Title: Bulk-boundary correspondence for 3D symmetry-protected topological phases

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Abstract:

Bulk-boundary correspondence for 3D symmetry-protected topological phases

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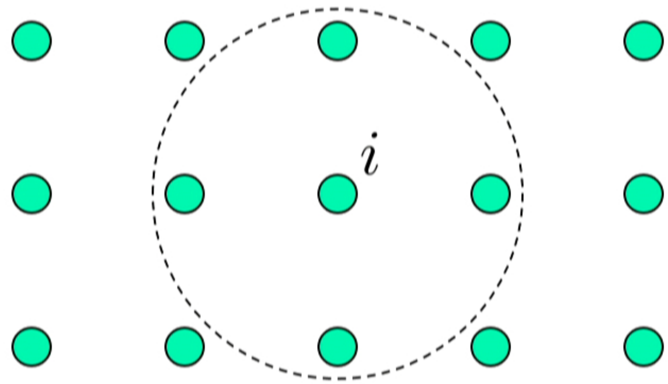
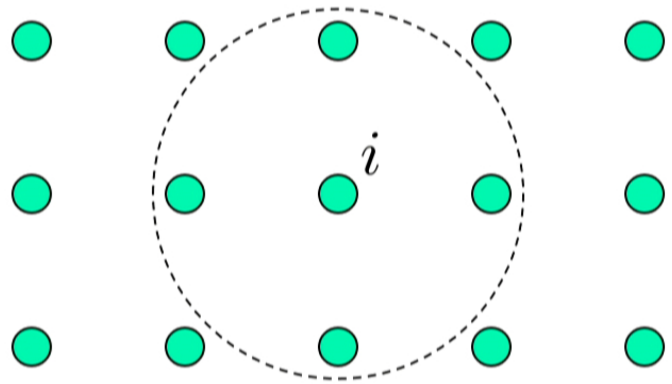
Quantum many-body systems



Hilbert space: $\mathcal{V} = V \otimes V \otimes V \otimes \cdots$

Hamiltonian: $H = \sum_i H_i$

Quantum many-body systems



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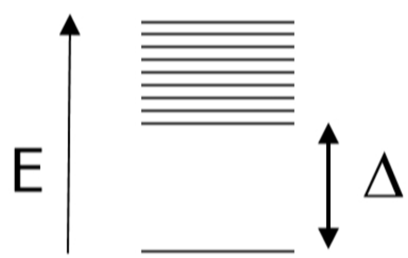
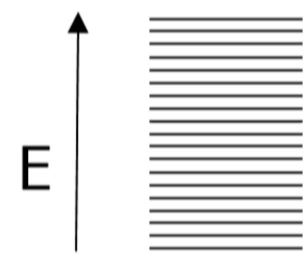
Hamiltonian: $H = \sum_i H_i$

