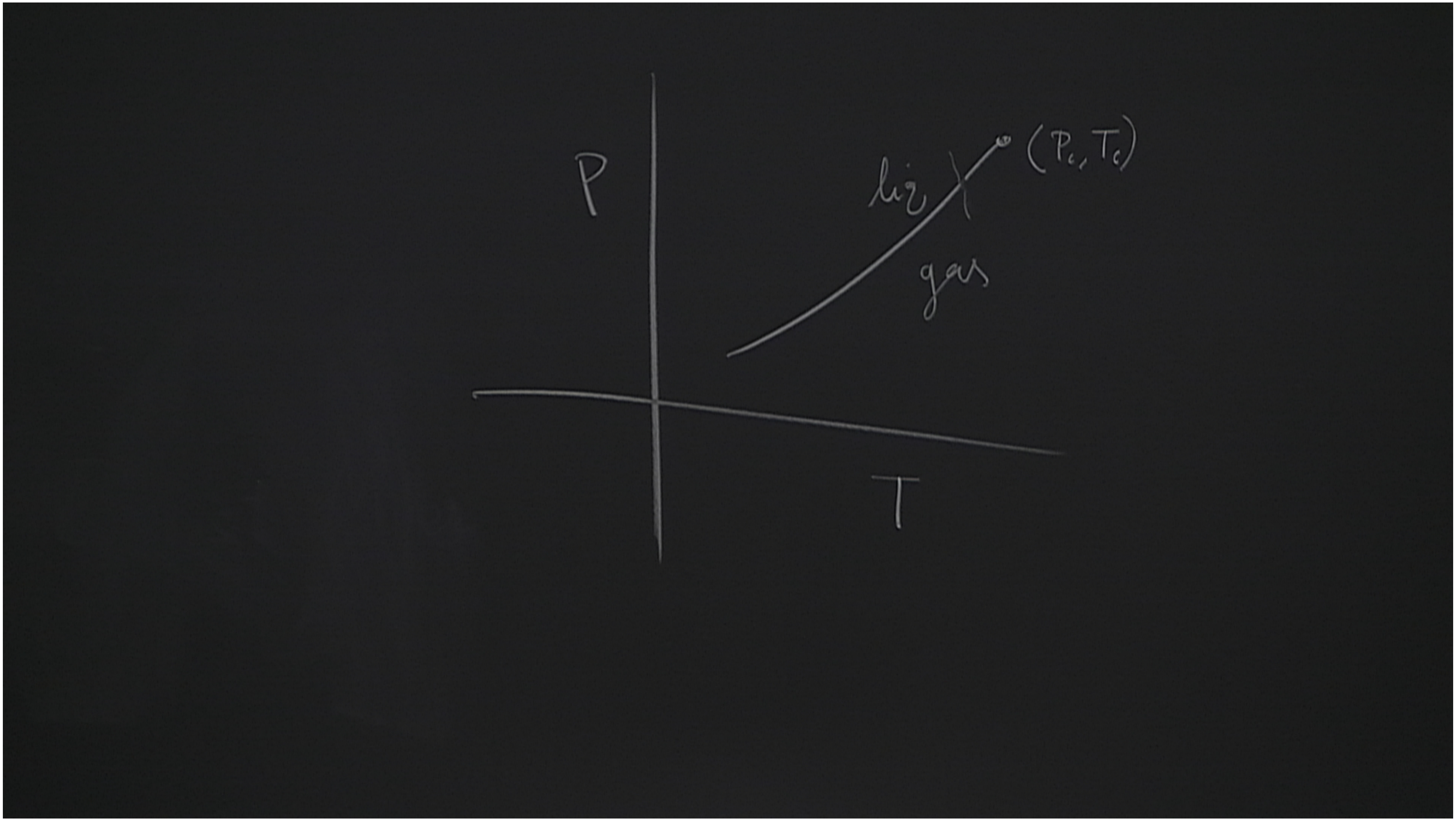
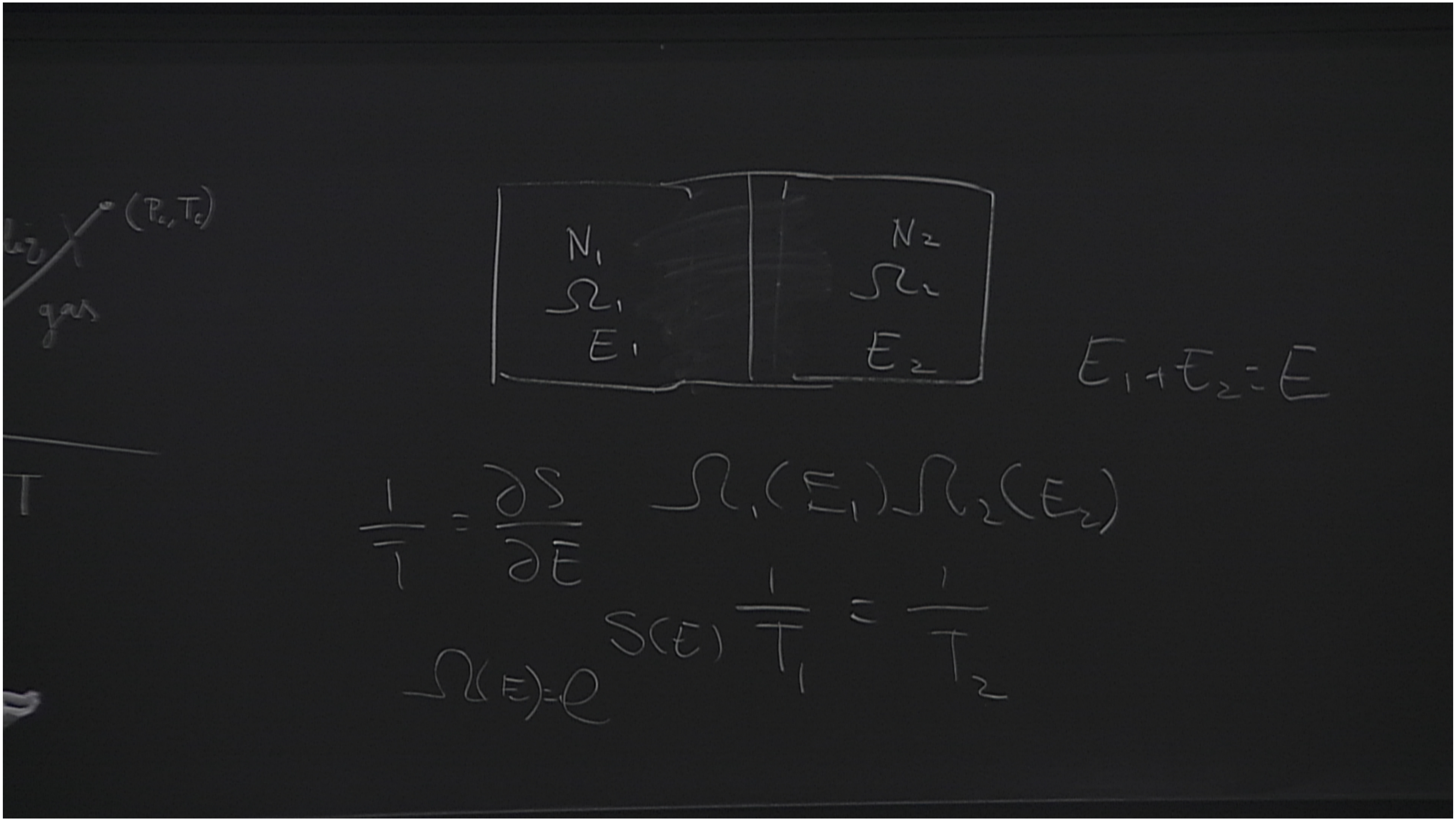
Title: PSI 2015/2016 Statistical Mechanics - G. Baskaran - Lecture 2

Date: Oct 14, 2015 10:45 AM

URL: http://pirsa.org/15100087

Abstract:


P
liq
gas
(Pc, Tc)
T


(P_c, T_c)
gas
T
N_1
Ω_1
E_1
N_2
Ω_2
E_2
$E_1 + E_2 = E$
$\frac{1}{T} = \frac{\partial S}{\partial E}$
$\Omega_1(E_1)\Omega_2(E_2)$
$\Omega(E) = e^{S(E)}$
$\frac{1}{T_1} = \frac{1}{T_2}$

phase space, Hilbert space

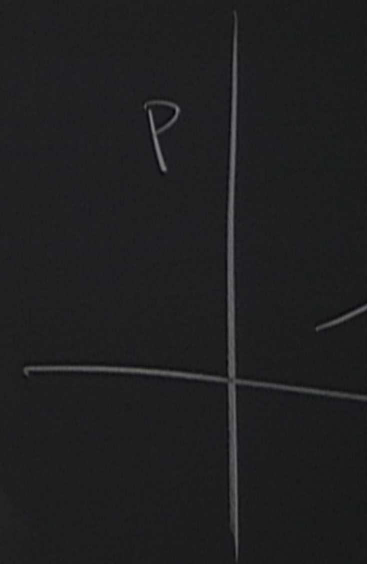
Liouville Theorem

Gibbs Distrbn.

