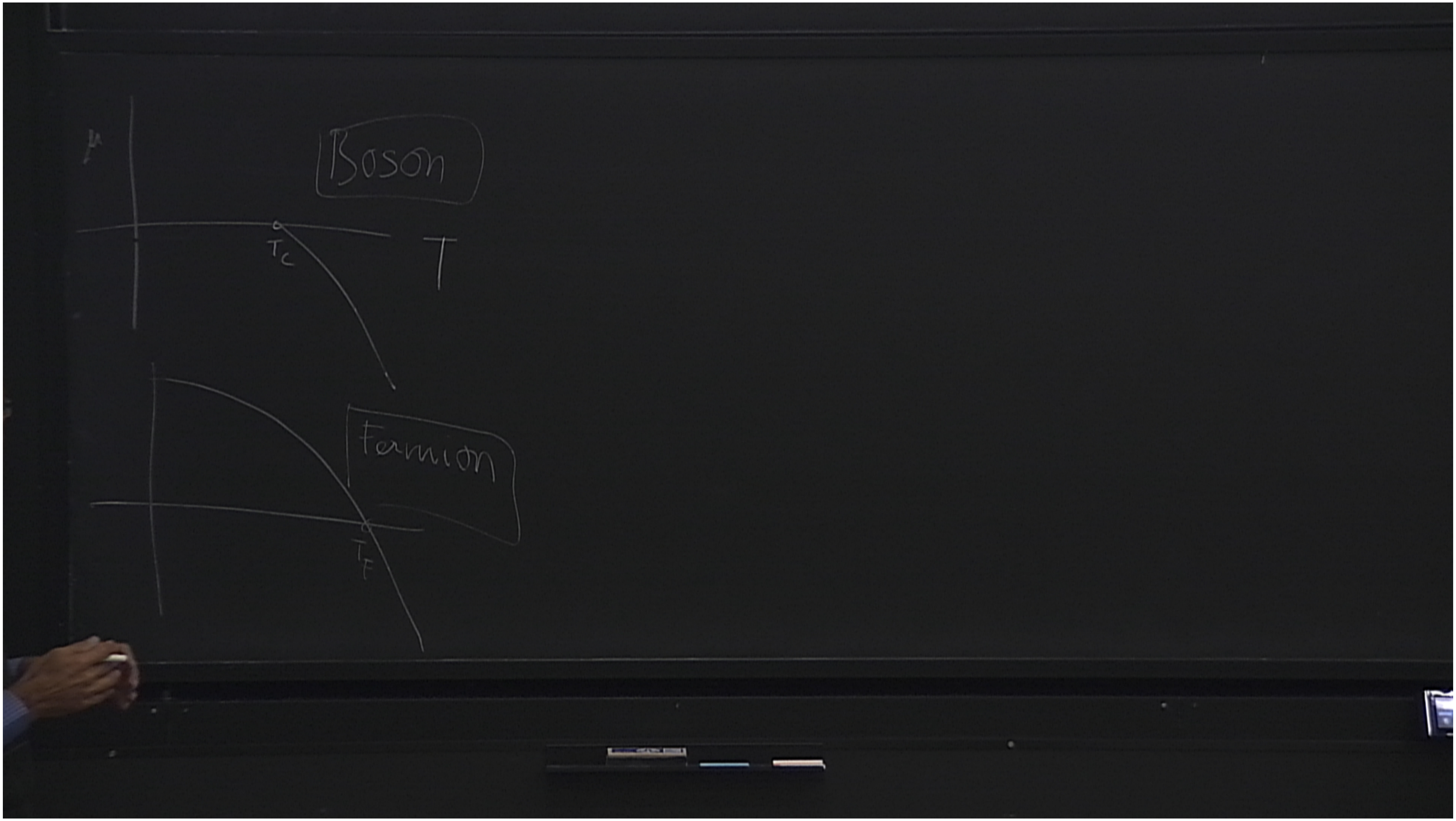
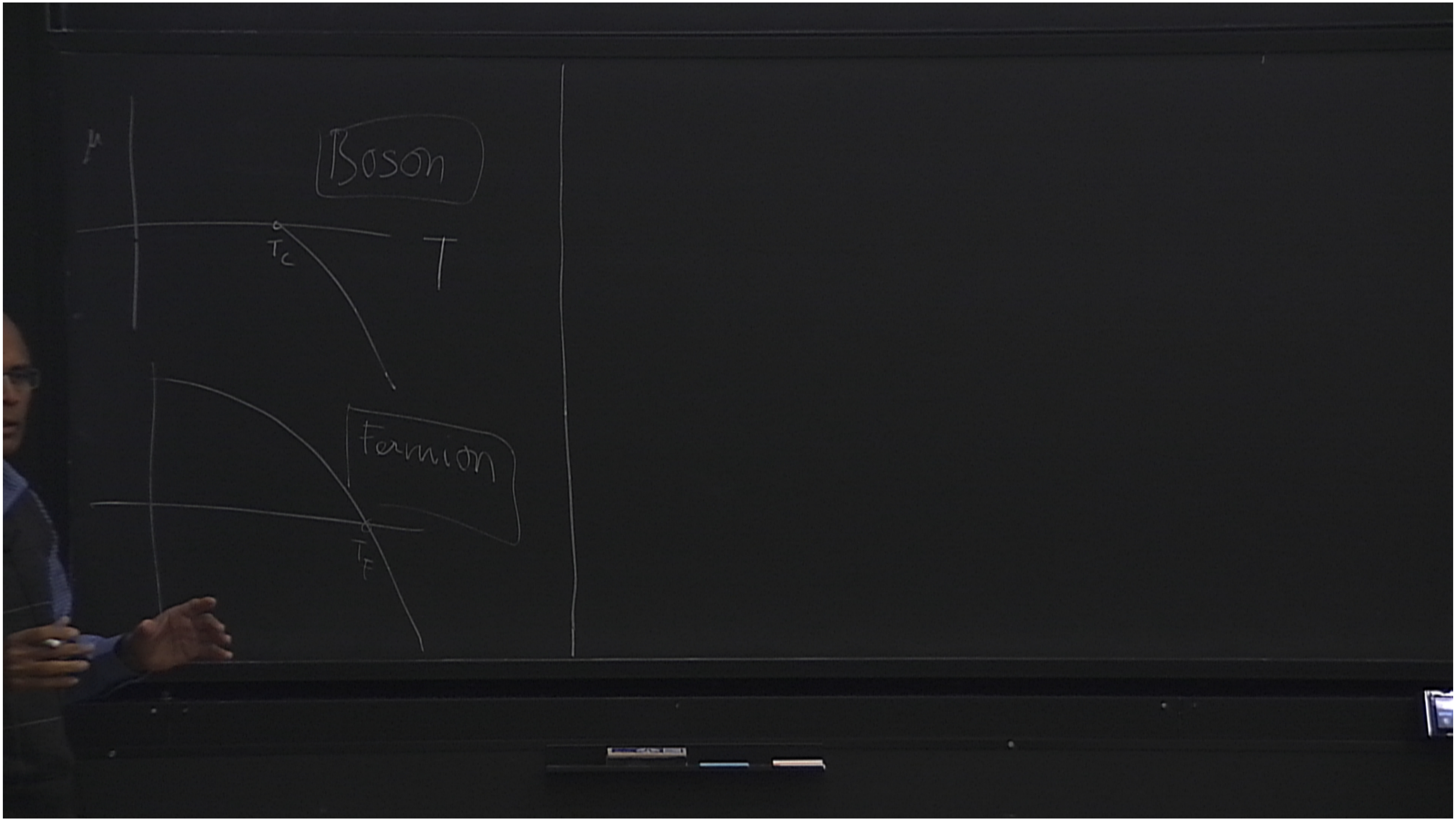
Title: PSI 2015/2016 Statistical Mechanics - G. Baskaran - Lecture 7

