

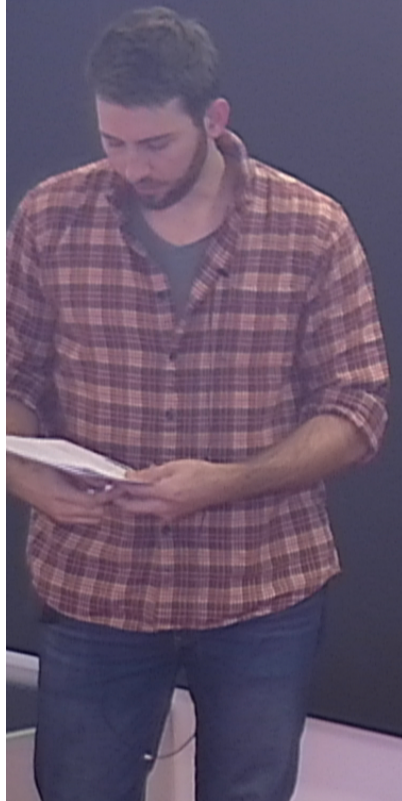
Title: Half-BPS boundary conditions in 3d N=4 theories

Date: Oct 08, 2015 01:30 PM

URL: <http://pirsa.org/15100071>

Abstract:

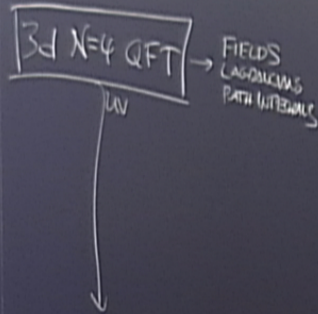
PREDICTIONS FOR DEFORMATION
OF HIC CONES FROM SCFT



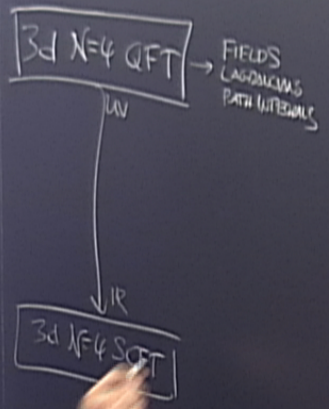
PREDICTIONS FOR DEQUANTIZATION
OF HK CONES FROM SCFT

3d N=4 QFT

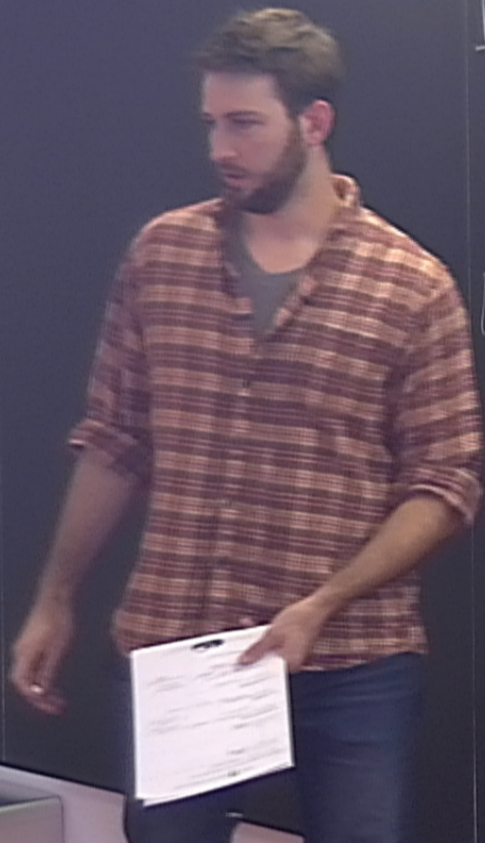
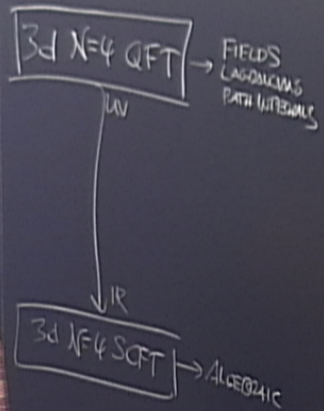
PREDICTIONS FOR DEQUANTIZATION
OF HK CONES FROM SCFT



PREDICTIONS FOR DEQUANTIZATION
OF HK CONES FROM SCFT



PREDICTIONS FOR DEQUANTIZATION
OF HK CONES FROM SCFT

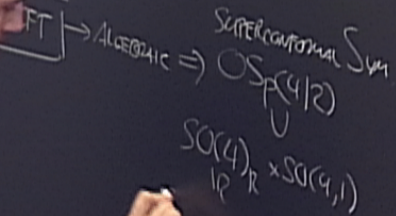
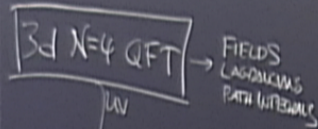


PREDICTIONS FOR DEFORMATION
OF HK CONES FROM SCFT

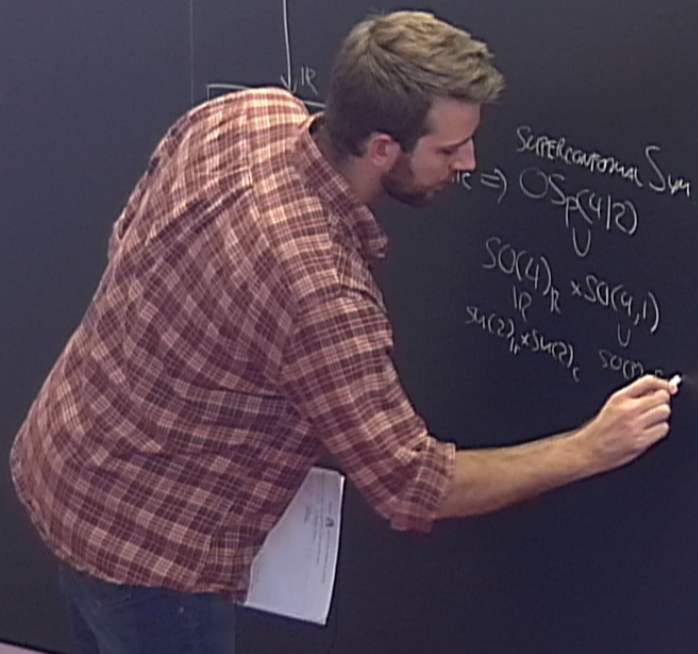
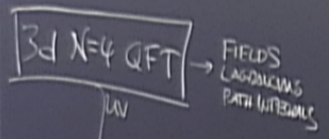
3d $N=4$ QFT \rightarrow FIELDS
LAGRANGIAN
PATH INTEGRALS

SCFT \rightarrow ALGEBRAIC \Rightarrow SUPERCONFORMAL SYM
 $OSp(4|2)$

PREDICTIONS FOR DEQUANTIZATION
OF HK CONES FROM SCFT

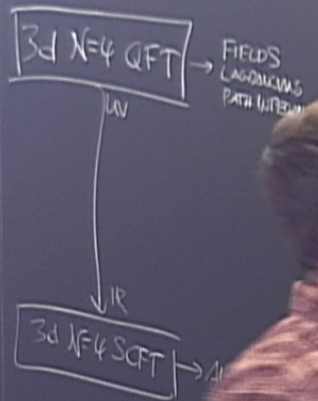


PREDICTIONS FOR DEQUANTIZATION
OF HK CONES FROM SCFT



SUPERCONFORMAL SYM
 \Rightarrow $OSp(4|2)$
 \cup
 $SO(4)_R \times SO(4)_1$
 \cup
 $su(2)_r \times su(2)_c$ $so(m)$

PREDICTIONS FOR DE QUANTIZATION
OF HK CONES FROM SCFT



LOCAL OPERATORS

$$\mathcal{O}_i(x) \leftarrow \begin{matrix} \text{UNITARY (H.W.)} \\ \text{IRREDUC. OF } \mathcal{O}_{Sp(4|2)} \end{matrix}$$

$S_{\mathcal{N}=4}$
(\mathcal{O}_i)
 $(S_{\mathcal{O}_i})$

PREDICTIONS FOR DEE QUANTIZATION
OF HK CONES FROM SCFT

3d N=4 QFT → FIELDS
LAGRANGIAN
PATH INTEGRALS

UV

IR

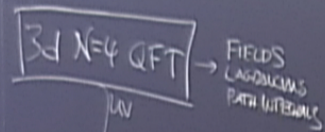
SCFT

ALGEBRAIC ⇒ SUPERCONFORMAL SYM
 $OSp(4|2)$
 \cup
 $SO(4)_R \times SO(4)_1$
 \cup
 $sp(2)_r \times sp(2)_c$ $SO(2) \times SO(2)_1$

