Title: BV-BFV Approach to General Relativity

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Abstract: <p>We analyse different classical formulations of General Relativity in the Batalin (Frad-<br>
kin) Vilkovisky framework with boundary, as a first step in the program of CMR [1] quantisation. Success and failure in satisfying the axioms will allow us to discriminate among the different descriptions, suggesting that some are more suitable than others in view of perturbative quantisation. Based on a joint work with A. Cattaneo [2, 3] we will present the details of the application of the BV-BFV formalism to the Einstein-Hilbert and Palatini-Holst formulations of General Relativity. We show that the two descriptions are no longer equivalent from this point of view, and we discuss possible interpretations of this result.<br>
<br>
[1] A. S. Cattaneo, P. MnÃ«v, N. Reshetikhin, Classical BV theories on manifolds with boundary, Comm. Math. Phys. 332, 2, 535-603 (2014); A.S. Cattaneo, P. MnÃ«v, N. Reshetikhin, Perturbative quantum gauge theories on manifolds with boundary, arXiv:1507.01221<br>
[2] A. S. Cattaneo, M. Schiavina, BV-BFV analysis of General Relativity. Part I: Einstein Hilbert action, arXiv:1509.05762 (2015).<br>
[3] A. S. Cattaneo, M. Schiavina, BV-BFV analysis of General Relativity. Part II: Palatini Holst action, in preparation; A. S. Cattaneo, M. Schiavina, On time, in preparation.</p>

# BV-BFV approach to GR

joint work with A.S. Cattaneo

## Plan of the Talk

- Lagrangian field theories with boundary - Axiomatisation
- The BV-BFV formalism - Batalin (Fradkin) Vilkovisky
- Overview on CMR quantisation - Cattaneo Mnëv Reshetikhin
- GR: Einstein-Hilbert action - arXiv:1509.05762
- GR: Palatini-Holst action - in preparation
- Outlook

# Lagrangian field theories with boundary

General framework for (regular) Lagrangian field theories on manifolds with boundary.

- To a space time $M$ assign a space of classical fields $F_M$ and an action functional $S^{cl}_M$.
- $\partial M \neq \emptyset \longrightarrow \delta S^{cl}_M$ splits in a bulk and boundary term.
- Bulk term $\longrightarrow$ Euler Lagrange equations.
  Boundary term $\longrightarrow$ 1-form $\widetilde{\alpha}$ on the boundary (Noether form).
  $\widetilde{F}_{\partial M}$, restrictions of fields and jets. (pre-boundary)
- If the theory is regular $(\widetilde{F}_{\partial M}, \delta\widetilde{\alpha})$ symplectic manifold.
  Space of boundary fields.

# Lagrangian field theories with boundary

Simple example: $S_M^{cl} = \int\limits_M \partial_\mu \phi \partial^\mu \phi \, \mathbf{dx}$ with $F_M = C^\infty(M)$.

$$\delta S_M^{cl} = -\int\limits_M \Delta\phi \, \delta\phi \, \mathbf{dx} + \int\limits_{\partial M} \partial_n \phi \, \delta\phi \, \mathbf{dx}$$

with $\partial_n$ normal derivative. Interpret as

$$\tilde{\alpha} = \int\limits_{\partial M} J_\phi \, \delta\phi \, \mathbf{dx}$$

with $J_\phi = \partial_n \phi\big|_{\partial M}$ and $(\phi, J_\phi) \in \tilde{F}_{\partial M}$. Then $\tilde{\omega} = \delta\tilde{\alpha}$ is symplectic.

$$\tilde{\pi}_M : F_M \longrightarrow \tilde{F}_{\partial M} \qquad (1)$$

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$$\widetilde{\pi}_M \colon \mathcal{F}_M \longrightarrow \widetilde{\mathcal{F}}_{\partial M} \qquad (1)$$

