Title: Time reversal invariant gapped boundaries of the double semion state

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Abstract:

Time reversal invariant gapped boundaries of the double semion state

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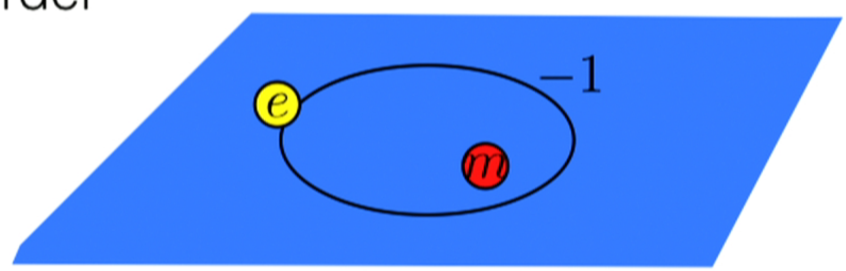
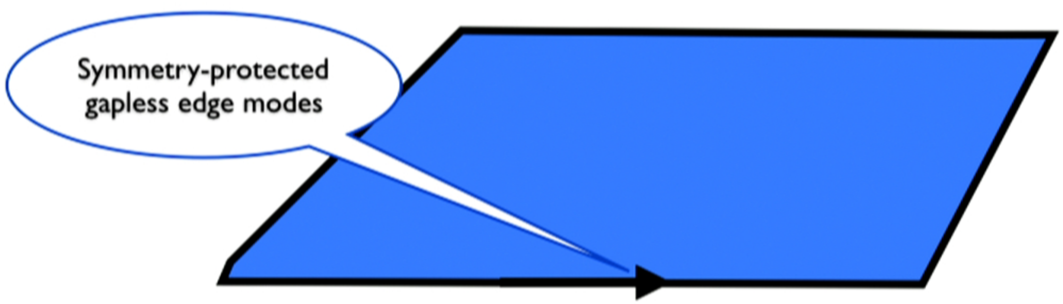
Overview:

Two kinds of “fractionalization”:

•Topological Order

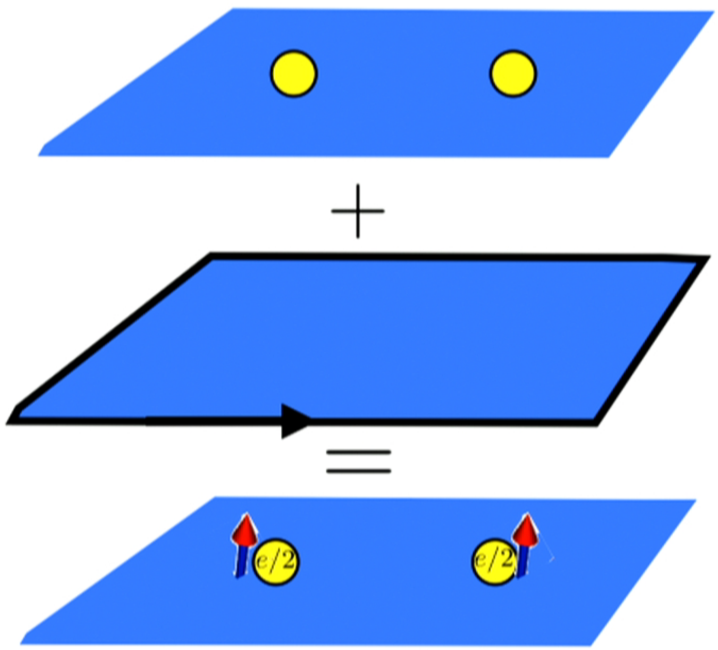


•Symmetry Protection (SPT)




Symmetry
protection
Anyons in 2D

How to combine these?
Symmetry Enriched Topological phases



Anyons in 2D

Symmetry protection

Anyons in 2D with fractional symmetry

- FQHE (fractional charge)
- Spin liquid (fractional spin)

The question

2D SPT with discrete symmetry $\mathbb{Z}_2 \times \mathbb{Z}_2^T$ → Gauge $\mathbb{Z}_2$ → 2D SET

- This is certainly one way to get a 2D SET
- But is the mapping one to one?