μ-Canonical

Canonical

& Grand Canon~.



$E_1 + E_2 = E$

$$\boxed{6N \quad (\vec{p}_i, \vec{q}_i)}$$

Ergodic hypothesis (Molecular chaos)

$$\boxed{E_0 = H(\vec{p}, \vec{q})}$$

6N-1 dim

$(\vec{p}_{i0}, \vec{q}_{i0})$

N_2

Ω_2

E_2

Gibbs

$E_1 + E_2 = E$

$(E_1)\,\Omega_2(E_2)$

$= \frac{1}{T}$

$6N$ $(\vec{p}_i, \vec{q}$

Ergodic hypotheses
(Molecular chao

$\boxed{E_0 = H(\vec{p}, \vec{q})}$

$6N$

$6N-1$

$$\boxed{\rho(p,q)}$$

$$\downarrow$$

$$\boxed{e^{-\beta H(p,q)}}$$

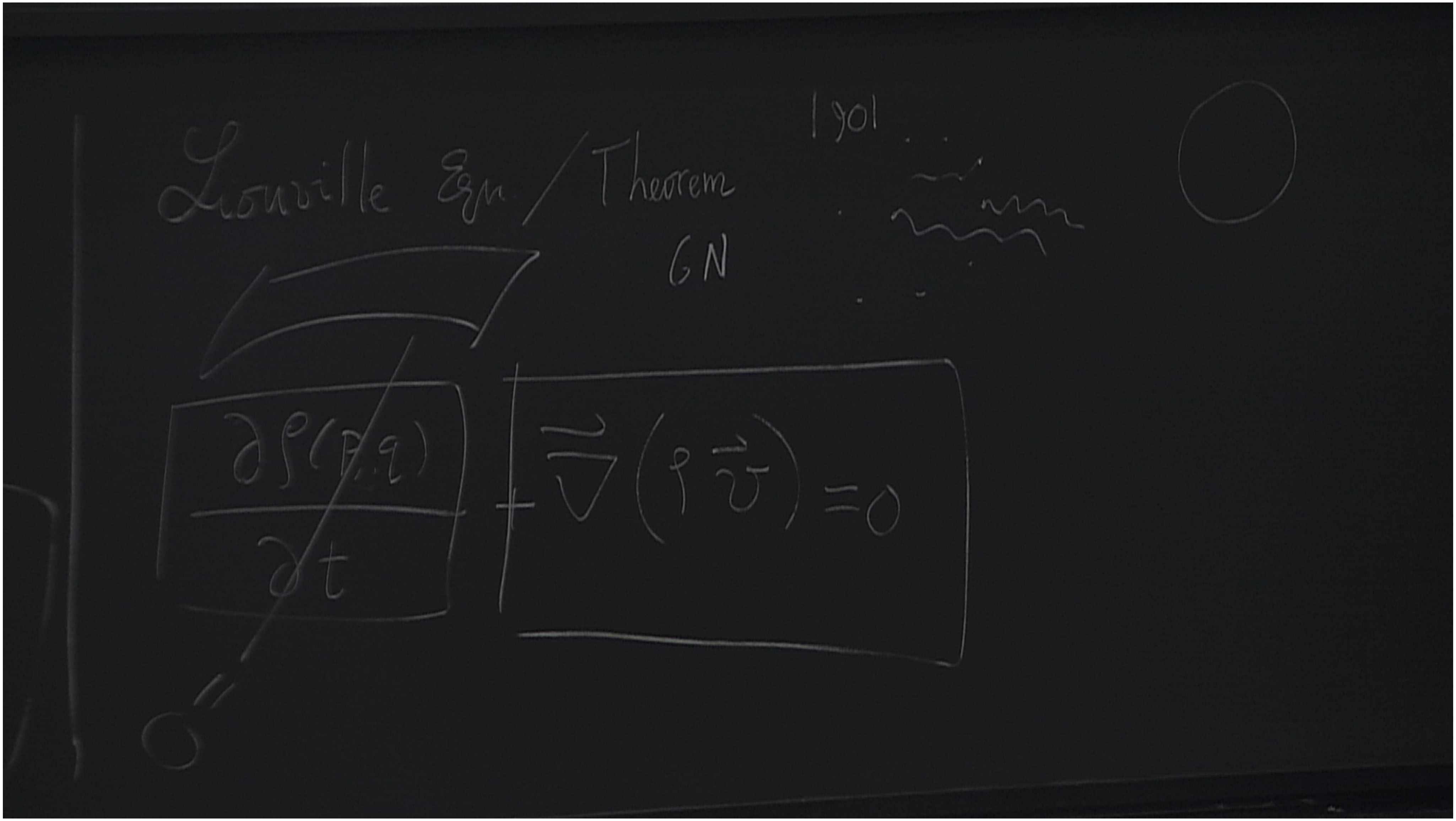
Liouville Eqn / Theorem

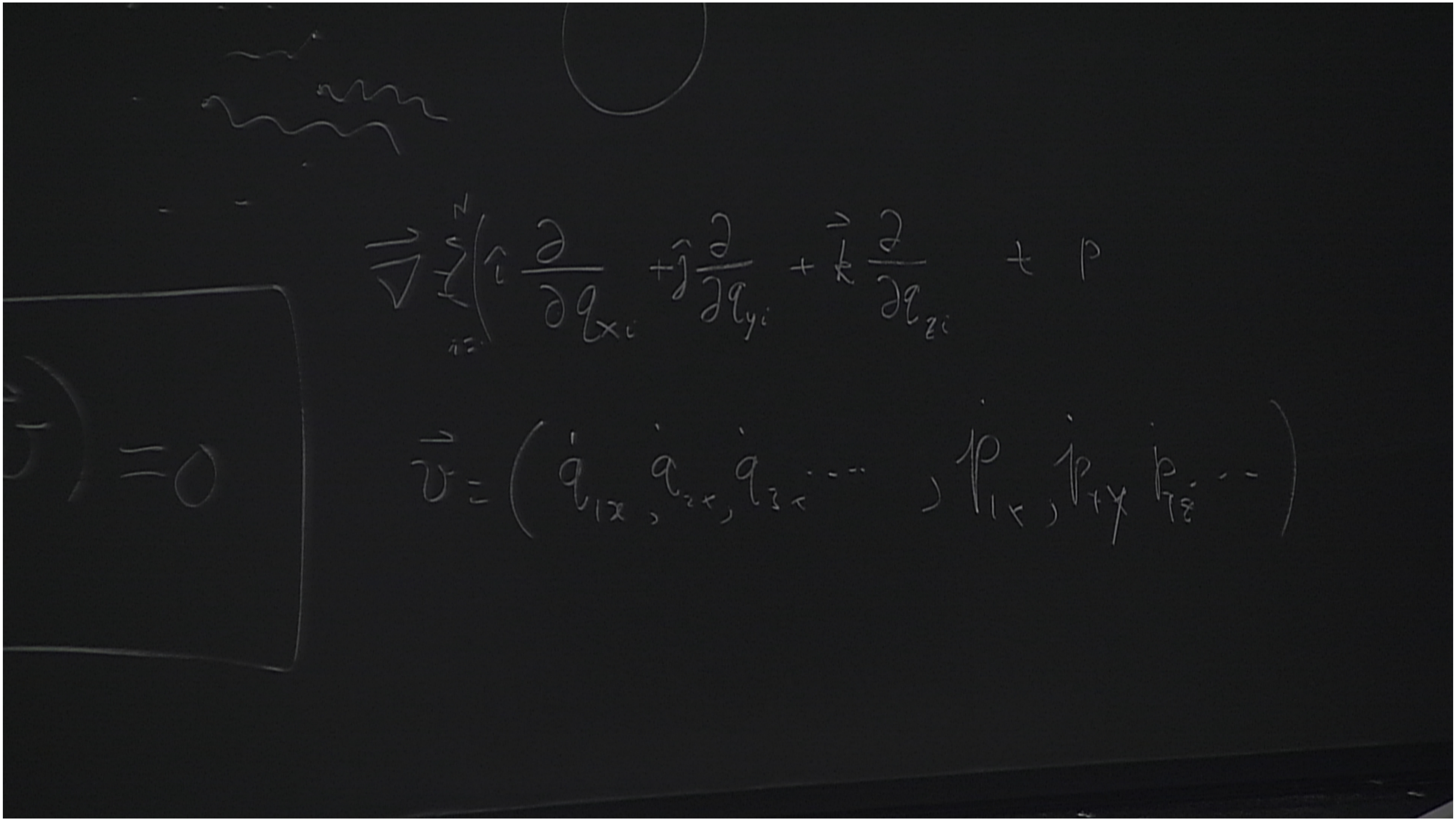
Liouville Eqn. / Theorem

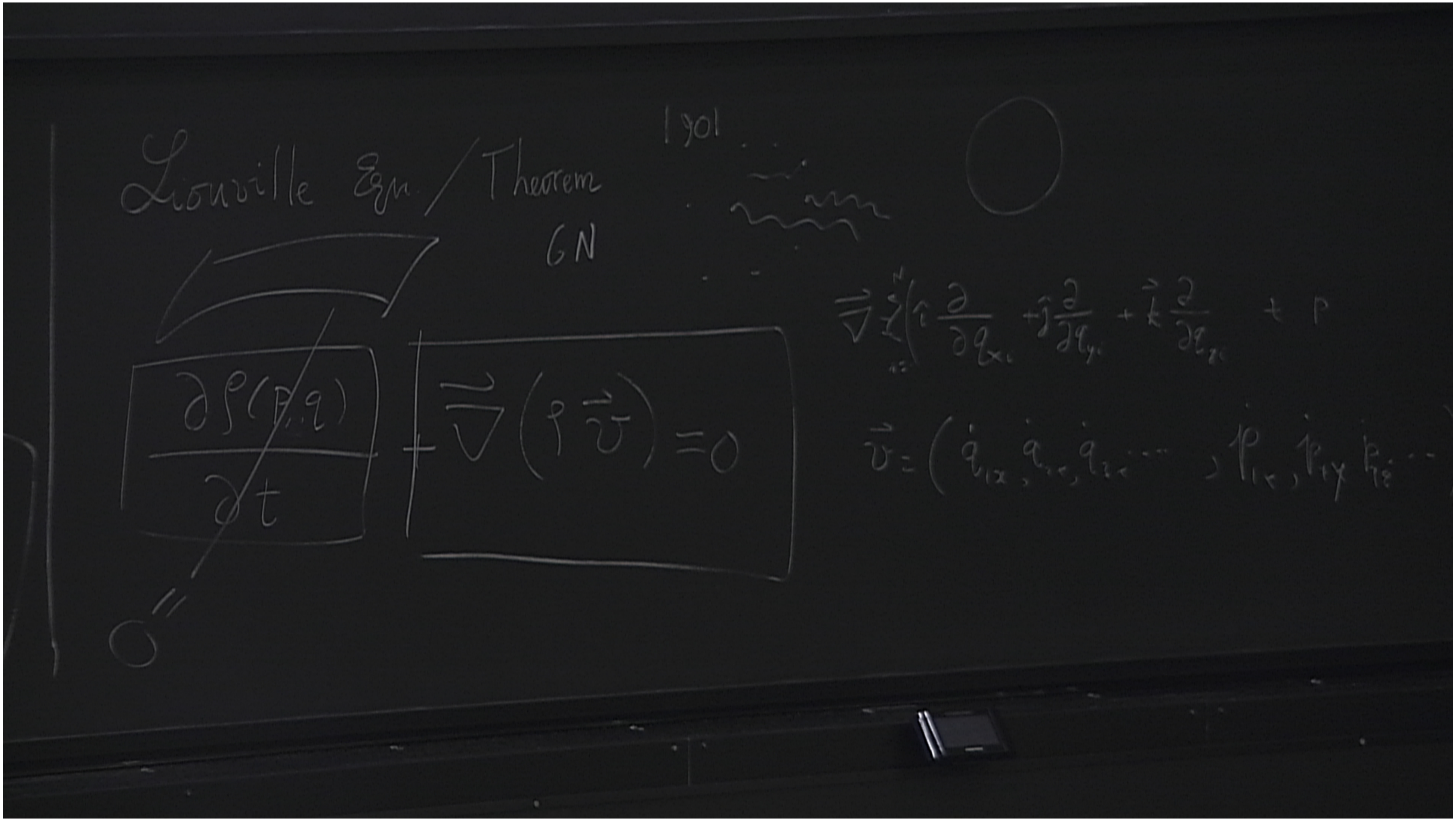
1901