Two possibilities for energy spectrum

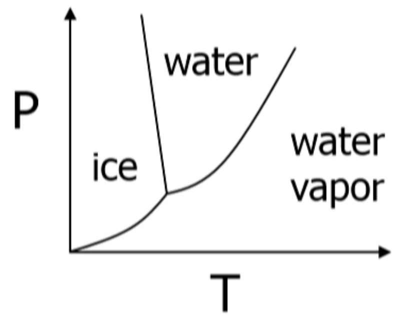


Gapped

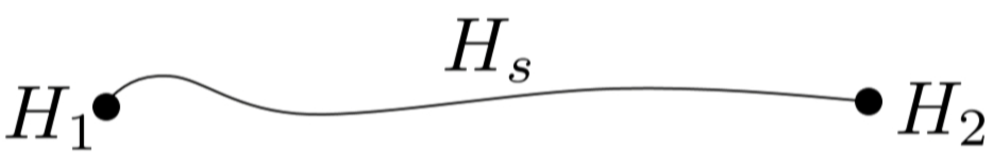


Gapless

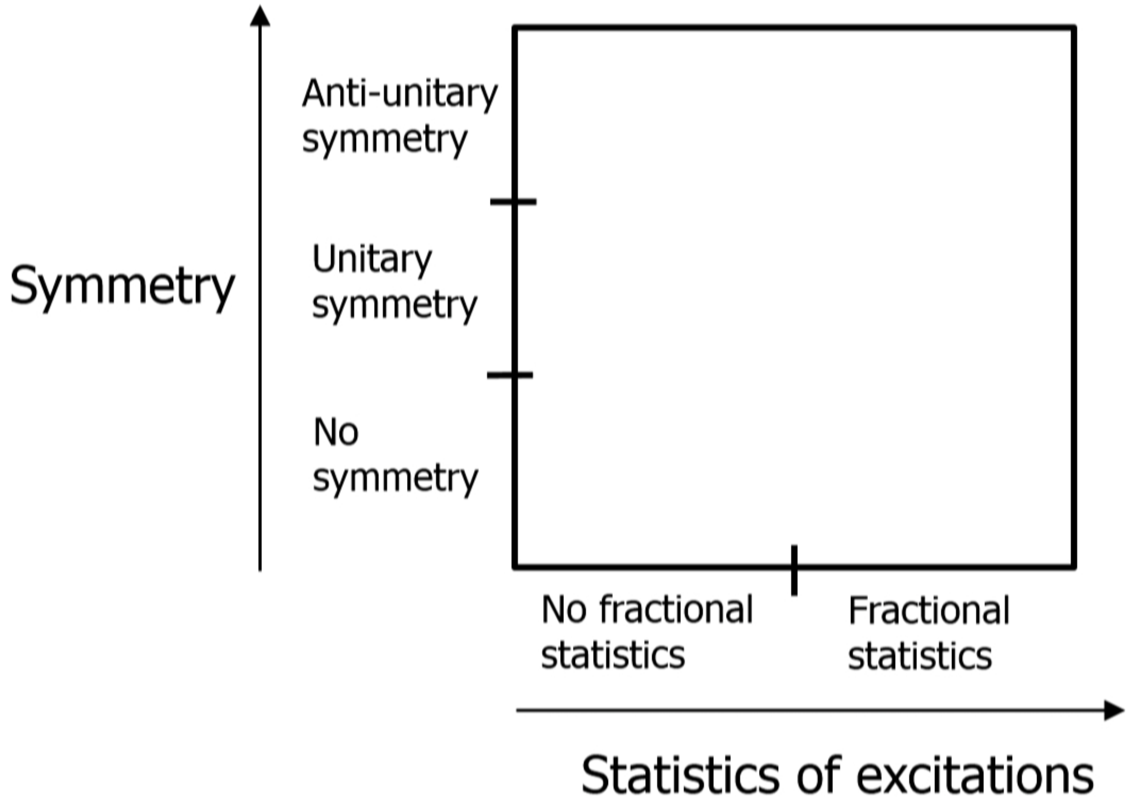
Quantum phases



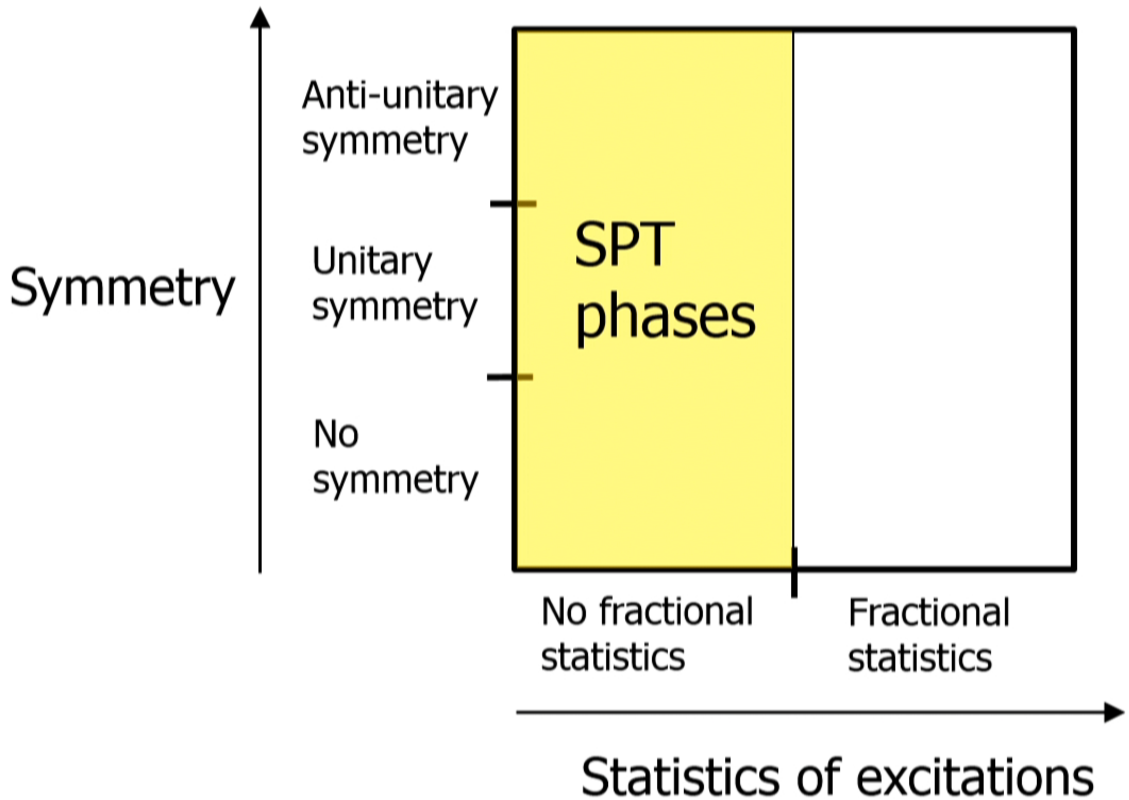
H_1, H_2 **belong to the same phase** if they can be smoothly connected without closing the energy gap:



Different kinds of gapped phases



Different kinds of gapped phases



H belongs to an **SPT phase** if it has:

- Some set of (unbroken) symmetries $\{S_1, S_2, ...\}$, i.e.

$$[S_1, H] = [S_2, H] = ... = 0$$

- "Short-range entangled" ground state $|\Psi\rangle$, i.e. there exists a local unitary U with

$$\begin{aligned} U|\Psi\rangle &= |\Psi_{prod}\rangle \\ &\equiv |\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle \otimes \cdots \end{aligned}$$

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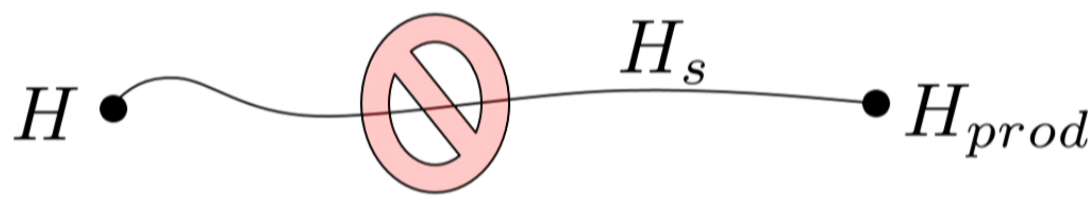
- SPT phase is **nontrivial** if it cannot be connected to product state without breaking symmetry or closing gap:



Examples

- Topological insulators (2D/3D, $U(1) \times$ TRS)

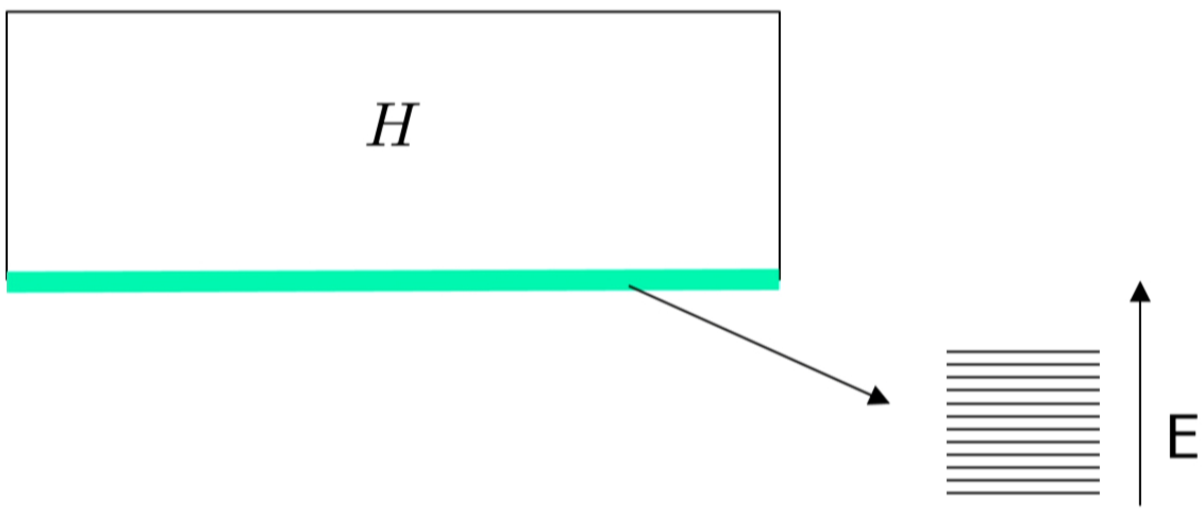
  (Hasan, Kane, RMP, 2010)

- Haldane spin-1 chain (1D, TRS)

  (Haldane, 1983)

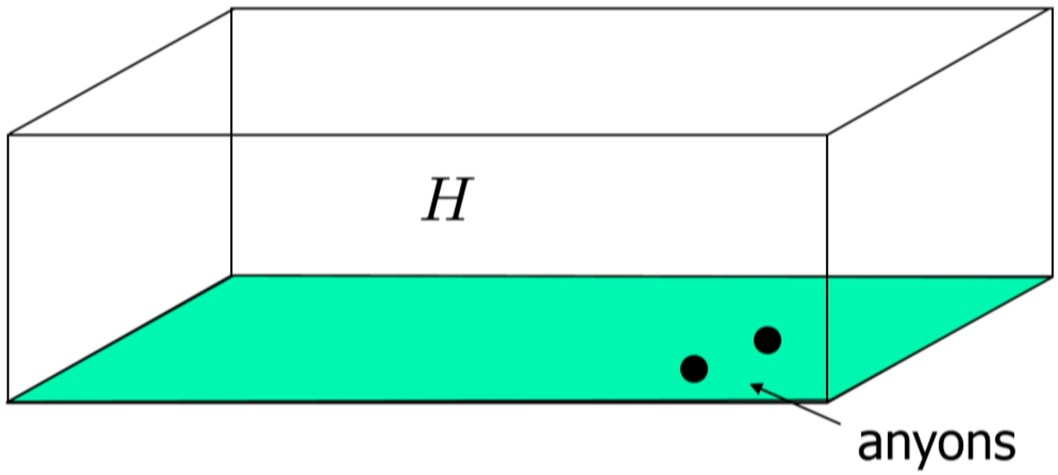
- Many others…

- Nontrivial SPT phases typically (always?) have "protected" gapless boundaries:



Meaning of "protected" in 3D case

Surface cannot be both gapped and symmetric, unless it supports excitations with fractional statistics.