Date: Oct 21, 2015 10:45 AM

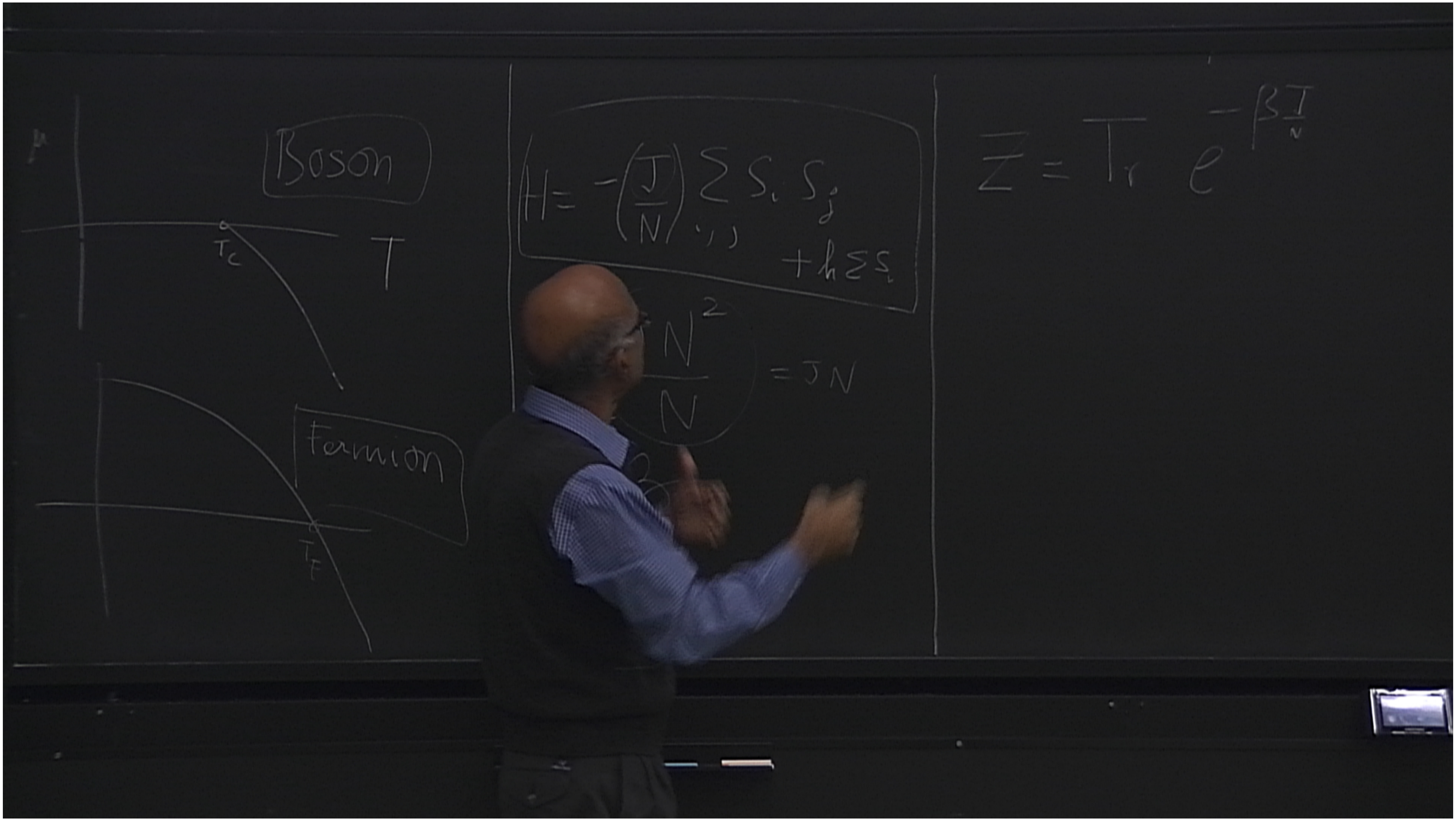
URL: http://pirsa.org/15100092

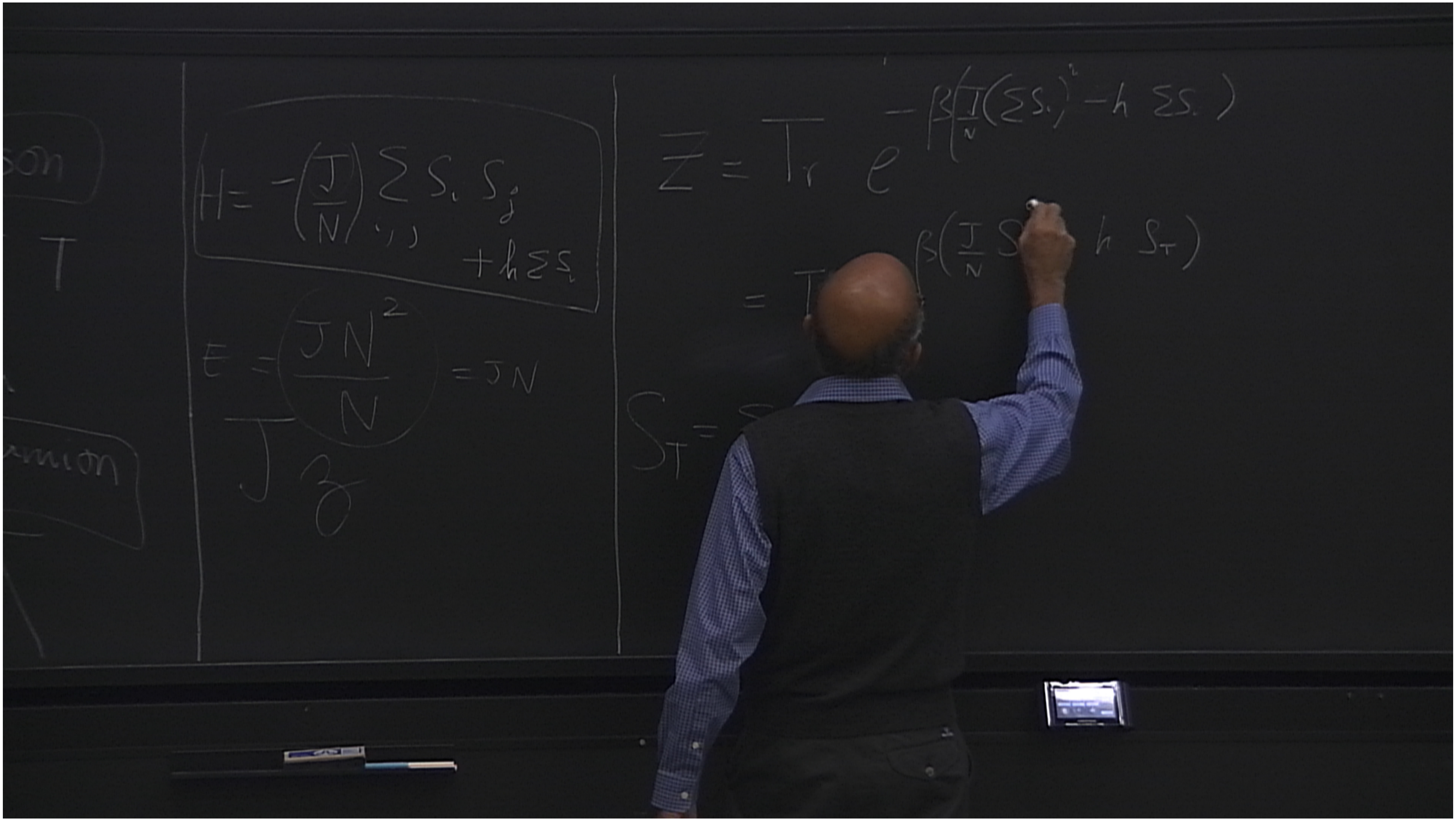
Abstract:


