Title: (3+1)-TQFTs from G-crossed braided fusion categories and their lattice realization

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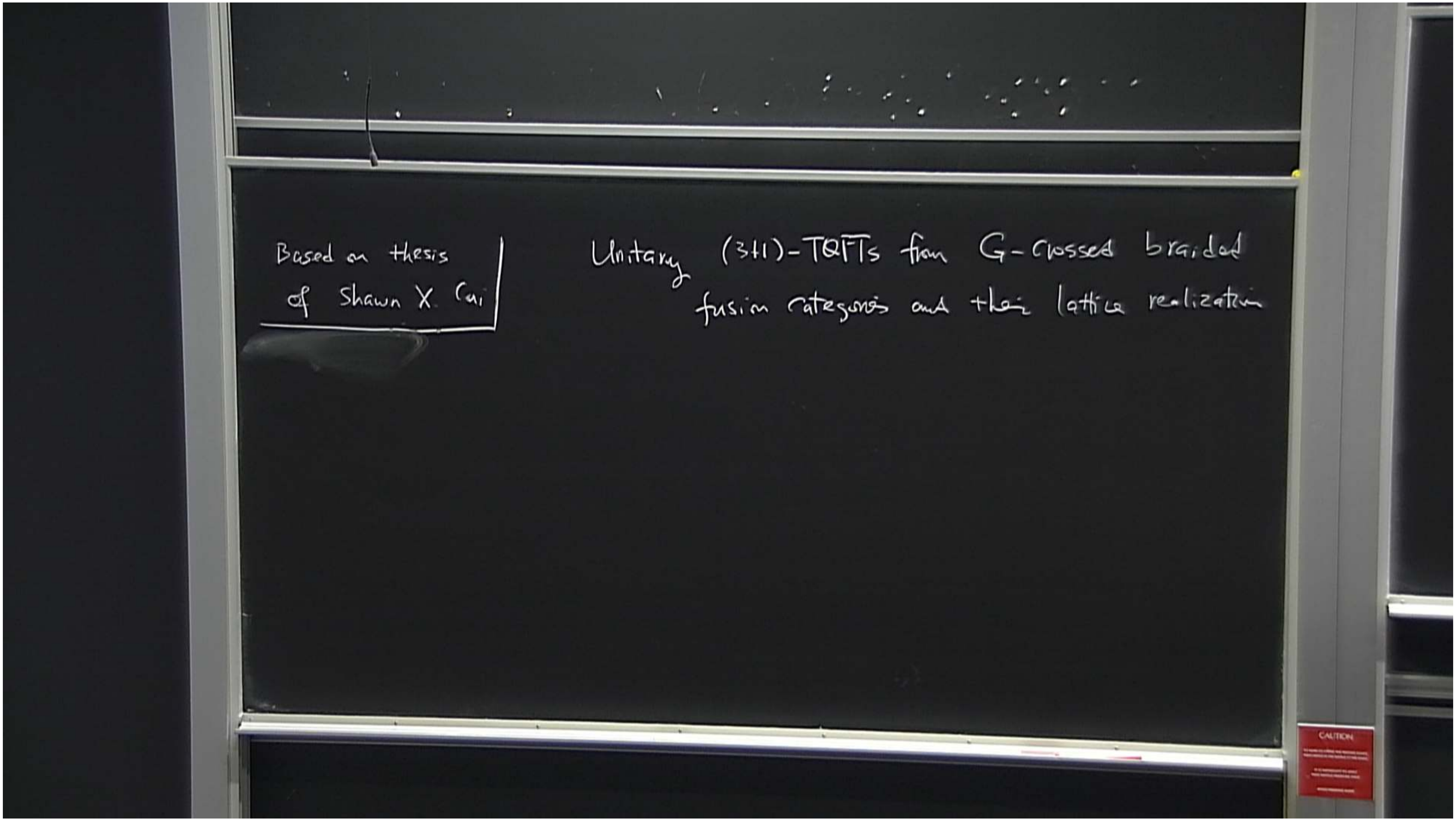
Abstract: Unitary fusion categories are the algebraic input for the Turaev-Viro (TV) type TQFTs in (2+1)-dimensions and their Hamiltonian realization for the Levin-Wen model. We are interested in a generalization to unitary 2-fusion category for (3+1)-dimensions. Mackaay's spherical 2-categories are not general enough to include interesting examples such as the G-crossed braided fusion categories and general homotopy 2-types. We will discussion new (3+1)-TQFTs with G-crossed braided fusion categories as input, and their lattice realization based on the thesis work of Shawn X. Cui.


Based on thesis
of Shawn X. Cui
Unitary (3+1)-TQFTs from G-crossed braided
fusion categories and their lattice realization

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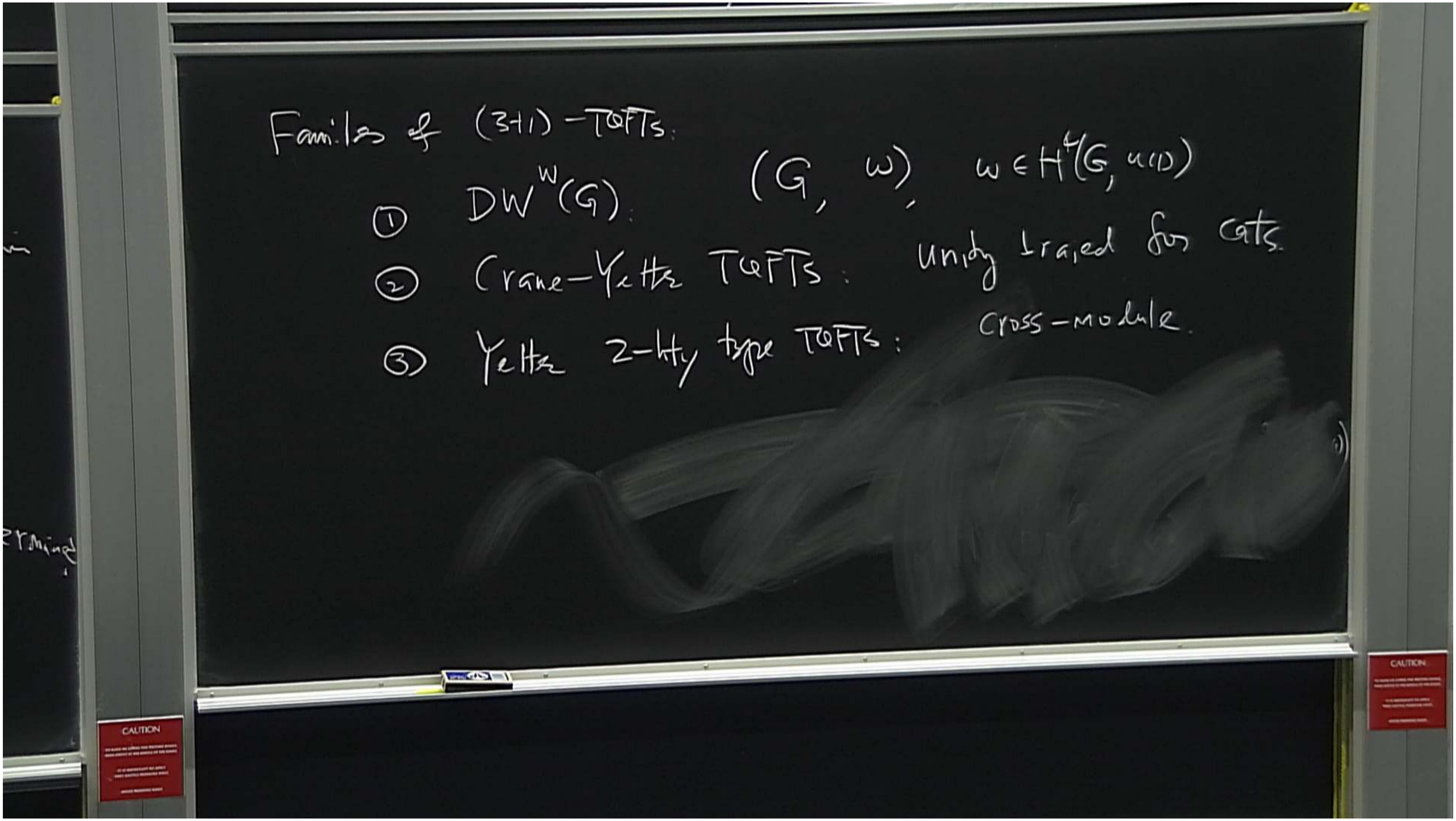
Unitary (3+1)-TQFTs from G-crossed braided fusion categories and their lattice realization

Goal: 3D analogue of MTC / spherical fusion cat

- spherical 2-cat. Mackaay: too limited
- conj: all such TQFTs are hty determined (+ ferm + symm)


Families of (3+1)-TQFTs:
① DW^ω(G): (G, ω), ω ∈ H^4(G, U(1))
② Crane-Yetter TQFTs: unitary braided fus cats.
③ Yetter 2-hty type TQFTs: cross-module.

