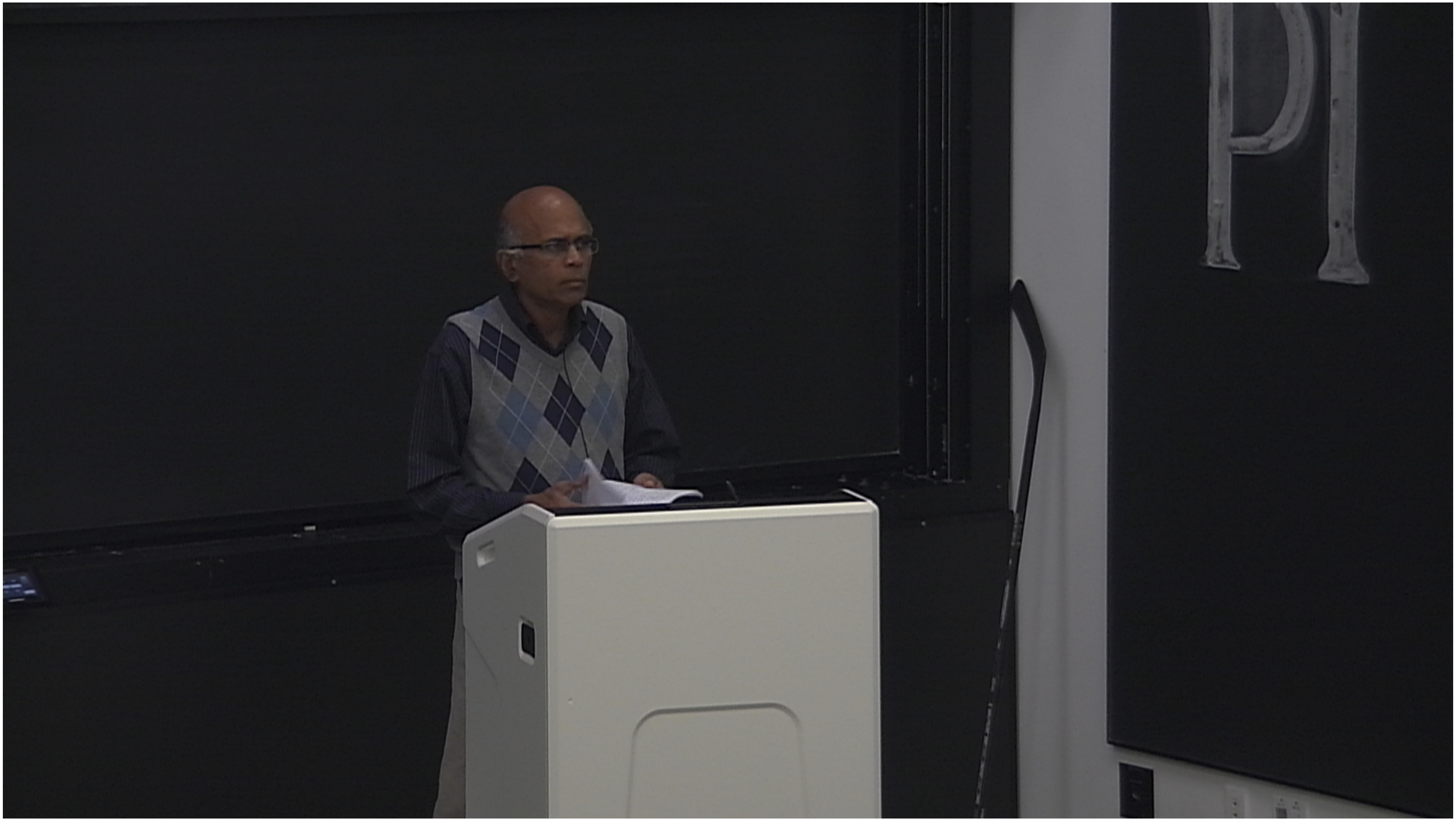
Title: PSI 2015/2016 Statistical Mechanics - G. Baskaran - Lecture 4

Date: Oct 16, 2015 10:45 AM

URL: http://pirsa.org/15100089

Abstract:



Classical & Quantum

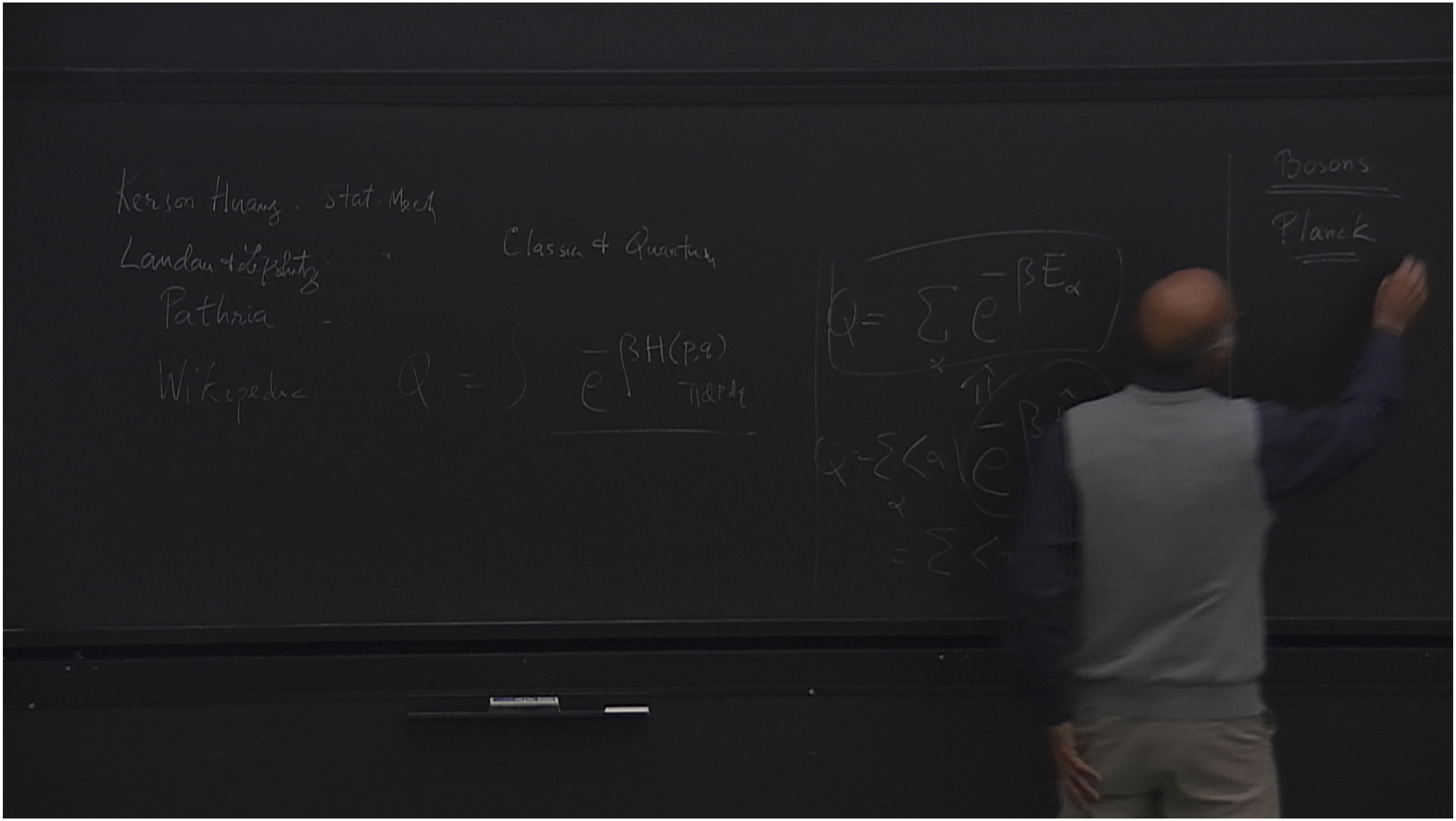
$$Q = \int \frac{e^{-\beta H(p,q)} \prod dp\, dq}{}$$