Two basic questions

Given H belonging to a 3D SPT phase:

1. What bulk properties of H determine whether it has a protected surface?

2. What is relationship between bulk and surface properties of H?

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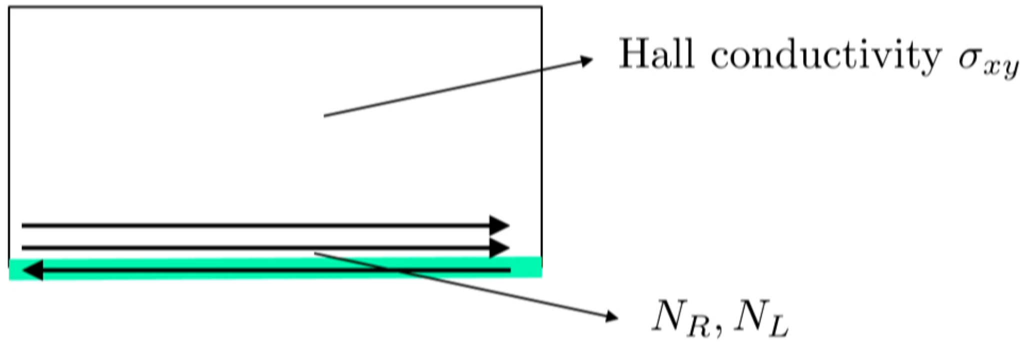
"Bulk-boundary correspondence"

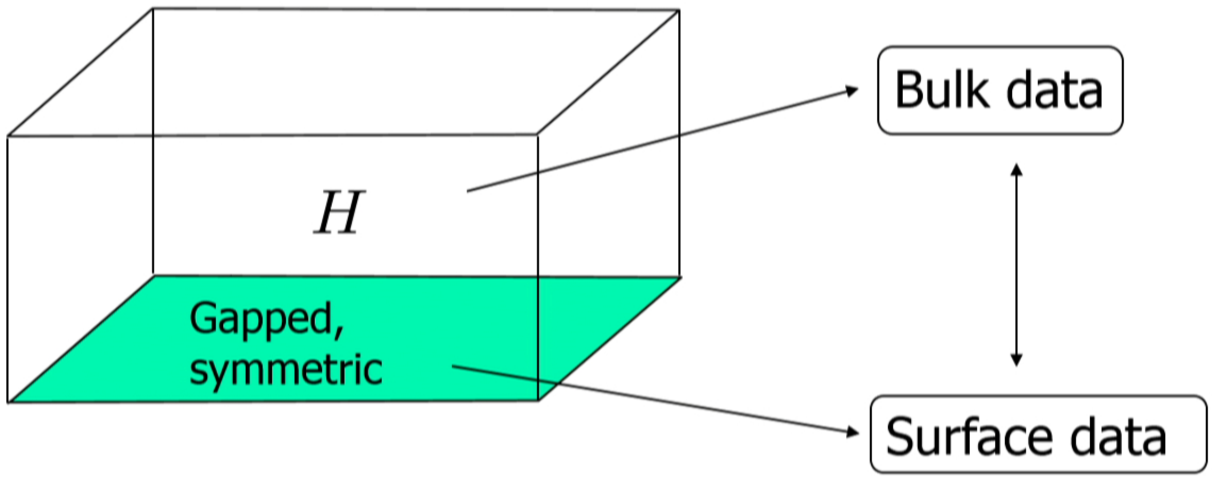
Analogy

Bulk-boundary correspondence for 2D band insulators with U(1) symmetry:



$$\sigma_{xy} = N_R - N_L$$

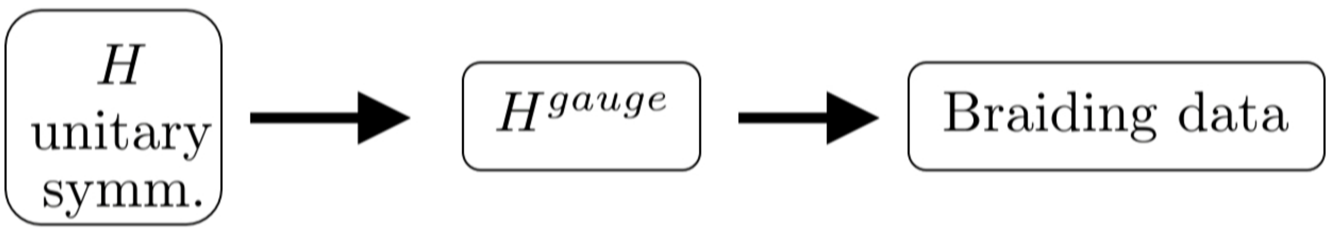
This talk



Focus on simple case

- Symmetries are **unitary** and form **Abelian** group $G = (\mathbb{Z}_N)^K$
- Surface anyons are Abelian and form group $A = (\mathbb{Z}_N)^M$
- Symmetry does not permute surface anyons
- Constituent particles are bosons

Recipe for finding bulk and surface data



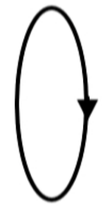
Excitations after gauging: bulk

Charges

Characterized by gauge charge:

$$q = (q_1, ..., q_K),$$

$$q_i = (\text{integer}) \pmod{N}$$



Vortex loops

Characterized by gauge flux:

$$\phi = (\phi_1, ..., \phi_K),$$

$$\phi_i = \frac{2\pi}{N}(\text{integer}) \pmod{2\pi}$$

Excitations after gauging: bulk

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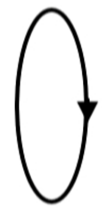

Vortex loops

Characterized by gauge flux:

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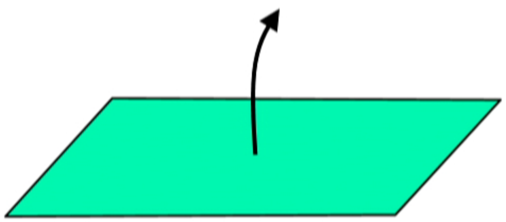
Excitations after gauging: surface

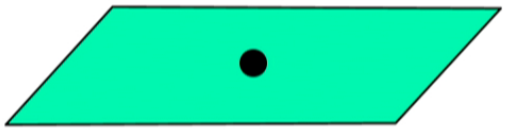


Vortex lines

Characterized by gauge flux:

$$\phi = (\phi_1, ..., \phi_K),$$

$$\phi_i = \frac{2\pi}{N}(\text{integer}) \quad (\text{mod } 2\pi)$$



Surface anyons

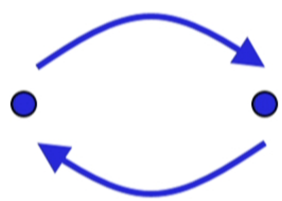
Characterized by "anyonic flux":