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③ Yetter 2-type TQFTs: cross-module

Kapustin– Quinn

(π_1, π_2) $\quad \partial : \pi_2(X, A) \longrightarrow \pi_1(A)$

Thm: Given $\mathcal{B}$ = unitary braided fus cat

G = a finite gp

Let $\mathcal{B}_G^\times = \bigoplus_{g \in G} \mathcal{B}_g$ be a unitary G-crossed braided

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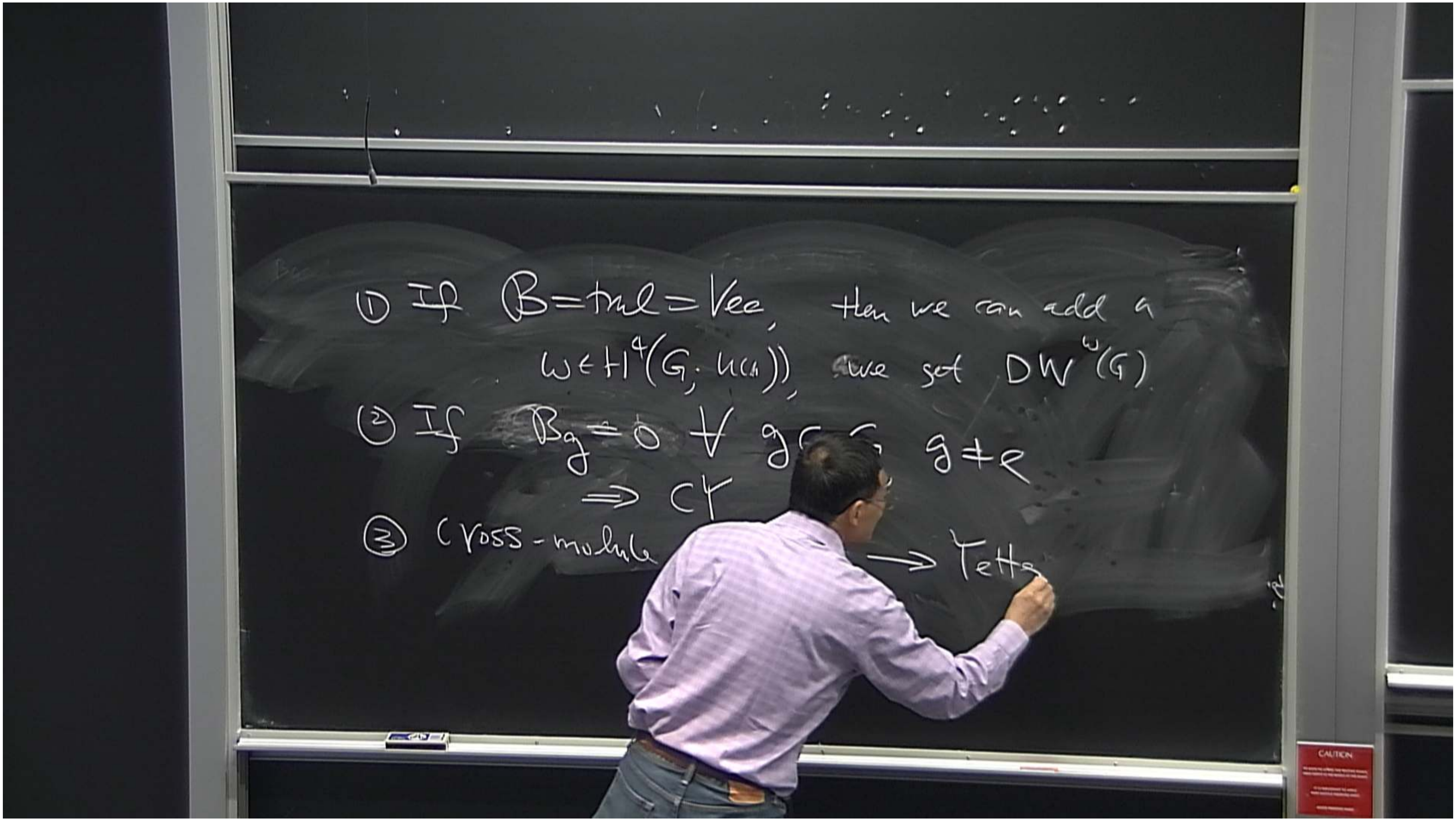
G = a finite grp

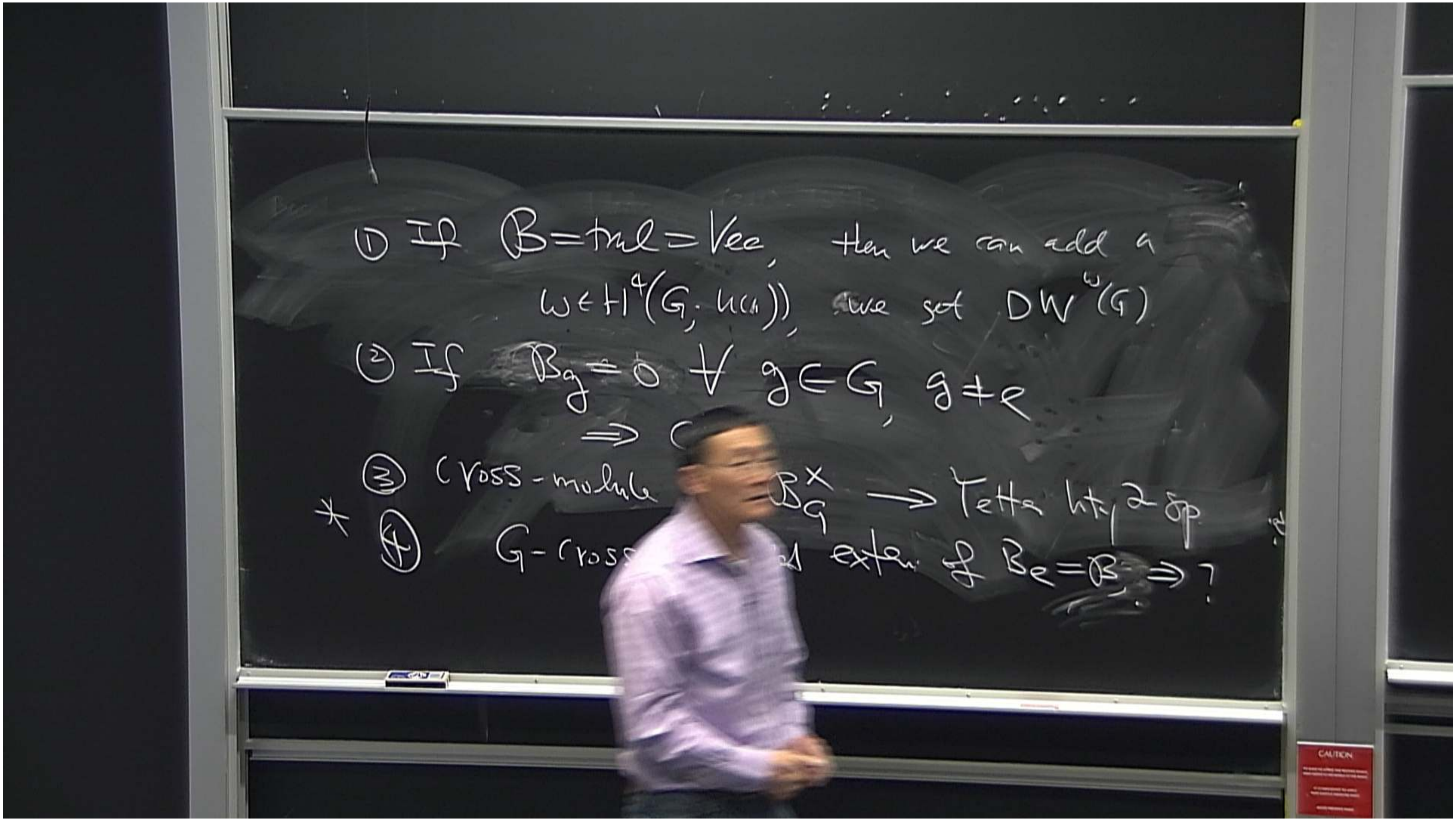
Let $\mathcal{B}_G^\times = \bigoplus_{g \in G} \mathcal{B}_g$ be a $\underline{\underline{\text{unitary}}}$ G-crossed braided fus cat with a categorical G-action

Then $\exists$ an associated (3+1)-TQFTs. Moreover

① If $\mathcal{B} = \mathrm{tml} = \mathrm{Vec}$, then we can add a $\omega \in H^4(G, U(1))$, we get $DW^\omega(G)$

② If G


① If B = tml = Vec, then we can add a
ω ∈ H⁴(G, U(1)), we get DW^ω(G)
② If B_g = 0 ∀ g ∈ G, g ≠ e
⇒ CY
③ Cross-module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⇒
③ Cross-module
* ④ G-cross
exten. of B_e = B ⇒ ?

① If $\mathcal{B} = \text{triv} = \text{Vec}$, then we can add a $\omega \in H^4(G, U(1))$, we get $DW^{\omega}(G)$

② If $\mathcal{B}_g = 0$ $\forall g \in G$, $g \neq e$
$\Rightarrow$ CY

③ Cross-module $\longrightarrow \mathcal{B}_G^{\times} \longrightarrow$ Tetra hty 2-gp

* ④ G-crossed braided exten. of $\mathcal{B}_e = \mathcal{B} \Rightarrow$?

