Title: Many-body localization and thermalization in disordered Hubbard chains

Date: Oct 26, 2015 03:30 PM

URL: http://pirsa.org/15100082

Abstract: <p>In this talk, I will revise some of the aspects that lead isolated interacting quantum systems to thermalize.</p>

<p>In the presence of disorder, however, the thermalization process fails resulting in a phenomena where </p>

<p>transport is suppressed known as many-body localization. Unlike the standard Anderson localization for </p>

<p>non-interacting systems, the delocalized (ergodic) phase is very robust against disorder even for moderate</p>

<p>values of interaction. Another interesting aspect of the many-body localization phase is that under the time</p>

<p>evolution of the quenched disorder, information present in the initial state may survive for arbitrarily long times.</p>

<p>This was recently used as a probe of many-body localization of ultracold fermions in optical lattices</p>

<p>with quasi-periodic disorder [1]. Here, we will stress that this analysis may suffer from substantial finite-size effects </p>

<p>after comparing with the numerical results in one-dimensional Hubbard chains [2].</p>

<p> </p>

<p>References:</p>

<p>[1] - M.Schreiber, S. S. Hodgman,. P. Bordia,.H. P. LÃ¼schen, M. H. Fischer, R. Vosk, E. Altman, U. Schneider, I. Bloch, Science 349, 842 (2015)</p>

<p>[2] - Rubem Mondaini and Marcos Rigol, Phys. Rev. A 92, 041601(R) (2015)</p>

Many-body localization and thermalization in disordered Hubbard chains

Rubem Mondaini and Marcos Rigol
The Pennsylvania State University

Perimeter Insitute, October 26 (2015)

Outline

- Localization in the absence of interactions
  - Experimental realizations
- Interactions – Many body localization – Optical lattice experiment Science **349**, 842 (2015)
- Thermalization – ETH
- Ergodicity and random matrices
- Many-body localization in the 1d Hubbard model
- Further aspects of the Many body localization
- Summary

Introduction

Anderson localization – non-interacting particles

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

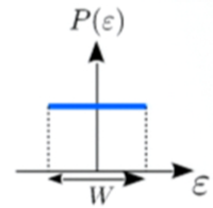
Absence of Diffusion in Certain Random Lattices

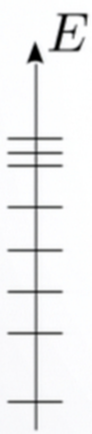
P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey

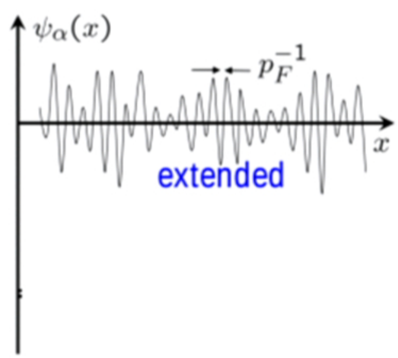
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

$$\hat{H} = -t \sum_{i,j} \hat{c}_i^\dagger \hat{c}_j + h.c. + \sum_i \varepsilon_i \hat{n}_i \qquad \varepsilon_i \in \left[-\frac{W}{2}, \frac{W}{2} \right]$$







Introduction

Anderson localization – non-interacting particles

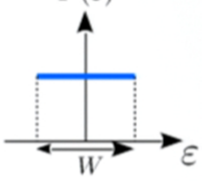
PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

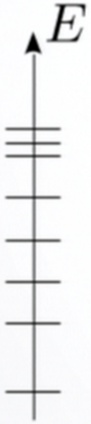
Absence of Diffusion in Certain Random Lattices

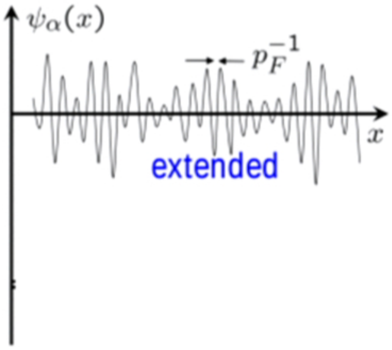
P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

$$\hat{H} = -t\sum_{i,j}\hat{c}_i^\dagger \hat{c}_j + h.c. + \sum_i \varepsilon_i \hat{n}_i \qquad \varepsilon_i \in \left[-\frac{W}{2}, \frac{W}{2}\right]$$







Introduction

Anderson localization – non-interacting particles

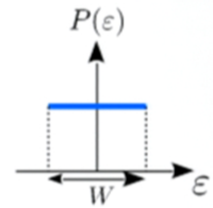
PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

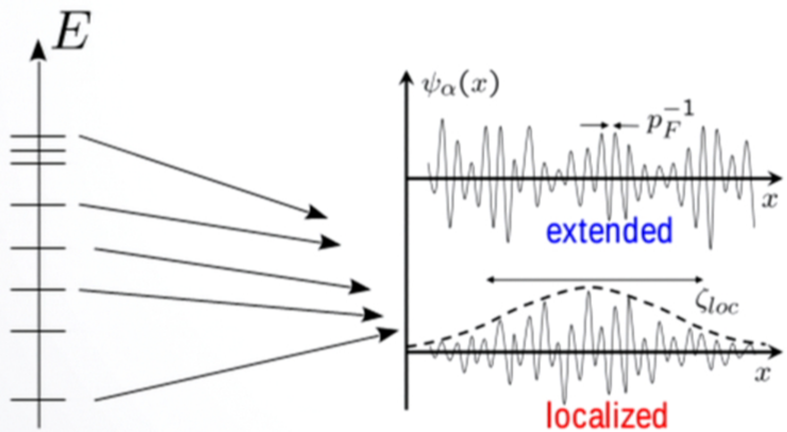
Absence of Diffusion in Certain Random Lattices

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

$$\hat{H} = -t\sum_{i,j}\hat{c}_i^\dagger\hat{c}_j + h.c. + \sum_i \varepsilon_i\hat{n}_i \qquad \varepsilon_i \in \left[-\frac{W}{2}, \frac{W}{2}\right]$$





Origin: enhanced backscattering due to quantum interference

$W_c = 0^+$ in 1D and 2D

Introduction

Anderson localization – experimental realizations

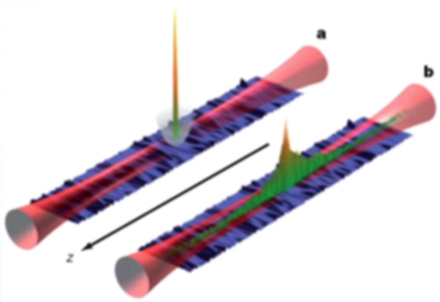
PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

BEC – optical lattices (^{87}Rb at low densities)



[Billy *et al*. Nature **453**, 891-894(2008) – Aspect's group]

Introduction

Anderson localization – experimental realizations

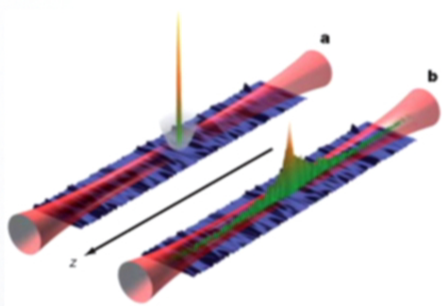
PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

BEC – optical lattices (^{87}Rb at low densities)



[Billy *et al*. Nature **453**, 891-894(2008) – Aspect's group]

Introduction

Anderson localization – experimental realizations

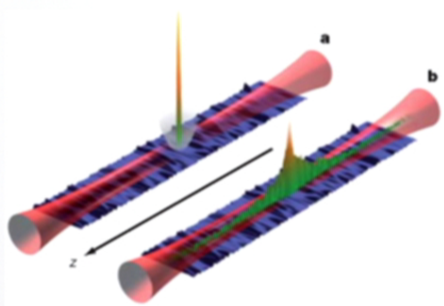
PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

BEC – optical lattices (^{87}Rb at low densities)



[Billy *et al*. Nature **453**, 891-894(2008) – Aspect's group]

Introduction

Anderson localization – experimental realizations

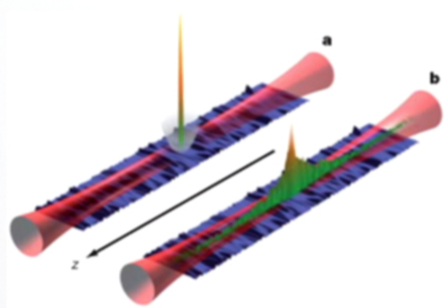
PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

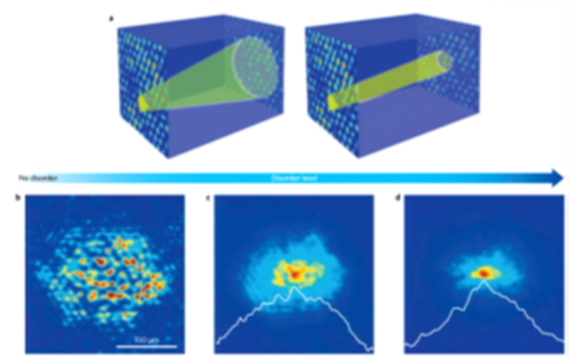
This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

BEC – optical lattices (^{87}Rb at low densities)



[Billy *et al*. Nature **453**, 891-894(2008) – Aspect's group]

Anderson localization in photonic lattices



[Schwartz *et al*. Nature **446**, 52 (2007)]

Introduction

Anderson localization – non-interacting particles – Aubry-André Model

$$\hat{H} = -t \sum_{i,j} \hat{c}_i^\dagger \hat{c}_j + \sum_i \varepsilon_i \hat{n}_i \qquad \varepsilon_i = \frac{\Delta}{2}\cos(2\pi\beta i + \phi)$$

[Roati et al. Nature **453**, 895 (2008) – Inguscio's group]

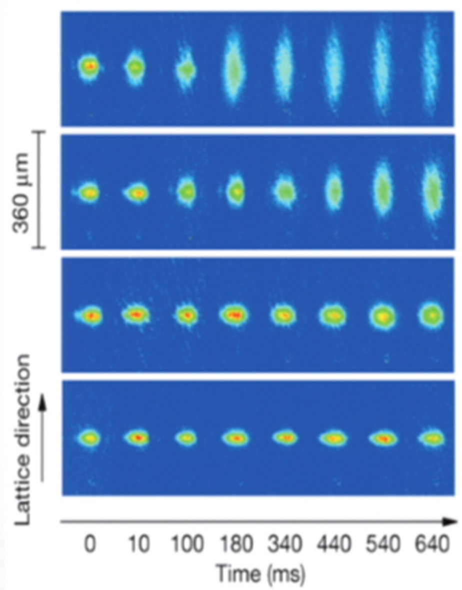
Introduction

Anderson localization – non-interacting particles – Aubry-André Model

$$\hat{H} = -t\sum_{i,j}\hat{c}_i^{\dagger}\hat{c}_j + \sum_i \varepsilon_i \hat{n}_i \qquad \varepsilon_i = \frac{\Delta}{2}\cos(2\pi\beta i + \phi)$$

$\Delta_c = 1$

delocalized localized Δ

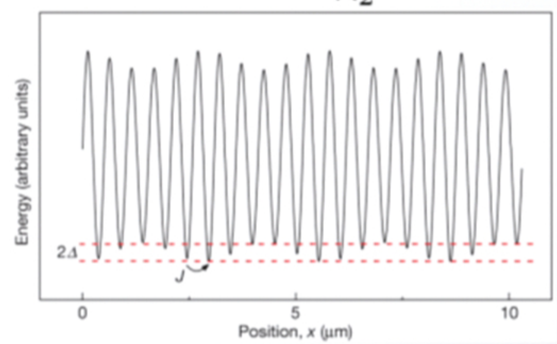


[Roati et al. Nature **453**, 895 (2008) – Inguscio's group]