μ
Boson
T_c
T
Fermion
T_F

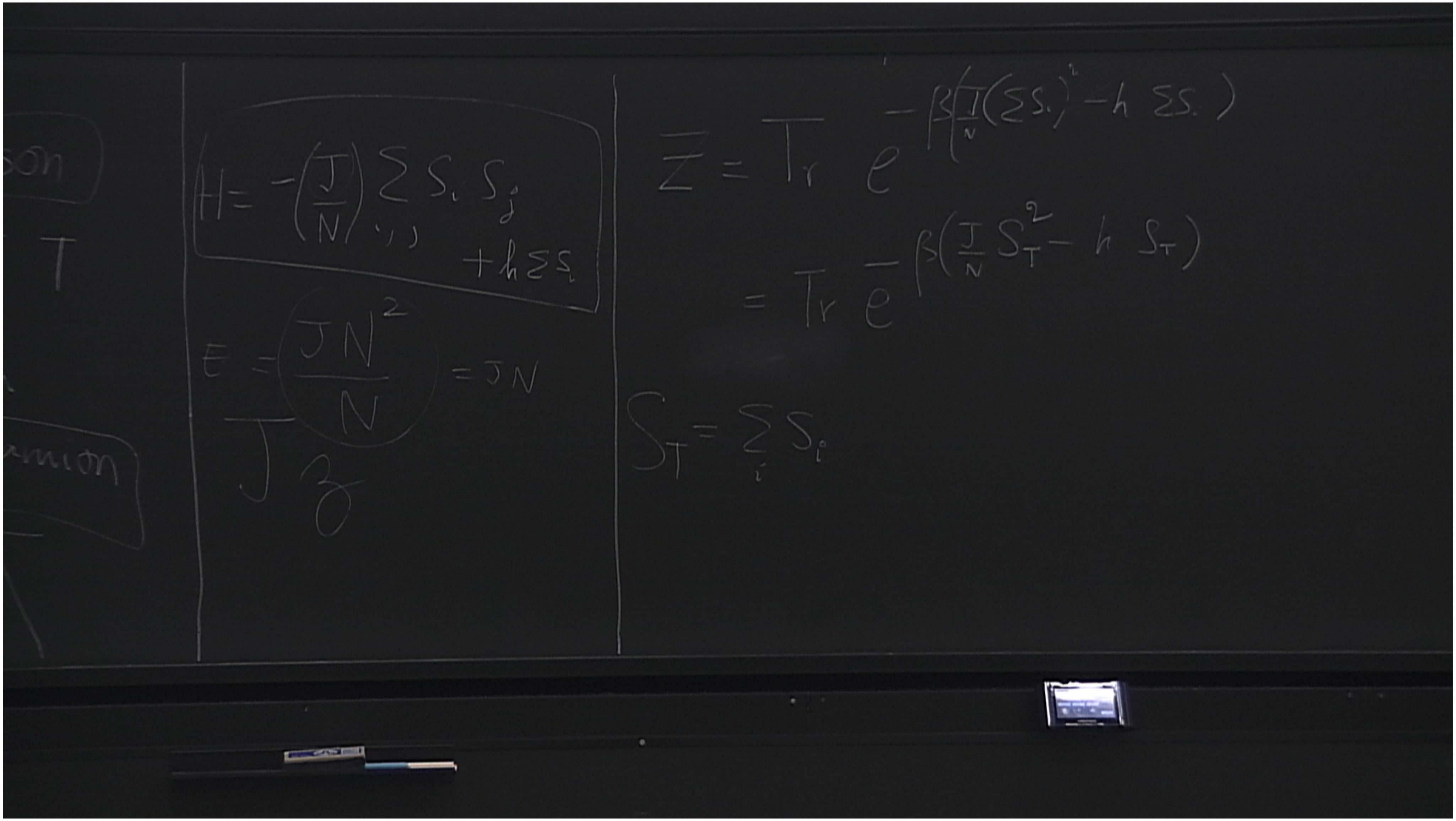


μ
Boson
T_c
T
Fermion
T_F


Boson
T_c
T
Fermion
T_F
$H = -\left(\frac{J}{N}\right) \sum_{i,j} S_i S_j + h \sum S_i$
$= J N$
$Z = \mathrm{Tr}\, e^{-\beta H}$



H = -(J/N) Σ S_i S_j
+ h Σ S_i
E = (JN)^2/N = JN
Z = Tr e^{-β(J/N (Σ S_i)^2 - h Σ S_i)}
S_T =


son
T
rmion
$H = -\left(\frac{J}{N}\right) \sum_{i,j} S_i S_j + h \sum S_i$
$E = \frac{JN^2}{N} = JN$
$Z = \mathrm{Tr}\, e^{-\beta\left(\frac{J}{N}\left(\sum S_i\right)^2 - h \sum S_i\right)}$
$= \mathrm{Tr}\, e^{-\beta\left(\frac{J}{N} S_T^2 - h S_T\right)}$
$S_T = \sum_i S_i$

$\Sigma S_i)$

$_T)$

$-2ax)$

$dx = e^{+\frac{a^2}{2\sigma^2}}$

$$Z = \sqrt{\frac{\beta J}{2\pi N}} \int_{\mathbb{R}} dt \; e^{-\frac{N}{2\beta J} t^2 + (h\beta + t) S_T}$$