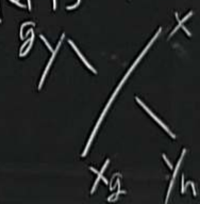
$(G, u(1))$
ied fus cats.
-module
Quin
$\to \pi_1(A)$

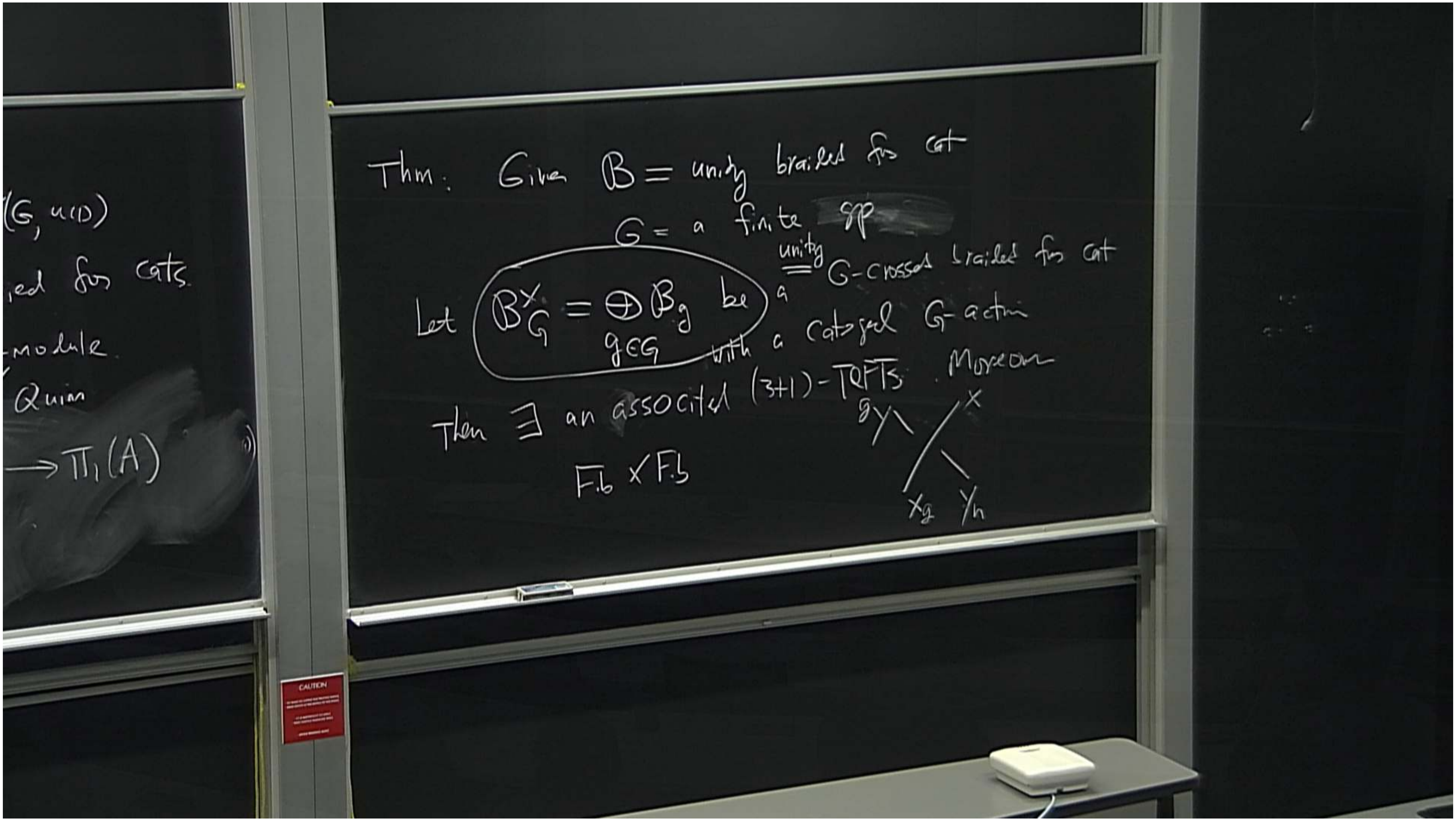
Thm: Given $\mathcal{B}$ = unitary braided fus cat
G = a finite gp

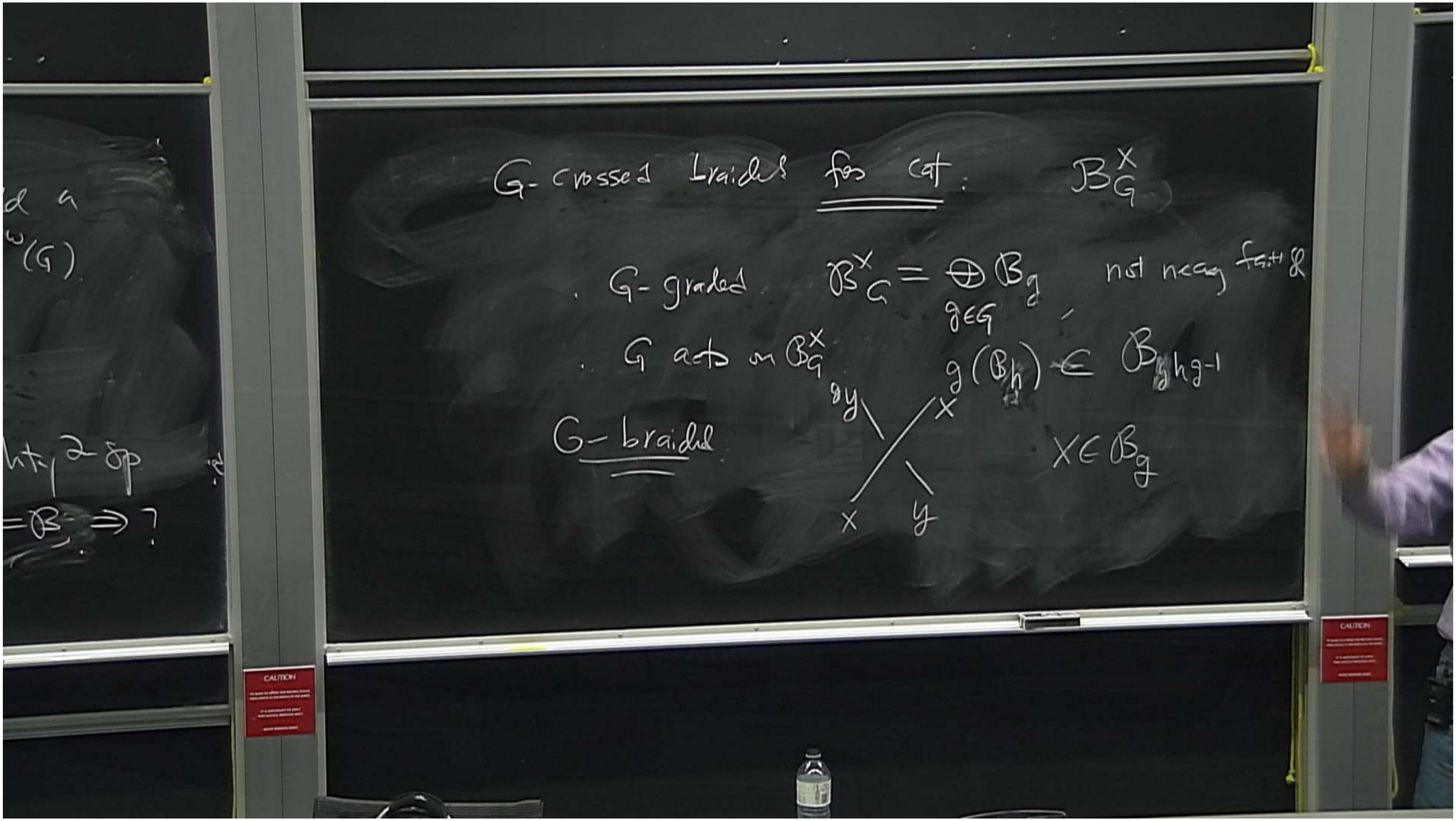
Let $\mathcal{B}_G^\times = \bigoplus_{g \in G} \mathcal{B}_g$ be a $\underline{\underline{\text{unitary}}}$ G-crossed braided fus cat with a categorical G-action

Then $\exists$ an associated (3+1)-TQFTs. Moreover

F.b $\times$ F.b




Thm: Given B = unitary braided fus cat
G = a finite gp
Let B^×_G = ⊕_{g∈G} B_g be a unitary G-crossed braided fus cat
with a categorical G-action
Then ∃ an associated (3+1)-TQFTs. Moreover
Fib × Fib
(G, u(1))
ied fus cats
-module
Quin
→ π_1(A)


G-crossed braided fus cat: $\mathcal{B}_G^\times$
G-graded $\mathcal{B}_G^\times = \bigoplus_{g\in G} \mathcal{B}_g$ not nec faithful
G acts on $\mathcal{B}_G^\times$ $g(\mathcal{B}_h) \in \mathcal{B}_{ghg^{-1}}$
G-braided
${}^g y$ x
x y
$x \in \mathcal{B}_g$