BEC – optical lattices

(^{39}K at low densities – $U \approx 10^{-5}J$)

$$\beta = \frac{\lambda_1}{\lambda_2}$$



Outline

- Localization in the absence of interactions
  - Experimental realizations
- Interactions – Many body localization – Optical lattice experiment Science **349**, 842 (2015)
- Thermalization – ETH
- Ergodicity and random matrices
- Many-body localization in the 1d Hubbard model
- Further aspects of the Many body localization
- Summary

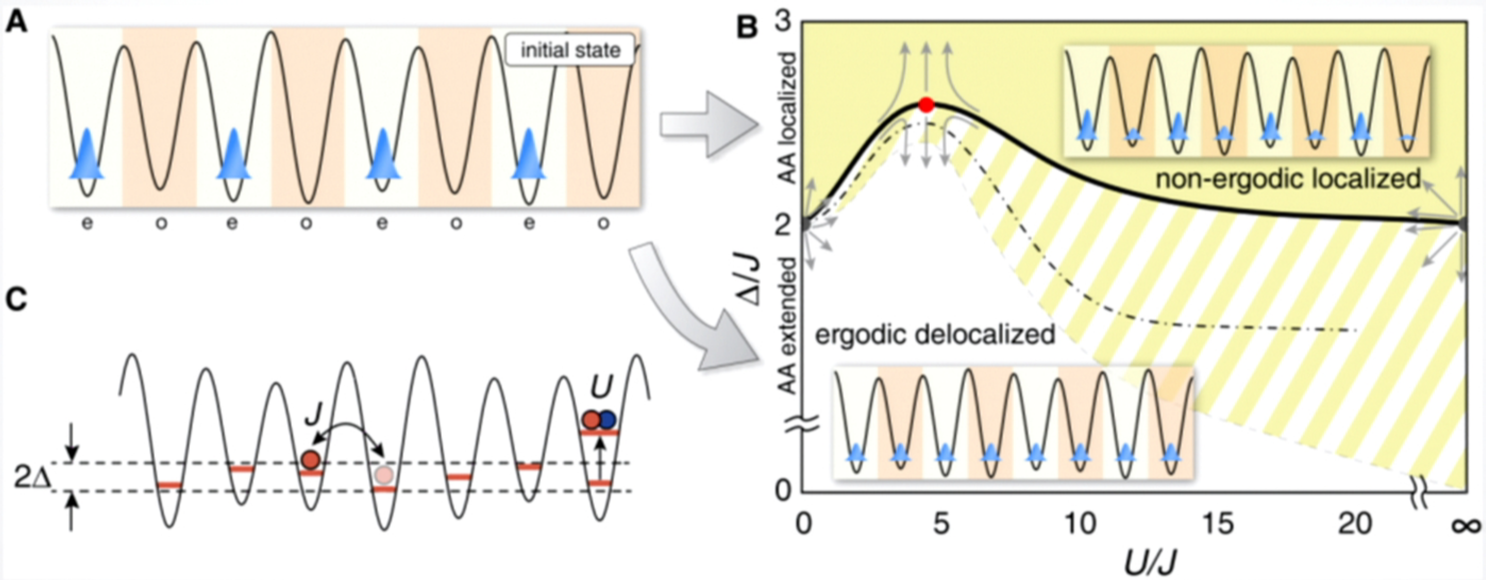
Interactions+ disorder?

Ergodicity breakdown – Many body localization

Ultracold fermionic atoms in optical lattices

$$^{40}K \rightarrow |F; m_F\rangle = \left|\frac{9}{2}; \frac{-9}{2}\right\rangle \text{ and } \left|\frac{9}{2}; \frac{-7}{2}\right\rangle$$

Memory of the initial state in the localized phase



[Michael Schreiber *et al* – Bloch's group] - Science **349** (6250): 842-845 (2015)

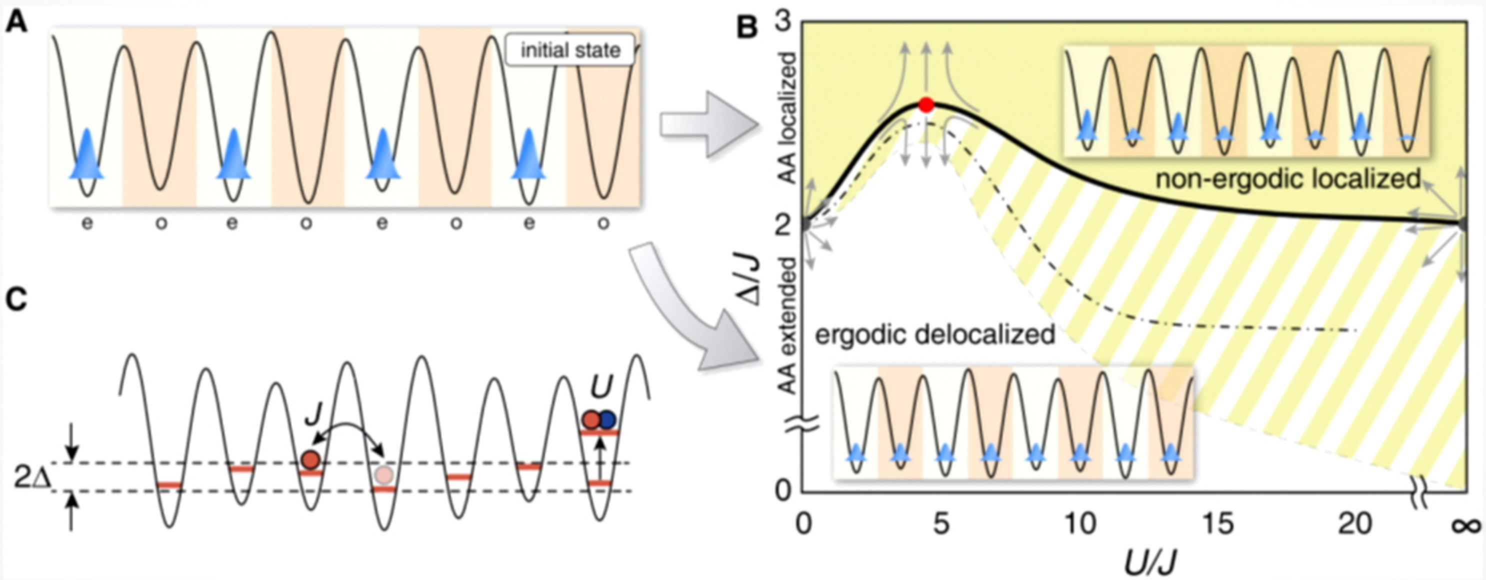
Interactions+ disorder?

Ergodicity breakdown – Many body localization

Ultracold fermionic atoms in optical lattices

$$^{40}K \rightarrow |F; m_F\rangle = \left|\frac{9}{2}; \frac{-9}{2}\right\rangle \text{ and } \left|\frac{9}{2}; \frac{-7}{2}\right\rangle$$

Memory of the initial state in the localized phase



[Michael Schreiber *et al* – Bloch's group] - Science **349** (6250): 842-845 (2015)

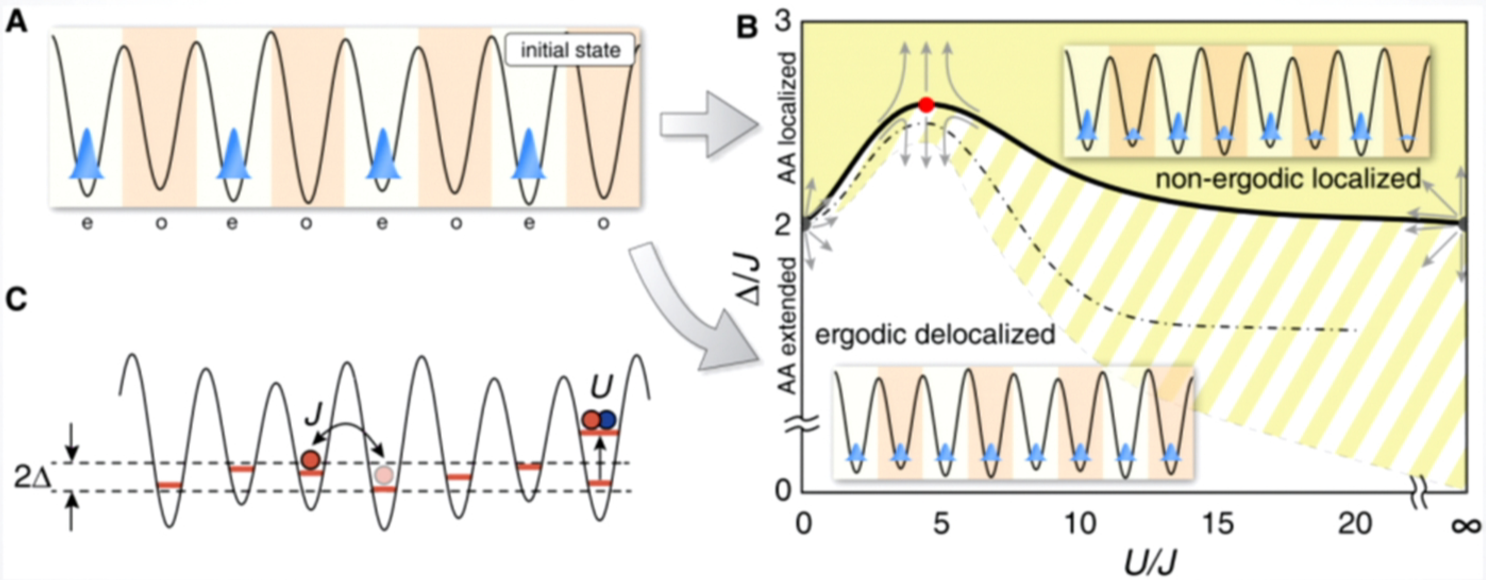
Interactions+ disorder?