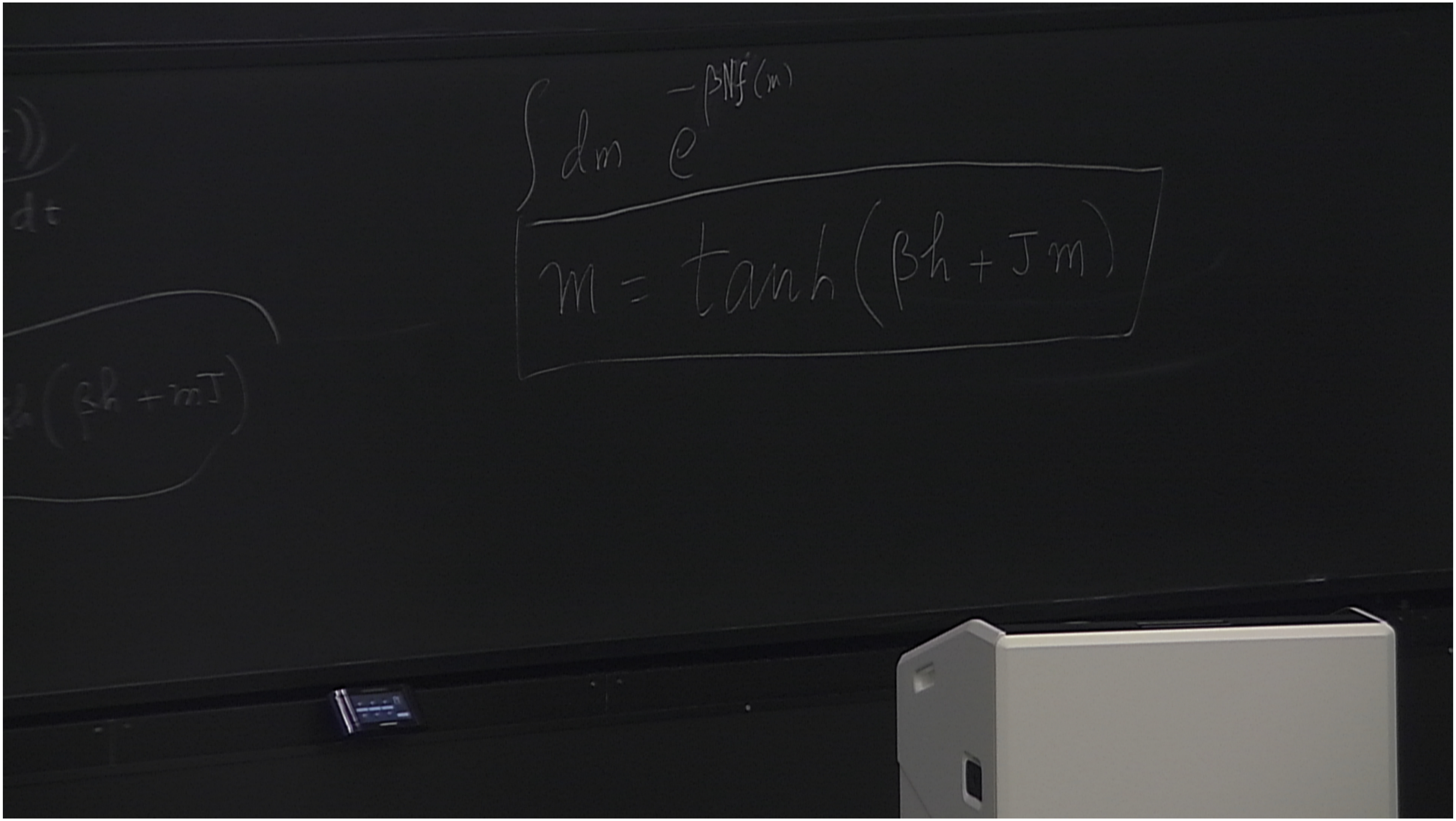
$$= \quad \cdots \quad \mathrm{Tr}\, e^{-(h\beta+t)\sum S_i}$$

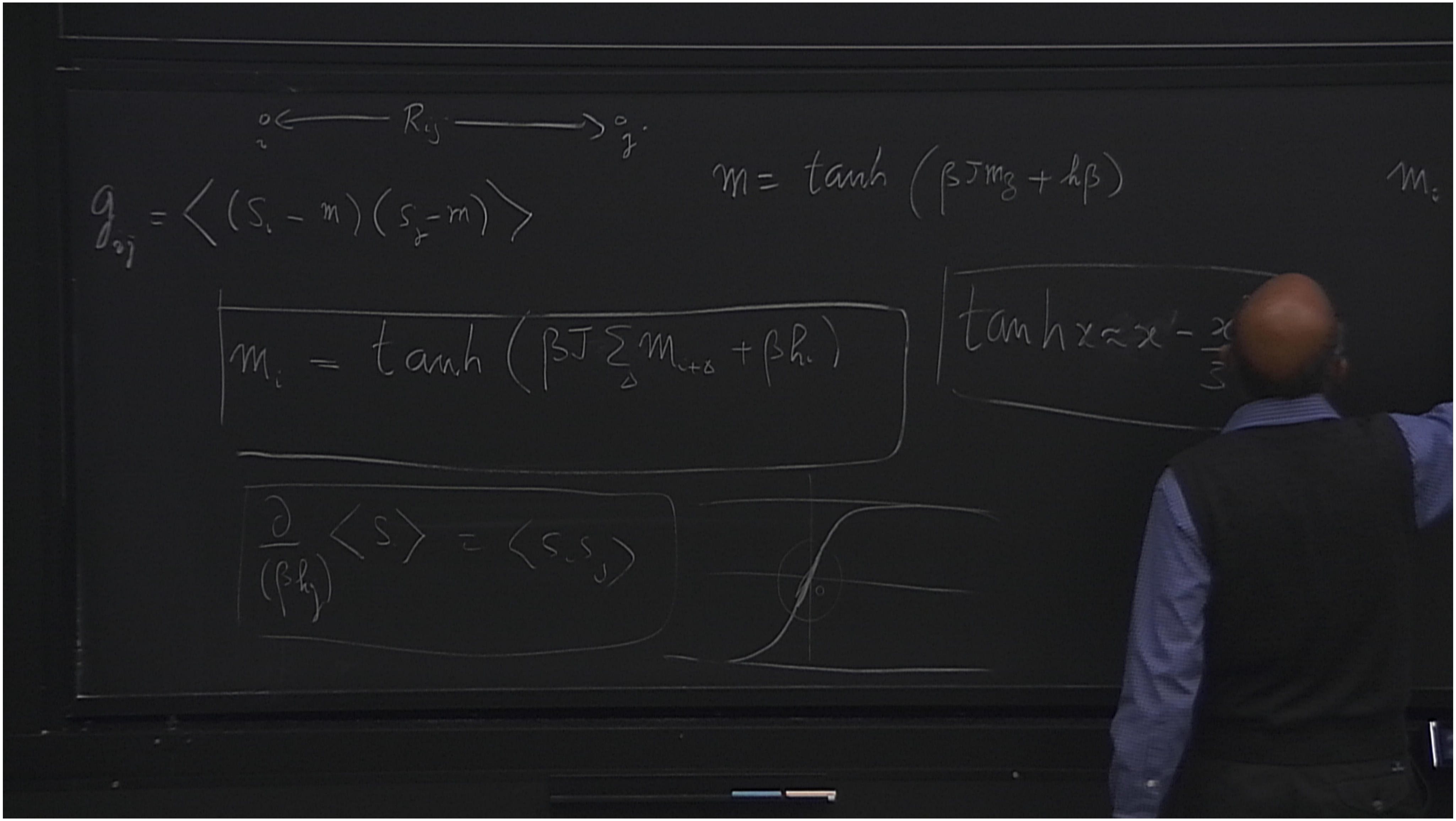
$$= \left(2\cosh(\beta h + t)\right)^N$$