LOCAL OPERATORS

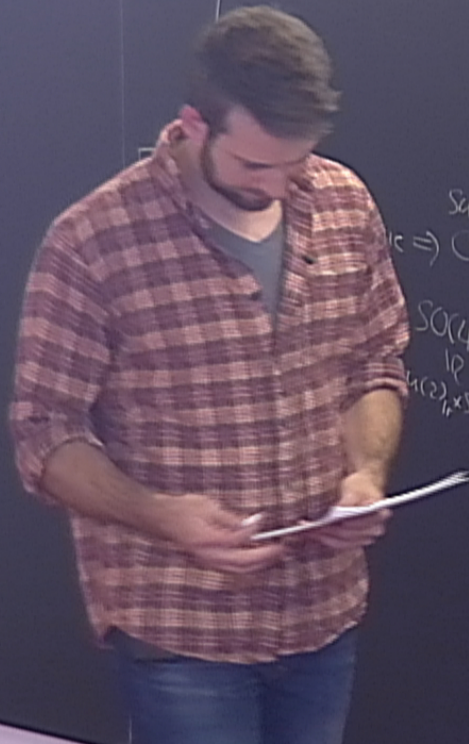
$\mathcal{O}_i(x) \leftarrow$ UNITARY (H.W.)
 IRREDS OF $OSp(4|2)$ (Δ, j, r, c)

PREDICTIONS FOR DEF QUANTIZATION
OF HK CONES FROM SCFT



LOCAL OPERATORS

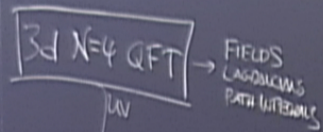
$$\mathcal{O}_i(x) \leftarrow \begin{matrix} \text{UNITARY (H.W.)} \\ \text{IRREDS OF } \mathfrak{osp}(4|2) \end{matrix} (\Delta, j, r, c)$$



Supercritical Sym

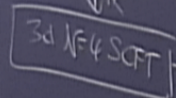
$$\begin{aligned} &\Rightarrow \mathfrak{osp}(4|2) \\ &\cup \\ &\mathfrak{so}(4)_R \times \mathfrak{so}(4)_I \\ &\cup \\ &\mathfrak{so}(2)_R \times \mathfrak{su}(2)_C \quad \mathfrak{so}(2) \times \mathfrak{so}(2)_I \end{aligned}$$

PREDICTIONS FOR DEF QUANTIZATION
OF HK CONES FROM SCFT



UV

IR



ALGEBRAIC ⇒ SUPERCONFORMAL SYM
 $OSp(4|2)$
 \cup
 $SO(4)_R \times SO(4)_1$
 \cup
 $sp(2)_r \times su(2)_c \cup so(2) \times so(1)$

LOCAL OPERATORS

$\mathcal{O}_i(x) \leftarrow$ UNITARY (H.W.)
 IRREDS OF $OSp(4|2)$ (Δ, j, r, c)
 (KINEMATICS)



PREDICTIONS FOR DEF QUANTIZATION
OF HK CONES FROM SCFT

3d N=4 QFT → FIELDS
CLASSICALS
PATH INTEGRALS

UV

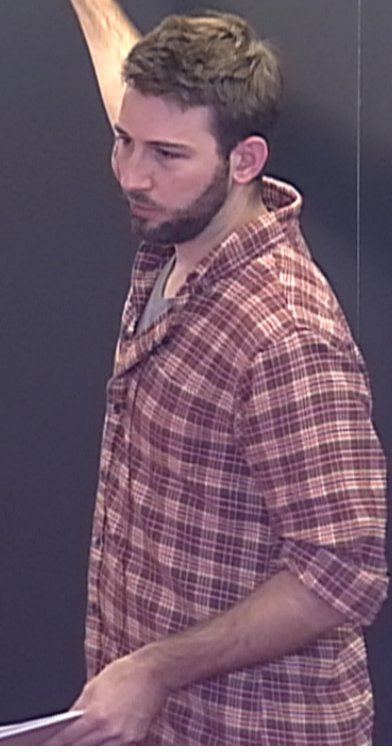
IR

3d N=4 SCFT

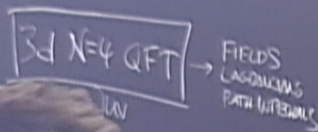
ALGEBRAIC ⇒ SUPERCONFORMAL Sym
 $OSp(4|2)$
 \cup
 $SO(4)_R \times SO(9,1)$
 \cup
 $Sp(2)_R \times SU(2)_C \times SO(3) \times SO(1,1)$

LOCAL OPERATORS

$\mathcal{O}_i(x) \leftarrow$ UNITARY (H.V.)
 TRACES OF $OSp(4|2)$ (Δ, j, r, r_c)
 (KINEMATICS)



PREDICTIONS FOR DEF QUANTIZATION
OF HK CONES FROM SCFT



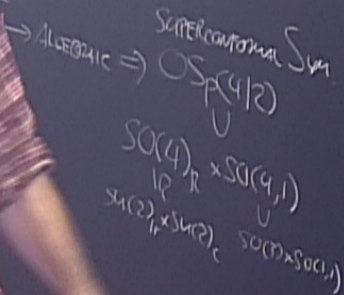
LOCAL OPERATORS

$\mathcal{O}_i(x) \leftarrow$ UNITARY (CHIR)
 TRIPLES OF $OSp(4|2)$ (Δ, j, r, c)

(KINEMATICS)

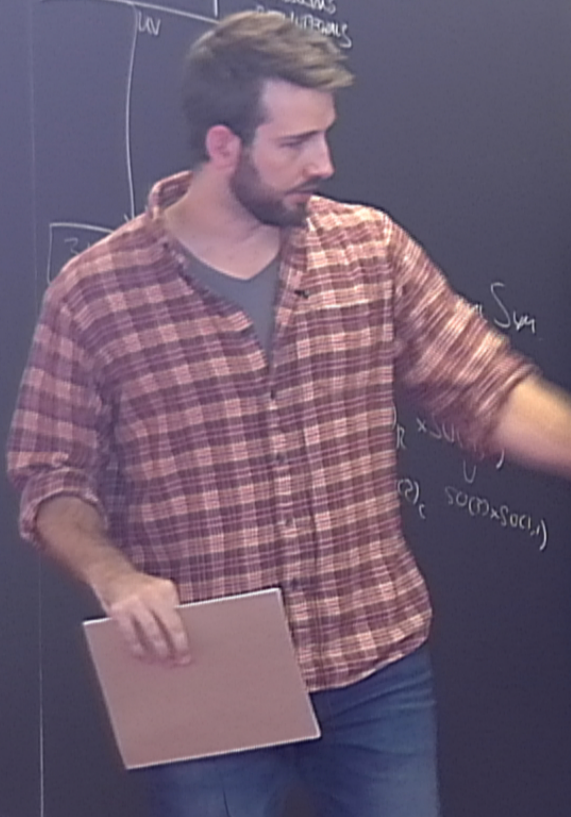
OPE'S & CORRELATORS

$\mathcal{O}_i(x) \mathcal{O}_j(y)$



PREDICTIONS FOR DEF QUANTIZATION
OF HK COMES FROM SCFT

\exists N=4 QFT \rightarrow FIELDS
LAGRANGIANS
DIFFERENTIALS



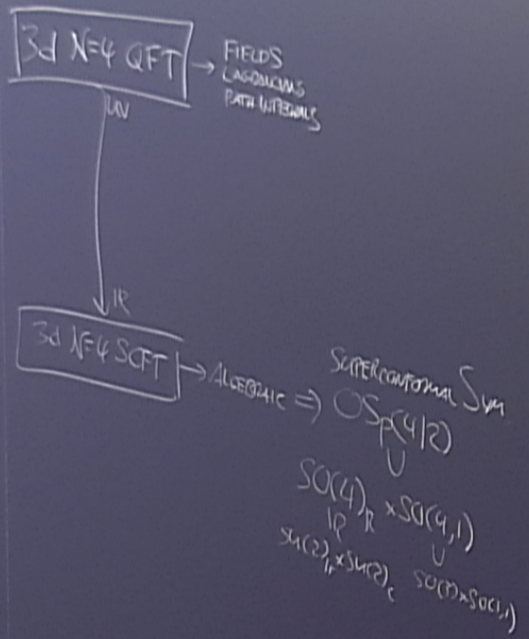
LOCAL OPERATORS

$\mathcal{O}_i(x) \leftarrow$ UNITARY (H.W.)
IRREDS OF $\mathcal{O}_{Sp}(4|2)$ $(\Delta_i, j_i, r_i, c_i)$
(KINEMATICS)