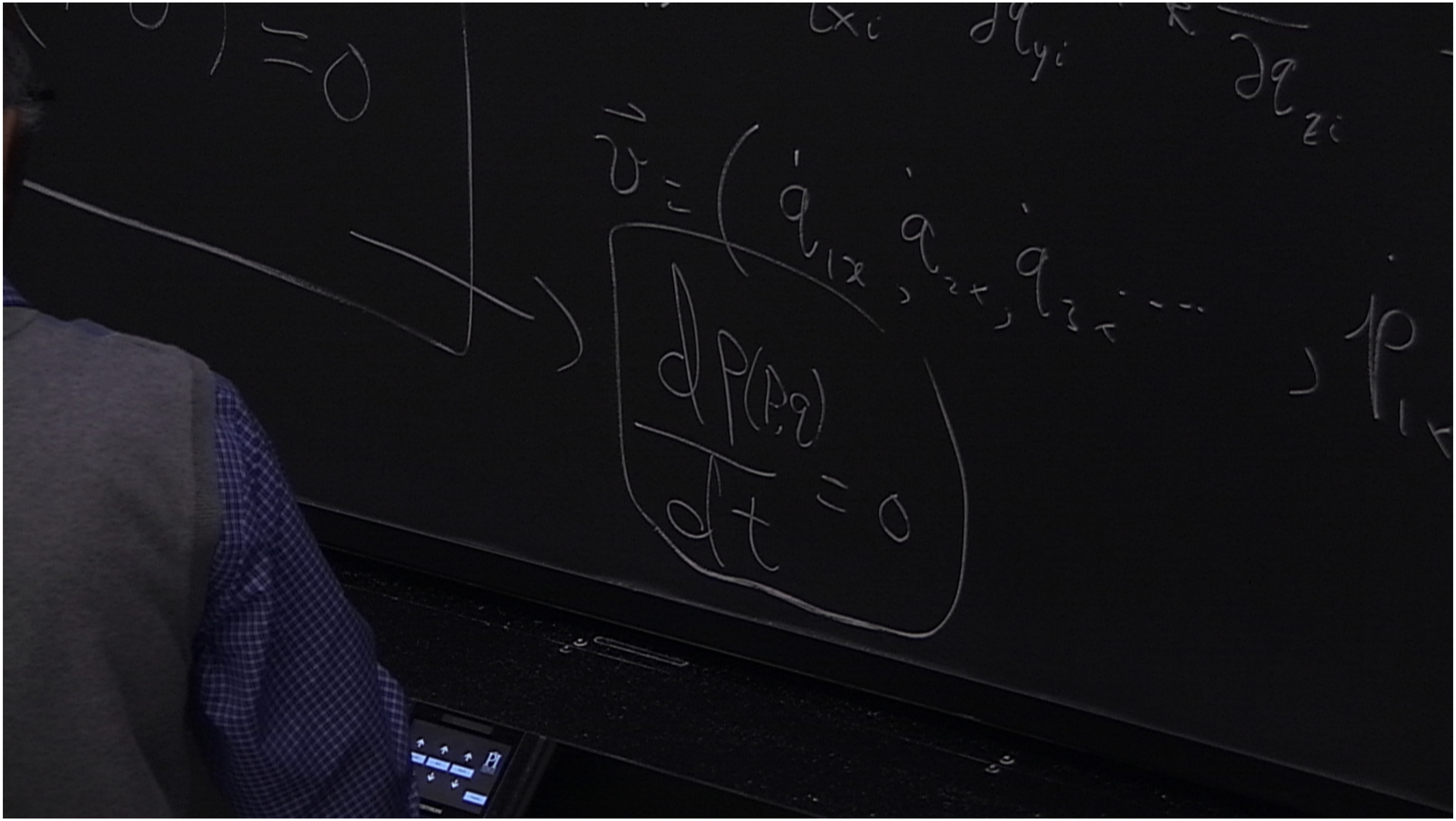
GN

$$\frac{\partial \rho(p,q)}{\partial t} + \vec{\nabla}\left(\rho \vec{v}\right) = 0$$


Liouville Eqn. / Theorem
1901
GN
$\frac{\partial \rho(p,q)}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$
$= 0$


$\vec{\nabla} = \sum_{i=1}^{N} \left(\hat{i} \frac{\partial}{\partial q_{xi}} + \hat{j} \frac{\partial}{\partial q_{yi}} + \hat{k} \frac{\partial}{\partial q_{zi}} + p \right.$
$= 0$
$\vec{v} = \left(\dot{q}_{1x}, \dot{q}_{2x}, \dot{q}_{3x} \dots, \dot{p}_{1x}, \dot{p}_{1y}, \dot{p}_{1z} \dots \right)$


Liouville Eqn. / Theorem
1901
GN
∂ρ(p,q)/∂t + ∇(ρ v) = 0
v = (q̇_1x, q̇_1y, q̇_1z, ..., ṗ_1x, ṗ_1y, ṗ_1z ...)