# Lagrangian field theories with boundary

- Regular theories $\longrightarrow$ Gauge theories.
- Usual problems of gauge fixing and degenerate critical locus.
- Standard approach to gauge theories is BRST Becchi Rouet Stora Tyutin. Gauge fixing as operation in cohomology .
- Generalise to BV to treat more general symmetry distributions. Gauge fixing as integration over Lagrangian submanifolds.
- BV also allows compatible treatment of boundary data. BV-BFV formalism

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- Generalise to BV to treat more general symmetry distributions.
  Gauge fixing as integration over Lagrangian submanifolds.
- BV also allows compatible treatment of boundary data.
  BV-BFV formalism

# BV formalism

Classical gauge theory: assign to $M$

- space of classical fields $F_M$
- classic action functional $S_M^{cl}$
- a distribution $D_M$ on $F_M$, representing symmetries.

$$X \in \Gamma(D_M) \Longleftrightarrow L_X S_M^{cl} = 0$$

$D_M$ Lie algebra action $\longrightarrow$ everywhere involutive.
"BRST-like" theory in the BV setting.

Embed all data in a *Graded Symplectic setting*
Ghosts and Antifields.

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# BV formalism
## No boundary

BV framework: fiber coordinates of $D_M$ have degree 1.
Construct the *shifted cotangent bundle* $\mathcal{F}_M = T^*[-1]D_M[1]$. Let $(\cdot,\cdot)$ be the canonical Poisson structure, on $\mathcal{F}_M$.
Encode symmetries in a degree 1 vector field $Q_M$ on $\mathcal{F}_M$ , such that $[Q_M, Q_M] = 0$ (Cohomological).

**Theorem**
*If the theory is BRST-like, then the action* $S_M = S_M^{cl} + \langle \Phi^\dagger, Q_M \Phi \rangle$ *is such that, if* $\partial M = \emptyset$

$$(S_M, S_M) = 0 \qquad \text{(CME)}$$

The Classical Master Equation.
**Question:** what if I have boundaries?

# BV-BFV formalism
## $\partial M \neq \emptyset$

$\mathcal{F}_M$ endowed with $(-1)$-symplectic form $\Omega_M$. Failure of (CME):

$$\iota_{Q_M}\Omega_M = \delta S_M + \widetilde{\pi}_M^* \widetilde{\alpha}_M$$

with $\widetilde{\alpha} \in \Omega^1(\widetilde{\mathcal{F}}_{\partial M})$ and surjective submersion

$$\widetilde{\pi}_M \colon \mathcal{F}_M \longrightarrow \widetilde{\mathcal{F}}_{\partial M}$$

restriction of fields and normal jets to the boundary.
(pre-boundary)

Consider $\widetilde{\omega} = \delta\widetilde{\alpha}$. This form is *degenerate*.
**Question:** is it presymplectic?

# BV-BFV formalism

Assume $\mathrm{Ker}(\widetilde{\omega})$ subbundle on $\widetilde{\mathcal{F}}_{\partial M}$ and that we can perform symplectic reduction.

**Theorem (Cattaneo Mnev Reshetikhin)**

*The BV data $(\mathcal{F}_M, S_M, Q_M, \Omega_M)$ induces the following:*

- *A surjective submersion $\pi_M : \mathcal{F}_M \longrightarrow \mathcal{F}^\partial_{\partial M}$, to the symplectic reduction of $\widetilde{\mathcal{F}}_{\partial M}$.*
- *A degree 1 vector field $Q^\partial$ such that $d\pi_M Q_M = Q^\partial_{\partial M}$, and $[Q^\partial_{\partial M}, Q^\partial_{\partial M}] = 0$.*
- *A (0)-symplectic form $\omega^\partial_{\partial M}$ on $\mathcal{F}^\partial_{\partial M}$ with $\widetilde{\omega} = \pi_M^* \omega^\partial_{\partial M}$*
- *A degree 1 functional $S^\partial_{\partial M}$ such that*

$$\iota_{Q^\partial_{\partial M}} \omega^\partial_{\partial M} = \delta S^\partial_{\partial M}$$

*satifying (CME).*

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$$\iota_{Q^{\partial}_{\partial M}} \omega^{\partial}_{\partial M} = \delta S^{\partial}_{\partial M}$$

*satifying* (CME).