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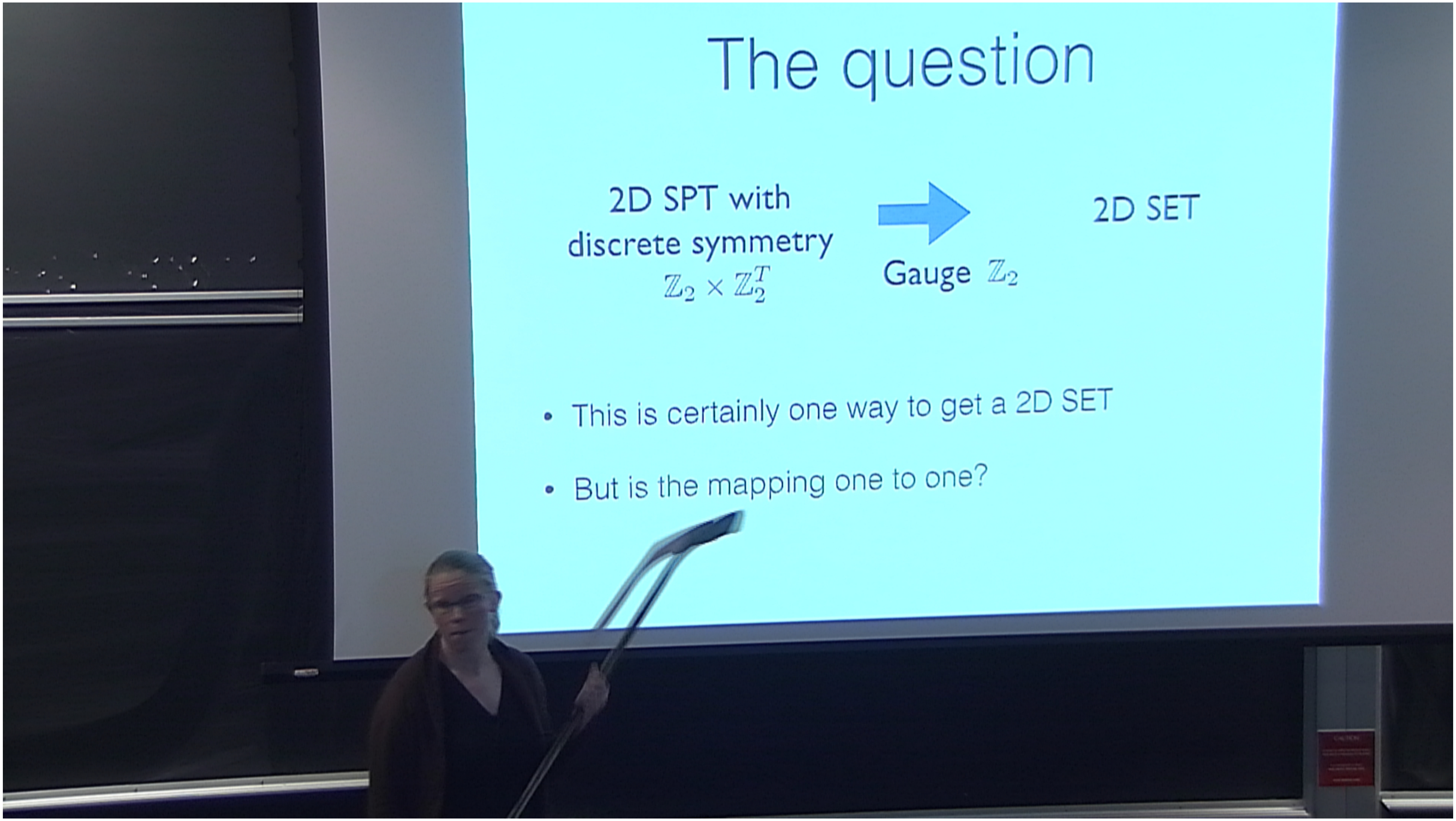
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Gauge $\mathbb{Z}_2$
2D SET
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• But is the mapping one to one?

Phases with $\mathbb{Z}_2 \times \mathbb{Z}_2^T$ symmetry

- Ordinary Ising paramagnet, $T^2(\text{vortex}) = 1$
- Ordinary Ising paramagnet, $T^2(\text{vortex}) = -1$
- Twisted Ising paramagnet $T^2(\text{vortex}) = 1$
  (Levin & Gu, '12)
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Gauging

Ordinary Ising paramagnet,
$T^2(\text{vortex}) = 1$

Gauge $\mathbb{Z}_2$ →

Toric code
$(T^2 = 1)$

Ordinary Ising paramagnet,
$T^2(\text{vortex}) = -1$

Gauge $\mathbb{Z}_2$ →

Toric code
$T^2(\text{vortex}) = -1$

- Two different SET phases!

Gauging

Twisted Ising paramagnet, $T^2(\text{vortex}) = 1$

Gauge $\mathbb{Z}_2$

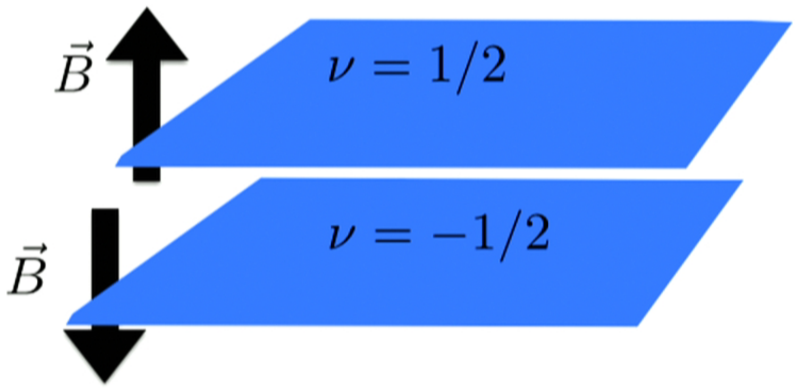
Twisted Ising paramagnet, $T^2(\text{vortex}) = -1$

Gauge $\mathbb{Z}_2$

Same bulk phase!

(with different boundaries…)

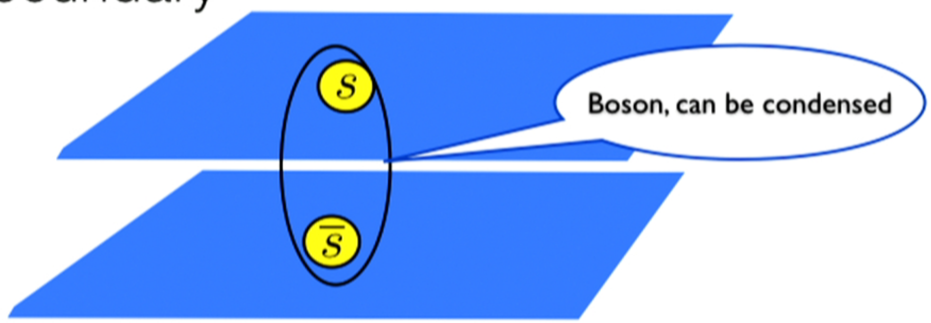
What's the difference?