$\vec{v} = (\dot{q}_{1x}, \dot{q}_{2x}, \dot{q}_{3x}, \dots, \dot{p}_{1x}$
$\boxed{\frac{d\rho(p,q)}{dt} = 0}$

$$\vec{\nabla}(\rho\vec{v}) = \left(\sum_i \frac{\partial\rho}{\partial\vec{q}_i}\dot{q}_i + \frac{\partial\rho}{\partial\vec{p}_i}\dot{p}_i\right) + \rho\left(\sum_i \frac{\partial\dot{\vec{q}}_i}{\partial\vec{q}_i} + \sum\frac{\partial\dot{p}_i}{\partial\vec{p}_i}\right)$$

$$\dot{\vec{q}}_i = \frac{\partial H}{\partial\vec{q}_i}$$

$$\dot{\vec{p}}_i = -\frac{\partial H}{\partial\vec{p}_i}$$

$$\rho(E, \vec{P}, \vec{M})$$

$$\boxed{\frac{d\rho(\vec{p}, q)}{dt} = 0}$$

$$\ln \rho_{12} = \ln \rho_1 + \ln \rho_2$$

$$\Downarrow$$

$$\rho_{12} = \rho_1 \rho_2 =$$

$$\boxed{\rho = \alpha + \beta E + \vec{\gamma} \cdot \vec{P} + \vec{\delta} \cdot \vec{M}}$$

$$\ln \rho_{12} = \ln \rho_1 + \ln \rho_2$$

$$\rho_{12} = \rho_1 \rho_2 =$$

$$\ln \rho = \alpha + \beta E + \vec{\gamma} \cdot \vec{P} + \vec{\delta} \cdot \vec{M}$$

$$\frac{\partial \dot{\vec q}_i}{\partial \vec q_i} + \sum \frac{\partial \dot{\vec p}_i}{\partial \vec p_i} \Big)$$