Fib × Fib

$X_g \quad Y_h$

Ex: Tambara-Yamagami: G abelian finite

$$TY(G) = \{ g \in G \} \cup \{ X \} \qquad G = \mathbb{Z}_3$$

$$gX = X$$

$$X^2 = \bigoplus_{g \in G} g$$

$\mathrm{Fib} \times \mathrm{Fib}$

$X_g \quad Y_h$

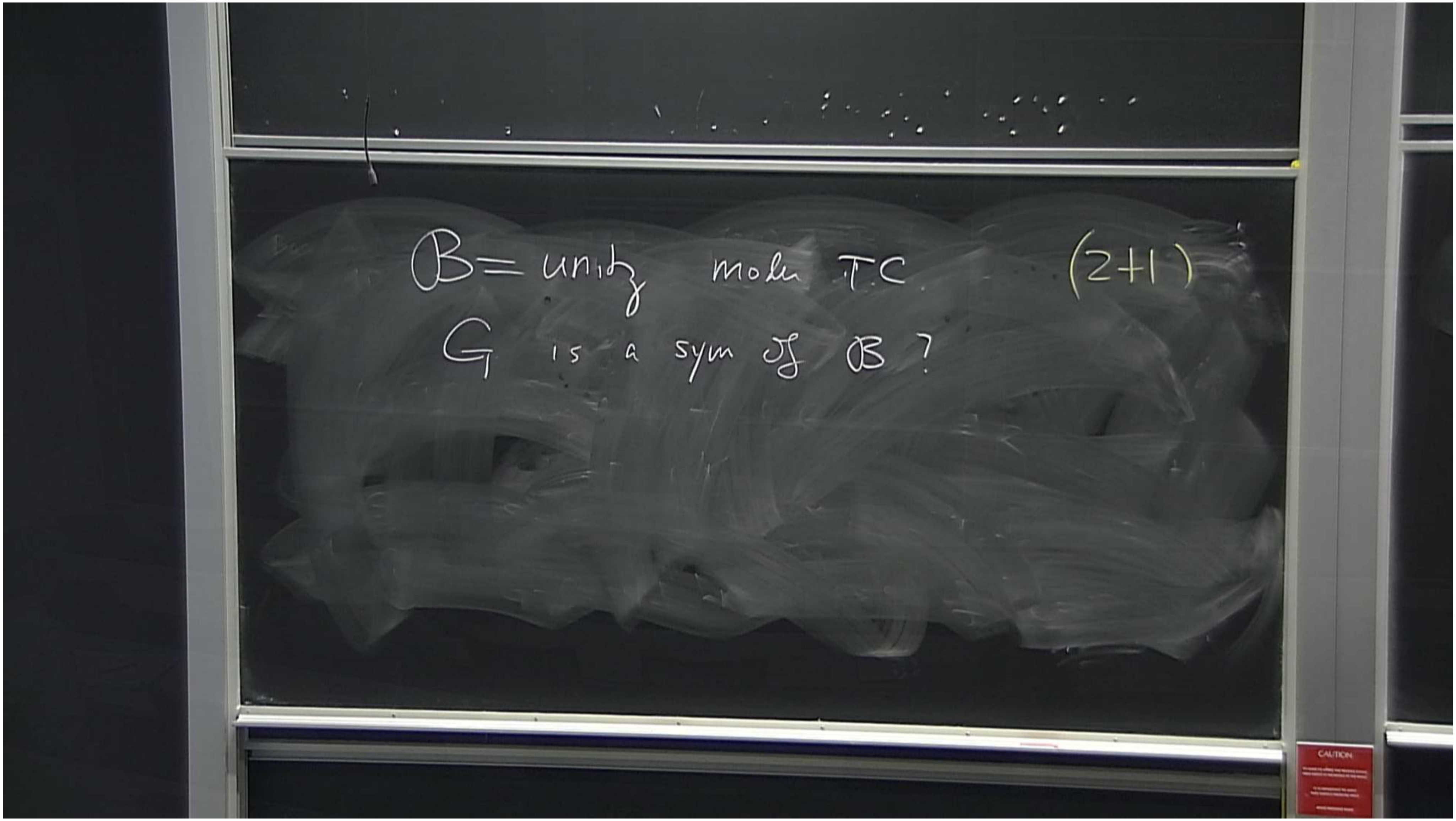
Ex: Tambara-Yamagami : G abelian finite

$$TY(G) = \{ g \in G \} \cup \{ X \} \qquad G = \mathbb{Z}_3 \times \{1, \epsilon\}$$

$0, 1, 2$

$$gX = X$$

$$X^2 = \bigoplus_{g \in G} g$$



B = unitary modular T.C
(2+1)
G is a sym of B ?

$\mathcal{B}$ = unitary modular TC (2+1)

G is a sym of $\mathcal{B}$?

Def:

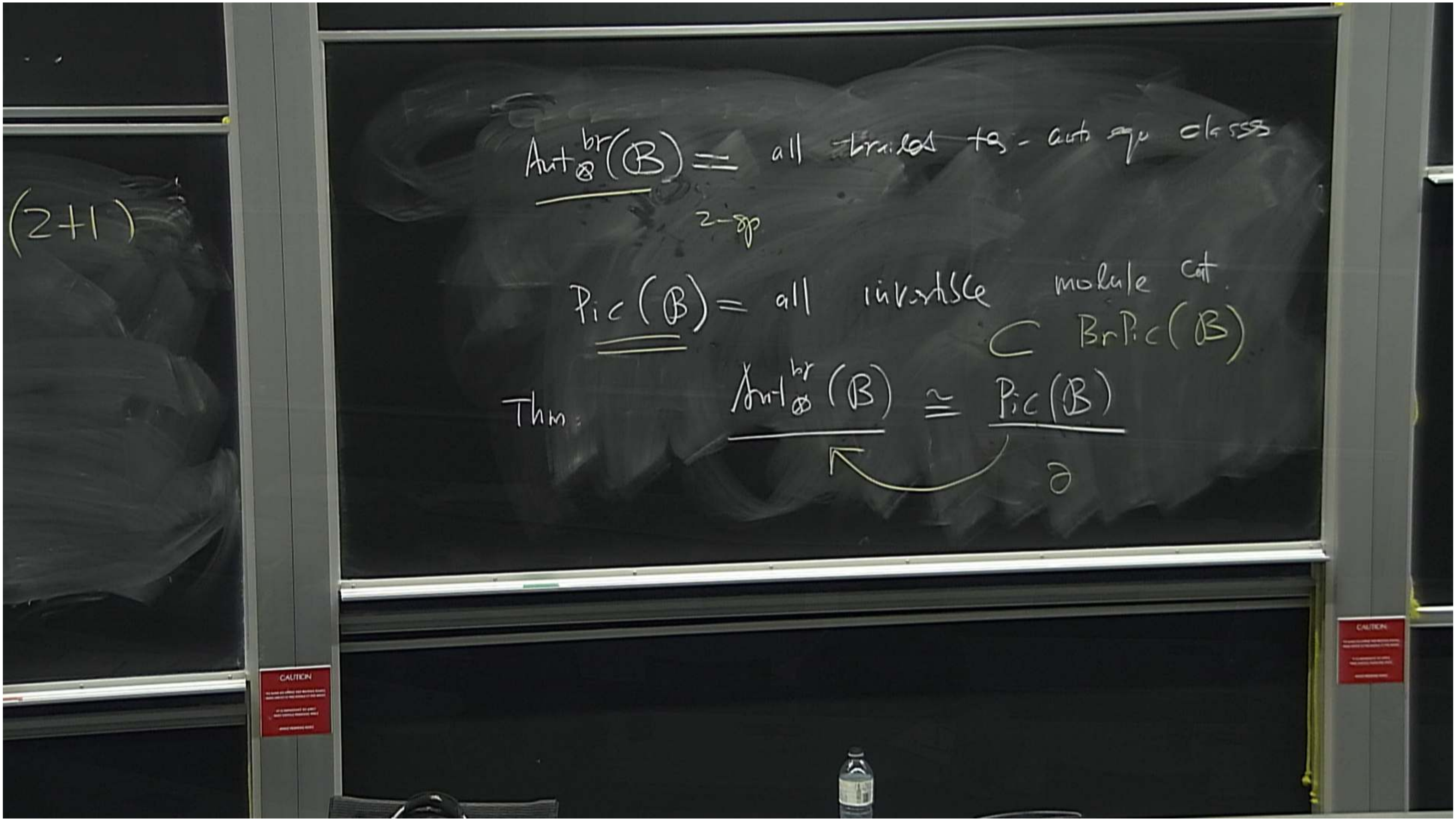
(2+1)

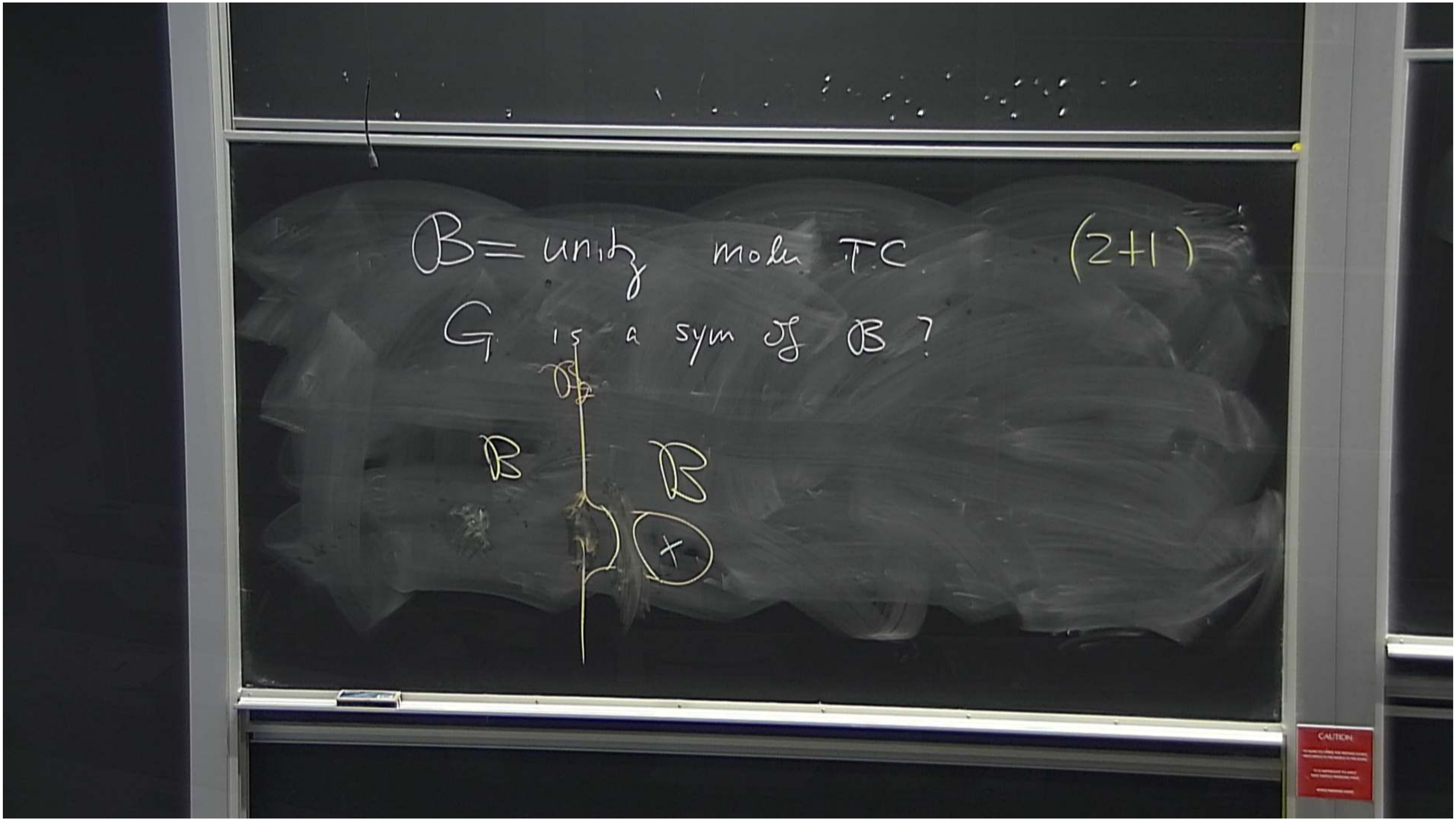
$\mathrm{Aut}^{br}_{\otimes}(\mathcal{B})$ = all braided tensor-auto equiv classes