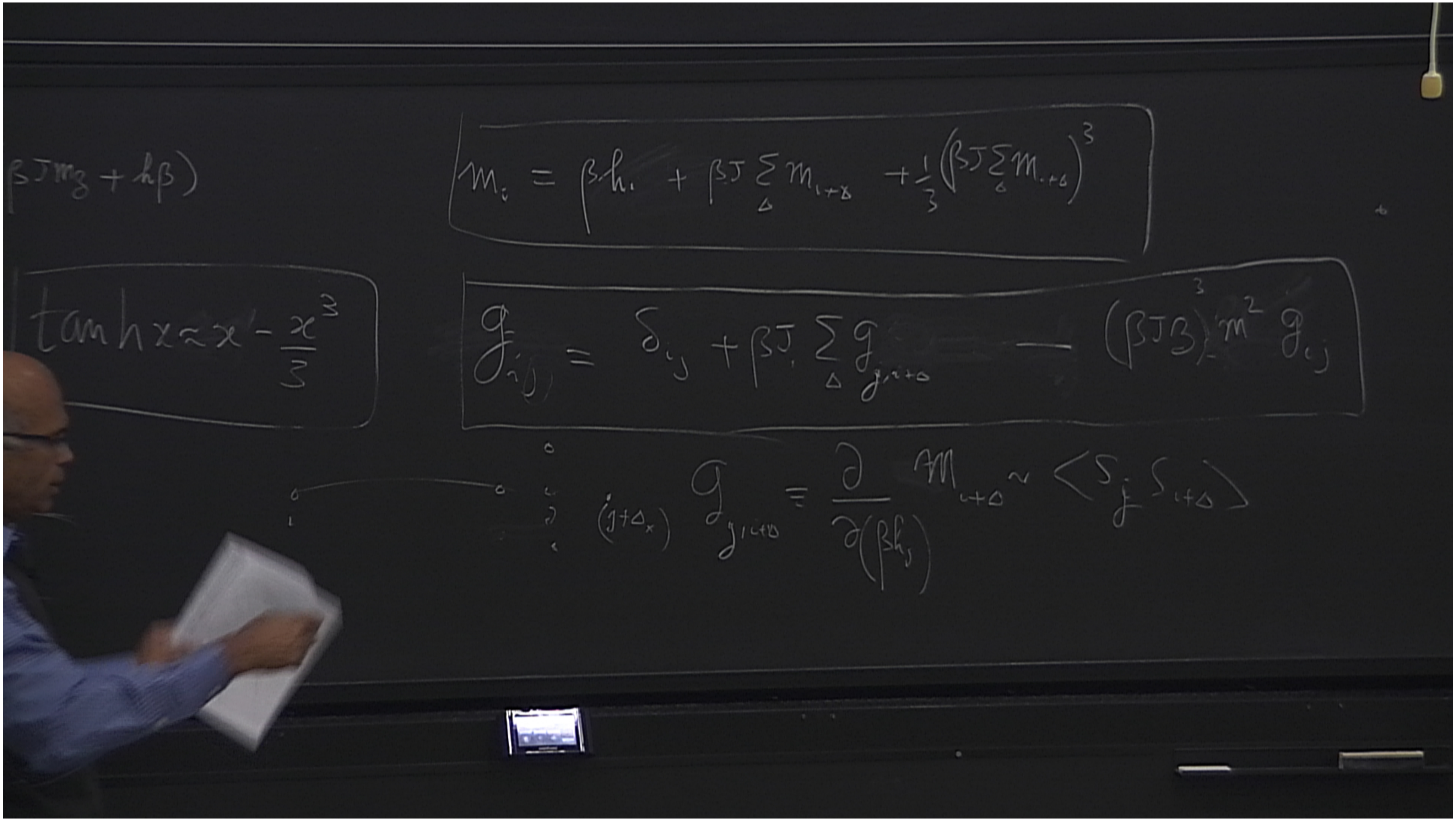
$$Z = \sqrt{\frac{J\beta}{2\pi N}} \int dt \; e^{-N\left(\frac{t^2}{\beta J} - kT \log \cosh(\beta h + t)\right)} \, dt$$

