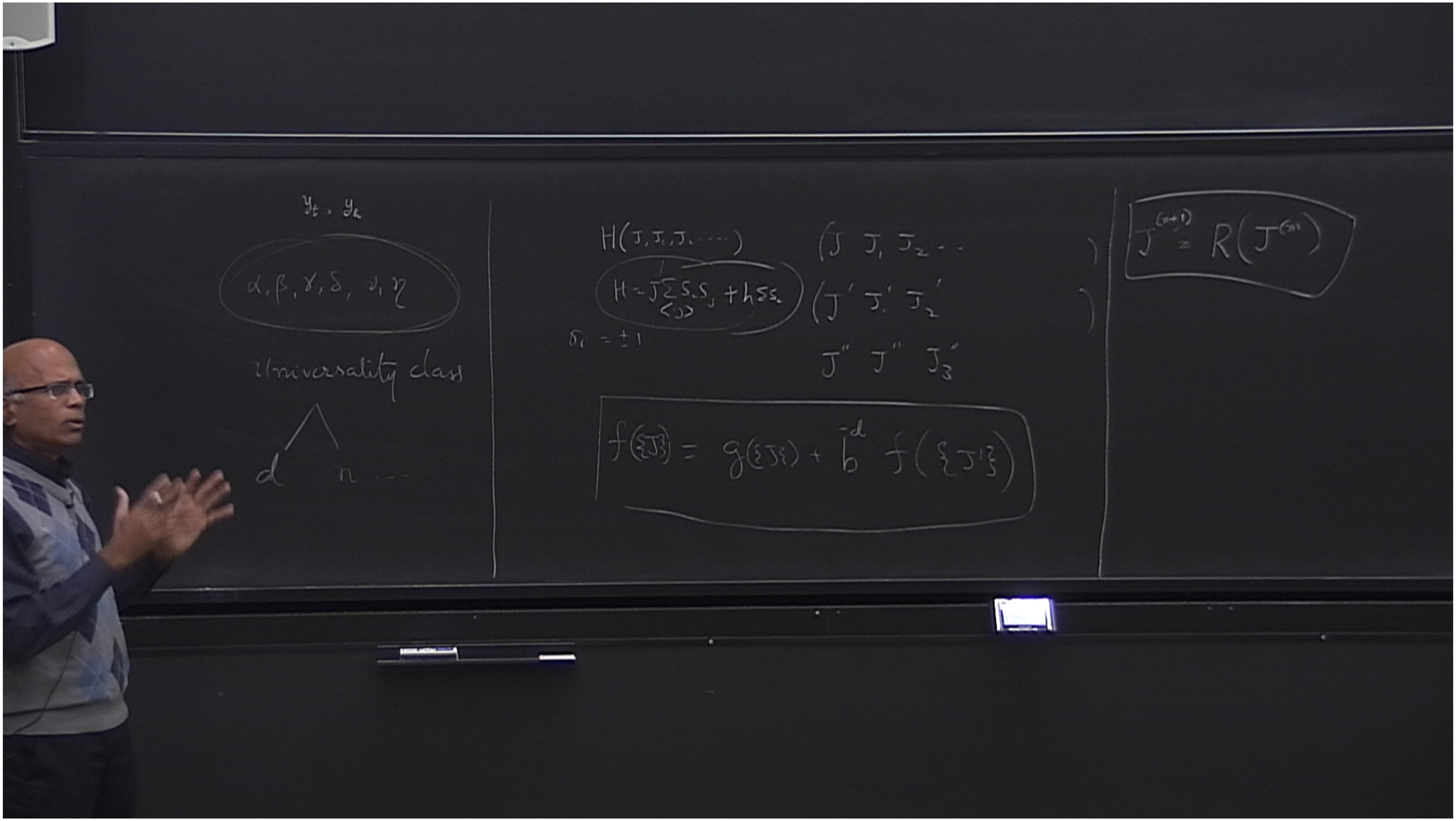
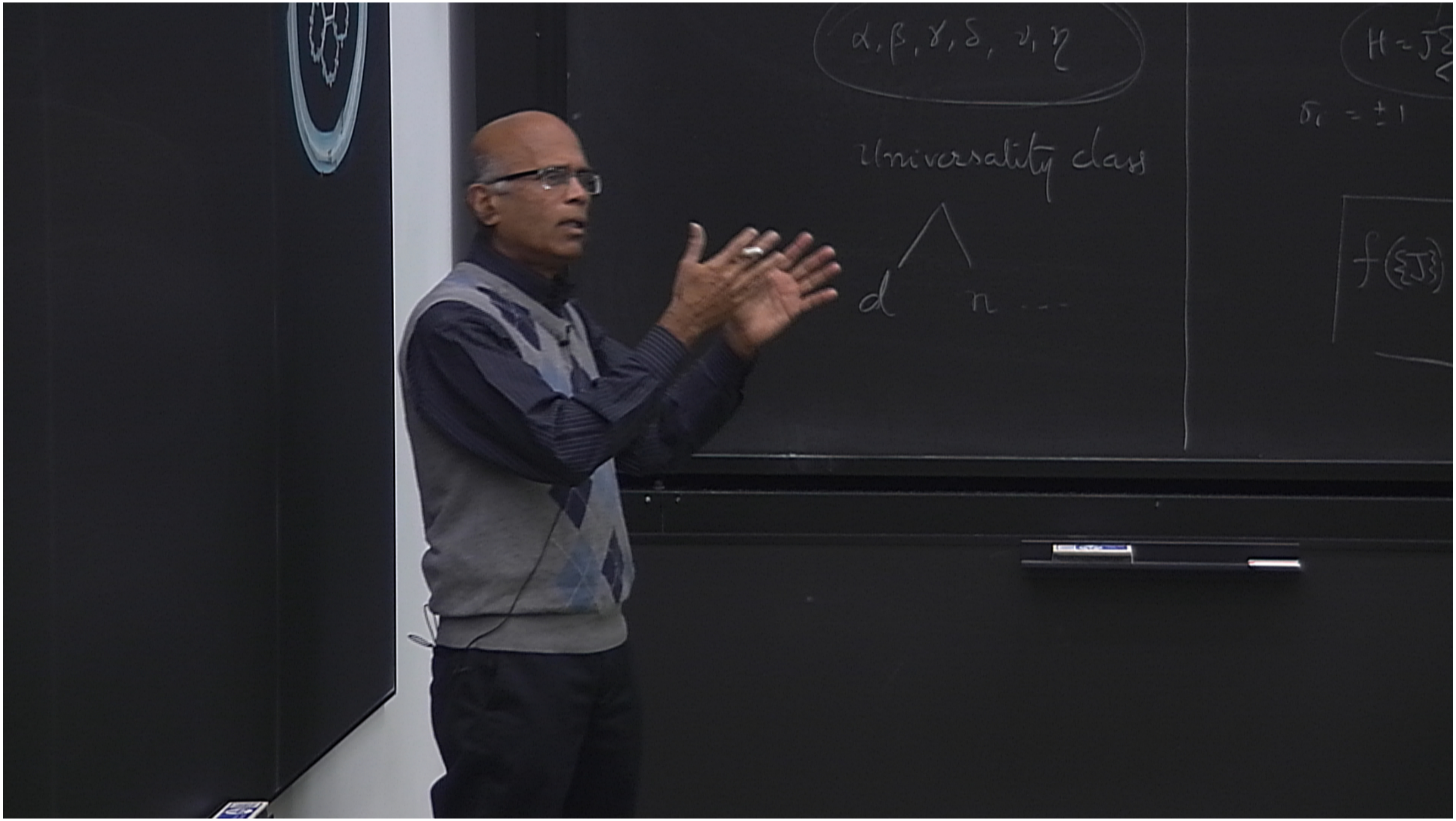
Title: PSI 2015/2016 Statistical Mechanics - G. Baskaran - Lecture 10

Date: Oct 26, 2015 10:45 AM

URL: http://pirsa.org/15100095

Abstract:


y_t , y_h
α, β, γ, δ, ν, η
Universality class
d
n . . .
H(J, J₁, J₂ . . .)
H = -J Σ S_i S_j + h Σ S_i
<ij>
S_i = ±1
(J J₁ J₂ . . .)
(J′ J₁′ J₂′)
J″ J″ J₃″
f({J}) = g({J}) + b^{-d} f({J′})
J^{(n+1)} = R(J^{(n)})