$$\ln \rho_{12} = \ln \rho_1 + \ln \rho_2$$

$$\dot{\vec q}_i = \frac{\partial H}{\partial \vec q_i}$$

$$\dot{\vec p}_i = -\frac{\partial H}{\partial \vec p_i}$$

$$\rho_{12} = \rho_1 \rho_2 =$$

$$\ln \rho = \alpha + \beta E + \vec\gamma \cdot \vec P + \vec\delta \cdot$$

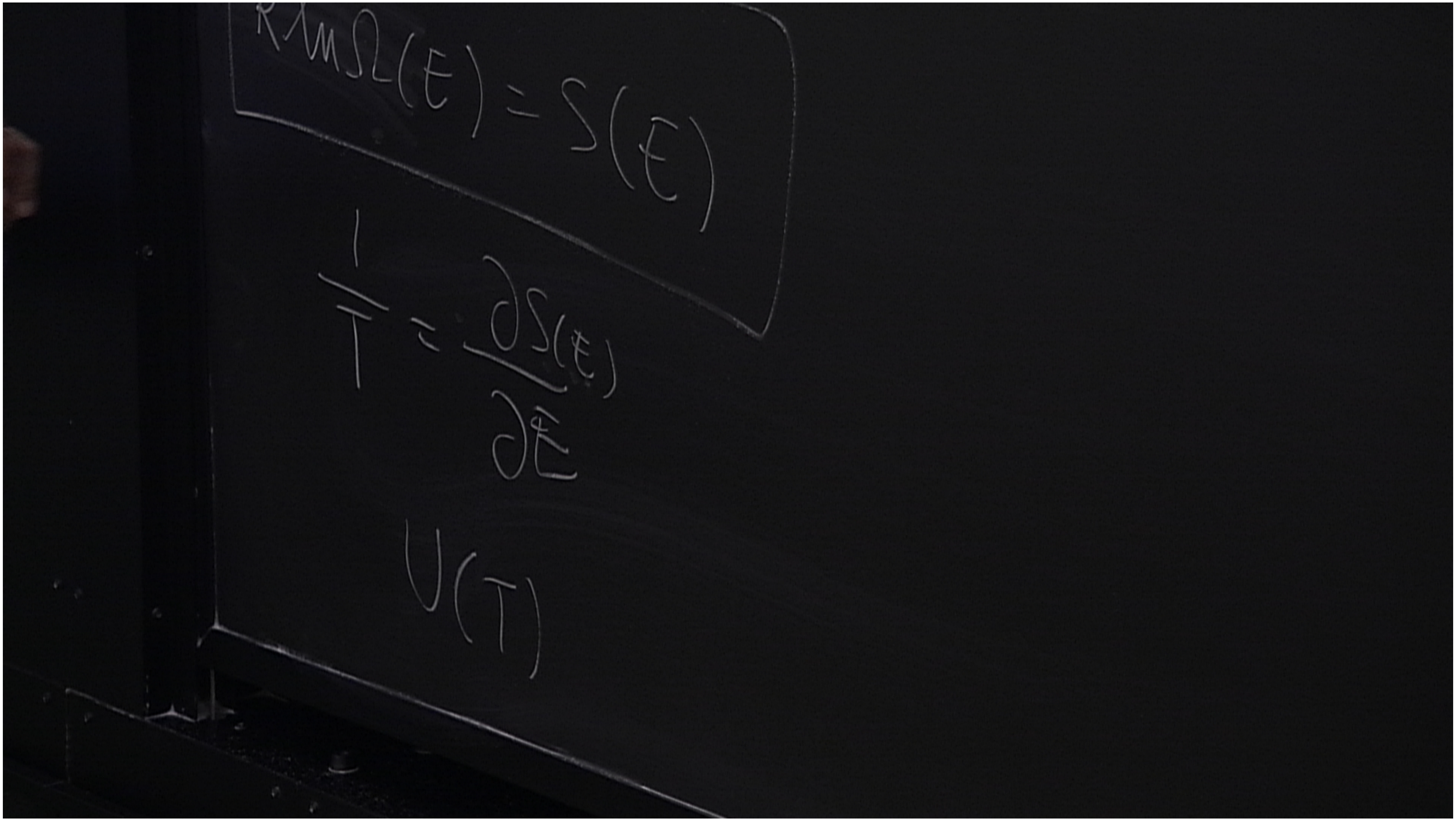
$$\ln \rho = \alpha + \beta E$$

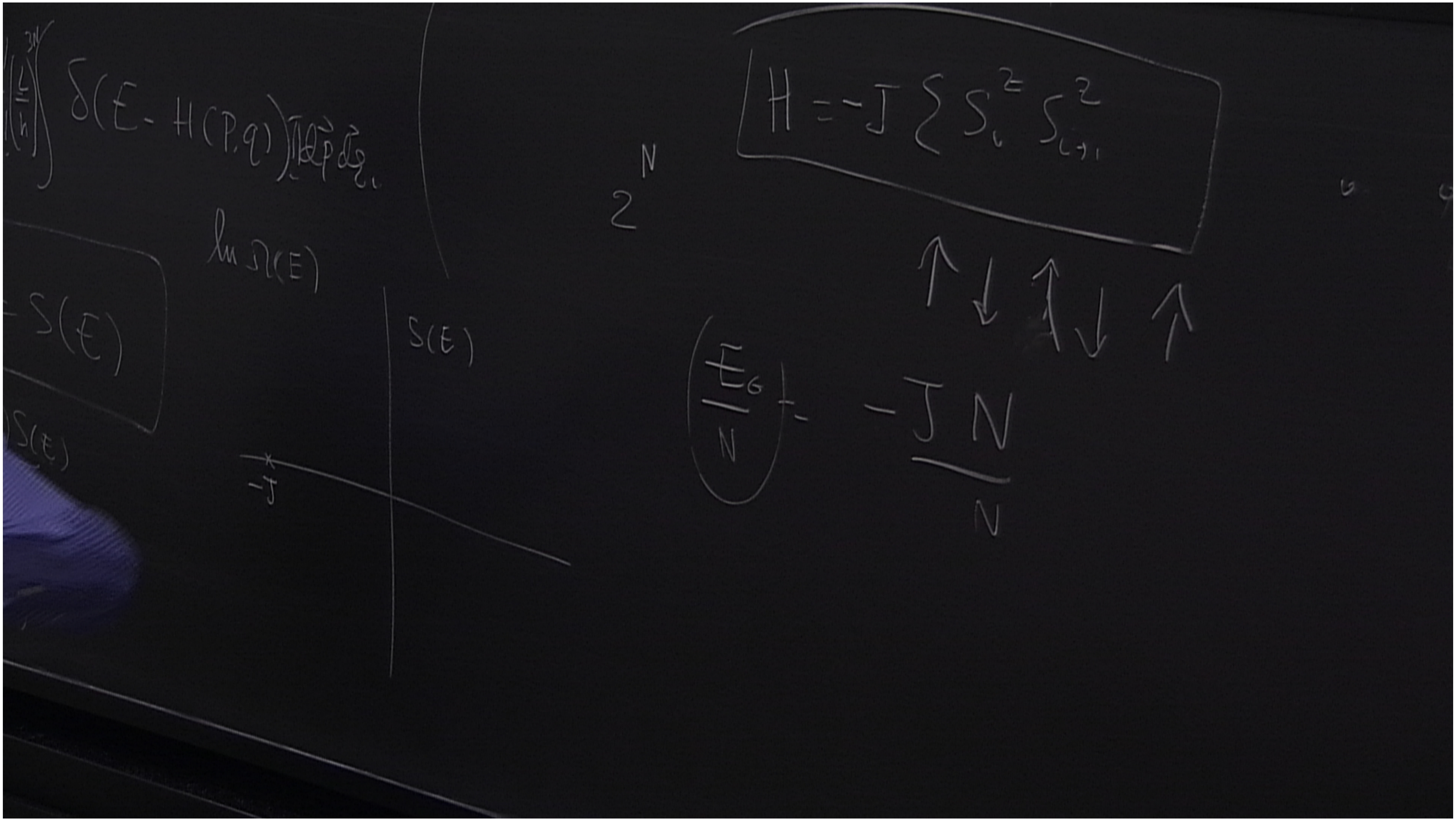
μ-Canonical

$$\boxed{\Omega(E)} = \frac{1}{N!}\left(\frac{L}{h}\right)^{3N} \int \delta(E - H(p,q)) \prod d\vec{p}\, d\vec{q}.$$

$$\boxed{k \ln \Omega(E) = S(E)}$$

$$\frac{1}{T} = \frac{\partial S(E)}{\partial E}$$


$k \ln \Omega(E) = S(E)$
$\frac{1}{T} = \frac{\partial S(E)}{\partial E}$
$U(T)$

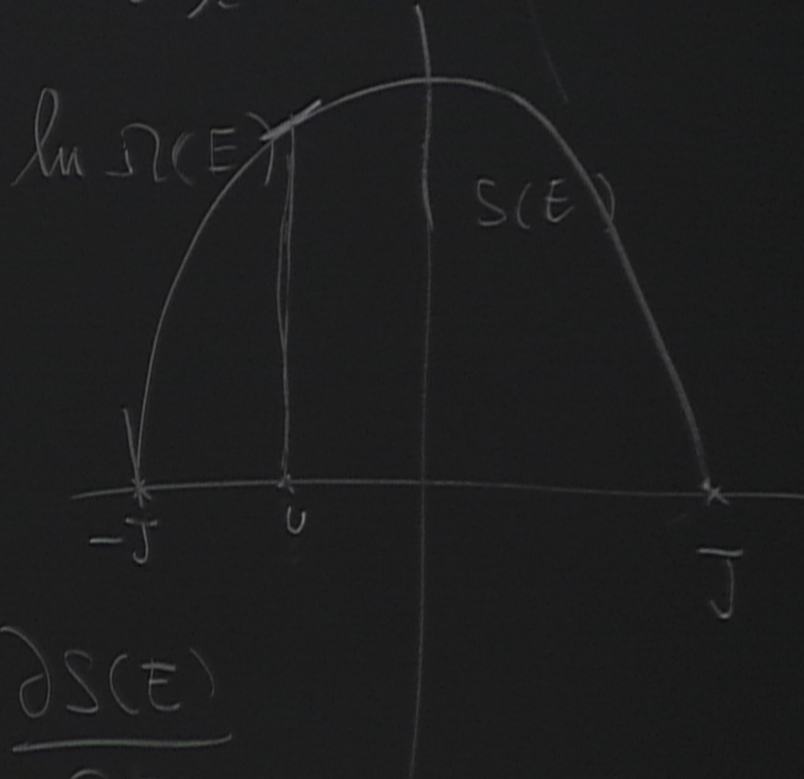

δ(E - H(P,q))
ln Ω(E)
S(E)
S(E)
S(E)
-J
2^N
H = -J Σ S_i^z S_{i+1}^z
E_G / N = -J N / N

2

$$\boxed{\Omega(E)} = \frac{1}{N!}\left(\frac{L}{h}\right)^{3N} \int \delta(E - H(p,q)) \prod d\vec{p}\, d\vec{q}$$

