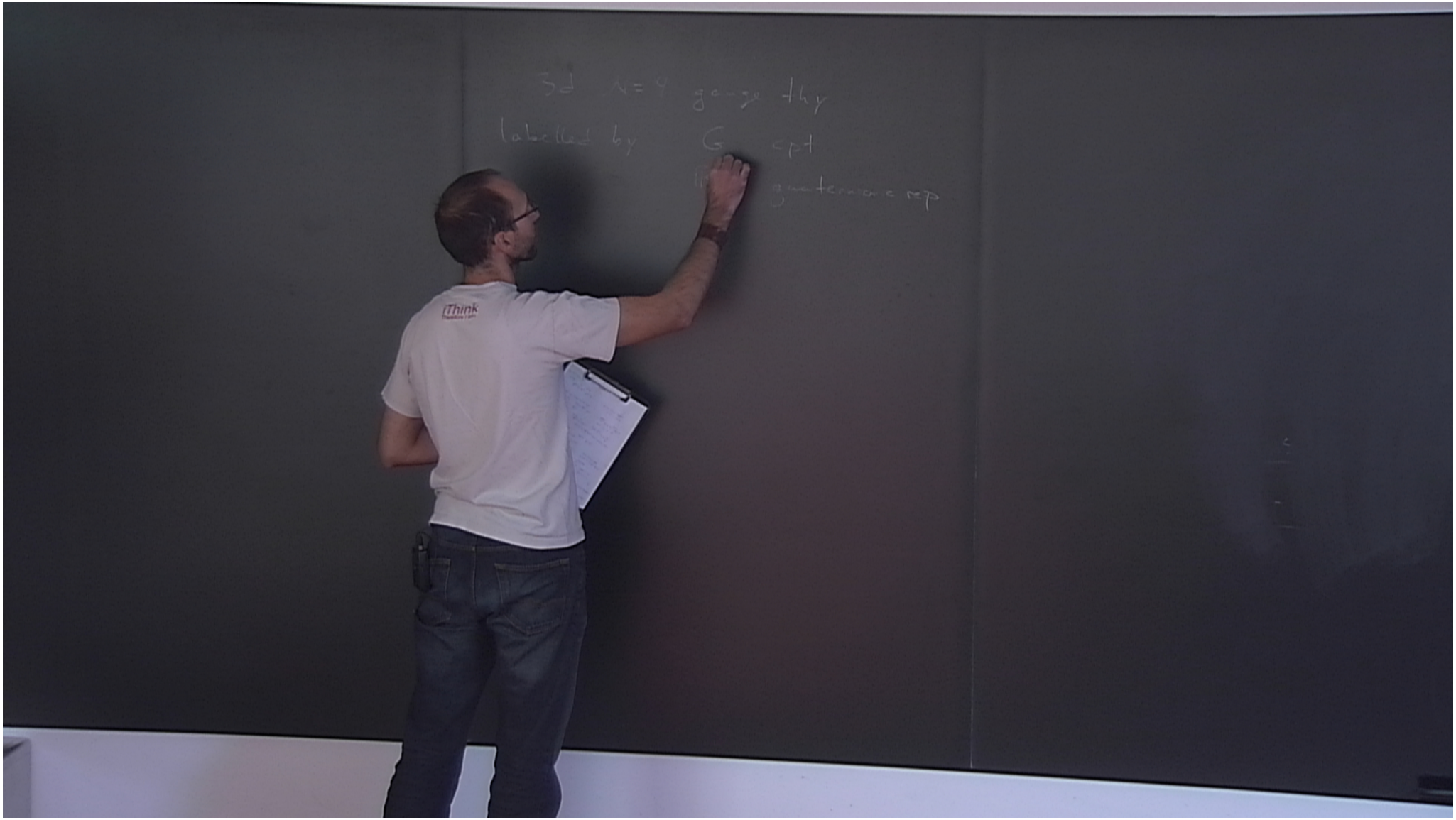
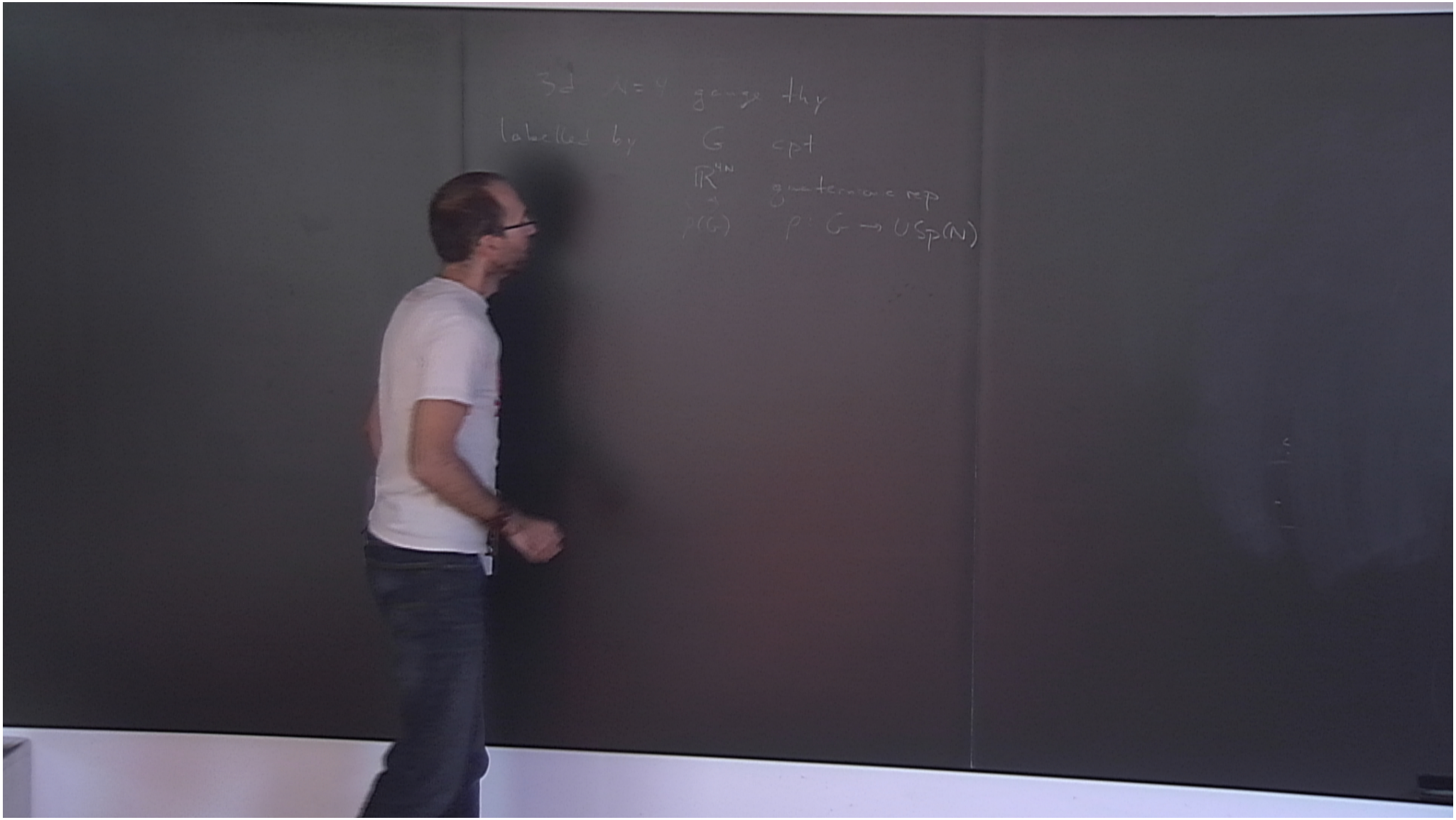
Title: Boundaries and D-modules in 3d N=4 theories

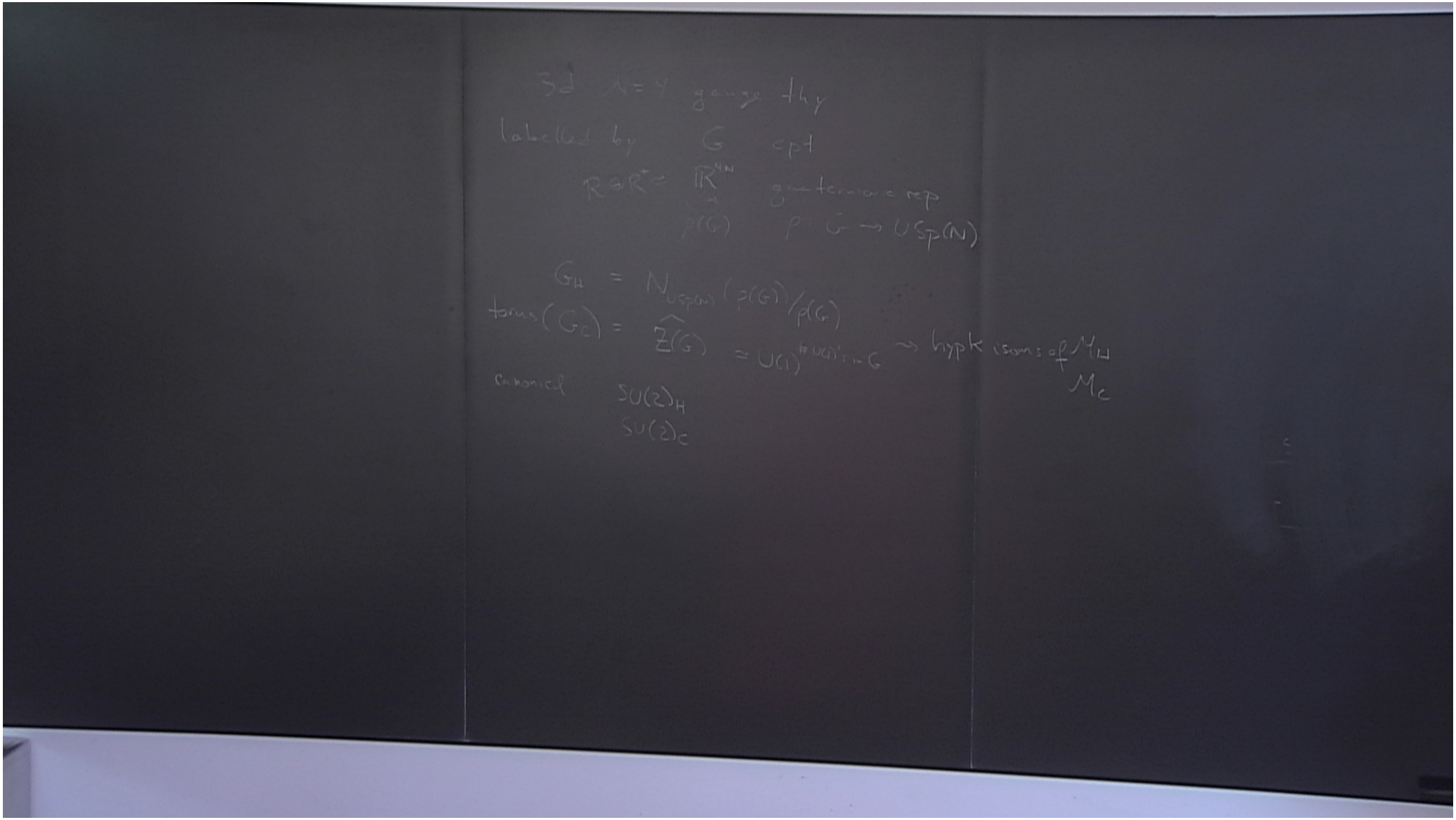
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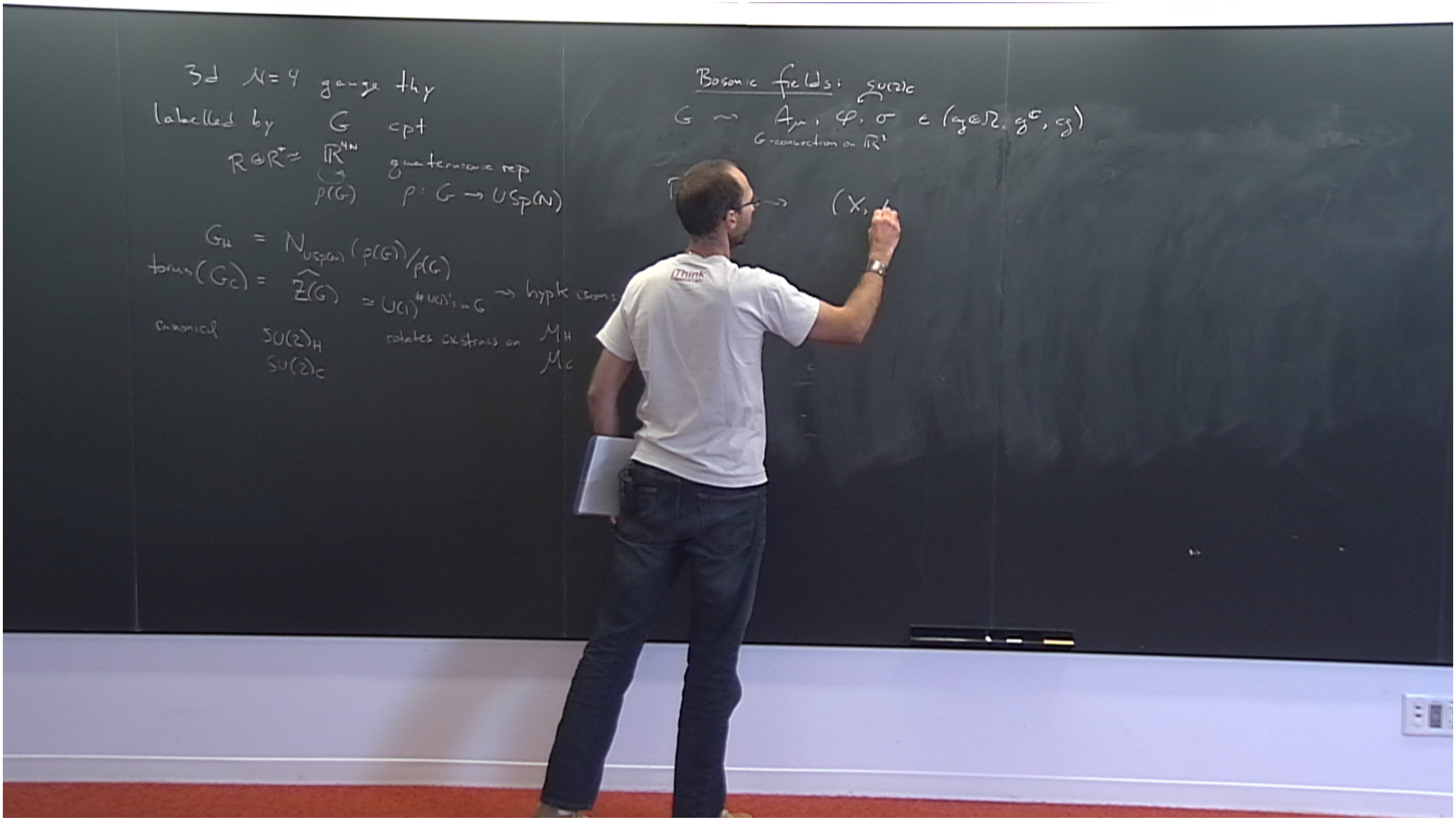
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Abstract: After a quick review of the Higgs and Coulomb branches of 3d N=4 theories, I'll introduce some simple classes of boundary conditions and explain how they lead to (pairs of) modules for certain (pairs of) quantum algebras. I will focus on abelian theories, for which the relevant boundary conditions/modules can be described using the geometry of (pairs of) hyperplane arrangements. From this, the simplest examples of symplectic-dual modules will arise.




3d N=4 gauge thy
labelled by G cpt
$\mathbb{R}^{4N}$
$\rho(G)$ $\rho: G \to USp(N)$


3d N=4 gauge thy
labelled by G cpt
R⊕R̄ = ℝ^{4N} symplectic rep
ρ(G) ρ G → USp(N)
G_H = N_{USp(N)}(ρ(G))/ρ(G)
torus(G_C) = Ẑ(G) = U(1)^{# U(1)'s in G}
→ hyptorus of M_H
M_C
canonical
SU(2)_H
SU(2)_C


3d N=4 gauge thy
labelled by G cpt
R⊕R* ≃ ℝ^{4N}
ρ(G)
quaternionic rep
ρ : G → USp(N)
Bosonic fields: SU(2)c
G ⇝ A_μ, φ, σ ∈ (g⊗ℝ, g^C, g)
G-connection on ℝ^3
(X,
G_H = N_{USp(N)}(ρ(G))/ρ(G)
torus(G_C) = Ẑ(G) = U(1)^{#U(1)'s in G}
hypk
canonical
SU(2)_H
SU(2)_C
rotates cx structs on
M_H
M_C
Think

3d $\mathcal{N}=4$ gauge thy

labelled by G cpt

$R \oplus R^* = \mathbb{R}^{4N}$ quaternionic rep

$\rho(G)$ $\rho: G \to USp(N)$

$G_H = N_{USp(N)}(\rho(G))/\rho(G)$

$\text{torus}(G_C) = \widehat{Z(G)} = U(1)^{\#U(1)'s \text{ in } G}$ $\rightsquigarrow$ hyptl isoms of $\mathcal{M}_H$, $\mathcal{M}_C$