α, β, γ, δ, ν, η
Universality class
d
n ...
σᵢ = ±1

$H(J, J_1, J_2, \cdots)$

$H = -J \sum_{\langle ij \rangle} S_i S_j + h \sum S_i$

$S_i = \pm 1$

$(J \quad J_1 \quad J_2 \cdots$

$(J' \quad J_1' \quad J_2'$

$J'' \quad J'' \quad J_3''$

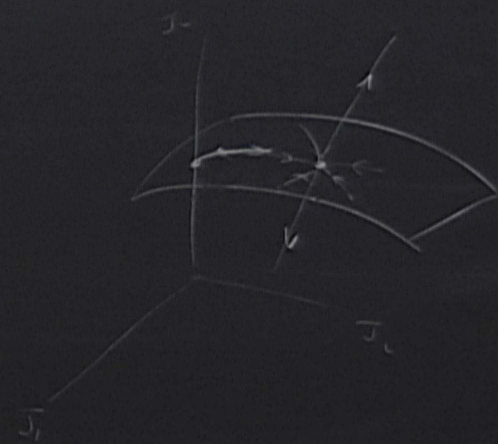
$$\boxed{f(\{J\}) = g(\{J\}) + b^{-d} f(\{J'\})}$$

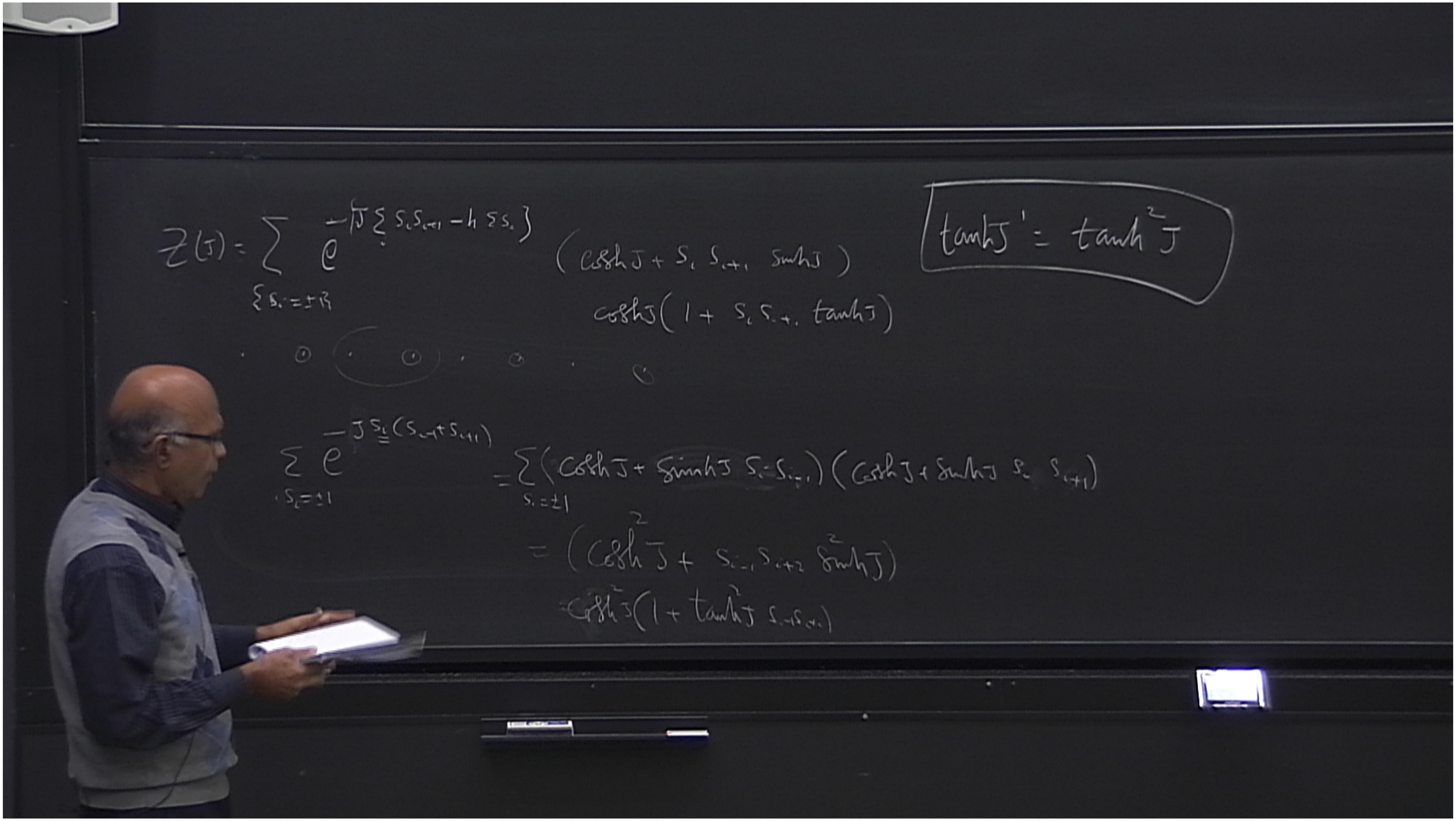
$$\boxed{J^{(n+1)} = R(J^{(n)})}$$

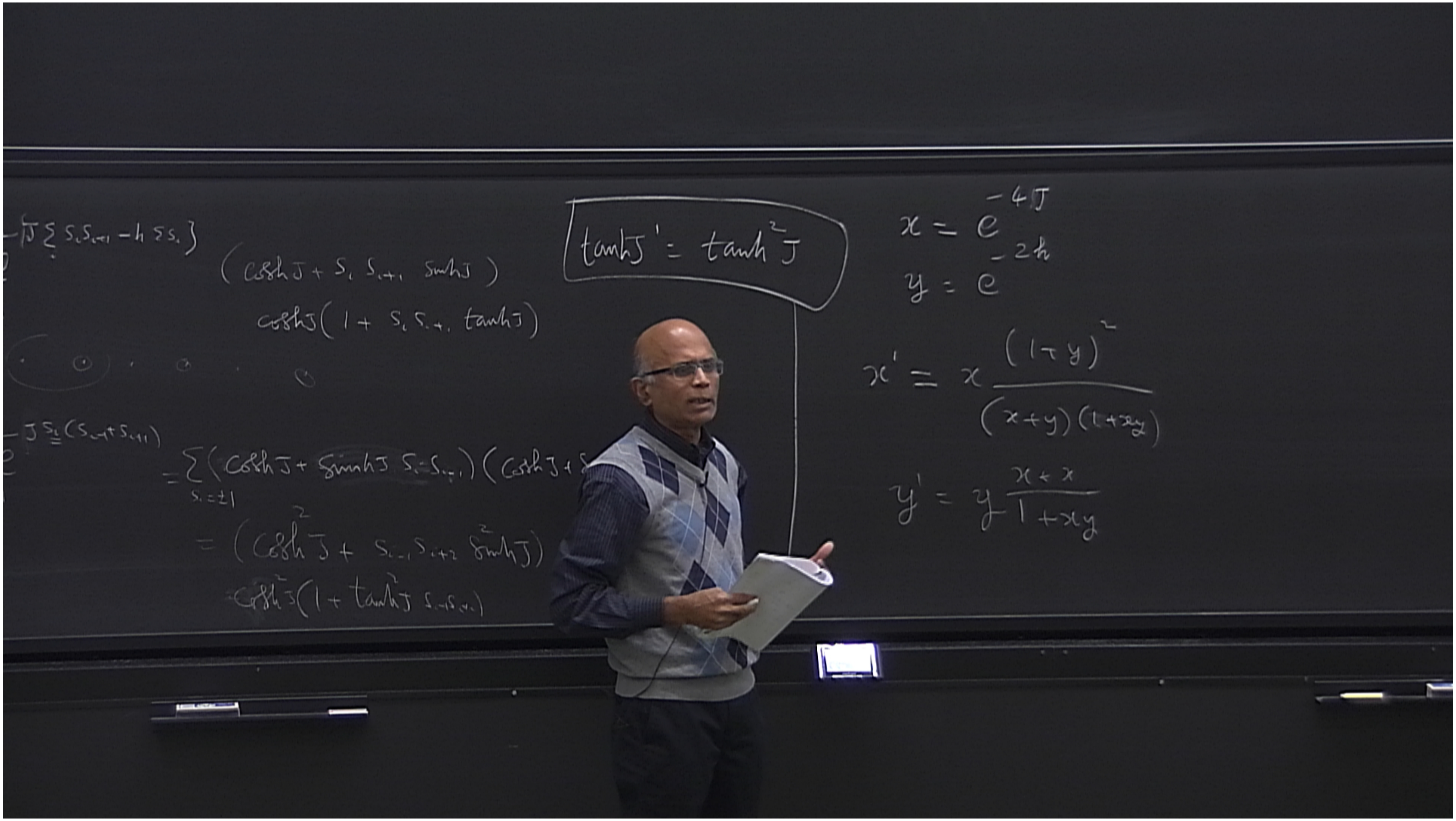
$x_{n+1} = f(x_n)$

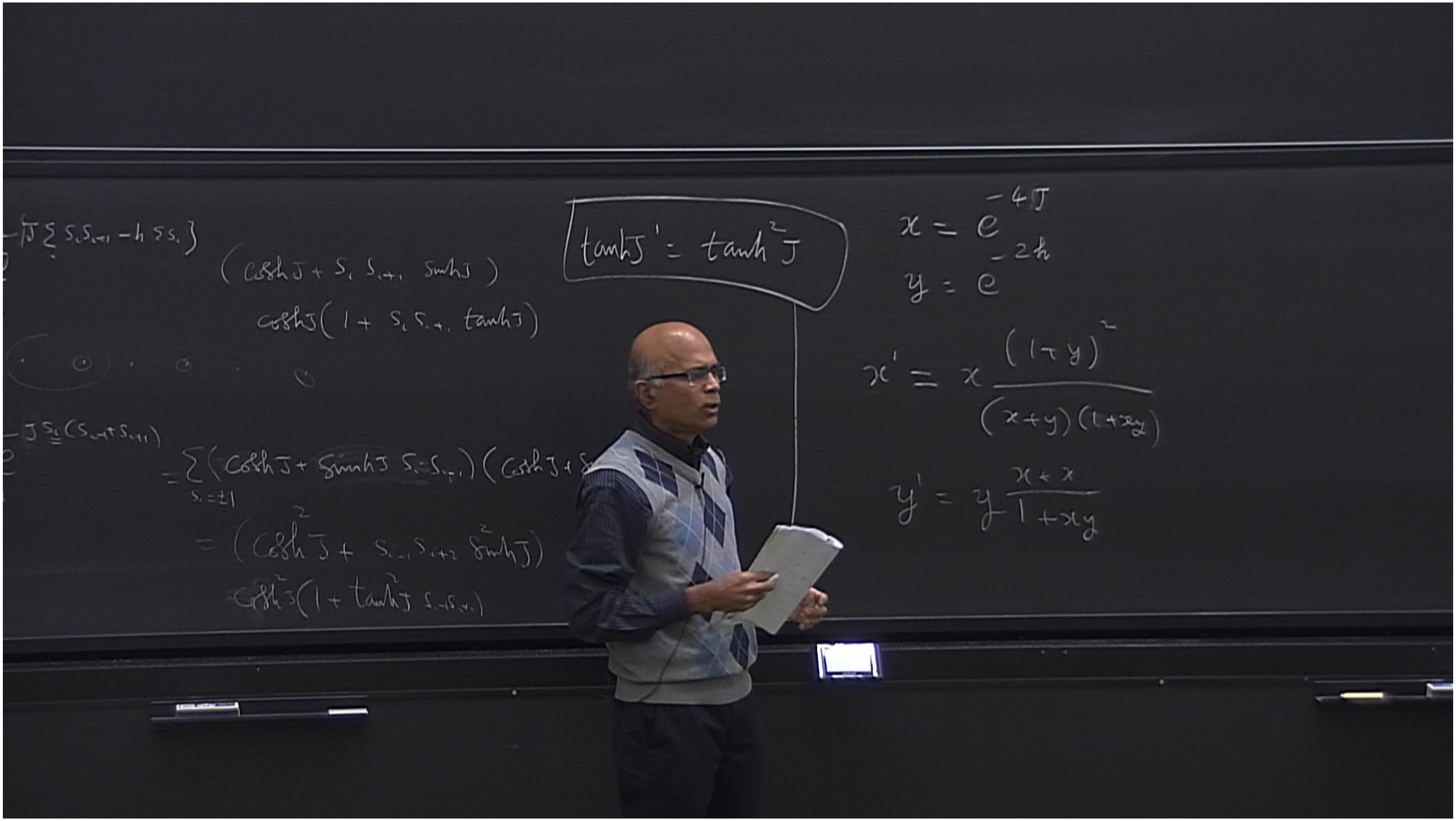
$$\boxed{x_{n+1} = \lambda x_n (1 - x_n)}$$

Lorenz




$Z(J) = \sum_{\{s_i = \pm 1\}} e^{-\beta J \sum_i s_i s_{i+1} - h \sum s_i}$
$(\cosh J + s_i s_{i+1} \sinh J)$
$\cosh J (1 + s_i s_{i+1} \tanh J)$