# BV-BFV formalism

The induced data on the boundary $(\mathcal{F}^{\partial}_{\partial M}, S^{\partial}_{\partial M}, Q^{\partial}_{\partial M}, \omega^{\partial}_{\partial M})$ defines what is usually called a BFV structure.

Cohomological resolution of (coisotropic) submanifolds (constraints in the phase space), modulo gauge symmetry.

We defined a BV-BFV theory on $M$.
If $\exists \alpha^{\partial}_{\partial M}$ such that $\omega^{\partial}_{\partial M} = \delta \alpha^{\partial}_{\partial M}$ the theory is Exact.

Advantage of the BV-BFV machinery: compatibility formulas

$$\begin{aligned} \iota_{Q_M} \Omega_M &= \delta S_M + \pi^*_M \alpha^{\partial}_{\partial M} \\ (S_M, S_M) &= 2\pi^* S^{\partial}_{\partial M} \end{aligned} \qquad \text{(MCME)}$$

in view of **quantisation** and **axiomatisation**.

## Quantisation Overview

### BV Setting, $\partial M = \emptyset$

BV formalism provides a framework for gauge fixing and control over gauge fixing independence.

**Definition (finite dimensions)**

BV Operator: $\Delta$ on a $(-1)$-symplectic graded manifold $(F, \omega)$ such that

- $\Delta^2 = 0$
- $\Delta(fg) = \Delta f g \pm f \Delta g \pm (f, g)_\omega$

Gauge fixing: integration over Lagrangian submanifolds $\mathcal{L}$.
BV theorems:

$$f = \Delta g \implies \int_{\mathcal{L}} f = 0$$

$$\Delta f = 0 \implies \int_{\mathcal{L}} f \text{ invariant under deformations of } \mathcal{L}$$

# Quantisation Overview

## BV Setting

$\Delta$-exactness for $f = e^{\frac{i}{\hbar}S_M} \iff$ Quantum Master Equation:

$$\frac{1}{2}(S_M, S_M)_\omega - i\hbar\Delta S_M = 0 \qquad \text{(QME)}$$

solve order by order in $\hbar$ (if possible). $S_M$ depends on $\hbar$.
Lowest order is (CME).

**Remark**

(CME) *makes sense in $\infty$-dimensions, even if $\Delta$ is not well defined. Path integrals and $\Delta$ defined via perturbative expansions.*

What happens when $\partial M \neq \emptyset$ and (CME) fails?

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# Quantisation Overview
## BV-BFV

Assume: exact BV-BFV theory. First step: quantise the boundary symplectic manifold.

- General assumption (CMR): Polarisation in $\mathcal{F}^{\partial}_{\partial M}$ such that $\mathcal{F}_M = \mathcal{Y} \times \mathcal{B}$, with $\mathcal{B}$ space of leaves of the polarisation. Hilbert space: functions on $\mathcal{B}$.
- Simple example on $T^*N$: polarisation separates $p$ and $q$ coordinates, $\mathcal{B} = N$.
- Idea: Quantise the boundary action $S^{\partial}$ to a boundary operator $\mathrm{D} = S^{\partial}(q, -i\hbar\frac{\partial}{\partial q})$.
  **Modified Quantum Master Equation**:

$$\hat{\mathrm{D}} e^{\frac{i}{\hbar} S_M} = \left(\hbar^2 \Delta + \mathrm{D}\right) e^{\frac{i}{\hbar} S_M} = 0 \qquad \text{(MQME)}$$

and $\hat{\mathrm{D}}^2 = 0$.