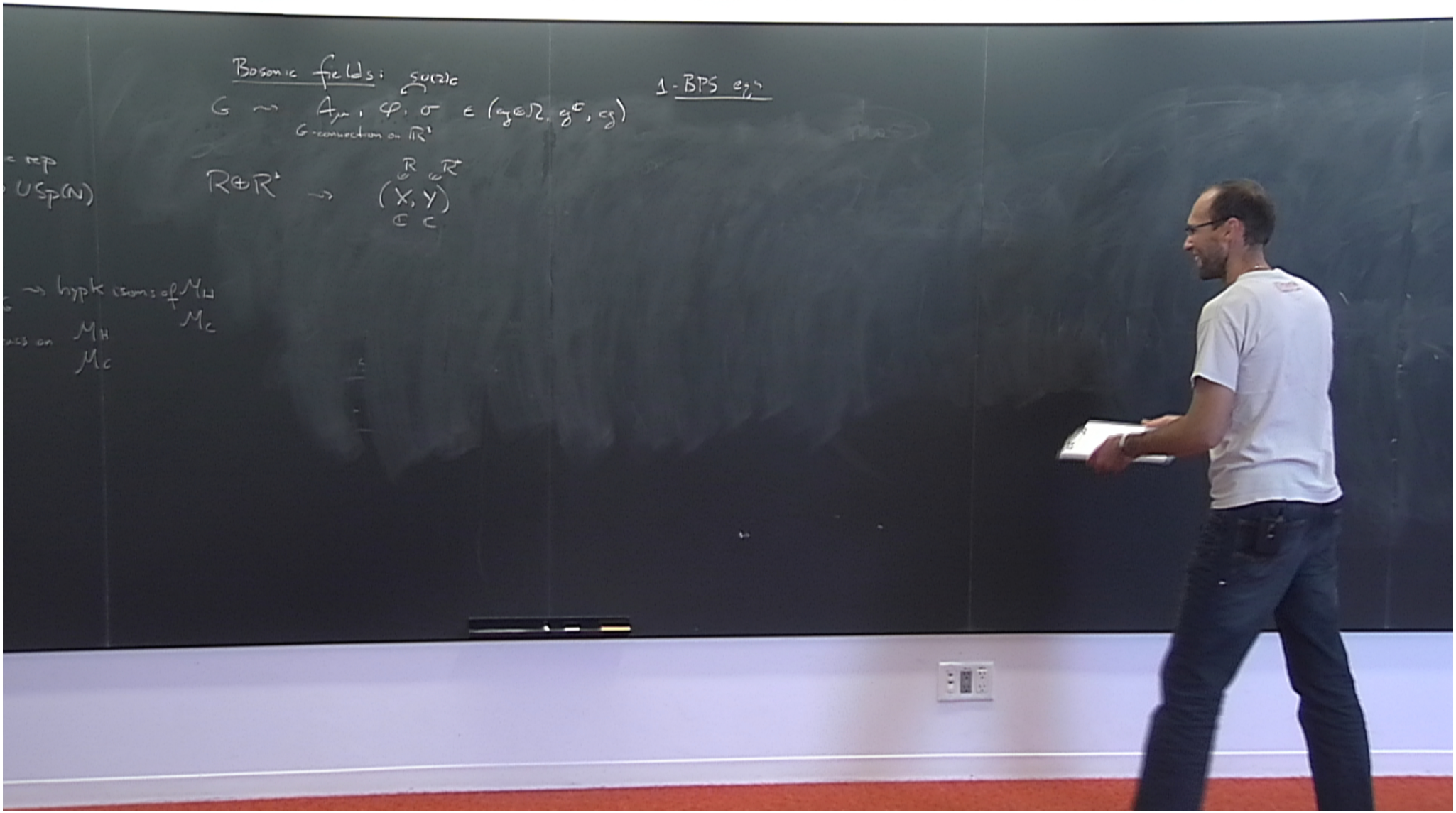
canonical $SU(2)_H$, $SU(2)_C$ rotates cx structs on $\mathcal{M}_H$, $\mathcal{M}_C$

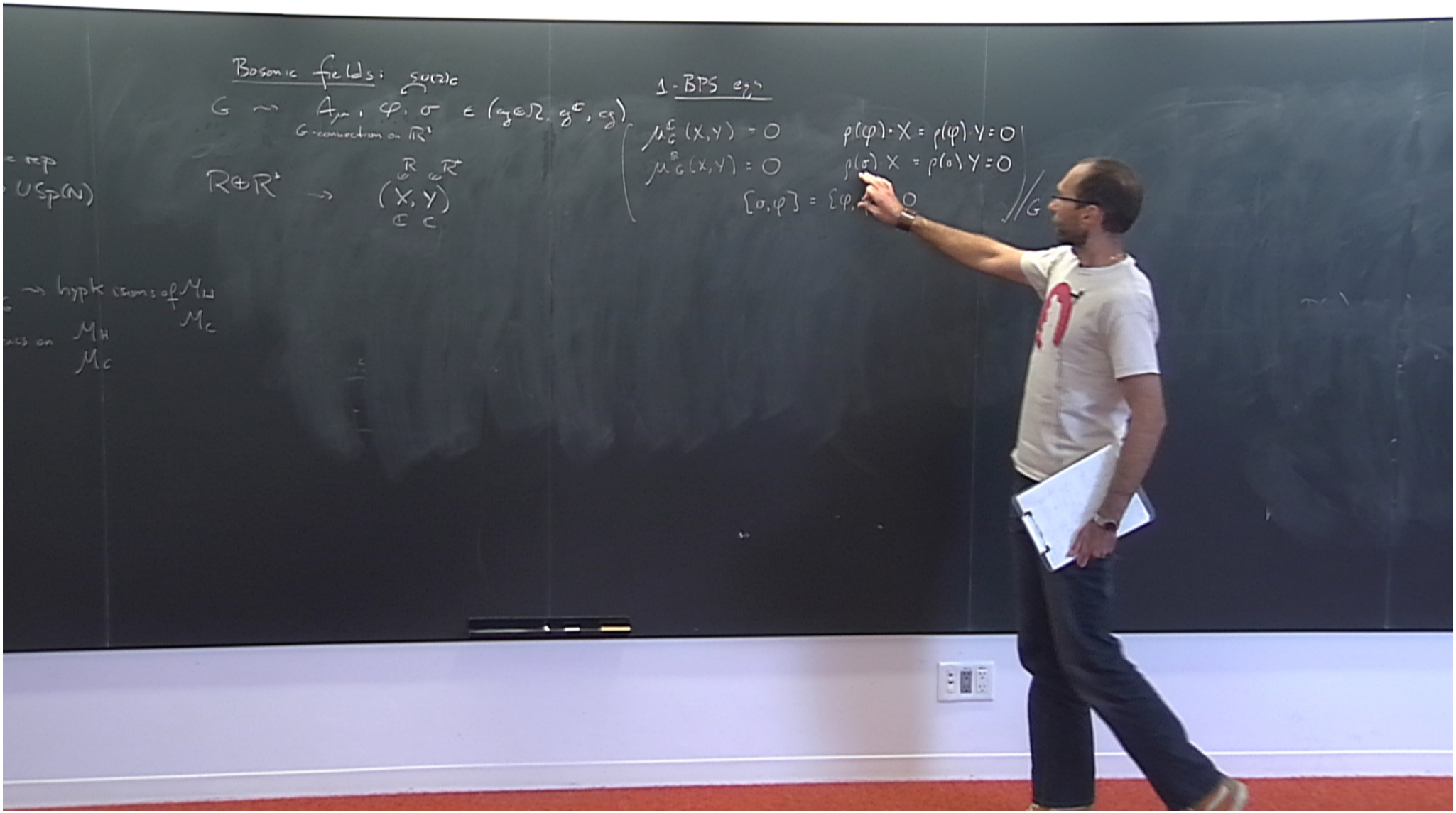
Bosonic fields: $SU(2)_C$

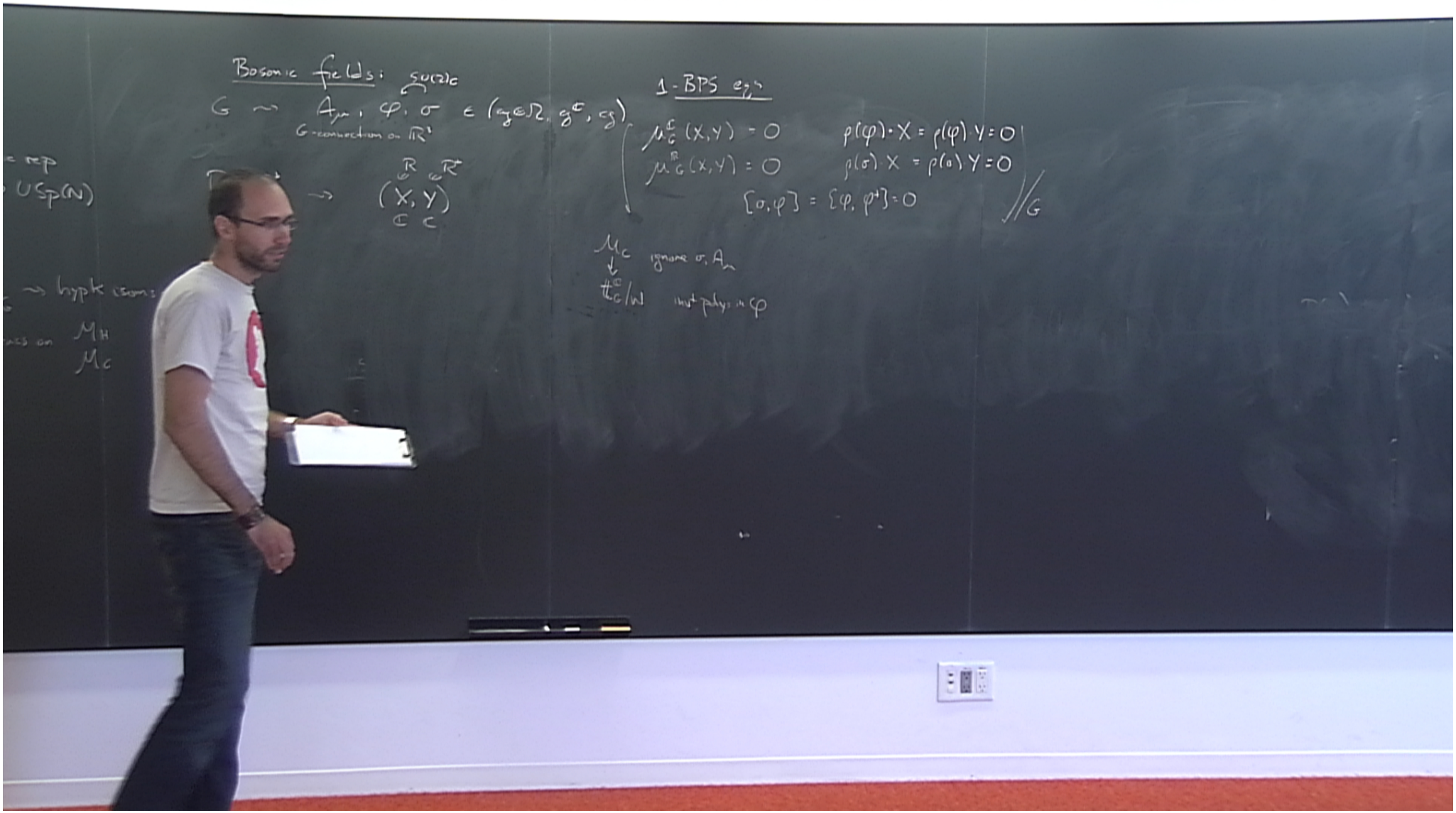
$G \rightsquigarrow A_\mu, \varphi, \sigma \in (\mathfrak{g} \otimes \mathbb{R}, \mathfrak{g}^{\mathbb{C}}, \mathfrak{g})$

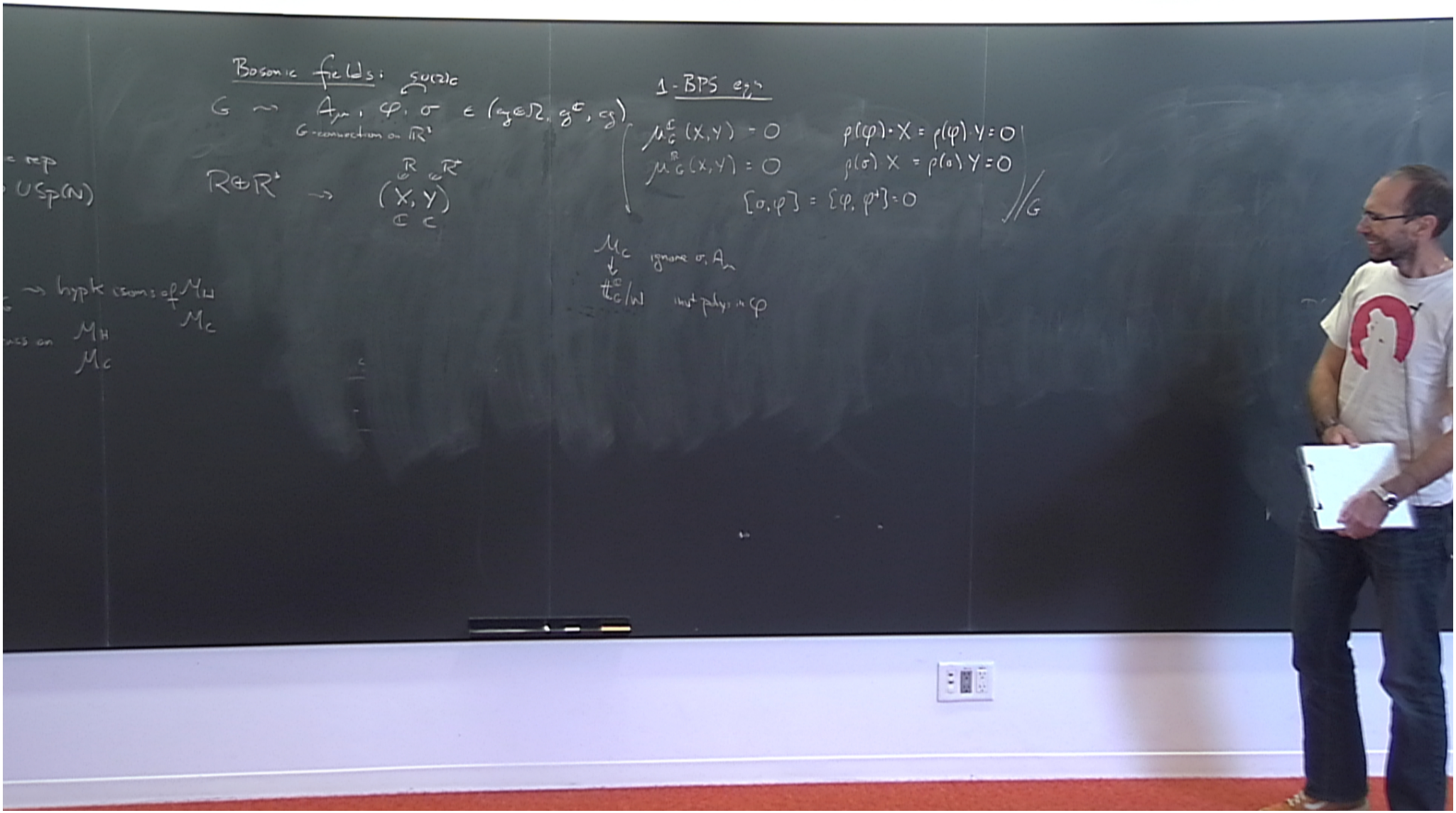
G-connection on $\mathbb{R}^3$

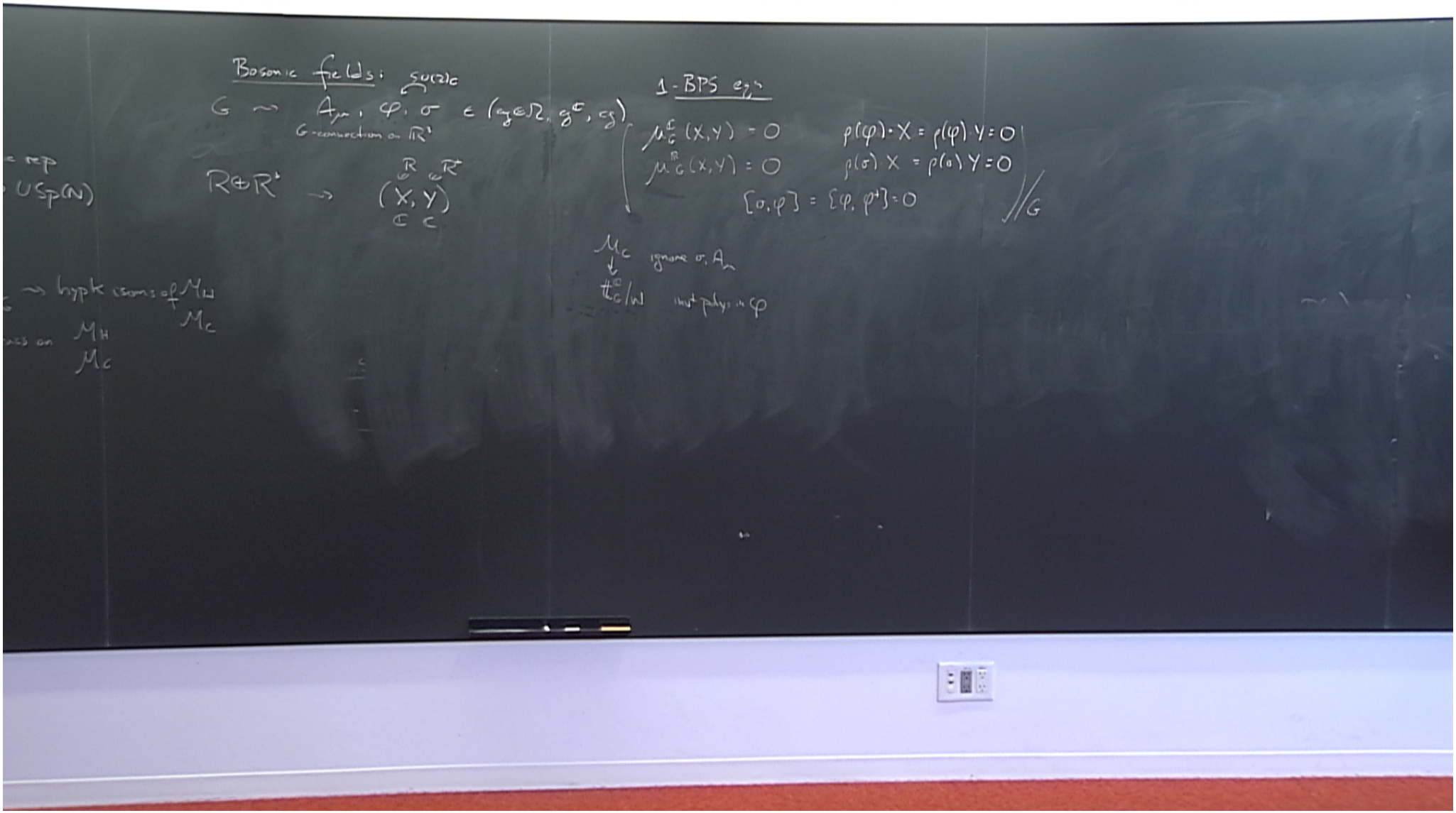
$R \oplus R^* \to (X, Y)$, $X \in R$, $Y \in R^*$