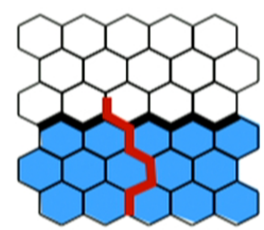


Gauging the twisted Ising paramagnet: doubled semion model

Where can we see $T^2 = \pm 1$?

- Bulk $\mathbf{T}|s\rangle \Rightarrow |\bar{s}\rangle$
- (Gapped) boundary
  (Levin ' 13)





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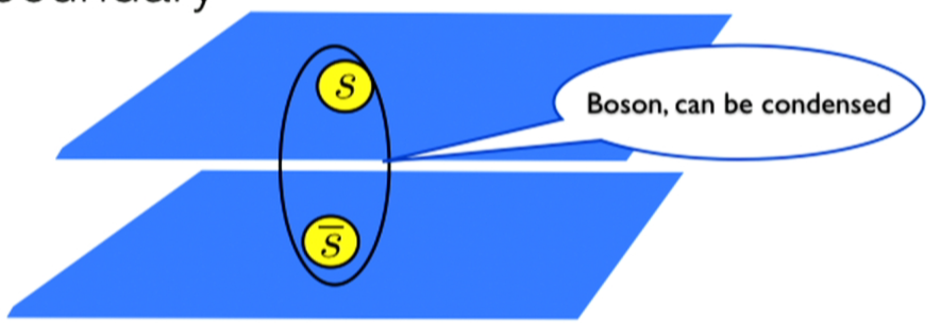
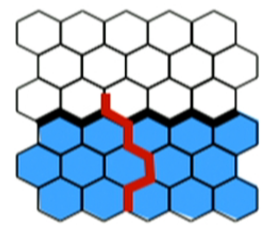


$$\mathbf{T}|s\rangle \Rightarrow |\bar{s}\rangle = |b \times s\rangle$$



- $T^2 = \pm 1$ at the boundary!

Bulk phases vs boundary conditions

- $T^2 = \pm 1$ at the boundary!

- Does this indicate distinct bulk phases
  - For the twisted Ising paramagnet? **Yes**
  - For the doubled semion model? **No**

Phases with $\mathbb{Z}_2 \times \mathbb{Z}_2^T$ symmetry

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Outline of the rest of the talk

- 2 kinds of Bose condensates = 2 kinds of boundaries $(\mathbf{T}^2 = \pm 1)$
- Why gauging matters
- Explicit construction
- Trijunctions and symmetry-protected degeneracies

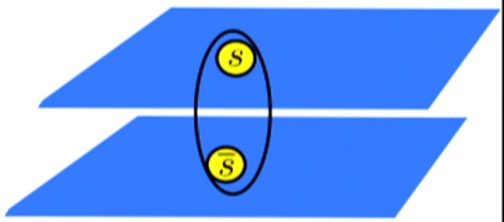
2 kinds of Bose condensates

$$|\Psi_{\text{BEC}}\rangle = \sum_N \left(e^{i\theta}\right)^N \sum_{x_i = \text{position of } i^{th} \text{ boson}} |x_1, \ldots x_{2N}\rangle$$