Ergodicity breakdown – Many body localization

Ultracold fermionic atoms in optical lattices

$$^{40}K \rightarrow |F; m_F\rangle = \left|\frac{9}{2}; \frac{-9}{2}\right\rangle \text{ and } \left|\frac{9}{2}; \frac{-7}{2}\right\rangle$$

Memory of the initial state in the localized phase



[Michael Schreiber *et al* – Bloch's group] - Science **349** (6250): 842-845 (2015)

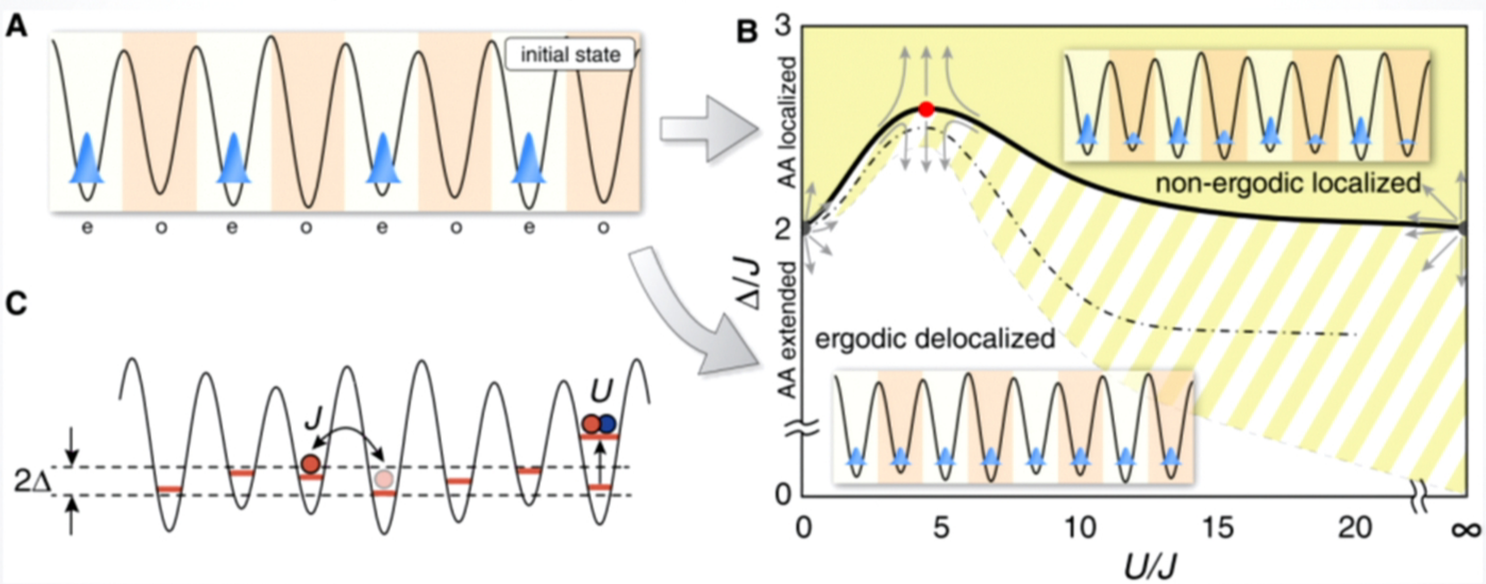
Interactions+ disorder?

Ergodicity breakdown – Many body localization

Ultracold fermionic atoms in optical lattices

$$^{40}K \rightarrow |F;m_F\rangle = \left|\frac{9}{2};\frac{-9}{2}\right\rangle \text{ and } \left|\frac{9}{2};\frac{-7}{2}\right\rangle$$

Memory of the initial state in the localized phase



[Michael Schreiber *et al* – Bloch's group] - Science **349** (6250): 842-845 (2015)

Outline

- Localization in the absence of interactions
  - Experimental realizations
- Interactions – Many body localization – Optical lattice experiment Science **349**, 842 (2015)
- Thermalization – ETH
- Ergodicity and random matrices
- Many-body localization in the 1d Hubbard model
- Further aspects of the Many body localization
- Summary

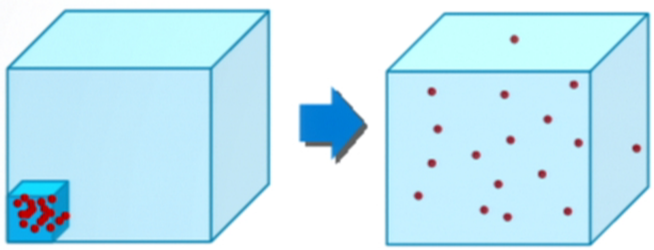
Thermalization - ETH

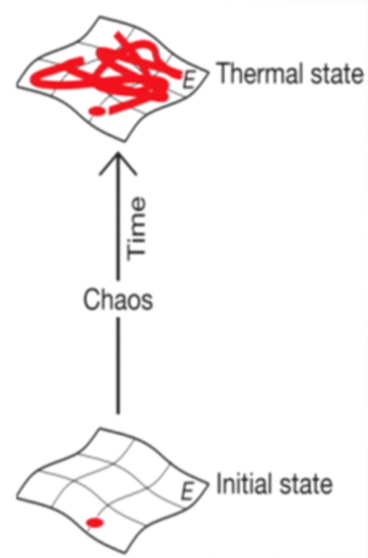
Isolated systems out of equilibrium:

$t = 0\!: H \quad \rightarrow \quad t = 0^{+}\!: H'$

Microcanonical ensemble

- Classical system:





- Non-linear chaotic evolution
- Observables equilibrate and thermalize
- No memory of the initial state

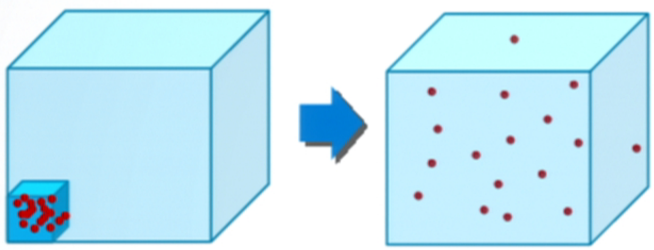
Thermalization - ETH

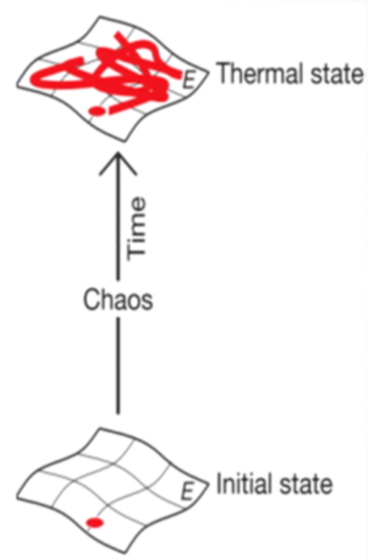
Isolated systems out of equilibrium:

$t = 0: H \quad \rightarrow \quad t = 0^{+}: H'$

Microcanonical ensemble

- Classical system:





- Non-linear chaotic evolution
- Observables equilibrate and thermalize
- No memory of the initial state

Thermalization - ETH

Isolated systems out of equilibrium:

$t = 0\colon H \quad \rightarrow \quad t = 0^{+}\colon H'$

Quantum system:

$$|\psi(0)\rangle \rightarrow |\psi(t)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle = \sum_{\alpha} c_{\alpha} e^{-iE_{\alpha}t}|\alpha\rangle$$: Unitary evolution

$$\langle\hat{O}(t)\rangle \equiv \langle\psi(t)|\hat{O}|\psi(t)\rangle = \sum_{\alpha,\beta} c_{\alpha}^{*} c_{\beta} e^{i(E_{\alpha}-E_{\beta})t} O_{\alpha\beta}$$

$$= \sum_{\alpha} |c_{\alpha}|^{2} O_{\alpha\alpha} + \sum_{\alpha,\beta\neq\alpha} c_{\alpha}^{*} c_{\beta} e^{i(E_{\alpha}-E_{\beta})t} O_{\alpha\beta}$$:Thermalization?

Thermalization - ETH

Isolated systems out of equilibrium:

$t = 0 \colon H \quad \rightarrow \quad t = 0^{+} \colon H'$

Quantum system:

$$|\psi(0)\rangle \rightarrow |\psi(t)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle = \sum_{\alpha} c_{\alpha} e^{-iE_{\alpha}t}|\alpha\rangle$$: Unitary evolution

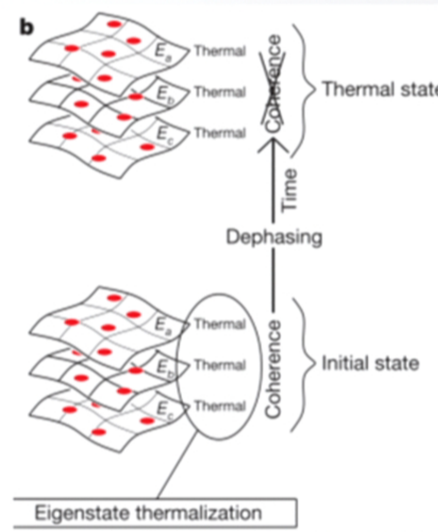
$$\langle \hat{O}(t)\rangle \equiv \langle \psi(t)|\hat{O}|\psi(t)\rangle = \sum_{\alpha,\beta} c_{\alpha}^{*} c_{\beta} e^{i(E_{\alpha}-E_{\beta})t} O_{\alpha\beta}$$

$$= \sum_{\alpha} |c_{\alpha}|^{2} O_{\alpha\alpha} + \sum_{\alpha,\beta\neq\alpha} c_{\alpha}^{*} c_{\beta} e^{i(E_{\alpha}-E_{\beta})t} O_{\alpha\beta}$$:Thermalization?

Thermalization - ETH

Isolated systems out of equilibrium:

$t = 0\colon H \quad \rightarrow \quad t = 0^{+}\colon H'$



[Rigol *et al.* Nature **452**, 7189 (2008)]

Quantum system:

$$|\psi(0)\rangle \rightarrow |\psi(t)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle = \sum_{\alpha} c_\alpha e^{-iE_\alpha t}|\alpha\rangle$$: Unitary evolution

$$\langle \hat{O}(t)\rangle \equiv \langle \psi(t)|\hat{O}|\psi(t)\rangle = \sum_{\alpha,\beta} c_\alpha^* c_\beta e^{i(E_\alpha - E_\beta)t} O_{\alpha\beta}$$

$$= \sum_{\alpha} |c_\alpha|^2 O_{\alpha\alpha} + \sum_{\alpha,\beta\neq\alpha} c_\alpha^* c_\beta e^{i(E_\alpha - E_\beta)t} O_{\alpha\beta}$$:Thermalization?

$$t \rightarrow \infty : \overline{\langle \hat{O}\rangle} \overset{!}{=} \sum_{\alpha} |c_a|^2 O_{\alpha\alpha} \overset{?}{=} \langle \hat{O}\rangle_{\text{microcanonical}} = \frac{1}{N_{E_0,}} \sum_{\alpha,|E_0 - E_\alpha|<\Delta E} O_{\alpha\alpha}$$

ETH

$$\langle \alpha|\hat{O}|\alpha\rangle = \langle \hat{O}\rangle_{\text{ME}}\left(E_\alpha\right)$$

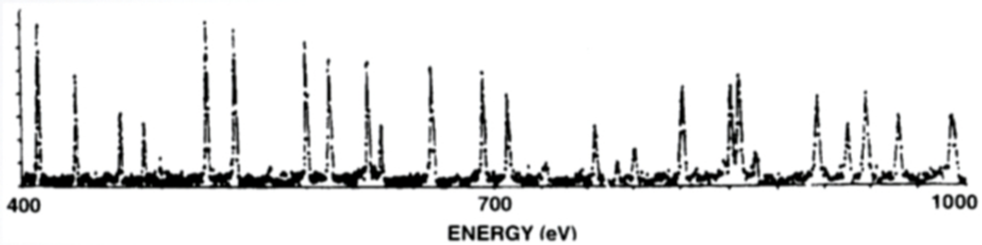
[Deutsch, J. M. PRA43, 2046 (1991)] & [Srednicki, M. PRE 50, 888 (1994)]

Outline

- Localization in the absence of interactions
  - Experimental realizations
- Interactions – Many body localization – Optical lattice experiment Science **349**, 842 (2015)
- Thermalization – ETH
- Ergodicity and random matrices
- Many-body localization in the 1d Hubbard model
- Further aspects of the Many body localization
- Summary

