Title: The Hawking-Hartle No Boundary Proposal in Causal Set Quantum Gravity

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Abstract: <p>The Hartle-Hawking (HH) no-boundary proposal provides a Euclidean path integral prescription for a measure on the space of all possible initial conditions. Apart from saddle point and minisuper-space calculations, it is hard to obtain results using the unregulated path integral. A promising choice of spacetime regularisation comes from the causal set (CST) approach to quantum gravity. Using analytic results as well as Markov Chain Monte Carlo and numerical integration methods we obtain the HH wave function in a theory of non-perturbative 2d CST. We find that the wave function is sharply peaked with the peak geometry changing discretely with "temperature". In the low temperature regime the peak corresponds to causal sets which have no continuum counterpart but exhibit physically interesting behaviour. They show a rapid spatial expansion with respect to the discrete proper time as well as a high degree of spatial homogeneity due to extensive overlap of the causal past. While our results are limited to 2 dimensions they provide a concrete example of how quantum gravity could explain the initial conditions for our observable universe.</p>

The Hartle-Hawking Wave Function in 2d Causal Set Quantum Gravity

Sumati Surya

Raman Research Institute

Perimeter Institute
Oct 2015

Outline

- A Rapid Review of Causal Set Theory (CST)
- A Continuum-Inspired Dynamics
- The Hartle-Hawking Wave Function in CST
- A Restriction to 2d
- Some Tantalising Results

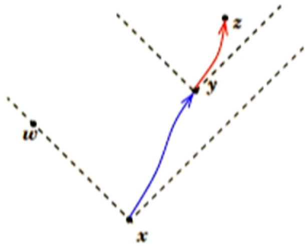
In collaboration with Lisa Glaser: `arXiv:1410.8775`

The Causal Set Hypothesis

– L.Bombelli, J.Lee, D. Meyer and R. Sorkin, PRL 1987

CST has two fundamental building blocks:

- The Causal Structure Poset $(M, \prec)$



  - M is the *set* of events.
  - $\prec$ is:
    - Acyclic: $x \prec y$ and $y \prec x \Rightarrow x = y$
    - Reflexive: $x \prec x$
    - Transitive: $x \prec y, y \prec z \Rightarrow x \prec z$

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- Fundamental Spacetime Discreteness:

  V has $n \sim V/V_p$ fundamental spacetime atoms.

Be Wise – discretise! – Mark Kac

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$\Downarrow$

The underlying structure of spacetime is a *causal set* or locally finite poset $(C, \prec)$

The Causal Set Hypothesis

Hawking-King-McCarthy-Malament Theorem

Causal Structure + Volume Element = Spacetime

The CST Hypothesis

Causal Structure $\rightarrow$ Partially Ordered Set

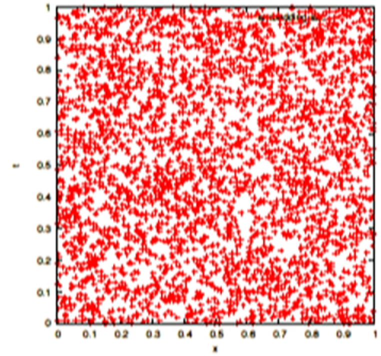
Spacetime Volume $\rightarrow$ Number

Order + Number $\sim$ Spacetime geometry

The Causal Set Hypothesis

- Spacetime emerges as a "random lattice" generated via a Poisson process:

$$P_V(n) \equiv \frac{1}{n!} \exp^{-\rho V} (\rho V)^n, \quad < N >= \rho V$$



- Local Lorentz invariance: there are no preferred directions

– L.Bombelli, J.Henson, R. Sorkin, Mod.Phys.Lett. 2009

- Non-locality: A causal set need not be a fixed valency graph.

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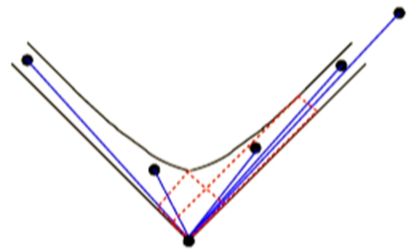


A Continuum Inspired Dynamics for Causal sets

- First principles: Quantum sequential growth using the *quantum measure formulation.*

  –F. Dowker, S. Johnston, S. Surya, J.Phys., 2010

  –R.Sorkin, arXiv:1104.0997

  –R.Sorkin and S. Surya, work in progress

- Continuum Inspired Dynamics:

$$Z_\Omega = \sum_{c \in \Omega} \exp^{\frac{i}{\hbar} S(c)}$$

  - $S(C)$ is the *Benincasa-Dowker action* which is the analog of the Einstein-Hilbert action in CST.

    –D. Benincasa and F. Dowker, Phys.Rev.Lett. 104 2010

  - Ω is a sample space of causal sets (e.g.: the set of all past-finite causal sets)

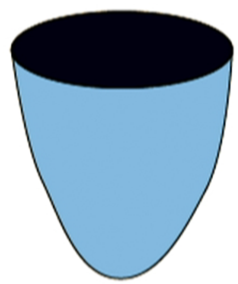
Analytic Continuation

$$Z_\Omega = \sum_{c\in\Omega} \exp^{\frac{i}{\hbar}S(c)} \longrightarrow Z_\Omega = \sum_{c\in\Omega} \exp^{-\frac{\beta}{\hbar}S(c)}$$

- Analytic continuation of a parameter β: $i\beta \to -\beta$
- Space of Configurations Ω is unchanged.
- MCMC for Ω_N:
  - $\beta = 0$:
    - In the $N \to \infty$ limit the *Kleitman-Rothschild* posets dominate.
    - The onset of the asymptotic regime occurs for $N > 80$
      – J. Henson, D. Rideout, R. Sorkin and S.Surya, arXiv:1504.05902, 2015
  - First steps being taken to study $\beta \neq 0$ – a challenge!
- Restriction to 2d orders: $\Omega_{2d} \subset \Omega_N$
  - $\beta = 0$ dominated by 2d Minkowski spacetime. –G. Brightwell, J. Henson, S.Surya, Class.Quant.Grav. 25, 2008
  - Phase transition from continuum to a crystalline phase. –S. Surya, Class.Quant.Grav. 29, 2012
  - Scaling near β_c suggests robust large N behaviour. –L. Glaser, D. O'Connor and S. Surya, work in progress

The Hartle-Hawking Prescription in CST

- Continuum Proposal: $\Psi_0(h_{ab}, \Sigma) = A \sum_M \int dg \ \exp^{-\frac{1}{\hbar} I_E(g)}, \qquad \partial M = \Sigma, g|_\Sigma = h$
  - Path integral over Riemannian geometries on M.
  - M is compact with a "final" boundary Σ.
  - Initial spatial "zero" geometry, *"a single point, which captures the idea of a universe emerging from nothing."'*



- CST Proposal: $\Psi_0^{(N)}(\mathcal{N}_f, \beta) \equiv A \sum_{c \in \Omega_N} \exp^{-\frac{1}{\hbar} \beta S(c)}, \qquad |Max(C)| = \mathcal{N}_f$

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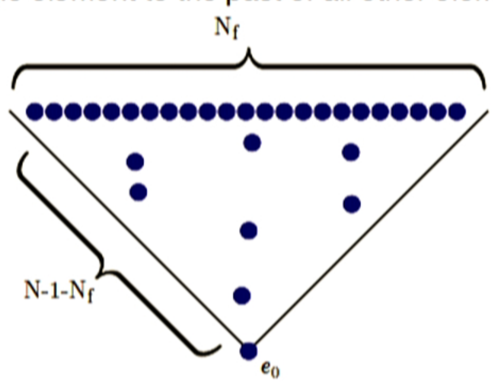
  - The sum is over "discrete Lorentzian" geometries or causal sets.
  - $c \in \Omega_N$ is of finite cardinality, and the final *maximal antichain* $\mathcal{A}_f$ is such that $|\mathcal{A}_f| = \mathcal{N}_f$.
  - Initial spatial geometry is a single element to the past of all other elements in c.