$\boxed{\tanh J' = \tanh^2 J}$
$\sum_{s_i = \pm 1} e^{-J s_i (s_{i-1} + s_{i+1})}$
$= \sum_{s_i = \pm 1} (\cosh J + \sinh J \, s_i s_{i-1})(\cosh J + \sinh J \, s_i s_{i+1})$
$= (\cosh^2 J + s_{i-1} s_{i+2} \sinh^2 J)$
$= \cosh^2 J (1 + \tanh^2 J \, s_{i-1} s_{i+1})$


tanh J' = tanh² J
x = e^{-4J}
y = e^{-2h}
x' = x (1+y)² / ((x+y)(1+xy))
y' = y (x+x) / (1+xy)
(cosh J + S_i S_{i+1} sinh J)
cosh J (1 + S_i S_{i+1} tanh J)
= (cosh² J + S_{i-1} S_{i+2} sinh² J)
cosh² J (1 + tanh² J S_{i-1} S_{i+1})


tanhJ' = tanh²J
x = e^{-4J}
y = e^{-2h}
x' = x (1+y)² / ((x+y)(1+xy))
y' = y (x+x) / (1+xy)
(coshJ + s_i s_{i+1} sinhJ)
coshJ(1 + s_i s_{i+1} tanhJ)
= (cosh²J + s_{i-1} s_{i+2} sinh²J)
cosh²J(1 + tanh²J s_{i-1} s_{i+1})