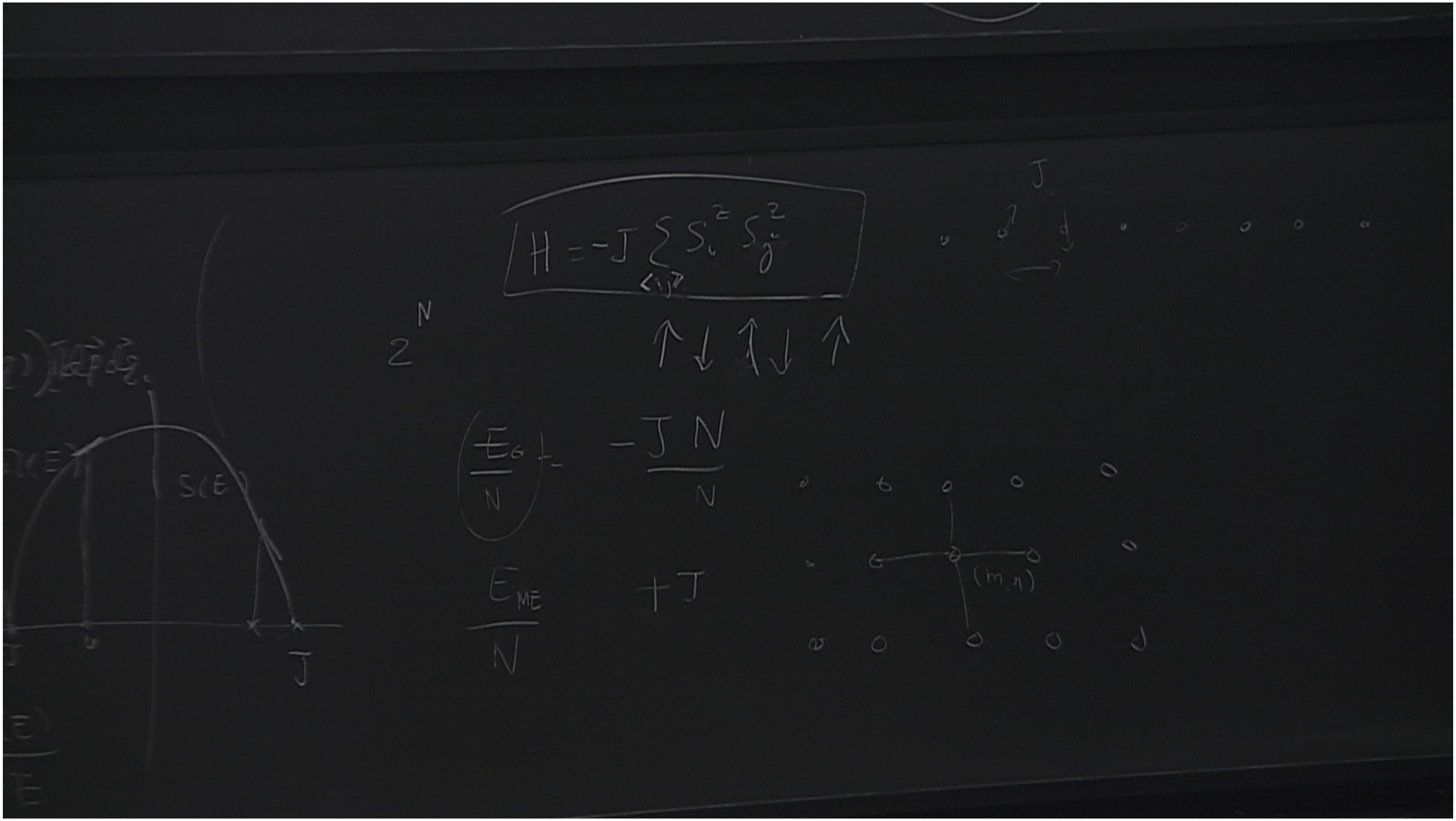
$$\boxed{k \ln \Omega(E) = S(E)}$$

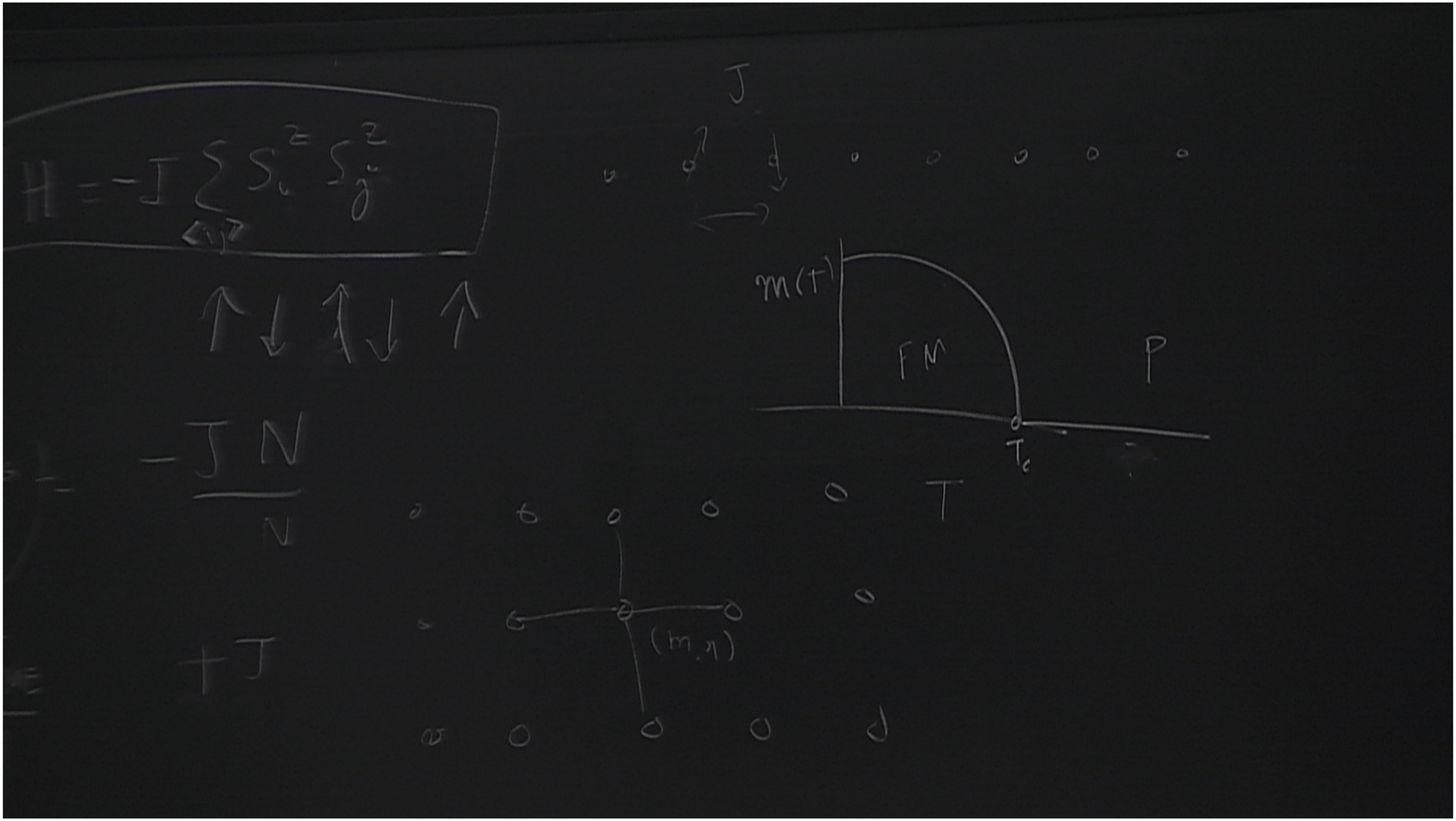
$$\frac{1}{T} = \frac{\partial S(E)}{\partial E}$$



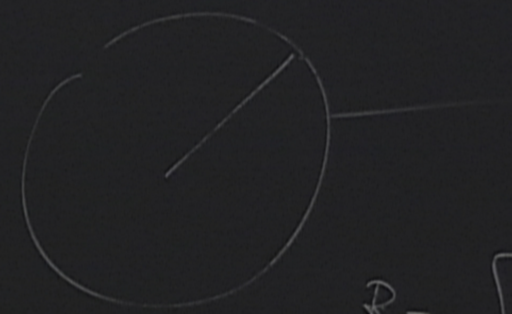
$$\frac{1}{T} = \frac{\partial S(E)}{\partial E}$$

$U(T)$


$H = -J \sum_{\langle ij \rangle} S_i^z S_j^z$
2^N
$\frac{E_G}{N} = -\frac{JN}{N}$
$\frac{E_{ME}}{N}$ $+J$
S(E)
J
(m,n)


$H = -J \sum_{\langle ij \rangle} S_i^z S_j^z$
J
m(T)
FM
P
T_c
T
$-\frac{JN}{N}$
(m,n)
+J

$$\Gamma(E) = \left(\frac{L}{h}\right)^{3N} \frac{1}{N!} V^N \boxed{\int_{\left(\sum \frac{p_i^2}{2m}\right) < E} \prod_i d\vec{p}_i}$$



$$R = \sqrt{2mE}$$

$$\frac{\partial \Gamma(E)}{\partial E} = \Omega(E)$$

$$\Omega_n = \frac{\pi^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2}+1\right)} R^n$$

Sakur-Tetrode - - -

$$S(E,V) = Nk \log\left(V \left(\frac{4\pi}{3} \frac{m}{h^3} \frac{E}{N} \right)^{3/2} \right)$$

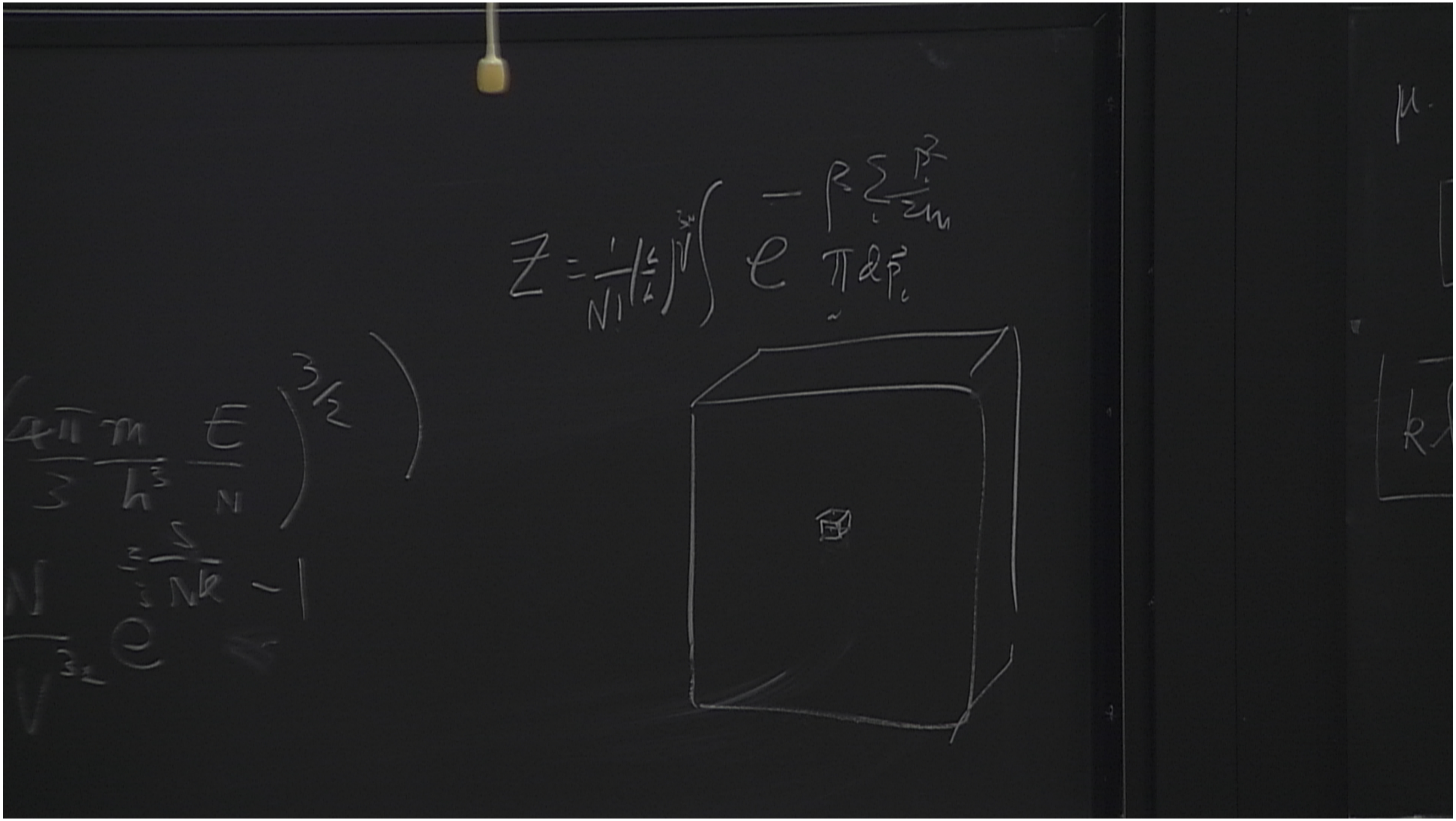
$$U(S,V) = \left(\frac{3h^2}{4\pi m} \right) \frac{N}{V^{3/2}} e^{\frac{2}{3}\frac{S}{Nk} - 1}$$