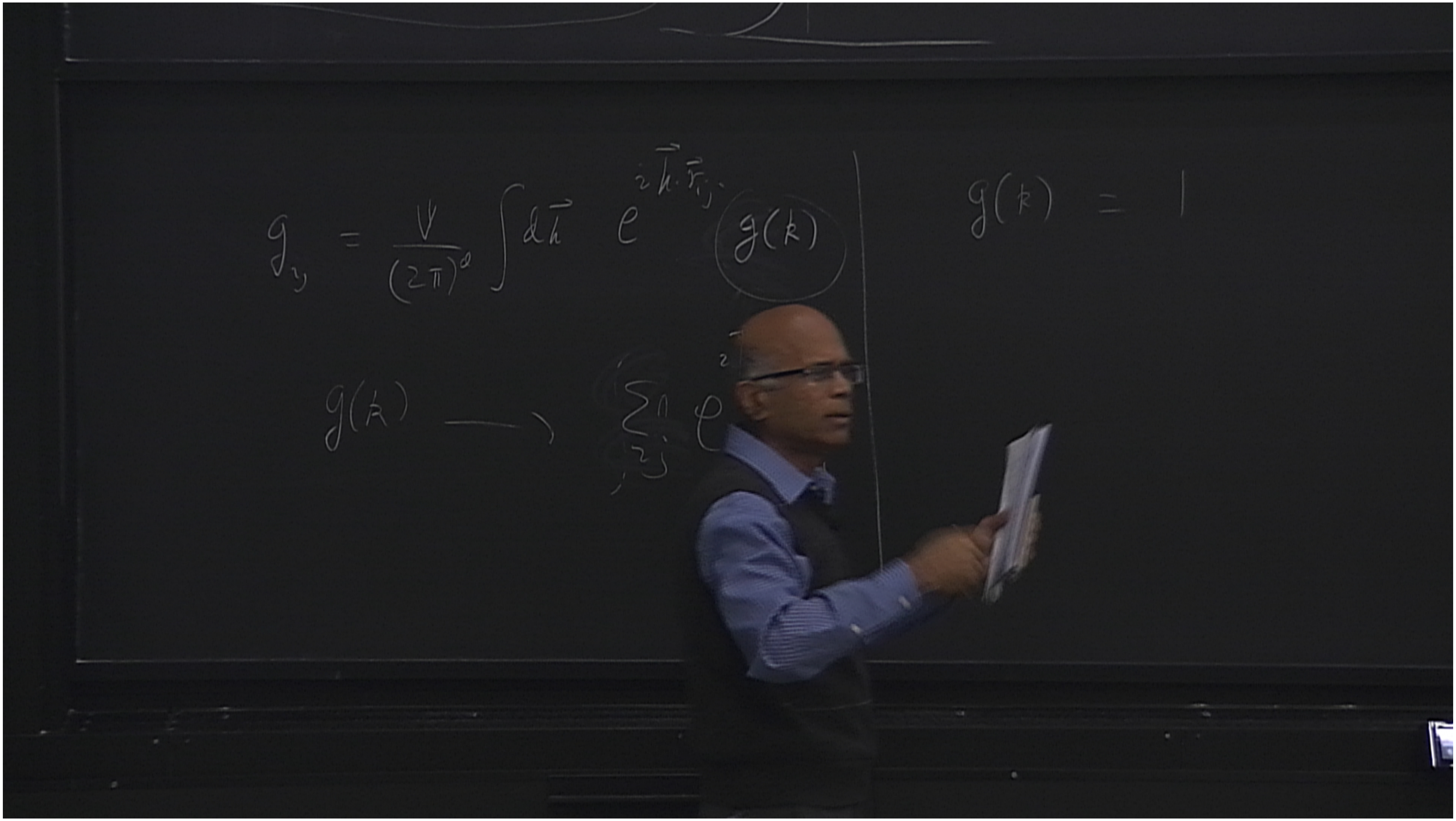
$$\boxed{\frac{t}{\beta J} = m}$$

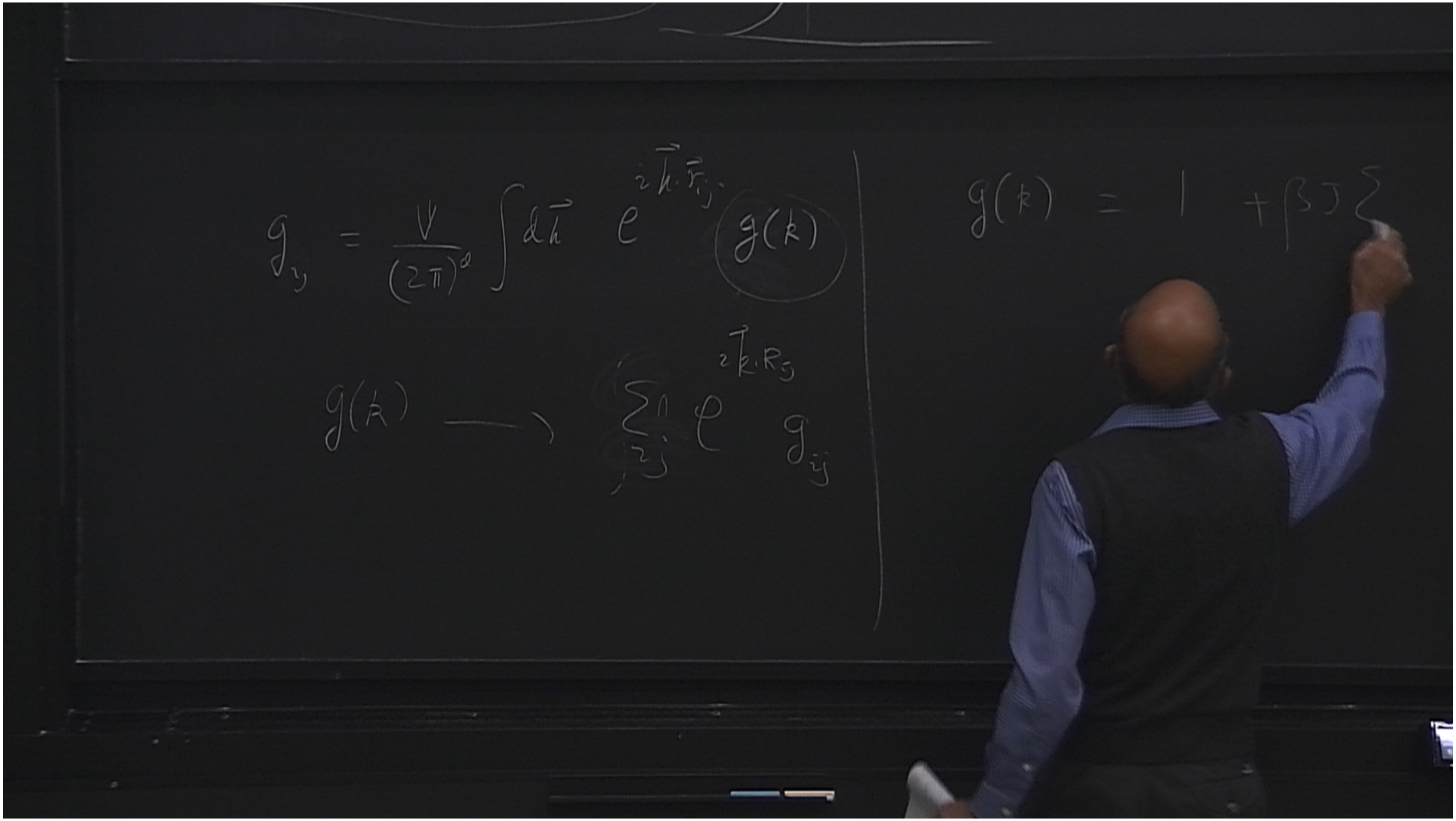
$$= \int dm \; e^{-N\overbrace{\left(J m^2 - kT \log \cosh(\beta h + mJ)\right)}^{F(m)}}$$

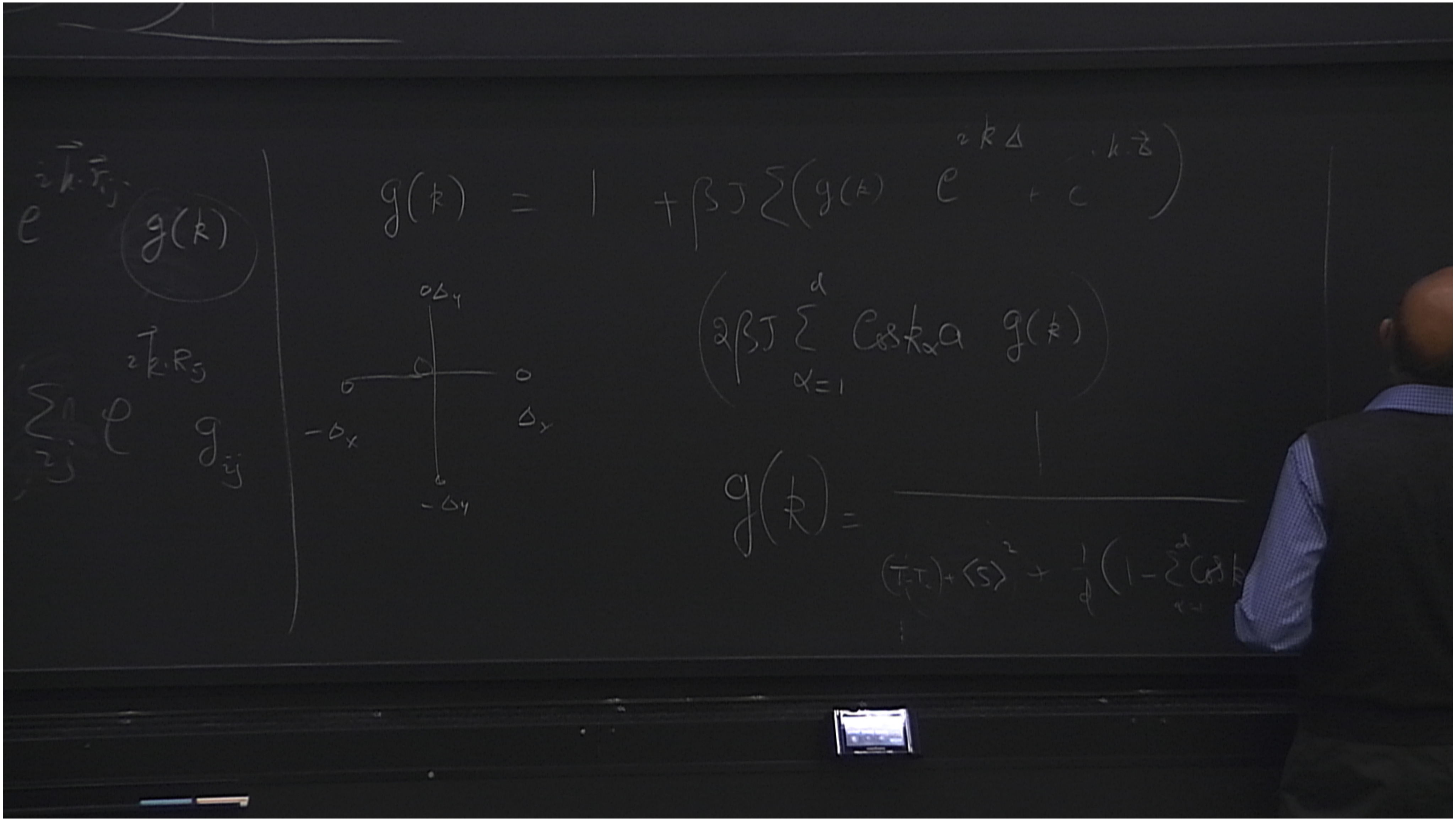

$\int dm \, e^{-\beta N f(m)}$
$\boxed{m = \tanh(\beta h + J m)}$
$(\beta h + mJ)$