OPE'S & CORRELATORS

$$\mathcal{O}_i(x) \mathcal{O}_j(y) \sim \sum_k C_{ij}^k(x,y) \mathcal{O}_k(z)$$

PREDICTIONS FOR DEF QUANTIZATION
OF HK CONES FROM SCFT



LOCAL OPERATORS

$\mathcal{O}_i(x) \leftarrow$ UNITARY (H.V.)
IRREDS OF $OSp(4|2)$ (Δ, j, r, r_c)

(KINEMATICS)

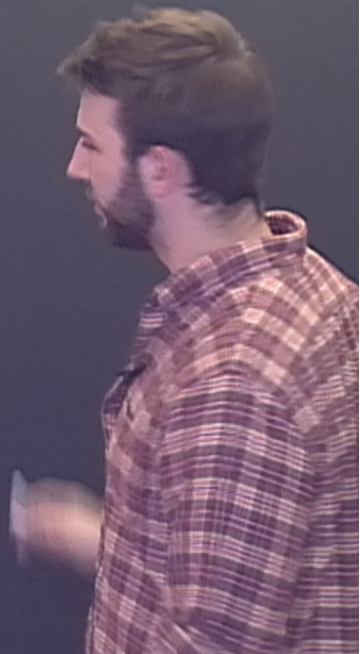
OPE'S & CORRELATORS

$\mathcal{O}_i(x) \mathcal{O}_j(y) \sim \sum_k C_{ij}^k(x,y) \mathcal{O}_k(z)$

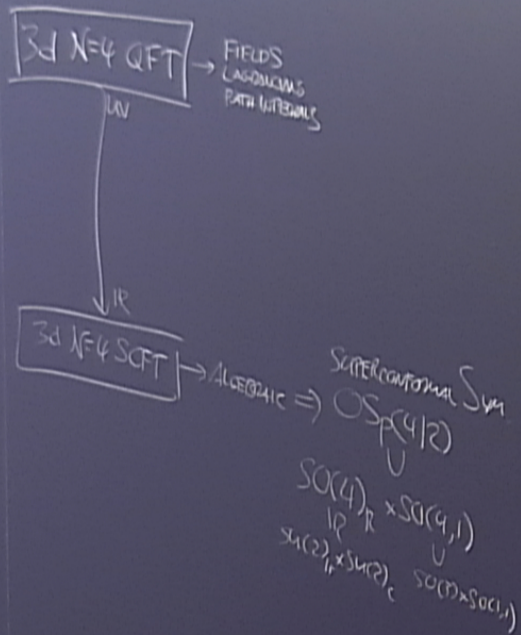
↑ PURE NUMBERS

$\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle = \frac{C_{ijk}}{(x-y)^{\Delta_i + \Delta_j - \Delta_k}}$

$\langle \mathcal{O}_i \mathcal{O}_j \rangle = \frac{\eta_{ij}}{|x-y|^{2\Delta}}$ ⇒ $\langle \mathcal{O}_i \dots \mathcal{O}_n \rangle$ FIXED



PREDICTIONS FOR DEF QUANTIZATION
OF HK CONES FROM SCFT



LOCAL OPERATORS

$\mathcal{O}_i(x) \leftarrow$ UNITARY (H.W.)
 TRIGS OF $OSp(4|2)$ (Δ, j, r, r_c)

(KINEMATICS)

OPE'S & CORRELATORS

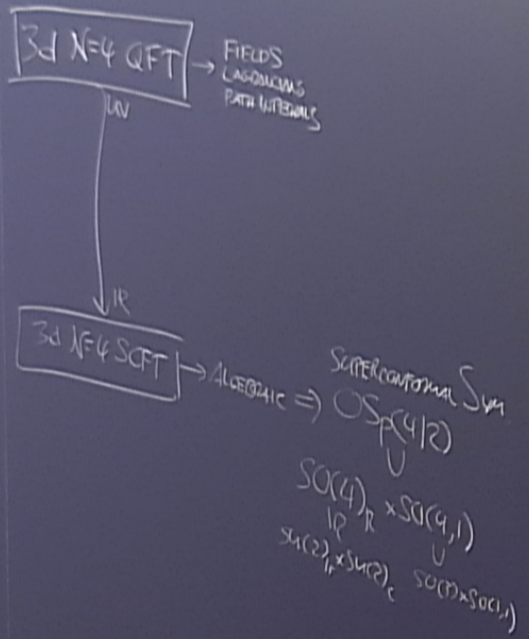
$\mathcal{O}_i(x) \mathcal{O}_j(y) \sim \sum_k C_{ijk} (x-y)^{\Delta_k - \Delta_i - \Delta_j}$

\uparrow PURE NUMBERS (ASSOCIATIVE)

$\langle \mathcal{O}_i^{\omega} \mathcal{O}_j^{\rho} \mathcal{O}_k^{\sigma} \rangle = \frac{C_{ijk}}{(x-y)^{\Delta_i + \Delta_j + \Delta_k}}$

$\langle \mathcal{O}_i \mathcal{O}_j \rangle = ?$

PREDICTIONS FOR DEF QUANTIZATION
OF HK CONES FROM SCFT



LOCAL OPERATORS

$\mathcal{O}_i(x) \leftarrow$ UNITARY (H.V.)
IRREDS OF $OSp(4|2)$ (Δ, j, r, r_c)

(KINEMATICS)

OPE'S & CORRELATORS

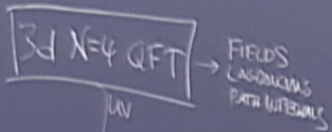
$\mathcal{O}_i(x) \mathcal{O}_j(y) \sim \sum_k C_{ij}^{(k)} \mathcal{O}_k(z)$ (ASSOCIATIVE)

↑ PURE NUMBERS

$\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle = \frac{C_{ijk}}{(x-y)^{\Delta_i + \Delta_j + \Delta_k}}$

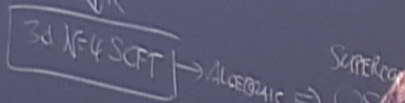
$\langle \mathcal{O}_i \mathcal{O}_j \rangle = \frac{\delta_{ij}}{|x-y|^{2\Delta_i}}$ ⇒ $\langle \mathcal{O}_i \dots \mathcal{O}_n \rangle$ FIXED

PREDICTIONS FOR DE-QUANTIZATION OF HK CONES FROM SCFT



UV

IR



LOCAL OPERATORS

$\mathcal{O}_i(x) \leftarrow$ UNITARY (H.W.)
IRREPS OF $OSp(4|2)$ (Δ, j, r, r_2)
(KINEMATICS)

E's & COEFFICIENTS

$\mathcal{O}_i(x) \mathcal{O}_j(y) \sim \sum_k C_{ij}^{(k)}(x,y) \mathcal{O}_k(z)$
↑ PURE NUMBERS

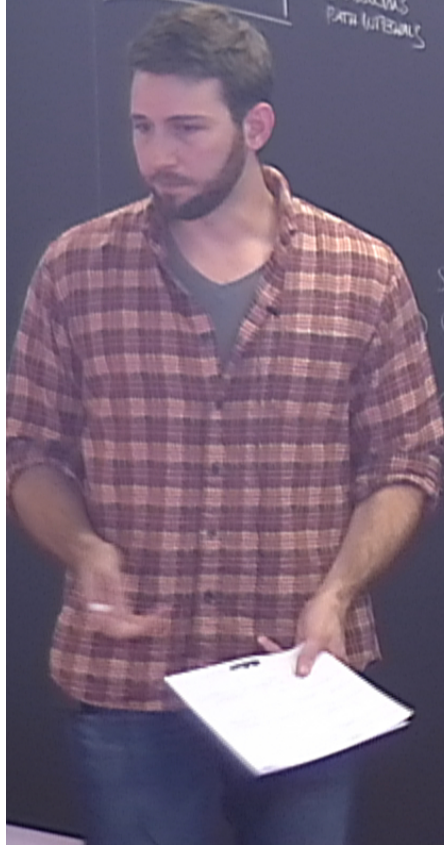
(ASSOCIATIVE)