2-gp

$\mathrm{Pic}(\mathcal{B})$ = all invertible module cat

$\subset \mathrm{BrPic}(\mathcal{B})$


(2+1)
$\mathrm{Aut}^{br}_{\otimes}(\mathcal{B})$ = all braided ⊗-auto equiv classes
2-gp
$\mathrm{Pic}(\mathcal{B})$ = all invertible module cat
$\subset \mathrm{BrPic}(\mathcal{B})$
Thm. $\mathrm{Aut}^{br}_{\otimes}(\mathcal{B}) \cong \mathrm{Pic}(\mathcal{B})$
∂


B = unitary modular TC
(2+1)
G is a sym of B ?
B
B
B

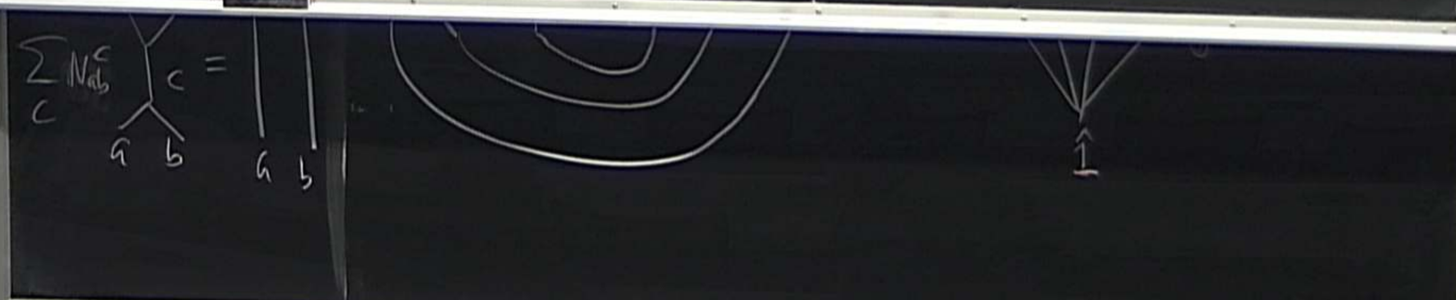
$\mathcal{B}$ = unitary modular T.C (2+1)

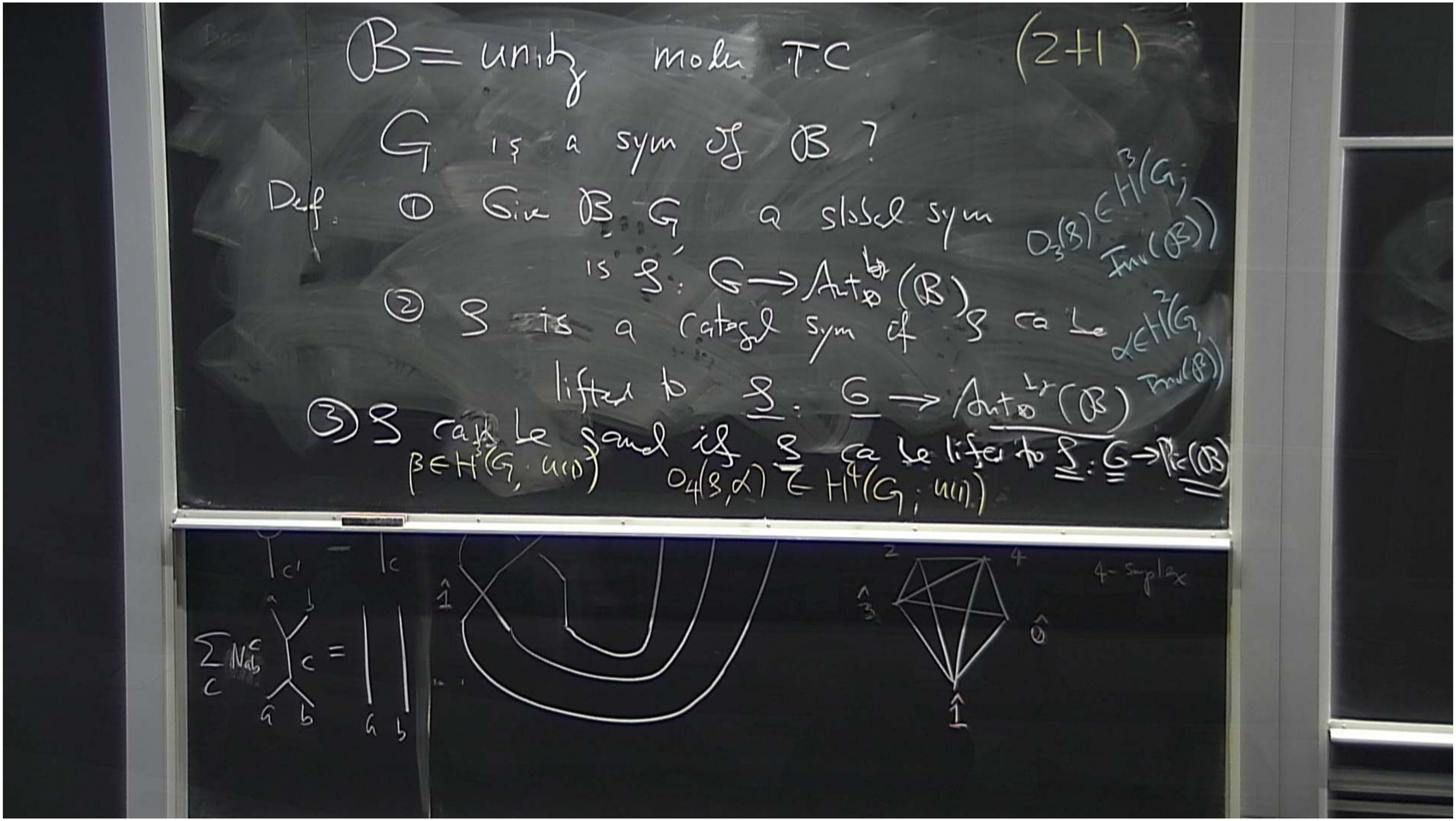
G is a sym of $\mathcal{B}$?

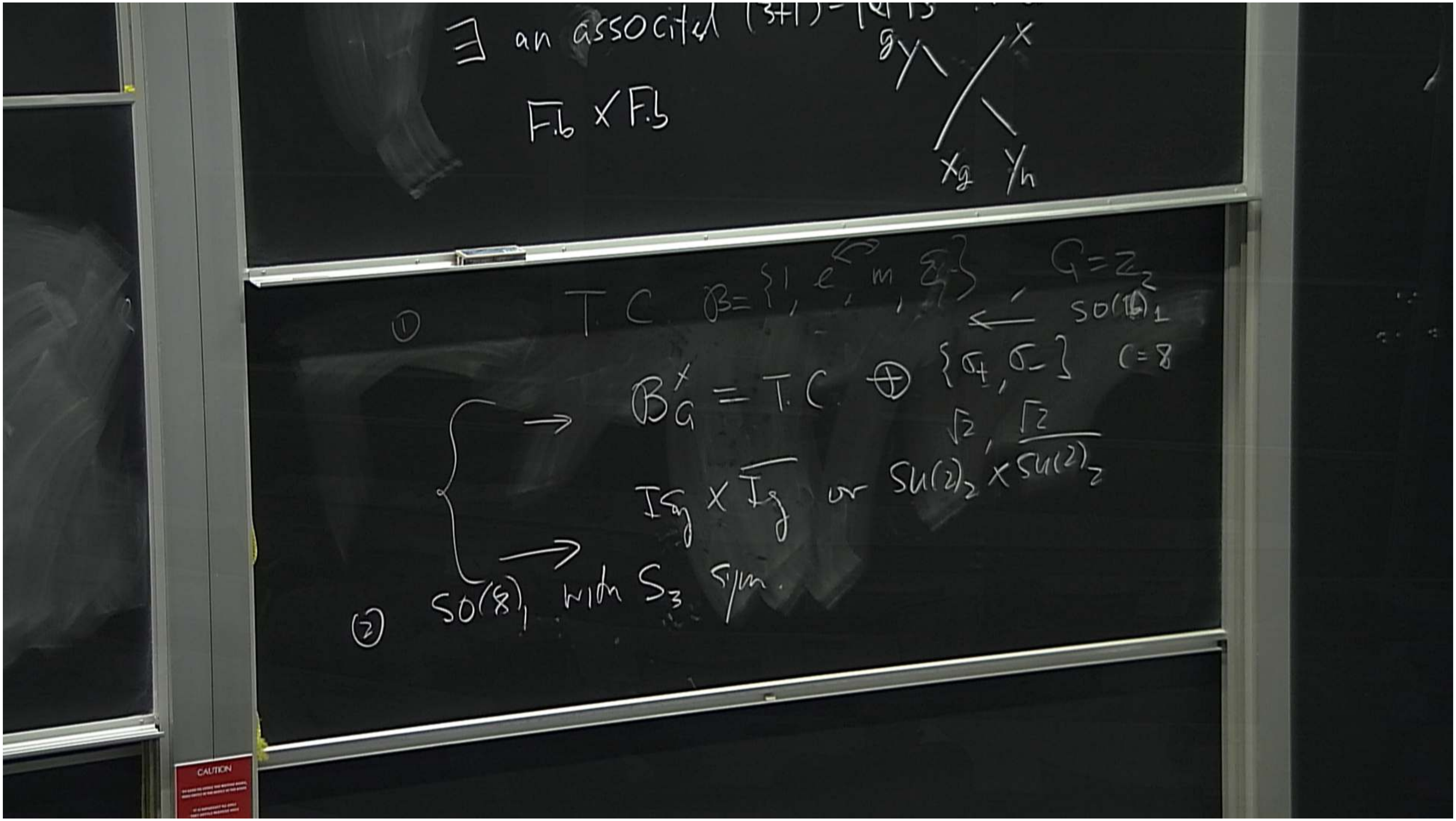
Def. ① Given $\mathcal{B}$, G a global sym is $\rho : G \to \mathrm{Aut}^{br}_{\otimes}(\mathcal{B})$