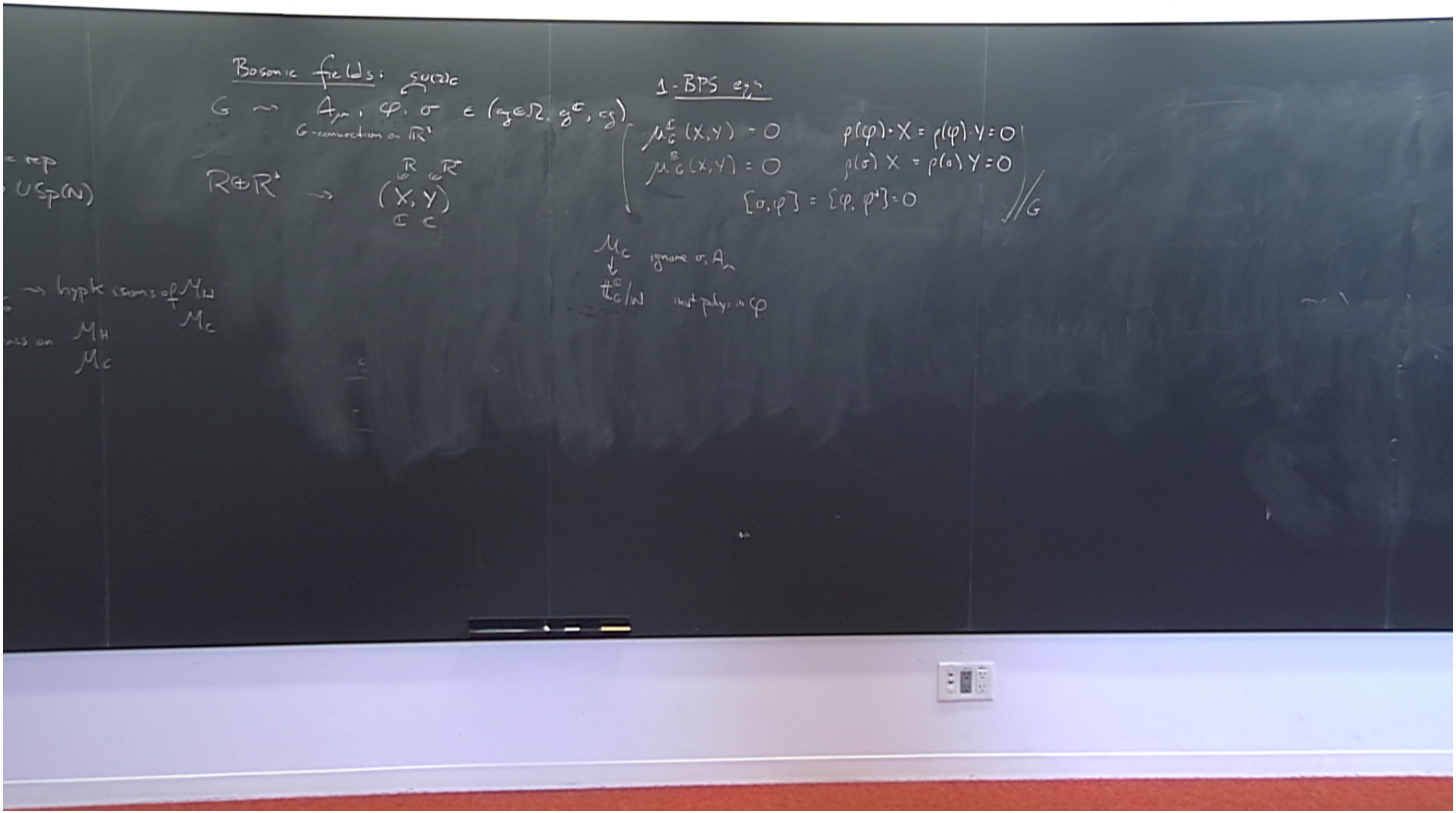

Bosonic fields:
SU(2)c
G ~ A_μ, φ, σ ∈ (g⊕R, g^C, g)
G-connection on R^3
1-BPS eq^n
R⊕R^3 → (X, Y)
R R
C C
USp(N)
~ hyptic susm: of M_H
M_C
M_H
M_C

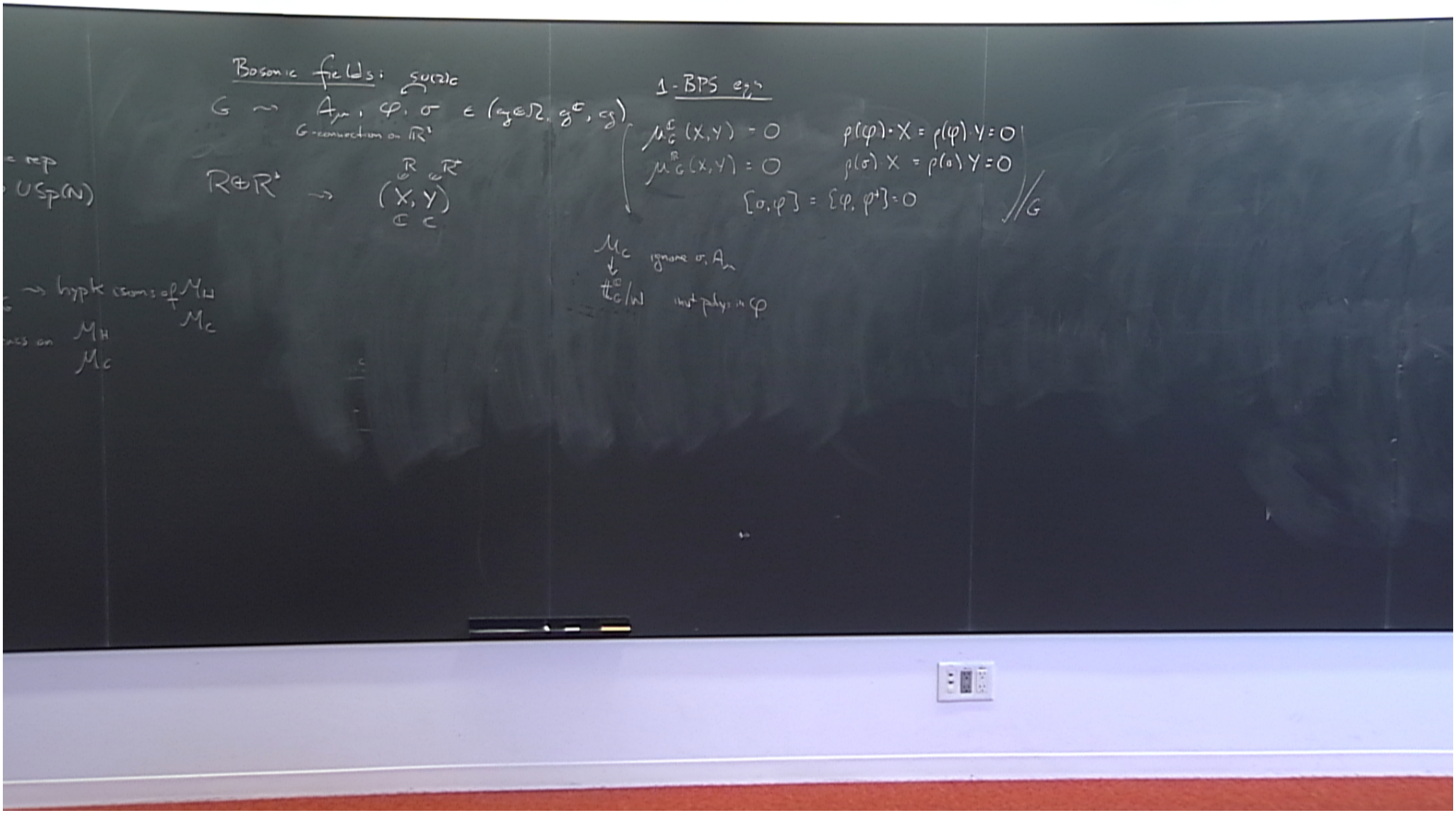

Bosonic fields:
SU(2)c
G ~ A_μ, φ, σ ∈ (g⊗R, g^c, g)
G-connection on R^3
R⊕R^3 → (X, Y)
1-BPS eqn
μ^c_G(X,Y) = 0
μ^R_G(X,Y) = 0
ρ(φ)·X = ρ(φ) Y = 0
ρ(σ) X = ρ(σ) Y = 0
[σ,φ] = [φ,
0
//G
hypk
M_H
M_C

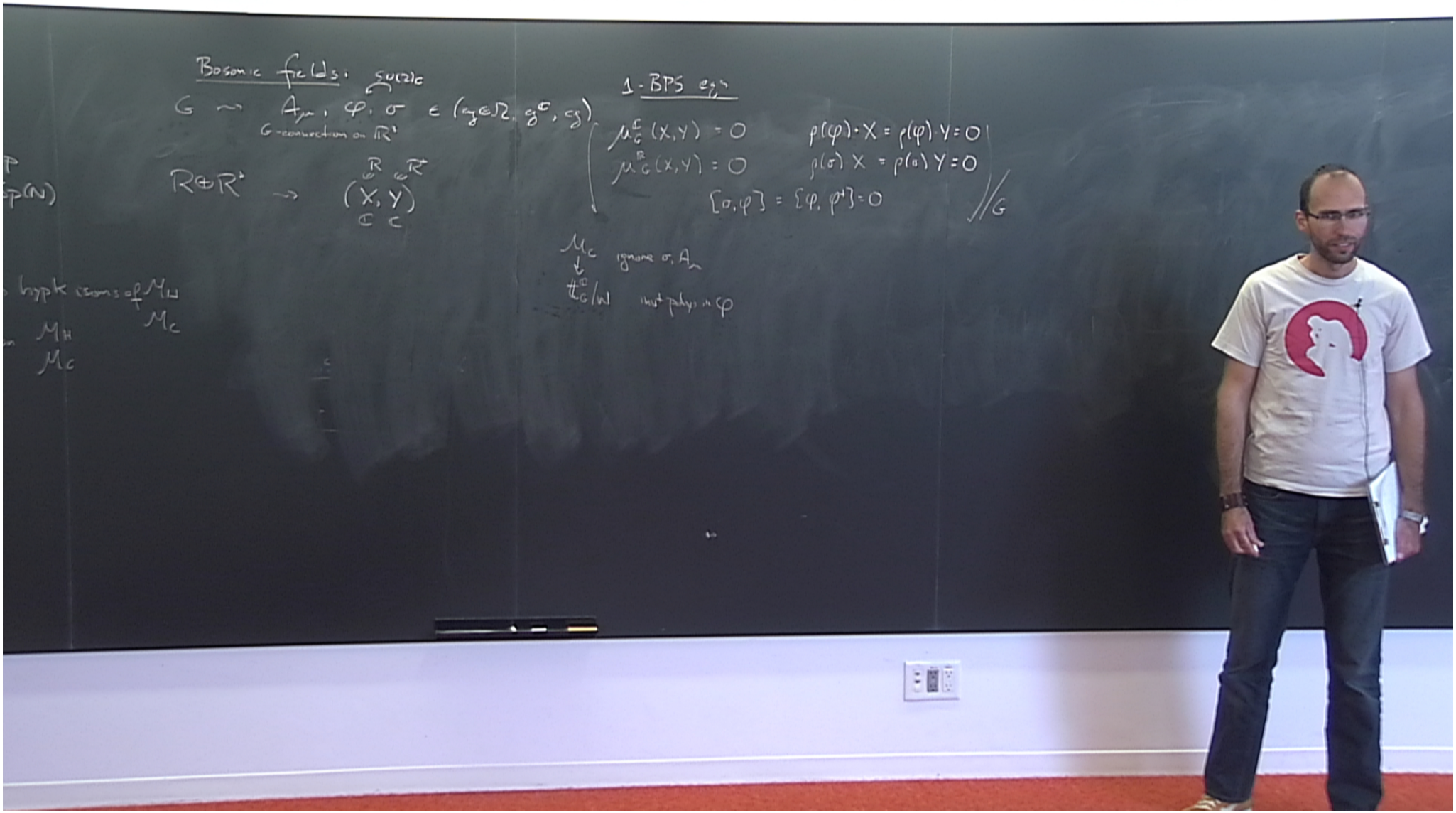

Bosonic fields:
SU(2)c
G ~ A_μ, φ, σ ∈ (g⊗Ω, g^c, g)
G-connection on R^3
R R
(X, Y)
C C
1-BPS eq's
μ^C_G(X,Y) = 0
μ^R_G(X,Y) = 0
ρ(φ)·X = ρ(φ) Y = 0
ρ(σ) X = ρ(σ) Y = 0
[σ,φ] = [φ, φ†] = 0
//G
M_C
ignore σ, A_μ
t^W_G/W
inv. poly's in φ
USp(N)
hypk
M_H
M_C

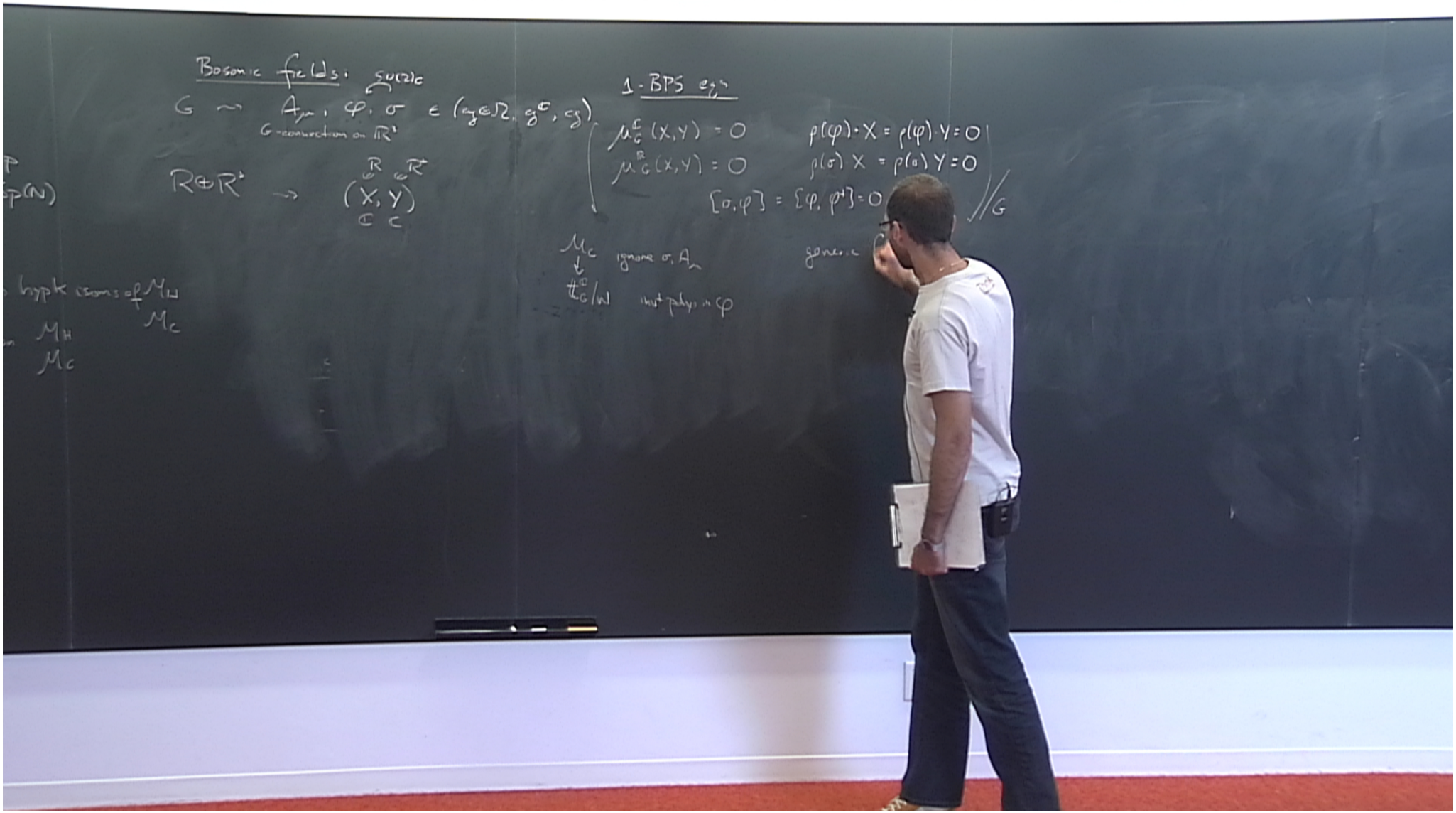

Bosonic fields:
SU(2)c
G ~ A_μ, φ, σ ∈ (g⊗R, g^C, g)
G-connection on R^3
R⊕R^3 → (X, Y)
1-BPS eq^n
μ^C_G(X,Y) = 0
μ^R_G(X,Y) = 0
ρ(φ)·X = ρ(φ)Y = 0
ρ(σ)X = ρ(σ)Y = 0
[σ,φ] = [φ, φ†] = 0
//G
M_C
ignore σ, A_μ
t^W_G/W
inv. poly. in φ
USp(N)
hyptic cons. of M_H
M_C
M_H
M_C

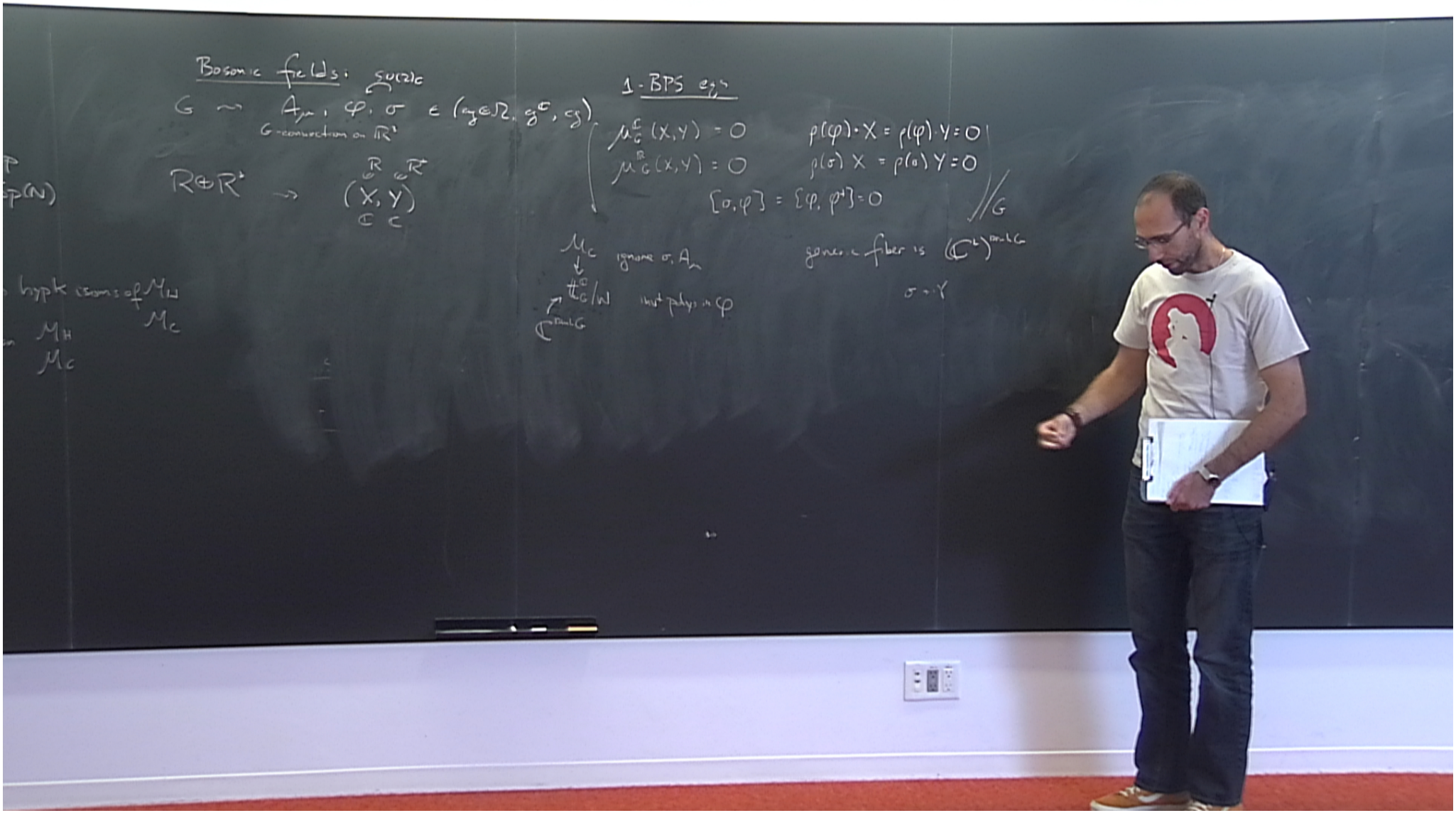

Bosonic fields:
SU(2)c
G ~ A_μ, φ, σ ∈ (g⊗R, g^c, g)
G-connection on R^3
R⊕R^3 → (X, Y)
R R
C C
1-BPS eq^n
μ_C^R(X,Y) = 0
μ_G^R(X,Y) = 0
ρ(φ)·X = ρ(φ) Y = 0
ρ(σ) X = ρ(σ) Y = 0
[σ,φ] = [φ, φ^+] = 0
//G
M_C
ignore σ, A_μ
t_C^∨/W
inv. poly's in φ
USp(N)
→ hyptn'l coords of M_H
M_C
M_H
M_C