2d **CST**

$$\Psi_0^{(N)}(\mathcal{N}_f, \beta) \equiv A \sum_{c \in \Omega_{2d}(N)} \exp^{-\frac{1}{\hbar}\beta S_{2d}(c)}$$

- $\Omega_{2d}(N) \subset \Omega(N)$: Sample space of N-element 2d-orders.
- $S_{2d}(c)$: 2d Benincasa-Dowker Action.

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Ω_{2d} the sample space of 2d Orders

- A 2d order is an *intersection of two linear orders*
  - Base Set: $S = (1, \ldots, N)$.
  - $u_i, v_i \in S = (1, \ldots, N)$, $i = 1, \ldots, N$.
  - $U = (u_1, u_2, \ldots u_i, \ldots u_j, \ldots u_N)$, $V = (v_1, v_2, \ldots v_i, \ldots v_j, \ldots v_N)$ are *total orders or chains.*
  - 2d order $C = U \cap V$: $e_i = (u_i, v_i) \prec e_j = (u_j, v_j)$ iff $u_i < u_j$ and $v_i < v_j$.
- Examples:
  - Discretisation of 2d (conformally flat) spacetimes.
  - Non continuum like 2d orders
  - Sprinklings into finite regions of ${}^2\mathbb{M}$ are 2d *random orders*

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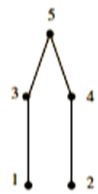
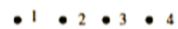
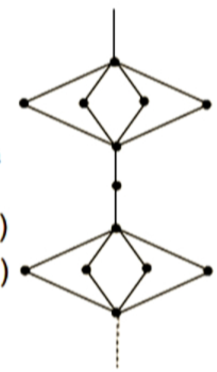
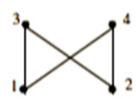


L = (1,3,2,4,5)
M= (2,4,1,3,5)



L = (1,2,3,4)
M= (4,3,2,1)





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Ω_{2d} the sample space of 2d Orders

- A 2d order is an *intersection of two linear orders*
- Examples:
  - Discretisation of 2d (conformally flat) spacetimes.
  - Non continuum like 2d orders
  - Sprinklings into finite regions of $^2\mathbb{M}$ are 2d *random orders*
    - U and V chosen randomly and independently from S.
    - Random 2d orders ($\sim {}^2\mathbb{M}$) dominate the uniform distribution.

–Peter Winkler, Order 1, 317, (1985), El-Zahar and N.W. Sauer, Order 5, 239, (1988)
–G. Brightwell, J. Henson, S.Surya, Class.Quant.Grav. 25, 2008

The 2d Benincasa-Dowker Action for a Causal Set

$$\frac{1}{\hbar}S(\epsilon) = 4\epsilon\left(N - 2\epsilon\sum_{n=0}^{N-2} N_n f(n,\epsilon)\right)$$

- Mesoscale $l_k >> l_p$, $\epsilon = \left(\frac{l_p}{l_k}\right)^2 \in (0,1]$
- $f(n,\epsilon) = (1-\epsilon)^n - 2\epsilon n(1-\epsilon)^{n-1} + \frac{1}{2}\epsilon^2 n(n-1)(1-\epsilon)^{n-2}$
- N_n: $\#$ of n-element order intervals
- $\epsilon = 1$: $\frac{1}{\hbar}S^{(2)}(C) = N - 2N_0 + 4N_1 - 2N_2$

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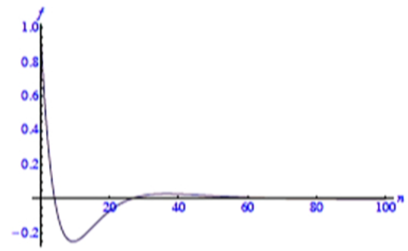

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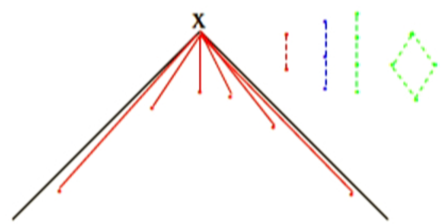

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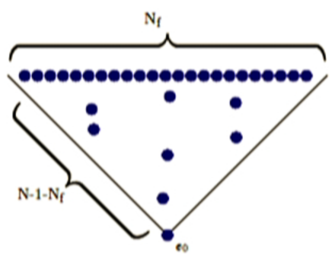
The 2d Hartle-Hawking Wave Function: Some Analytic Results

$$\Psi_0^{(N)}(\mathcal{N}_f, \beta) \equiv A \sum_{c \in \Omega_{2d}} \exp^{-\frac{1}{\hbar}\beta S_{2d}(c)}, \qquad \mathcal{N}_f = N - p$$



- $\mathbf{p = 1}$:
  - $N_0 = N - 1$, $N_i = 0 \, \forall \, i > 0$.
  - $\frac{1}{\hbar} S_{2d}(N, \epsilon) = 2\epsilon N(1 - 2\epsilon) + 4\epsilon^2$
  - $\Psi_0(N - 1) = A \exp^{-\beta R}$, $R = 2\epsilon N(1 - 2\epsilon) + 4\epsilon^2$

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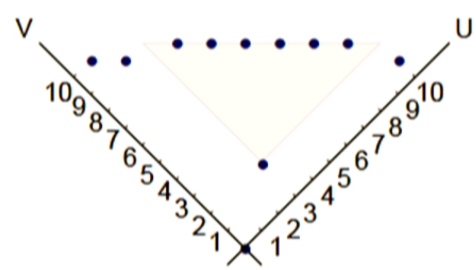

• $\mathbf{p = 2}$:

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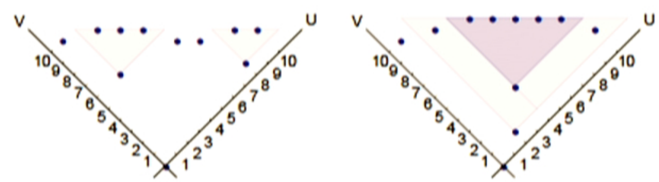

- **p = 3** :
  - antichain
  - chain

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- antichain

$$\Psi_0^{(a,i)}(N-3) = A \exp^{-\beta R} \sum_{\ell_1=1}^{N-3} \sum_{\ell_2=1}^{N-3} \sum_{m=m_0}^{m_f} (N-2-\ell_1-\ell_2+m) \exp^{\beta P m} \exp^{\beta Q(\ell_1+\ell_2)}$$

- chain

$$\Psi_0^{(a,ii)}(N-3) = A \exp^{-\beta R} \sum_{\ell_1=1}^{(N-3-1)} \sum_{\ell_2=1}^{(N-3-\ell_1)} \sum_{\tilde{m}=0}^{(N-3-\ell_1-\ell_2)} (N-2-\ell_1-\ell_2-\tilde{m}) \exp^{\beta Q(\ell_1+\ell_2)}$$

$$P = 24\epsilon^4, \quad m_0 = max(1, \ell_1+\ell_2-N+3), \quad m_f = min(\ell_1, \ell_2)$$

The 2d Hartle-Hawking Wave Function: Some Analytic Results

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- **p** $>$ **3** : Analytically challenging/impossible!