$i \longleftarrow R_{ij} \longrightarrow j$
$g_{ij} = \langle (S_i - m)(S_j - m) \rangle$
$m = \tanh(\beta J m z + h\beta)$
$m_i = \tanh(\beta J \sum_\delta m_{i+\delta} + \beta h_i)$
$\tanh x \approx x -$
$\frac{\partial}{\partial(\beta h_j)} \langle S_i \rangle = \langle S_i S_j \rangle$

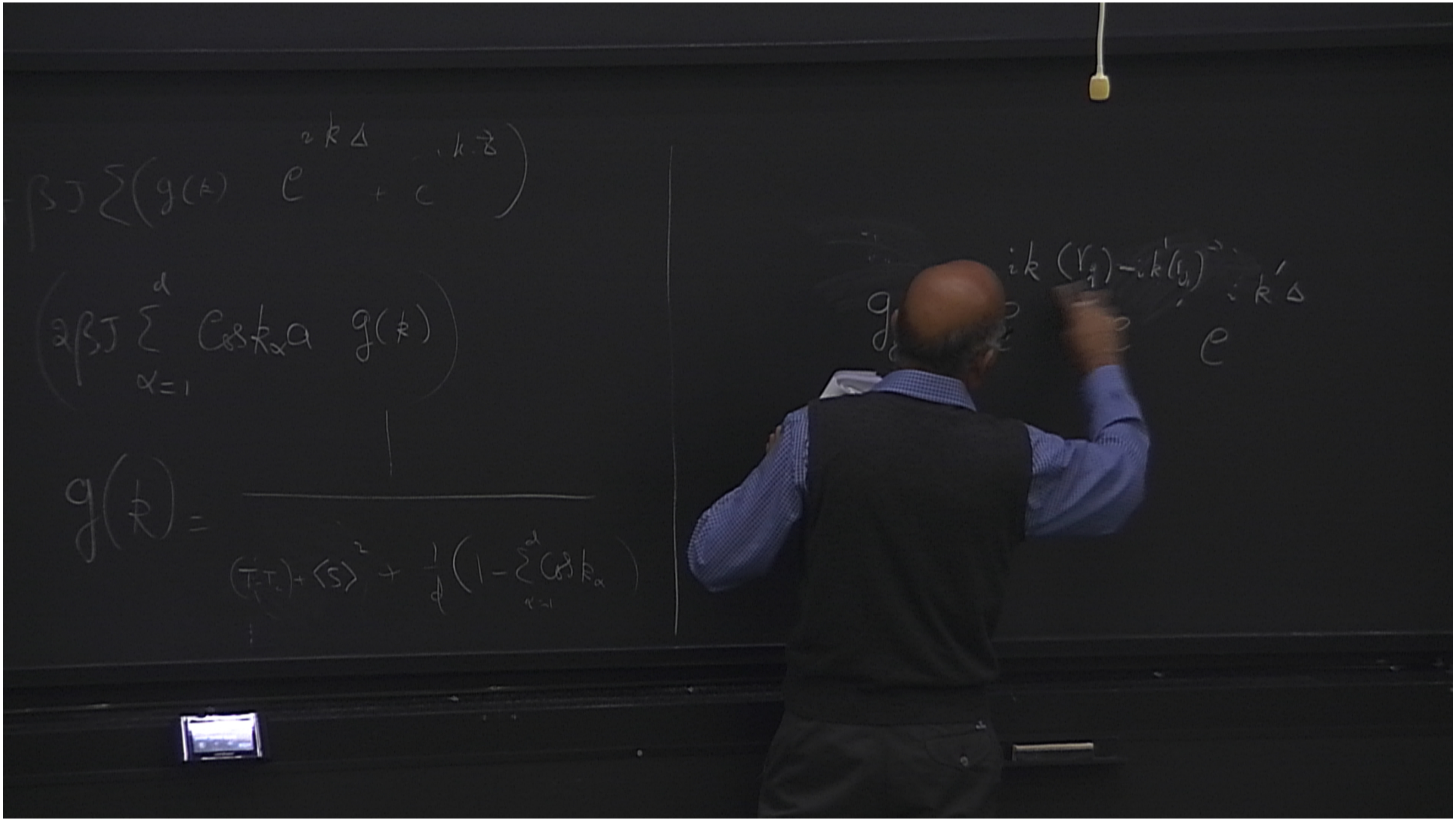

$\beta J m_3 + h\beta)$
$m_i = \beta h_i + \beta J \sum_\Delta m_{i+\Delta} + \frac{1}{3}\left(\beta J \sum_\Delta m_{i+\Delta}\right)^3$
$\tanh x \approx x - \frac{x^3}{3}$
$g_{ij} = \delta_{ij} + \beta J \sum_\Delta g_{j,i+\Delta} - (\beta J 3)^3 m^2 g_{ij}$
$(j+\Delta_x)$
$g_{j,i+\Delta} = \frac{\partial}{\partial(\beta h_j)} m_{i+\Delta} \sim \langle S_j S_{i+\Delta} \rangle$


g(k) = 1


$g_{ij} = \frac{V}{(2\pi)^d} \int d\vec{k} \; e^{i\vec{k}\cdot\vec{r}_{ij}} \; g(k)$
$g(k) \longrightarrow \sum_{ij} e^{i\vec{k}\cdot R_{ij}} \; g_{ij}$
$g(k) = 1 + \beta J \{$