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# Quantisation Overview

## BV-BFV

Gauge fixing is controlled by the cohomology of $\hat{\mathrm{D}}$: choose lagrangian submanifold $\mathcal{L} \in \mathcal{Y}$ and define

$$\Psi_{\mathcal{L}} = \int_{\mathcal{L}} e^{\frac{i}{\hbar} S_M} \in C^\infty(\mathcal{B}) \tag{2}$$

the state associated to the boundary. Changing the Lagrangian submanifold is a $\hat{\mathrm{D}}$-exact term, and the state itself is $\hat{\mathrm{D}}$-closed. The Hilbert space is the cohomology of $\hat{\mathrm{D}}$ in degree zero.

### Remark

*The key information is the compatibility between bulk and boundary:* (MQME)

# General Relativity

**Questions:** what about General Relativity? Does it satisfy the BV-BFV axioms? If yes, can we use the CMR approach to quantisation?

Test the formalism on different formulations:

- Einstein Hilbert action ✓
- Palatini Holst action ⚠
- (non-Chiral) Plebanski action ⚠
- MacDowell Mansouri action ∼ ✓

Comparing EH and PH already teaches us a lot.

## Einstein Hilbert Action

Variational problem for the EH action on a $d+1$-dimensional Manifold

$$S_{EH} = \int_M \sqrt{-g}(R[g] - 2\Lambda) \tag{3}$$

$g$ any Lorentzian metric, $\Lambda$ the cosmological constant.
Symmetries are given by $\mathrm{Diffeo}(M)$ and, infinitesimally, by $\Gamma(TM)$.

The theory is BRST-like. Therefore we have $\xi \in \Gamma[1](TM)$:

$$Qg = L_\xi g \qquad Q\xi = \frac{1}{2}[\xi, \xi]$$

The BV-extended action on $T^*[-1](\mathcal{PR}_{(d,1)}(M) \oplus \Gamma[1]TM)$ is

$$S_{EH}^{BV} = S_{EH} + \int_M g^\dagger L_\xi g + \frac{1}{2}\int_M \iota_{[\xi,\xi]}\xi^\dagger \tag{4}$$