$$x = e^{-4J}$$

$$y = e^{-2h}$$

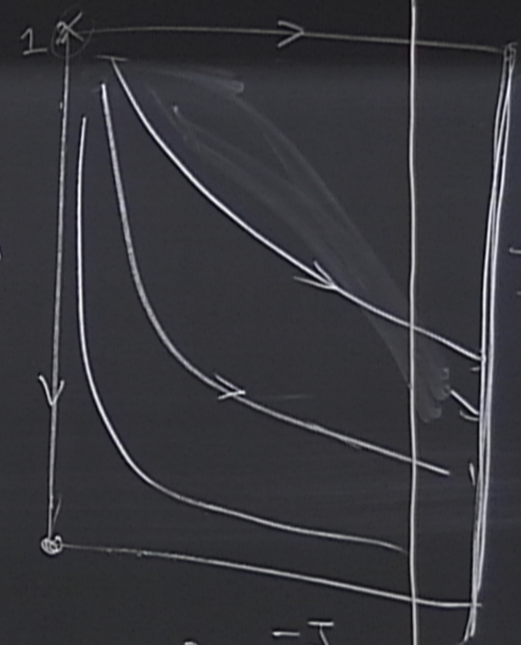
$$x' = x\,\frac{(1+y)^2}{(x+y)(1+xy)}$$

$$y' = y\,\frac{x+x}{1+xy}$$

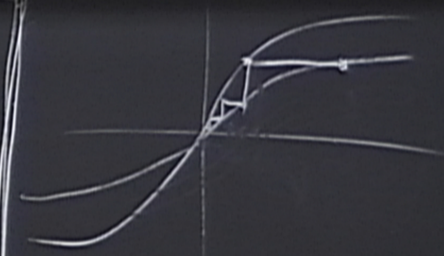
$(0,1)$

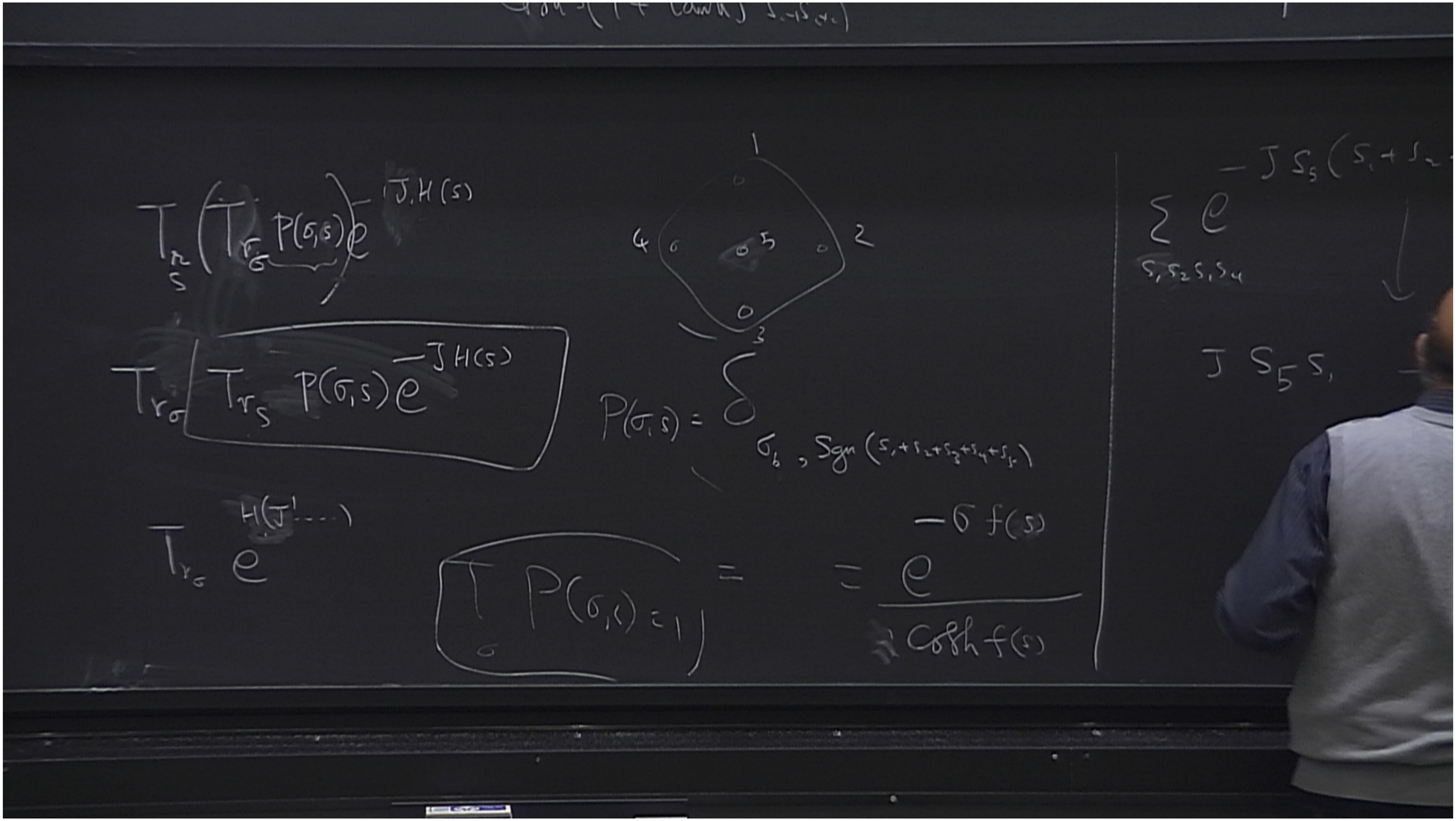
$(1,1)$

$y = e^{-h}$



$x = e^{-J}$




Tr_s (Tr_σ P(σ,s)) e^{-J H(s)}
Tr_σ [Tr_s P(σ,s) e^{-J H(s)}]
Tr_σ e^{H(J' ...)}
4 2 5 1 3
P(σ,s) = δ_{σ_5, Sgn(s_1+s_2+s_3+s_4+s_5)}
Tr_σ P(σ,s) = 1
= e^{-σ f(s)} / 2 cosh f(s)
Σ_{s_1 s_2 s_3 s_4} e^{-J S_5 (S_1 + S_2 ...}
J S_5 S_1