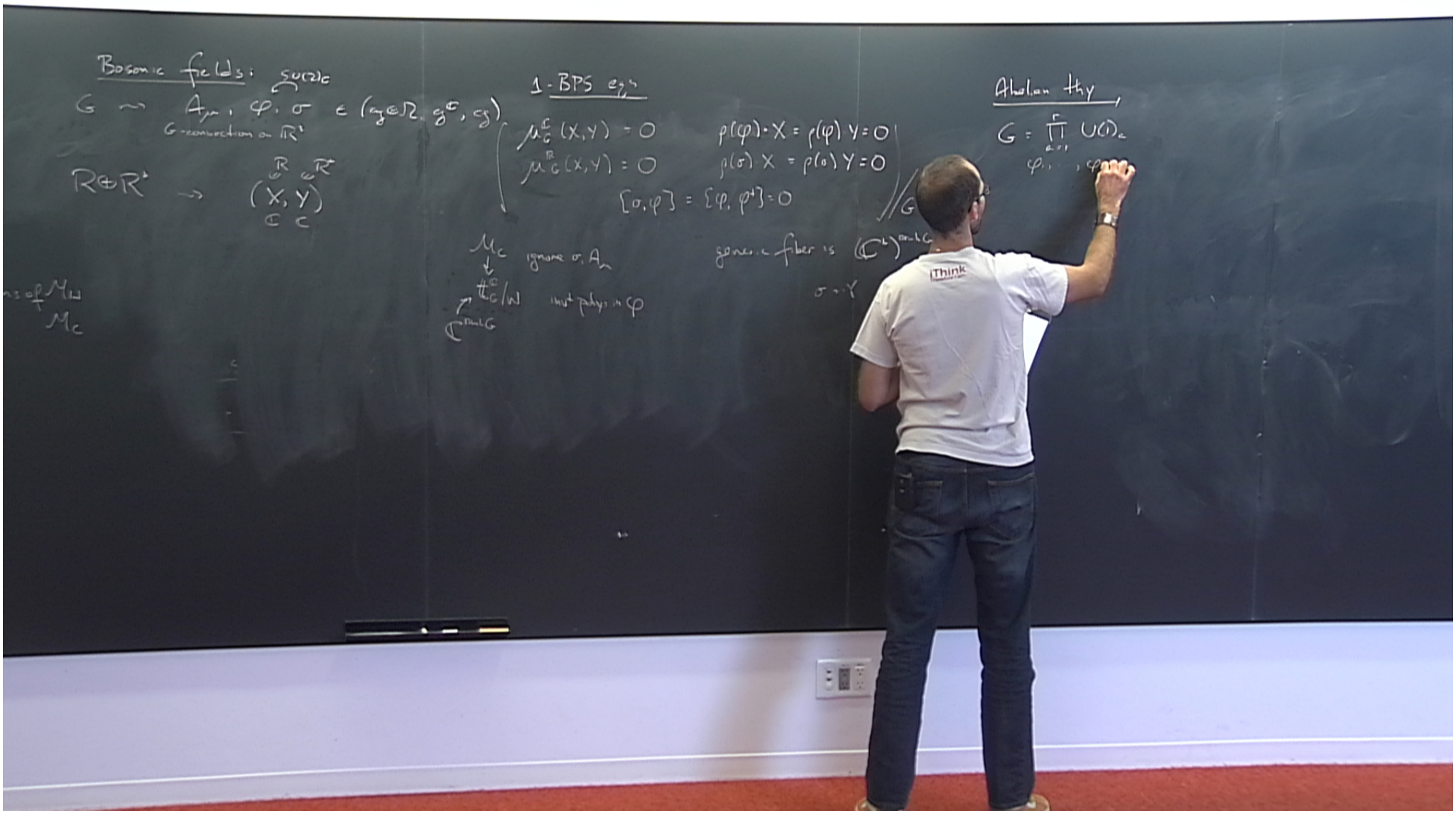

Bosonic fields: SU(2)C
G ~ A_μ, φ, σ ∈ (g⊗R, g^C, g)
G-connection on R^3
R⊕R^3 → (X, Y)
1-BPS eq^n
μ_C^R(X,Y) = 0
μ_G^R(X,Y) = 0
ρ(φ)·X = ρ(φ)Y = 0
ρ(σ)X = ρ(σ)Y = 0
[σ,φ] = [φ, φ^+] = 0
//G
M_C
ignore σ, A_μ
t_C^0/W
inst poly in φ
Usp(N)
→ hypk isom: of M_H
M_C
M_H
M_C

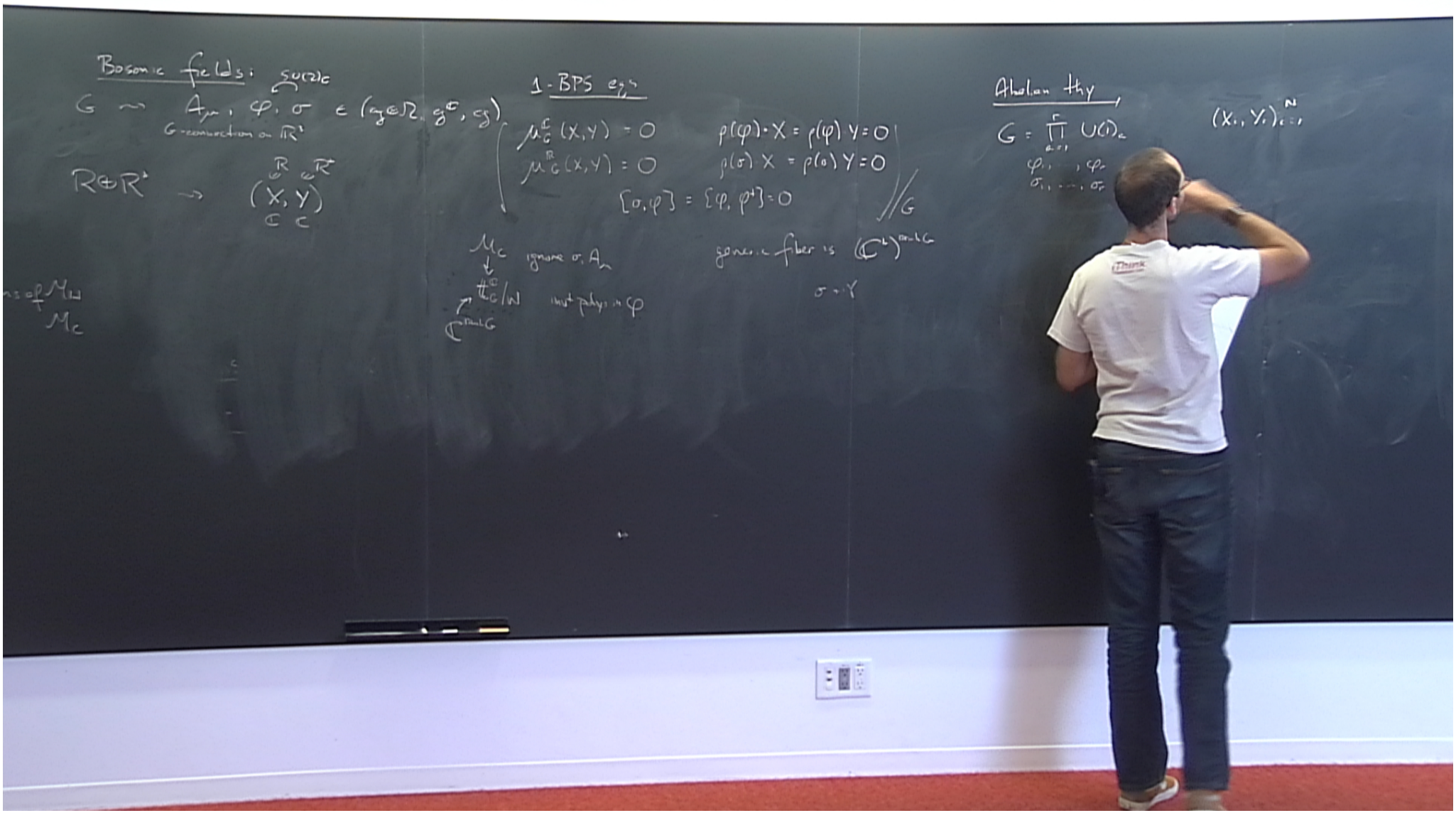

Bosonic fields:
SU(2)C
G ~ A_μ, φ, σ ∈ (g⊗R, g^C, g)
G-connection on R^3
R⊕R^3 → (X, Y)
1-BPS eq^n
μ_C^R(X,Y) = 0
μ_G^R(X,Y) = 0
ρ(φ)·X = ρ(φ)Y = 0
ρ(σ)X = ρ(σ)Y = 0
[σ,φ] = [φ, φ^+] = 0
//G
M_C
ignore σ, A_μ
t_C^C/W
inv poly's in φ
Usp(N)
→ hypk struct of M_H
M_C
M_H
M_C

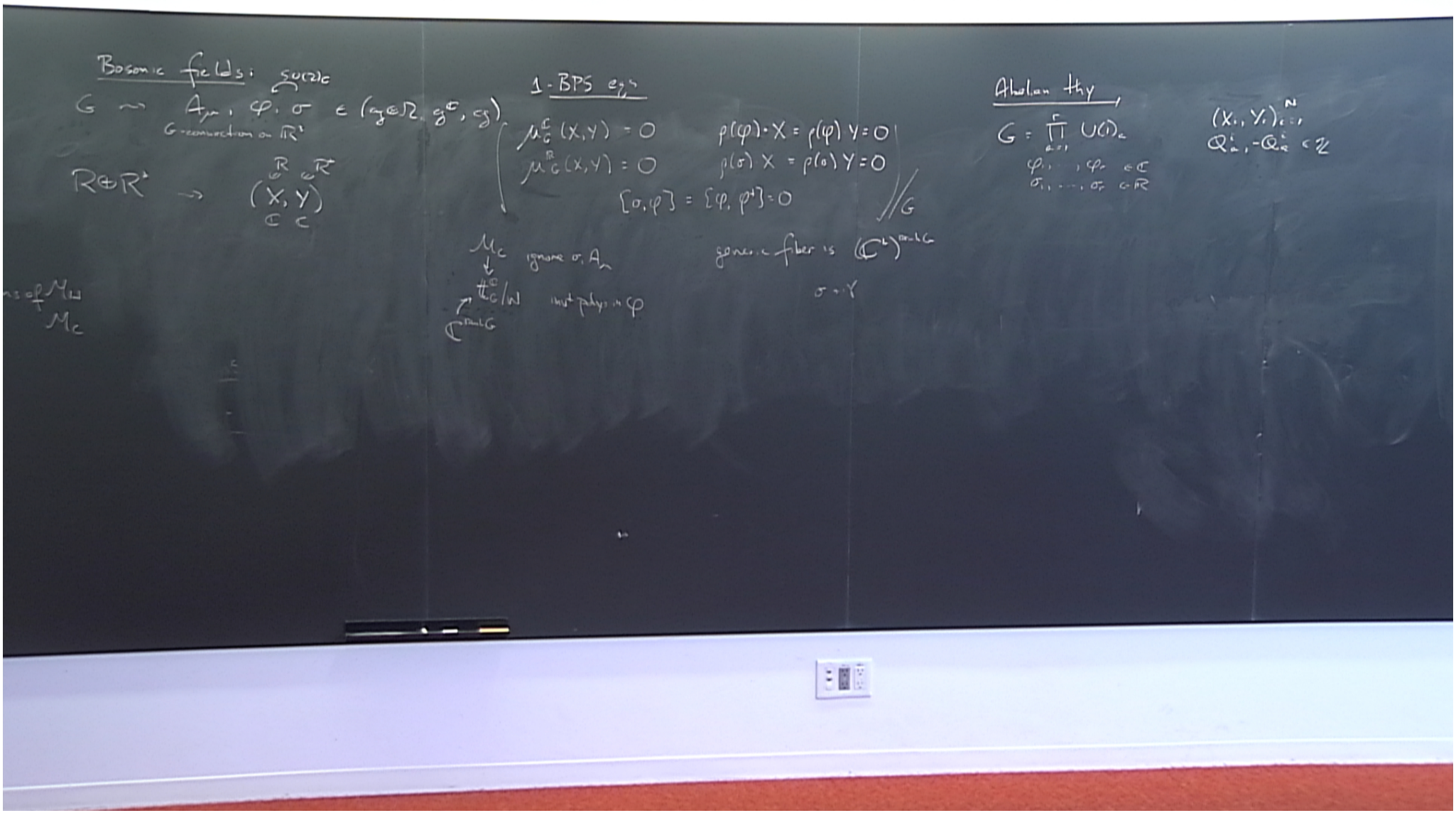

Bosonic fields: SU(2)C
G ~ A_μ, φ, σ ∈ (g∈Lie G, g^C, g)
G-connection on R^1
R⊕R^1 → (X, Y)
1-BPS eq^ns
μ_C^C(X,Y) = 0
μ_C^R(X,Y) = 0
ρ(φ)·X = ρ(φ)·Y = 0
ρ(σ) X = ρ(σ) Y = 0
[σ,φ] = [φ, φ^+] = 0
//G
M_C
ignore σ, A_μ
t_C^W/W
inv. poly's in φ
hypk isom's of M_H
M_H
M_C

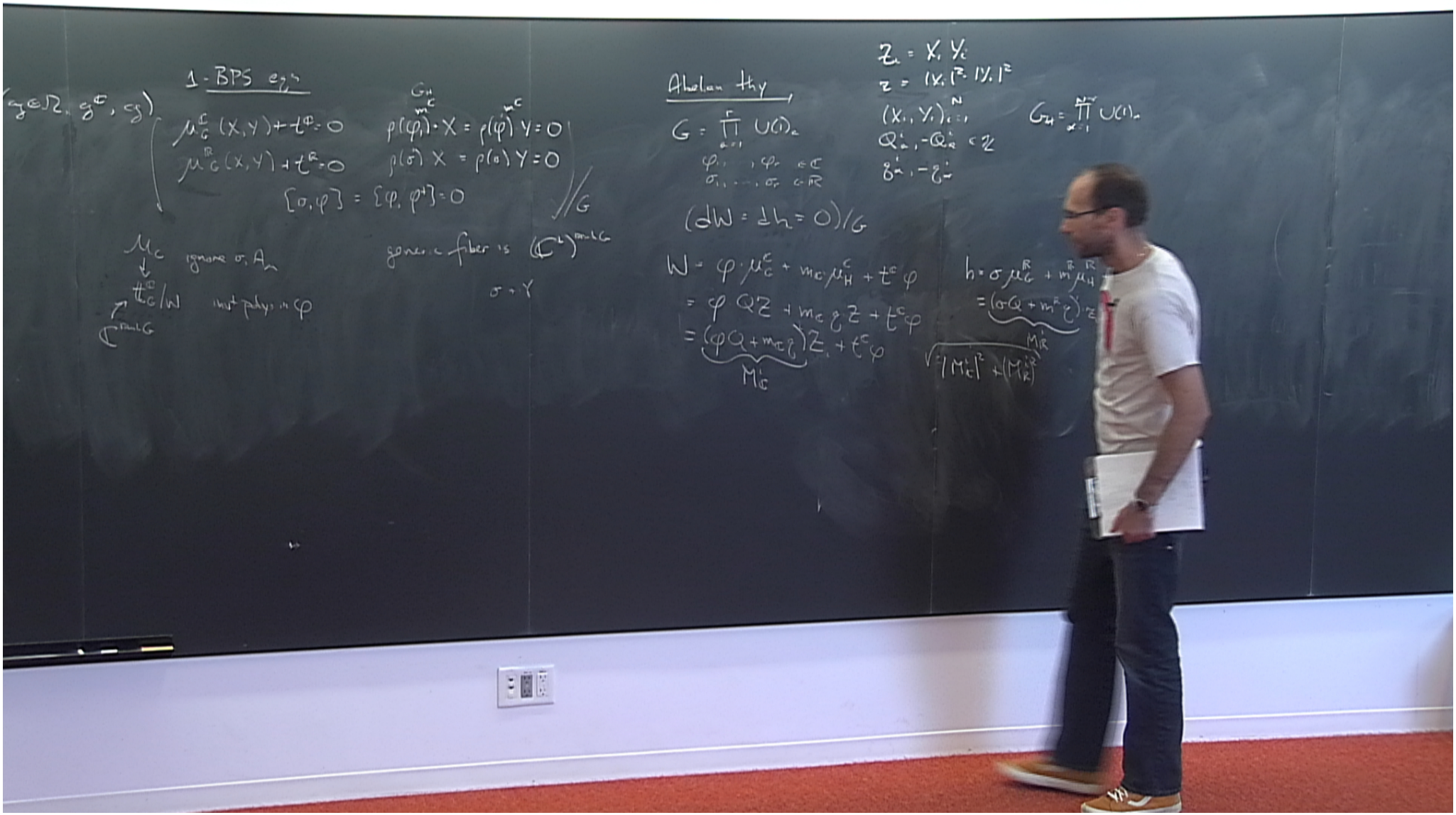

Bosonic fields: SU(2)c
G ~ A_μ, φ, σ ∈ (e_g ∈ R, g^a, c_g)
G-connection on R^1
R⊕R^1 → (X, Y)
1-BPS eq^n
μ_C^C(X,Y) = 0
μ_C^R(X,Y) = 0
ρ(φ)·X = ρ(φ)·Y = 0
ρ(σ)X = ρ(σ)Y = 0
[σ,φ] = [φ,φ^+] = 0
M_C
ignore σ, A_μ
t_C^W/W
inv. poly. in φ
hypk mfds of M_H
M_H
M_C
//G