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# Einstein Hilbert Action

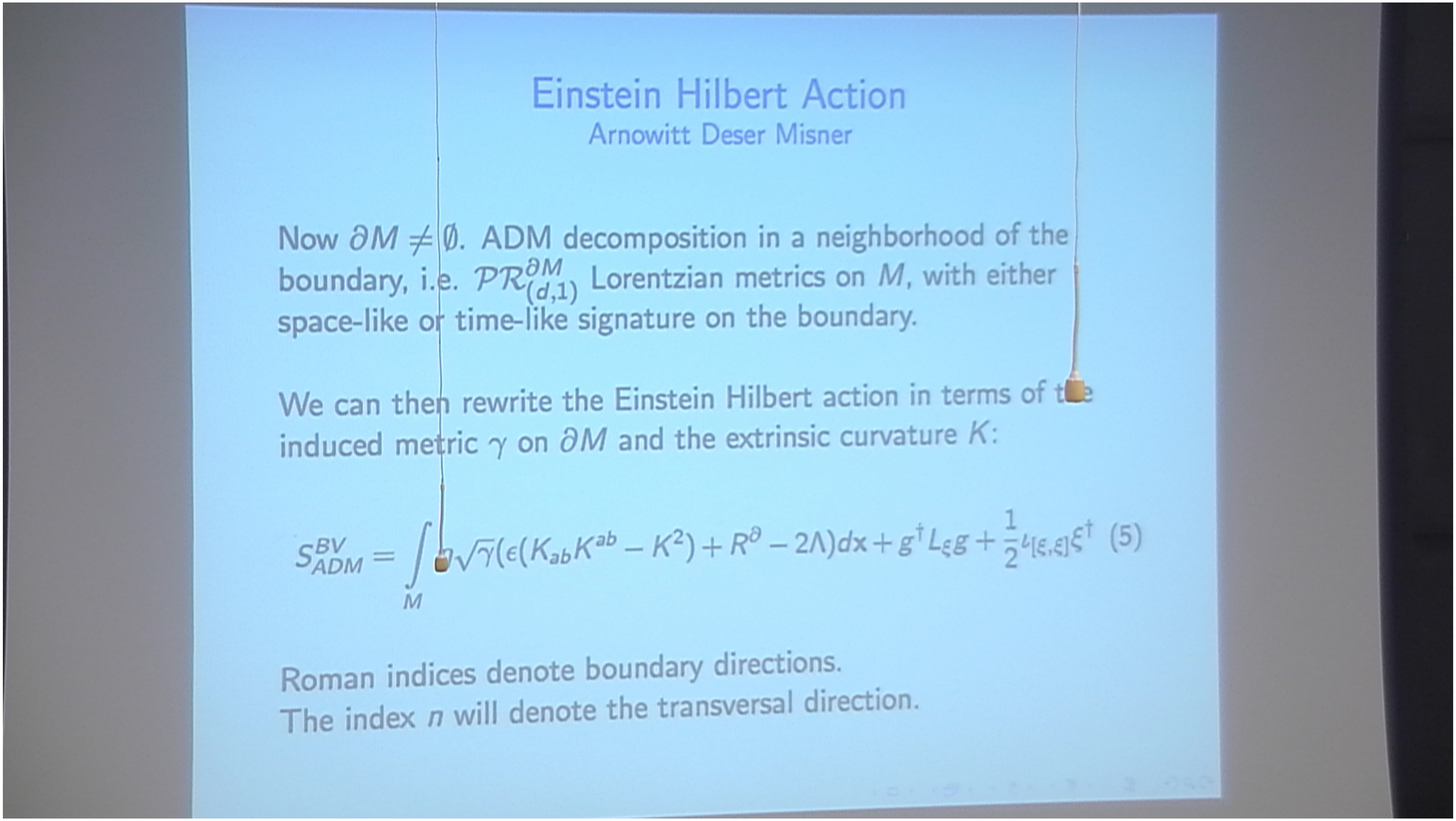
## Arnowitt Deser Misner

Now $\partial M \neq \emptyset$. ADM decomposition in a neighborhood of the boundary, i.e. $\mathcal{PR}^{\partial M}_{(d,1)}$ Lorentzian metrics on $M$, with either space-like or time-like signature on the boundary.

We can then rewrite the Einstein Hilbert action in terms of the induced metric $\gamma$ on $\partial M$ and the extrinsic curvature $K$:

$$S^{BV}_{ADM} = \int_M \partial\sqrt{\gamma}(\epsilon(K_{ab}K^{ab} - K^2) + R^{\partial} - 2\Lambda)dx + g^{\dagger} L_{\xi} g + \frac{1}{2}\iota_{[\xi,\xi]}\xi^{\dagger} \quad (5)$$

Roman indices denote boundary directions.
The index $n$ will denote the transversal direction.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# Einstein Hilbert Action

**Theorem ($d+1 \neq 2$)**

*The BV data for the Einstein Hilbert theory of GR in the boundary ADM decomposition yields an exact BV-BFV structure. The boundary action and two form read*

$$S^{\partial} = \int_{\partial M} \left\{ \frac{\epsilon}{\sqrt{\gamma}} \left( \boldsymbol{\Pi}^{ab}\boldsymbol{\Pi}_{ab} - \frac{1}{d-1}\mathrm{Tr}\boldsymbol{\Pi}^2 \right) + \sqrt{\gamma}\left(R^{\partial} - 2\Lambda\right) + \epsilon\partial_a\left(\xi^a\varphi_n\right) \right\}\xi^n$$

$$- \int_{\partial M} \epsilon\gamma^{ab}\varphi_b\partial_a\xi^n\xi^n + \left\{ \partial_c\left(\gamma^{cd}\boldsymbol{\Pi}_{da}\right) + \left(\partial_a\gamma^{cd}\right)\boldsymbol{\Pi}_{cd} - \epsilon\partial_c\left(\xi^c\varphi_a\right) \right\}\xi^a.$$