$$\boxed{P = \frac{NkT}{V}}$$

$$Z = \frac{1}{N!}\left(\frac{L}{h}\right)^{3N} \int e^{-\beta \sum_i \frac{\vec{p}_i^{\,2}}{2m}} \prod_i d\vec{p}_i$$

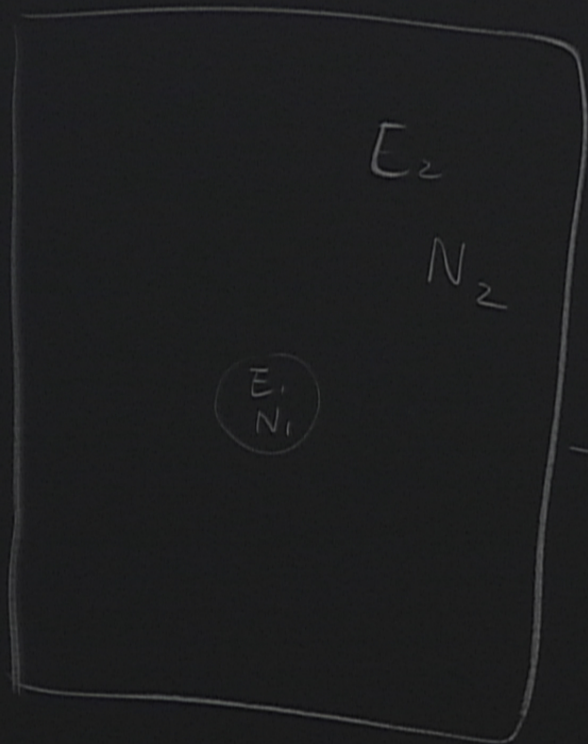
$$\left(\frac{4\pi}{3}\frac{m}{h^3}\frac{E}{N}\right)^{3/2}$$

$$e^{\frac{2}{3}\frac{S}{Nk} - 1}$$


$Z = \frac{1}{N!}\left(\frac{L}{h}\right)^{3N} \int e^{-\beta \sum_i \frac{\vec{p}_i^2}{2m}} \prod_i d\vec{p}_i$

$E_2 \gg E_1$

$N_2 \gg N_1$



$E = E_1 + E_2$

$\left(\prod d\vec{p}_i\, d\vec{q}_i\right) \quad (\vec{p}_i, \vec{q}_i)$

$\Omega_1(\vec{p}_i, \vec{q}_i) = \left(\prod d\vec{p}_i\, d\vec{q}_i\right) \Omega_2(E_2)$

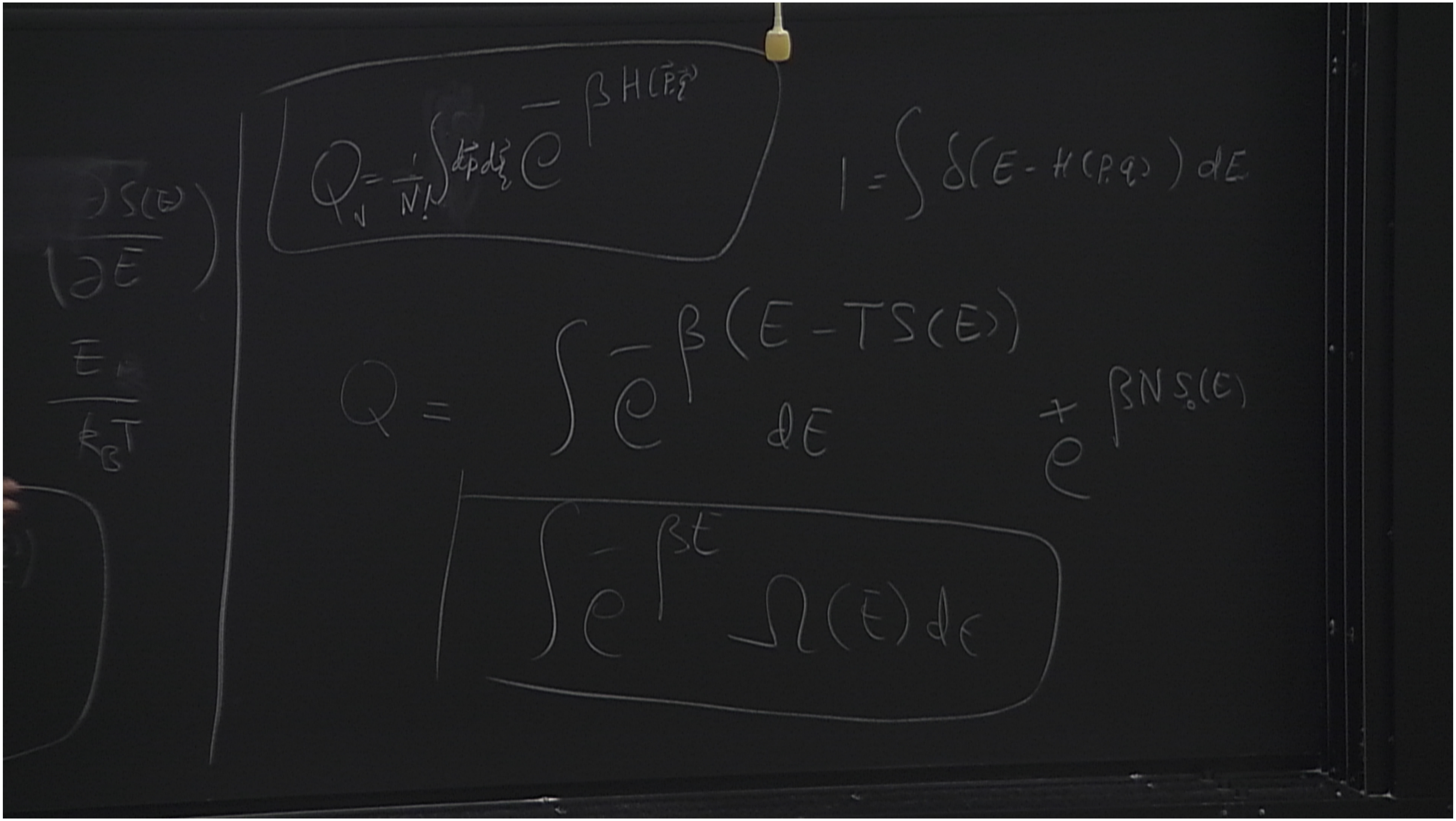
$\left(\qquad \right) e^{S_2(E - E_1)}$

$$S_2(E - E_1) \simeq S_2(E) + (E_1)\left(\frac{\partial S(E)}{\partial E}\right)$$