$$\sum_{s_1 s_2 s_3 s_4} e^{-J s_5 (s_1 + s_2 + s_3 + s_4)}$$

$$J s_5 s_1 \longrightarrow$$

$$J_4 s_1 s_2 s_3 s_4$$

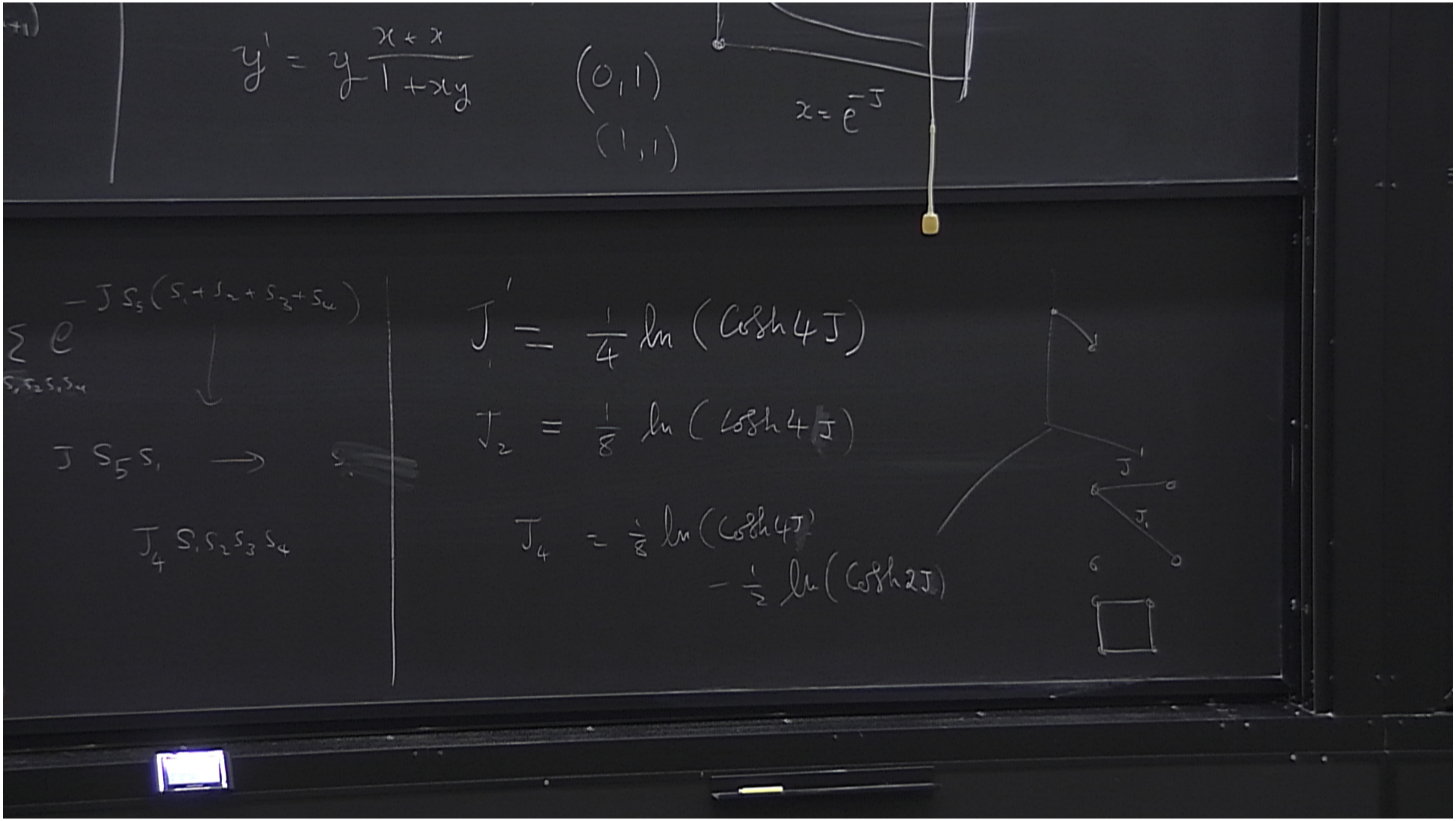
$$J' =$$

$$J_2 =$$

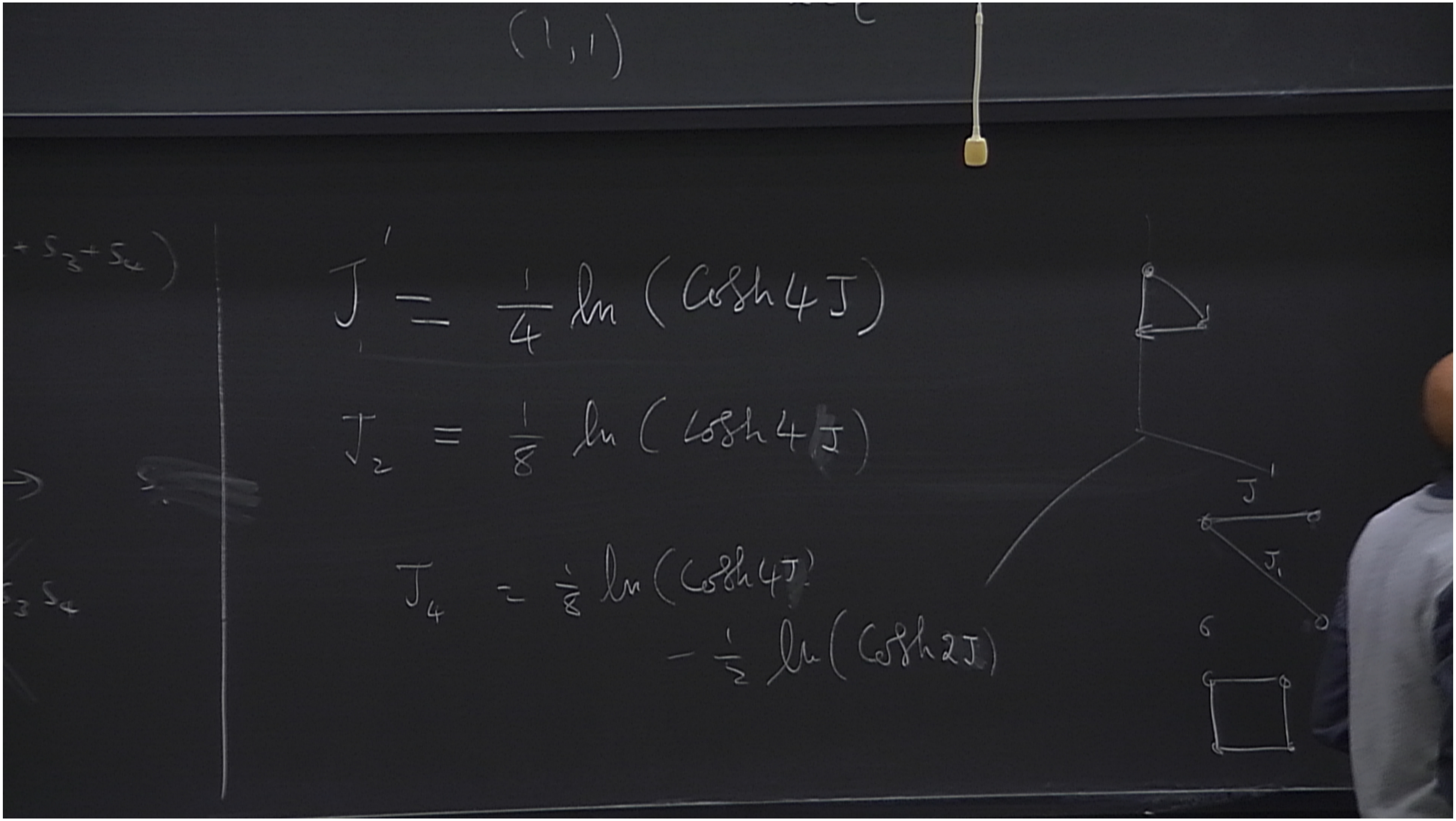
$$J_4 =$$

$$+ s_2 + s_3 + s_4 + s_5)$$

$$- \sigma f(s)$$



$y' = y\frac{x+x}{1+xy}$
$(0,1)$
$(1,1)$
$x = e^{-J}$
$\sum_{s_1 s_2 s_3 s_4} e^{-J s_5 (s_1+s_2+s_3+s_4)}$
$J s_5 s_1 \longrightarrow$
$J_4 s_1 s_2 s_3 s_4$
$J' = \frac{1}{4} \ln(\cosh 4J)$
$J_2 = \frac{1}{8} \ln(\cosh 4J)$
$J_4 = \frac{1}{8} \ln(\cosh 4J) - \frac{1}{2} \ln(\cosh 2J)$
J'
J_1

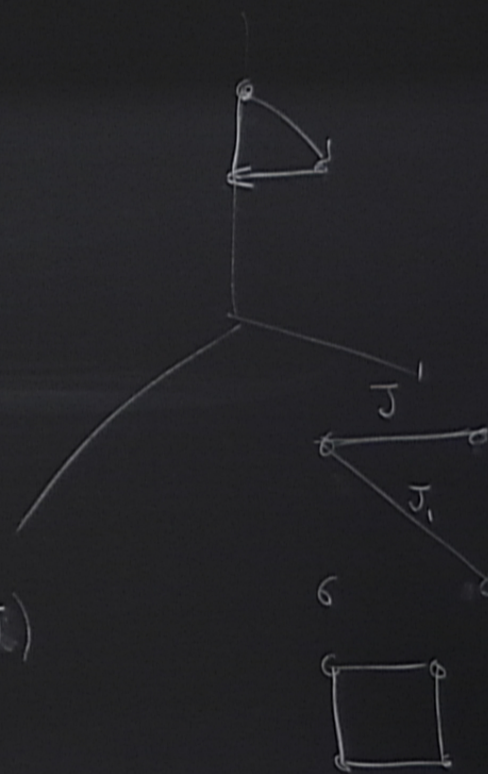

(1, 1)
$J' = \frac{1}{4} \ln(\cosh 4J)$
$J_2 = \frac{1}{8} \ln(\cosh 4J)$
$J_4 = \frac{1}{8} \ln(\cosh 4J) - \frac{1}{2} \ln(\cosh 2J)$
J'
J_1