$\mathcal{O}_1 \cdot \mathcal{O}_3$
 $\mathcal{O}_2 \cdot \mathcal{O}_4$

$\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle_{SIRCF}$

PREDICTIONS FOR DE-QUANTIZATION
OF HK CONES FROM SCFT

3d N=4 QFT → FIELDS
CLASSICALS
PATH INTEGRALS



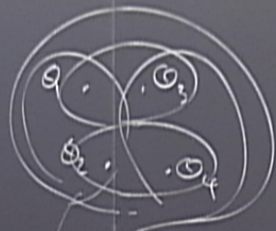
SUPERCONFORMAL SYM
OSp(4|2)
SO(4) × SO(3,1)
SU(2) × SO(3) × SO(2)

LOCAL OPERATORS

$$\mathcal{O}_i(x) \leftarrow \text{UNITARY (H.W.)}$$

$$\text{IRREPS OF } OSp(4|2) \quad (\Delta, j, r, r_2)$$

(KINEMATICS)



OPE's & CORRELATORS

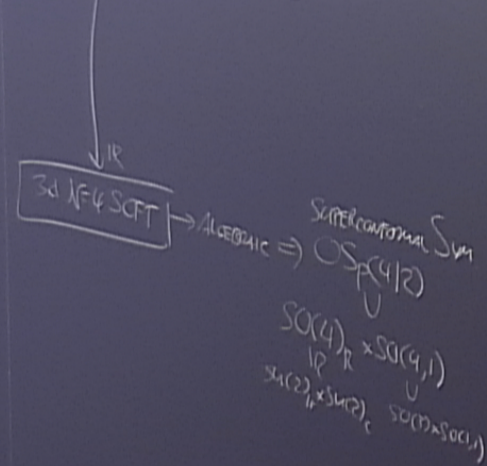
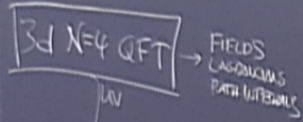
$$\mathcal{O}_i(x) \mathcal{O}_j(y) \sim \sum_k \binom{k}{i, j} \mathcal{O}_k(z) \quad (\text{ASSOCIATIVE})$$

↑
PURE NUMBERS

$$\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle = \frac{C_{ijk}}{(x-y)^{\Delta_i + \Delta_j + \Delta_k}}$$

$$\langle \mathcal{O}_i \mathcal{O}_j \rangle = \frac{2\delta_{ij}}{|x-y|^{2\Delta_i}} \Rightarrow \langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle \text{ SIX REG}$$

PREDICTIONS FOR DE QUANTIZATION OF HK CONES FROM SCFT



LOCAL OPERATORS

$\mathcal{O}_i(x) \leftarrow$ UNITARY (H.W.)
TRACE OF $OSp(4|2)$ (Δ, j, r, r_2)

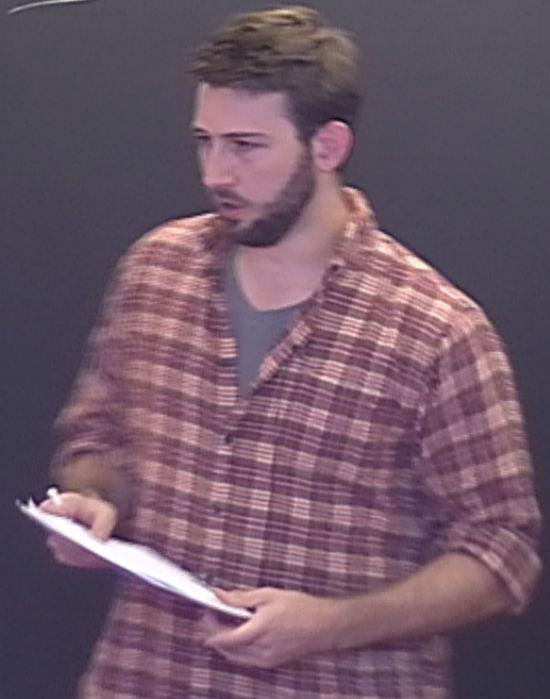
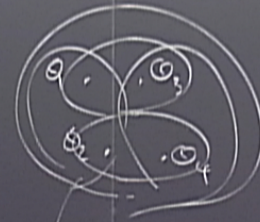
(KINEMATICS)

OPE'S & CORRELATORS

$\mathcal{O}_i(x) \mathcal{O}_j(y) \sim \sum_k C_{ij}^k(x,y) \mathcal{O}_k(z)$ (ASSOCIATIVE)

↑ PURE NUMBERS

$\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle = \frac{C_{ij}^k}{(x-y)^{\Delta_i+\Delta_j-\Delta_k}}$
 $\langle \mathcal{O}_i \mathcal{O}_j \rangle = \frac{2\delta_{ij}}{|x-y|^{2\Delta}}$ ⇒ $\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle$ Fixed



PREDICTIONS FOR DEFORMATIONS
OF HK CONES FROM SCFT

3d N=4 QFT → FIELDS
LAGRANGIANS
PATH INTEGRALS

UV

IR

3d N=4 SCFT

ALGEBRAIC ⇒ SUPERCONFORMAL SYM

$OSp(4|2)$

$SO(4)_R \times SO(4)_1$

$U(1)_F$
 $SU(2)_L \times SU(2)_C$ $SO(2) \times SO(4)$

LOCAL OPERATORS

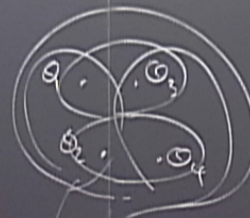
$\mathcal{O}_i(x) \leftarrow$ UNITARY (H.W.)
IRREPS OF $OSp(4|2)$ (Δ, j_1, r, r_c)

(KINEMATICS)

OPE'S & COEFFICIENTS

$\mathcal{O}_i(x) \mathcal{O}_j(y) \sim \sum_k C_{ij}^k(x-y) \mathcal{O}_k$
↑
PURE NUMBERS

$\langle \mathcal{O}_i^{(p)} \mathcal{O}_j^{(q)} \rangle = \frac{C_{ijk}}{(x-y)^{\Delta_i + \Delta_j - \Delta_k}}$
 $\langle \mathcal{O}_i \mathcal{O}_j \rangle = \delta_{ij} \Rightarrow$



ASSOCIATIVE

BPS operators

"HIGGS" $\Delta = r_1$

"COULOMBS"

PREDICTIONS FOR DEFORMATIONS
OF HK CONES FROM SCFT

3d N=4 QFT → FIELDS
LAGRANGIAN
PATH INTEGRALS

UV

IR

3d N=4 SCFT → ALGEBRAIC ⇒ SUPERCON

LOCAL OPERATORS

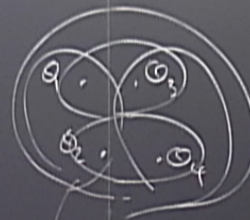
$$\mathcal{O}_i(x) \leftarrow \text{UNITARY (H.W.)} \\ \text{IRREPS OF } \mathcal{O}_{Sp(4|2)} (\Delta, j, r, r_c)$$

(KINEMATICS)

OPE'S & CORRELATORS

$$\mathcal{O}_1(x) \mathcal{O}_2(y) \sim \sum_k C_k^{(x,y)} \mathcal{O}_k(z) \quad (\text{ASSOCIATIVE})$$

↑
PURE NUMBERS



$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle = \frac{C_{12k}}{(x-y)^{\Delta_1 + \Delta_2 - \Delta_k}} \Rightarrow \langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle \text{ fixed}$$

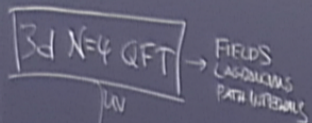
BPS operators

"HIGGS" $\Delta = r_1, r_2 = j = 0$

"COULOMB" $\Delta = r_2, r_1 = j = 0$

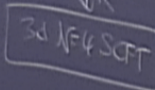
R-symmetry h.c.s
annihilated by half supercharges

PREDICTIONS FOR DEFORMATIONS OF HK COMES FROM SCFT



UV

IR



ALGEBRAIC ⇒ SUPERCONFORMAL SYM
 $OSp(4|2)$
 $SO(4)_R \times SO(4)_1$
 $SO(2)_R \times SU(2)_C \times SO(2) \times SO(3)_1$

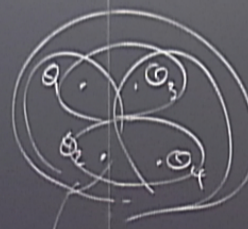
LOCAL OPERATORS

$\mathcal{O}_i(x) \leftarrow$ UNITARY (H.W.)
 IRREPS OF $OSp(4|2)$ (Δ, j, r, r_c)

(KINEMATICS)

