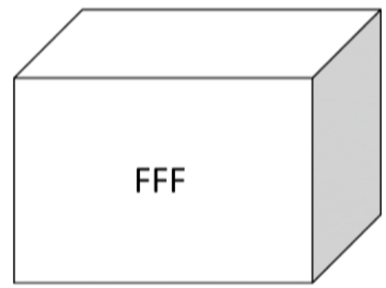
$$\{1, e, m, \epsilon\}$$

$$T_e^2 = -1, \quad T_m^2 = -1$$



$$\{1, f_1, f_2, f_3\}$$

$$T_{f_i}^2 = 1$$



$$Z_{\mathrm{eTmT}} = (-1)^{\int w_1^4}$$

$$Z_{\mathrm{eTmT}}(\mathbb{RP}^4) = -1$$

$$Z_{\mathrm{FFF}} = (-1)^{\chi}$$

$$Z_{\mathrm{FFF}}(\mathbb{CP}^2) = -1$$

S-duality, θ=π

$$L = -\frac{1}{4e^2} f_{\mu\nu} f^{\mu\nu} + \bar{\psi} i \gamma^\mu (\partial_\mu - i a_\mu)\psi - m\bar{\psi}\psi, \quad m < 0, \quad |m| \to \infty$$

- To show S_f: $T \leftrightarrow CT$, $e \leftrightarrow \frac{4\pi}{e}$
  need to compare partition functions on all manifolds, including non-orientable
- Assuming only ambiguity is a bosonic SPT phase

  $\mathbb{RP}^4$ and $\mathbb{CP}^2$

Dirac fermion on non-orientable manifold

$$L = \bar{\psi}\gamma^\mu(\partial_\mu + i\omega_\mu - ia_\mu)\psi + m\bar{\psi}\psi$$

CT symmetry: $u(1) \times CT$ - require Pin_c structure

$$\lim_{m\to\infty} \frac{Z_\psi(-m)}{Z_\psi(m)} = e^{2\pi i\eta}$$

$$\eta = \frac{1}{2}\left(\sum_{\lambda>0}\lambda^{-s} - \sum_{\lambda<0}\lambda^{-s} + N_0\right)$$

$$\eta = \frac{1}{8}n$$

Dirac fermion on non-orientable manifold

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CT symmetry: $u(1) \times CT$ - require Pin_c structure

$$\lim_{m\to\infty} \frac{Z_{\psi}(-m)}{Z_{\psi}(m)} = e^{2\pi i\eta}$$ - Pin_c bordism invariant

$$\eta = \frac{1}{2}\left(\sum_{\lambda>0}\lambda^{-s} - \sum_{\lambda<0}\lambda^{-s} + N_0\right)$$

$$\eta = \frac{1}{8}n \qquad \mathbb{Z} \to \mathbb{Z}_8$$

Dirac fermion on non-orientable manifold

$$L = \bar{\psi}\gamma^\mu(\partial_\mu + i\omega_\mu - ia_\mu)\psi + m\bar{\psi}\psi$$

T symmetry: $u(1) \rtimes T$ - require $\mathrm{Pin}_{\tilde{c}}$ structure

$$\frac{Z_\psi(-m)}{Z_\psi(m)} = (-1)^{N_0/2}$$ - $\mathrm{Pin}_{\tilde{c}}$ bordism invariant

N_0 - number of zero modes of "doubled" Dirac operator

Dirac fermion on non-orientable manifold

$$L = \bar{\psi}\gamma^{\mu}(\partial_{\mu} + i\omega_{\mu} - ia_{\mu})\psi + m\bar{\psi}\psi$$

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N_0 - number of zero modes of "doubled" Dirac operator

$\mathbb{Z}_2$ classification

- On orientable manifold,

$$e^{2\pi i\eta} = (-1)^{N_0/2} = (-1)^{\frac{1}{2(2\pi)^2}\int f\wedge f - \frac{1}{8}\sigma}$$