$$\omega^{\partial} = \epsilon \int_{\partial M} \delta\gamma^{ab}\delta\boldsymbol{\Pi}_{ab} + \delta\boldsymbol{\xi}^{\rho}\delta\vartheta_{\rho}$$

with

$$\pi_M : \begin{cases} \Pi_{lm} &= \frac{\sqrt{\gamma}}{2}\left(\tilde{J}_{lm} - \gamma_{lm}\gamma^{ij}\tilde{J}_{ij}\right) \\ \varphi_n &= -2\left\{\eta g^{\dagger nn} - \frac{\epsilon}{2}\eta^{-1}\left(\beta^a\chi_a - \chi_n\right)\xi^n\right\} \\ \varphi_a &= 2\,\gamma_{ab}\left\{g^{\dagger bn} + \gamma^{ba}\beta_a g^{\dagger nn} - \frac{\epsilon}{2}\gamma^{ba}\chi_a\xi^n\right\} \\ \xi^b &= \xi^b + \gamma^{ba}\beta_a\xi^n \\ \xi^n &= \eta\,\xi^n \\ \gamma_{ab} &= \gamma_{ab} \end{cases}$$

the projection to boundary fields, and

$$\tilde{J}_{lm} = \left\{\eta^{-1}\left(J_{lm} - 2\nabla_{(l}\beta_{m)}\right) - \frac{2\epsilon}{\sqrt{\gamma}}\left(\gamma_{al}\gamma_{bm} - \frac{1}{d-1}\gamma_{lm}\gamma_{ab}\right)g^{\dagger ab}\xi^n \right.$$

$$\left. - \frac{4}{\sqrt{\gamma}}\,\epsilon\left(\beta_{(l}\gamma_{m)b} - \frac{1}{d-1}\gamma_{lm}\beta_b\right)g^{\dagger bn}\xi^n - \frac{2\epsilon}{\sqrt{\gamma}}\left(\beta_{(l}\beta_{m)} - \frac{1}{d-1}\gamma_{lm}\beta_b\beta^b\right)g^{\dagger nn}\xi^n\right\}$$

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## Canonical Analysis and Boundary Gauge Symmetry

Extract relevant information from $S^\partial$:
(projected) canonical constraints

$$\left.\frac{\delta S^\partial}{\delta \xi^n}\right|_{gh=0} = \frac{\epsilon}{\sqrt{\gamma}}\left(\Pi^{ab}\Pi_{ab} - \frac{1}{d-1}\mathrm{Tr}\Pi^2\right) + \sqrt{\gamma}\left(R^\partial - 2\Lambda\right) \equiv \mathcal{H}$$

$$\left.\frac{\delta S^\partial}{\delta \xi^a}\right|_{gh=0} = -\,\partial_c\left(\gamma^{cd}\Pi_{da}\right) - (\partial_a\gamma^{cd})\Pi_{cd} \equiv \mathcal{H}_a$$

and residual gauge symmetry on the boundary:

$$(Q^\partial)_\gamma = \frac{2}{\sqrt{\gamma}}(\Pi - \frac{\gamma}{d-1}\mathrm{Tr}\Pi)\xi^n + L_{\xi^\partial}\gamma$$

$$(Q^\partial)_{\xi^\partial} = \xi^n\gamma^{-1}\nabla\xi^n + \frac{1}{2}[\xi^\partial, \xi^\partial]$$

$$(Q^\partial)_{\xi^n} = L_{\xi^\partial}\xi^n$$

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# Palatini Holst action

The PH tetrad formulation of gravity is a variational problem for the action

$$S_{PH} = \int_M \hat{T}_\gamma \left[ e \wedge e \wedge F_\omega \right] + \Lambda \mathrm{Tr}(e^4) \tag{6}$$

The fields are a (co-)tetrad $e \colon TM \longrightarrow \mathcal{V}$ and a principal $SO(3,1)$ connection $\omega$. Internal space $(V, \eta)$, and $\{u_i\}$ basis of eigenvectors of $\eta$:

$$\hat{T}_\gamma [u_i \wedge u_j \wedge u_k \wedge u_l] = \left[ \epsilon_{ijkl} + \frac{2}{\gamma} \eta_{i(k} \eta_{l)j} \right] . \tag{7}$$

$\gamma$ is the Barbero-Immirzi parameter.

