Title: Realizing anomalous anyonic symmetries at the surfaces of 3d gauge theories

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Abstract:

Realizing anomalous anyonic symmetries on the surfaces of 3d gauge theories

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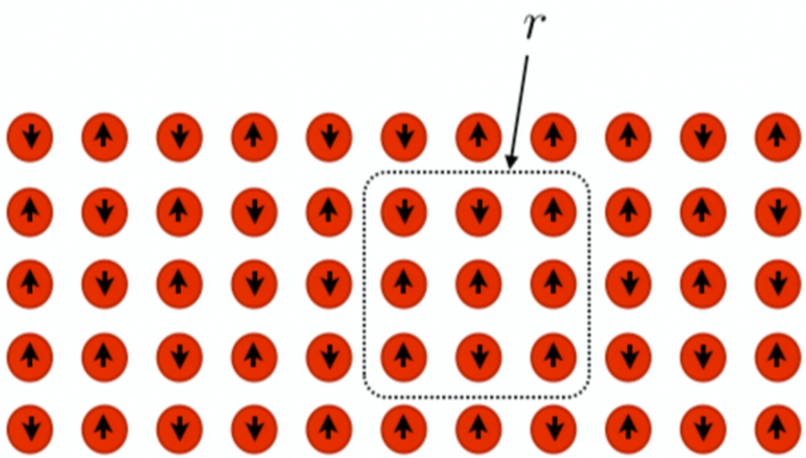
Quantum systems

Hilbert space: $\mathcal{H} = \otimes_{\mathrm{sites\,i}} \mathcal{H}_i$

Hamiltonian: $H = \sum_r H_r$



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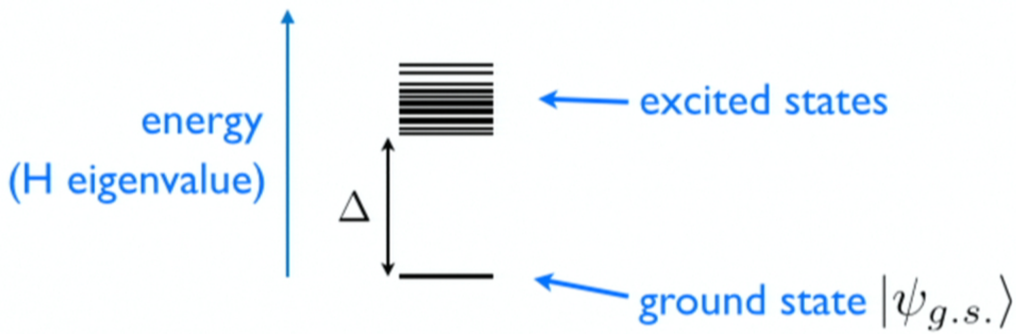
Gapped quantum systems

- Diagonalize H:



- low temperature: physics of the ground state

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Symmetric gapped quantum systems

extra assumptions:

- "symmetry" group G - for us, always finite
- each $\mathcal{H}_i$ forms a unitary representation of G:

$$\rho_i(g) : \mathcal{H}_i \to \mathcal{H}_i$$

- Hamiltonian commutes with $\rho(g) \equiv \otimes_i \rho_i(g)$:

$$[\rho(g), H] = 0$$

- no spontaneous symmetry breaking

Two threads:

1) 2+1 spacetime dimensions:

- phases described by unitary modular tensor categories (UMTCs) $\mathcal{A}$ (and central charge)
- symmetry enriched phases described by braided G-crossed categories extending $\mathcal{A}$

existence of such extensions can be **obstructed**

(Etingof, Niksych, Ostrik; Barkeshli, Bonderson, Cheng, Wang; Fidkowski,Lindner, Kitaev)

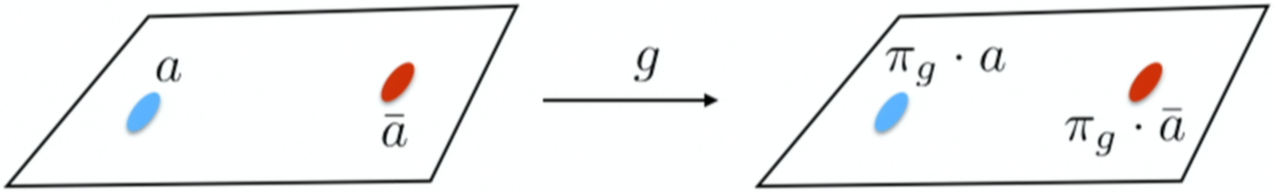
2) 3+1 spacetime dimensions:

- phases contain particle-like and loop-like excitations
- can characterize symmetry enriched phases by specifying symmetry fractionalization on both particles and **loops**

2+1d: Anyonic symmetry

- group map from G to permutations of anyons

$$g \to \pi_g$$



- more precisely, group map $\hat{\pi}$ from G to Γ_1 = group of braided autoequivalences of $\mathcal{A}$ up to isomorphisms

braided autoequivalence = invertible braided tensor functor from $\mathcal{A}$ to itself

2+1d: Anyonic symmetry

Q: Can every such $\hat{\pi}$ be realized in a physical system?