Numerical Simulations

$$\Psi_0(\mathcal{N}_f) = A\mathcal{Z}_\beta(\mathcal{N}_f) = A\mathcal{Z}_0(\mathcal{N}_f)\exp\left(-\int_0^\beta d\beta' \langle S_{\beta'}(\mathcal{N}_f)\rangle\right)$$

- Calculation of $\langle S_\beta(\mathcal{N}_f)\rangle$ using MCMC methods.
- Numerical Integration: $\int_0^\beta d\beta' \langle S_{\beta'}(\mathcal{N}_f)\rangle$
- Estimation of $\mathcal{Z}_0(\mathcal{N}_f)$.
- Normalise to get A.
- Calculations performed for $N = 50, \epsilon = 0.12, 0.5, 1$.

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Calculating $\langle S_\beta(\mathcal{N}_f)\rangle$ using Markov Chain Monte Carlo Methods

The Move:

- $U = (u_1, u_2, \ldots u_i, \ldots u_j, \ldots u_N)$, $V = (v_1, v_2, \ldots v_i, \ldots v_j, \ldots v_N)$
- Pick a pair (u_i, v_i) and (u_j, v_j) at random and exchange: $u_i \leftrightarrow u_j$
- $U' = (u_1, u_2, \ldots u_j, \ldots u_i, \ldots u_N)$, $V' = (v_1, v_2, \ldots v_i, \ldots v_j, \ldots v_N)$
- Example:

  $u_2 \leftrightarrow u_3$: $U = (1, 2, 3, 4)$, $V = (1, 2, 3, 4) \longrightarrow U' = (1, 3, 2, 4)$, $V' = (1, 2, 3, 4)$

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Expectation values for Covariant Observables

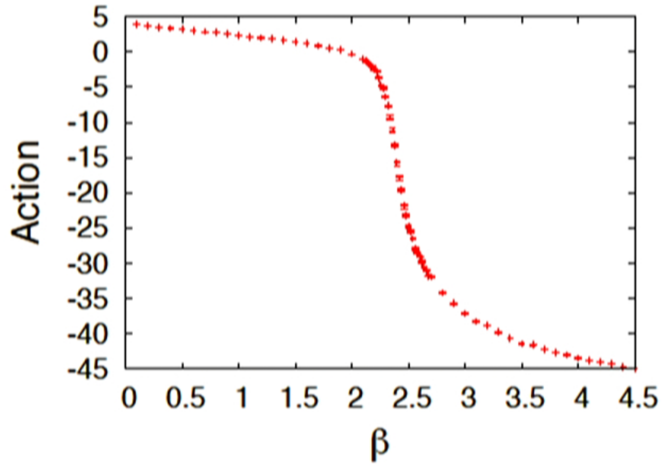
Covariance $\sim$ Label invariance

- Ordering Fraction: $\chi = 2r/N(N-1)$
  r: actual number of relations in the causal set, $N(N-1)/2$: maximum number of possible relations
- Dimension: Spacetime dimension v/s poset dimension
  In 2d Myrheim-Meyer dimension $d_{MM} = \chi^{-1}$
- Action ($\sim$ energy):
  $S(\epsilon)/\hbar = 4N\epsilon \times \left(1 - 2\frac{\epsilon}{N}\sum_{n=0}^{N-2} N_n f(n,\epsilon)\right)$
- N_n: Abundance of n-order intervals
- Height: Length of the longest chain $\sim$ longest time-like distance
- Time asymmetry: Difference in number of minimal and maximal elements

Unconstrained $\mathcal{N}_f$: A Phase Transition

–S. Surya, Class.Quant.Grav. 29, 2012

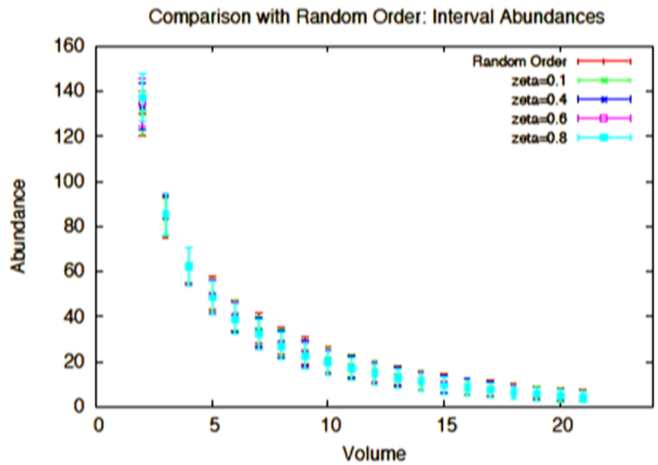
$\langle S_\beta(\mathcal{N}_f)\rangle$ vs β for $N = 50, \epsilon = 0.12$



Continuum Phase

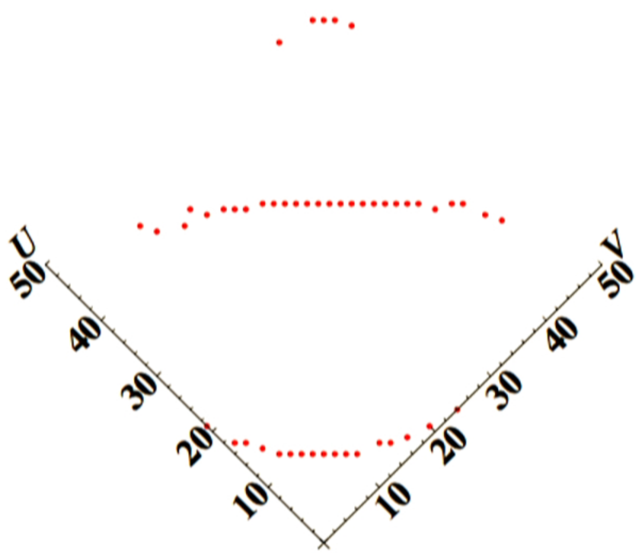
For $\epsilon = 0.12$, $\beta = 0.1$:

- Ordering Fraction: $< \chi >= 0.498 \pm 0.045$. $\Rightarrow$ $< d_{MM} > \sim 2$.
- Height: $< h >= 10.217 \pm 1.401$ (*Height of* $V = 50$ *Minkowski interval is* $\sqrt{100} = 10$)
- Time Asymmetry: $< TA >= -0.007 \pm 2.411$
- Action: $< S > /\hbar = 3.845 \pm 1.256$
- Abundance of Intervals:



Continuum Phase closely resembles the random 2D order aka the Minkowski interval

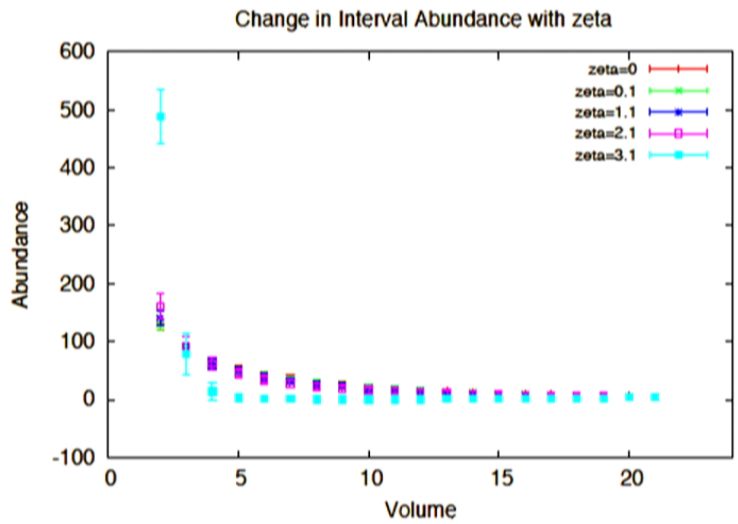
Crystalline Phase



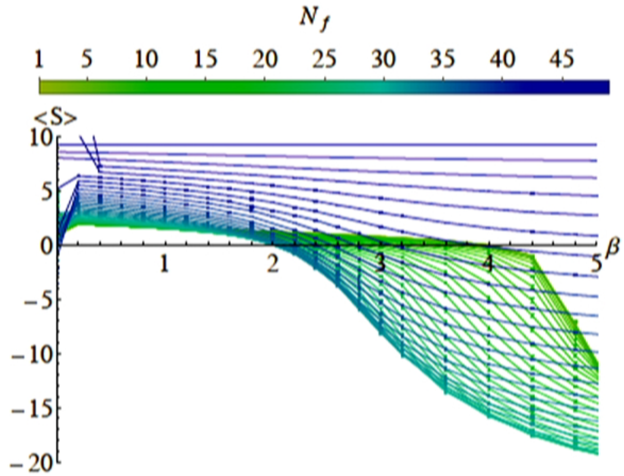
Crystalline Phase

For $\epsilon = 0.12$, $\beta = 3.1$

- Ordering Fraction: $<\chi>= 0.589 \pm 0.001$. $\Rightarrow$ $<d_{MM}>\sim 1.7$.
- Height: $<h>= 4.631 \pm 0.860$
- Time Asymmetry: $<TA>= -1.327 \pm 5.156$
- Action: $<S>/\hbar = -38.000 \pm 3.197$
- Abundance of Intervals:



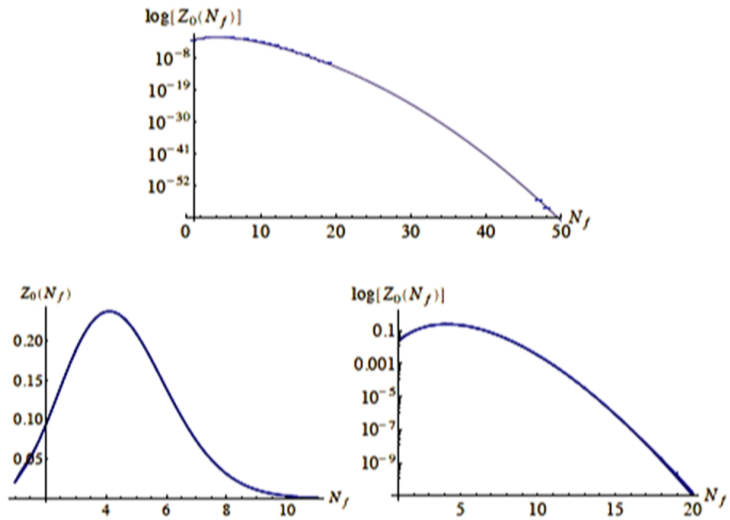
The HH boundary condition: fixing $\mathcal{N}_f$



- Phase transition for smaller $\mathcal{N}_f$.
- As N_f increases to $N-1$, the phase transition is wiped out
- Minimum value of β_c at $\mathcal{N}_f \sim 30$: $\beta_c(\mathcal{N}_f)$ is not a monotonic function.

Estimating $\mathcal{Z}_0(\mathcal{N}_f)$

- $\mathcal{Z}_0 = \sum_{\mathcal{N}_f} \mathcal{Z}_0(\mathcal{N}_f)$: dominated by 2d random orders.
- Simulate 1.138×10^{10} 2d random orders
- Generate frequency profile for $\mathcal{N}_f$ up to 19



$$f(x) = (a + hx^m + (e + fx + gx^2 + jx^3 + kx^{5.5}))\ln(x)e^{-b(x+d)^2}$$

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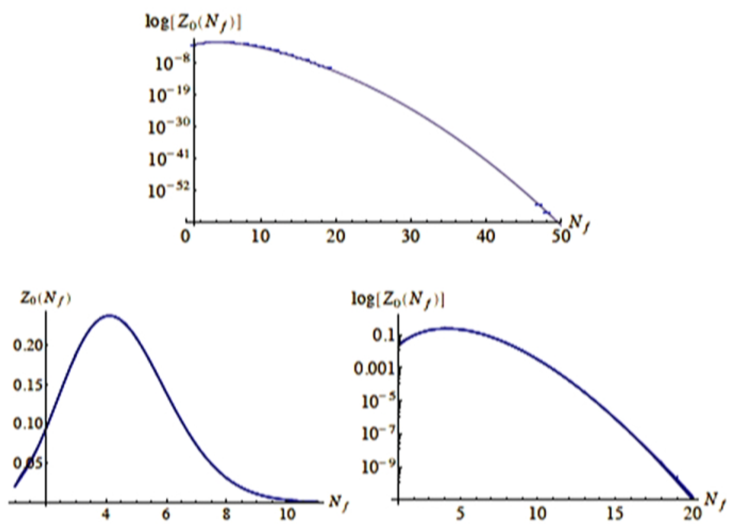

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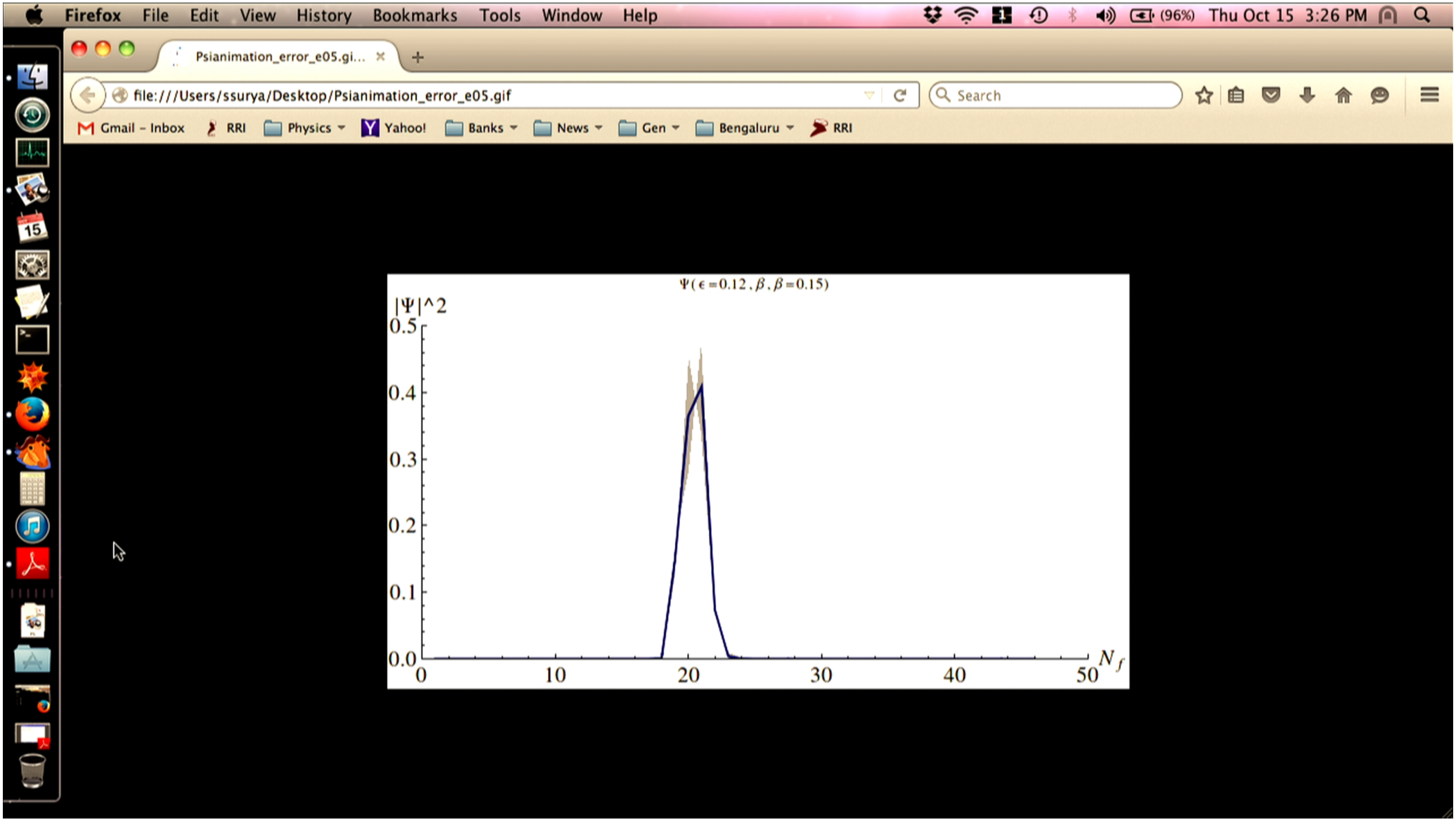