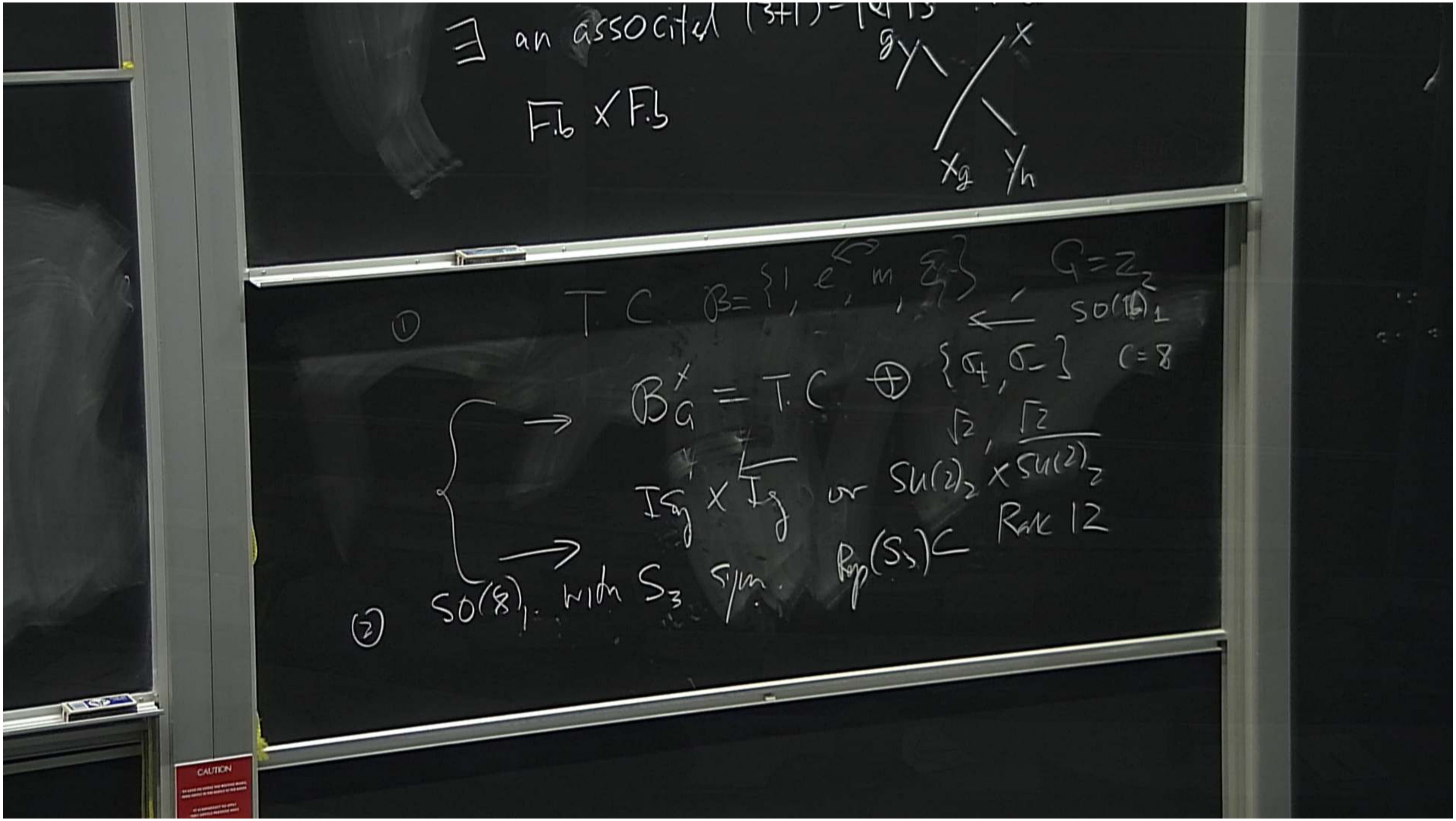
② ρ is a categorical sym if ρ can be lifted to $\underline{\rho} : \underline{G} \to \underline{\mathrm{Aut}^{br}_{\otimes}(\mathcal{B})}$

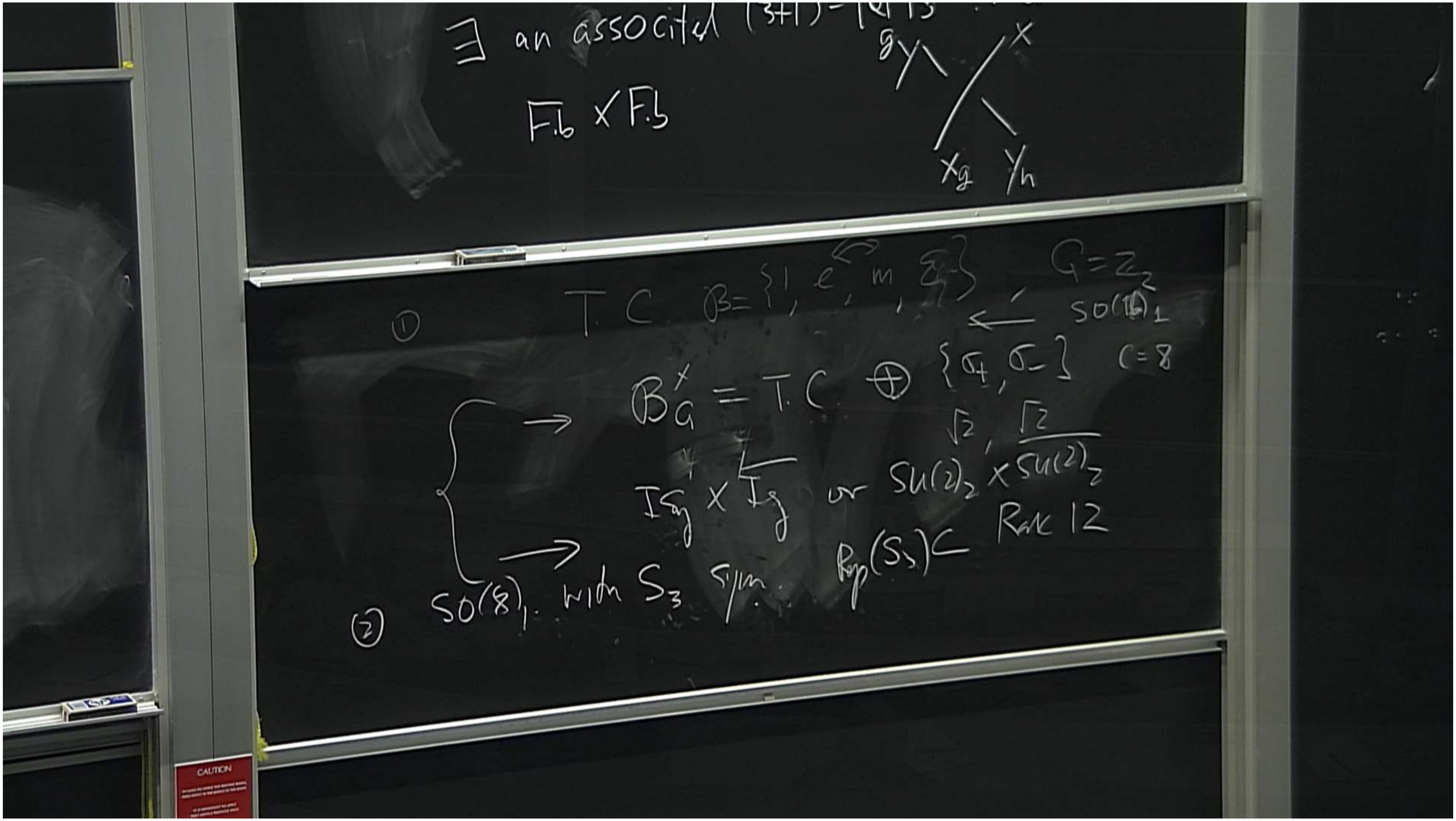
③ ρ can be gauged if $\underline{\rho}$ can be lifted to $\underline{\underline{\rho}} : \underline{\underline{G}} \to \underline{\underline{\mathrm{Pic}(\mathcal{B})}}$