A: No - there is an obstruction in $H^3(G, \mathcal{A}_{\text{inv}})$

Argument: if $\hat{\pi}$ could be realized, then we could gauge G. The result would be a larger theory with anyons of $\mathcal{A}$ but also with fluxes of G. This is is a braided G-crossed extension of $\mathcal{A}$. However, not every $\hat{\pi}$ can be realized by such an extension.

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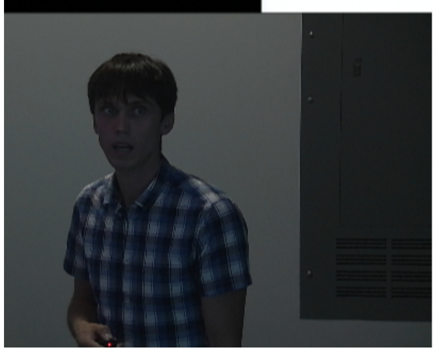

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Example:

- Eilenberg-Maclane obstruction: H a finite non-abelian group with center Z

$$1 \to H \to E \to G \to 1$$

$$\Downarrow$$

$$G \to \mathrm{Aut(H)/Inn(H)} \leftarrow \mathrm{Out}(H)$$

$$g \to [x \to \bar{g}x\bar{g}^{-1}]$$

but reverse <u>not true</u>

obstruction valued in $H^3(G, Z)$

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Example (con't):

Now take $\mathcal{A} = \mathcal{Z}(H)$

↑ quantum double of H; physically, the discrete H gauge theory

Then a map from G to Out(H) induces a map

$$\hat{\pi} : G \to \Gamma_1(\mathcal{A})$$

The obstruction to realizing this symmetry physically in a discrete H gauge theory is then given by

$$H^3(G, Z) \to H^3(G, \mathcal{A}_{\text{inv}})$$

induced by $Z \subset \mathcal{A}_{\text{inv}}$

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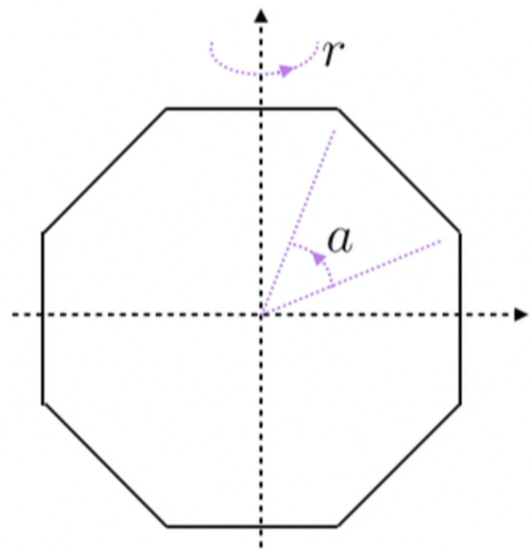
Concrete example:

$H = \mathbb{D}_{16}$

$$r^2 = a^8 = 1$$
$$rar = a^{-1}$$

$Z = \mathbb{Z}_2 = \{1, a^4\}$



$G = \mathbb{Z}_2 = \{1, \sigma\}$

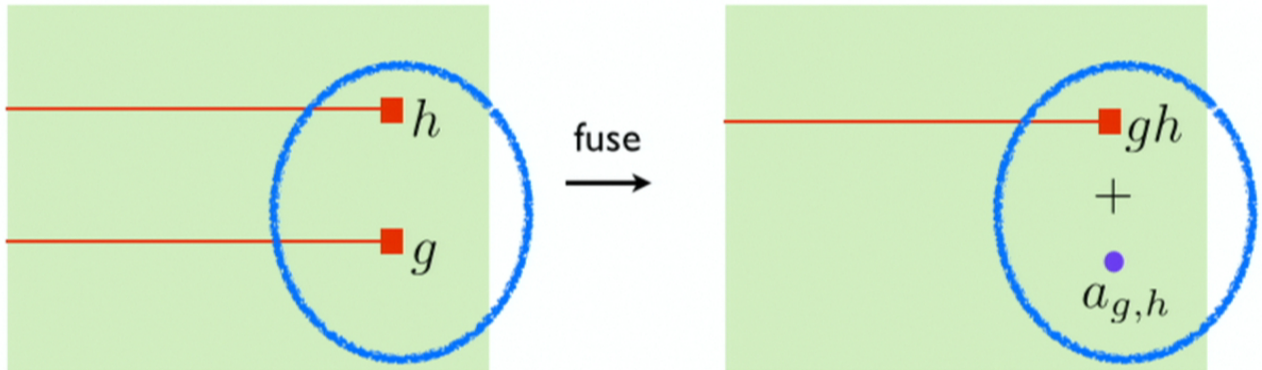
σ acts by

$$r \to ra$$
$$a \to a^5$$

(note that σ^2 is conjugation by a^{-3})

Earlier story:

Even if $\hat{\pi}$ is trivial, can have non-trivial G-flux fusion:



ambiguity in definition of G-fluxes:

$$a_{g,h} \to a_{g,h} + b_g + b_h - b_{gh}$$

associativity of G-flux fusion:

$$a_{g,h} - a_{fg,h} + a_{f,gh} - a_{f,g} = 0$$

thus $[a] \in H^2(G, \mathcal{A}_{\text{inv}})$

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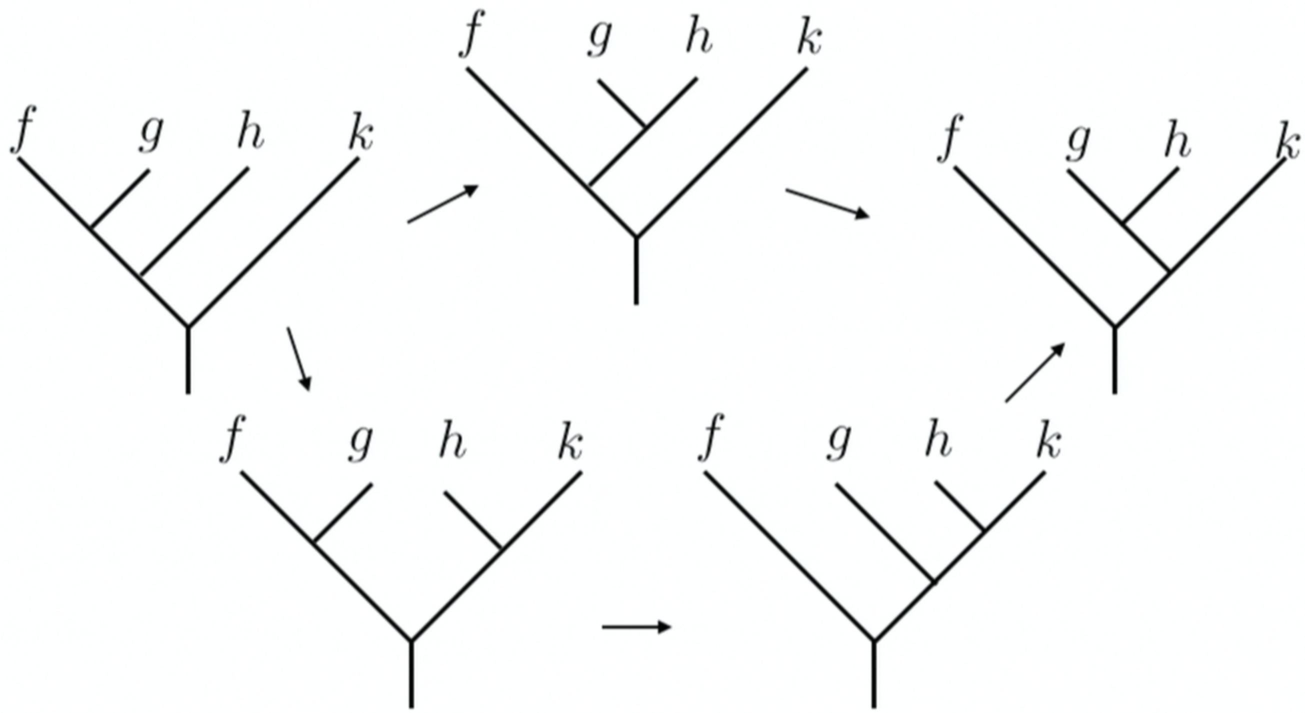
Not all G-flux fusion patterns are allowed