Simpler problem: bosonic matter, θ=0

$$Z^b_{CT}(e) = \frac{1}{\mathrm{Vol}(\mathcal{G})} \sum_L \int Da_\mu \, e^{-S[a]}$$ $u(1) \times CT$

$$Z^b_T(e) = \frac{1}{\mathrm{Vol}(\mathcal{G})} \sum_{\tilde{L}} \int Da_\mu \, e^{-S[a]}$$ $u(1) \rtimes T$

$$S[a] = \frac{1}{4e^2} \int d^4x \sqrt{g} f_{\mu\nu} f^{\mu\nu}$$

$$Z^b_{CT}(e) \stackrel{?}{=} Z^b_T\left(\frac{2\pi}{e}\right)$$

Simpler problem: bosonic matter, θ=0

$$\frac{Z^b_{CT}(e)}{Z^b_T(2\pi/e)} = \frac{2\left(\det\mathcal{N}^{2,+}\right)^{-1/2}\left(\det\mathcal{N}^{1,+}\right)^{1/2}}{\sqrt{V(M)}\,(\det\mathcal{N}^{1,-})^{1/2}}\frac{|\mathrm{Tor}(H^2(M,\mathbb{Z}))|}{|\mathrm{Tor}(H_2(M,\mathbb{Z}))|}$$

$$\times \quad \frac{\det'_+\Delta^0}{\det_-\Delta^0}\left(\frac{\det'_+\Delta^1}{\det'_-\Delta^1}\right)^{-1/2}$$

$\mathcal{N}^{i,\pm}$ - inner product on H^i_{dR}

Simpler problem: bosonic matter, θ=0

$$\frac{\bar{Z}^b_{CT}(e)}{\bar{Z}^b_T(2\pi/e)} = \frac{T^{RS}(M)}{T^c(M)}$$

Thanks to E. Witten!

- Analytic Ray-Singer torsion

$$T^{RS}(M) = (\det{}_+\Delta^4)^{-1}\left(\det{}'_+\Delta^3\right)^{1/2}(\det{}'_+\Delta^1)^{-1/2}\det{}'_+\Delta^0$$

- Reidemeister torsion

$$\begin{aligned} T^c(M) \quad &= \quad \frac{|\mathrm{Tor}(H^3(M,\mathbb{Z}))|}{|\mathrm{Tor}(H^4(M,\mathbb{Z}))||\mathrm{Tor}(H^2(M,\mathbb{Z}))|} \times \\ &\times \quad \left(\det\mathcal{N}^{3,+}\right)^{-1/2}\left(\det\mathcal{N}^{2,+}\right)^{1/2}\left(\det\mathcal{N}^{1,+}\right)^{-1/2}\left(\det\mathcal{N}^{0,+}\right)^{1/2} \end{aligned}$$

Simpler problem: bosonic matter, θ=0

$$\frac{\bar{Z}^b_{CT}(e)}{\bar{Z}^b_T(2\pi/e)} = \frac{T^{RS}(M)}{T^c(M)} \quad = 1$$

Cheeger, Muller (77-78)

Thanks to E. Witten!

- Analytic Ray-Singer torsion

$$T^{RS}(M) = (\det{}_+\Delta^4)^{-1}\left(\det{}'_+\Delta^3\right)^{1/2}(\det{}'_+\Delta^1)^{-1/2}\det{}'_+\Delta^0$$

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$$\times \left(\det\mathcal{N}^{3,+}\right)^{-1/2}\left(\det\mathcal{N}^{2,+}\right)^{1/2}\left(\det\mathcal{N}^{1,+}\right)^{-1/2}\left(\det\mathcal{N}^{0,+}\right)^{1/2}$$

Back to fermions

$$Z_{CT}(e) = \frac{1}{\mathrm{Vol}(\mathcal{G})} \sum_{\mathrm{Pin}_c} \int Da_\mu \, e^{-S[a]} e^{2\pi i \eta}$$

$$Z_T(e) = \frac{1}{\mathrm{Vol}(\mathcal{G})} \sum_{\mathrm{Pin}_{\tilde{c}}} \int Da_\mu \, e^{-S[a]} (-1)^{N_0/2}$$

$$S[a] = \frac{1}{4e^2} \int d^4x \sqrt{g} f_{\mu\nu} f^{\mu\nu}$$

$$Z_{CT}(e) \stackrel{?}{=} Z_T\left(\frac{4\pi}{e}\right)$$

Back to fermions

$$\frac{Z^f_{cl,T}(4\pi/e)}{Z^f_{cl,CT}(e)} \overset{?}{=} 2^{(1-b_1+b_3)/2}(\det\mathcal{N}^{2,+})^{1/2}\frac{|\mathrm{Tor}(H_2(M,\mathbb{Z}))|}{|\mathrm{Tor}(H^2(M,\mathbb{Z}))|}\left(\frac{e^2}{4\pi}\right)^{-b_2/2}$$