$$Q = \sum_\alpha e^{-\beta E_\alpha}$$

$$\Uparrow$$

$$Q = \sum_\alpha \langle \alpha | e^{-\beta \hat{H}} | \alpha \rangle$$

$$= \sum \langle n | e^{-\beta \hat{H}} | n \rangle$$


Kerson Huang . Stat. Mech
Landau + Lifshitz
Pathria
Wikipedia
Classical + Quantum
Bosons
Planck

Kerson Huang : Stat. Mech

Landau + Lifshitz

Pathria ...

Wikipedia

Classical + Quantum

$$Q = \int e^{-\beta H(p,q)} \prod dp\, dq$$

$$\boxed{Q = \sum_\alpha e^{-\beta E_\alpha}}$$

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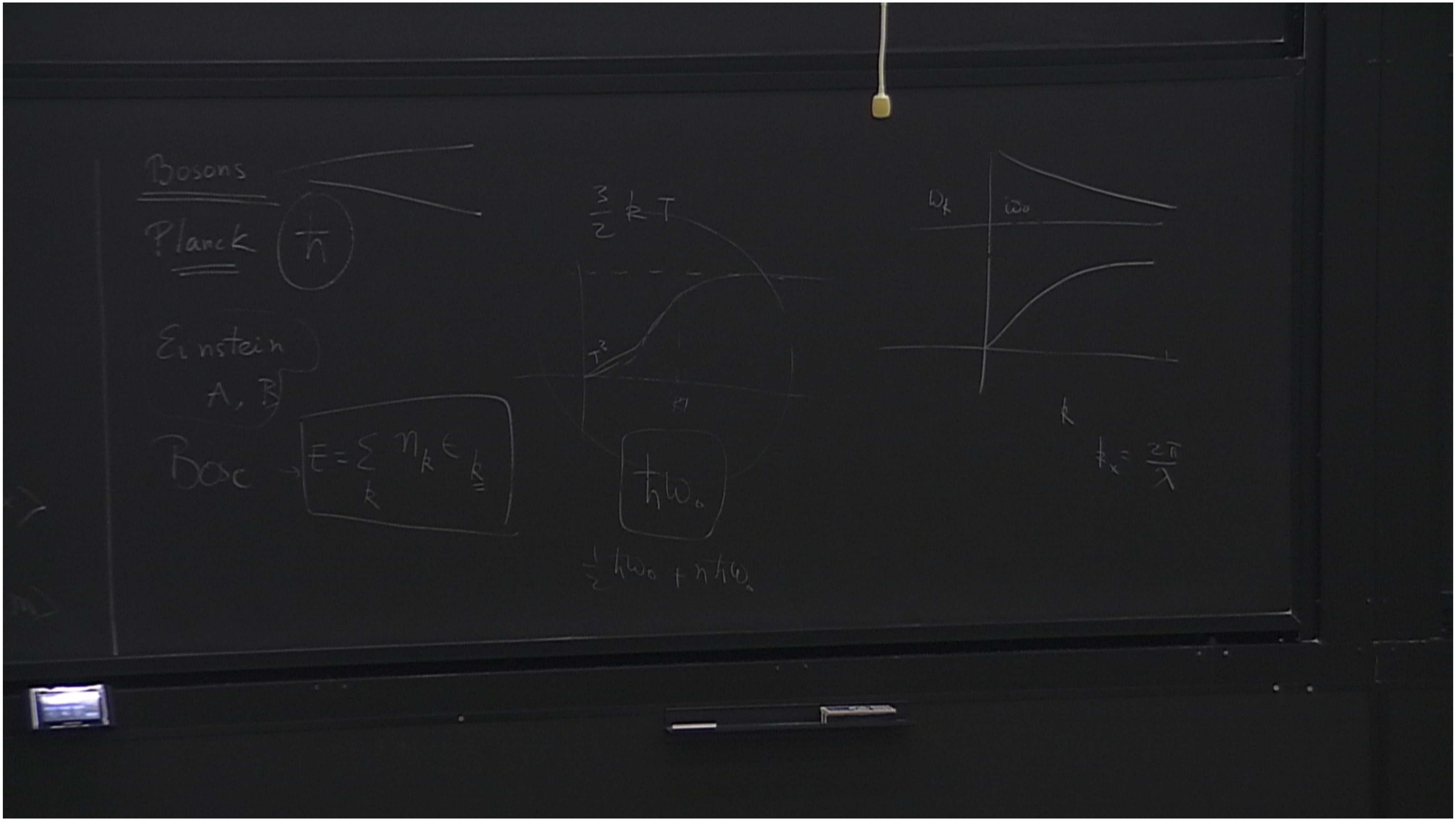
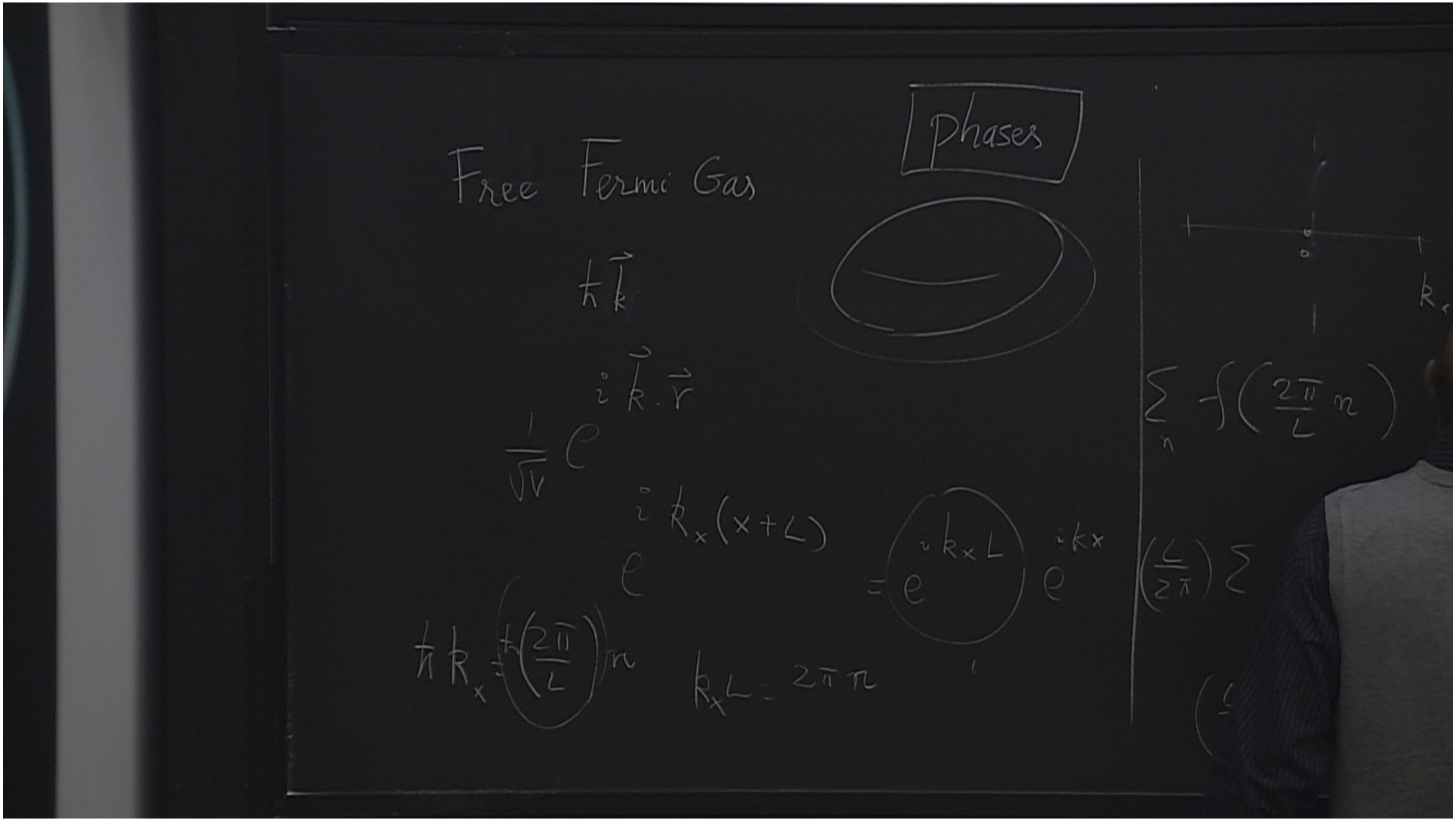
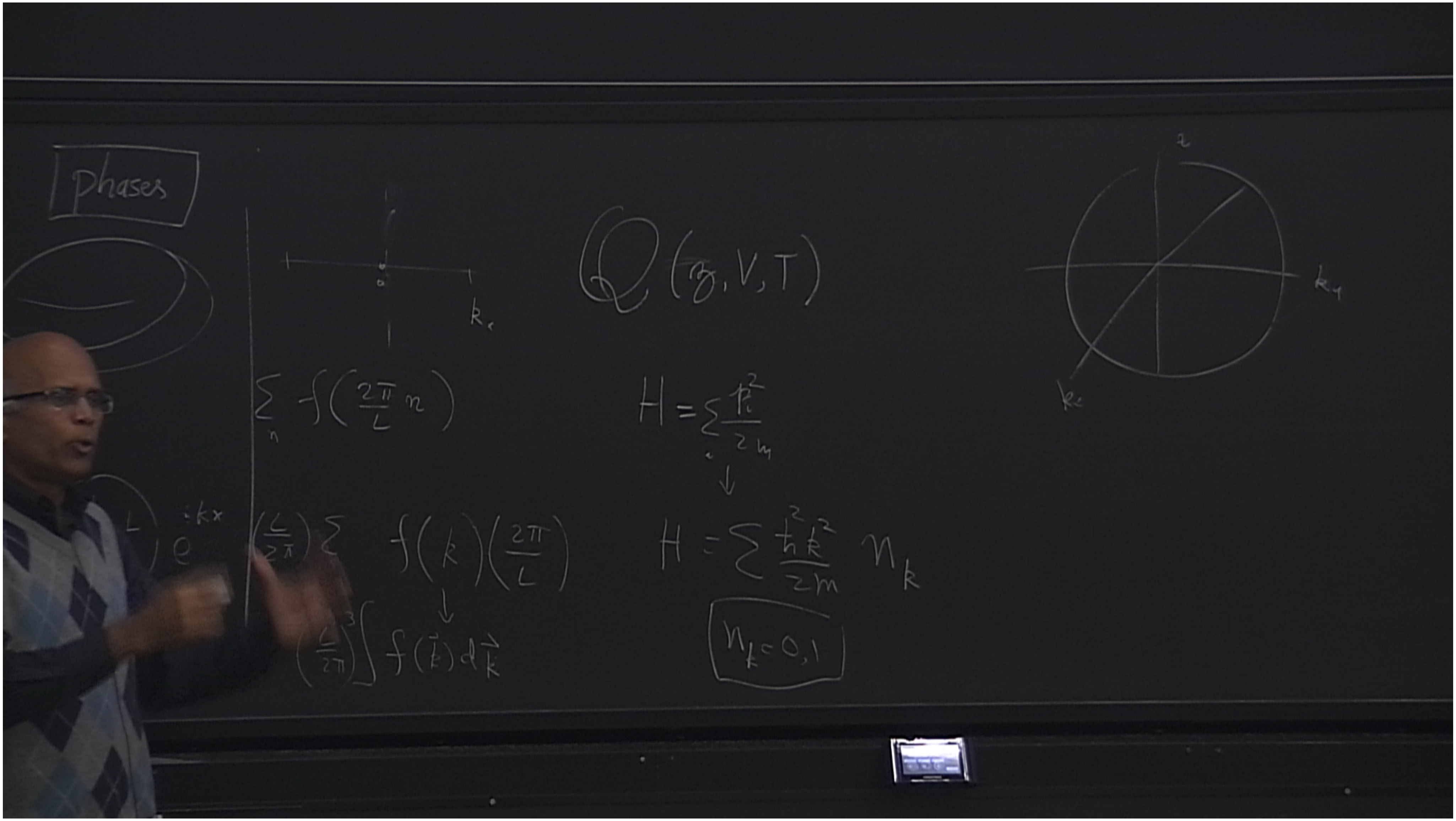
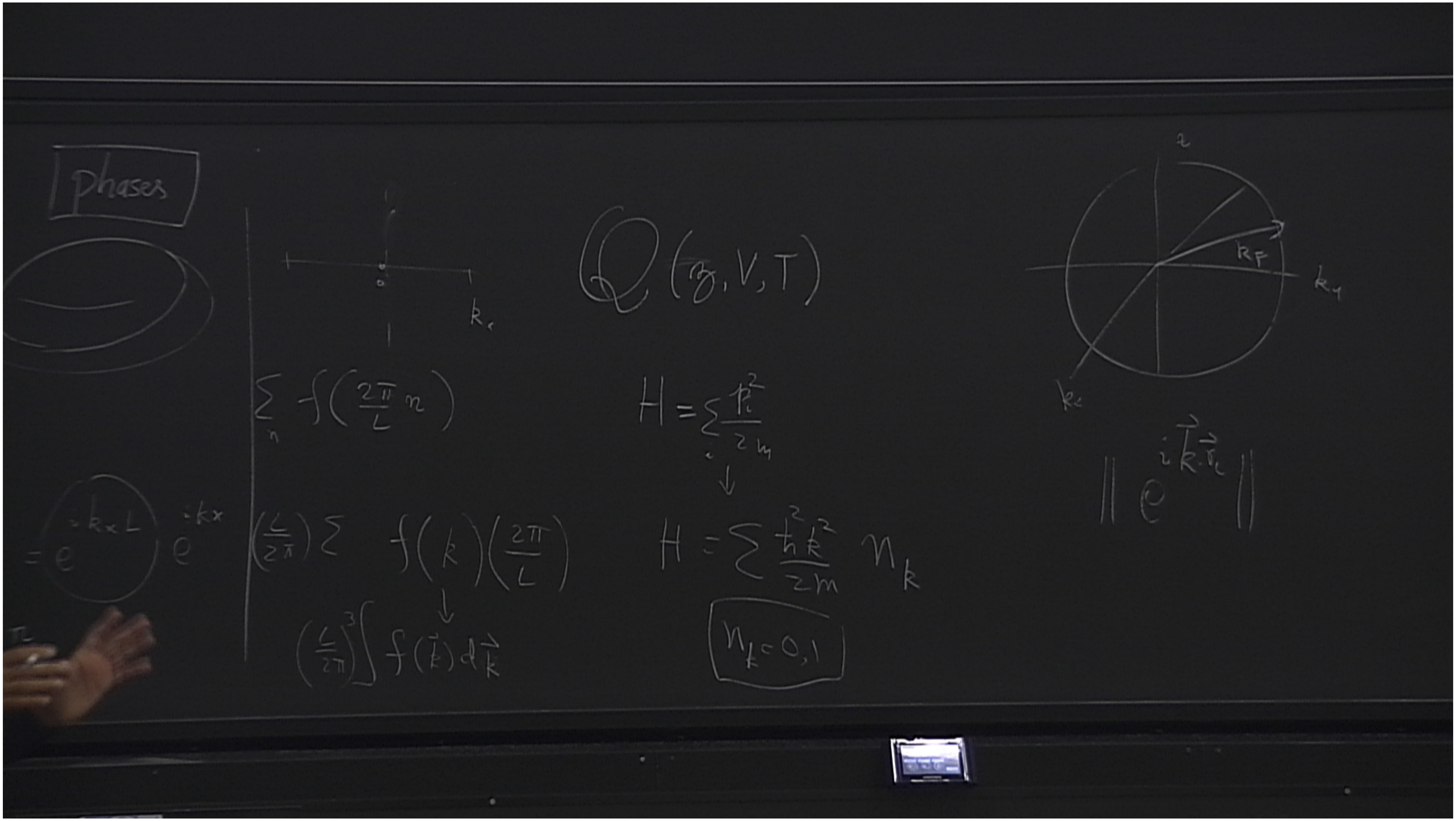
$$Q = \sum_\alpha \langle \alpha | e^{-\beta \hat{H}} | \alpha \rangle$$