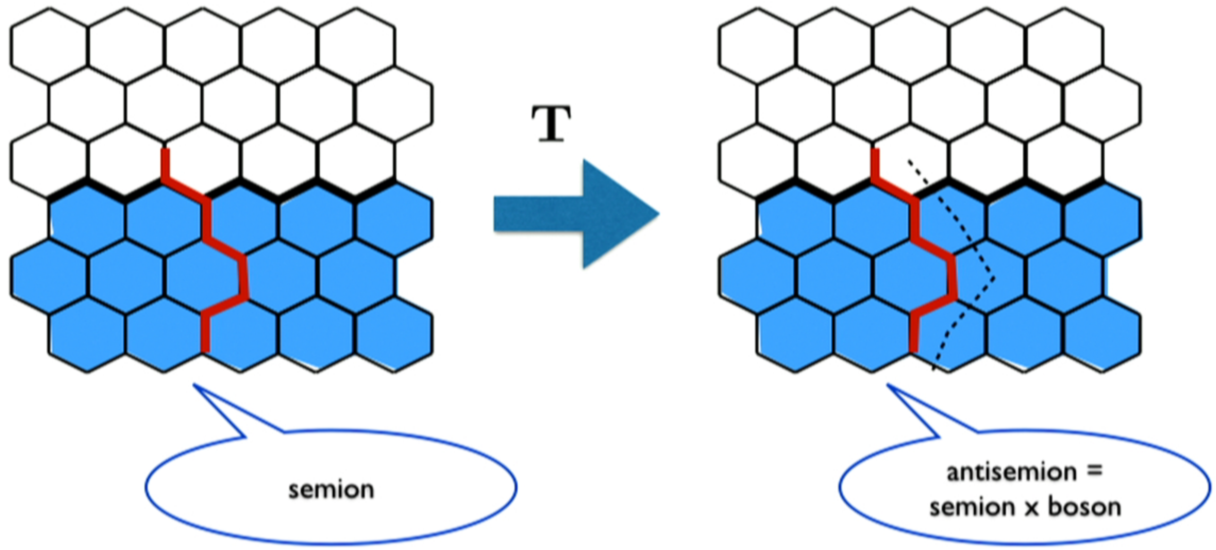
Phase factor for each pair

N boson pairs

- Time reversal: $\theta = 0, \pi$



2 kinds of Bose condensates



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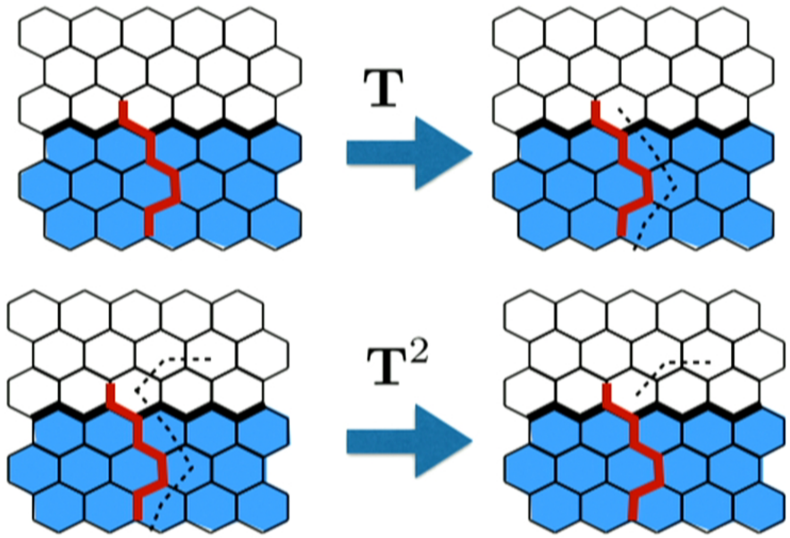

- $\mathbf{T}^2$ creates 2 bosons → $\mathbf{T}^2 = e^{i\theta}$

The importance of gauging

$$\mathcal{L}_{\text{edge}} = \frac{2}{4\pi}\partial_x\phi_1\partial_t\phi_1 - \frac{2}{4\pi}\partial_x\phi_2\partial_t\phi_2 - g\cos\left(2\phi_1 - 2\phi_2 + \theta\right)$$

Gapping Term

$$\mathbb{Z}_2^T : \begin{cases} \phi_1 \to \phi_2 \ , \quad \phi_2 \to \phi_1 & \mathbf{T}^2 = 1 \\ \phi_1 \to \phi_2 \ , \quad \phi_2 \to \phi_1 + \pi & \mathbf{T}^2 = -1 \end{cases}$$

- 2 possible time-reversal transformations, depending on which condensate (i.e. what gapping term we add to the edge)

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$$\mathbb{Z}_2 : \phi_1 \to \phi_1 + \pi \ , \quad \phi_2 \to \phi_2$$

- 2 possible time-reversal transformations are related by $\mathbb{Z}_2$ transformation
- If $\mathbb{Z}_2$ is global, they are different (2 distinct SPT's)
- If $\mathbb{Z}_2$ is gauged, this is just a gauge transformation and they must be the same (only 1 SET!)

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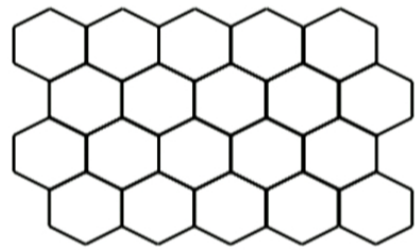
Doubled semion model

(Levin & Wen)

$$H = -\sum_v A_v + \sum_P B_P$$

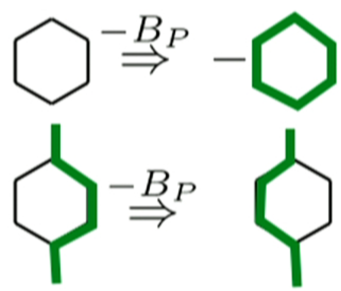


- Vertices:

$$A_v = \prod \sigma^z$$

Plaquettes:

$$B_p = \prod \sigma^x \times \text{Phase terms}$$



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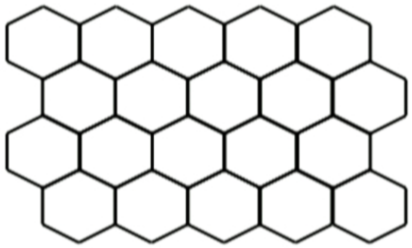
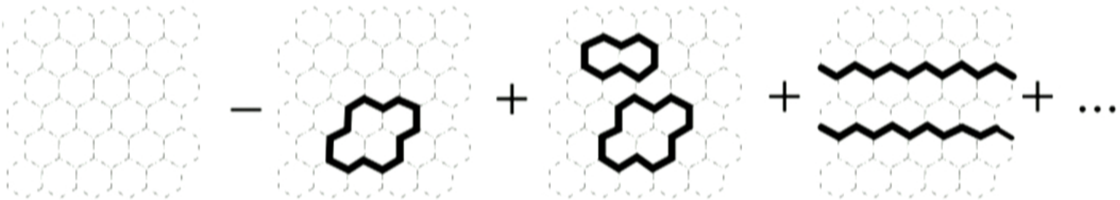


$$|\Psi_0\rangle = \sum_{\mathcal{C}=\{\text{loops}\}} (-1)^{N_{\text{loops}}} |\mathcal{C}\rangle$$