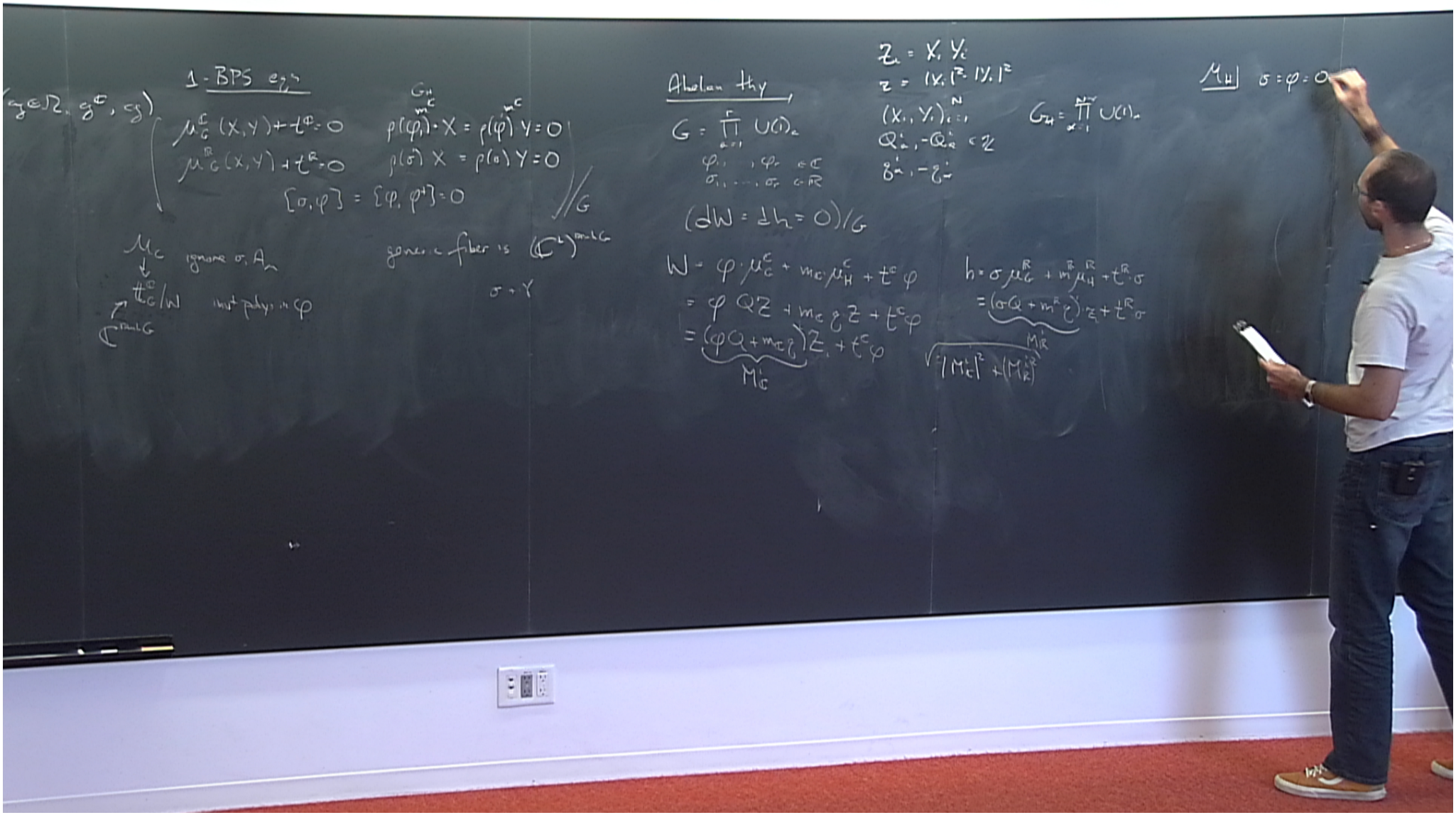

Bosonic fields:
1-BPS eqns
hypk
hypk
Pirsa: 15100066
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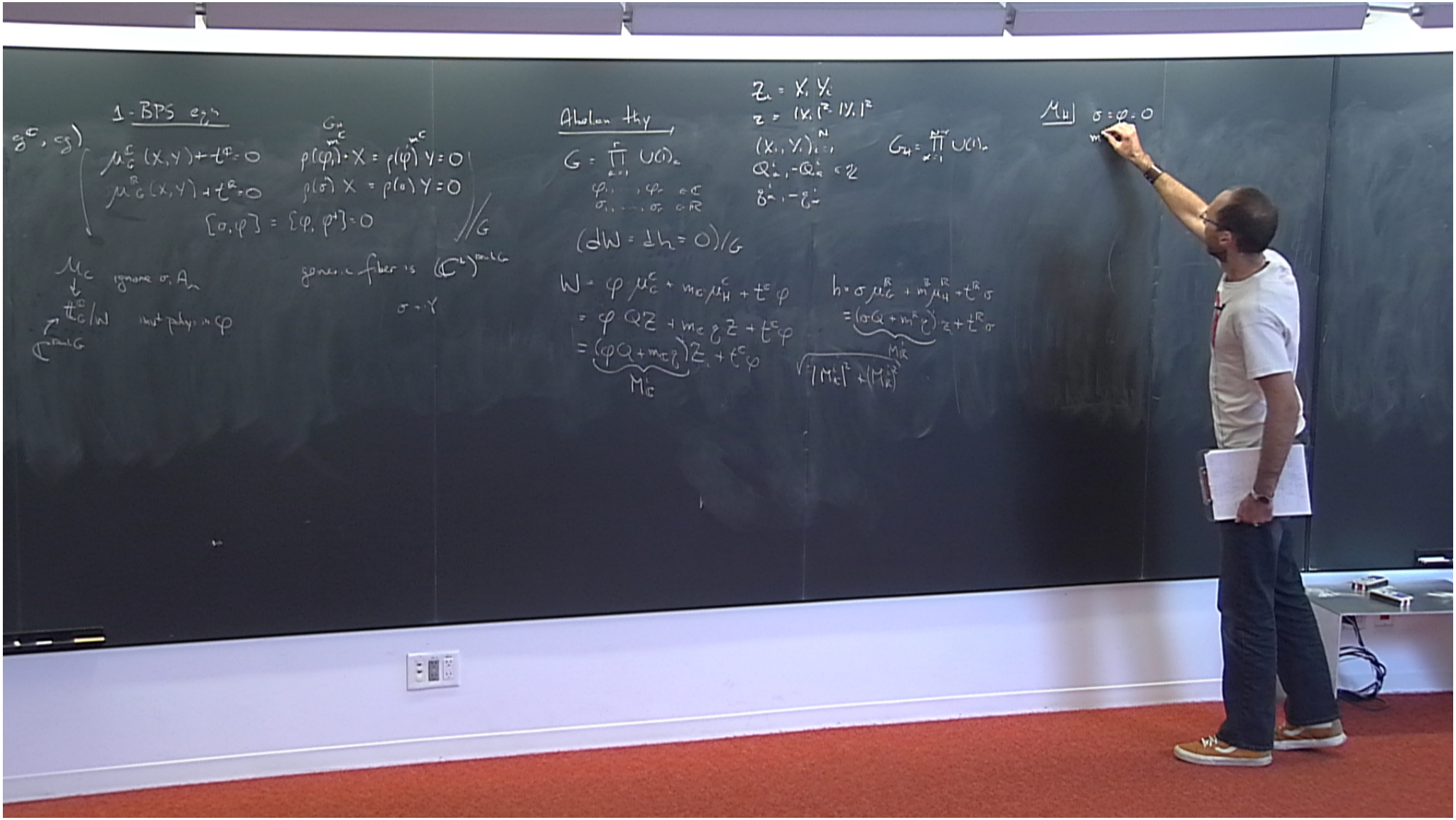

Bosonic fields:
1-BPS eq^n
Abelian thy
1-BPS eq^n
Think

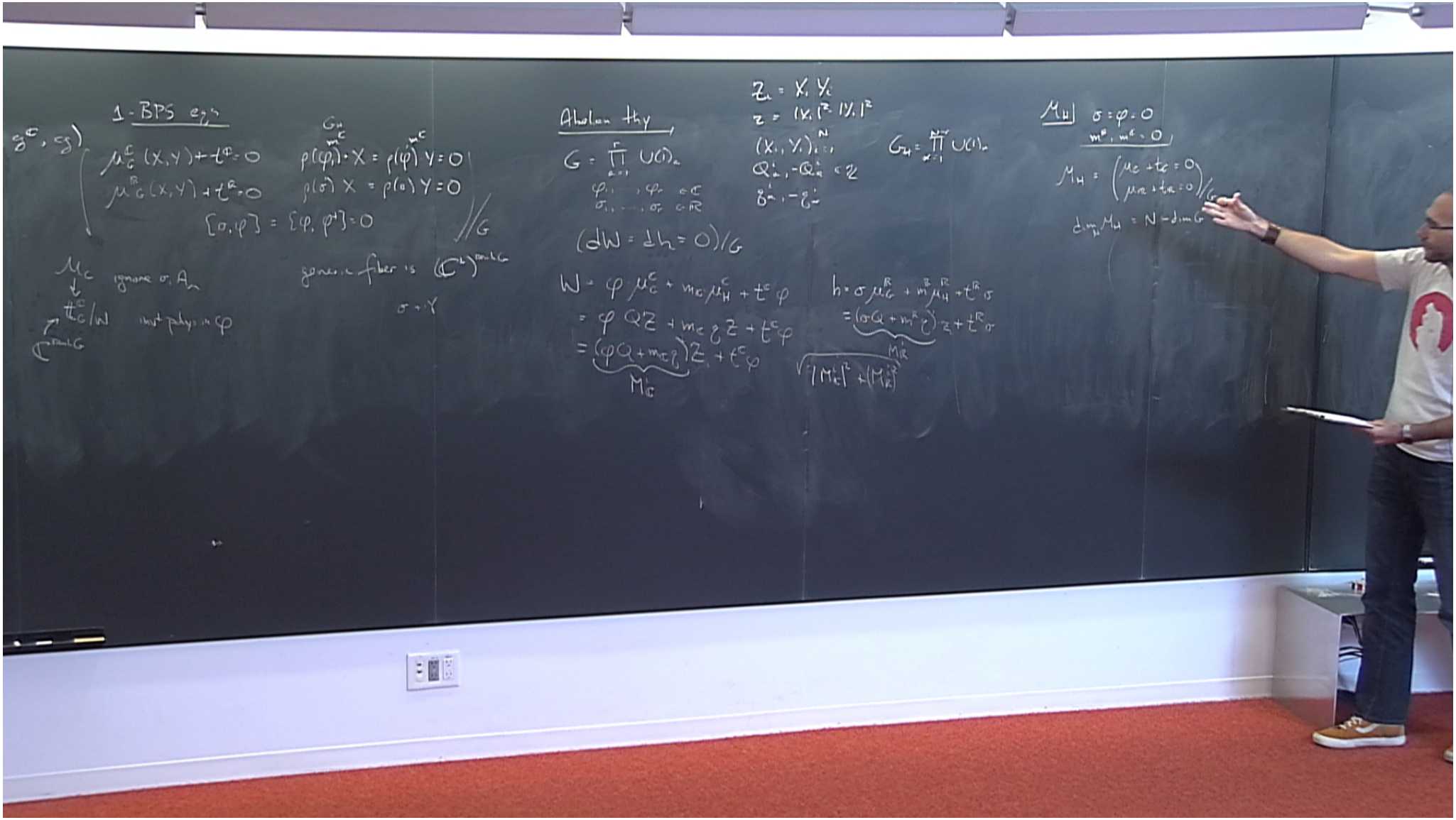

Bosonic fields:
1-BPS eqns
Abelian thy

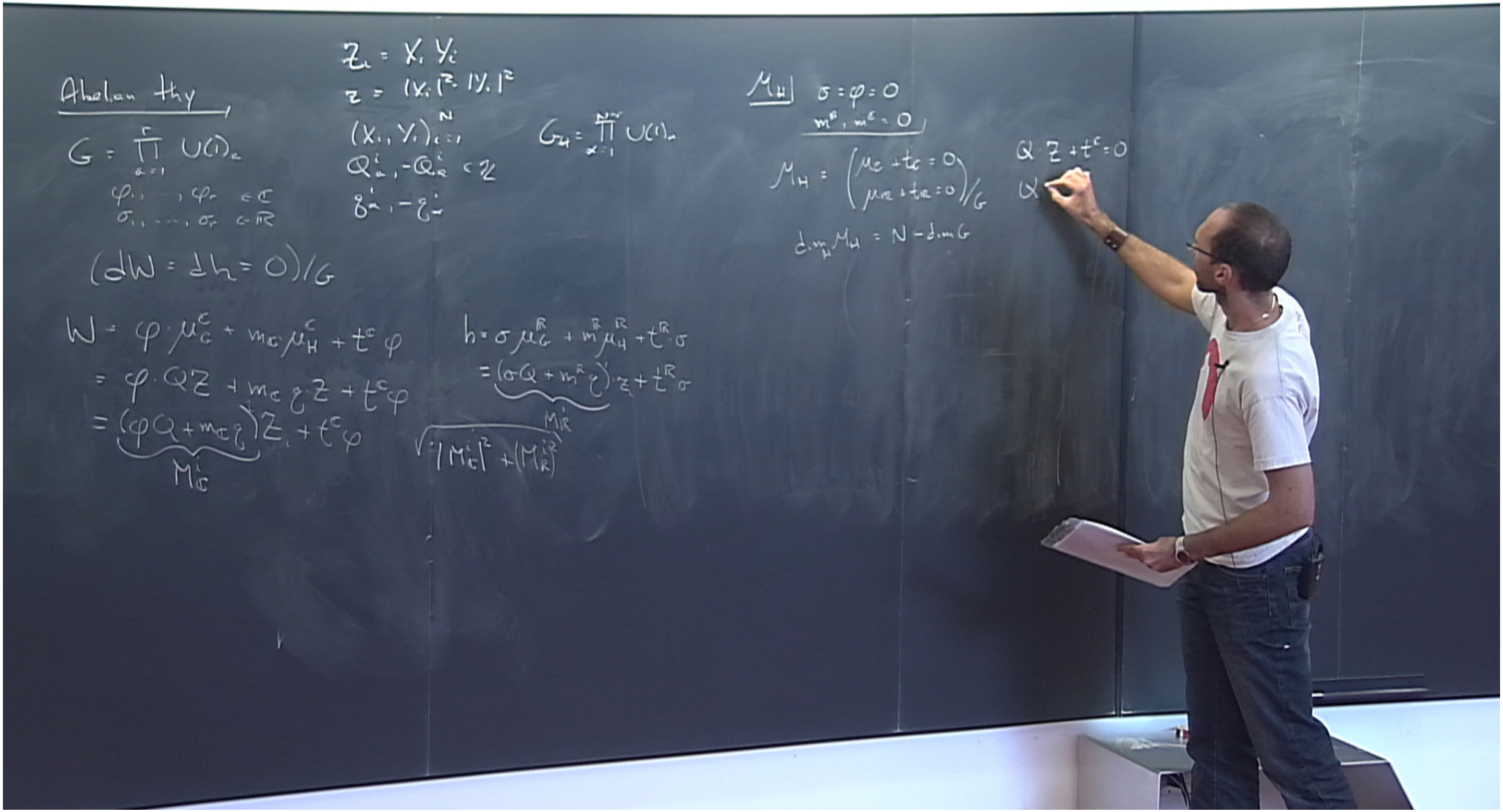

Bosonic fields:
G ~ A_μ, φ, σ ∈ (adj ∈ Lie G, g^C, g)
G-connection on R^3
R⊕R^3 → (X, Y)
1-BPS eq^n
μ_G^C(X,Y) = 0
μ_G^R(X,Y) = 0
ρ(φ)·X = ρ(φ)Y = 0
ρ(σ)X = ρ(σ)Y = 0
[σ,φ] = [φ,φ^†] = 0
/G
M_C
ignore σ, A_μ
t_G^C/W
inv. poly's in φ
generic fiber is (C^*)^{rk G}
σ ↔ Y
Abelian thy
G = ∏_{a=1}^r U(1)_a
φ_1, ..., φ_r ∈ C
σ_1, ..., σ_r ∈ R
(X_i, Y_i)_{i=1}^N
Q_a^i, -Q_a^i ∈ Z

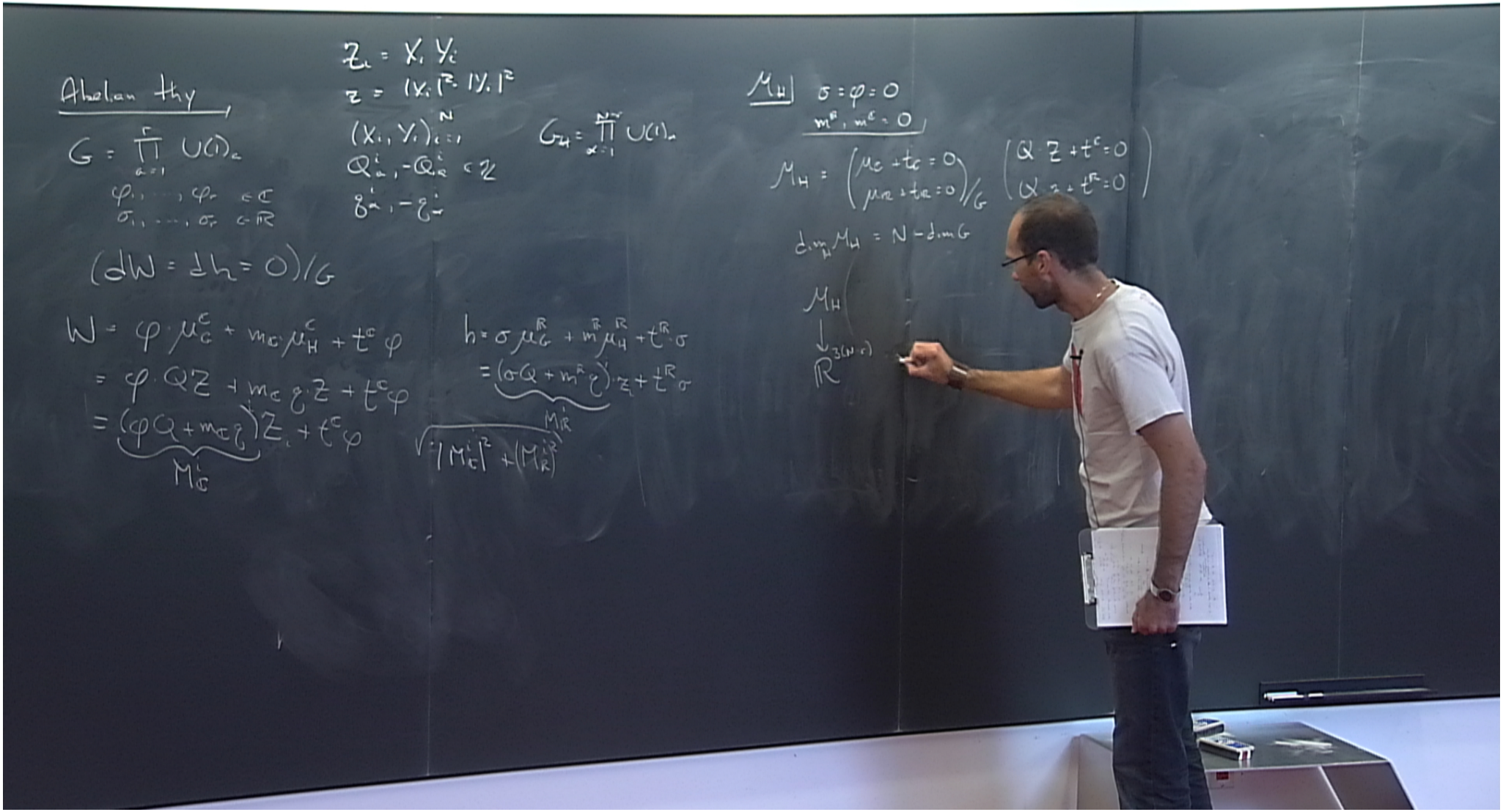

1-BPS eqns
Abelian thy
$Z_i = X_i Y_i$
$z = |X_i|^2 - |Y_i|^2$
$(X_i, Y_i)_{i=1}^N$
$Q_a^i, -Q_R^i \in \mathbb{Z}$
$\zeta_a^i, -\zeta_R^i$
$G = \prod_{a=1}^r U(1)_a$
$G_H = \prod_{i=1}^{N_f} U(1)_i$
$\varphi_1, \ldots, \varphi_r \in \mathbb{C}$
$\sigma_1, \ldots, \sigma_r \in \mathbb{R}$
$(dW = 0 = h)/G$
$W = \varphi \cdot \mu_G^C + m_C \cdot \mu_H^C + t^C \varphi$
$= \varphi Q Z + m_C \zeta Z + t^C \varphi$
$= (\varphi Q + m_C \zeta) Z_i + t^C \varphi$
M_C^i
$h = \sigma \mu_G^R + m_R^i \mu_H^R$
$= (\sigma Q + m_R^i \zeta^i) \cdot z$
M_R^i
$\sqrt{|M_C^i|^2 + (M_R^i)^2}$
$\mathcal{M}_C$
t_G^C/W
$\mathbb{C}^{rank G}$
ignore σ, A_μ
inst. poly. in φ
generic fiber is $(\mathbb{C}^*)^{n-rk G}$
$\sigma \to \infty$