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Ψ(ϵ=0.12, β, β=0.15)
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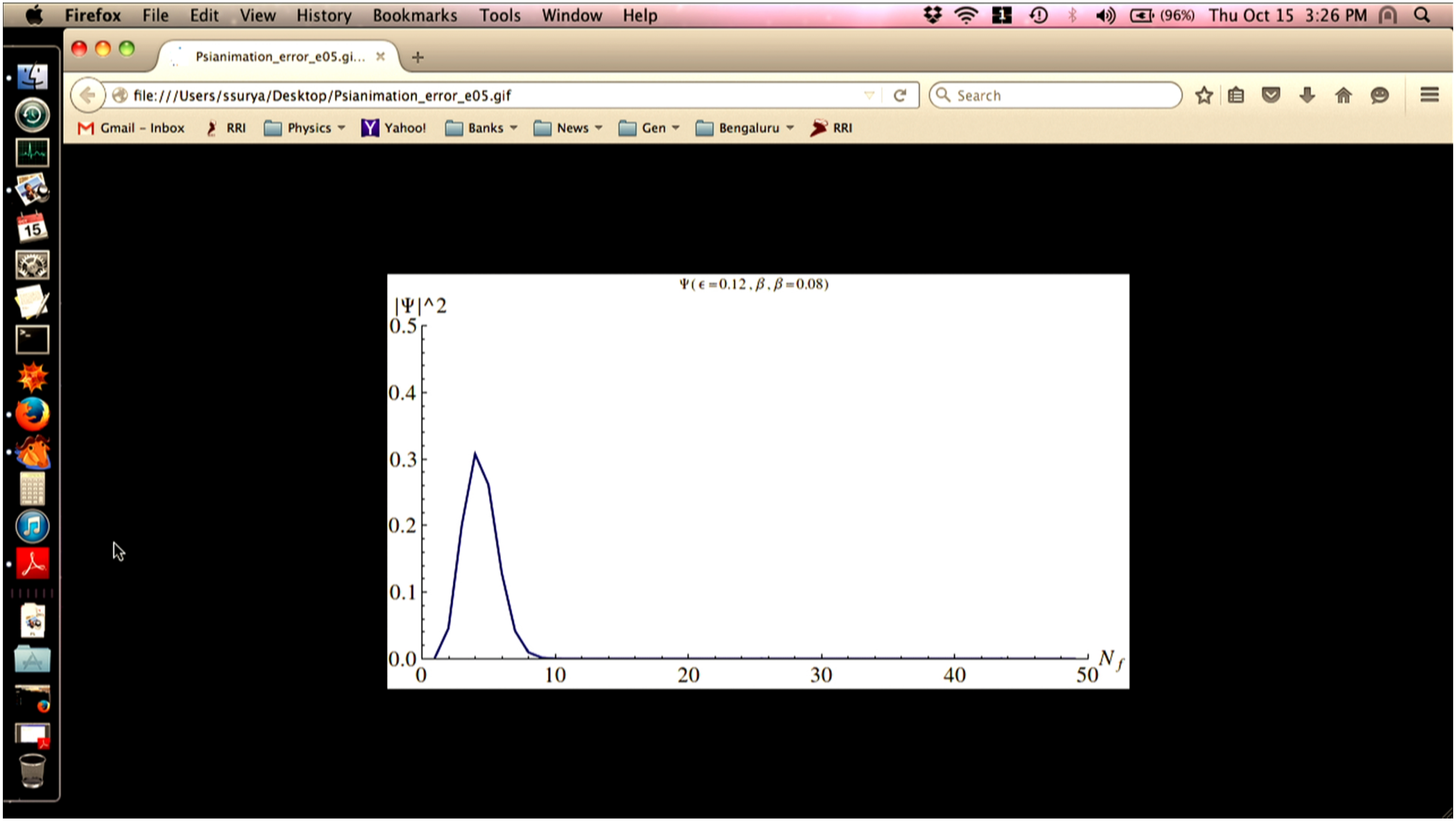
N_f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