$$x = (x_1, ..., x_M),$$

$$x_i = (\text{integer}) \quad (\text{mod } N)$$

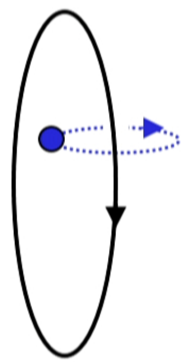
Braiding statistics of bulk excitations

Charge-charge



$$\theta = 0$$

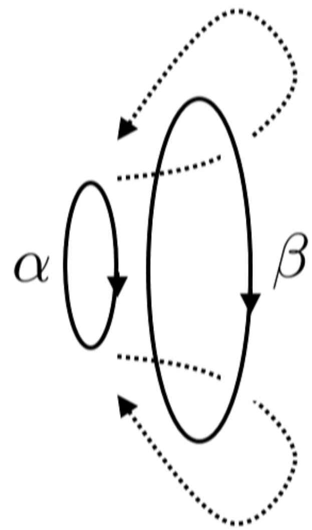
Charge-loop



$$\theta = q \cdot \phi$$

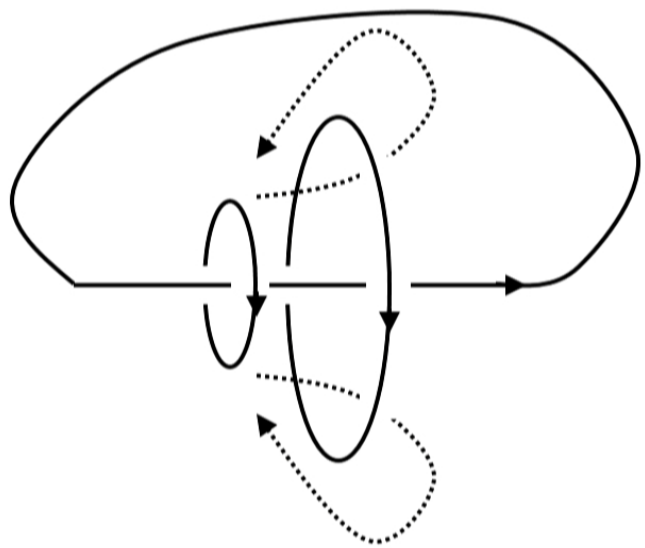
Braiding statistics of bulk excitations

Loop-loop



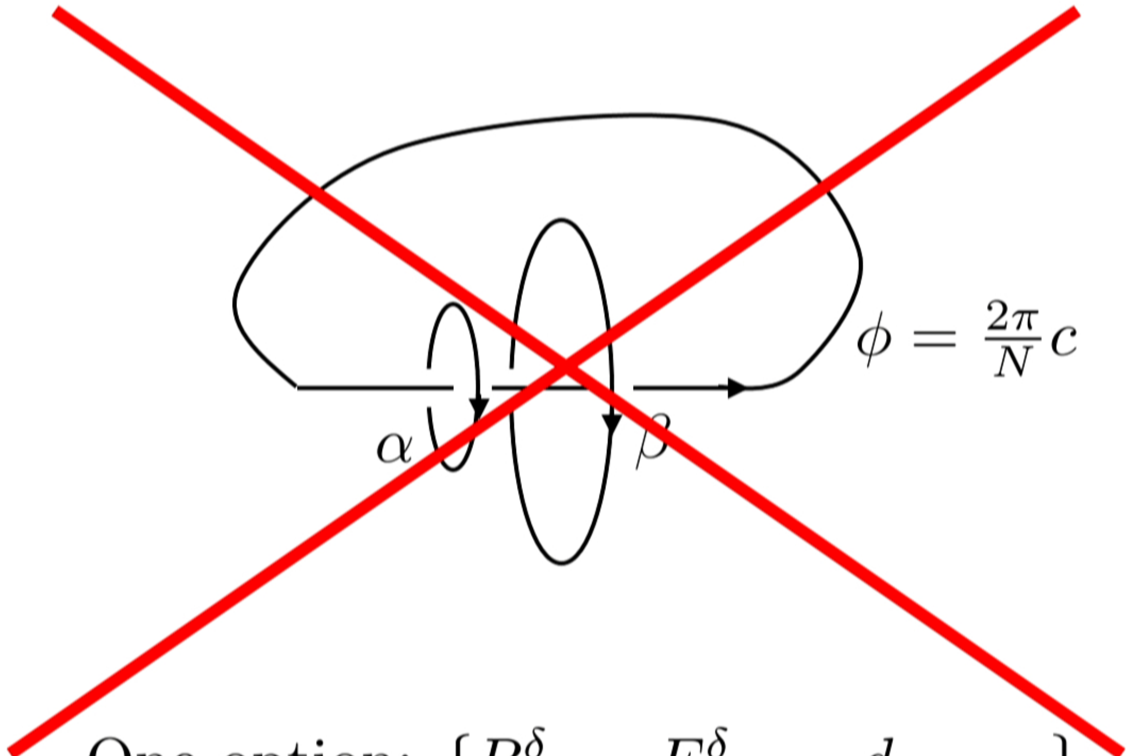
$$\theta_{\alpha\beta} = ?$$

Braiding statistics of bulk excitations



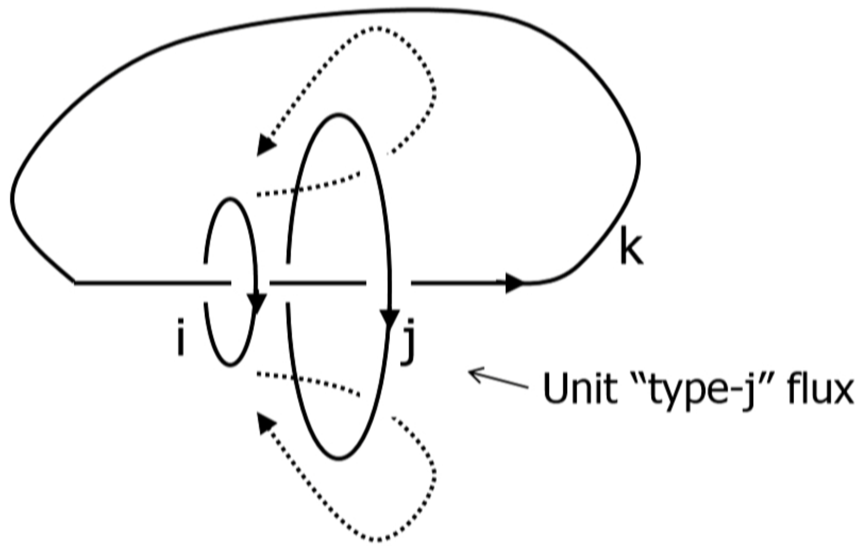
(C. Wang, ML, 2014)
(Jiang, Mesaros, Ran, 2014)

How to describe braiding data?

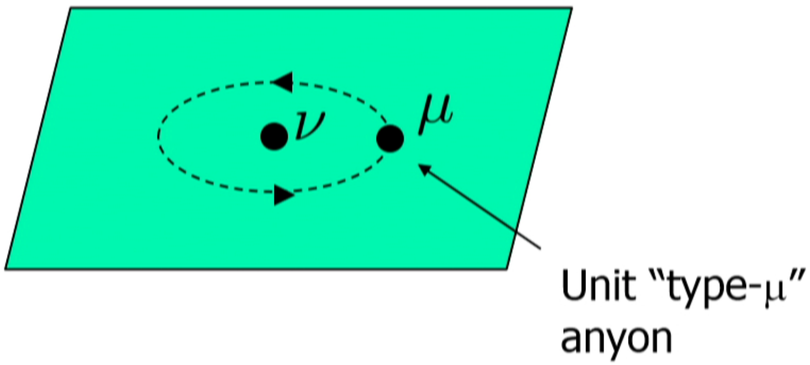


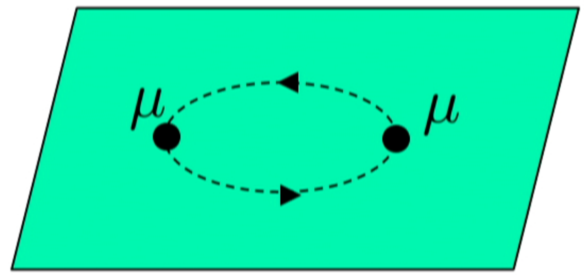
One option: $\{R^{\delta}_{\alpha\beta,c}, F^{\delta}_{\alpha\beta\mu,c}, d_{\alpha,c}, \ldots\}$