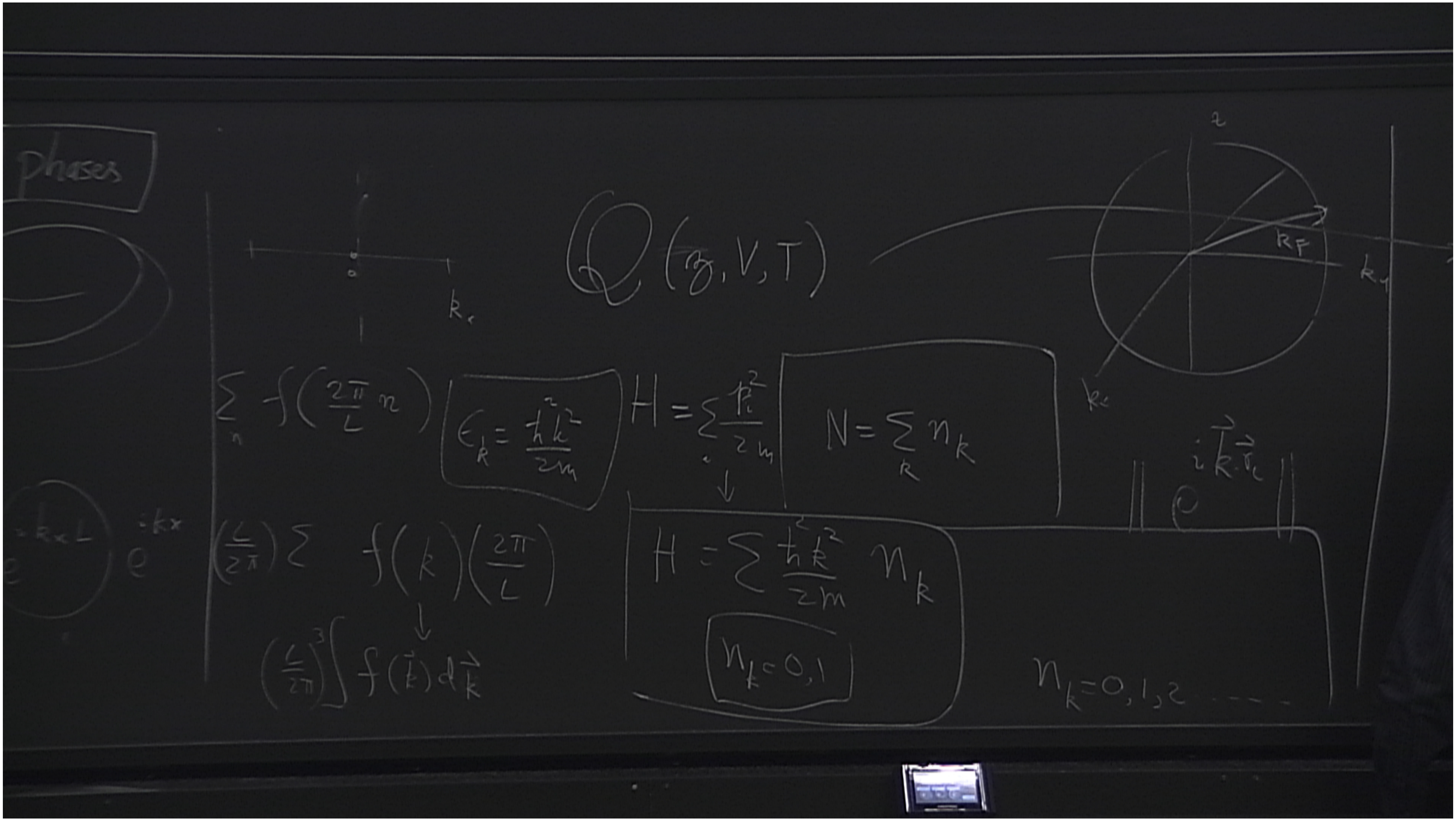
$$= \sum \langle n | e^{-\beta \hat{H}} | n \rangle$$