Extra term proportional to $\gamma$ is a boundary term.

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$\gamma$ is the Barbero-Immirzi parameter.
Extra term proportional to $\gamma$ is a boundary term.

# Palatini Holst action

Symmetries are given by diffeomorphisms and internal gauge.

**Proposition**

*Let $P \longrightarrow M$ be a $G$ principal bundle and let $A$ be a connection on it. Consider any degree 1 vector field $\xi$ on $M$, and any associated vector bundle $\mathcal{V}$ with typical fiber the $\mathfrak{g}$ module $V_{\mathfrak{g}}$, denote by $\rho$ the representation. Let $c \in \Omega^0[1](M, \mathrm{ad}P)$ be a degree 1 function and define $Q$ a vector field on the graded manifold*

$$\mathcal{A}_P \oplus \Omega^\bullet(M, \mathcal{V}) \oplus \Gamma(T[1]M) \oplus \Omega^0[1](M, \mathrm{ad}P)$$

*by*

$$Q\,A = \iota_\xi F_A - d_\omega c \qquad Q\,\Phi = L^\omega_\xi \Phi - \rho(c)\Phi$$
$$Q\,c = \tfrac{1}{2}\iota_\xi \iota_\xi F_A - \tfrac{1}{2}[c,c] \qquad Q\,\xi = \tfrac{1}{2}[\xi,\xi] \tag{8}$$

*Then $[Q,Q] = 0$, and $QS_{PH} = 0$ when $\Phi = e$.*

# Palatini Holst action

So the BV-extended action reads:

$$
\begin{aligned}
S^{BV}_{PH} = \int_M & T_\gamma(e \wedge e \wedge F_\omega) + \mathrm{Tr}\left\{ (\iota_\xi F_\omega - d_\omega c)\,\omega^\dagger - \left(L^\omega_\xi e - [c, e]\right) e^\dagger \right\} \\
& + \frac{1}{2}\int_M \mathrm{Tr}\left\{ (\iota_\xi \iota_\xi F_\omega - [c, c])\, c^\dagger \right\} + \int_M \frac{1}{2}\iota_{[\xi,\xi]}\xi^\dagger
\end{aligned} \tag{9}
$$

And we have

**Theorem**

*Such a BV action on a manifold with boundary M does not define a BV-BFV theory.*

The pre-boundary 2-form fails to be pre-symplectic.
No compatibility between bulk and BFV resolution of constraints.
CMR procedure as it is cannot be adopted.

# Interpretation and Some Questions

The interpretation of this result is still open.
It states a fundamental difference of the BV theories.
**Question:** to what extent should we consider *classically equivalent* theories to be actually *physically equivalent*?
Quantisation takes into account what happens outside of the critical locus.
**Question:** what about Half Shell Localisation: $d_\omega e = 0$?
This is possibly even more singular. Also it raises questions about how to consistently implement Lagrange multipliers.
**Question:** Ho do other theories of gravity behave?
We have analysed other theories and the situation is complex.
Apparently the better behaved non-metric theory is MacDowell-Mansouri gravity (in progress).