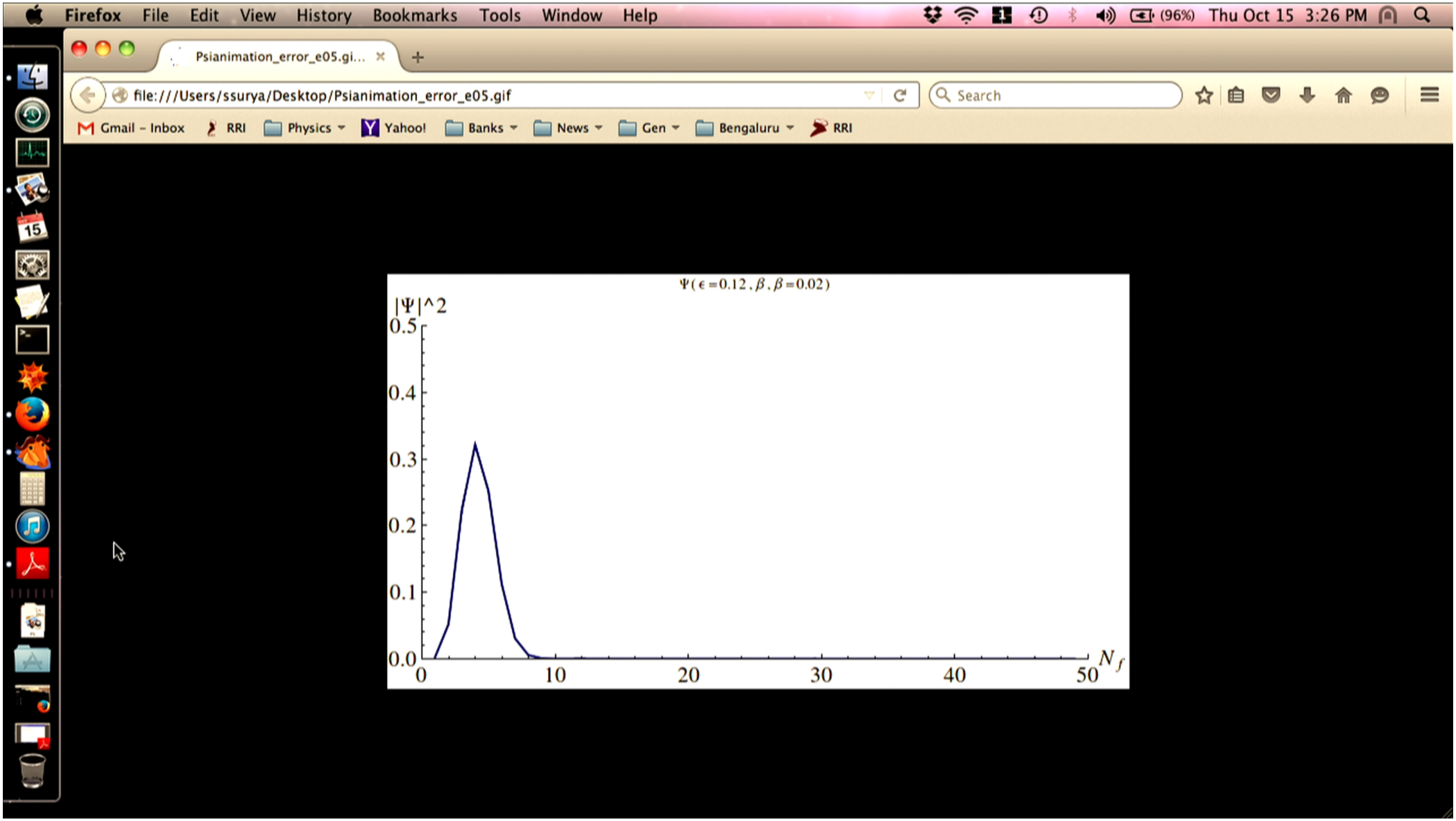
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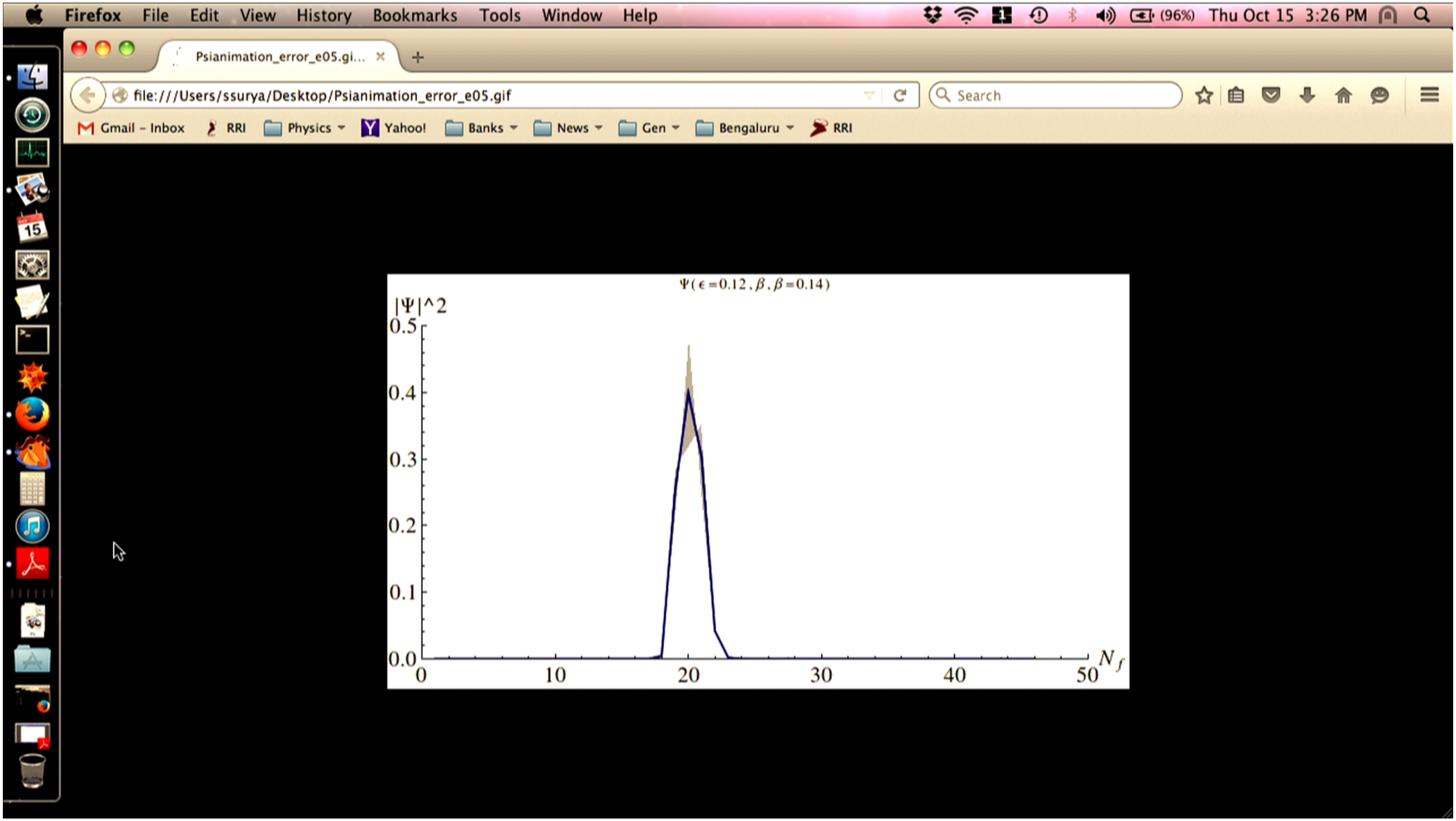
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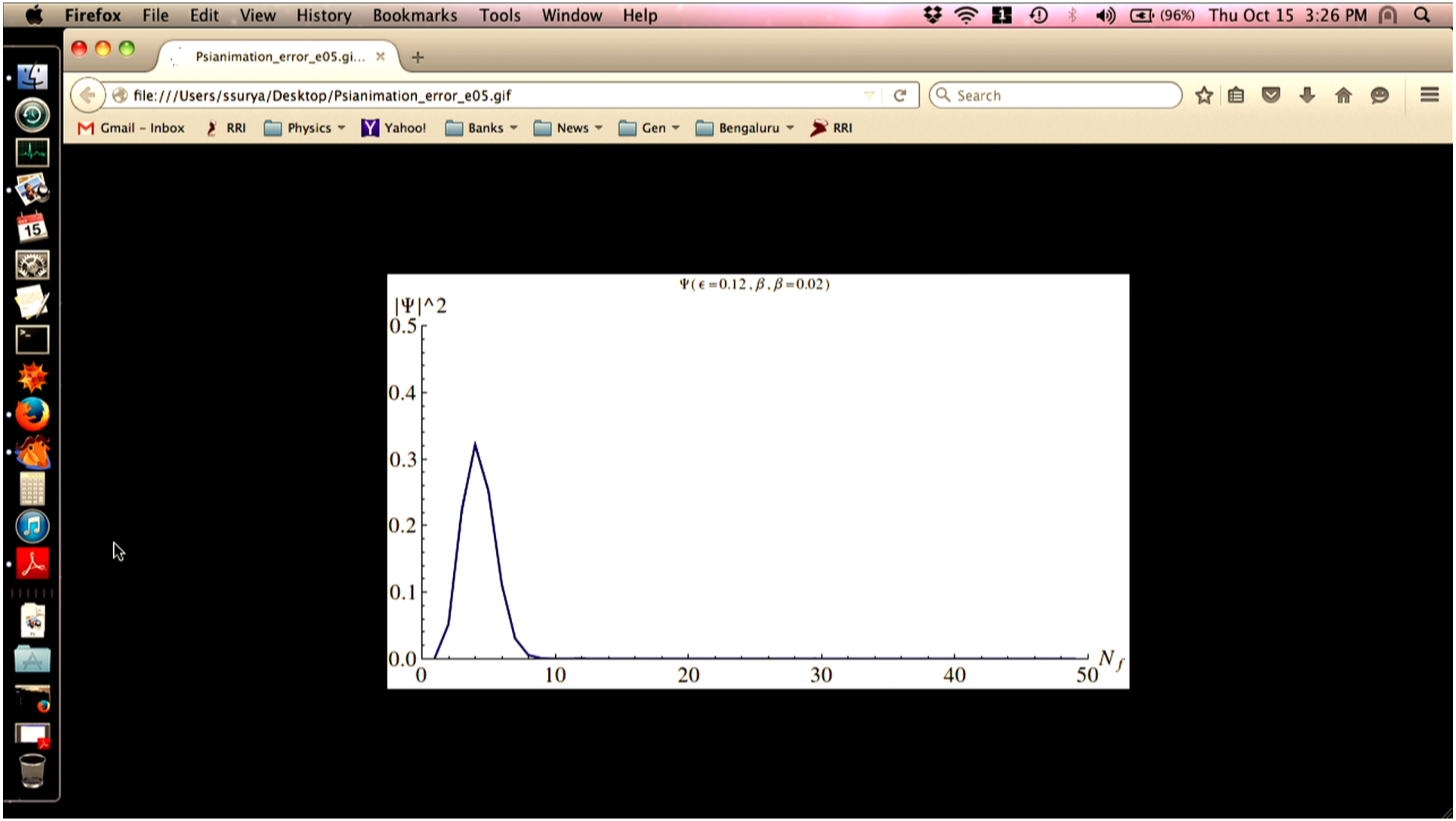