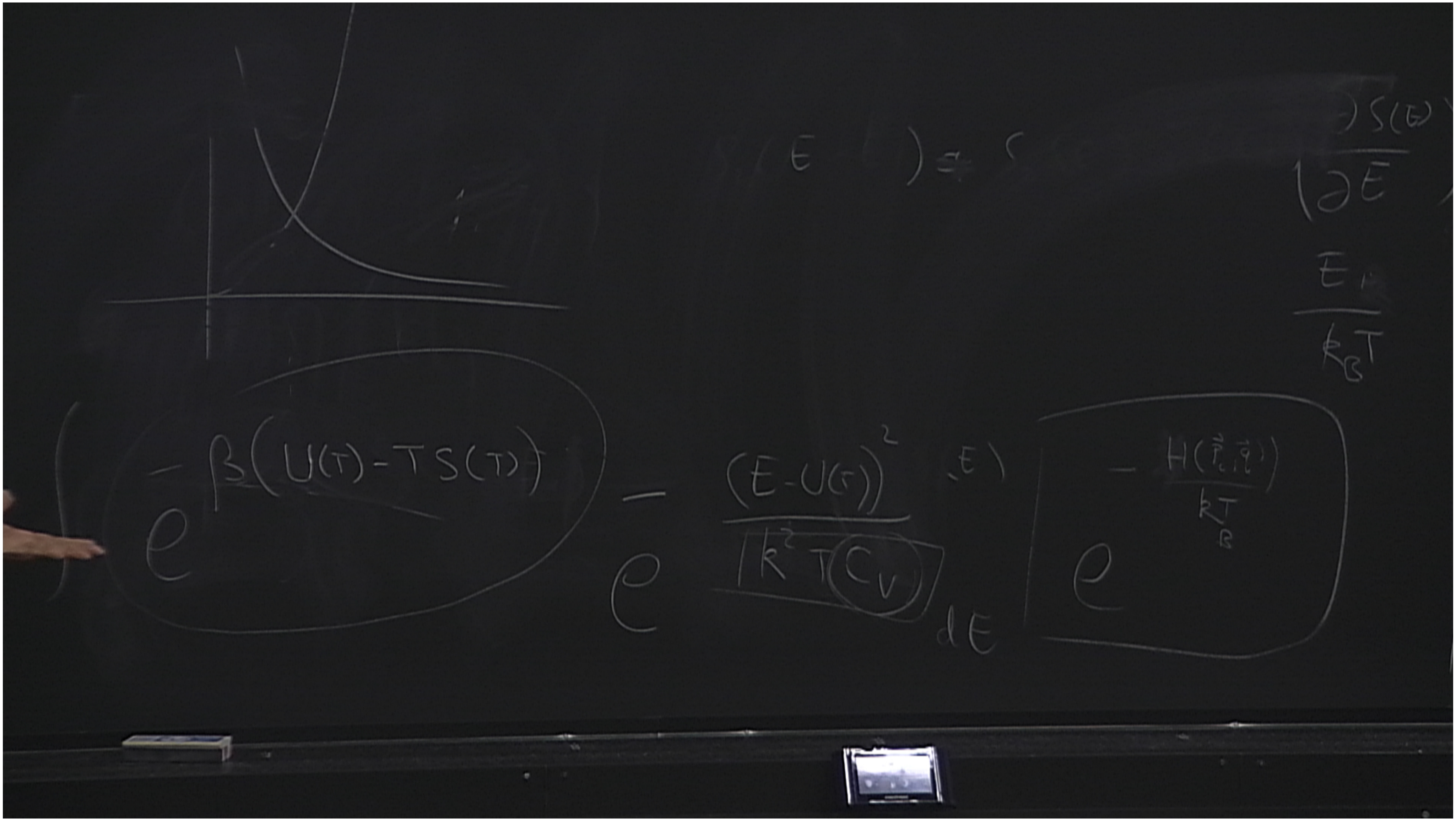
$$\frac{E_1}{k_B T}$$

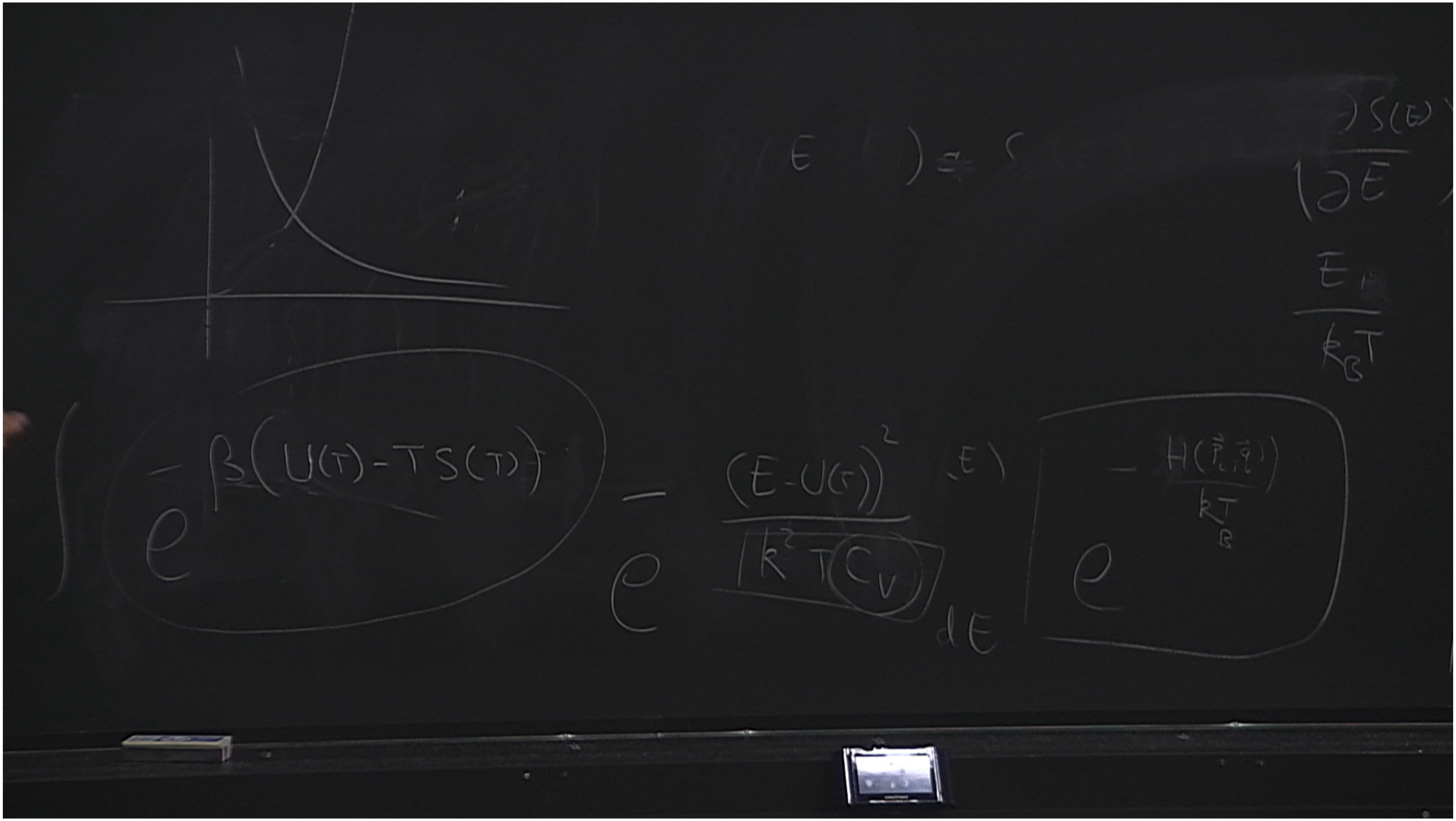
$$\left(\prod_i d\vec{p}_i\, d\vec{q}_i\right) e^{S_2(E)} \boxed{e^{-\frac{H(\vec{p}_i,\vec{q}_i)}{k_B T}}}$$

$$\boxed{Q_N = \frac{1}{N!}\int d\vec{p}\, d\vec{q}\; e^{-\beta H(\vec{p},\vec{q})}}$$



$Q_N = \frac{1}{N!}\int d\vec{p}\, d\vec{q}\, e^{-\beta H(\vec{p},\vec{q})}$
$1 = \int \delta(E - H(p,q))\, dE$
$Q = \int e^{-\beta(E - TS(E))}\, dE$
$e^{-\beta N s_0(E)}$
$\int e^{-\beta E}\, \Omega(E)\, dE$
$\frac{\partial S(E)}{\partial E}$
$\frac{E}{k_B T}$


$e^{-\beta(U(T)-TS(T))}$
$e^{-\frac{(E-U(T))^2}{k^2 T C_V}} dE$
$e^{-\frac{H(\vec{p}_i,\vec{q}_i)}{k_B T}}$
$\frac{E_i}{k_B T}$


$e^{-\beta(U(T)-TS(T))}$
$e^{-\frac{(E-U(T))^2}{k^2 T C_V}}$
dE
$e^{-\frac{H(\vec{p}_i,\vec{q}_i)}{k_B T}}$
$\frac{E}{k_B T}$