$$H^2(G,A) \to H^4(G,U(1))$$



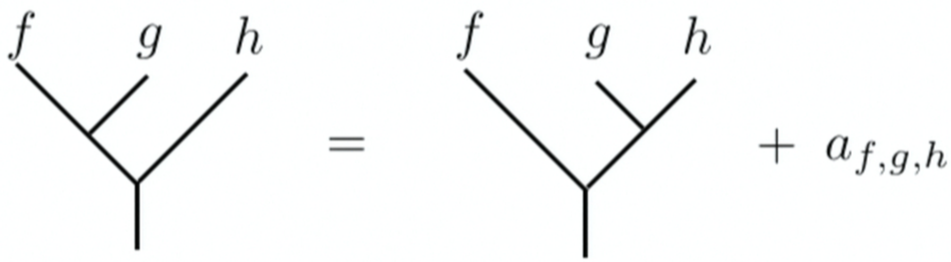
Physical interpretation: can be realized at surface of non-trivial 3+1d SPT

(Chen, Gu, Liu, Wen, et al.)

(Chen, Burnell, Vishwanath, Fidkowski)

Back to current story:

Can interpret $H^3(G, \mathcal{A}_{\text{inv}})$ class as ambiguity in fusion associativity of G-fluxes:



- Suggests that in any surface realization, $a_{f,g,h}$ should be allowed to go into the 3+1d bulk

- In the Eilenberg-Maclane class of examples, this suggests that 3+1d bulk should be a Z gauge theory

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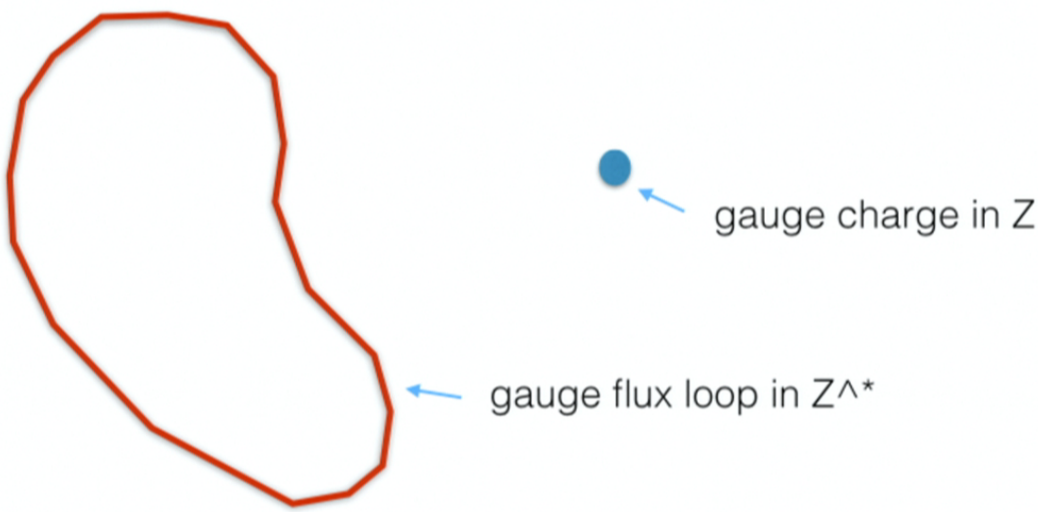
Second thread: symmetry enriched phases in 3+1d

3+1d bulk: Z^* gauge theory



- braiding gauge charge around gauge flux gives U(1) phase

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Second thread: symmetry enriched phases in 3+1d

Symmetry fractionalization:

1) symmetry G may permute gauge charges / fluxes

2) charges / fluxes may carry fractional quantum numbers of G

3) flux loops may behave as edges of 2+1d SPTs

no analog in 2+1d symmetry enriched phases

(Captured in 3-loop braiding statistics)

(Levin, Wang)