$$Z^f_{cl,CT}(e) = \sum_{\mathrm{Pin}_c} \exp\left(-\frac{1}{2e^2}\langle f,f\rangle + 2\pi i\eta\right)$$

$$Z^f_{cl,T}(e) = \sum_{\mathrm{Pin}_{\tilde{c}}} \exp\left(-\frac{1}{2e^2}\langle f,f\rangle + \frac{\pi i N_0}{2}\right)$$

$$\Delta f = 0$$

Back to fermions

$$\frac{Z^f_{cl,T}(4\pi/e)}{Z^f_{cl,CT}(e)} \stackrel{?}{=} 2^{(1-b_1+b_3)/2}(\det\mathcal{N}^{2,+})^{1/2}\frac{|\mathrm{Tor}(H_2(M,\mathbb{Z}))|}{|\mathrm{Tor}(H^2(M,\mathbb{Z}))|}\left(\frac{e^2}{4\pi}\right)^{-b_2/2}$$

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$$Z^f_{cl,T}(e) = \sum_{\mathrm{Pin}_{\tilde{c}}} \exp\left(-\frac{1}{2e^2}\langle f, f\rangle + \frac{\pi i N_0}{2}\right)$$

$$\Delta f = 0$$

Back to fermions, RP4

$$Z^f_{cl,CT}(e) = \sum_{\mathrm{Pin}_c} \exp\left(-\frac{1}{2e^2}\langle f, f\rangle + 2\pi i\eta\right) = e^{2\pi i/8} + e^{-2\pi i/8} = \sqrt{2}$$

$$Z^f_{cl,T}(e) = \sum_{\mathrm{Pin}_{\tilde{c}}} \exp\left(-\frac{1}{2e^2}\langle f, f\rangle + \frac{\pi i N_0}{2}\right) = 1$$

$$\frac{Z^f_{cl,T}(4\pi/e)}{Z^f_{cl,CT}(e)} = 2^{(1-b_1+b_3)/2}(\det\mathcal{N}^{2,+})^{1/2}\frac{|\mathrm{Tor}(H_2(M,\mathbb{Z}))|}{|\mathrm{Tor}(H^2(M,\mathbb{Z}))|}\left(\frac{e^2}{4\pi}\right)^{-b_2/2}$$

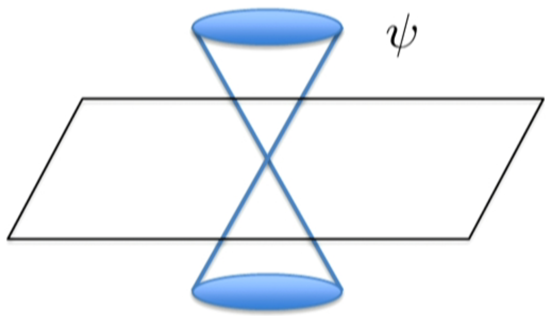
$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$\frac{1}{\sqrt{2}} \qquad \sqrt{2} \qquad 1 \qquad \frac{1}{2} \qquad 1$$

YES!

Surface "duality"

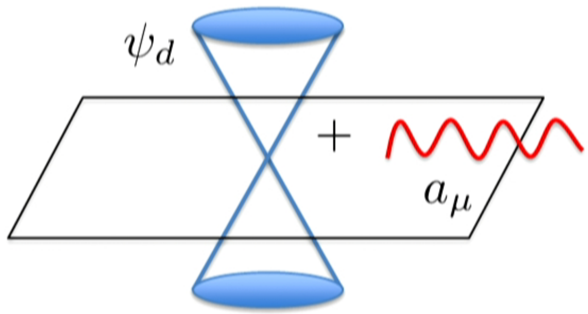
- Particle-vortex duality of single Dirac cone on TI surface