OPE'S & COEFFICIENTS

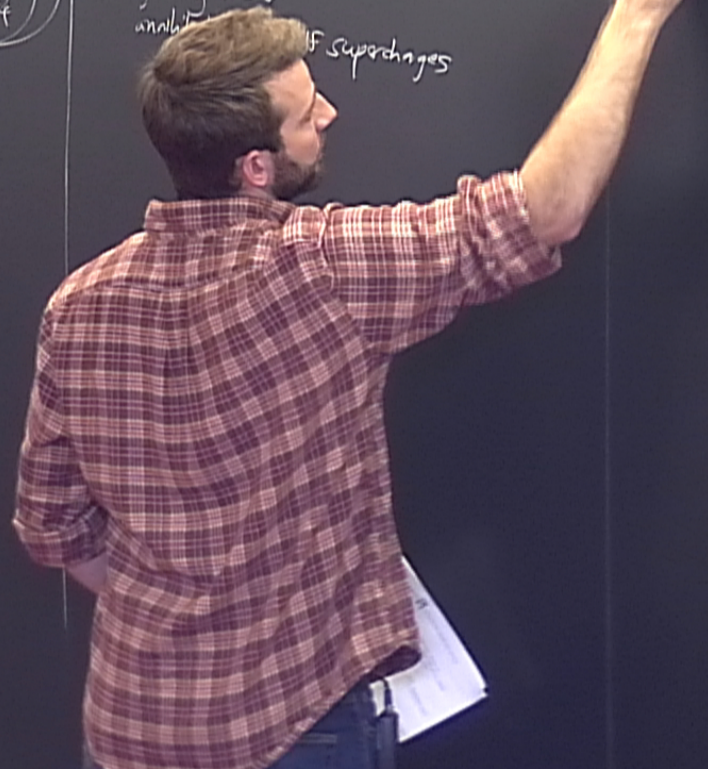
$\mathcal{O}_i(x) \mathcal{O}_j(y) \sim \sum_k C_{ij}^k(x-y) \mathcal{O}_k(y)$ (ASSOCIATIVE)



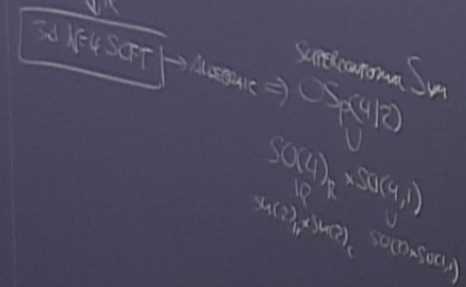
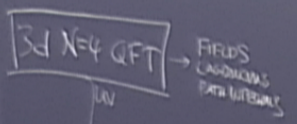
$\langle \mathcal{O}_i^{(p)} \mathcal{O}_j^{(q)} \rangle = \frac{C_{ijk}}{(x-y)^{\Delta_i + \Delta_j - \Delta_k}}$
 $\langle \mathcal{O}_i \mathcal{O}_j \rangle = \frac{C_{ij}}{|x|^{2\Delta}}$ ⇒ $\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle$ FIXED

BPS operators

"HIGGS" $\Delta = r_1, r_2 = j = 0$
 "COULOMB" $\Delta = r_2, r_1 = j = 0$ } FORM COMMUTATIVE
 R-symmetry h.c.s
 annihilates \mathbb{F} supercharges



PREDICTIONS FOR DE QUANTIZATION OF HK CONES FROM SCFT



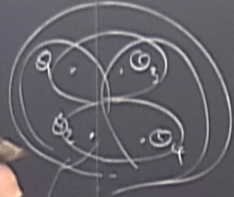
LOCAL OPERATORS

$\mathcal{O}_i(x) \in$ FINITELY (H.W.) IRREPS OF $OSP(4|2)$ (Δ, j_1, r_1, r_2)
(KINEMATICS)

OPE'S & CORRELATORS

$\mathcal{O}_i(x)\mathcal{O}_j(y) \sim \sum_k C_{ij}^k(x,y) \mathcal{O}_k(z)$
↑
FUNCTION

$\langle \mathcal{O}_i^{\Delta_i, j_i, r_i} \mathcal{O}_j^{\Delta_j, j_j, r_j} \rangle = \frac{C_{ij}^k}{(x-y)^{\Delta_i + \Delta_j - \Delta_k}}$
 $(0,0) = \frac{2}{1-x}$

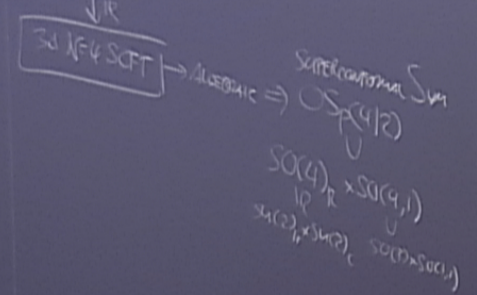
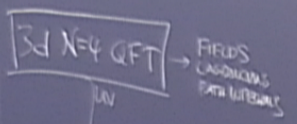


BPS operators

"HIGGS" $\Delta = r_1, r_2 = j = 0$
"COULOMB" $\Delta = r_2, r_1 = j = 0$
R-symmetry h.c.s annihilated by half supercharges

- FORM COMMUTATIVE RINGS
- short distance OPE
 - OPE modulo \mathbb{Q} -exact

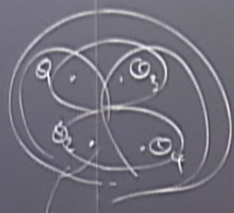
PREDICTIONS FOR DE QUANTIZATION OF HK CONES FROM SCFT



LOCAL OPERATORS

$\mathcal{O}_i(x) \in$ LINEARLY (H.W.) IRREDS OF $\mathcal{O}_{Sp(4|2)}$ $(\Delta, j; r, r_2)$

(KINEMATICS)



OPE'S & CORRELATORS

$\mathcal{O}_i(x) \mathcal{O}_j(y) \sim \sum_k \langle \mathcal{O}_k(y) \rangle$ (ASSOCIATIVE)

↑ PURE NUMBERS

$\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle = \frac{C_{ijk}}{(x_{ij})^{\Delta_i} (x_{jk})^{\Delta_j} (x_{ki})^{\Delta_k}}$

$(0,0) = \sum_{ij} \frac{C_{ij}}{|x|^{2\Delta}}$ ⇒ $\langle \mathcal{O}_1, \dots, \mathcal{O}_n \rangle$ fixed

BPS operators

"HIGGS" $\Delta = r_1, r_2 = j = 0$

"COULOMB" $\Delta = r_2, r_1 = j = 0$

FORM COMMUTATIVE RINGS

- short distance OPE
- OPE modulo \mathbb{Q} -exact

R-symmetry h.c.s annihilated by half supercharges

CHIRAL RING \Leftrightarrow COORDINATE RING ON A HK CONE

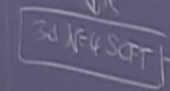
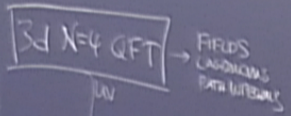
$(\Delta, r) \Leftrightarrow \mathbb{C}^*$ grading

$SU(2)_R \Leftrightarrow SU(2)$ ISOMETRY

$F_i F_j = C_{ij}^k F_k \Rightarrow C_{ij}^k = C_{[ij]^k}^{rk} \quad |k| = |i| + |j|$

QUESTION

PREDICTIONS FOR DE QUANTIZATION OF HK CONES FROM SCFT

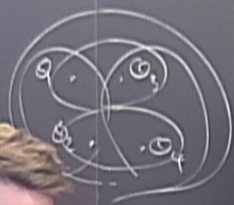


Supercanonical Sym
 $SO(4) \times SO(4,1)$
 $SO(2,2) \times SO(4)$
 $SO(2,1) \times SO(3)$

LOCAL OPERATORS

$\mathcal{O}_i(x) \sim$ primary (h,w) irreps of $OSp(4|2)$ $(\Delta, j; r, r_2)$

(KINEMATICS)



OPE'S & CORRELATORS

$\mathcal{O}_i(x) \mathcal{O}_j(y) \sim \sum_k C_{ij}^{(k)} \mathcal{O}_k(z)$
 PURE NUMERICAL

$(\mathcal{O}_1, \mathcal{O}_2) = \frac{C_{12}}{r^{2\Delta_1 + 2\Delta_2}}$
 $(\mathcal{O}, \mathcal{O}) = 2$