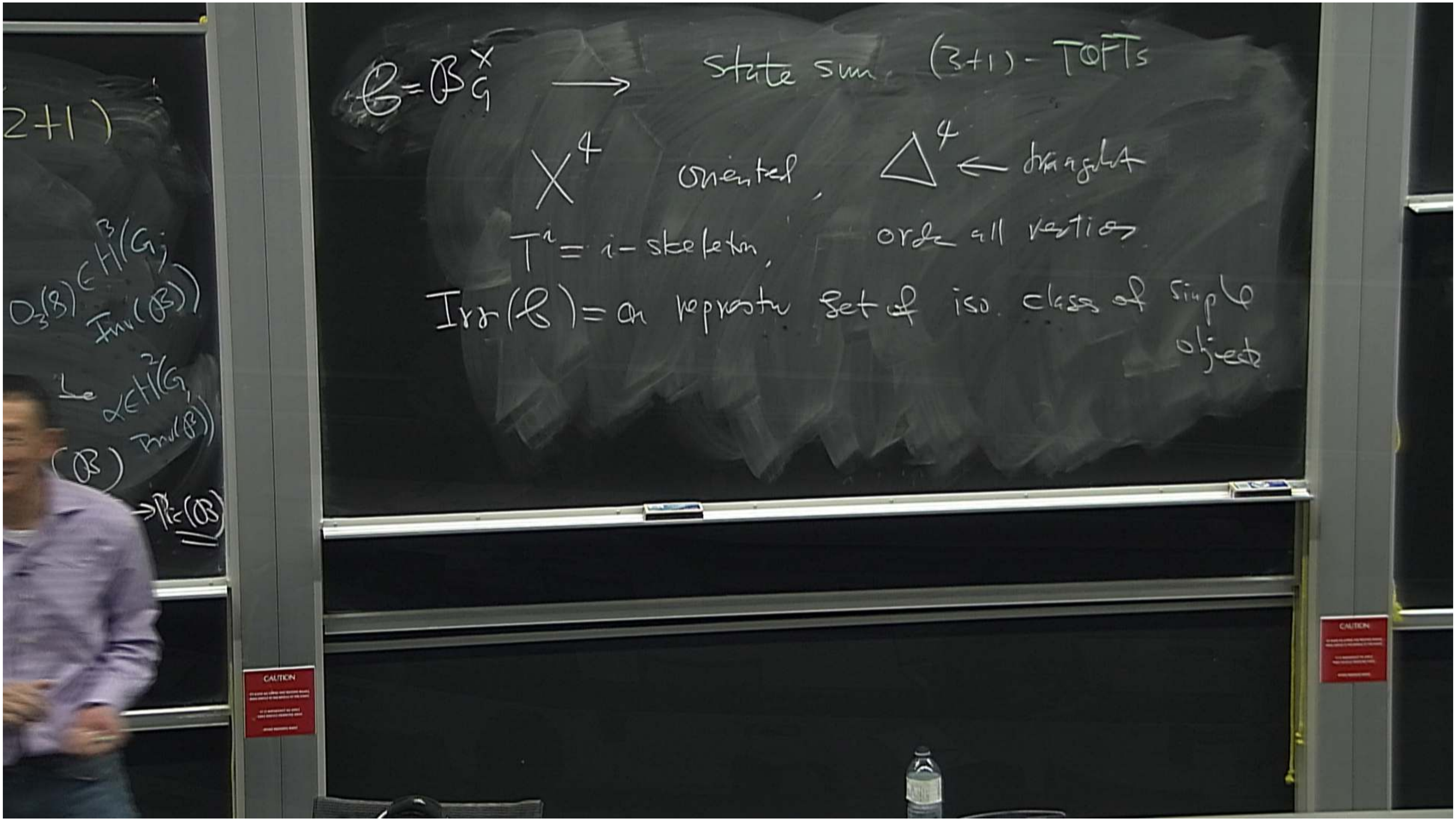



B = unitary modular T.C
(2+1)
G is a sym of B ?
Def. ① Given B, G a global sym
is ρ: G → Aut⊗br(B)
$O_3(\rho) \in H^3(G; \mathrm{Inv}(B))$
② ρ is a categorical sym if ρ can be
lifted to ρ: G → Aut⊗br(B)
$\alpha \in H^2(G, \mathrm{Inv}(B))$
③ ρ can be gauged if ρ can be lifted to ρ: G → Pic(B)
$\beta \in H^3(G, U(1))$
$O_4(\rho, \alpha) \in H^4(G; U(1))$
4-simplex


∃ an associted (3+1)-
F.b × F.b
g
x
x_g
y_h
① T.C B = {1, e, m, ε} , G = Z_2
← SO(16)_1
B^×_G = T.C ⊕ {σ+, σ−}
c = 8
√2, √2
Isg × Isg or SU(2)_2 × SU(2)_2
② SO(8)_1 with S_3 sym.


∃ an associted
F.b × F.b
g
x
x_g
y_h
① T.C $\mathcal{B} = \{1, e, m, \varepsilon\}$
$G = \mathbb{Z}_2$
$SO(16)_1$
$\{\sigma_+, \sigma_-\}$
$c = 8$
$\mathcal{B}^{\times}_{G} = T.C \oplus \{\sigma_+, \sigma_-\}$
$\sqrt{2}, \sqrt{2}$
$Isg \times Isg$ or $SU(2)_2 \times \overline{SU(2)_2}$
Rank 12
$Rep(S_3) \subset$
② $SO(8)_1$ with S_3 sym.


∃ an associated (3+1)
F.b × F.b
T.C B = {1, e, m, ε}
G = Z2
SO(16)1
{σ+, σ−}
c = 8
B×G = T.C ⊕ {σ+, σ−}
√2, √2
Isg × Isg or SU(2)2 × SU(2)2
Rank 12
Rep(S3) ⊂
① ②
SO(8)1 with S3 sym.



(2+1)
$\mathcal{B} = \mathcal{B}_G^X \longrightarrow$ state sum (3+1)-TQFTs
X^4 oriented, $\Delta^4 \leftarrow$ triangulat
T^i = i-skeleton, order all vertices
$Irr(\mathcal{B})$ = a representative set of iso. class of simple objects

(2+1)

$O_3(\mathcal{B}) \in H^3(G; \mathrm{Inv}(\mathcal{B}))$

a Le $\alpha \in H^2(G, \mathrm{Inv}(\mathcal{B}))$

$\otimes^{br}(\mathcal{B})$

to $\underline{\underline{f}} : \underline{\underline{G}} \to \mathrm{Pic}(\mathcal{B})$