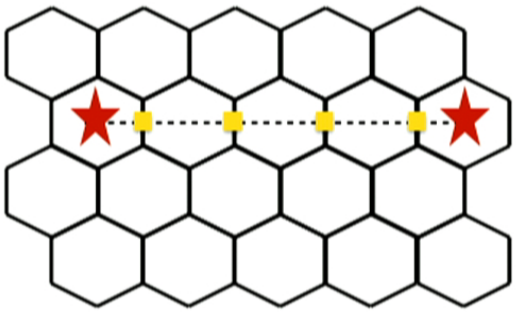


Doubled semion model

$$B_p = \prod \sigma^x \times \text{Phase terms}$$

- Boson pair creation: (squares to +1)

  ■ $= \sigma^z$



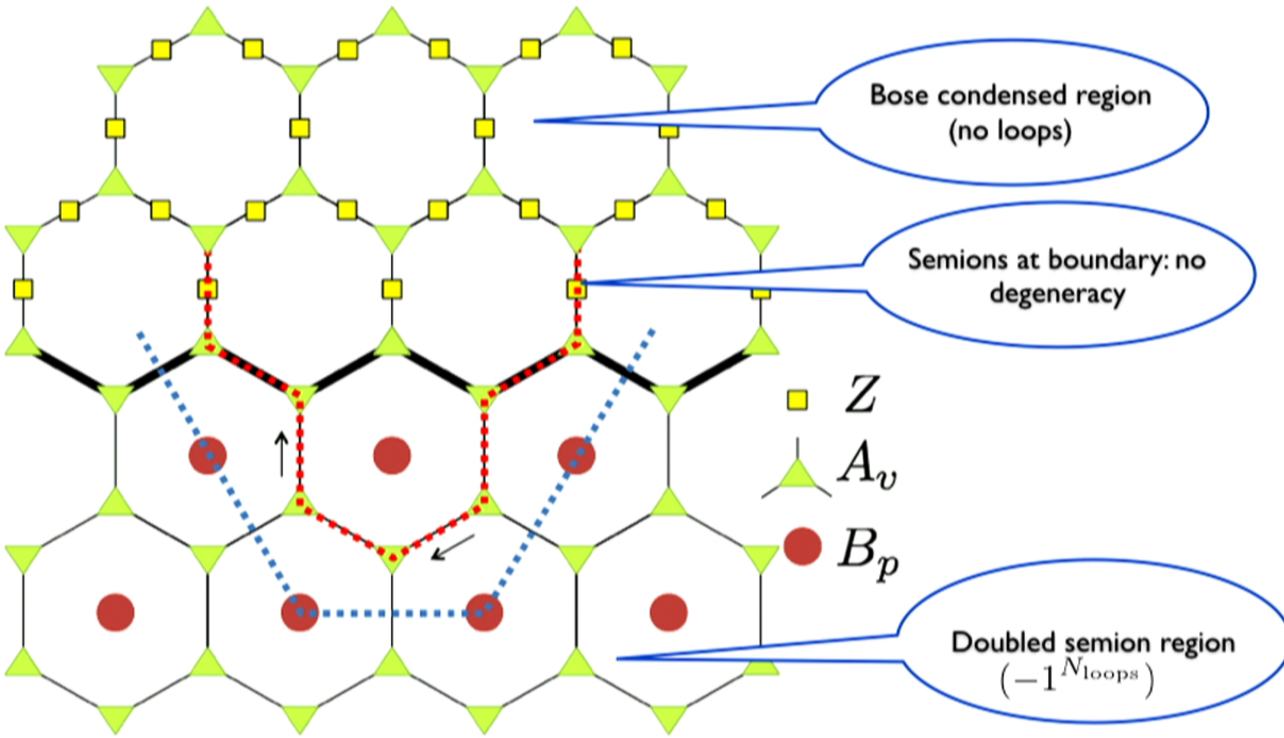
- Boson condensation (1):