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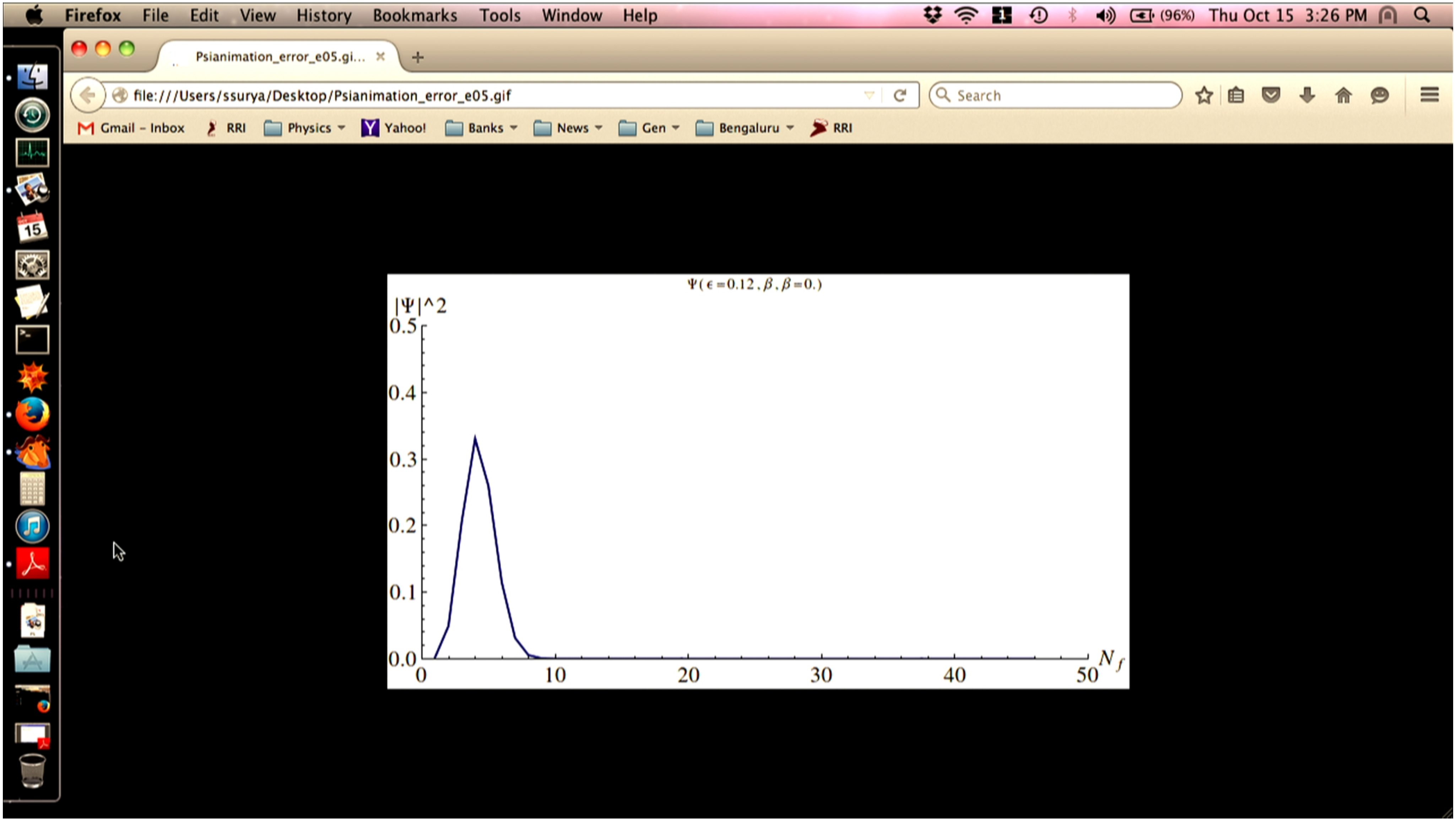
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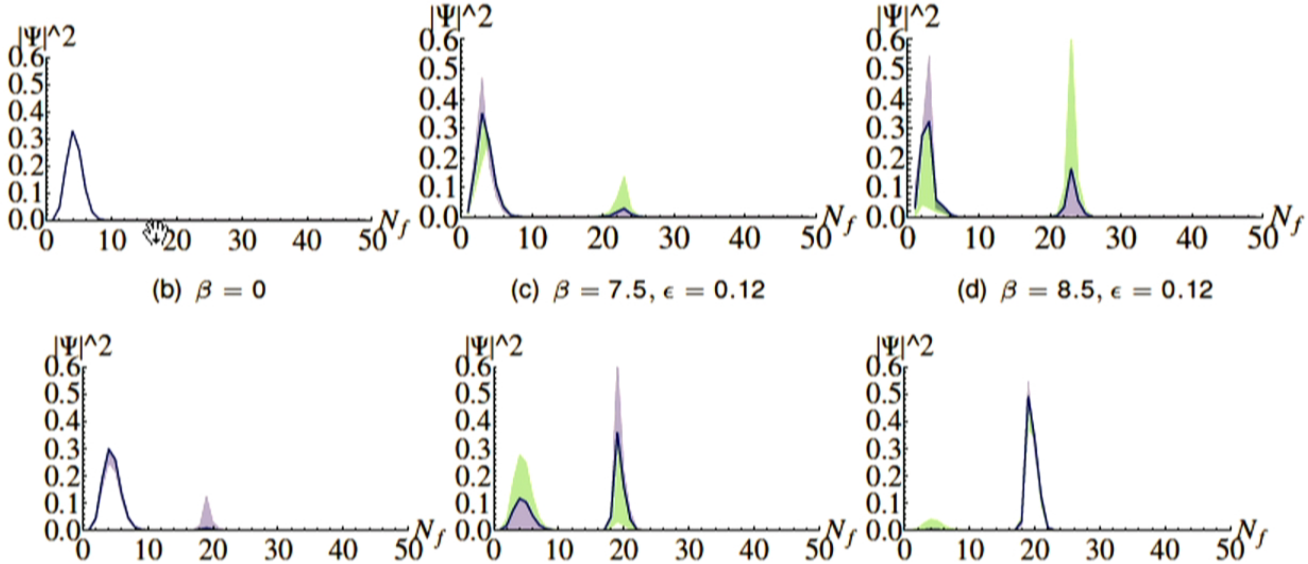
The Hartle-Hawking Wave Function.