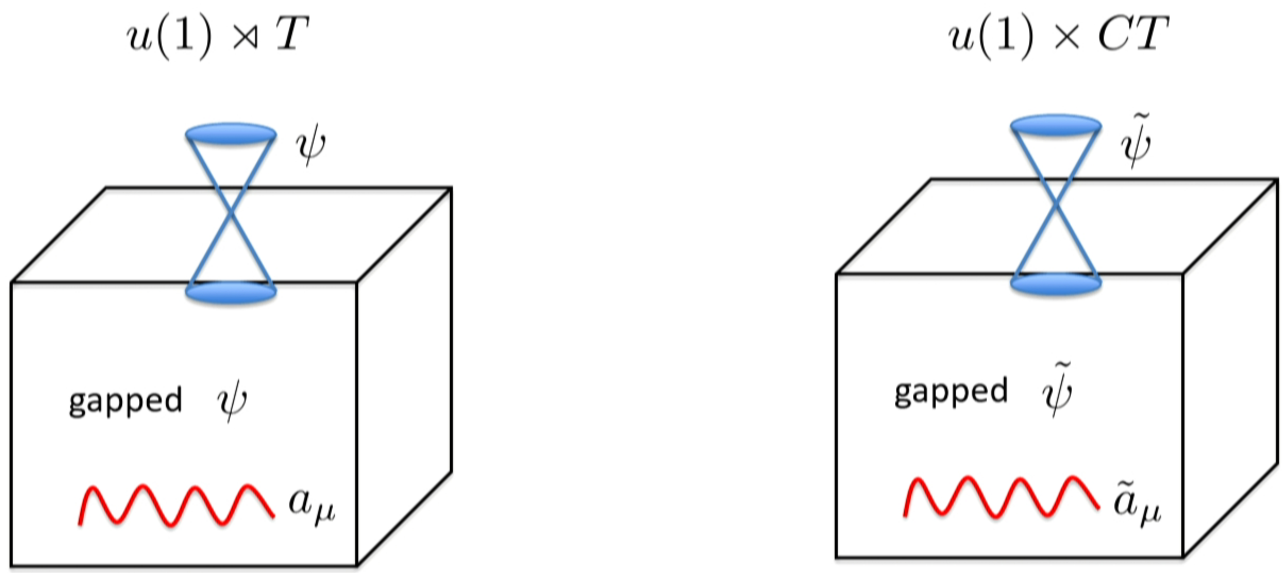
$$L_{free} = \bar{\psi}_e \gamma^\mu (\partial_\mu - i A_\mu^{ext}) \psi_e$$

$\updownarrow$

$$L_{QED_3} = \bar{\psi}_d \gamma^\mu (\partial_\mu - i a_\mu) \psi_d + \frac{1}{4g^2} f_{\mu\nu}^2 + \frac{i}{4\pi} A_\mu^{ext} \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda$$



Surface ``duality"



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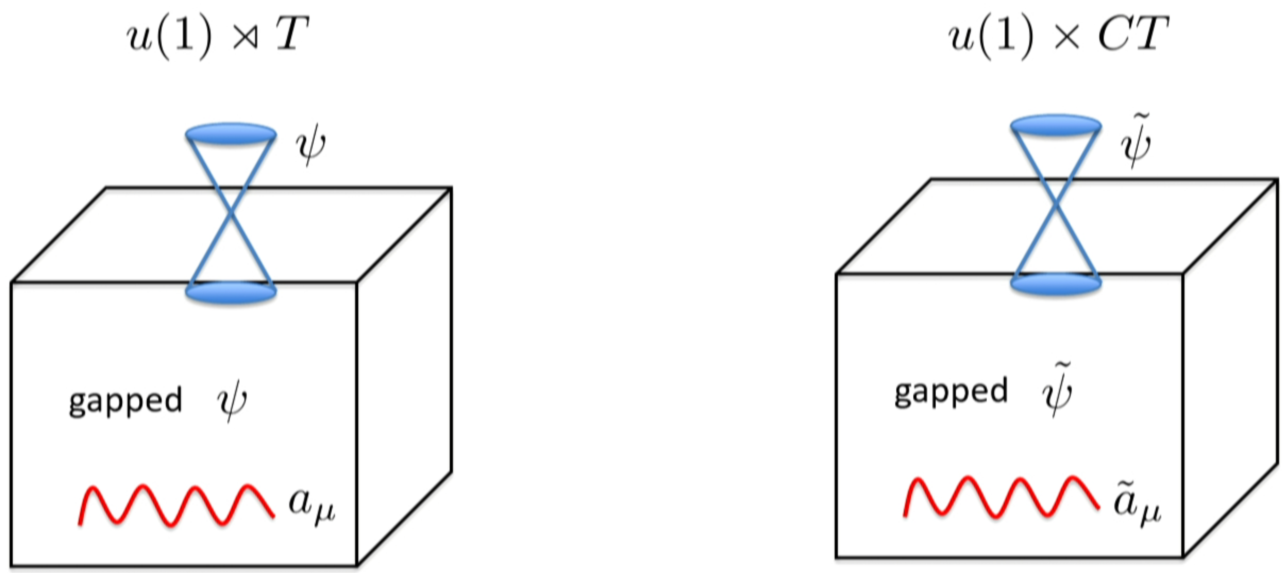
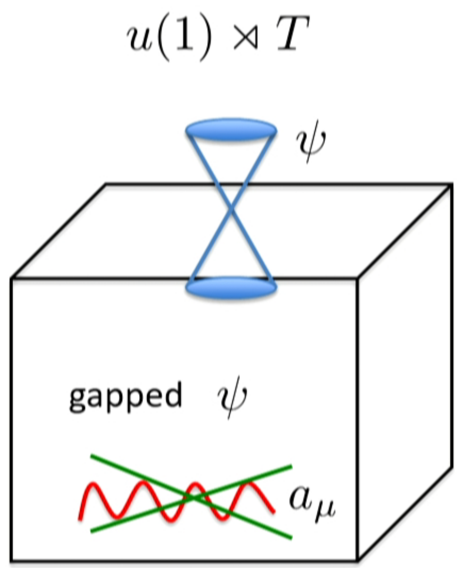