g(k)
2βJ Σ_{α=1}^{d} cos k_α a g(k)
g(k) =



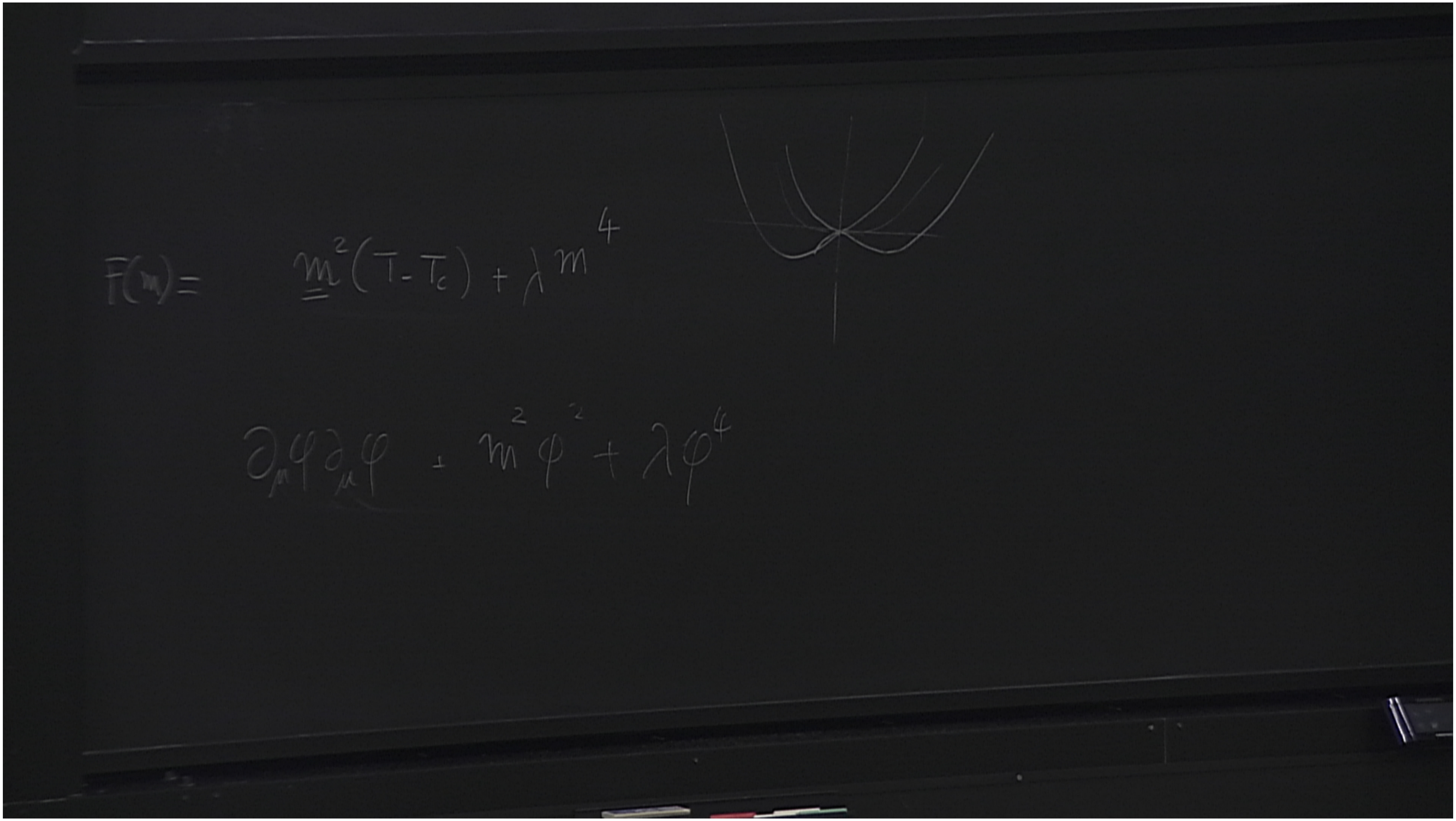
$$-\beta J \sum \left(g(k)\; e^{ik\Delta} + e^{ik\cdot z} \right)$$

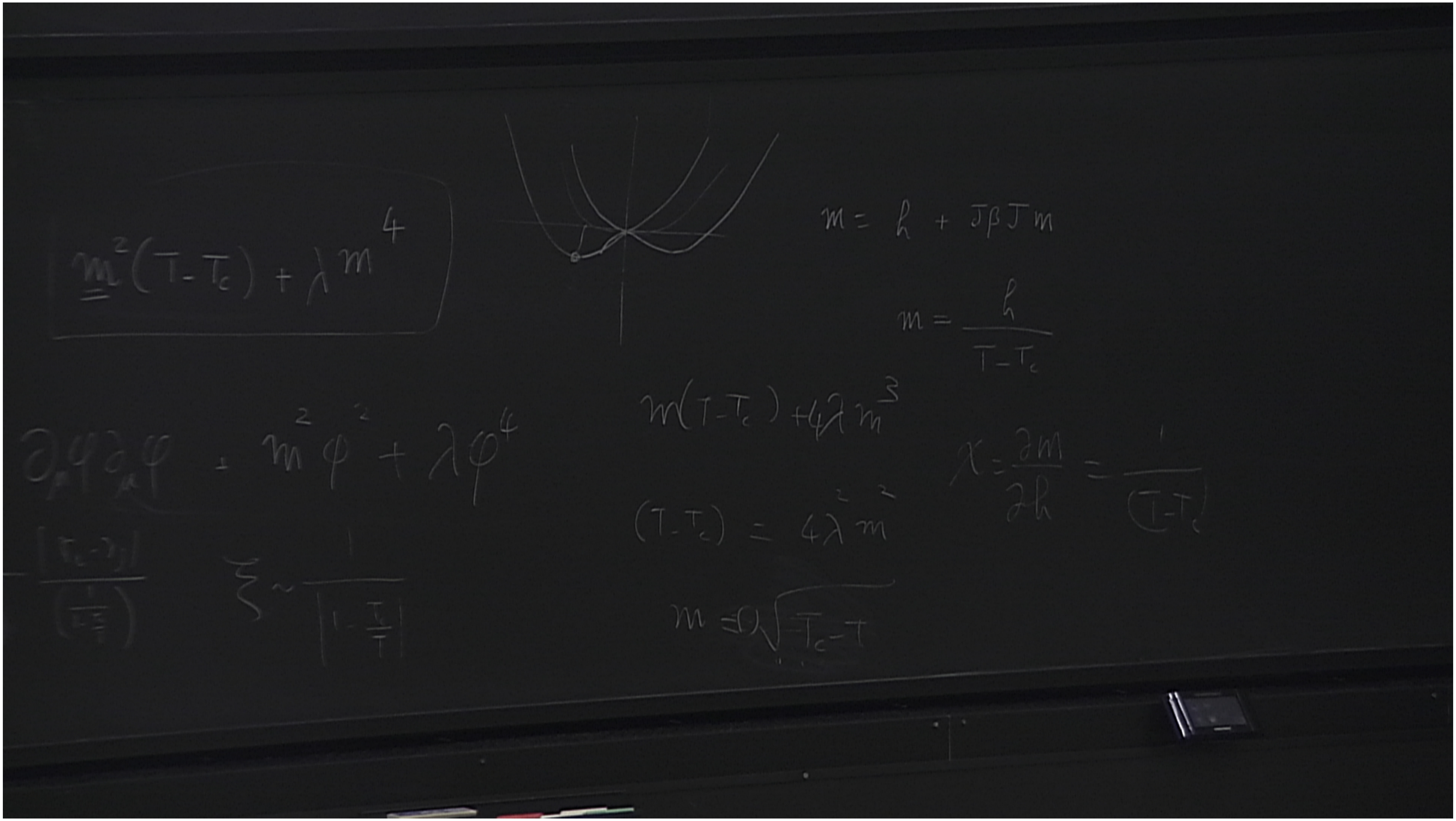
$$\left(2\beta J \sum_{\alpha=1}^{d} \cos k_\alpha a \; g(k) \right)$$

$$g(k) = \frac{1}{(T_i - T_c) + \langle S \rangle^2 + \frac{1}{d}\left(1 - \sum_{\alpha=1}^{d} \cos k_\alpha\right)}$$

$$g_{g,\, r+\Delta}\; e^{ik(r_1) - ik(r_1)}\; e\; e^{ik'\Delta}$$

$$\delta_{kk'}$$


$F(m) = m^2(T - T_c) + \lambda m^4$
$\partial_\mu \varphi \partial_\mu \varphi + m^2 \varphi^2 + \lambda \varphi^4$


m²(T - Tc) + λm⁴
m = h + JβJm
m = h / (T - Tc)
m(T - Tc) + 4λm³
∂μφ∂μφ + m²φ² + λφ⁴
χ = ∂m/∂h = 1/(T - Tc)
(T - Tc) = 4λ²m²
ξ ~ 1/|1 - Tc/T|
m = 0, √(Tc - T)