Ergodicity and random matrices

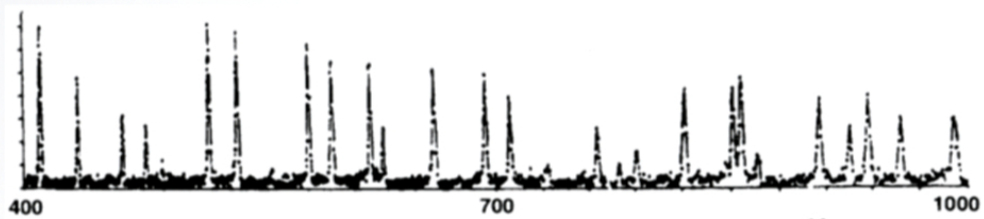
Experimental data for excitation spectra of heavy nuclei:



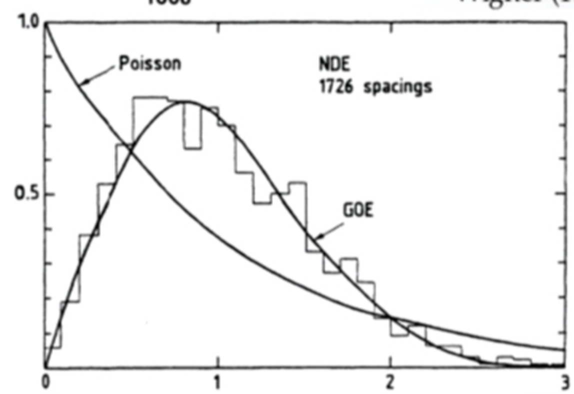
Wigner (1956)

Ergodicity and random matrices

Experimental data for excitation spectra of heavy nuclei:



Wigner (1956)



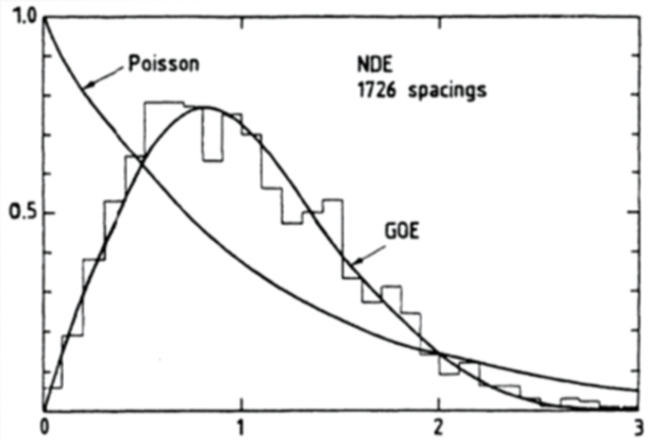
- Hopeless to predict the exact eigenvalues and eigenstates of complex quantum-mechanical systems
- Hamiltonian matrix looks random in non-fine tuned basis…
- Successive energy levels of large nuclei have universal spacing statistics that can be described by random matrices.
- Gap distribution from 2x2 random matrices:

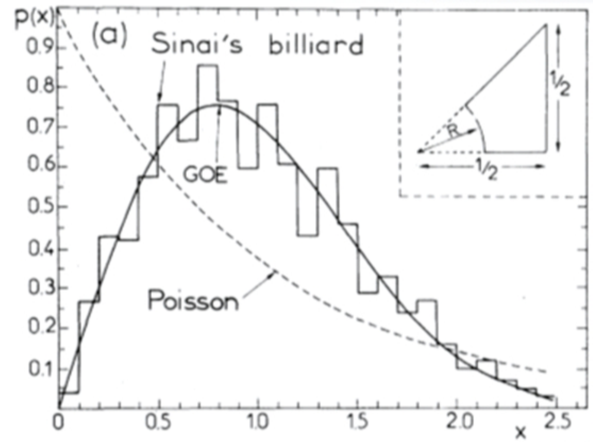
$$P(\omega) = \frac{\omega}{2\sigma^2} exp\left[-\frac{\omega^2}{4\sigma^2}\right]$$

$$\omega = E_2 - E_1$$

Ergodicity and random matrices

- Comparison with other quantum mechanical problems:



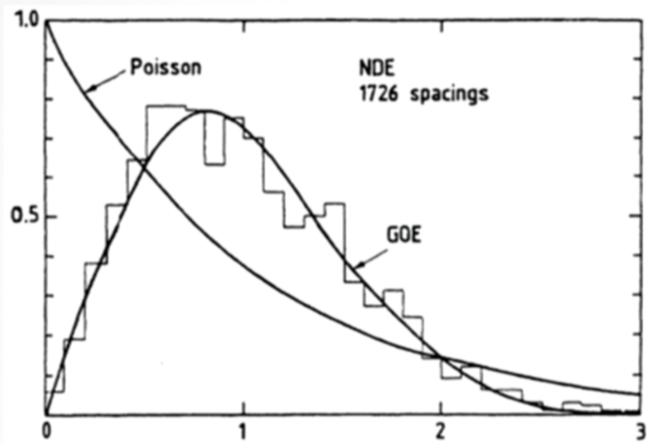


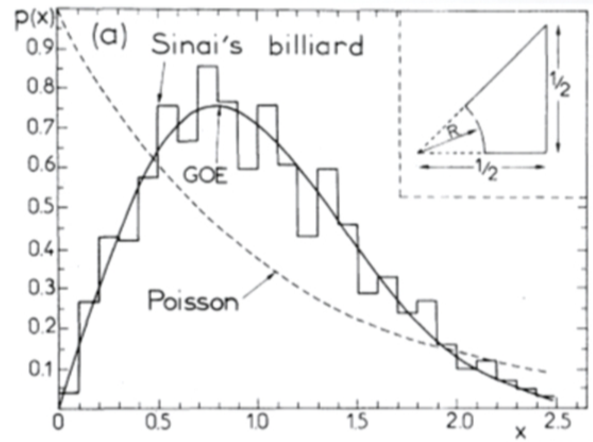
  - Level fluctuations of the quantum Sinai's billiard are consistent with the predictions of the **GOE** of random matrices at high-energies (semi-classical approximation).
  - Level fluctuation laws are universal – defining property of quantum chaotic systems

- Ergodic systems (with time-reversal symmetry): GOE $P(\omega) = \frac{\omega}{2\sigma^2} exp\left[-\frac{\omega^2}{4\sigma^2}\right]$
- Non-ergodic: Poisson $P(\omega) = exp[-\omega]$

Ergodicity and random matrices

- Comparison with other quantum mechanical problems:



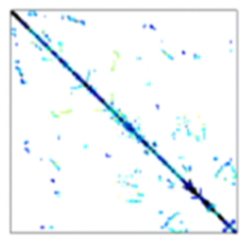


  - Level fluctuations of the quantum Sinai's billiard are consistent with the predictions of the **GOE** of random matrices at high-energies (semi-classical approximation).
  - Level fluctuation laws are universal – defining property of quantum chaotic systems

- Ergodic systems (with time-reversal symmetry): GOE $P(\omega) = \frac{\omega}{2\sigma^2} exp\left[-\frac{\omega^2}{4\sigma^2}\right]$
- Non-ergodic: Poisson $P(\omega) = exp[-\omega]$

Ergodicity and random matrices - eigenvectors

- Hamiltonian matrices in typical many-body problems are **sparse** and **not random**



- Do the **eigenvectors** still follow the predictions of random matrices→ **random unit vectors**?
- Information entropy (measure of delocalization in a given basis):

$$S_m = -\sum_i |c_m^i|^2 \ln |c_m^i|^2 \qquad |m\rangle = \sum_i c_m^i |i\rangle$$

$$S_{GOE} = \ln(0.48\mathcal{D}) + \mathcal{O}(1/\mathcal{D})$$

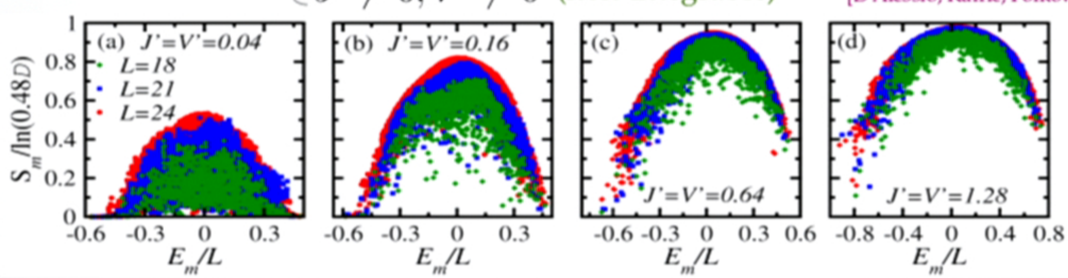
- Example: interacting spinless fermions in 1D (non-integrable)

$$\hat{H} = \sum_i^L \left[-J(\hat{c}_i^\dagger \hat{c}_{i+1} + h.c.) + V\left(\hat{n}_i - \frac{1}{2}\right)\left(\hat{n}_{i+1} - \frac{1}{2}\right)\right] + \sum_i^L \left[-J'(\hat{c}_i^\dagger \hat{c}_{i+2} + h.c.) + V'\left(\hat{n}_i - \frac{1}{2}\right)\left(\hat{n}_{i+2} - \frac{1}{2}\right)\right]$$

$J' = 0, V' = 0$ (integrable) → basis

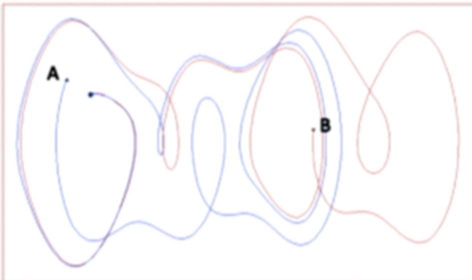
$J' \neq 0, V' \neq 0$ (non-integrable)

[D'Alessio, Kafric, Polkovnikov, Rigol, arXiv 1509.06411]

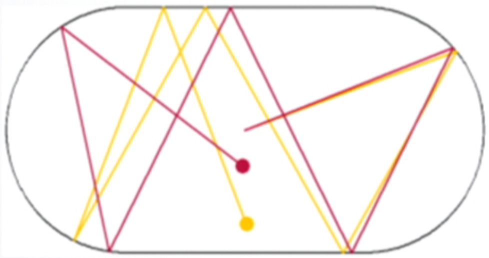


Chaos – exponentially departing trajectories

Classical scenario



[https://hcurocks.files.wordpress.com/2014/08/untitled1.png]

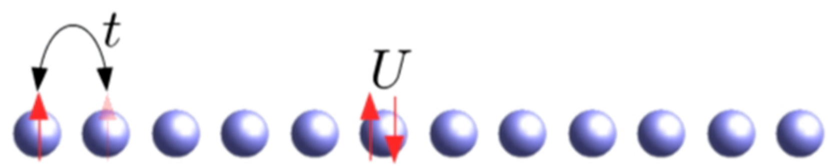


[Scholarpedia and Wikimedia]

The 1d Hubbard model

$$\hat{H} = -t\sum_{i,\sigma}\left(\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i+1,\sigma} + h.c.\right) + U\sum_{i}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

OBC:



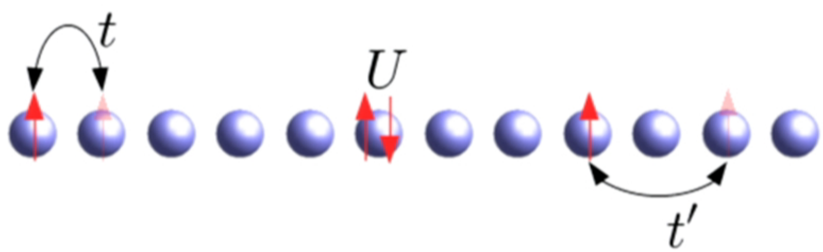
The 1d Hubbard model

$$\hat{H} = -t\sum_{i,\sigma}\left(\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i+1,\sigma} + h.c.\right) + U\sum_{i}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

$$-t'\sum_{i,\sigma}\left(\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i+2,\sigma} + h.c.\right)$$

Integrable → Bethe Ansatz solution → lack of thermalization

OBC:

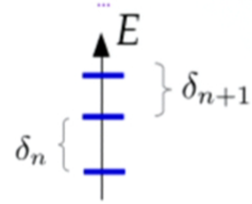


Ergodicity analysis – clean system

- Adjacent gap analysis:

[V. Oganesyan, D. Huse PRB **75**, 155111 (2007)]
[A. Pal and D. Huse PRB **82**, 174411 (2010)]
[B. Tang, D. Iyer, and M. Rigol, PRB **91**, 161109(R) (2015)]
...