Introduce electron ψ_e charged under global $U(1)_{ext} \rtimes T$ symmetry. Put into a trivial insulator.

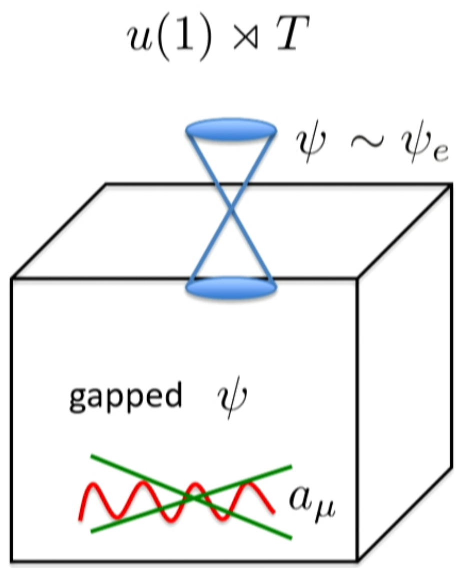
Higgs transition



Condense $\psi_e^\dagger \psi$

Higgs effect

Higgs transition



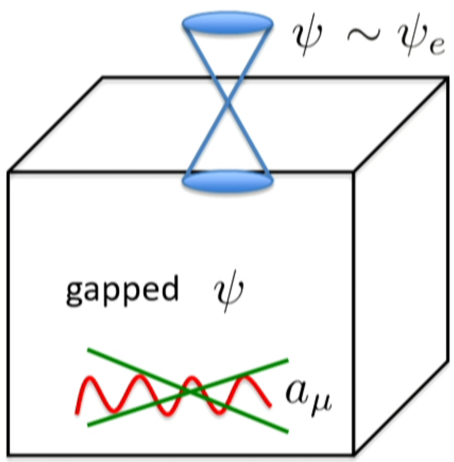
Condense $\psi_e^\dagger \psi$

Higgs effect

Result: ordinary TI!

Dual picture

$u(1) \rtimes T$



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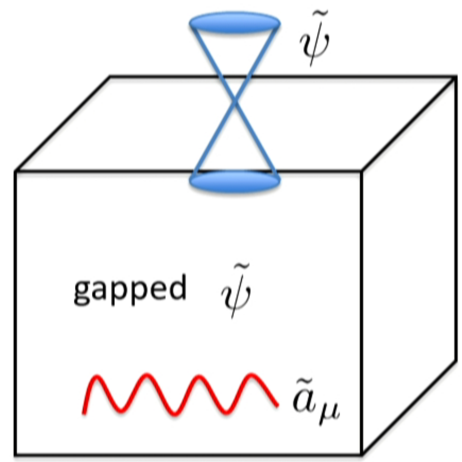
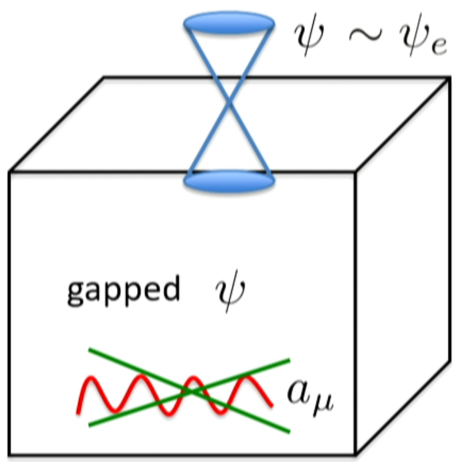
$u(1) \times CT$