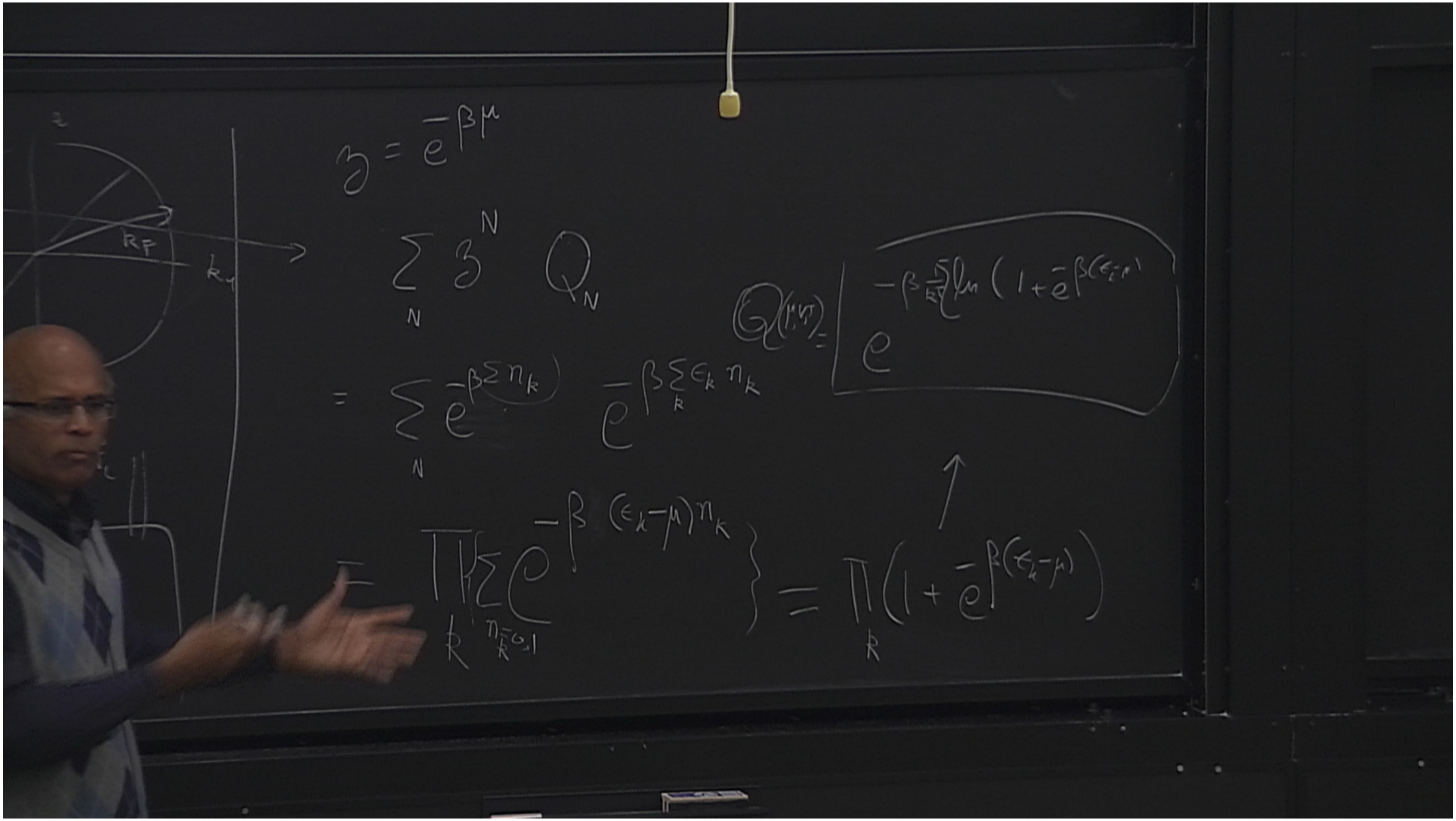

Bosons
Planck
Einstein
A, B
Bose
E = Σ n_k ε_k
3/2 kT
ħω₀
1/2 ħω₀ + nħω₀
ω₀
k
k_x = 2π/λ