$$H = -\sum_v A_v + h \sum_P B_P - J \sum_e \sigma_e^z$$

$$\overset{J/h \to \infty}{\Rightarrow} -\sum_e \sigma_e^z$$

Trivial ground state with no loops

Gapped boundary 1



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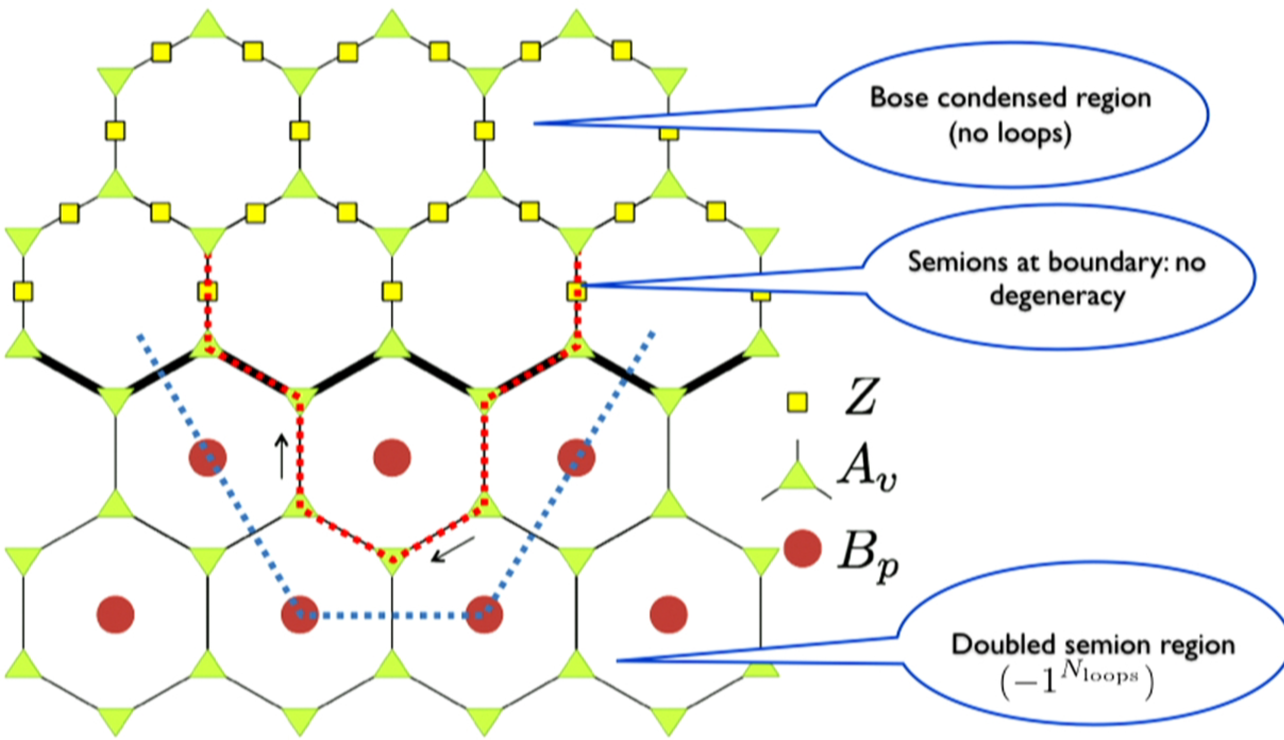
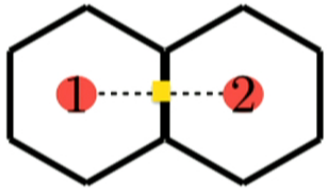


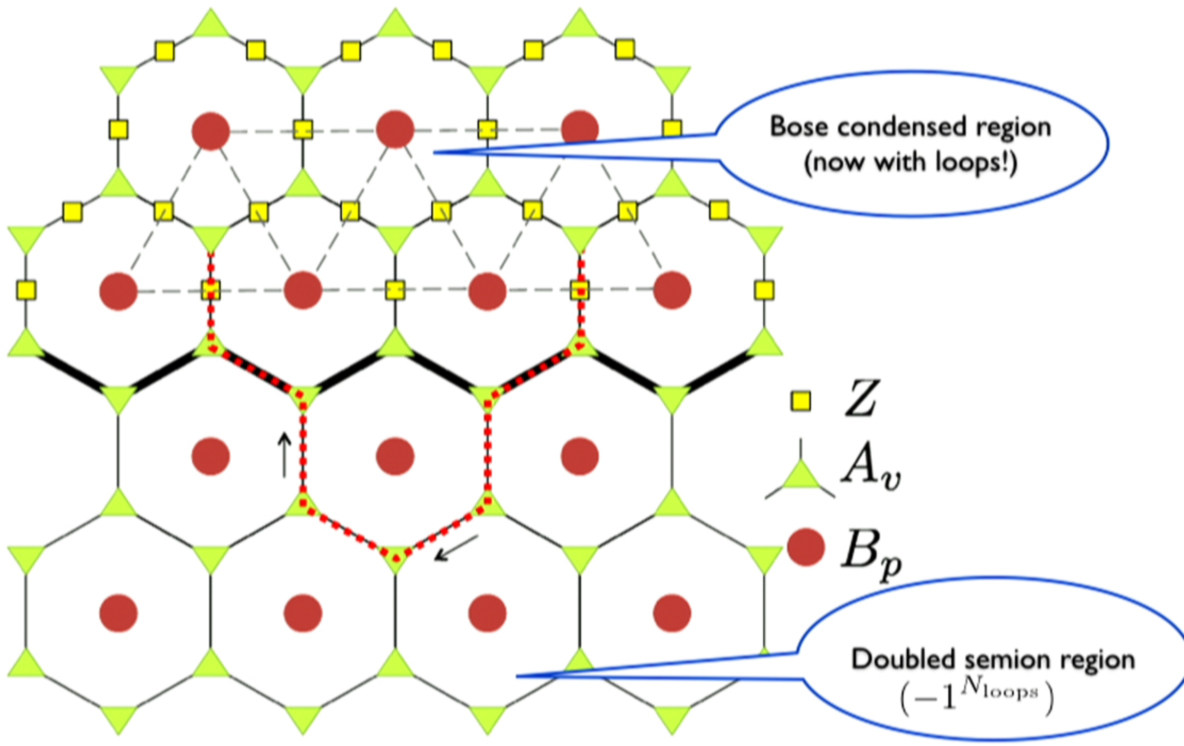
Gapped boundary 2



$B_{P_1} \sigma^z_{12} B_{P_2}$

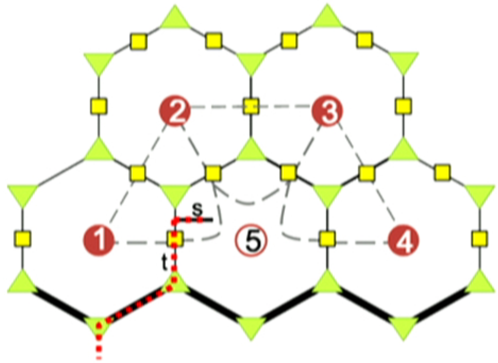
- squares to +1
- Boson pair creation/ annihilation: (-1)
- Boson hopping: +1