Condense $\psi_e^\dagger \times$ monopole^2

Dual picture

$u(1) \rtimes T$



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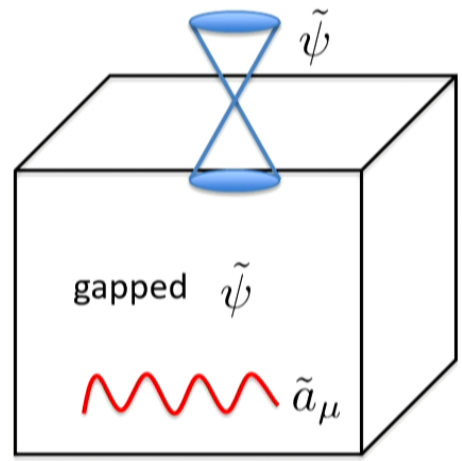

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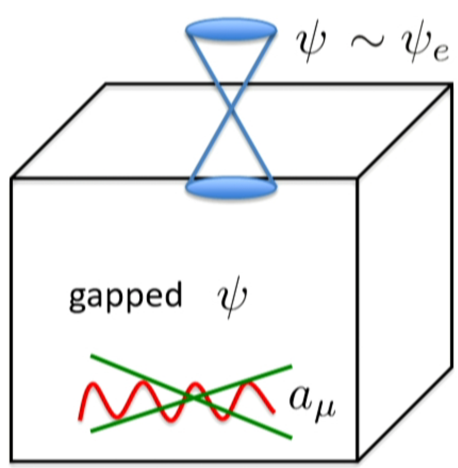

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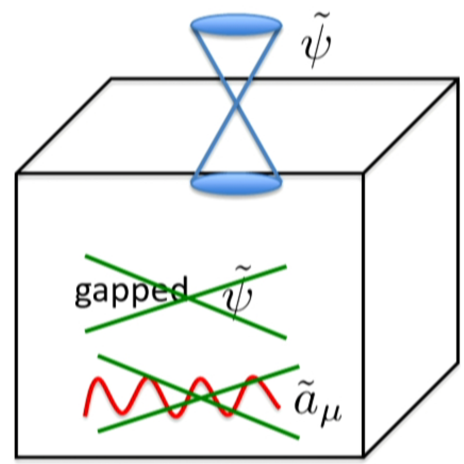

Condense $\psi_e^\dagger \times$ monopole^2

Bulk confinement

Surface: QED_3

Surface "duality"

- Particle-vortex duality of single Dirac cone on TI surface

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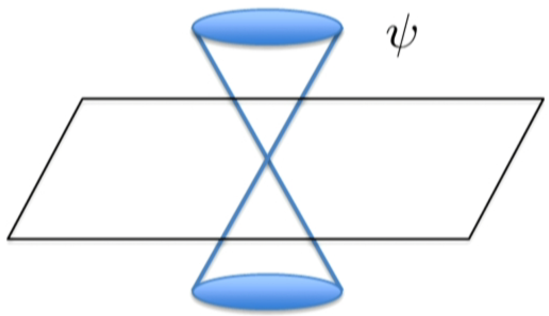
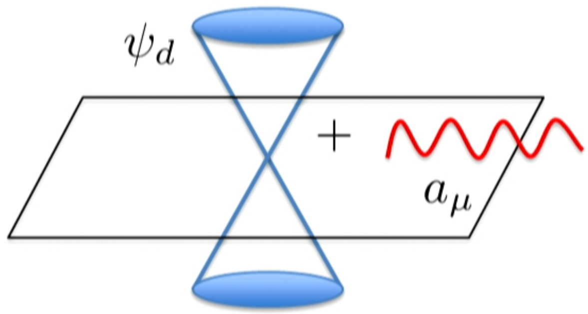


$\updownarrow$

$$L_{QED_3} = \bar{\psi}_d \gamma^\mu (\partial_\mu - i a_\mu) \psi_d + \frac{1}{4g^2} f_{\mu\nu}^2 + \frac{i}{4\pi} A^{ext} \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda$$