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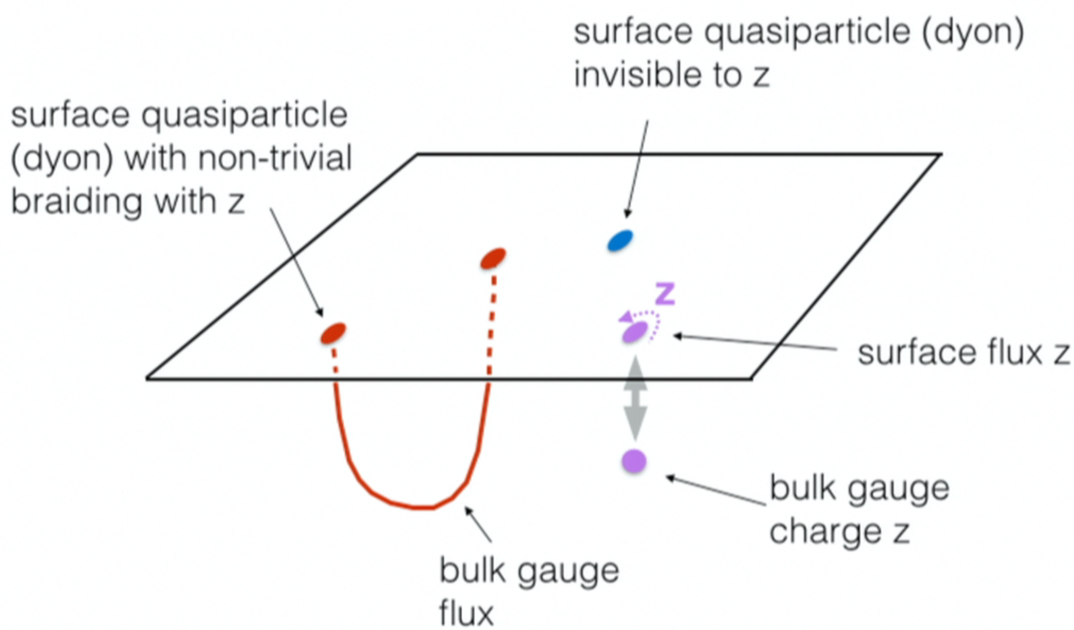
Putting together the 2+1d and 3+1d theories:

- identify bulk gauge charges with surface Z fluxes

- this binds certain surface quasiparticles to gauge flux endpoints



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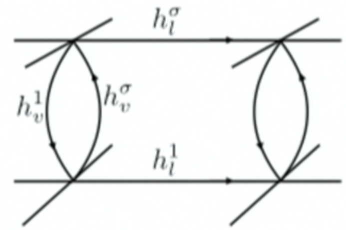
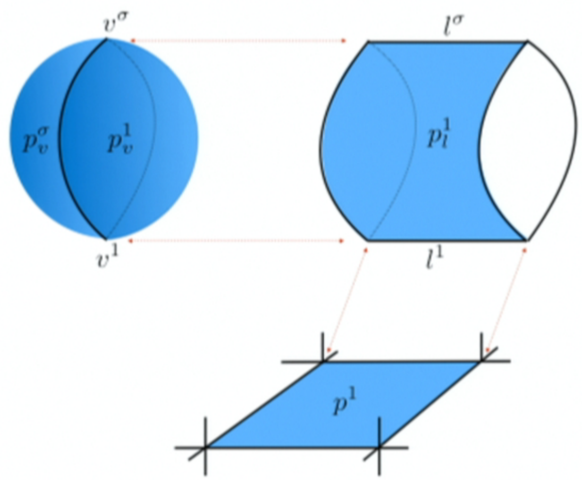
Exactly solved model

- surface lattice model:



(Hermele)

- 3d bulk model: dual representation of Z^* gauge theory in terms of fluctuating 2d surfaces:



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Subtleties

- Our 3d bulk model also has 'trivial' gapped surfaces corresponding to surface condensate of gauge charges

- General correspondence

$$H^3(G, Z) \to H^3(G, \mathcal{A}_{\text{inv}})$$
$$Z \to \mathcal{A}_{\text{inv}}$$

not necessarily one to one map:
e.g. Z=Z_2, U(1)_8 surface

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Conclusions

- Two obstructions in construction of braided G-crossed extension

$$H^3(G, \mathcal{A}_{\text{inv}})$$

system realizeable at surface of 3d symmetry enriched phase with fractionalization along gauge flux loops

$$H^4(G, U(1))$$

system realizeable at surface of 3+1d SPT

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Extensions

1) General UMTCs with action by braided autoequivalences?

- can $H^3(G, \mathcal{A}_{\text{inv}})$ obstruction always be taken to be valued in bosons?

2) 3+1d bulk with fermionic gauge charges?

- maybe useful for describing 3+1d fermionic SPTs?

- e.g. superconductors in Cartan class DIII

3) Relation to higher form symmetry framework

(Kapustin, Seiberg et al.)