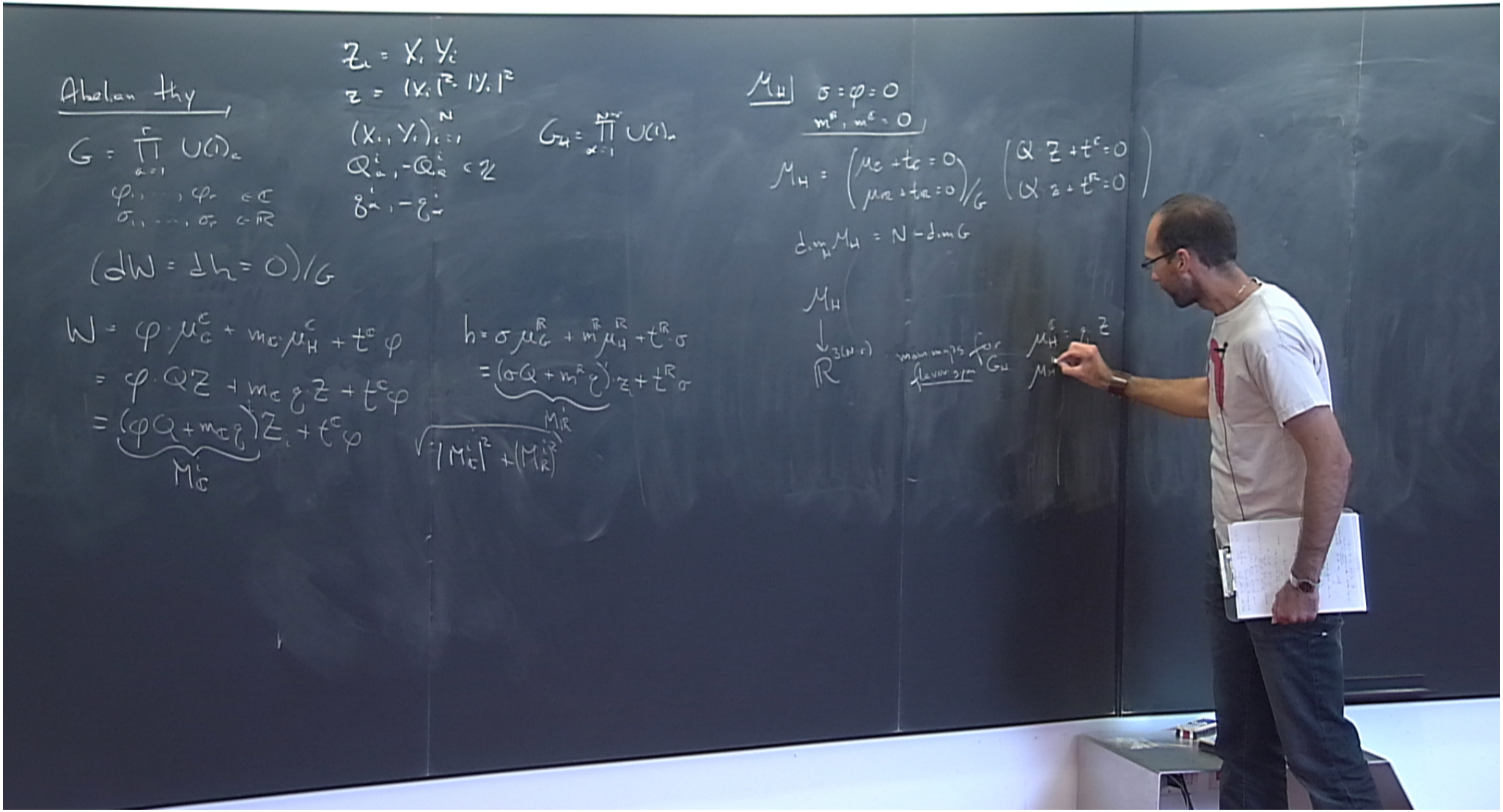

1-BPS eqns
Abelian thy
$Z_i = X_i Y_i$
$z = |X_i|^2 - |Y_i|^2$
$G = \prod_{a=1}^{r} U(1)_a$
$G_H = \prod_{a=1}^{N-r} U(1)_a$
$\mu_H \quad \sigma = \varphi = 0$
$W = \varphi \cdot \mu_C^C + m_C \cdot \mu_H^C + t^C \varphi$
$= \varphi Q Z + m_C \rho Z + t^C \varphi$
$= (\varphi Q + m_C \rho) Z_i + t^C \varphi$
M_C
$h = \sigma \mu_C^R + m^R \mu_H^R + t^R \sigma$
$= (\sigma Q + m^R \rho) \cdot z + t^R \sigma$
M_R
$\sqrt{|M_C^i|^2 + (M_R^i)^2}$
$\mu_C^C(X,Y) + t^C = 0$
$\mu_C^R(X,Y) + t^R = 0$
$[\sigma, \varphi] = [\varphi, \varphi^\dagger] = 0$
$/\!/G$
$\mathcal{M}_C$
ignore σ, A_μ
inst. poly. in φ
t_G^C/W
$\mathbb{C}^{m-\mathrm{rk}G}$
generic fiber is $(\mathbb{C}^*)^{m-\mathrm{rk}G}$
$\sigma \to \gamma$
$(dW = dh = 0)/G$

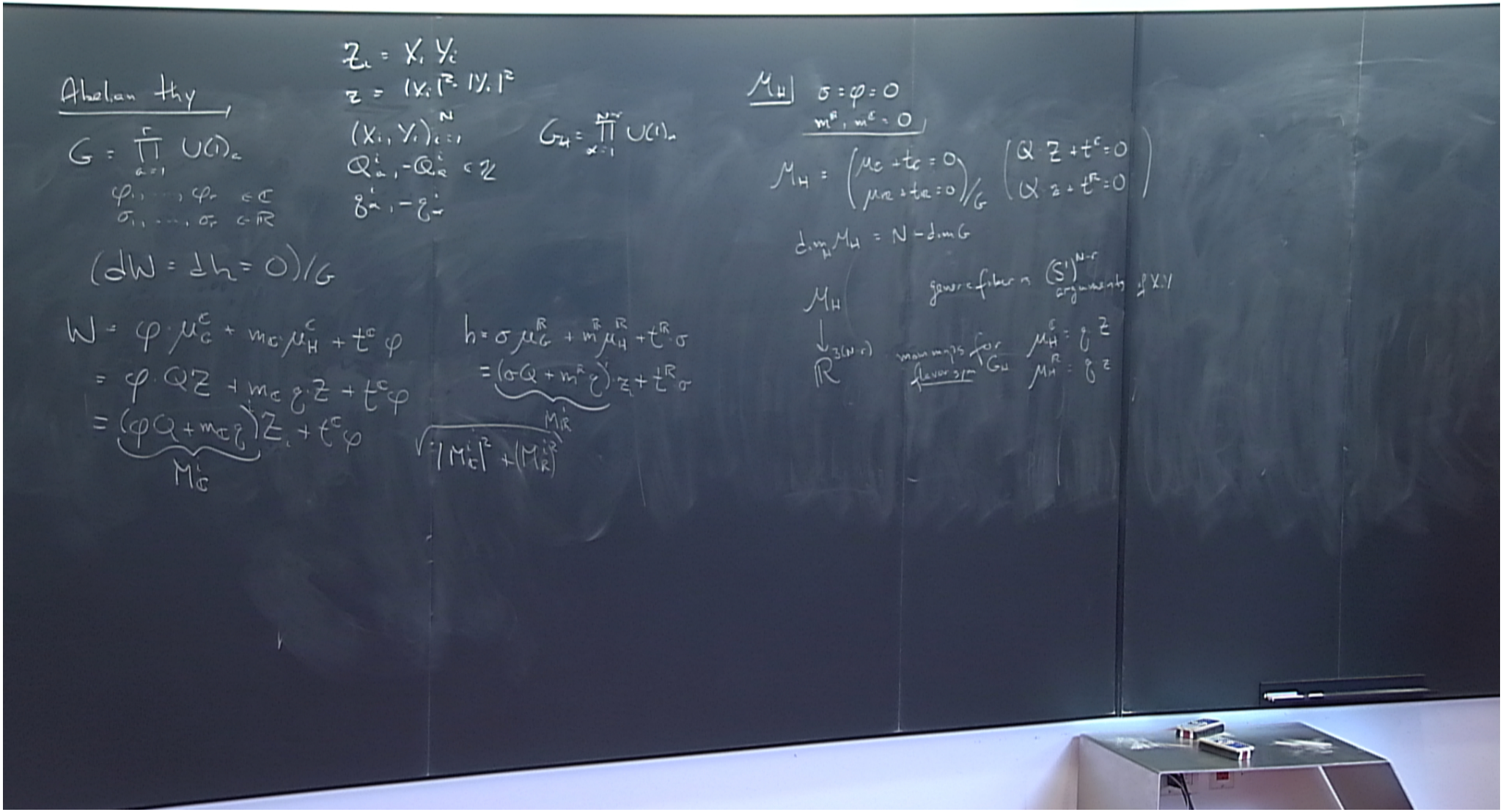

1-BPS eqns
Abelian thy
$\mathcal{M}_H$ $\sigma = \varphi = 0$
$G = \prod_{a=1}^{r} U(1)_a$
$(dW = dh = 0)/G$
$W = \varphi \mu_C^C + m_C \mu_H^C + t^C \varphi$
$= \varphi Q Z + m_C \zeta Z + t^C \varphi$
$= (\varphi Q + m_C \zeta) Z + t^C \varphi$
$h = \sigma \mu_C^R + m^R \mu_H^R + t^R \sigma$
$= (\sigma Q + m^R \zeta) Z + t^R \sigma$
$Z_i = X_i Y_i$
$z = |X_i|^2 - |Y_i|^2$
$G_H = \prod_{a=1}^{N-r} U(1)_a$
$\mathcal{M}_C$ ignore σ, A_m
t_C^C/W Weyl poly in φ
generic fiber is $(\mathbb{C}^*)^{N-r-G}$
$\sigma \to Y$

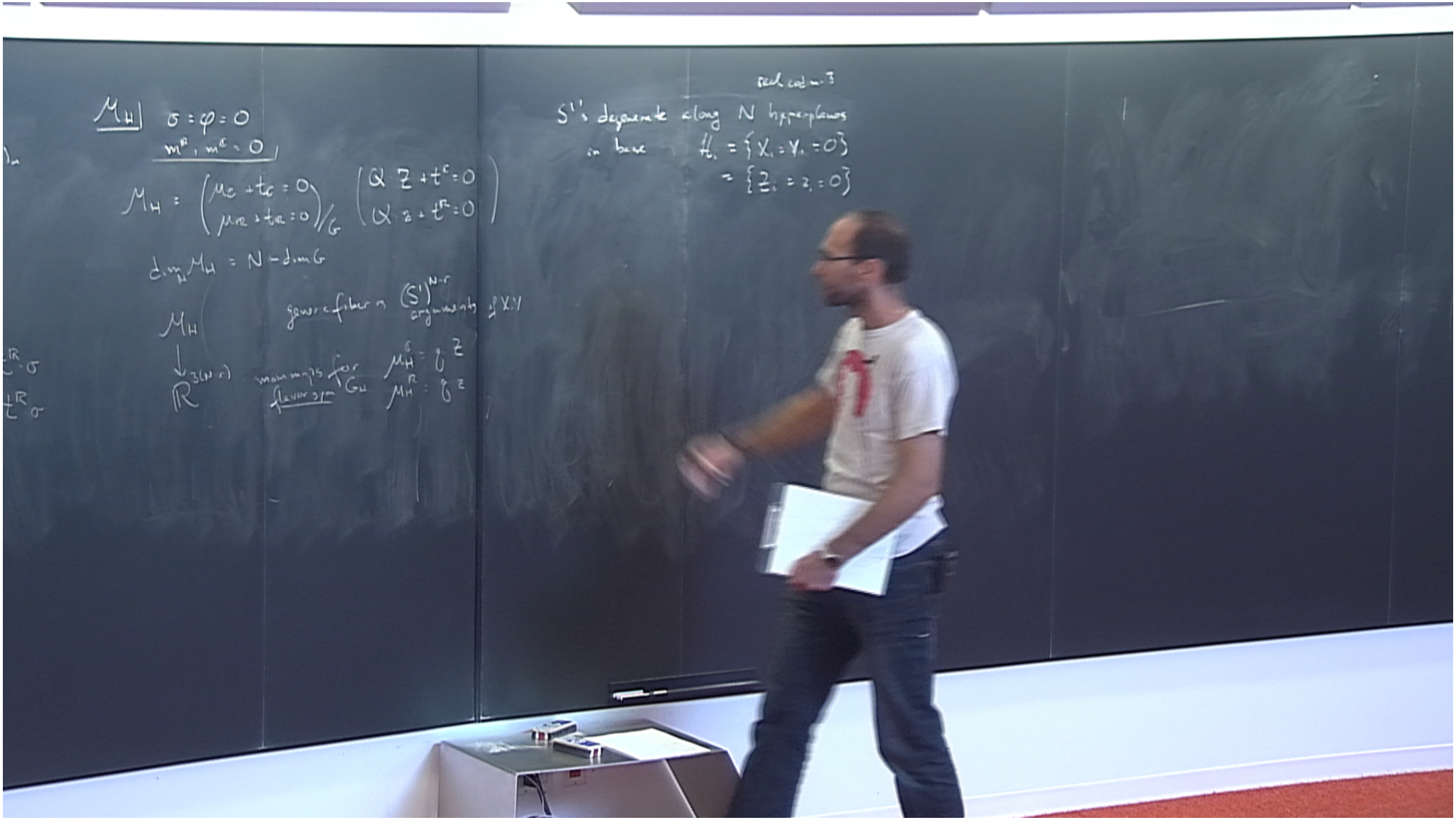

1-BPS eqns
Abelian thy
$\mathcal{M}_H$
$\sigma = \varphi = 0$
$m^c, m^R = 0$
$\mathcal{M}_H = \left(\begin{matrix} \mu_c + t_c = 0 \\ \mu_R + t_R = 0 \end{matrix}\right)/G$
$\dim_{\mathbb{H}} \mathcal{M}_H = N - \dim G$
$G = \prod_{a=1}^{r} U(1)_a$
$(dW = dh = 0)/G$
$W = \varphi \mu_C^c + m_c \mu_H^c + t^c \varphi$
$= \varphi Q^c Z + m_c \zeta Z + t^c \varphi$
$= (\underbrace{\varphi Q + m_c \zeta}_{M_C^i}) Z_i + t^c \varphi$
$h = \sigma \mu_C^R + m^R \mu_H^R + t^R \sigma$
$= (\underbrace{\sigma Q + m^R \zeta}_{M_R^i}) Z + t^R \sigma$
$\sqrt{|M_C^i|^2 + (M_R^i)^2}$
$Z_i = X_i Y_i$
$z = |X_i|^2 - |Y_i|^2$
$(X_i, Y_i)_{i=1}^{N}$
$Q_a^i, -Q_a^i \in \mathbb{Z}$
$\zeta^a, -\zeta^a$
$G_H = \prod_{a=1}^{N-r} U(1)_a$
$\rho(\varphi_i) \cdot X = \rho(\varphi) Y = 0$
$\rho(\sigma) X = \rho(\sigma) Y = 0$
$[\sigma, \varphi^\dagger] = [\varphi, \varphi^\dagger] = 0$
$//G$
$\mathcal{M}_C$
ignore σ, A_m
t_C^c/W
$(\mathbb{C}^*)^{\text{rank}\,G}$
general fiber is $(\mathbb{C}^*)^{N-\text{rank}\,G}$
$\sigma \to Y$