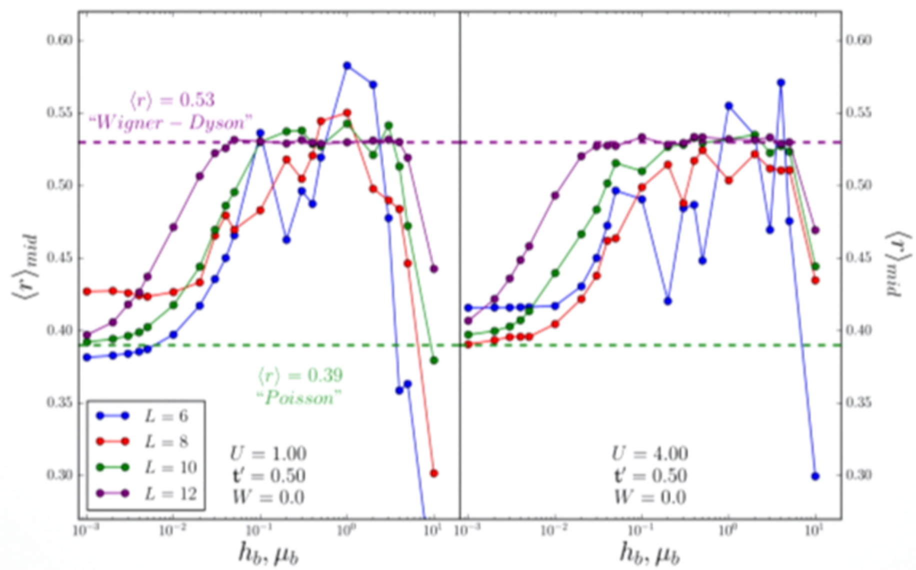
$$r_n = \frac{\min(\delta_n, \delta_{n+1})}{\max(\delta_n, \delta_{n+1})} \quad \text{where} \quad \delta_n = E_{n+1} - E_n$$



$$\langle r \rangle_{GOE} \approx 0.53$$

$$\langle r \rangle_{Poisson} \approx 0.39$$

- Symmetry breaking fields allow ergodicity:

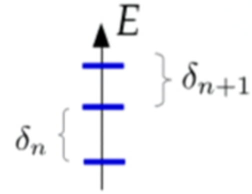


Ergodicity analysis – clean system

- Adjacent gap analysis:

[V. Oganesyan, D. Huse PRB **75**, 155111 (2007)]
[A. Pal and D. Huse PRB **82**, 174411 (2010)]
[B. Tang, D. Iyer, and M. Rigol, PRB **91**, 161109(R) (2015)]
...

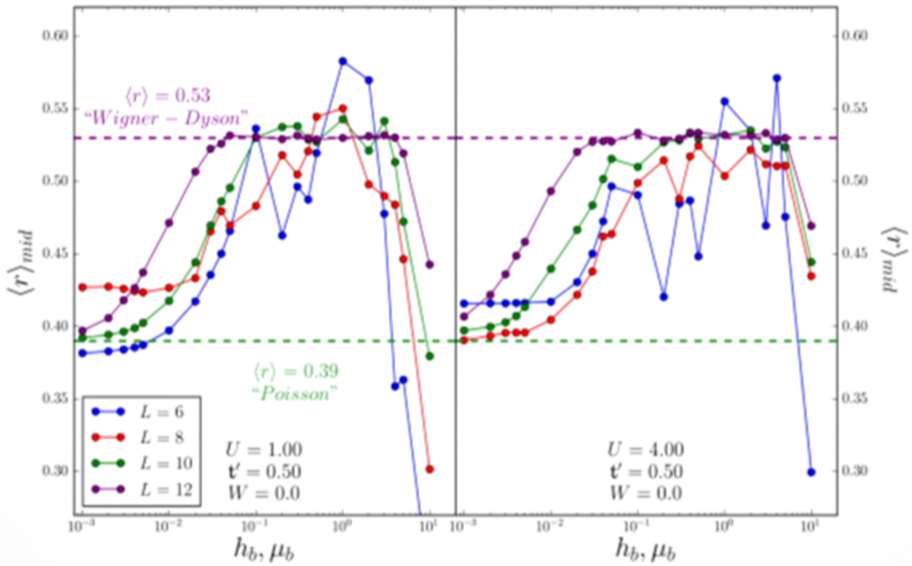
$$r_n = \frac{\min(\delta_n, \delta_{n+1})}{\max(\delta_n, \delta_{n+1})} \quad \text{where} \quad \delta_n = E_{n+1} - E_n$$



$\langle r \rangle_{GOE} \approx 0.53$

$\langle r \rangle_{Poisson} \approx 0.39$

- Symmetry breaking fields allow ergodicity:

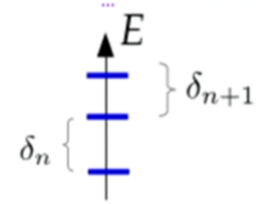


Ergodicity analysis – clean system

- Adjacent gap analysis:

[V. Oganesyan, D. Huse PRB **75**, 155111 (2007)]
[A. Pal and D. Huse PRB **82**, 174411 (2010)]
[B. Tang, D. Iyer, and M. Rigol, PRB **91**, 161109(R) (2015)]
...

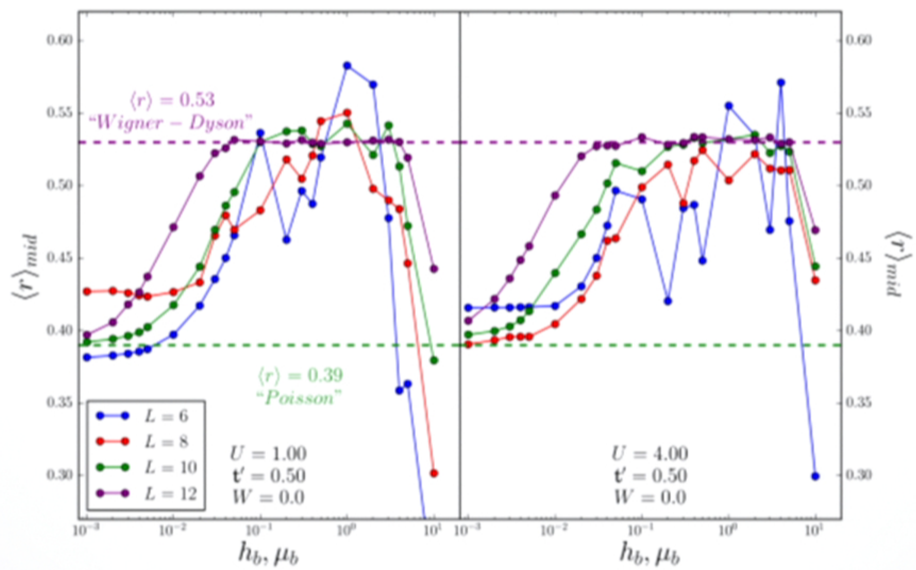
$$r_n = \frac{\min(\delta_n, \delta_{n+1})}{\max(\delta_n, \delta_{n+1})} \quad \text{where} \quad \delta_n = E_{n+1} - E_n$$



$$\langle r \rangle_{GOE} \approx 0.53$$

$$\langle r \rangle_{Poisson} \approx 0.39$$

- Symmetry breaking fields allow ergodicity:



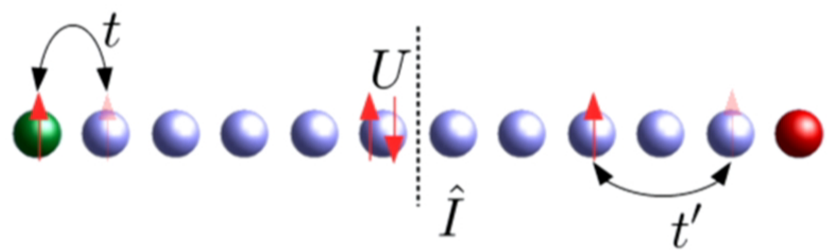
The 1d Hubbard model

$$\hat{H} = -t\sum_{i,\sigma}\left(\hat{c}^\dagger_{i,\sigma}\hat{c}_{i+1,\sigma} + h.c.\right) + U\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

$$-t'\sum_{i,\sigma}\left(\hat{c}^\dagger_{i,\sigma}\hat{c}_{i+2,\sigma} + h.c.\right) + h_b(\hat{n}_{1,\uparrow} - \hat{n}_{1,\downarrow}) + \mu_b(n_{L,\uparrow} - \hat{n}_{L,\downarrow})$$

Integrable → Bethe Ansatz solution → lack of thermalization

OBC:



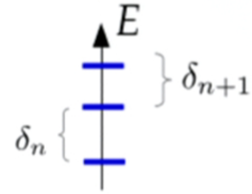
SU(2) and spatial inversion symmetries... Non-ergodic.

Ergodicity analysis – clean system

- Adjacent gap analysis:

[V. Oganesyan, D. Huse PRB **75**, 155111 (2007)]
[A. Pal and D. Huse PRB **82**, 174411 (2010)]
[B. Tang, D. Iyer, and M. Rigol, PRB **91**, 161109(R) (2015)]
...