Bulk data



Surface data I

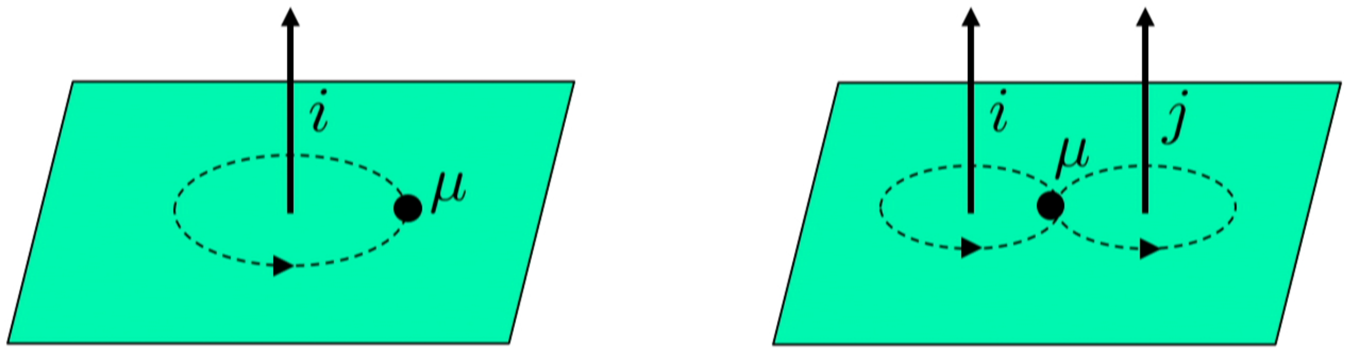




$$e^{i\Phi_{\mu\nu}} = (\text{Braid } \mu, \nu)$$

$$e^{i\Phi_{\mu}} = (\text{Exchange } \mu, \mu)$$

Surface data II

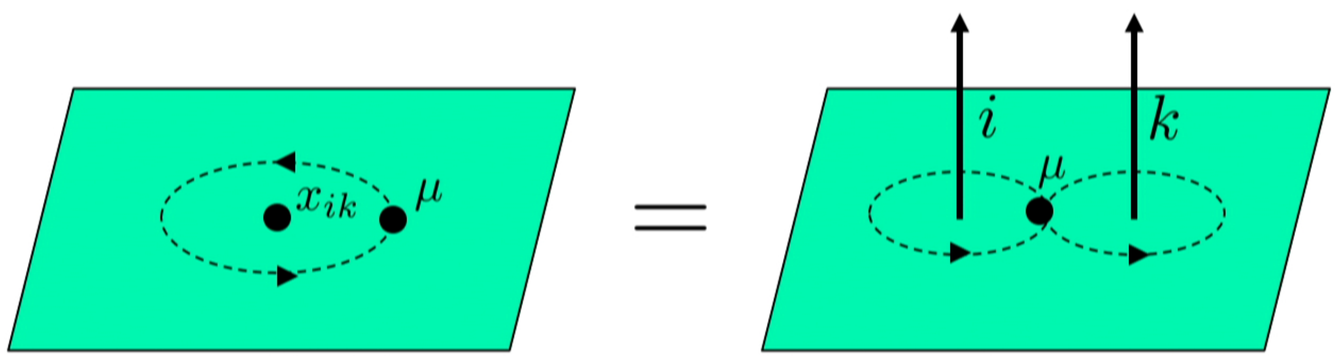


$$e^{i\Omega_{\mu i}} = (\text{Braid } \mu, i)^N$$

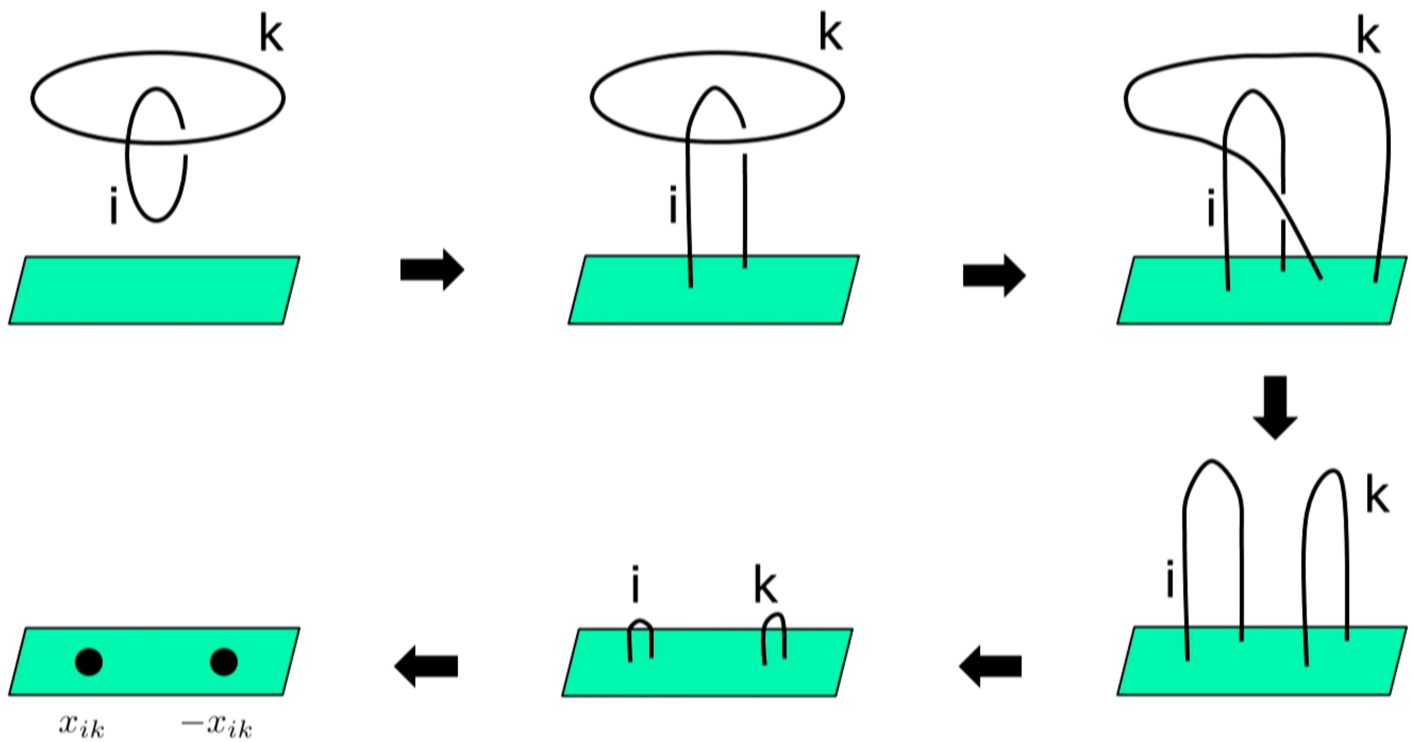
An additional surface quantity

Define integer tensor x^{ν}_{ik} by:

$$\sum_{\nu} x^{\nu}_{ik} \Phi_{\mu\nu} = \Omega_{ik\mu} \pmod{2\pi}$$



An additional surface quantity



Summary of data

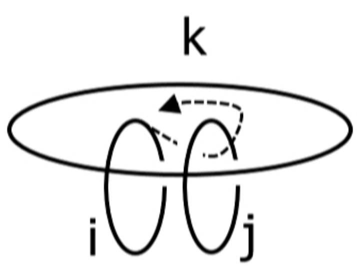
Bulk data

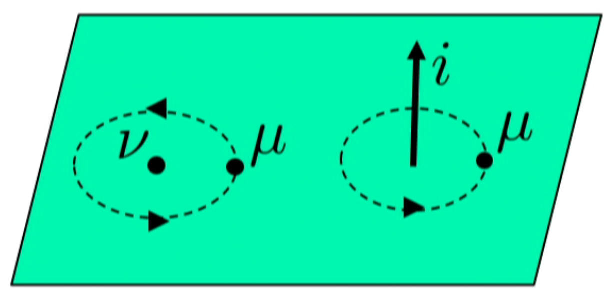
$$\{\Theta_{i,k}, \Theta_{ij,k}\}$$

Surface data

$$\{\Phi_\mu, \Phi_{\mu\nu}, \Omega_{i\mu}, \Omega_{ij\mu}\}$$

$$\{x^\mu_{ik}\}$$





(C. Wang, C.-H. Lin, ML, in preparation)