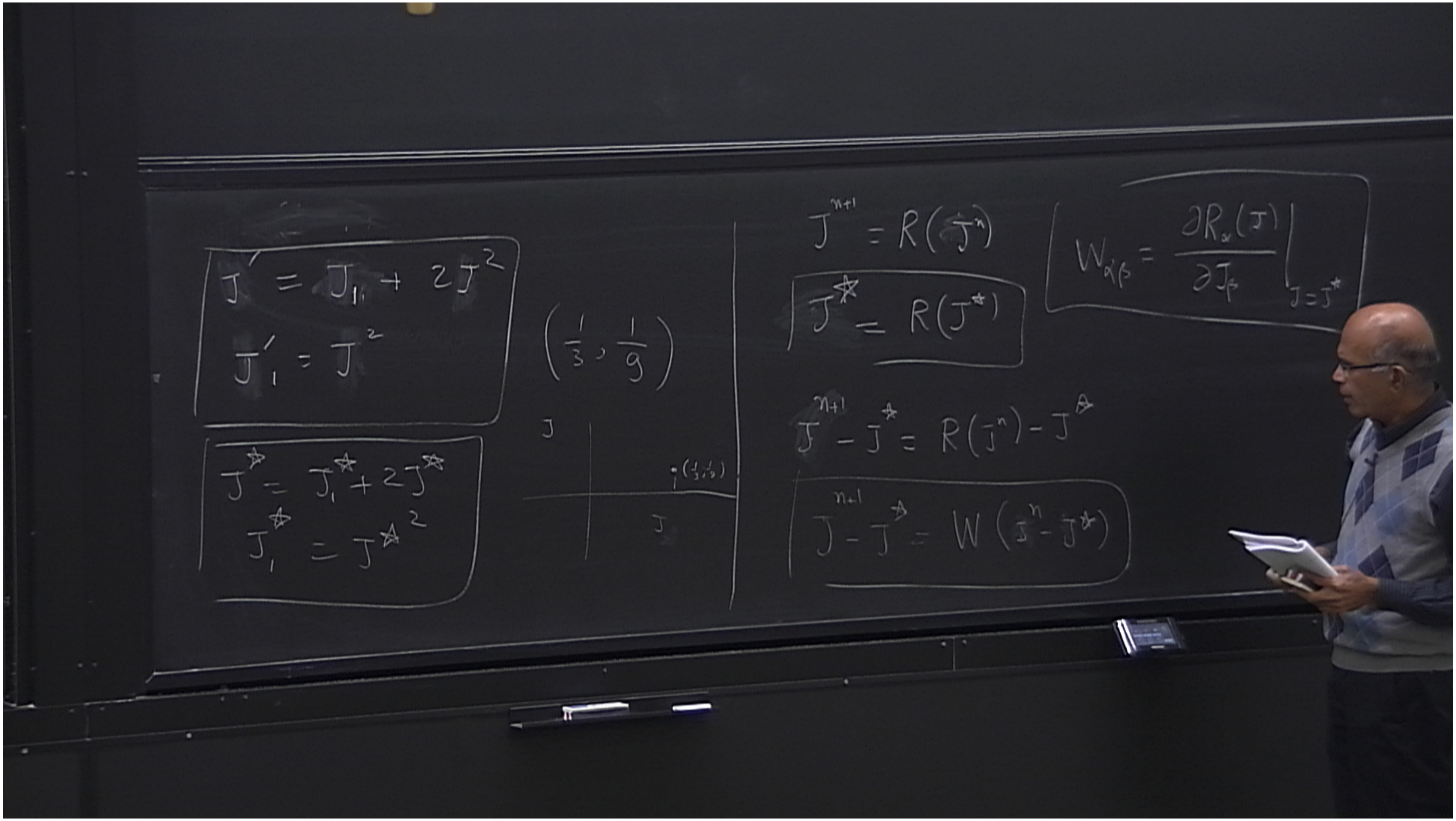
$(1,1)$

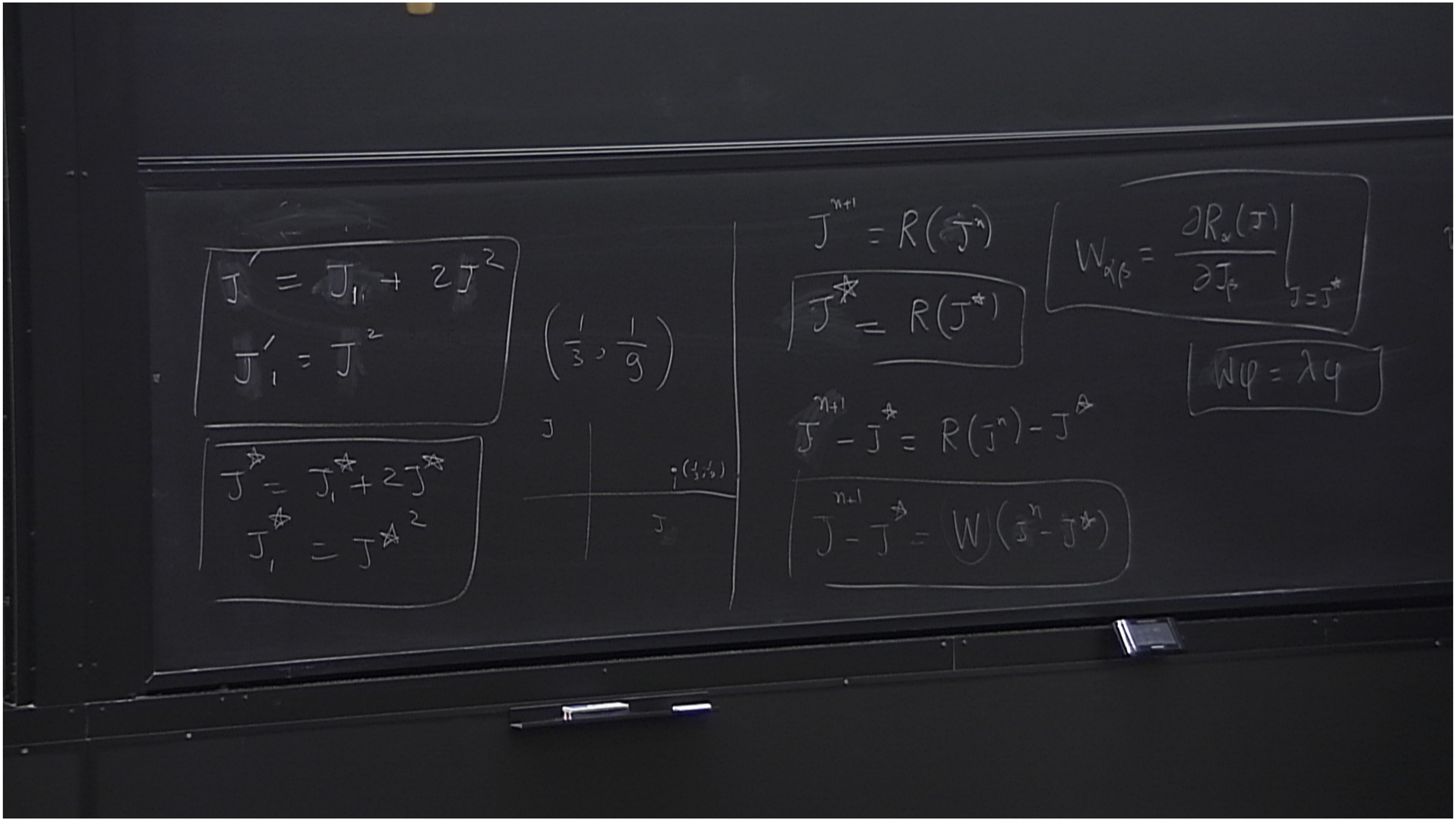
$$J' = \frac{1}{4}\ln(\cosh 4J)$$

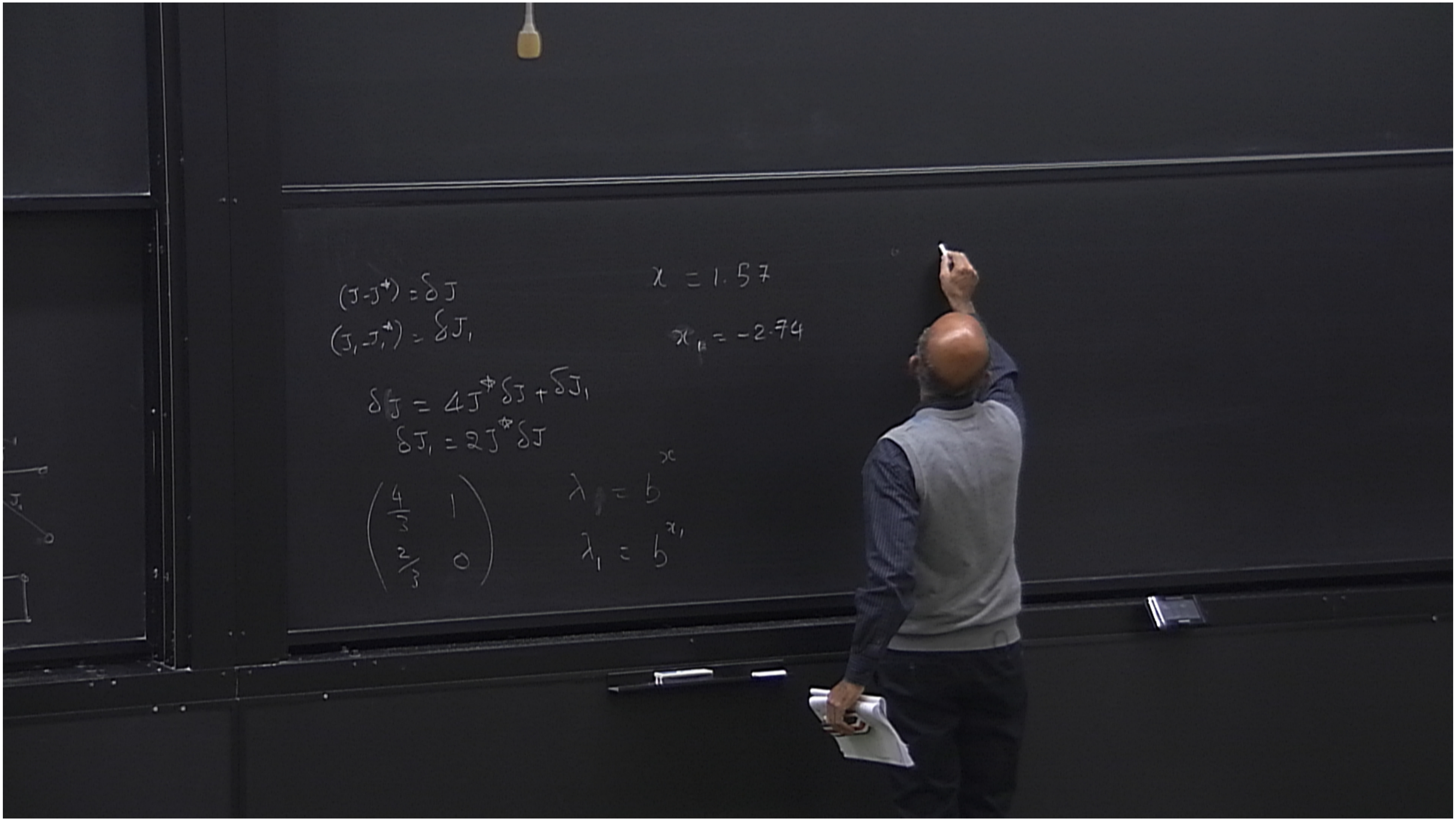