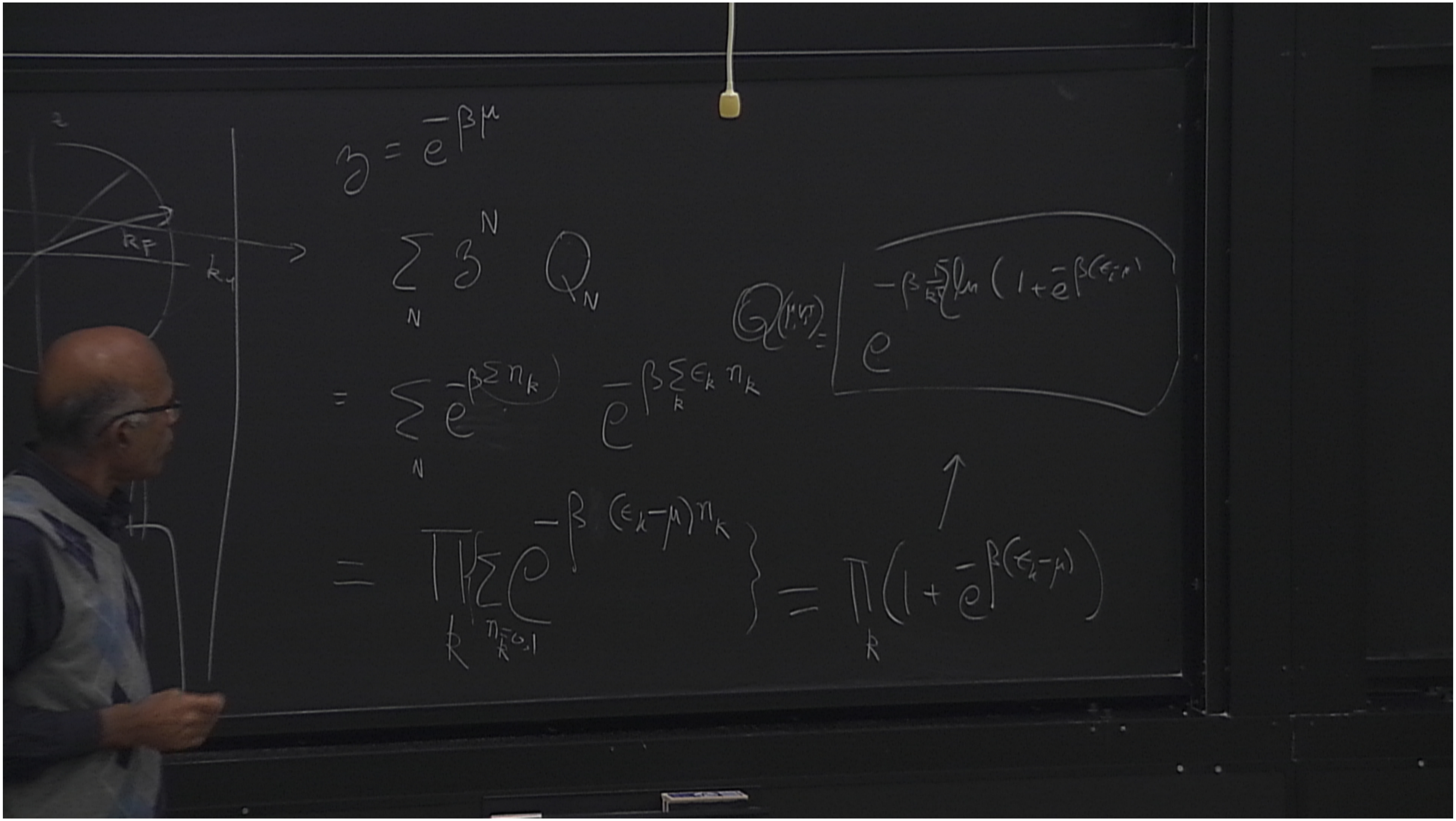

Phases
Free Fermi Gas
$\hbar \vec{k}$
$\frac{1}{\sqrt{V}} e^{i \vec{k} \cdot \vec{r}}$
$e^{i k_x (x+L)} = e^{i k_x L} e^{ikx}$
$\hbar k_x = \hbar \left(\frac{2\pi}{L}\right) n$
$k_x L = 2\pi n$
$\sum_n f\left(\frac{2\pi}{L} n\right)$
$\left(\frac{L}{2\pi}\right) \sum$