Gapped boundary 2



The semion at the edge

- Problem: now we can't terminate the semion string without breaking T, unless we also remove some terms from the Hamiltonian!
- Semions+ T = Kramers degeneracy!
- Can show: $T^2 = -1$



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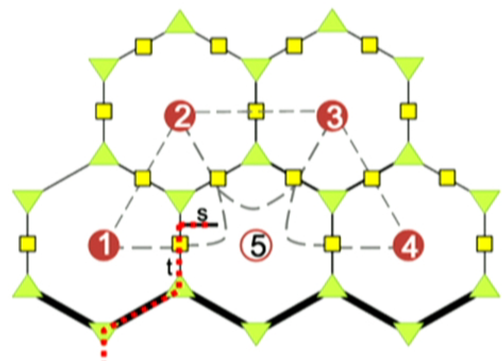
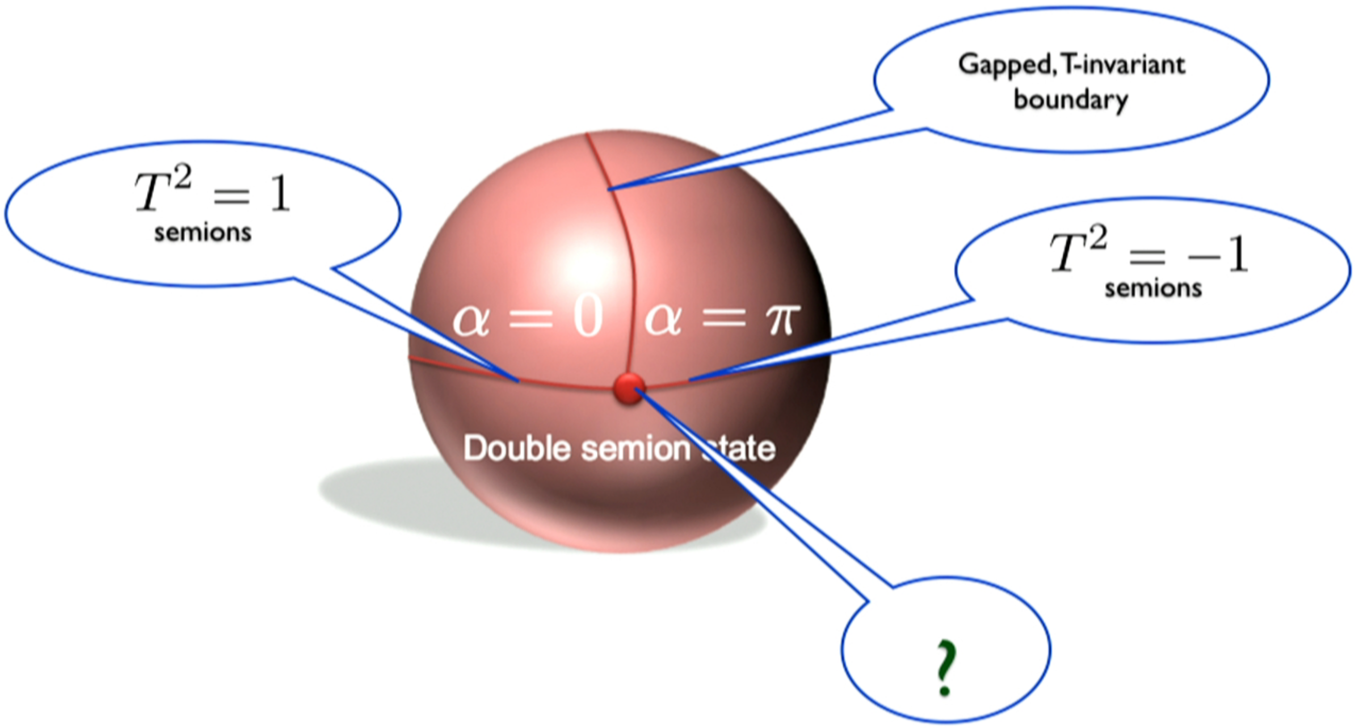
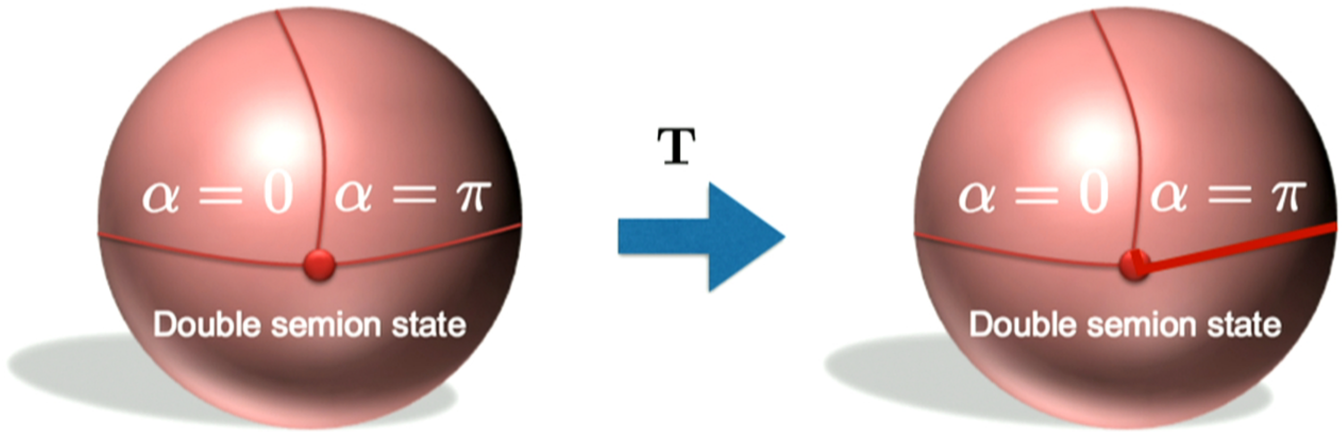