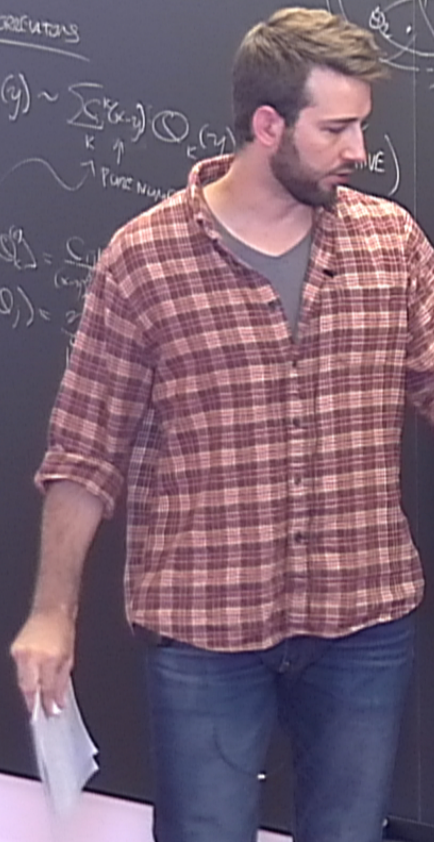
BPS operators

"HIGGS" $\Delta = r_1, r_2 = j = 0$
 "COULOMB" $\Delta = r_2, r_1 = j = 0$
 Form COMMUTATIVE RINGS
 - short distance OPE
 - OPE modulo \mathbb{Q} -exact
 R-symmetry h.c.s annihilated by half supercharges

CHIRAL RING \Leftrightarrow COORDINATE RING ON A HK CONE
 $(\Delta, r) \Leftrightarrow \mathbb{C}^*$ grading

$SU(2)_R \Leftrightarrow SU(2)$ ISOMETRY
 $\mathbb{C}^* \times SU(2)_R \Leftrightarrow \mathbb{C}^* \times SU(2)$

QUESTION
 $C_{ijk} = C_{jki}^{rk} \quad |k| = |i| + |j|$
 $C_{ijk} \quad ||i| - |j|| \leq |k| \leq |i| + |j|$



BPS operators

"Higgs" $\Delta = r_1, r_2 = j = 0$

"Coulomb" $\Delta = r_2, r_1 = j = 0$

Form commutative rings
 - short distance OPE
 - OPE modulo \mathbb{Q} -exact

Resonance
 annihilates h.c.f. supercharges

CRITICAL POINT

(Δ, r)

$S_{\text{eff}} \leftarrow$

$S_1, S_2 = C, F, S_k =$

QUESTION

ON A HIK CURVE

$|k| = |i| + |j|$

$|k| \leq |i| + |j|$

CLAIM (PHYSICS)

CAN FIND NILPOTENT $\mathbb{Q} = Q + \beta S$ S.T.

\mathbb{Q} -closed (not exact) local operators are twisted

versions of BPS ops.



$\hat{Q}(S) = \ominus$

operators

$\Delta = r_1, r_2 = j = 0$
 $\Delta = r_2, r_1 = j = 0$

Form commutative rings

- short distance OPE
- OPE modulo Q -exact

related by half-supersymmetries

CHIRAL RING \Leftrightarrow

$(\Delta, r) \Leftrightarrow$ HK cone

$SU(2)_R \Leftrightarrow$

$S^1 \times S^1 \times S^1 \Leftrightarrow$

QUESTION

CLAIM (PHYSICS)

CAN FIND NILPOTENT $\hat{Q}_s = Q + \beta S$ S.T.

- \hat{Q} -closed (not exact) local operators are twisted versions of BPS ops.



$$\hat{Q}(s) = \hat{Q}^+_{(s)} + s \hat{Q}^+_{(s)} + \dots + s^2 \hat{Q}^+_{(s)} + \dots + s^2 \hat{Q}^-_{(s)} + \dots$$

operators

$\Delta = r_1, r_2 = j = 0$
 $\Delta = r_2, r_1 = j = 0$

Form commutative rings
 • short distance OPE
 • OPE modulo Q -exact

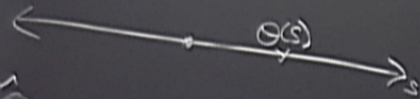
related by half-supersymmetries

CRITICAL POINT
 COORDINATE RING ON A HK CONE
 L^* grading
 $|k| = |i| + |j|$
 $|k| \leq |i| + |j|$

CLAIM (PHYSICS)

CAN FIND NILPOTENT $\mathbb{Q} = Q + \beta S$ S.T.

- \mathbb{Q} -closed (not exact) local operators are twisted versions of BPS ops.



$$\hat{Q}(s) = \mathbb{Q}_{(s)}^{+-+} + s \mathbb{Q}_{(s)}^{+--} + \dots + s^2 \mathbb{Q}_{(s)}^{-+-}$$

-OPE

DPS operators

"HIGGS" $\Delta = r_1 \quad r_2 = j = 0$
 "COULOMB" $\Delta = r_2 \quad r_1 = j = 0$

Form commutative rings
 - short distance OPE
 - OPE modulo \mathbb{Q} -exact

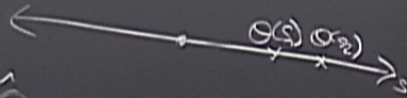
R-symmetry h.c.s
 annihilated by half supercharges

RING \Leftrightarrow COORDINATE RING ON A HK CONE
 $\mathbb{R}^2 \Leftrightarrow \mathbb{C}^*$ gradng
 \Rightarrow SU(2) ISOMETRY
 $\Rightarrow \sum_{|k|=|i|+|j|} c_{ij}^k z^k$
 $|i|-|j| \leq |k| \leq |i|+|j|$

CLAIM (PHYSICS)

CAN FIND NILPOTENT $\mathbb{Q}_S = Q + gS$ S.T.

\mathbb{Q} -closed (not exact) local operators are twisted versions of BPS ops.



$$\hat{Q}_S = Q_{(s)}^{++} + s^j Q_{(s)}^{+-} + \dots + s^j g^2 j Q_{(s)}^{--}$$

- OPE modul- \mathbb{Q} -exact is S-independent (but care about ordering)
 \Rightarrow

DPS operators

"HIGGS" $\Delta = r_1 \quad r_2 = j = 0$
 "COULOMB" $\Delta = r_2 \quad r_1 = j = 0$

Form commutative rings
 - short distance OPE
 - OPE modulo \mathbb{Q} -exact

R-symmetry h.c.s
 annihilated by half supercharges

CRITICAL RING \Leftrightarrow COORDINATE RING ON A HK CURVE

$(\Delta, r) \Leftrightarrow \mathbb{C}^*$ grading

$SU(2)_R \Leftrightarrow SU(2)$ ISOMETRY

$S_i S_j = C_{ij}^k S_k \Rightarrow C_{ij}^k = C_{ji}^k \quad |k| = |i| + |j|$

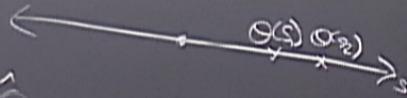
QUESTION:

$C_{ijk} \quad ||i|-|j|| \leq |k| \leq |i|+|j|$

CLAIM (PHYSICS)

CAN FIND NILPOTENT $\mathbb{Q}_S = Q + \beta S \quad S.T.$

\mathbb{Q} -closed (not exact) local operators are twisted versions of BPS ops.



$$\hat{Q}(s) = \mathbb{Q}^+ \mathbb{Q}^- + s \beta \mathbb{Q}^+ \mathbb{Q}^- + \dots + s^2 \beta^2 \mathbb{Q}^+ \mathbb{Q}^-$$

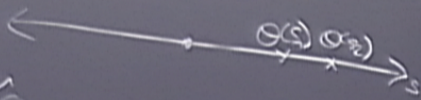
- OPE mod. \mathbb{Q} -exact is S-independent (but care about ordering)
 \Rightarrow Non-commutative Assoc. ALGEBRA (w/EVALUATION)

$\beta \rightarrow 0$ limit \Rightarrow chiral ring

CLAIM (PHYSICS)

CAN FIND NILPOTENT $\mathbb{Q}_S = Q + \beta S$ S.T.