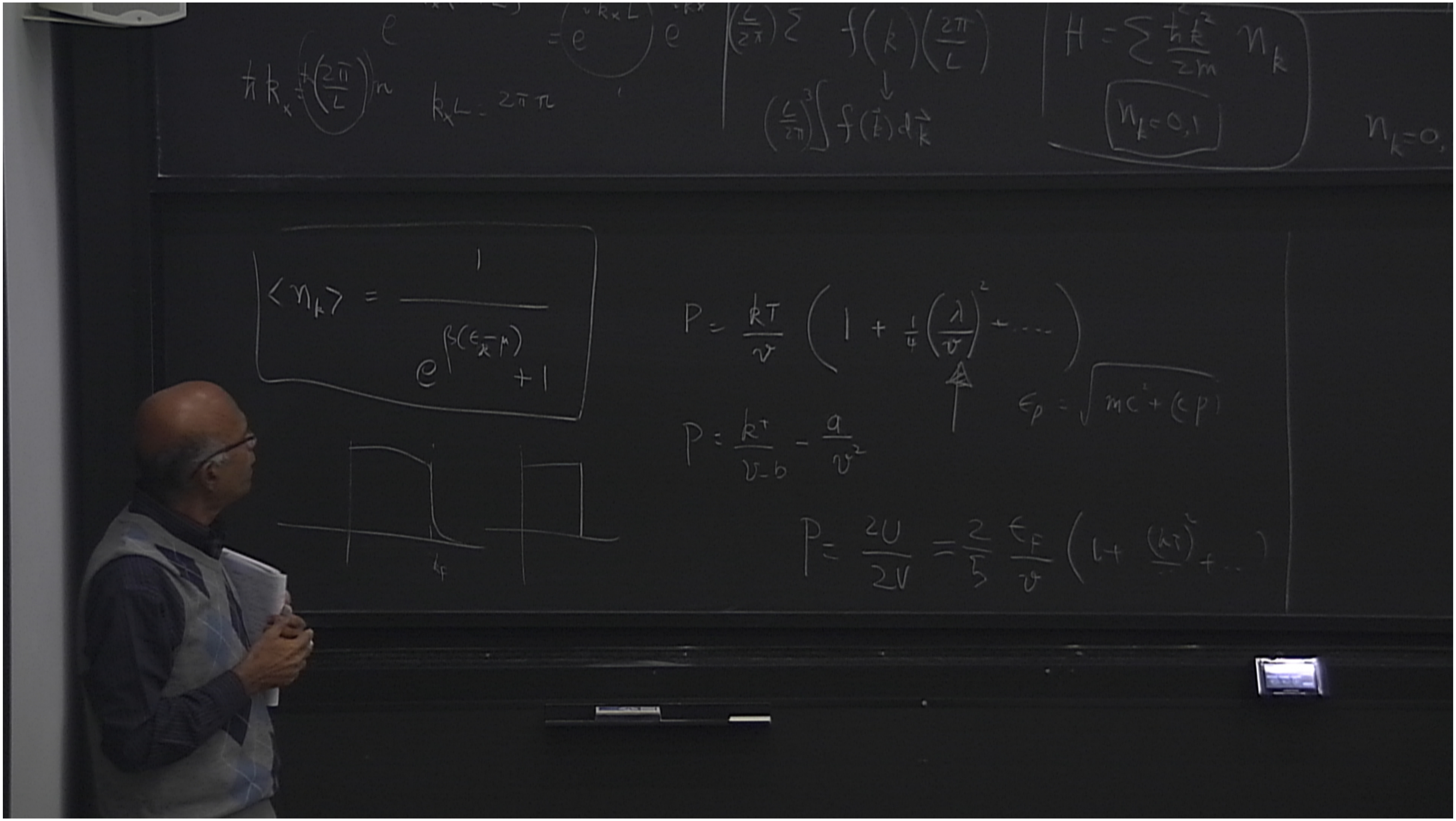

Phases
Q(z, V, T)
H = Σ p²/2m
H = Σ ħ²k²/2m n_k
n_k = 0, 1
k_x
k_y
k_z

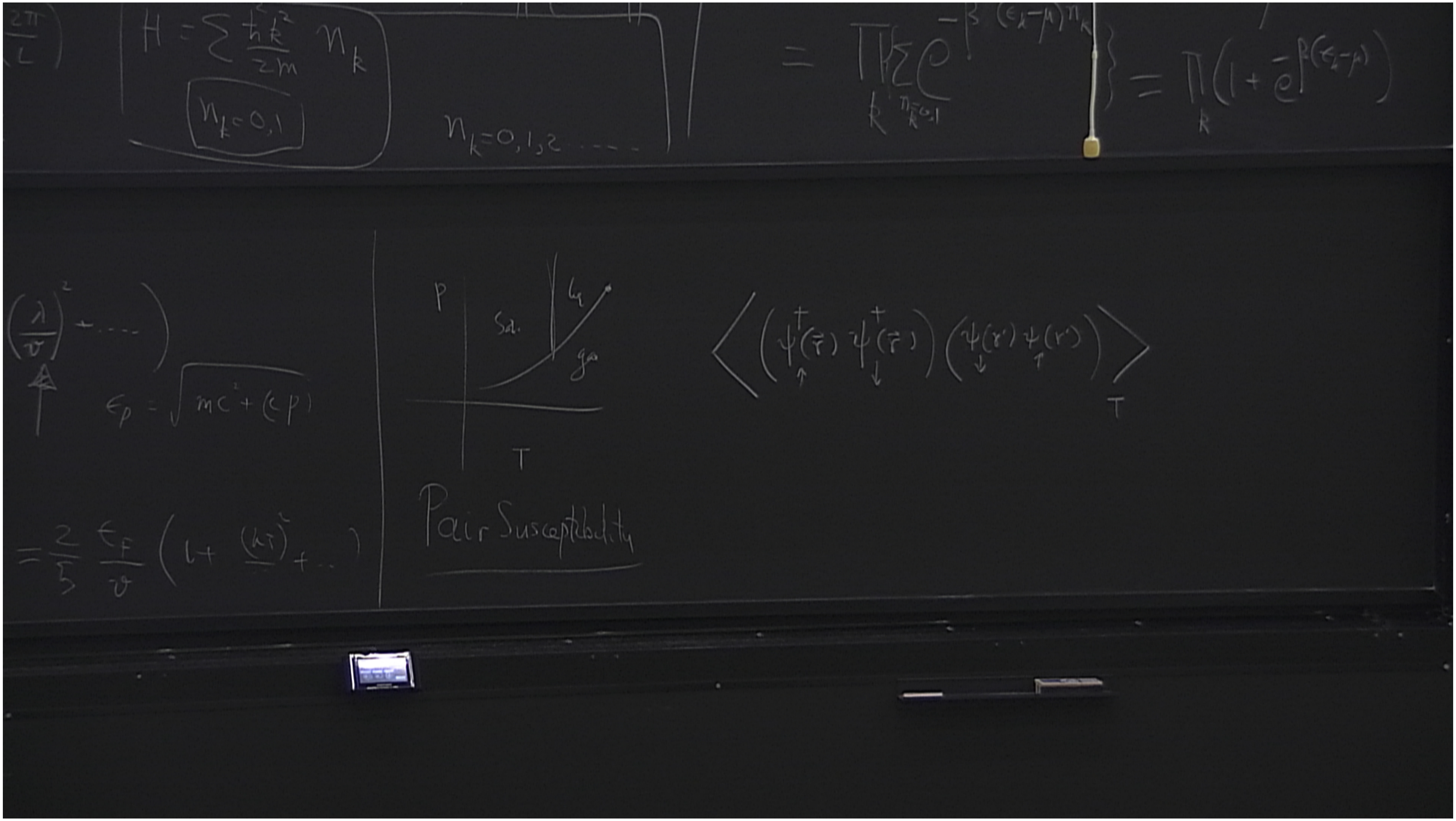

phases
$Q(z, V, T)$
$\sum_n f\left(\frac{2\pi}{L} n\right)$
$\left(\frac{L}{2\pi}\right) \sum f(k) \left(\frac{2\pi}{L}\right)$
$\left(\frac{L}{2\pi}\right)^3 \int f(\vec{k})\, d\vec{k}$
$H = \sum_i \frac{p_i^2}{2m}$
$H = \sum \frac{\hbar^2 k^2}{2m} n_k$
$n_k = 0, 1$
k_F
$\| e^{i \vec{k} \cdot \vec{r}_i} \|$


phases
e^{ikx}
k_x
$Q(z, V, T)$
k_F
$\sum_n f\left(\frac{2\pi}{L} n\right)$
$\epsilon_k = \frac{\hbar^2 k^2}{2m}$
$H = \sum_i \frac{p_i^2}{2m}$
$N = \sum_k n_k$
$e^{i\vec{k}\cdot\vec{r}_i}$
$\left(\frac{L}{2\pi}\right)\sum f(k)\left(\frac{2\pi}{L}\right)$
$H = \sum \frac{\hbar^2 k^2}{2m} n_k$
$\left(\frac{L}{2\pi}\right)^3 \int f(\vec{k})\, d\vec{k}$
$n_k = 0, 1$
$n_k = 0, 1, 2 \ldots$


k_F
k_y
$z = e^{-\beta\mu}$
$\sum_N z^N Q_N$
$= \sum_N e^{-\beta\sum n_k} e^{-\beta\sum_k \epsilon_k n_k}$
$= \prod_k \{ \sum_{n_k=0,1} e^{-\beta(\epsilon_k-\mu)n_k} \} = \prod_k \left(1 + e^{-\beta(\epsilon_k-\mu)}\right)$
$Q(\mu,T) = e^{-\beta\frac{1}{kT}\sum \ln\left(1+e^{-\beta(\epsilon_i-\mu)}\right)}$


k_F
$z = e^{-\beta\mu}$
$\sum_N z^N Q_N$
$= \sum_N e^{-\beta \sum n_k} e^{-\beta \sum_k \epsilon_k n_k}$
$= \prod_k \left\{ \sum_{n_k = 0,1} e^{-\beta (\epsilon_k - \mu) n_k} \right\} = \prod_k \left(1 + e^{-\beta(\epsilon_k - \mu)}\right)$
$Q(\mu, V, T) =$





$H = \sum \frac{\hbar^2 k^2}{2m} n_k$
$n_k = 0, 1$
$n_k = 0, 1, 2, \ldots$
$= \prod_k \sum_{n_k = 0,1} e^{-\beta(\epsilon_k - \mu) n_k} = \prod_k \left(1 + e^{-\beta(\epsilon_k - \mu)}\right)$
$\epsilon_p = \sqrt{mc^2 + (cp)^2}$
$= \frac{2}{5} \frac{\epsilon_F}{v} \left(1 + (kT)^2 + \ldots\right)$
P
T
Sol.
liq
gas
Pair Susceptibility
$\left\langle \left(\psi^\dagger_\uparrow(\vec r)\, \psi^\dagger_\downarrow(\vec r)\right) \left(\psi_\downarrow(r')\, \psi_\uparrow(r')\right) \right\rangle_T$