Part 3: Tri-junctions

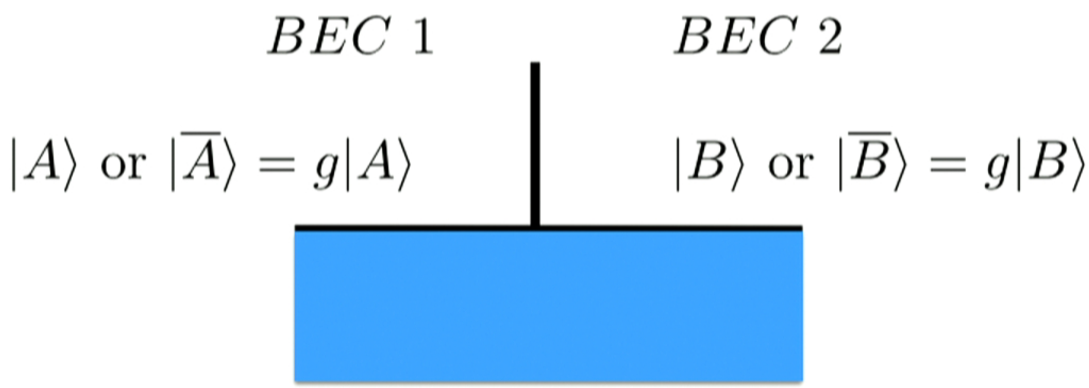


Tri-junctions



- Time reversal relates 2 states which are not connected by a local operator (but by an extended semion string)
- Tri-junctions with time-reversal have degeneracy (even without semions)

Tri-junctions



Gapped edges: $|AB\rangle + |\overline{AB}\rangle$

$|A\overline{B}\rangle + |\overline{A}B\rangle$

Tri-junctions

$$|A\rangle \equiv |\phi_1 - \phi_2 = 0\rangle \ , \quad |B\rangle \equiv |\phi_1 - \phi_2 = \pi/2\rangle$$

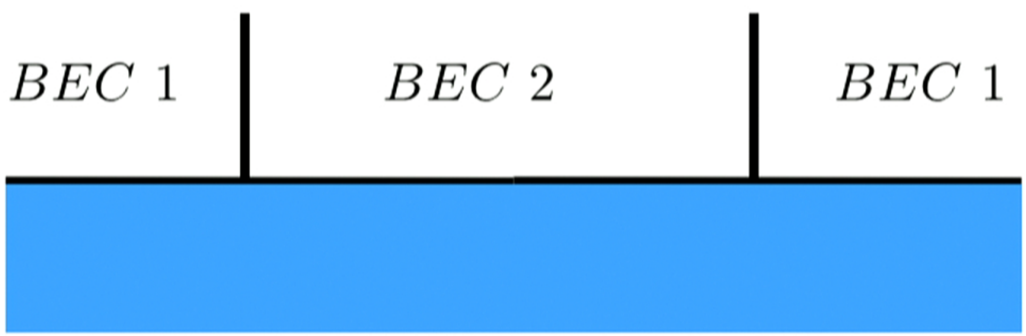
$$|\overline{A}\rangle \equiv |\phi_1 - \phi_2 = \pi\rangle \ , \quad |\overline{B}\rangle \equiv |\phi_1 - \phi_2 = -\pi/2\rangle$$

$$\mathbb{Z}_2^T : \begin{cases} \phi_1 \to \phi_2 \ , \quad \phi_2 \to \phi_1 \\ \phi_1 \to \phi_2 \ , \quad \phi_2 \to \phi_1 + \pi \end{cases} \qquad \begin{matrix} |A\rangle \to |A\rangle \ , \ |B\rangle \to |\overline{B}\rangle \\ |A\rangle \to |\overline{A}\rangle \ , \ |B\rangle \to |B\rangle \end{matrix}$$

Tri-junctions

$$\mathbf{T}\left(|A\overline{B}\rangle + |\overline{A}B\rangle\right) = |AB\rangle + |\overline{AB}\rangle$$

- 2-fold degeneracy for each pair of domain walls between BEC1 and BEC2
- Operator mapping between them: $|B\rangle \to g|B\rangle$ tunnels a semion between the two boundaries



Summary

- 2 SPT's -> 1 SET
- 2 "different" kinds of gapped boundary for the doubled semion model
- …which are in the same phase but lead to different time-reversal transformations of boundary semions
- (Gapped) domains between these 2 gapped boundaries have an extra T-protected degeneracy