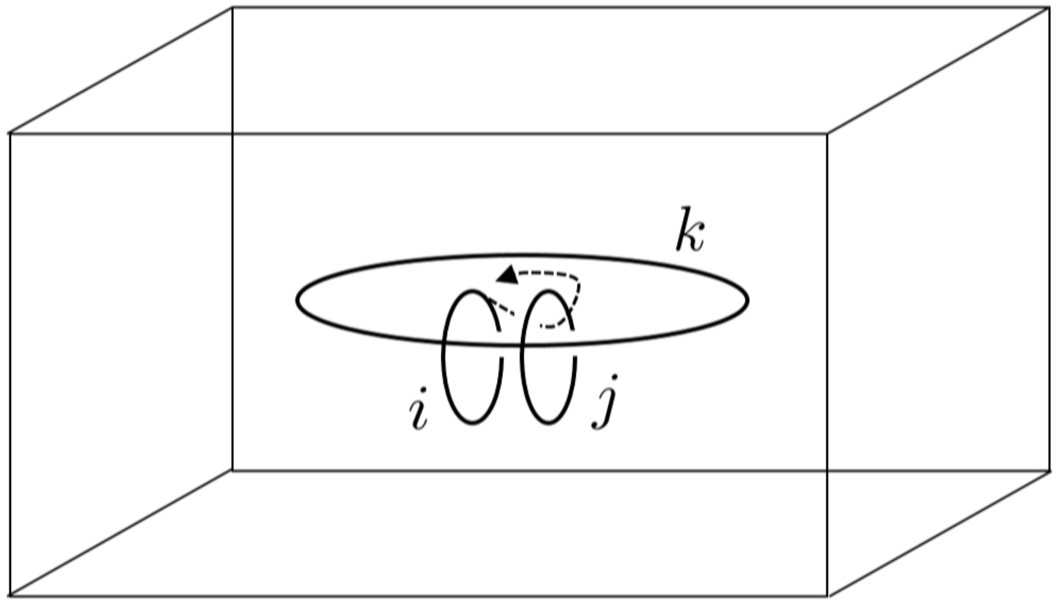
Main result

$$\Theta_{i,k} = \sum_\mu x^\mu_{ik}\Omega_{i\mu} + N\sum_\mu (x^\mu_{il})^2\Phi_\mu$$

$$\Theta_{ij,k} = \sum_\mu (x^\mu_{ik}\Omega_{j\mu} + x^\mu_{jk}\Omega_{i\mu}) + \frac{N(N-1)}{2}\sum_\mu (x^\mu_{jk} - x^\mu_{ik})\Omega_{ij\mu}$$

Corollary: If $\{\Theta_{i,k}, \Theta_{ij,k}\} \neq 0$ then surface is protected.

Deriving the correspondence



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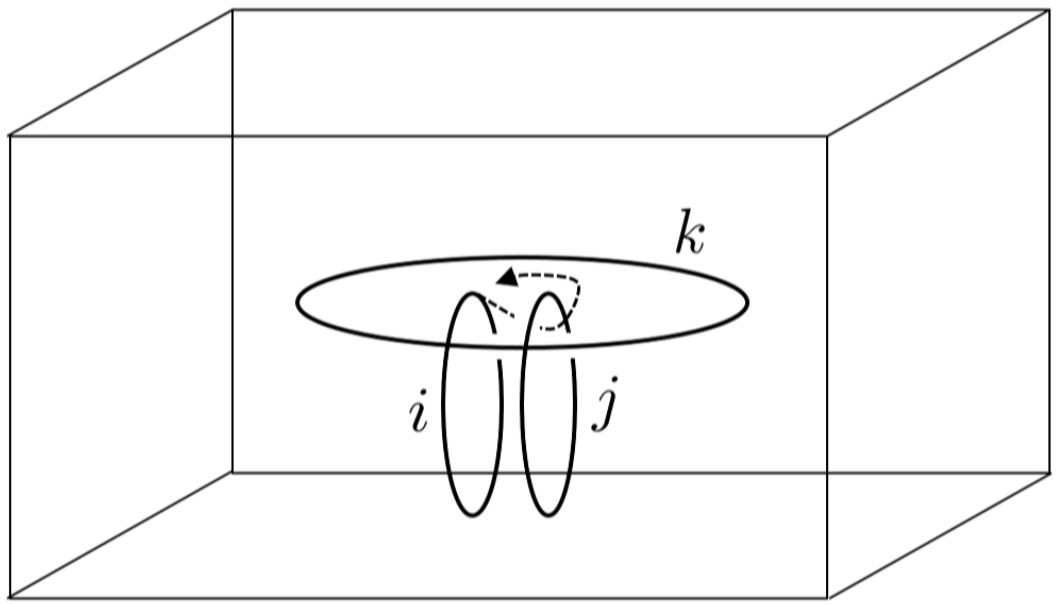

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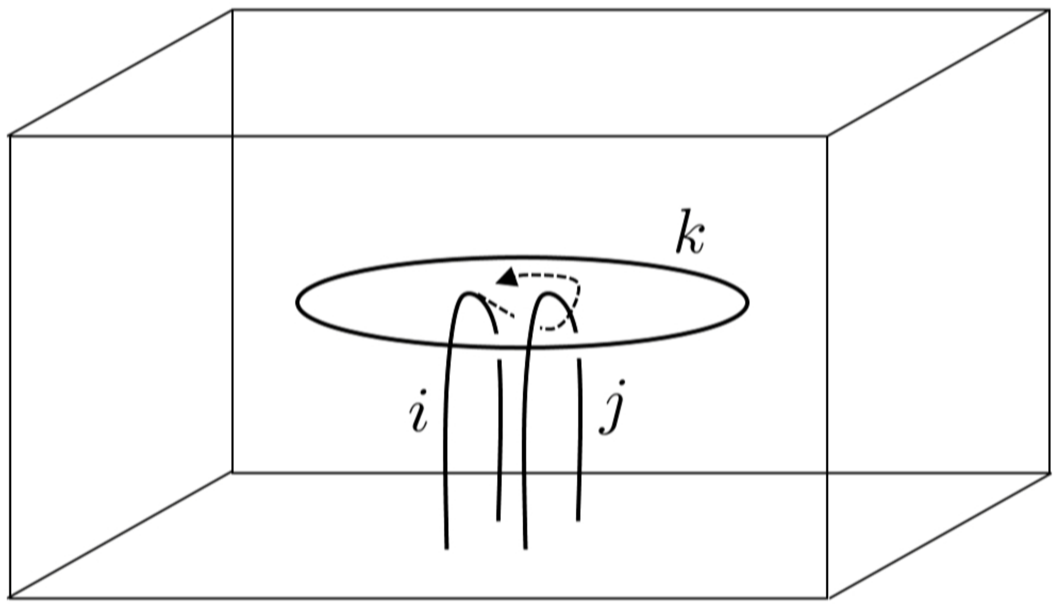

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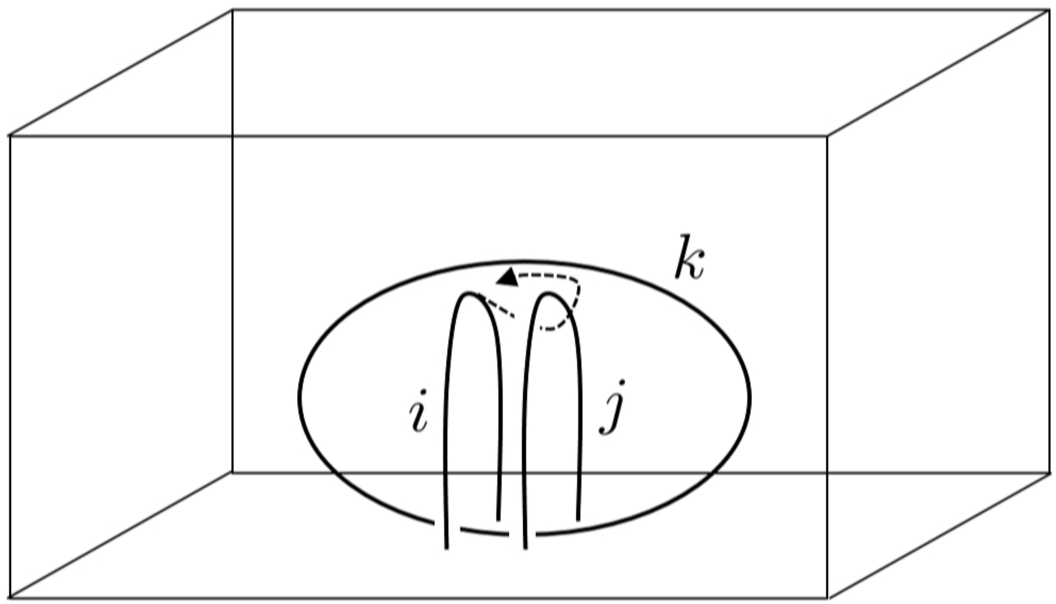