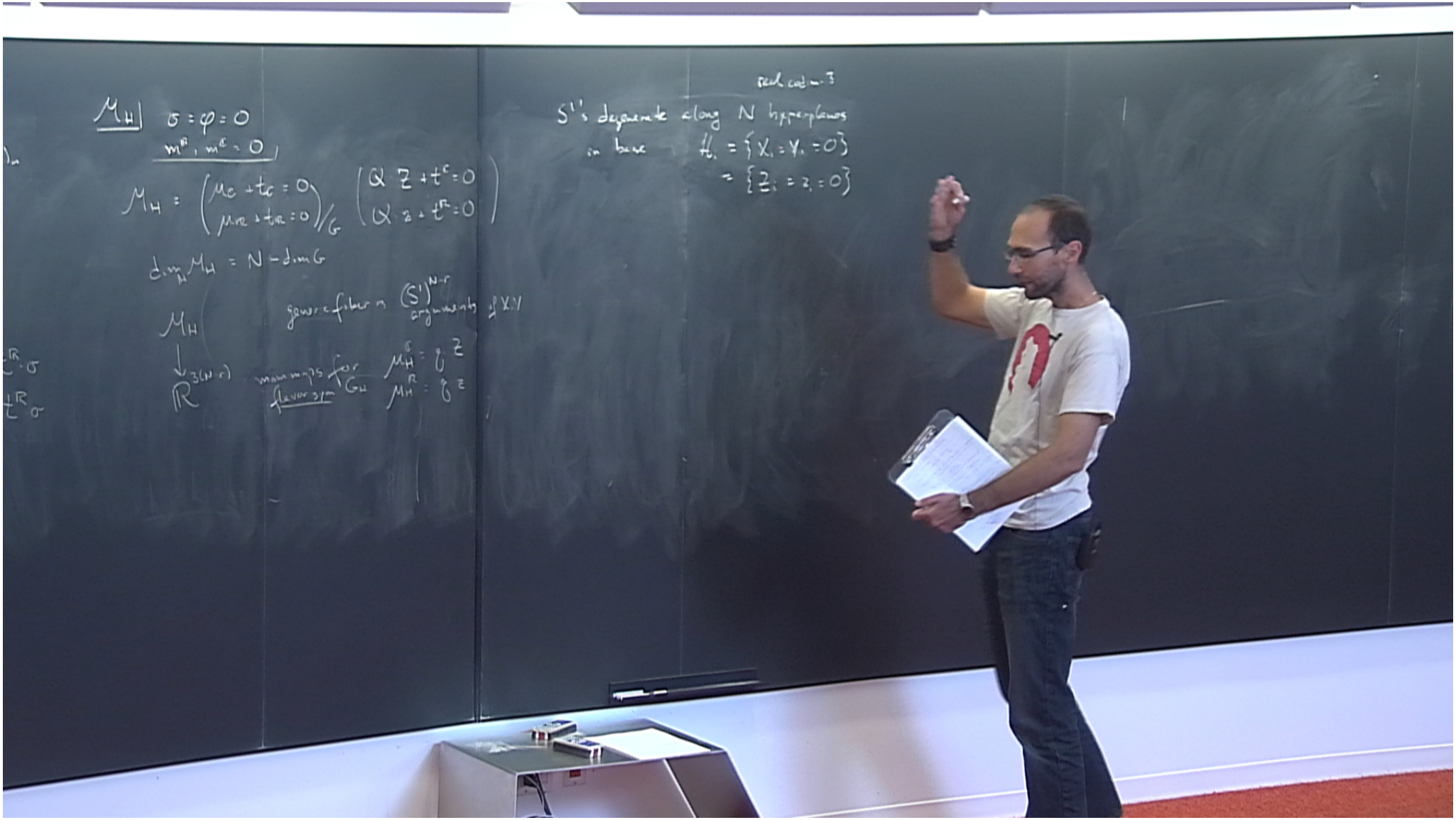

Abelian thy
$G = \prod_{a=1}^{r} U(1)_a$
$\varphi_1, \dots, \varphi_r \in \mathbb{C}$
$\sigma_1, \dots, \sigma_r \in \mathbb{R}$
$(dW = dh = 0)/G$
$W = \varphi \cdot \mu_G^{\mathbb{C}} + m_{\mathbb{C}} \mu_H^{\mathbb{C}} + t^{\mathbb{C}} \varphi$
$= \varphi \cdot QZ + m_{\mathbb{C}} \xi Z + t^{\mathbb{C}} \varphi$
$= (\varphi Q + m_{\mathbb{C}} \xi) Z_i + t^{\mathbb{C}} \varphi$
$M_{\mathbb{C}}^i$
$Z_i = X_i Y_i$
$z = |X_i|^2 - |Y_i|^2$
$(X_i, Y_i)_{i=1}^N$
$Q_a^i, -Q_a^i \in \mathbb{Z}$
$\xi_a^i, -\xi_a^i$
$G_H = \prod_{\alpha=1}^{N-r} U(1)_\alpha$
$h = \sigma \mu_G^{\mathbb{R}} + m^{\mathbb{R}} \mu_H^{\mathbb{R}} + t^{\mathbb{R}} \sigma$
$= (\sigma Q + m^{\mathbb{R}} \xi) \cdot z + t^{\mathbb{R}} \sigma$
$M_{\mathbb{R}}^i$
$\sqrt{|M_{\mathbb{C}}^i|^2 + (M_{\mathbb{R}}^i)^2}$
μ_H $\sigma = \varphi = 0$
$m^{\mathbb{R}}, m^{\mathbb{C}} = 0$
$\mu_H = \begin{pmatrix} \mu_{\mathbb{C}} + t_{\mathbb{C}} = 0 \\ \mu_{\mathbb{R}} + t_{\mathbb{R}} = 0 \end{pmatrix} /G$
$\dim_{\mu} \mu_H = N - \dim G$
$Q \cdot Z + t^{\mathbb{C}} = 0$
Q

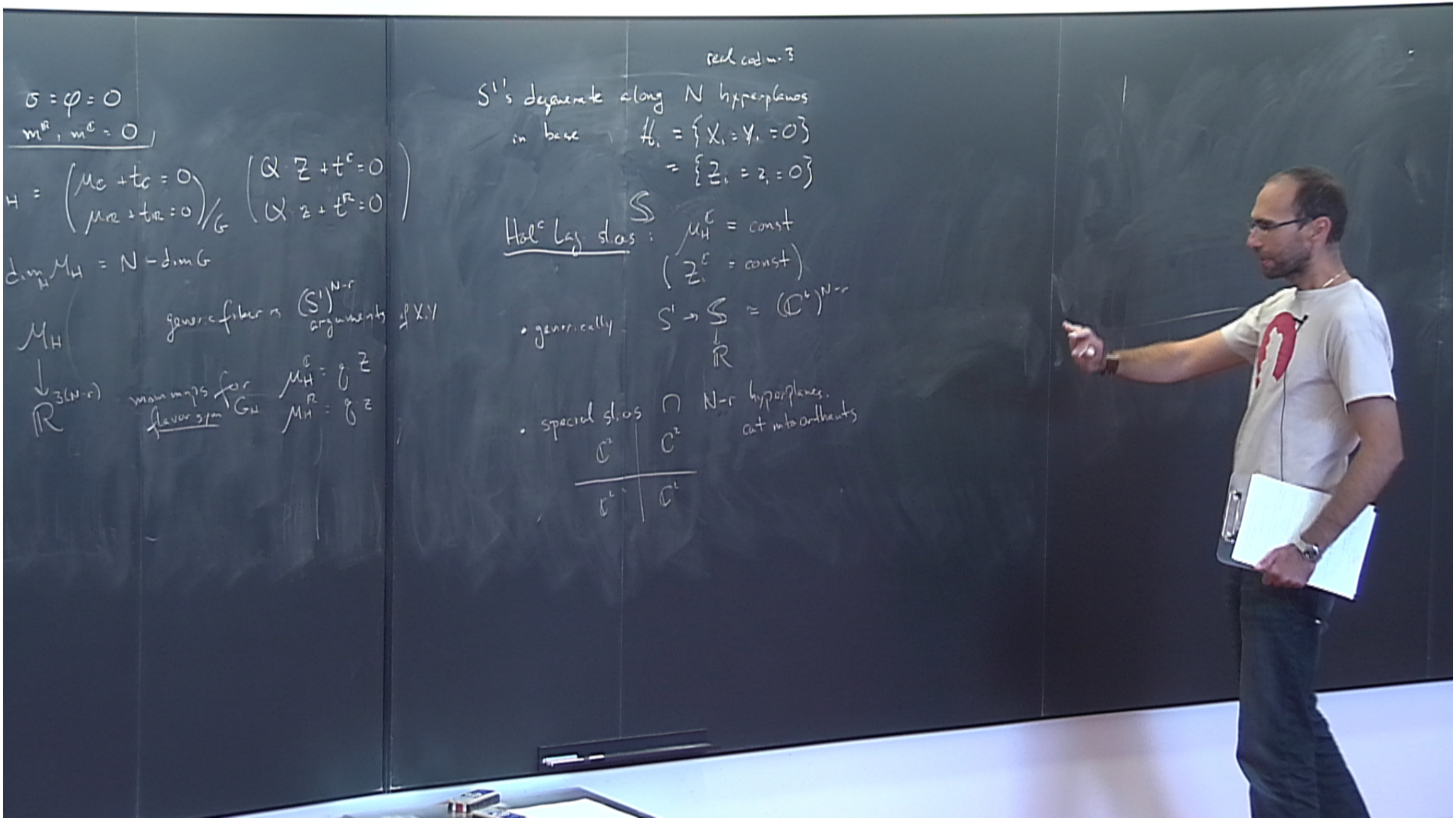

Abelian thy
$G = \prod_{a=1}^{r} U(1)_a$
$\varphi_1, \dots, \varphi_r \in \mathbb{C}$
$\sigma_1, \dots, \sigma_r \in \mathbb{R}$
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$= \varphi \cdot QZ + m_{\mathbb{C}} \cdot \xi \cdot Z + t^{\mathbb{C}} \varphi$
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$M_{\mathbb{C}}^i$
$Z_i = X_i Y_i$
$z = |X_i|^2 - |Y_i|^2$
$(X_i, Y_i)_{i=1}^{N}$
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$\xi_a^i, -\xi_a^i$
$G_H = \prod_{\alpha=1}^{N-r} U(1)_\alpha$
$h = \sigma \mu_G^{\mathbb{R}} + m^{\mathbb{R}} \mu_H^{\mathbb{R}} + t^{\mathbb{R}} \cdot \sigma$
$= (\sigma Q + m^{\mathbb{R}} \xi) \cdot z + t^{\mathbb{R}} \sigma$
$M_{\mathbb{R}}^i$
$\sqrt{|M_{\mathbb{C}}^i|^2 + (M_{\mathbb{R}}^i)^2}$
$\mathcal{M}_H$ $\sigma = \varphi = 0$
$m^{\mathbb{R}}, m^{\mathbb{C}} = 0$
$\mathcal{M}_H = \begin{pmatrix} \mu_{\mathbb{C}} + t_{\mathbb{C}} = 0 \\ \mu_{\mathbb{R}} + t_{\mathbb{R}} = 0 \end{pmatrix} /G$ $\begin{pmatrix} Q \cdot z + t^{\mathbb{C}} = 0 \\ Q \cdot z + t^{\mathbb{R}} = 0 \end{pmatrix}$
$\dim_{\mathbb{H}} \mathcal{M}_H = N - \dim G$
$\mathcal{M}_H$
$\downarrow_{3(N-r)}$
$\mathbb{R}$

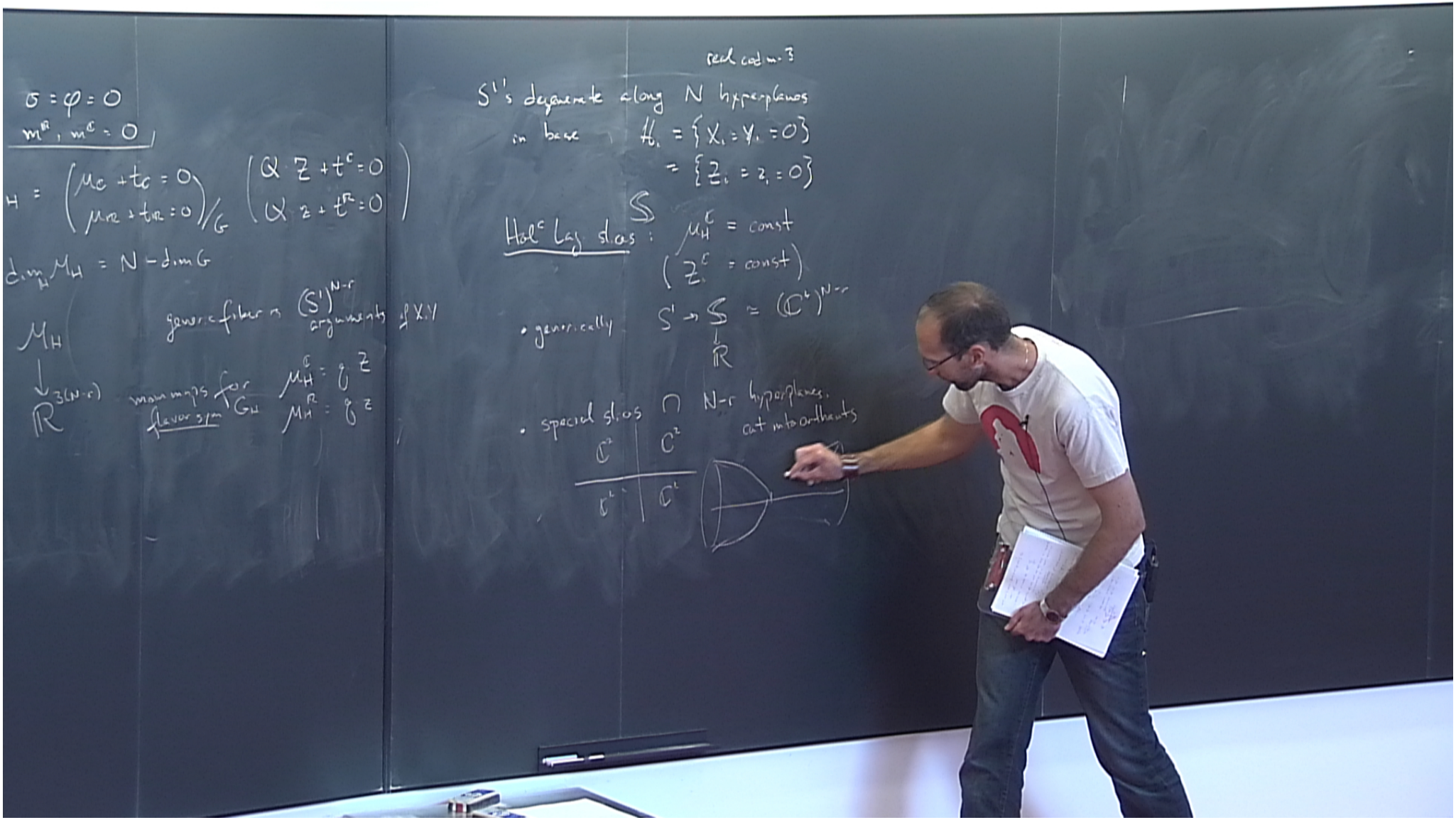

Abelian thy
$G = \prod_{a=1}^{r} U(1)_a$
$\varphi_1, \dots, \varphi_r \in \mathbb{C}$
$\sigma_1, \dots, \sigma_r \in \mathbb{R}$
$(dW = dh = 0)/G$
$W = \varphi \cdot \mu_G^{\mathbb{C}} + m_{\mathbb{C}} \cdot \mu_H^{\mathbb{C}} + t^{\mathbb{C}} \varphi$
$= \varphi \cdot QZ + m_{\mathbb{C}} \cdot \xi \cdot Z + t^{\mathbb{C}} \varphi$
$= (\varphi Q + m_{\mathbb{C}} \xi) Z_i + t^{\mathbb{C}} \varphi$
$M_{\mathbb{C}}^i$
$Z_i = X_i Y_i$
$z = |X_i|^2 - |Y_i|^2$
$(X_i, Y_i)_{i=1}^{N}$
$Q_a^i, -Q_a^i \in \mathbb{Z}$
$\xi_a^i, -\xi_a^i$
$G_H = \prod_{a=1}^{N-r} U(1)_a$
$h = \sigma \mu_G^{\mathbb{R}} + m^{\mathbb{R}} \mu_H^{\mathbb{R}} + t^{\mathbb{R}} \cdot \sigma$
$= (\sigma Q + m^{\mathbb{R}} \xi) \cdot z + t^{\mathbb{R}} \sigma$
$M_{\mathbb{R}}^i$
$\sqrt{|M_{\mathbb{C}}^i|^2 + (M_{\mathbb{R}}^i)^2}$
$\mathcal{M}_H$ $\sigma = \varphi = 0$
$m^{\mathbb{R}}, m^{\mathbb{C}} = 0$
$\mathcal{M}_H = \begin{pmatrix} \mu_{\mathbb{C}} + t_{\mathbb{C}} = 0 \\ \mu_{\mathbb{R}} + t_{\mathbb{R}} = 0 \end{pmatrix} /G \quad \begin{pmatrix} Q \cdot Z + t^{\mathbb{C}} = 0 \\ Q \cdot z + t^{\mathbb{R}} = 0 \end{pmatrix}$
$\dim_{\mathbb{H}} \mathcal{M}_H = N - \dim G$
$\mathcal{M}_H$
$\downarrow$
$\mathbb{R}^{3(N-r)}$
moduli for G_H
$\mu_H^{\mathbb{C}} = \varphi_i Z$
$\mu_H^{\mathbb{R}} = \dots$