$\mathcal{B} = \mathcal{B}_G^X \longrightarrow$ state sum (3+1)-TQFTs

X^4 oriented, $\Delta^4 \leftarrow$ triangulat

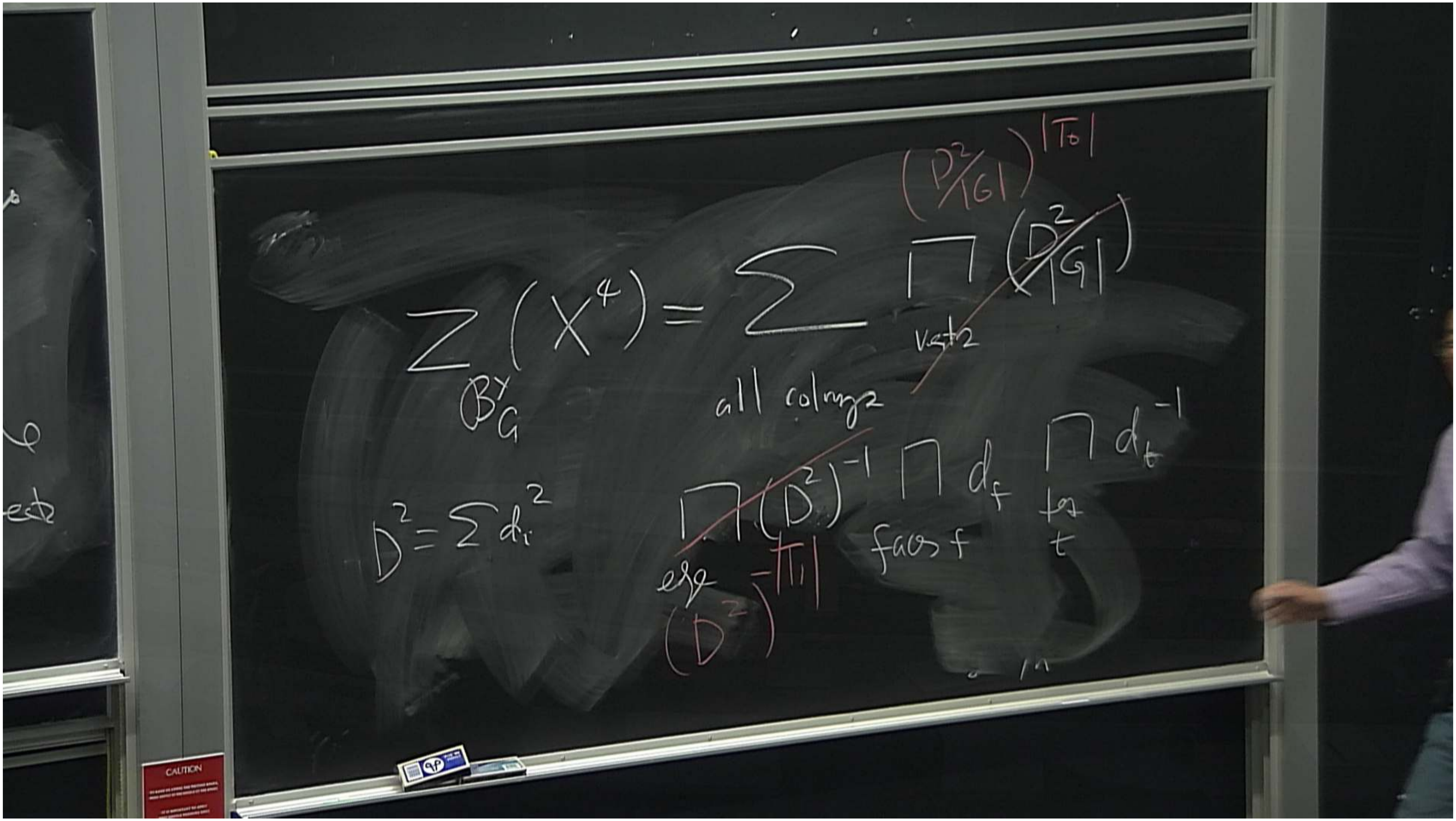
T^i = i-skeleton, order all vertices

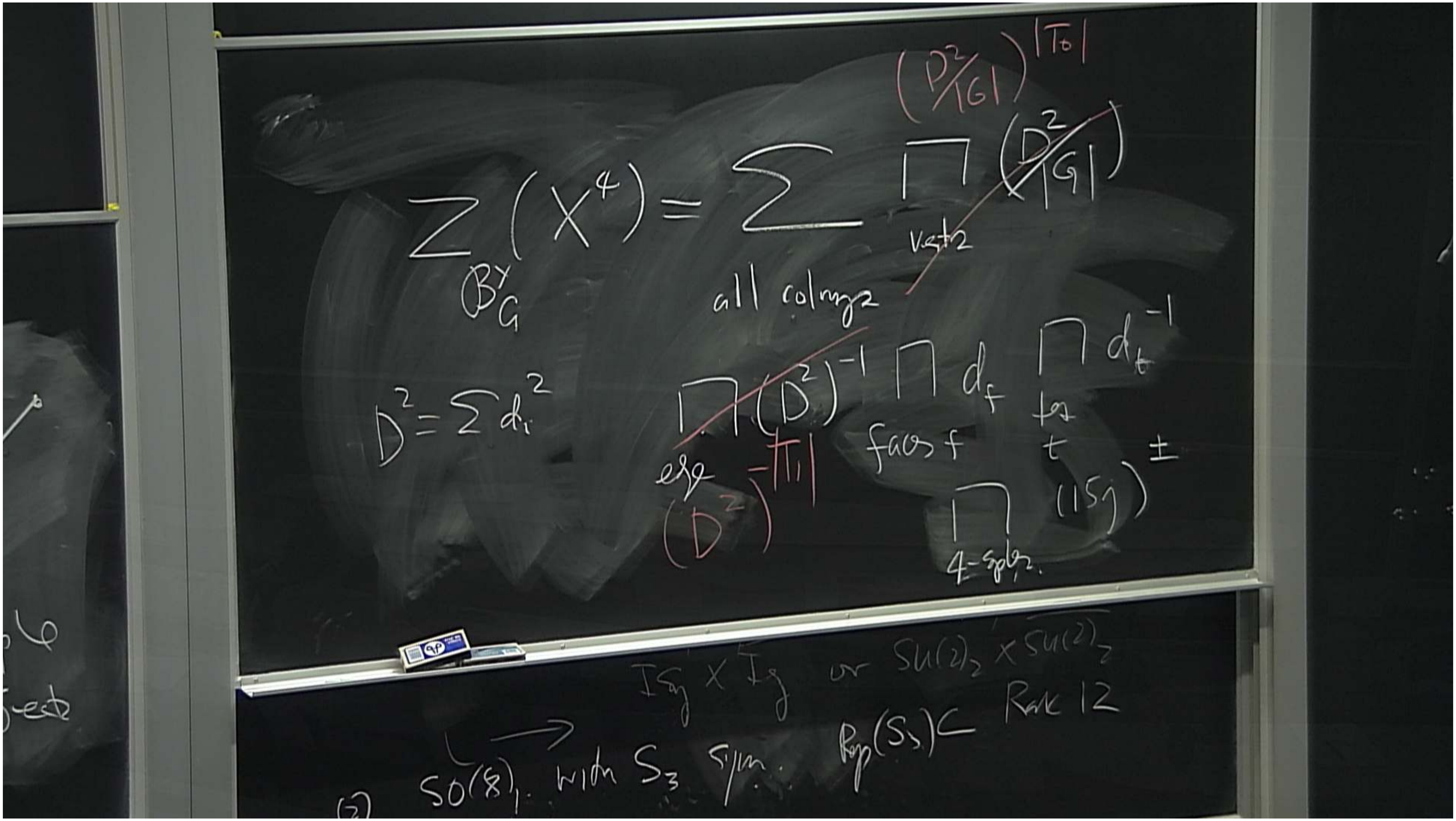
$\mathrm{Irr}(\mathcal{B})$ = a representative set of iso classes of simple objects

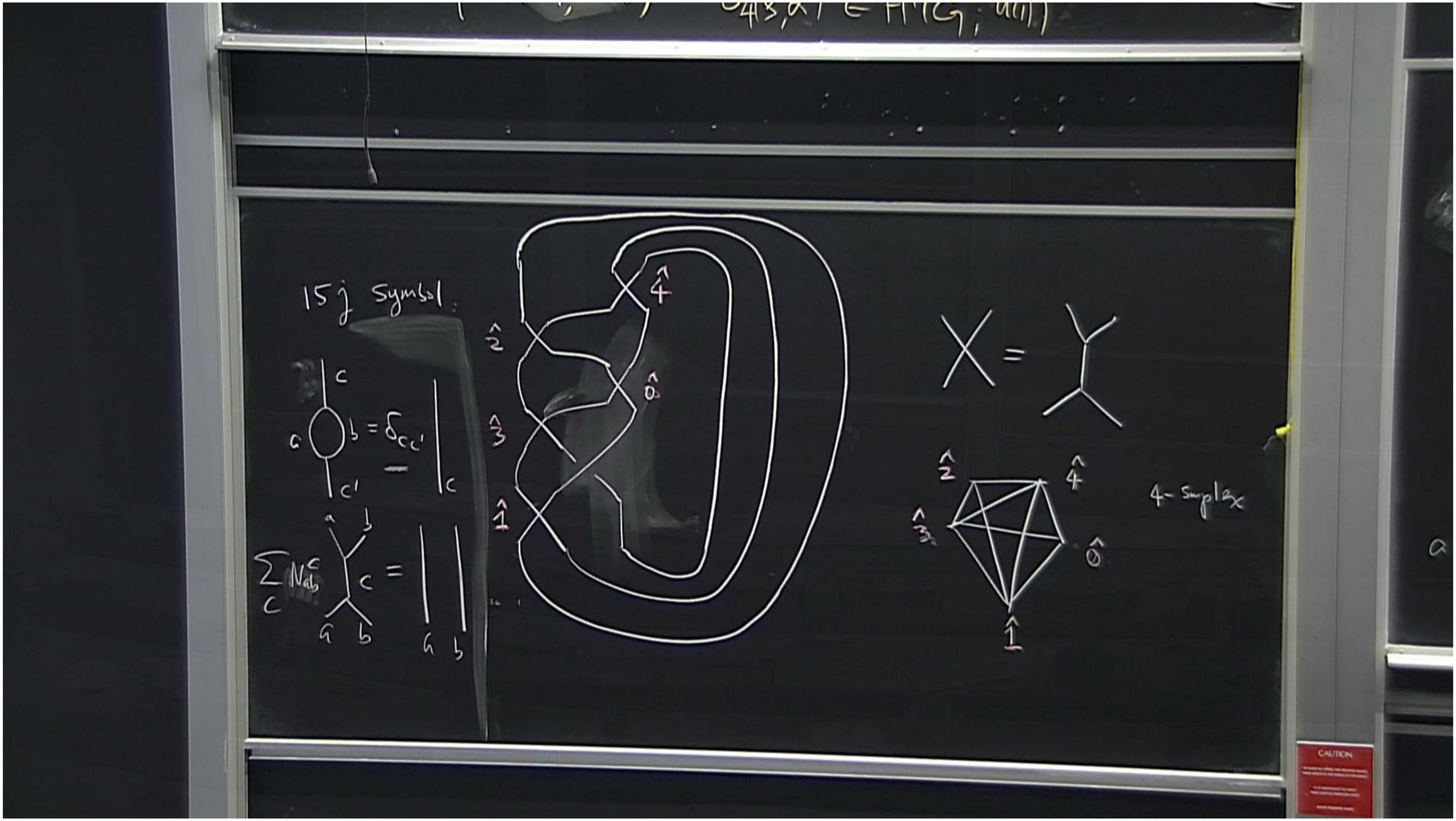
a coloring: label all edges by $g \in G$

all triangles by objects in $\mathrm{Irr}(\mathcal{B})$

all tetrahedra by ~~object in $\mathrm{Irr}(\mathcal{B})$~~.


$Z_{B^Y_G}(X^4) = \sum_{\text{all colorings}} \prod_{v \in t_2} \cancel{(D^2/|G|)}$
$(D^2/|G|)^{|T_0|}$
$\cancel{\prod_{\text{edge}} (D^2)^{-1}} \prod_{\text{faces } f} d_f \prod_{\text{tet } t} d_t^{-1}$
$(D^2)^{-|T_1|}$
$D^2 = \sum d_i^2$


$\left(\frac{D^2}{|G|}\right)^{|T_0|}$
$Z_{\mathcal{B}^Y_G}(X^4) = \sum_{\text{all colorings}} \prod_{\text{verts}} \left(\frac{D^2}{|G|}\right)$
$D^2 = \sum d_i^2$
$\prod_{\text{edges}} (D^2)^{-1} \prod_{\text{faces } f} d_f \prod_{\text{tets } t} d_t^{-1}$
$(D^2)^{-|T_1|}$
$\prod_{\text{4-splxs}} (15j)^{\pm}$
$I_{Sing} \times I_{Sing}$ or $SU(2)_2 \times SU(2)_2$
$Rep(S_3) \subset$ Rank 12
(2) $SO(8)_1$ with S_3 sym.


15j symbol:
= δcc'
ΣNabc
4-simplex