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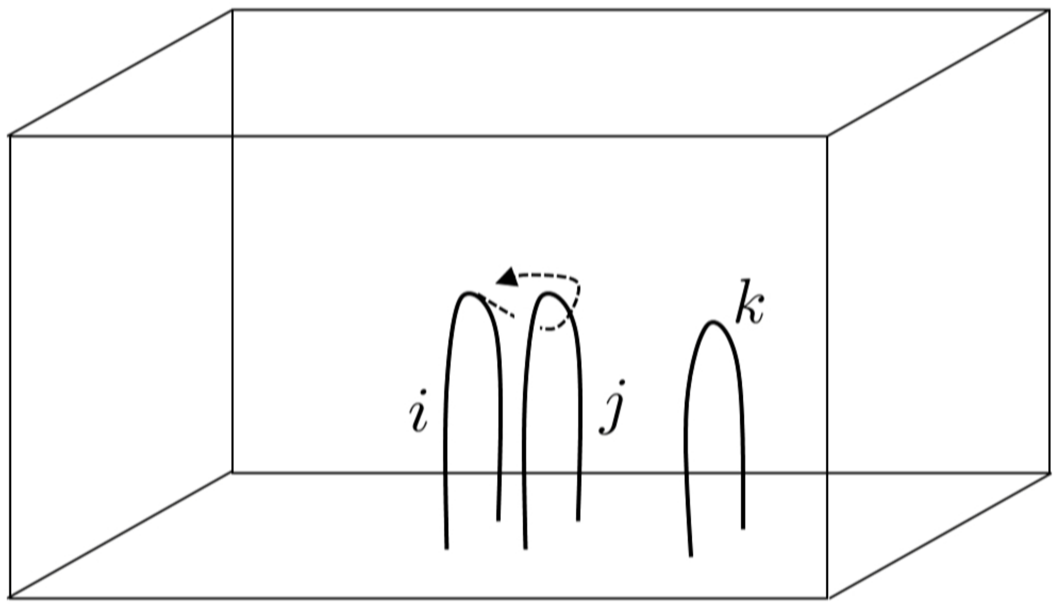

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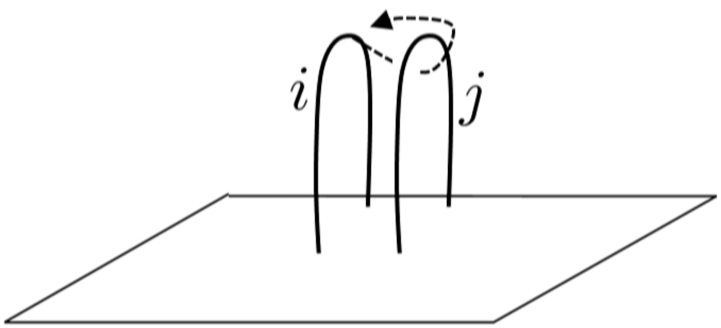

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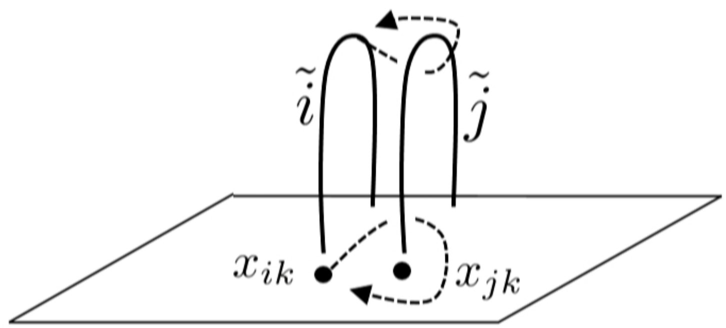

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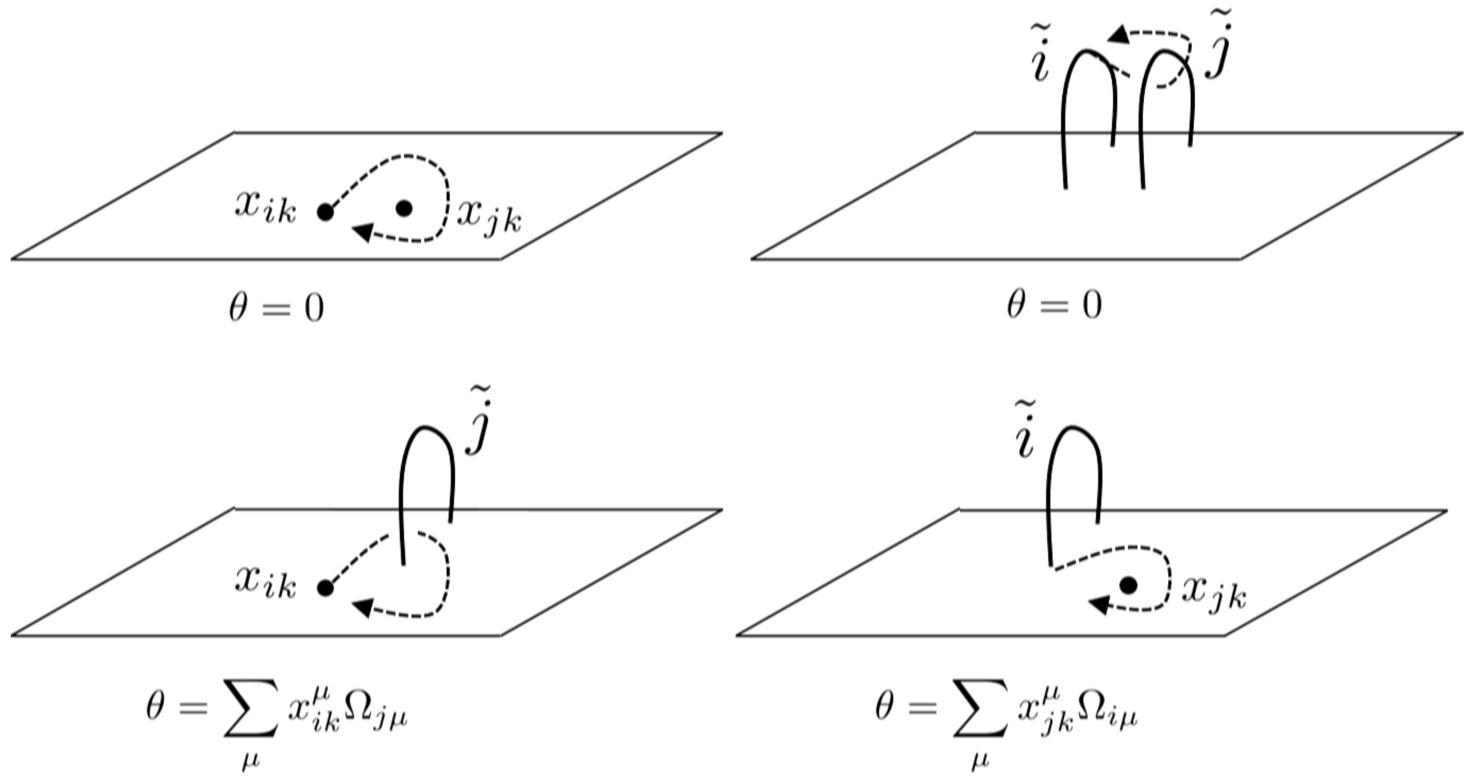


Deriving the correspondence

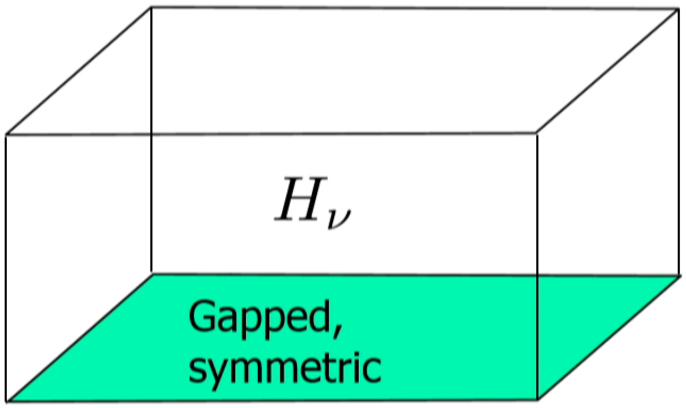
$$\Theta_{ij,k} = \sum_{\mu}(x^{\mu}_{ik}\Omega_{j\mu} + x^{\mu}_{jk}\Omega_{i\mu}) \ + \ \frac{N(N-1)}{2}\sum_{\mu}(x^{\mu}_{jk} - x^{\mu}_{ik})\Omega_{ij\mu}$$