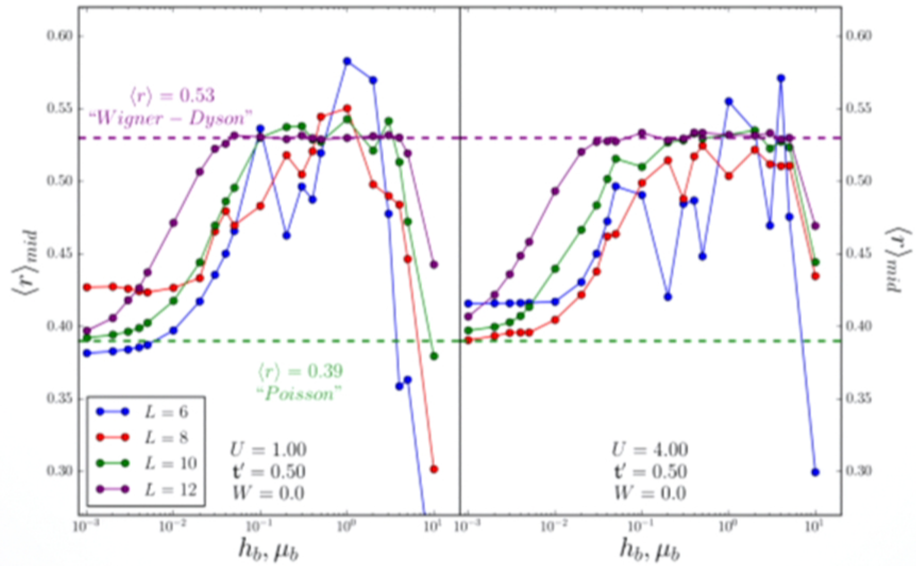
$$r_n = \frac{\min(\delta_n, \delta_{n+1})}{\max(\delta_n, \delta_{n+1})} \quad \text{where} \quad \delta_n = E_{n+1} - E_n$$



$$\langle r \rangle_{GOE} \approx 0.53$$

$$\langle r \rangle_{Poisson} \approx 0.39$$

- Symmetry breaking fields allow ergodicity:



Ergodicity analysis – disordered system

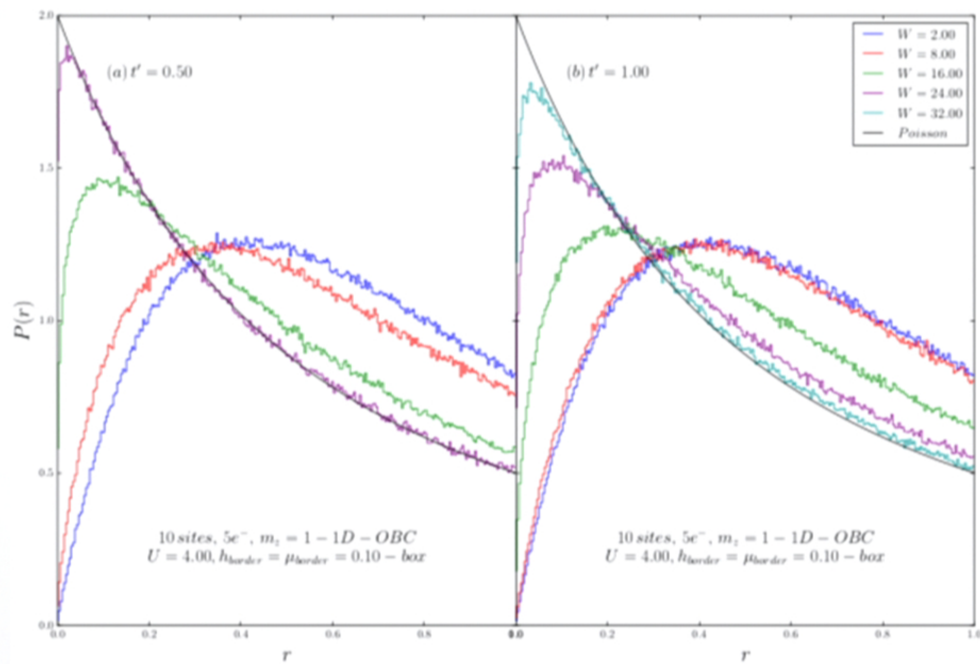
- Adjacent gap analysis:

[V. Oganesyan, D. Huse PRB **75**, 155111 (2007)]
[A. Pal and D. Huse PRB **82**, 174411 (2010)]
[B. Tang, D. Iyer, and M. Rigol, PRB **91**, 161109(R) (2015)]
...

$$r_n = \frac{\min(\delta_n, \delta_{n+1})}{\max(\delta_n, \delta_{n+1})} \quad \text{where} \quad \delta_n = E_{n+1} - E_n$$

$$\langle r \rangle_{GOE} \approx 0.53$$

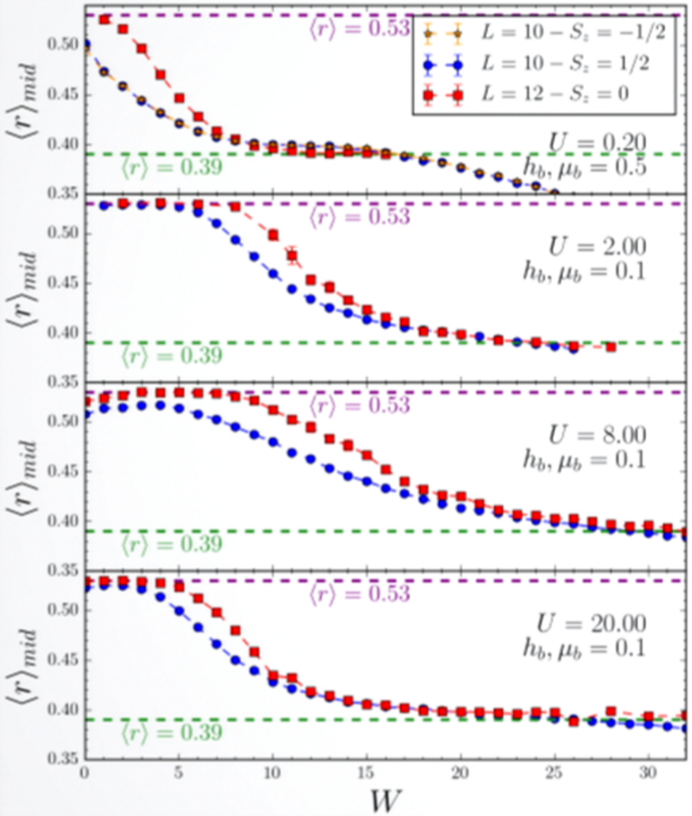
$$\langle r \rangle_{Poisson} \approx 0.39$$



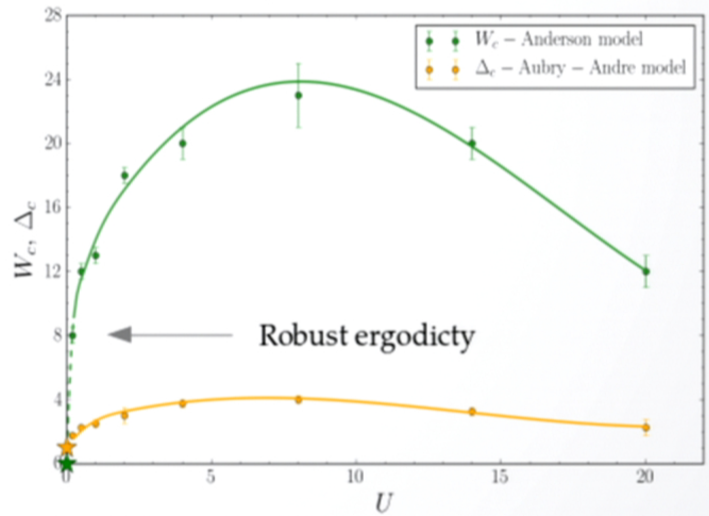
MBL – Anderson and Aubry-André models

Quarter-filling: $\rho = n_{el.}/L = 1/2$

$$\hat{H} = \hat{H}_{Hubbard+t'} + \hat{H}_{sb} + \sum_i \varepsilon_i \hat{n}_i$$



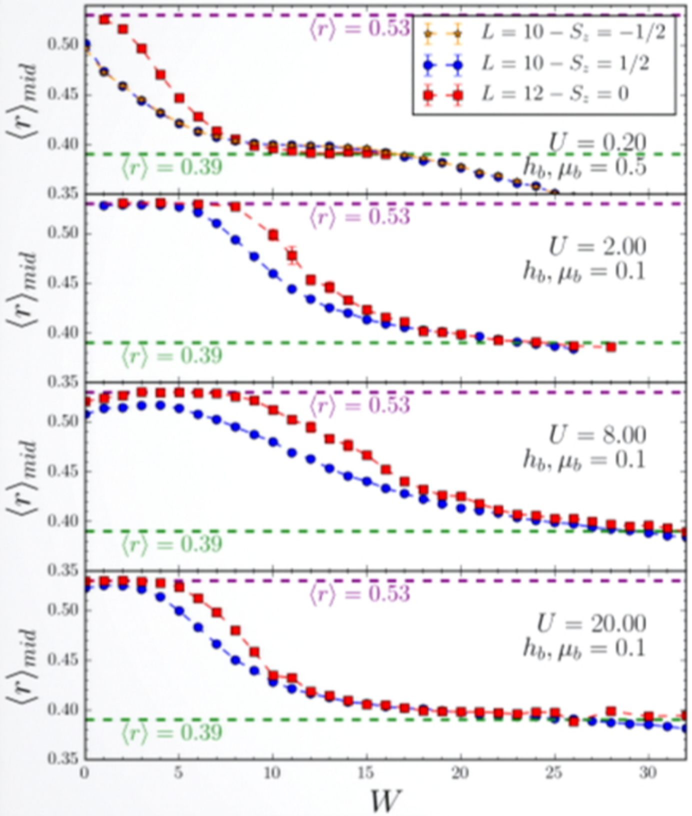
- Anderson: $\varepsilon_i \in \left[-\frac{W}{2}, \frac{W}{2}\right]$
- Aubry-André: $\varepsilon_i = \frac{\Delta}{2}\cos(2\pi\beta i + \phi)$ $\phi \in [-\pi, \pi]$
  - Non-monotonic
  - Non-interacting limit when $U \to \infty$ Spinless fermions



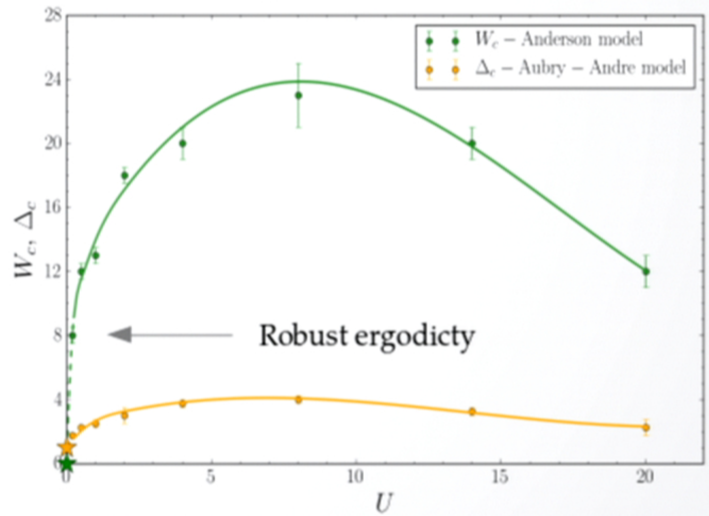
MBL – Anderson and Aubry-André models

Quarter-filling: $\rho = n_{el.}/L = 1/2$

$$\hat{H} = \hat{H}_{Hubbard+t'} + \hat{H}_{sb} + \sum_i \varepsilon_i \hat{n}_i$$



- Anderson: $\varepsilon_i \in \left[-\frac{W}{2}, \frac{W}{2}\right]$
- Aubry-André: $\varepsilon_i = \frac{\Delta}{2}\cos(2\pi\beta i + \phi) \quad \phi \in [-\pi, \pi]$
  - Non-monotonic
  - Non-interacting limit when $U \to \infty$ Spinless fermions

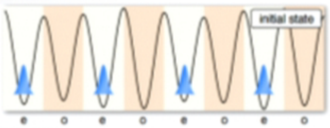


MBL – Anderson and Aubry-André models

Numerical experiment:

$$|\psi_0\rangle = |\uparrow 0 \downarrow 0 \uparrow ...\rangle \longrightarrow |\psi(0)\rangle \to |\psi(t)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle$$

$$\hat{H} = \hat{H}_{Hubbard+t'} + \hat{H}_{sb} + \sum_i \varepsilon_i \hat{n}_i$$



MBL manifest on the dynamics of the observables?