$$J_2 = \frac{1}{8}\ln(\cosh 4J)$$

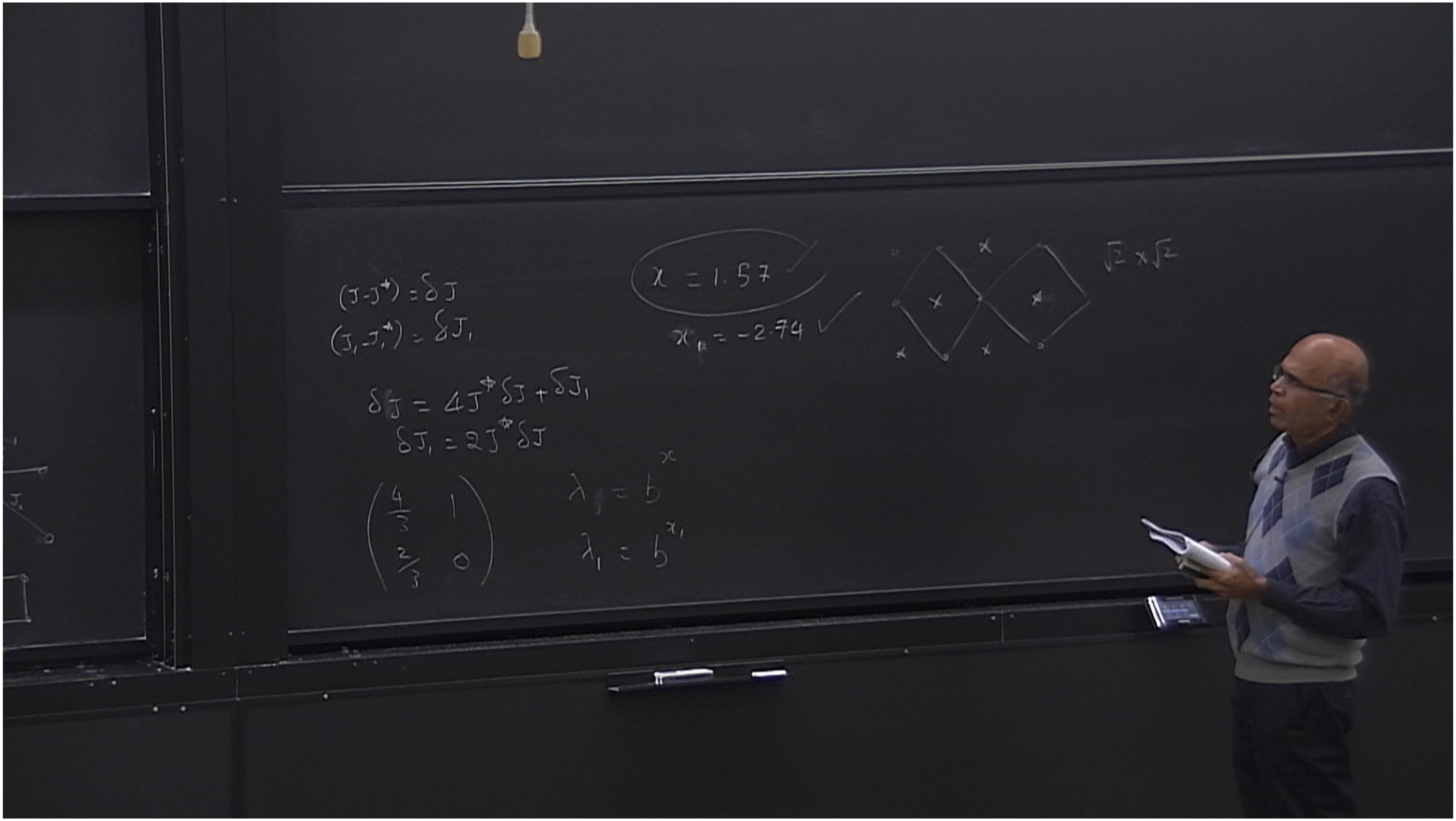
$$J_4 = \frac{1}{8}\ln(\cosh 4J) - \frac{1}{2}\ln(\cosh 2J)$$