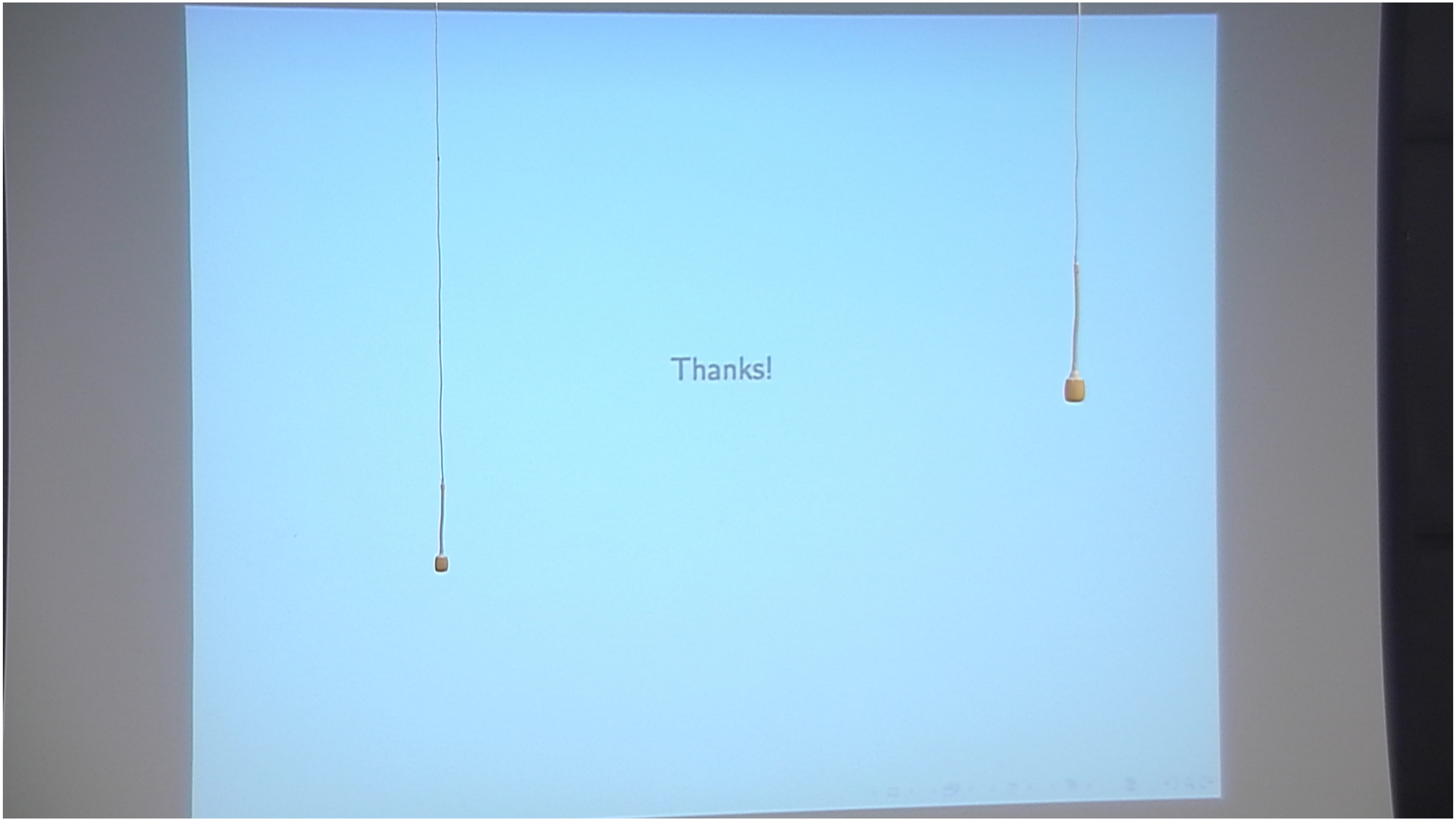
# Outlook

CMR axioms for a gauge theory to be BV-BFV distinguish between classically equivalent theories.

Diffeomorphism as an *external* gauge theory tells about *allowed* couplings and potentials.

Use this to construct BV-BFV equivalent theories of gravity.

First step towards CMR quantisation of gravity as a gauge theory.


Thanks!