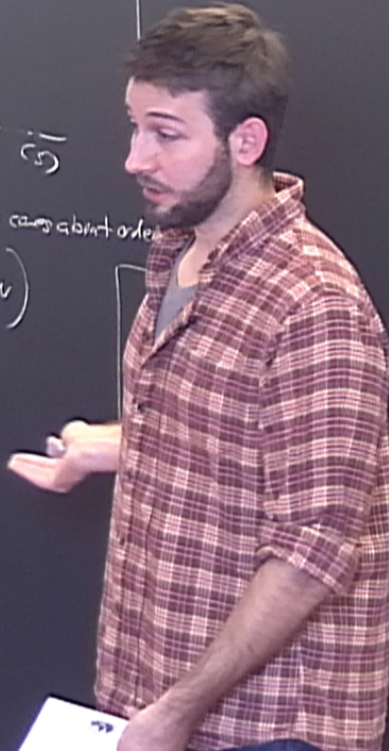
- \mathbb{Q} -closed (not exact) local operators are twisted versions of BPS ops.



$$\hat{\mathcal{O}}(s) = \mathcal{O}^{+-+}(s) + s g^i \mathcal{O}^{+--}(s) + \dots + s^{2j} g^{2j} \mathcal{O}^{-\bar{c}\bar{c}}(s)$$

- OPE modul- \mathbb{Q} -exact is S -indepent (but care about order)
- ⇒ Non-commutative, Assoc Algebra (G/EVALUATION)
- $g \rightarrow 0$ limit \Rightarrow chiral ring

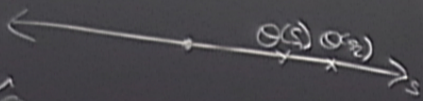
HOPE ASSOCIATIVITY WILL FIX Structure constants



CLAIM (PHYSICS)

CAN FIND NILPOTENT $\mathbb{Q}_S = Q + \beta S$ S.T.

- \mathbb{Q} -closed (not exact) local operators are twisted versions of BPS ops.



$$\hat{Q}(S) = Q^{+-+}(s) + \beta g Q^{+-}(s) + \dots + \beta^2 g^2 Q^{--}(s)$$

- OPE modul- \mathbb{Q} -exact is S-independent (but care about ordering)
- ⇒ Non-commutative, Assoc Algebra (G/EVALUATION)
- $\beta \rightarrow 0$ limit \Rightarrow chiral ring

HOPE ASSOCIATIVITY will fix structure constants

Assoc Alg problem

DEF QUANTIZATION OF HK cone

GOOD: CLASSIFICATION OF DEF'S $(H^2(M), \mathbb{C})$

BAD: WRONG CLASSIFICATION

ALGEBRA PROBLEM

M HYPERBOLIC CONE

A COORDINATE RING

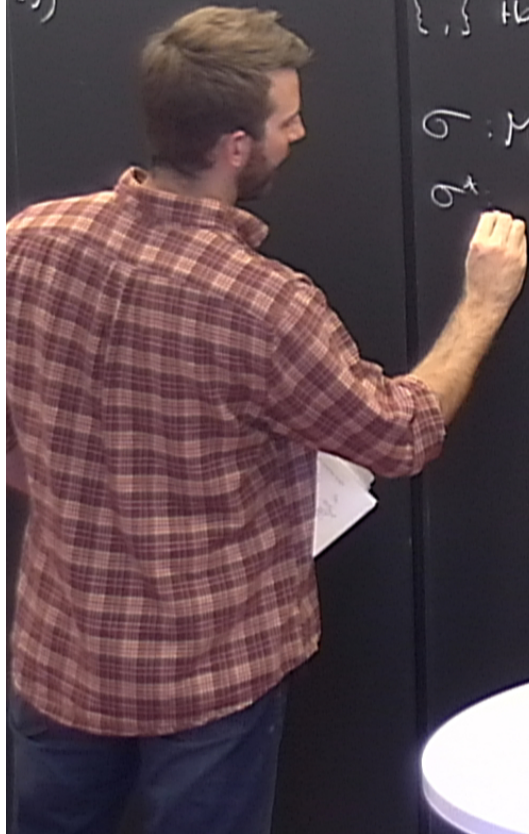
$\{, \}$ Holo. POISSON BRACKET
(deg -1)

$\sigma : M \rightarrow M$ (ROTATIONS BY π IN
 $SU(2)$)

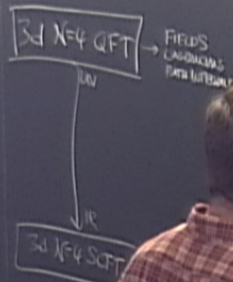
eg:

$\mathbb{C}^2/\mathbb{Z}_2$	$\mathbb{C}^2/\mathbb{Z}_3$
$(\mathbb{C}[X,Y,Z])/XY=Z^2$	$(\mathbb{C}[X,Y,Z])/XY=Z^3$
$\{X,Y\}=Z$	$\{X,Y\}=3Z^2$
$\{Z,X\}=X$	\longrightarrow "
$\{Z,Y\}=-Y$	\longrightarrow "

c)



PROCEEDINGS FOR DE QUANTIZATION
OF HK CONES FROM SCFT



\mathbb{C}^* -equivariant $*$ -product on A

$$f_{d_1} \star g_{d_2} = (fg)_{d_1+d_2} + \frac{\hbar}{2} \{f, g\}_{d_1+d_2} + \dots + (\hbar)^2 T_{d_1+d_2}$$

(ASSOCIATIVE)

classified up to $S_d \rightarrow S_d + \hbar S_{d-1} + \dots$

CONDITIONS

(1)

BPS operators

"HIGGS" $\Delta = r_1, r_2 = j = 0$
 "COULOMB" $\Delta = r_2, r_1 = j = 0$

FORM COMMUTATIVE (LW)
 - short distance OPE
 - OPE modulo \mathbb{Q}

\mathbb{R} -symmetry h.c.s
 annihilated by half supercharges

CHIRAL RING \Leftrightarrow COORDINATE RING ON A HK CONE

$(\Delta, r) \Leftrightarrow \mathbb{C}^*$ grading

$SU(2)_R \Leftrightarrow SU(2)$ ISOMETRY

$$S_i S_j = C_{ij}^k S_k \Rightarrow C_{ij}^k = C_{ji}^k \quad |k| = |i| + |j|$$

QUESTION

$$C_{ijk} \quad ||i|-|j|| \leq |k| \leq |i|+|j|$$

PREDICTIONS FOR DE QUANTIZATION OF HK CONES FROM SCFT

3d N=4 QFT → FIELDS
LAGRANGIAN
FOR LOCALITY

UV

3d N=4 SCFT → ALGEBRAIC

\mathbb{C}^* -equivariant $*$ -product on A

$$F_{d_1} \star F_{d_2} = (F_{d_1})_{d_1+d_2} + \frac{\hbar}{2} \{F_{d_1}, F_{d_2}\}_{d_1+d_2} + \dots + (\hbar^2) T_{d_1+d_2}^2$$

(ASSOCIATIVE)

classified up to $F_{d_1} \mapsto F_{d_1} + \hbar F_{d_1-1} + \dots$

CONDITIONS

(1) EVENNESS

form $\frac{\hbar}{2} \{F, F\}$ symmetric anti-sym.

BPS operators

"HIGGS" $\Delta = r_1, r_2 = j = 0$
 "COULOMB" $\Delta = r_2, r_1 = j = 0$

FORM COMMUTATIVE RING
 - short distance OPE
 - OPE modulo \mathbb{Q}

R-symmetry h.c.s
 annihilated by half supercharges

CHIRAL RING \Leftrightarrow COORDINATE RING ON A HK CONE

$(\Delta, r) \Leftrightarrow \mathbb{C}^*$ grading

$SU(2)_R \Leftrightarrow$ SU(2) ISOMETRY

$F_i F_j = C_{ij}^k F_k \Rightarrow C_{ij}^k = C_{ji}^k \quad |k| = |i| + |j|$

QUESTION

$C_{ijk} \quad ||i|-|j|| \leq |k| \leq |i|+|j|$

PREDICTIONS FOR DE QUANTIZATION OF HK COMES FROM SCFT

3d N=4 QFT → FIELDS CLASSICAL PER TITANS

UV

IR

3d N=4 SCFT

classical → $U(1) \times SU(2) \times SU(2)$

supersymmetry $Su(2)_R$
 $SO(4) \times SO(2)$
 $SO(2) \times SO(2)$

\mathbb{C} -equivariant \star -product on A

$$\mathcal{F}_{d_1} \star \mathcal{F}_{d_2} = (\mathcal{F}_{d_1})_{d_1, d_2} + \frac{\hbar}{2} \{ \mathcal{F}_{d_1}, \mathcal{F}_{d_2} \}_{d_1, d_2} + \dots + (\hbar)^{d_1+d_2} \tau_c$$

(ASSOCIATIVE)

$$\text{to } \mathcal{F}_d \mapsto \mathcal{F}_d + \hbar \mathcal{F}_{d-1} + \dots$$

form $\frac{\hbar}{\hbar}$ odd symmetric } contains $[P] \in H^2(\mathbb{R}^4, \mathbb{C})$
 $\frac{\hbar}{\hbar}$ even antisym

kept (usual) (truncation)