T-Pfaffian$_+$

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↓ Charge 2 Higgs: $\langle \psi_d^T C \psi_d \rangle \neq 0$

$U(1)_{ext} \rtimes T$ symmetric gapped topologically ordered surface T-Pfaffian$_+$

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$U(1)_{ext} \rtimes T$ symmetric gapped topologically ordered surface T-Pfaffian$_+$

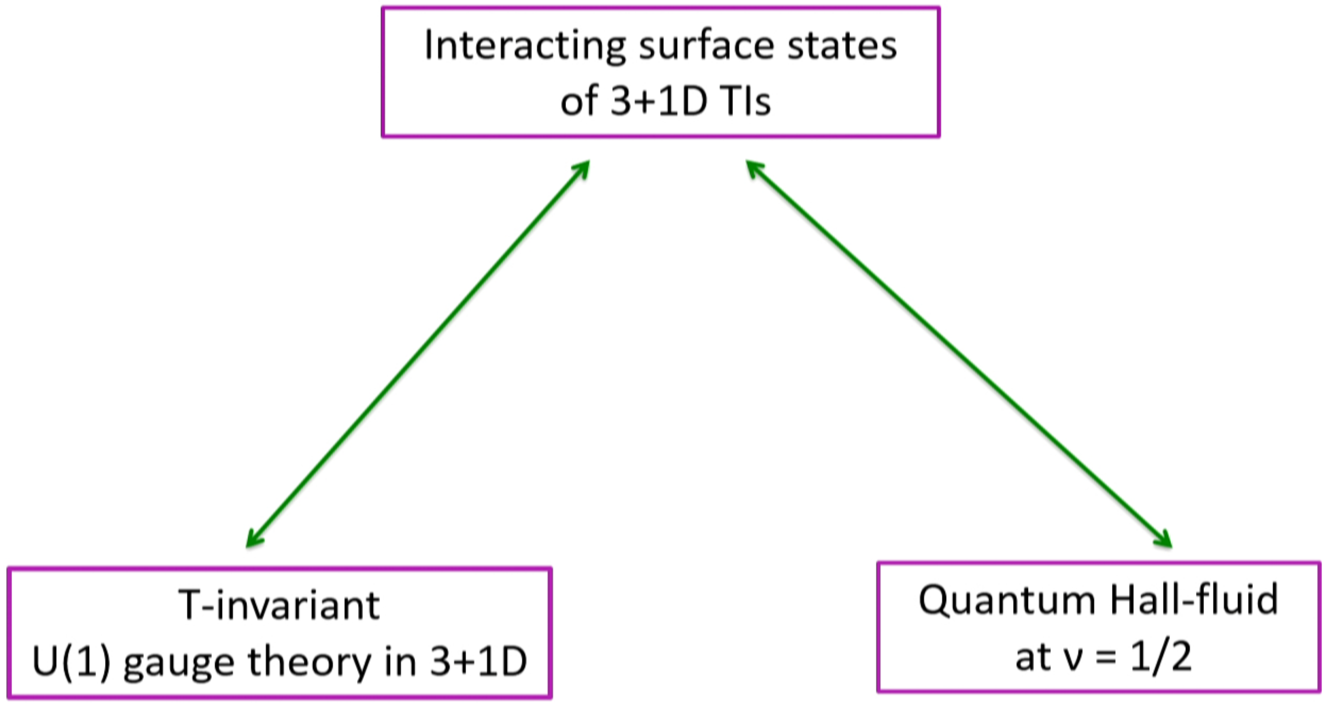
$$\mathrm{T-Pfaffian}_+ \subset \mathrm{Ising} \times U(1)_{-8}$$

$k \rightarrow$	0	1	2	3	4	5	6	7
I	1		$-i$		1		$-i$	
σ		1		-1		-1		1
ψ	-1		i		-1		i	
T^2	1	η		$-\eta$	-1	$-\eta$		η

$Q_{ext} = q/4$

$\eta = \pm 1$

P. Bonderson, C. Nayak, X.L. Qi (2013), X. Chen, L. Fidkowski, A. Vishwanath (2013)


Interacting surface states of 3+1D TIs
T-invariant U(1) gauge theory in 3+1D
Quantum Hall-fluid at ν = 1/2

Thank you!