Connection with cohomology

$\nu \in H^4(G, U(1))$ → Hamiltonian H_ν for SPT phase with symmetry group G

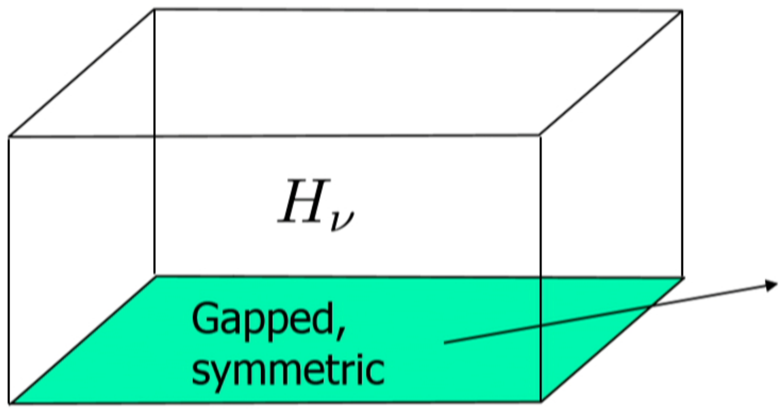
(Chen, Gu, Liu, Wen, 2011)

Connection with cohomology



(Chen, Burnell, Vishwanath, Fidkowski, 2014)

Connection with cohomology

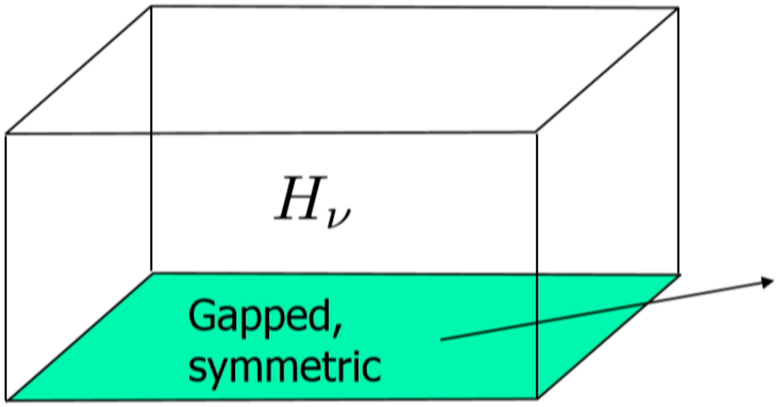


$$\{R_{xy}, F_{xyz}\}$$

$$\omega \in H^2(G, A)$$

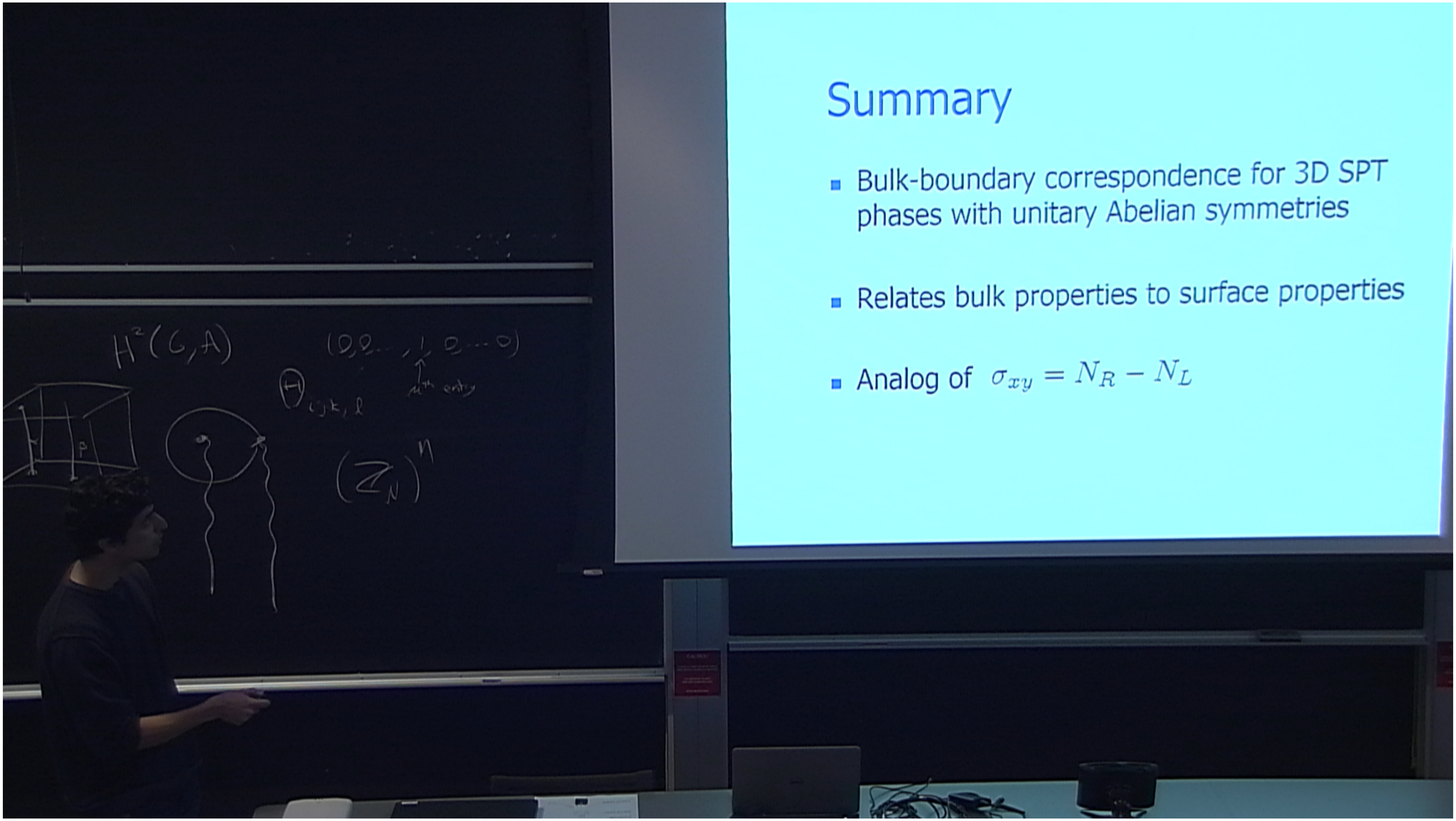
(Chen, Burnell, Vishwanath, Fidkowski, 2014)

Connection with cohomology



$$\{R_{xy}, F_{xyz}\}$$
$$\omega \in H^2(G, A)$$

$$\nu(f,g,h,k) \stackrel{?}{=} R_{\omega(h,k),\omega(f,g)} F_{\omega(g,h),\omega(f,gh),\omega(fgh,k)} F^{-1}_{\omega(g,h),\omega(gh,k),\omega(f,ghk)} \times F_{\omega(f,g),\omega(h,k),\omega(fg,hk)} F^{-1}_{\omega(f,g),\omega(fg,h),\omega(fgh,k)} \times F_{\omega(h,k),\omega(g,hk),\omega(f,ghk)} F^{-1}_{\omega(h,k),\omega(f,g),\omega(fg,hk)}$$


Summary
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Relates bulk properties to surface properties
Analog of $\sigma_{xy} = N_R - N_L$

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- Relates bulk properties to surface properties

- Analog of $\sigma_{xy} = N_R - N_L$