$k_x L = 2\pi n$

$\left(\frac{L}{2\pi}\right)^3 \int f(\vec{k})\, d\vec{k}$

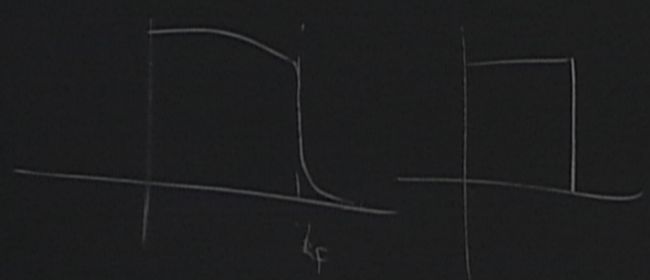
$n_k = 0, 1$

$$\boxed{\langle n_k \rangle = \frac{1}{e^{\beta(\epsilon_k - \mu)} + 1}}$$

$$P = \frac{kT}{v}\left(1 + \frac{1}{4}\left(\frac{\lambda}{v}\right)^2 + \cdots\right)$$

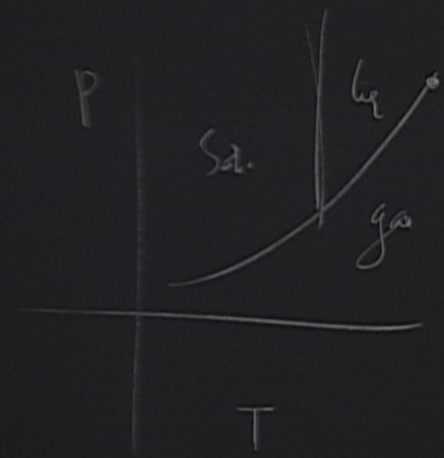
$$\epsilon_p = \sqrt{mc^2 + (cp)^2}$$

$$P = \frac{kT}{v-b} - \frac{a}{v^2}$$



$$P = \frac{2U}{2V} = \frac{2}{5}\frac{\epsilon_F}{v}\left(1 + (kT)^2 + \cdots\right)$$

$n_k = 0, 1, 2 \ldots\ldots$

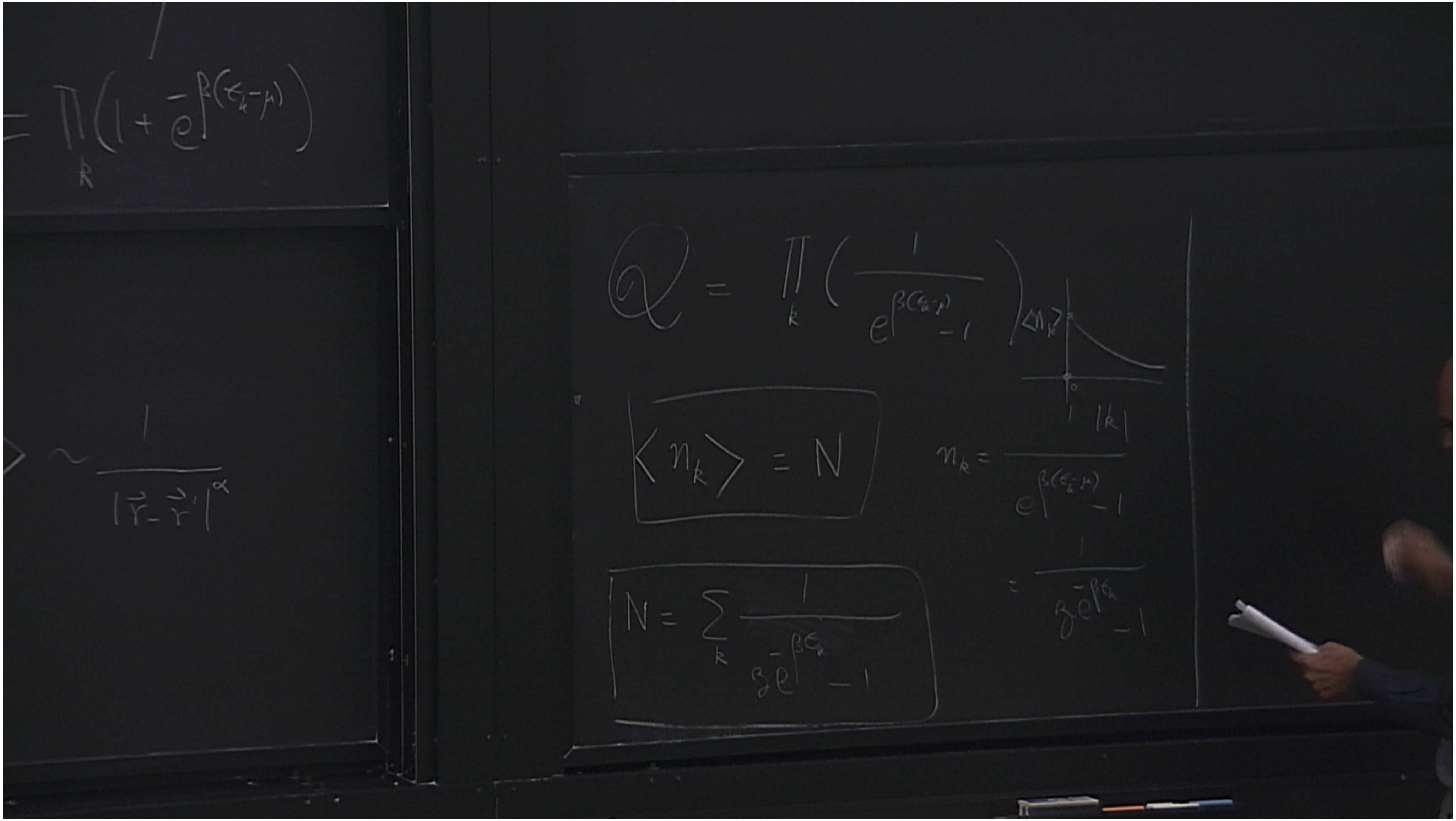


$$F(|\vec{r} - \vec{r}'|) = \left\langle \left(\psi_{\uparrow}(\vec{r})\, \psi_{\downarrow}(\vec{r})\right) \left(\psi^{\dagger}_{\downarrow}(r')\, \psi^{\dagger}_{\uparrow}(r')\right) \right\rangle_T \sim \frac{1}{|\vec{r} - \vec{r}'|^{\alpha}}$$

Pair Susceptibility

$$\int F(\vec{r})\, d\vec{r}$$

$$\frac{r^2}{r^{\alpha}}\, dr$$


$= \prod_k \left(1 + e^{-\beta(\epsilon_k - \mu)}\right)$
$\sim \frac{1}{|\vec{r} - \vec{r}'|^\alpha}$
$Q = \prod_k \left(\frac{1}{e^{\beta(\epsilon_k-\mu)} - 1} \right)$
$\langle n_k \rangle$
$|k|$
$\boxed{\langle n_k \rangle = N}$
$n_k = \frac{1}{e^{\beta(\epsilon_k-\mu)} - 1} = \frac{1}{z^{-1} e^{\beta\epsilon_k} - 1}$
$\boxed{N = \sum_k \frac{1}{z^{-1} e^{\beta\epsilon_k} - 1}}$