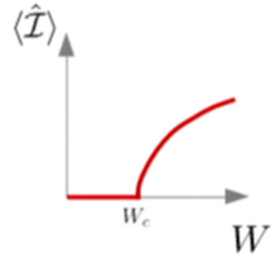
ETH breakdown

$$\overline{\langle \hat{O} \rangle} = \sum_{\alpha} |c_a|^2 O_{\alpha\alpha} \neq \langle \hat{O} \rangle_{\text{microcanonical}}$$

in the MBL phase...

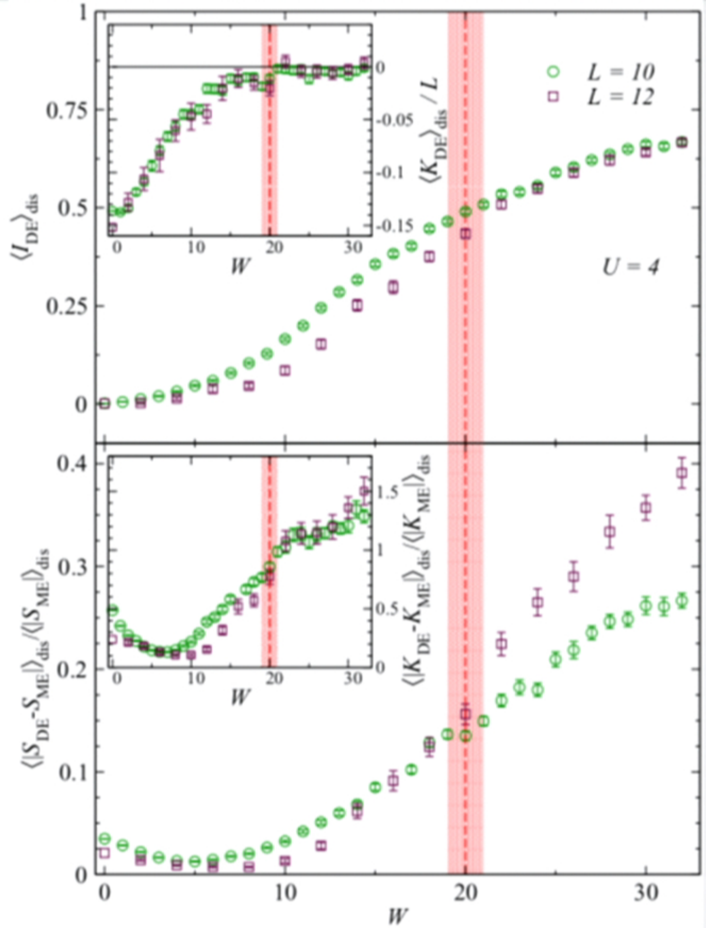
- Charge imbalance as an order parameter?

$L \to \infty$



- Large finite size effects

- $\langle K_{DE} \rangle \approx 0$ after $W \gtrsim W_c$



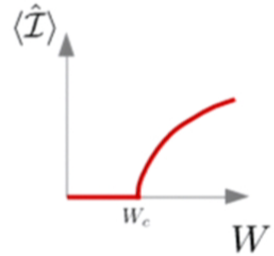
ETH breakdown

$$\overline{\langle \hat{O} \rangle} = \sum_{\alpha} |c_a|^2 O_{\alpha\alpha} \neq \langle \hat{O} \rangle_{\text{microcanonical}}$$

in the MBL phase...

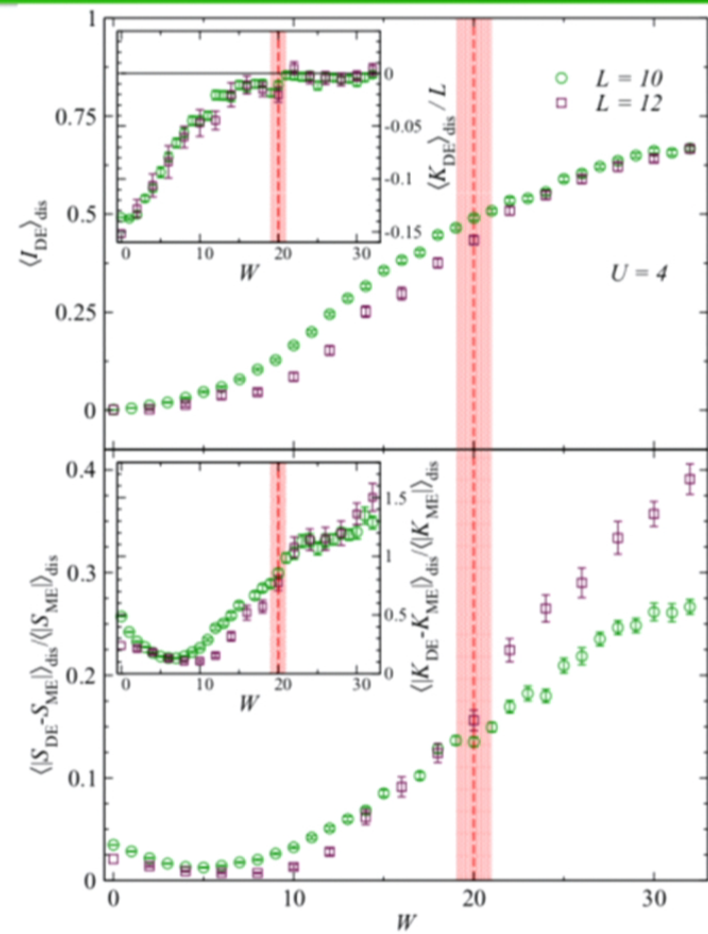
- Charge imbalance as an order parameter?

$L \to \infty$



- Large finite size effects

- $\langle K_{DE} \rangle \approx 0$ after $W \gtrsim W_c$



ETH breakdown

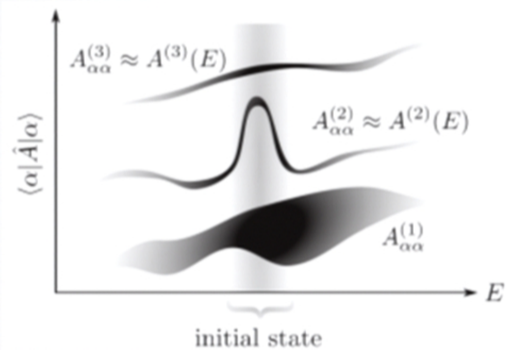
ETH ansatz: $O_{\alpha\beta} = O(\bar{E})\delta_{\alpha\beta} + e^{-S(\bar{E})/2} f_O(\bar{E},\omega) R_{\alpha\beta}$

[Srednicki, M. PRE 50, 888 (1994). ...]

$\bar{E} = (E_\alpha + E_\beta)/2$

$\omega = E_\alpha - E_\beta$

$O; f_O$: smooth functions



[Sorg *et al.* PRA **90**, 033606 (2014)]

$\langle\psi_0|\hat{H}|\psi_0\rangle = 0$

ETH breakdown

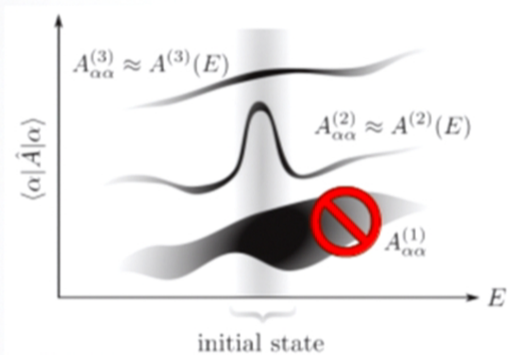
ETH ansatz: $O_{\alpha\beta} = O(\bar{E})\delta_{\alpha\beta} + e^{-S(\bar{E})/2} f_O(\bar{E},\omega) R_{\alpha\beta}$

[Srednicki, M. PRE 50, 888 (1994). ...]

$\bar{E} = (E_\alpha + E_\beta)/2$

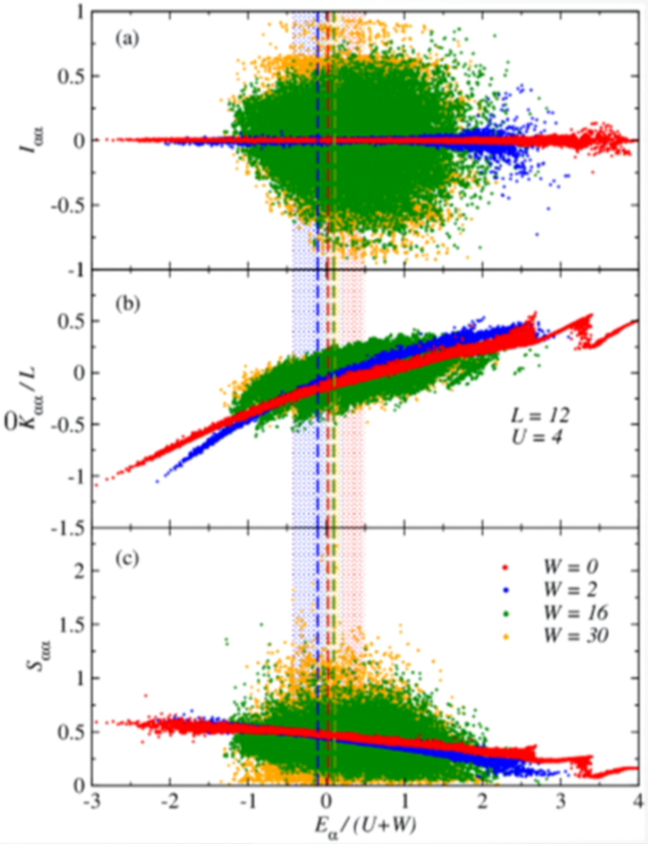
$\omega = E_\alpha - E_\beta$

$O; f_O$: smooth functions



[Sorg *et al.* PRA **90**, 033606 (2014)]

$\langle\psi_0|\hat{H}|\psi_0\rangle = 0$



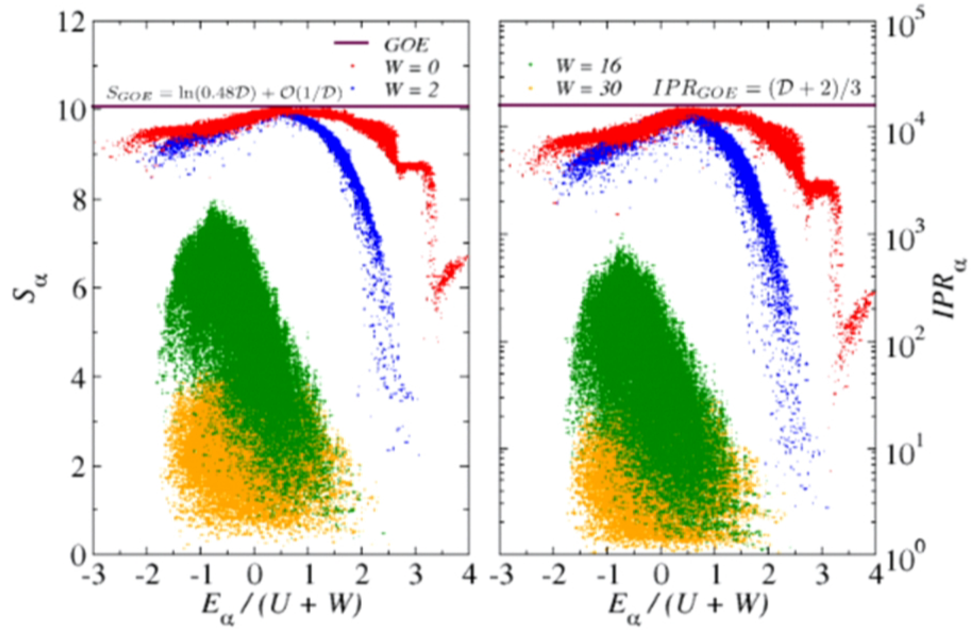
$$\hat{K} = -t\sum_{i,\sigma}\left(\hat{c}^\dagger_{i,\sigma}\hat{c}_{i+1,\sigma} + h.c.\right) - t'\sum_{i,\sigma}\left(\hat{c}^\dagger_{i,\sigma}\hat{c}_{i+2,\sigma} + h.c.\right)$$

$$\hat{S}_{AF} = \frac{1}{L}\sum_{i,j} e^{i\pi(i-j)}(\hat{n}_{i,\uparrow} - \hat{n}_{i,\downarrow})(\hat{n}_{j,\uparrow} - \hat{n}_{j,\downarrow})$$