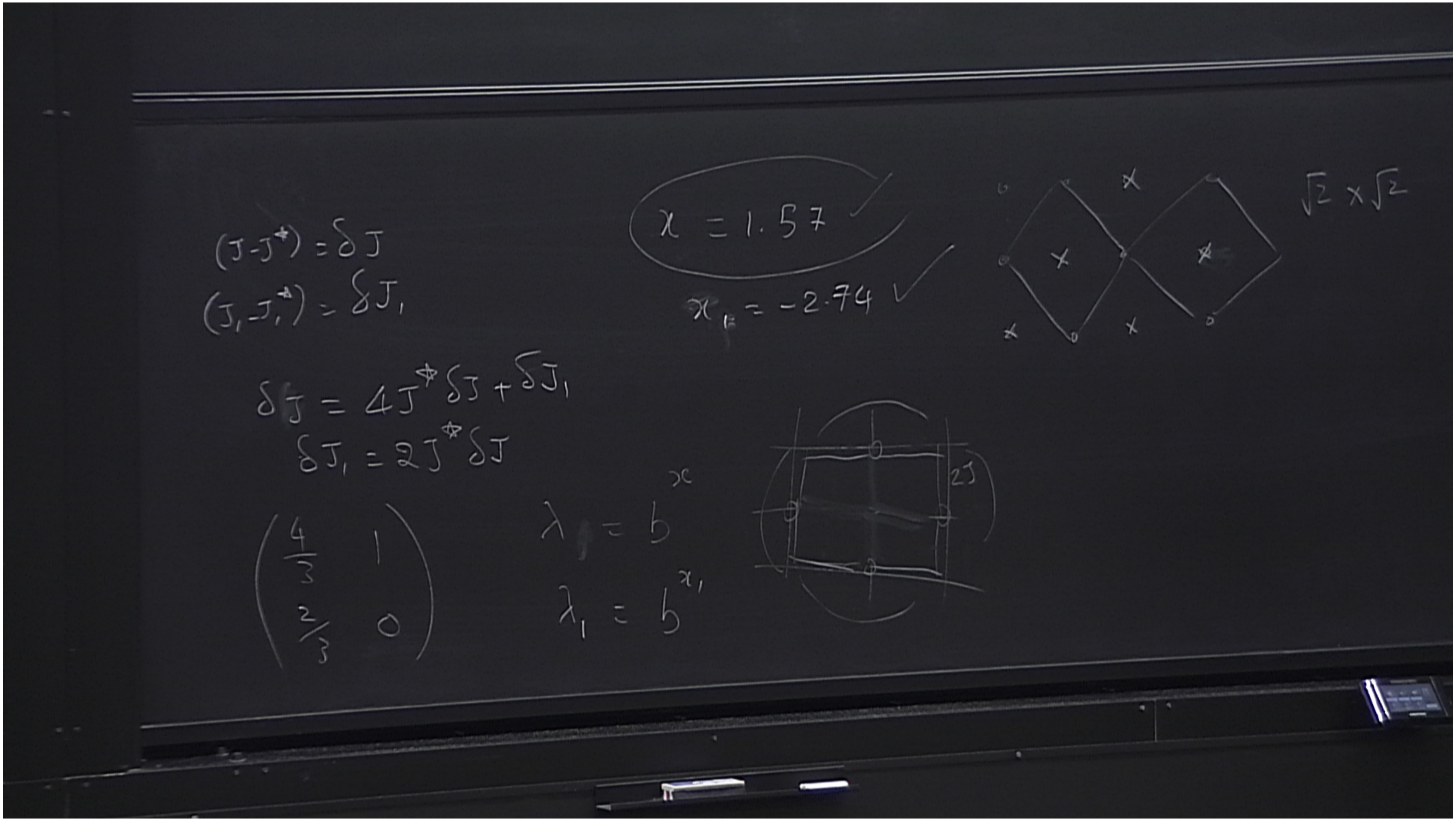


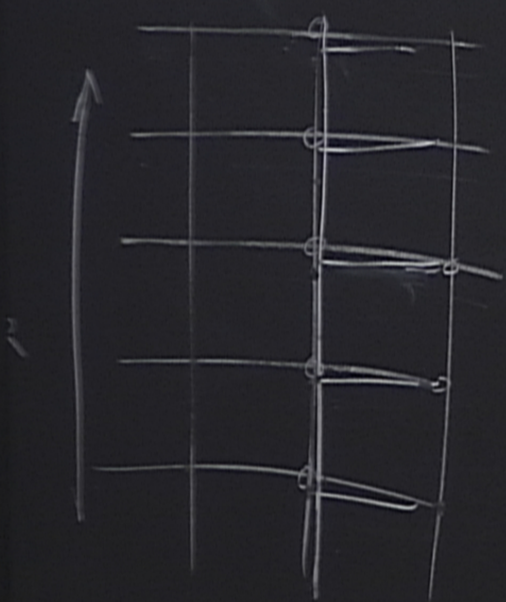

$J' = J_1 + 2J^2$
$J_1' = J^2$
$J^* = J_1^* + 2J^{*2}$
$J_1^* = J^{*2}$
$\left(\frac{1}{3}, \frac{1}{9}\right)$
$J^{n+1} = R(J^n)$
$J^* = R(J^*)$
$W_{\alpha\beta} = \frac{\partial R_\alpha(J)}{\partial J_\beta}\Big|_{J=J^*}$
$J^{n+1} - J^* = R(J^n) - J^*$
$J^{n+1} - J^* = W(J^n - J^*)$


$J' = J_1 + 2J^2$
$J_1' = J^2$
$J^* = J_1^* + 2J^{*2}$
$J_1^* = J^{*2}$
$\left(\frac{1}{3}, \frac{1}{9}\right)$
J
J
$\left(\frac{1}{3}, \frac{1}{9}\right)$
$J^{n+1} = R(J^n)$
$J^* = R(J^*)$
$W_{\alpha\beta} = \frac{\partial R_\alpha(J)}{\partial J_\beta}\Big|_{J=J^*}$
$W\varphi = \lambda\varphi$
$J^{n+1} - J^* = R(J^n) - J^*$
$J^{n+1} - J^* = W(J^n - J^*)$


(J-J*) = δJ
(J₁-J₁*) = δJ₁
δJ = 4J* δJ + δJ₁
δJ₁ = 2J* δJ
4/3 1
2/3 0
x = 1.57
x₁ = -2.74
λ = b^x
λ₁ = b^x₁


(J-J*) = δJ
(J1-J1*) = δJ1
δJ = 4J* δJ + δJ1
δJ1 = 2J* δJ
x = 1.57
x1 = -2.74
λ = b^x
λ1 = b^x1
√2 x √2


$(J - J^*) = \delta J$
$(J_1 - J_1^*) = \delta J_1$
$x = 1.57$
$x_1 = -2.74$
$\sqrt{2} \times \sqrt{2}$
$\delta J = 4J^* \delta J + \delta J_1$
$\delta J_1 = 2J^* \delta J$
$2J$
$\begin{pmatrix} \frac{4}{3} & 1 \\ \frac{2}{3} & 0 \end{pmatrix}$
$\lambda = b^{x}$
$\lambda_1 = b^{x_1}$



i

2^M

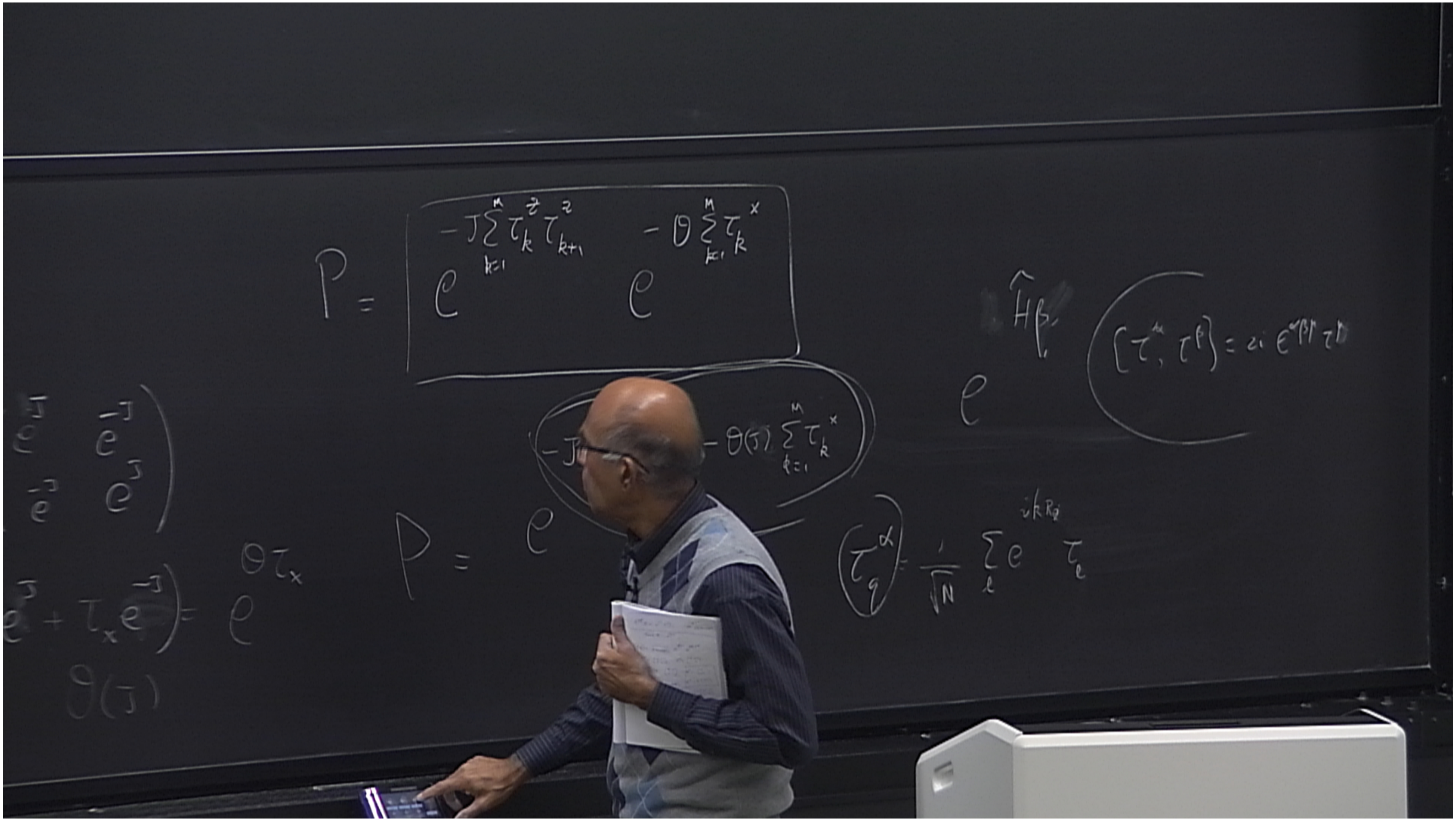
$$P_{\{s\}\{s_{k+1}\}} \; P = \mathrm{Tr}\, P^M$$

$$P =$$

$$\boxed{e^{-J\left(\sum_k s_k s_{k+1} + \sum s_k s'_k\right)}} \qquad \begin{pmatrix} e^{J} & e^{-J} \\ e^{-J} & e^{J} \end{pmatrix}$$

$$e^{-J\sum} \qquad \boxed{\tanh\theta = e^{-2J}} \qquad \left(I e^{J} + \tau_x e^{-J}\right) = e^{\theta \tau_x}$$

$$\theta(J)$$


$P = e^{-J\sum_{k=1}^{M}\tau_k^z\tau_{k+1}^z} \; e^{-\theta\sum_{k=1}^{M}\tau_k^x}$
$\tau_q^\alpha = \frac{1}{\sqrt{N}}\sum_\ell e^{ikR_\ell}\tau_\ell$

$$e^{\hat{H}_{p}}$$

$$[\tau^\alpha, \tau^\beta] = 2i\,\epsilon^{\alpha\beta\gamma}\tau^\gamma$$

$$P = e^{-J\sum_{k=1}^{n}\tau^z_k\tau^z_{k+1} - \theta(J)\sum_{k=1}^{n}\tau^x_k}$$

$$\tau^\alpha_q = \frac{1}{\sqrt{N}}\sum_\ell e^{ikR_\ell}\tau_\ell$$

$$= e^{\theta\tau_x}$$

$$\theta(J)$$