(b) $\beta = 0$

(c) $\beta = 7.5, \epsilon = 0.12$

(d) $\beta = 8.5, \epsilon = 0.12$

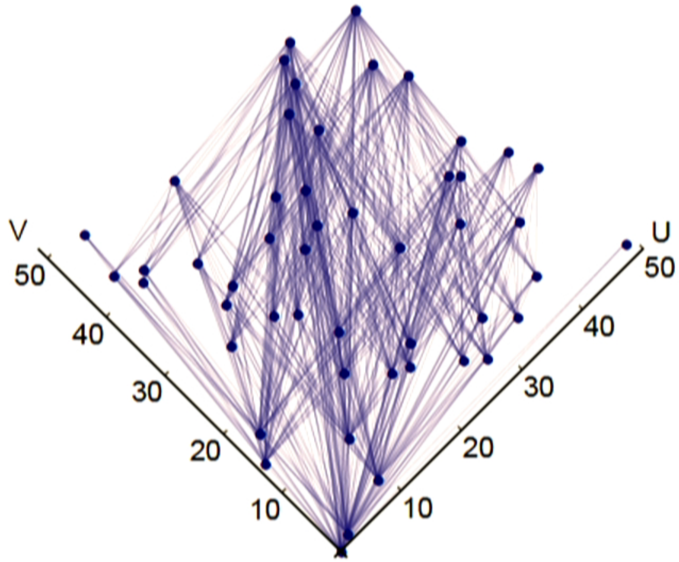
(e) $\beta = 0.10, \epsilon = 0.5$

(f) $\beta = 0.11, \epsilon = 0.5$

(g) $\beta = 0.12, \epsilon = 0.5$

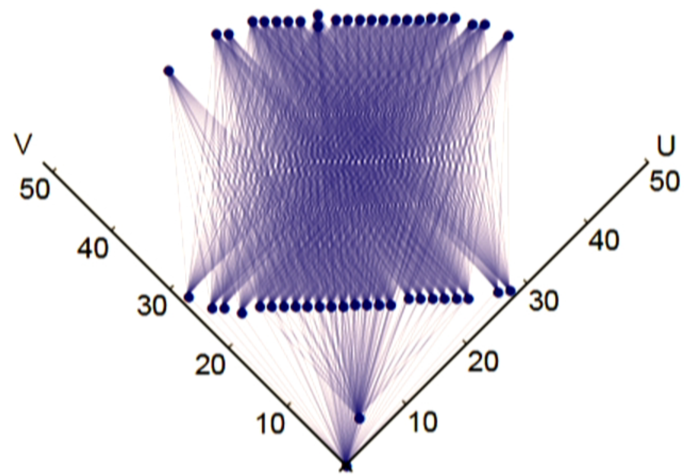
The Two Peaks

- The Peak at $\mathcal{N}_f \sim 4$



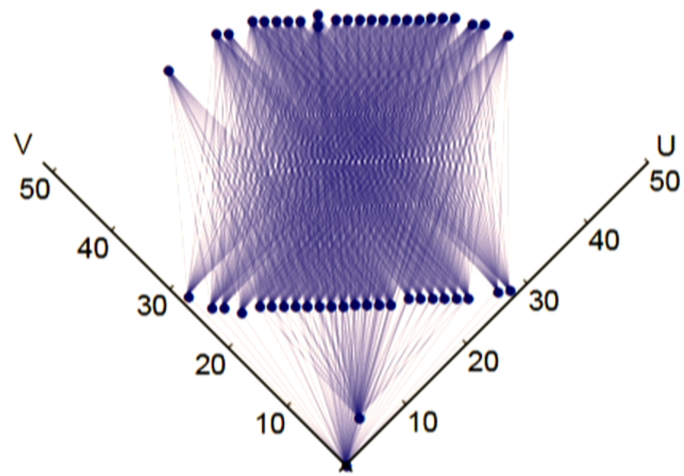
The Two Peaks

- The Peak at $\mathcal{N}_f \sim 23$



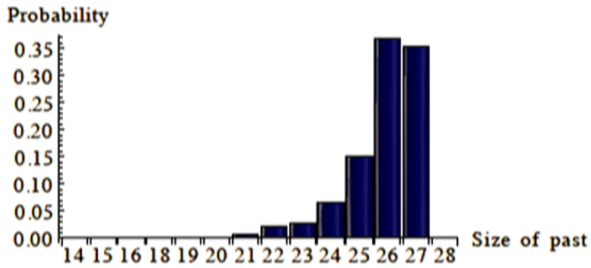
The Two Peaks

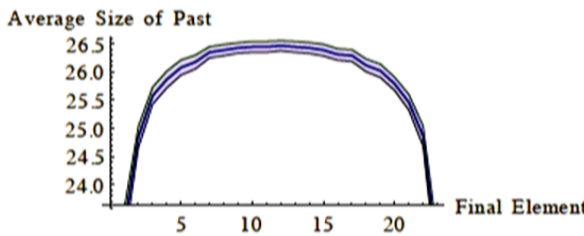
- **The Peak at** $\mathcal{N}_f \sim 23$



Features of the second peak geometry

- Rapid expansion from a single element to a large spatial slice: $\mathcal{N}_f/height \sim 6$.
- Homogeneity determined from causal past.





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- Non-manifold like.

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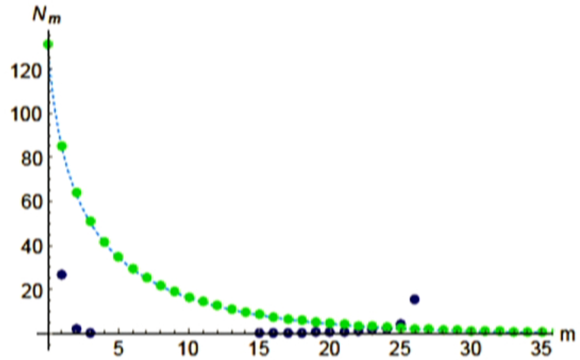

Speculations and Open Questions

- A concrete *illustration* of how physically interesting initial conditions can determined by a theory of quantum gravity. Importance of non-continuum structures.
- Comparison with DT and CDT: $\Psi_0^{DT}(L,\Lambda) \sim -\Lambda L^{-1/2}, \quad \Psi_0^{CDT}(L,\Lambda) = \exp^{-\sqrt{\Lambda}L}$
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  - 4d calculation could look very different
  - Spontaneous dimensional reduction: – Steven Carlip, AIP Conf.Proc. 1483 (2012)
  - "Graceful exit" ?
- Large N limit: Non-trivial scaling with respect to N, ϵ and β.
  – Lisa Glaser, Denjoe O'Connor and S.Surya, work in progress
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