ETH fails in the MBL phase

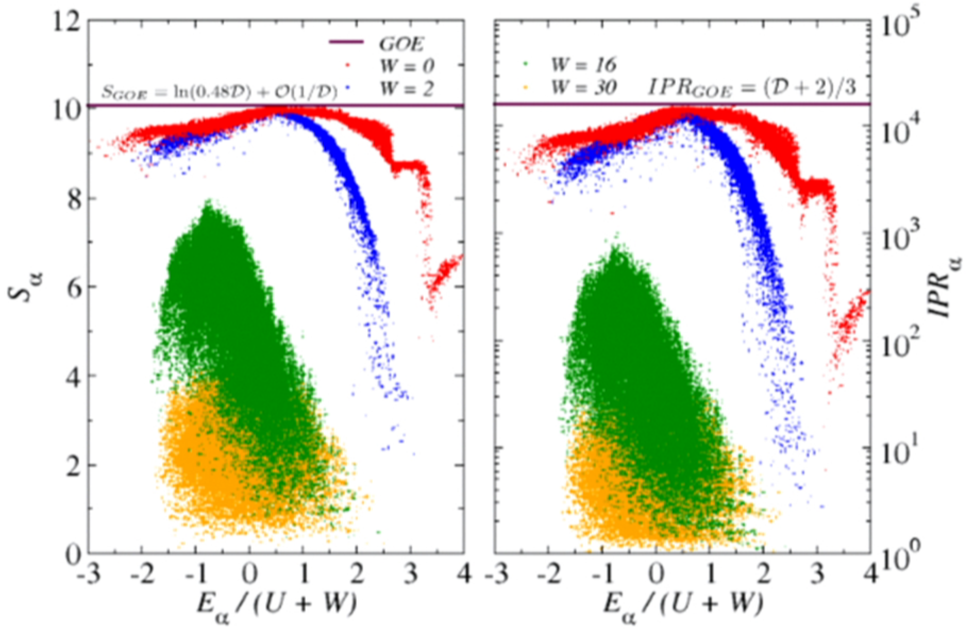
ETH breakdown

- Shannon entropy in the Fock basis: $S_\alpha = -\sum_i |c_\alpha^i|^2 \ln |c_\alpha^i|^2$ where $|\alpha\rangle = \sum_i c_\alpha^i |i\rangle$
- Inverse participation ratio: $IPR_\alpha = \frac{1}{\sum_{j=1}^{\mathcal{D}} |c_\alpha^j|^4}$



ETH breakdown

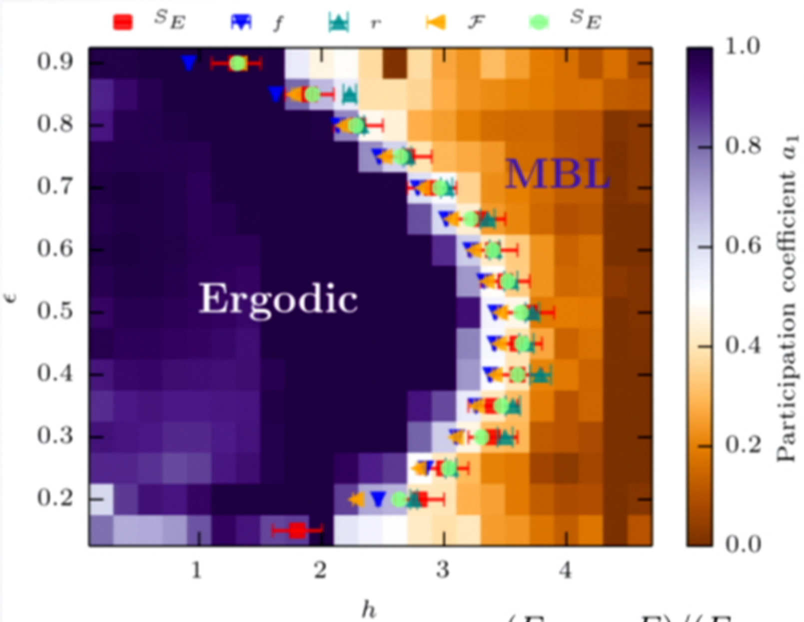
- Shannon entropy in the Fock basis: $S_\alpha = -\sum_i |c_\alpha^i|^2 \ln |c_\alpha^i|^2$ where $|\alpha\rangle = \sum_i c_\alpha^i |i\rangle$
- Inverse participation ratio: $IPR_\alpha = \frac{1}{\sum_{j=1}^{\mathcal{D}} |c_\alpha^j|^4}$



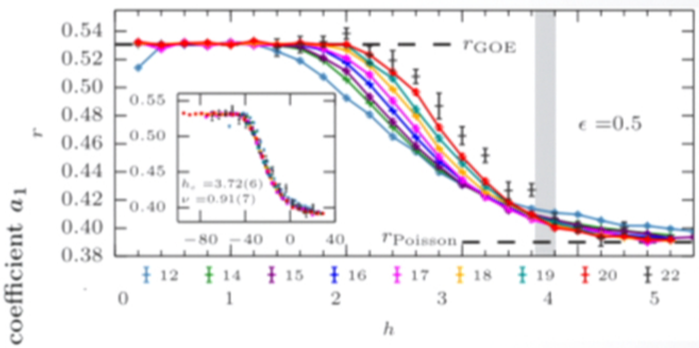
Other aspects of MBL

Many-body mobility edge – Disordered Heisenberg chain

$$\hat{H} = \sum_i \vec{S}_i \cdot \vec{S}_{i+1} - h_i S_z \qquad h_i \in [-h/2, h/2]$$



$\epsilon = (E_{\max} - E)/(E_{\max} - E_0)$



Scaling: $g(h - h_c)L^{1/\nu}$

- New computational thechniques for large system sizes → shift invert method...

[David J. Luitz, Nicolas Laflorencie, and Fabien Alet PRB 91, 081103(R) (2015)]

Other aspects of MBL

- Orthogonality catastrophe [P. W. Anderson, PRL **18**, 1049 (1967)]

  $|G\rangle$: Ground state of a metallic system (extended states)

  $|G'\rangle$: Ground-state after a **local** perturbation (adding an impurity)

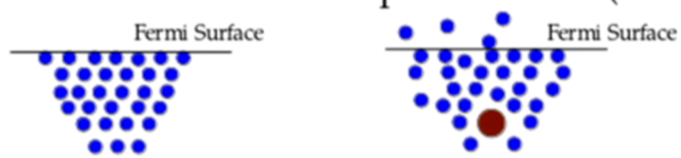
$$F \equiv |\langle G|G'\rangle| = L^{-\gamma}$$

Other aspects of MBL

- Orthogonality catastrophe [P. W. Anderson, PRL **18**, 1049 (1967)]

$|G\rangle$: Ground state of a metallic system (extended states)

$|G'\rangle$: Ground-state after a **local** perturbation (adding an impurity)

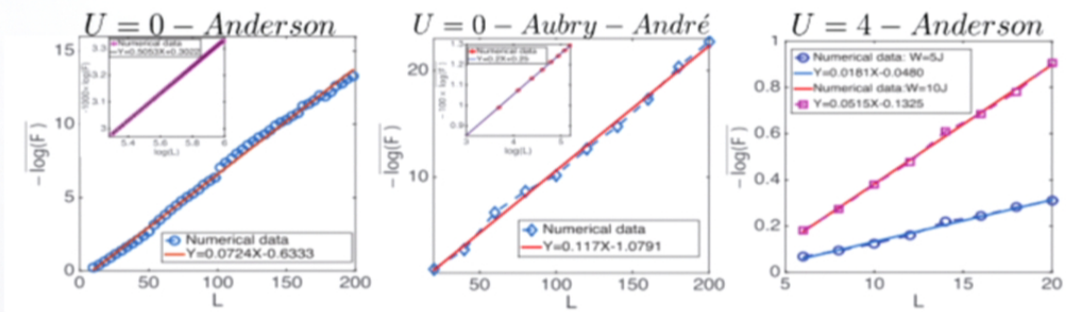


$$F \equiv |\langle G|G'\rangle| = L^{-\gamma}$$

- How about localized, or even many-body localized systems?

→ **Statistical** orthogonality catastrophe [V.Khemani,R.Nandkishore,S.L.Sondhi, Nature Phys. 2015] [D.-L. Deng, J. H. Pixley, X. Li, and S. Das Sarma, arXiv1508.01270]

$$\langle F\rangle = e^{-\beta L}$$



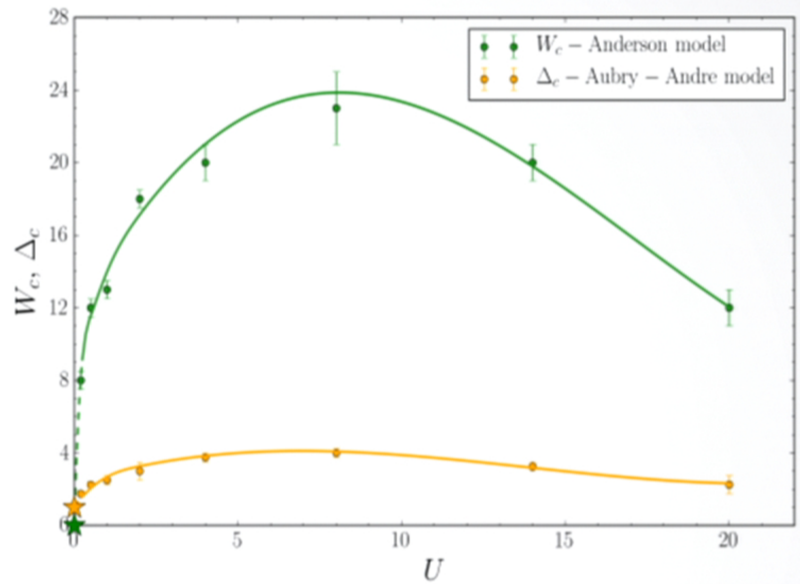
- Spinless fermions

SOC → diagnostic tool to identify localization?

Rubem Mondaini Post-doc @ Penn State | Many-body localization and thermalization in disordered Hubbard chains | 26.10.15 - Perimeter Institute

Summary

- Ergodicity is extremely robust against disorder in the Hubbard model



- Close to the MBL transition → large equilibration times → experimentally feasible?
- Scalings? Large system sizes… Computationally feasible?

* * *