C.T. $[P] \in H^2(\mathbb{R}^4, \mathbb{C})$

M : MINIMAL MULTISCALE OF SHRE UFAISSING

- G -symmetric
- no changes of basis
- truncation automatic
- can check unitarity

CLAIM (PHYSICS)

CAN FIND NILPOTENT

• \square -closed

←

$$\hat{\mathcal{O}}(s) = \mathcal{O}(s) + \dots$$

- OPE moduli

⇒ Non-commutative

• $\hbar \rightarrow 0$ limit ⇒

HOPE ASSOCIATIVITY STRUC

PRECONDITIONS FOR DE QUANTIZATION OF HK CORES FROM SCFT

3d N=4 QFT → FIELDS, LAGRANGIANS, OPERATORS

3d N=4 SCFT

Supersymmetric Sym
 $U(1) \times SO(4) \cong U(1) \times SU(2) \times SU(2)$
 $SO(4) \cong SO(3) \times SO(3)$
 $SO(3) \cong SU(2)$

\mathbb{C} -equivariant $*$ -product on A

$$\mathbb{F}_{d_1} \star \mathbb{F}_{d_2} = (\mathbb{F}_g)_{d_1, d_2} + \frac{1}{2} \{ \mathbb{F}_g \}_{d_1, d_2} + \dots + (\#) \mathbb{F}_c^{d_1, d_2}$$

(ASSOCIATIVE)

classified upto \mathbb{F}_g

CONDITIONS

(1) EVENNESS

(2)

contains $H^2(G, \mathbb{C})$

M : MINIMAL MULTIPLET CONTENT OF SIMPLE LIE ALGEBRA

→ G -symmetric

- no change of basis
- truncation automatic
- can check unitarity

$M: \mathbb{Z}^2 \rightarrow \mathbb{Z}$ (KLEINIAN SIG)

(EVENNESS + TRUNCATION) ⇒ finite dim'l space of M

UNITARITY

unique for $g \neq A_n$
 parity ⇒ unique for $g \in A_n$
 A_n family (h.m.s)

CLAIM (PHYSICS)

CAN FIND MULTIPLET

• \square -closed

$\hat{O}(s) = \hat{O}(s)^+$

- OPE moduli

⇒ Non-commutative

$\cdot \rightarrow 0$ limit ⇒

HOPE ASSOCIATIVITY STRUC

PROBLEMS FOR DE QUANTIZATION OF HK COMES FROM SCFT

3d N=4 QFT → FIELDS CLASSICAL PERS. EQUATIONS

3d N=4 SCFT → CLASSICAL ⇒ CSP

SO(4) → SO(3) × SO(3)
SO(2) × SO(2)

\mathbb{C} -equivariant \star -product on A

$$\mathcal{F}_{d_1} \star_{\hbar} \mathcal{F}_{d_2} = (\mathcal{F}_{d_1} \mathcal{F}_{d_2} + \frac{\hbar}{2} \{ \mathcal{F}_{d_1}, \mathcal{F}_{d_2} \} + \dots + (\frac{\hbar}{2})^{d_1+d_2} T_2)$$

(ASSOCIATIVE)

classified upto $\mathcal{F}_{d_1} \mapsto \mathcal{F}_{d_1} + \hbar \mathcal{F}_{d_1-1} + \dots$

CONDITIONS

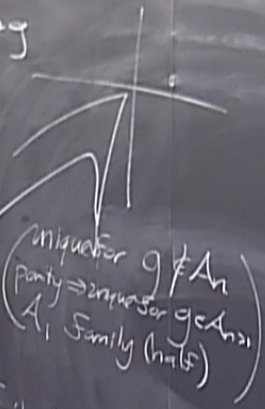
UNITARINESS

form $\frac{h}{\hbar}$ odd } contains $[P] \in H^2(G, \mathbb{C})$
 symmetric }
 anti-sym }
 (truncation)

$$C.T. [P] \times S > 0$$

M: MINIMAL MULTIPLET OF SIMPLE LIE ALGEBRA

- G-symmetric
- no change of basis
- truncation automatic
- can check unitarity



M: $\mathbb{Z}/2$ (KLEINIAN SIG)

unique for $g \neq A_n$
 parity \Rightarrow unique for $g \neq A_n$
 A_1 family (hms)

(EVENTS + POINTS) \Rightarrow finite dim space $\mathbb{Z}/2$

UNITARITY \Rightarrow sumy subspaces

CLAIM (PHYSICS)

CAN FIND NILPOTENT

• \square -closed

$\hat{\mathcal{O}}(S) = \mathcal{O}(S)$

- OPE mod \Rightarrow Non-commutative

$\cdot \rightarrow 0$ limit \Rightarrow

HOPE ASSOCIATIVITY STRUC

\mathbb{C} -equivariant $*$ -product on A

$$f_{d_1} * g_{d_2} = (fg)_{d_1+d_2} + \frac{1}{2} \{f, g\}_{d_1+d_2} + \dots + \frac{1}{(d_1+d_2)!} \{f, g\}_{d_1+d_2}^{(d_1+d_2)}$$

(ASSOCIATIVE)

classified up to $S_{d_1} \rightarrow S_{d_1} + \hbar S_{d_1-1} + \dots$

CONDITIONS

(1) EVENNESS

$$A_{\pm}^{\text{op}} = A_{\pm}$$

(2) $A_p * A_q \subset A_{p+q}$

(3) UNITARITY

form $\frac{\hbar}{\hbar}$ odd symmetric
 anti-symmetric
 C.T. [PG]

M. MINIMAL MILPOTENTIATION OF SIMPLE LIE ALGEBRAS

$\rightarrow G$ -symmetric

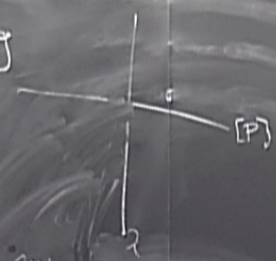
- no change of basis
- truncation automatic
- can check unitarity

M. \mathbb{C}^2/Γ (KLEINIAN SING)

(unique for $g \notin An$)
 parity \Rightarrow unique for $g \in An$
 A_1 Family (half)

(EVENNESS + TRUNCATION) \Rightarrow Finite dim'l space
 Δ/\hbar

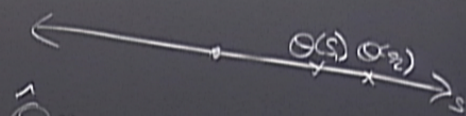
UNITARITY \Rightarrow Sum of subspaces



CLAIM (PHYSICS)

CAN FIND MILPOTIENT $\mathbb{Q}_\hbar = Q + \hbar S$ S.T.

- \mathbb{Q}_\hbar -closed (not exact) local quantities



$$\hat{Q}(s) = Q^+(s) + \hbar Q^+ Q^-(s) + \dots + S^2$$

- OPE modul- \mathbb{Q} -exact is S -independent
- \Rightarrow Non-commutative, Assoc ALGEBRA (Q/EV)
- $\hbar \rightarrow 0$ limit \Rightarrow chiral ring

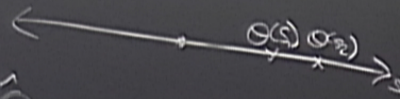
HOPE ASSOCIATIVITY will fix structure constants

Nilpotent orbit
 simple Lie algebra
 metric
 change of basis
 action automatic
 check unitarity
 (Kleinian group)
 (quaternion $g \in A_n$)
 parity \Rightarrow spinors for $g \in A_n$
 A_1 family (hats)
 \Rightarrow finite dim space
 \Rightarrow sumy subspaces
 UNITARITY \Rightarrow sumy subspaces

CLAIM (PHYSICS)

CAN FIND NILPOTENT $\mathbb{Q}_S = Q + gS$ S.T.

- \mathbb{Q} -closed (not exact) local quantities are twisted versions of BPS ops.



$$\hat{Q}(s) = \hat{Q}(s_1) + s g \hat{Q}(s_2) + \dots + s^{n-1} g^{n-1} \hat{Q}(s_n)$$

- OPE moduli \mathbb{Q} -exact is S -independent (but care about ordering)
- \Rightarrow Non-commutative, Assoc Algebra (G/EVALUATION)
- $g \rightarrow 0$ limit \Rightarrow chiral ring

HOPE ASSOCIATIVITY will fix structure constants

Assoc Alg problem

DEF. QUANTIZATION OF HK CONE

GOOD. CLASSIFICATION OF DEF'S $(H^2(X; \mathbb{C}))$

BAD. WRONG CLASSIFICATION

GOOD. EXTRA CONDITIONS