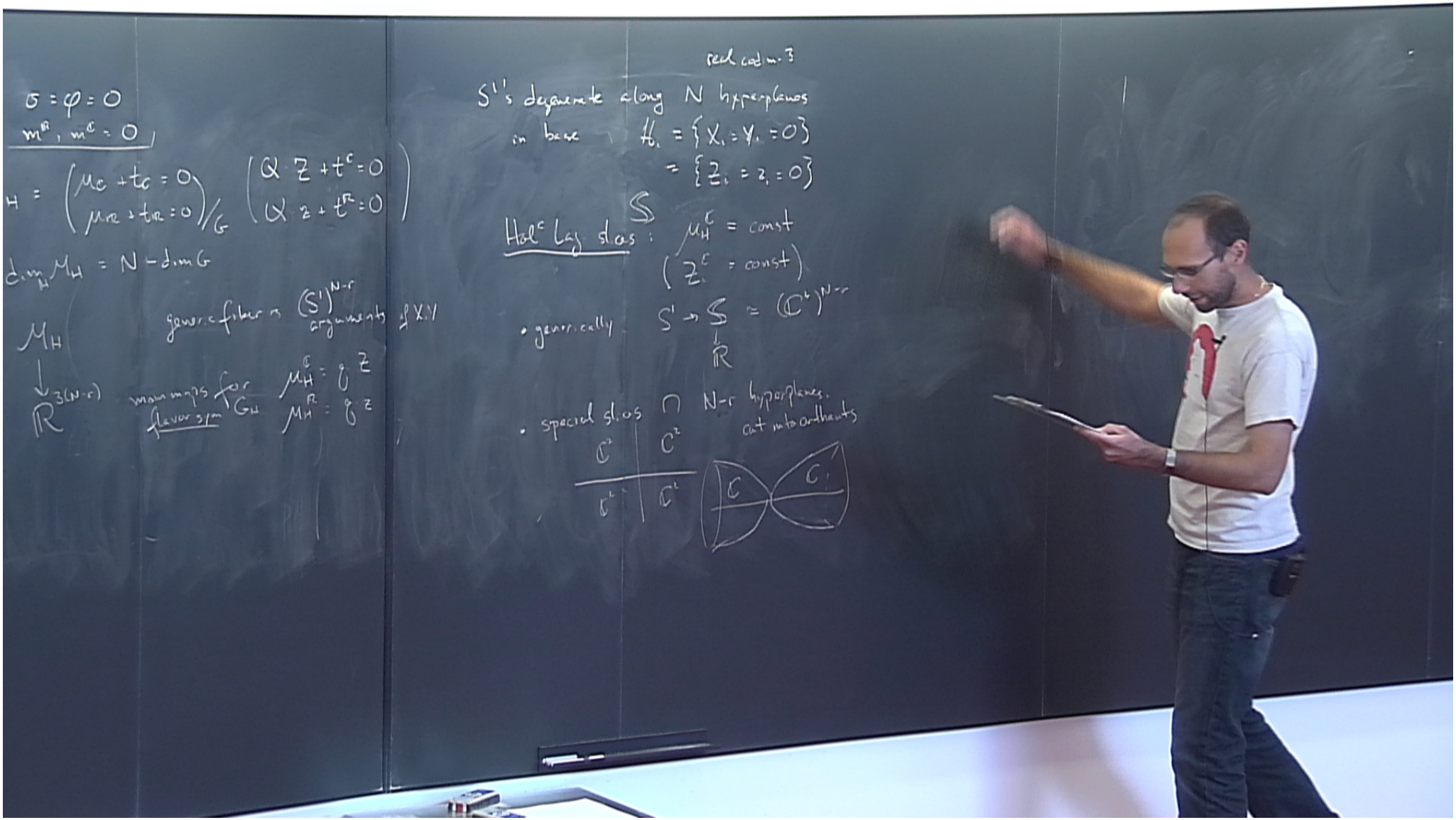

Abelian thy
$G = \prod_{a=1}^{r} U(1)_a$
$\varphi_1, \dots, \varphi_r \in \mathbb{C}$
$\sigma_1, \dots, \sigma_r \in \mathbb{R}$
$(dW = dh = 0)/G$
$Z_i = X_i Y_i$
$z = |X_i|^2 - |Y_i|^2$
$(X_i, Y_i)_{i=1}^{N}$
$Q_a^i, -Q_a^i \in \mathbb{Z}$
$\ell^i, -\ell^i$
$G_H = \prod_{a=1}^{N-r} U(1)_a$
$W = \varphi \cdot \mu_G^{\mathbb{C}} + m_{\mathbb{C}} \cdot \mu_H^{\mathbb{C}} + t^{\mathbb{C}} \varphi$
$= \varphi \cdot QZ + m_{\mathbb{C}} \ell \cdot Z + t^{\mathbb{C}} \varphi$
$= \underbrace{(\varphi Q + m_{\mathbb{C}} \ell)}_{M_{\mathbb{C}}^i} Z_i + t^{\mathbb{C}} \varphi$
$h = \sigma \cdot \mu_G^{\mathbb{R}} + m^{\mathbb{R}} \mu_H^{\mathbb{R}} + t^{\mathbb{R}} \cdot \sigma$
$= \underbrace{(\sigma Q + m^{\mathbb{R}} \ell)}_{M_{\mathbb{R}}^i} \cdot z + t^{\mathbb{R}} \sigma$
$\sqrt{|M_{\mathbb{C}}^i|^2 + (M_{\mathbb{R}}^i)^2}$
$\mathcal{M}_H$ $\sigma = \varphi = 0$
$m^{\mathbb{R}}, m^{\mathbb{C}} = 0$
$\mathcal{M}_H = \begin{pmatrix} \mu_{\mathbb{C}} + t_{\mathbb{C}} = 0 \\ \mu_{\mathbb{R}} + t_{\mathbb{R}} = 0 \end{pmatrix} /\!/ G \quad \begin{pmatrix} Q \cdot Z + t^{\mathbb{C}} = 0 \\ Q \cdot z + t^{\mathbb{R}} = 0 \end{pmatrix}$
$\dim_{\mathbb{H}} \mathcal{M}_H = N - \dim G$
$\mathcal{M}_H$ gauge fibers $(S^1)^{N-r}$ argument of X_i
$\downarrow_{3(N-r)}$
$\mathbb{R}^{3(N-r)}$ moment maps for G_H flavor $SU(2)_R$
$\mu_H^{\mathbb{C}} = \ell Z$
$\mu_H^{\mathbb{R}} = \ell z$


$\mathcal{M}_H$ $\sigma = \varphi = 0$
$m^R, m^C = 0$
$\mathcal{M}_H = \begin{pmatrix} \mu_C + t_C = 0 \\ \mu_R + t_R = 0 \end{pmatrix} /\!/_G \begin{pmatrix} Q\,Z + t^C = 0 \\ Q\,z + t^R = 0 \end{pmatrix}$
$\dim_{\mathbb{H}} \mathcal{M}_H = N - \dim G$
$\mathcal{M}_H$
$\downarrow$
$\mathbb{R}^{3(N-r)}$
generic fiber = $(S^1)^{N-r}$ arguments of X_i
massless for G_H
flavour 3^m
$\mathcal{M}_H^C = \{Z\}$
$\mathcal{M}_H^R = \{z\}$
each cod-3
S^1's degenerate along N hyperplanes
in base $H_i = \{X_i = Y_i = 0\}$
$= \{Z_i = z_i = 0\}$


$\mathcal{M}_H$ $\sigma = \varphi = 0$
$m^R, m^C = 0$
$\mathcal{M}_H = \begin{pmatrix} \mu_C + t_C = 0 \\ \mu_R + t_R = 0 \end{pmatrix} /\!/_G \begin{pmatrix} Q\,Z + t^C = 0 \\ Q\,z + t^R = 0 \end{pmatrix}$
$\dim_{\mathbb{H}} \mathcal{M}_H = N - \dim G$
$\mathcal{M}_H$
$\downarrow$
$\mathbb{R}^{3(N-r)}$
generic fiber = $(S^1)^{N-r}$ arguments of X_i
massless for G_H
flavor 3^m
$\mathcal{M}_H^C = \{Z\}$
$\mathcal{M}_H^R = \{z\}$
real codim = 3
S^1's degenerate along N hyperplanes in base
$H_i = \{X_i = Y_i = 0\}$
$= \{Z_i = z_i = 0\}$


$\sigma = \varphi = 0$
$m^R, m^C = 0$
$H = \left(\mu_C + t_C = 0\right) /\!/ G \quad \left(Q\,Z + t^C = 0,\ Q\,z + t^R = 0\right)$
$\dim_H \mathcal{M}_H = N - \dim G$
$\mathcal{M}_H$
$\downarrow$
$\mathbb{R}^{3(N-r)}$
generic fiber = $(S^1)^{N-r}$ arguments of X_i
flavour sym G_H moment maps for $\mu_H^C = Q\,Z$, $\mu_H^R = Q\,z$
real codim. 3
S^1's degenerate along N hyperplanes in base $H_i = \{X_i = Y_i = 0\} = \{Z_i = z_i = 0\}$
Hol^C Lag slice $\mathcal{S}$: $\mu_H^C = \text{const}$ $(Z_i^C = \text{const})$
• generically $S^1 \to \mathcal{S} \to \mathbb{R}$ $\simeq (\mathbb{C}^*)^{N-r}$
• special slices $\subset$ $N-r$ hyperplanes, cut into orthants
$\mathbb{C}^2$ | $\mathbb{C}^2$
$\mathbb{C}^2$ | $\mathbb{C}^2$


$\sigma = \varphi = 0$
$m^R, m^C = 0$
$H = \left(\mu_C + t_C = 0 \;\; \mu_{NR} + t_{NR} = 0\right)/G \quad \left(Q\,Z + t^C = 0 \;\; Q\,z + t^R = 0\right)$
$\dim_H \mathcal{M}_H = N - \dim G$
$\mathcal{M}_H$
$\downarrow$
$\mathbb{R}^{3(N-r)}$
generic fiber = $(S^1)^{N-r}$ arguments of X_i
mom maps for flavor sym G_H
$\mu_H^C = \zeta^C$
$\mu_H^R = \zeta^R$
real codim. 3
S^1's degenerate along N hyperplanes in base
$\mathcal{H}_i = \{X_i = Y_i = 0\}$
$= \{Z_i = z_i = 0\}$
Hol^C Lag sols: $\mathcal{S}$
$\mu_H^C = \text{const}$
$(Z_i^C = \text{const})$
• generically $S^1 \to \mathcal{S} \to \mathbb{R}$ $\simeq (\mathbb{C}^*)^{N-r}$
• special slices $\cap$ $N-r$ hyperplanes, cut into orthants
$\mathbb{C}^2$ | $\mathbb{C}^2$
$\mathbb{C}^2$ | $\mathbb{C}^2$


$\sigma = \varphi = 0$
$m^R, m^C = 0$
$H = \left(\mu_C + t_C = 0,\ \mu_R + t_R = 0\right)/G \quad \left(Q\,Z + t^C = 0,\ Q\,z + t^R = 0\right)$
$\dim_H \mathcal{M}_H = N - \dim G$
$\mathcal{M}_H$
$\downarrow$
$\mathbb{R}^{3(N-r)}$
generic fiber is $(S^1)^{N-r}$ arguments of X_i
mom maps for flavor sym G_H
$\mu_H^C = Q\,Z$
$\mu_H^R = Q\,z$
real codim. 3
S^1's degenerate along N hyperplanes in base
$H_i = \{X_i = Y_i = 0\}$
$= \{Z_i = z_i = 0\}$
Holo Lag slices: $\mathcal{S}$
$\mu_H^C = \text{const}$
$(Z_i^C = \text{const})$
• generically $S^1 \to \mathcal{S} \to \mathbb{R}$ $\simeq (\mathbb{C}^*)^{N-r}$
• special slices $\cap$ $N-r$ hyperplanes, cut into orthants
$\mathbb{C}^2$ | $\mathbb{C}^2$
$\mathbb{C}^2$ | $\mathbb{C}^2$
C
C'

$\sigma = \varphi = 0$

$m^R, m^C = 0$

$H = \left(\begin{matrix} \mu_C + t_C = 0 \\ \mu_R + t_R = 0 \end{matrix} \right) /\!/ G \quad \left(\begin{matrix} Q\,Z + t^C = 0 \\ Q\,z + t^R = 0 \end{matrix} \right)$

$\dim_H \mathcal{M}_H = N - \dim G$

$\mathcal{M}_H$ — generic fiber is $(S^1)^{N-r}$ winding arguments of X_i

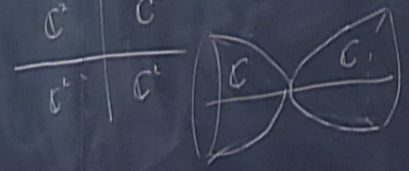
$\downarrow$ $\mathbb{R}^{3(N-r)}$ — moduli for flavor sym G_H — $\mu_H^C = \zeta^C$, $\mu_H^R = \zeta^R$

red cod. 3

S^1's degenerate along N hyperplanes in base $\mathcal{H}_i = \{X_i = Y_i = 0\} = \{Z_i = z_i = 0\}$

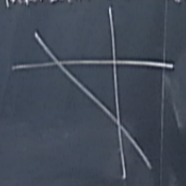
Hol^C Lag slice $\mathcal{S}$: $\mu_H^C = \text{const}$ $(Z_i^C = \text{const})$

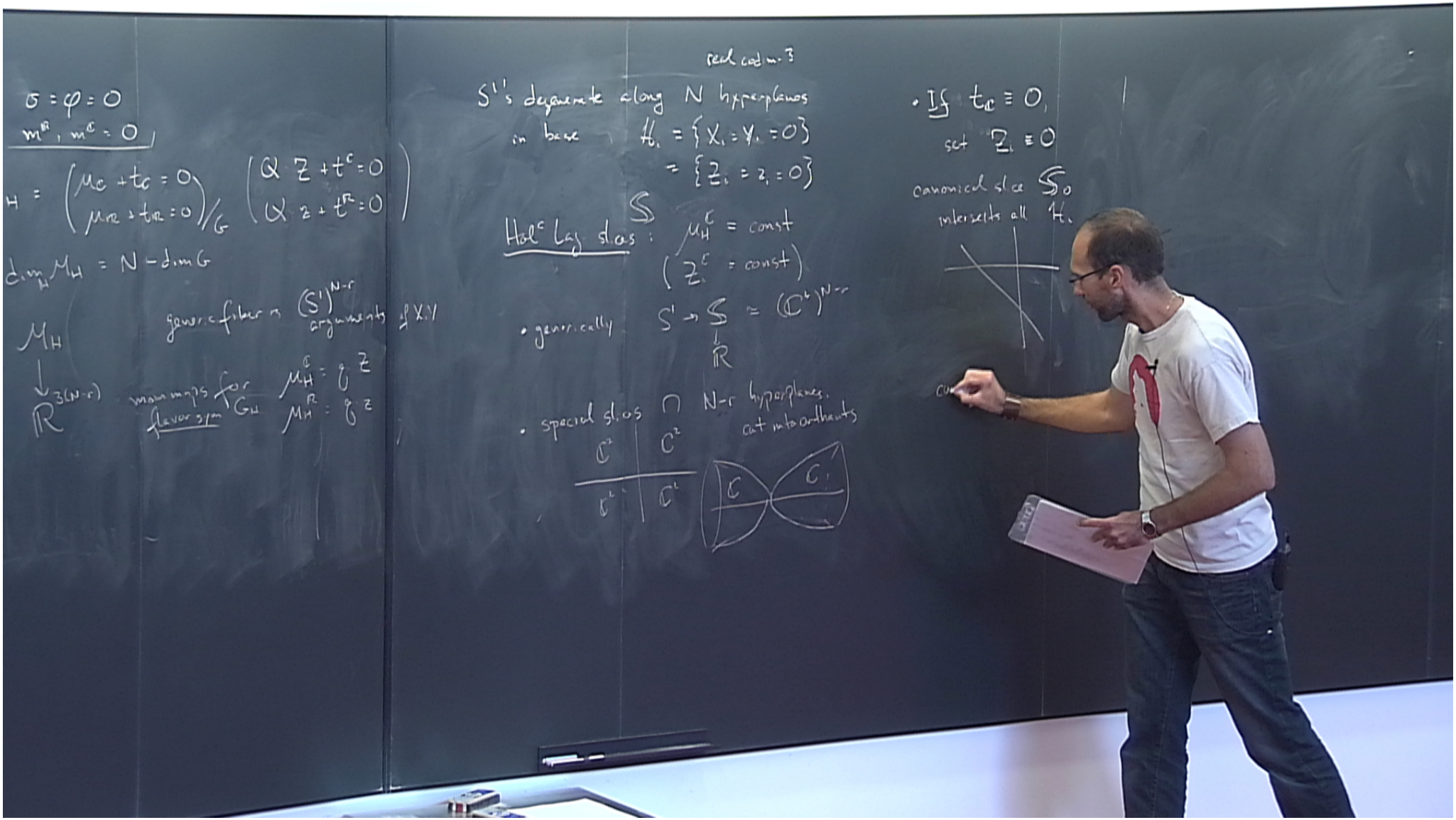
- generically $S^1 \to \mathcal{S} \to \mathbb{R}$ $\simeq (\mathbb{C}^*)^{N-r}$
- special slices $\cap$ $N-r$ hyperplanes, cut into orthants

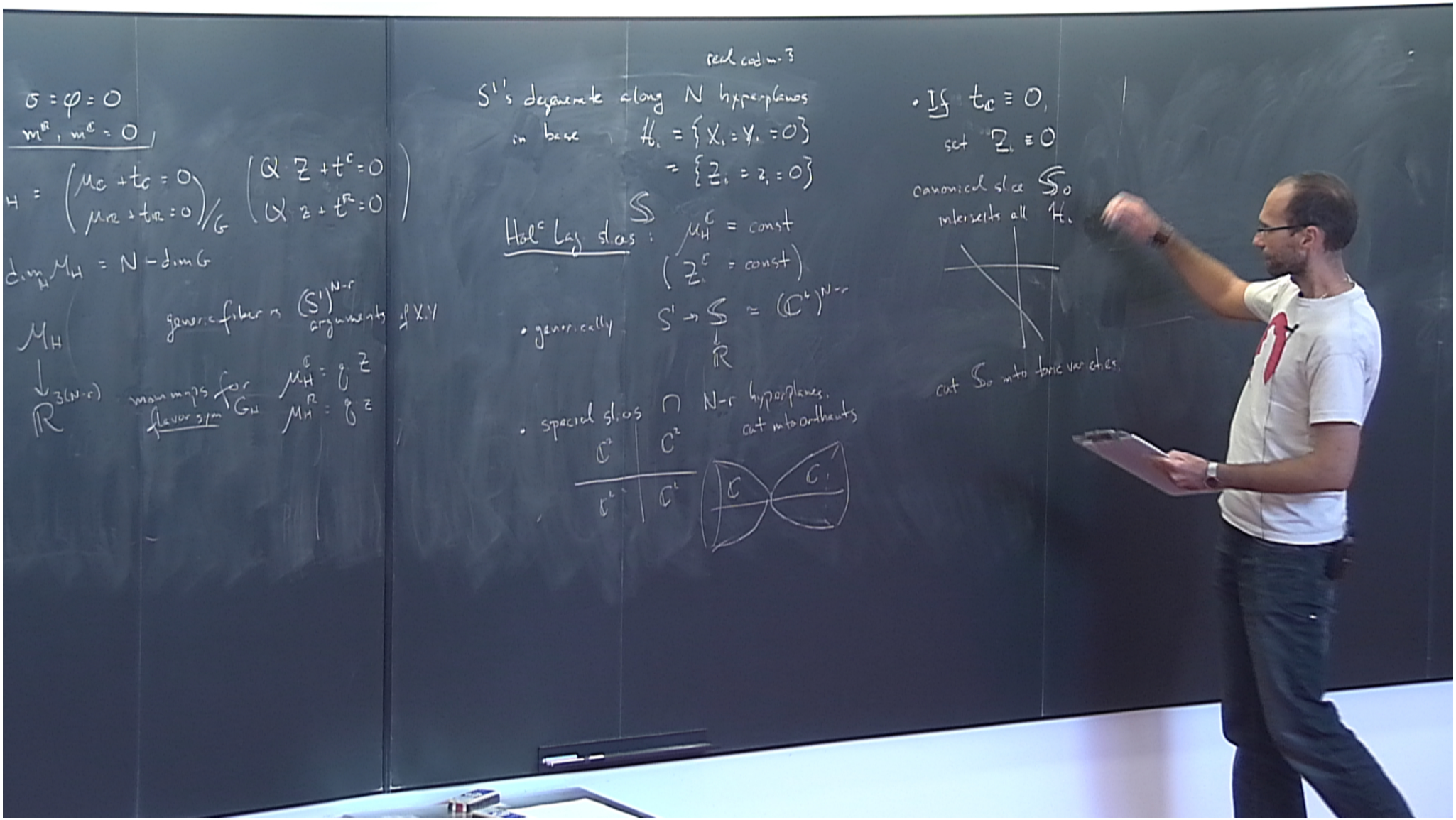


- If $t_C \equiv 0$, set $Z_i \equiv 0$

canonical slice $\mathcal{S}_0$ intersects all $\mathcal{H}_i$




$\sigma = \varphi = 0$
$m^R, m^C = 0$
$H = \left(\mu_C + t_C = 0\right)/G \quad \left(Q\,Z + t^C = 0,\ Q\,z + t^R = 0\right)$
$\dim_H \mathcal{M}_H = N - \dim G$
$\mathcal{M}_H$
$\mathbb{R}^{3(N-r)}$
generic fiber is $(S^1)^{N-r}$ arguments of X, Y
mom maps for flavor sym G_H
$\mu_H^C = g\,Z$
$\mu_H^R = g\,z$
real codim. 3
S^1's degenerate along N hyperplanes in base $\mathcal{H}_i = \{X_i = Y_i = 0\} = \{Z_i = z_i = 0\}$
Hol^C Lag slice $\mathcal{S}$: $\mu_H^C = \text{const}$ $(Z_i^C = \text{const})$
• generically $S^1 \to \mathcal{S} \to \mathbb{R}$ $\simeq (\mathbb{C}^*)^{N-r}$
• special slices $\cap$ $N-r$ hyperplanes, cut into components
$\mathbb{C}^2$ | $\mathbb{C}^2$
$\mathbb{C}^2$ | $\mathbb{C}^2$
$\mathbb{C}$ $\mathbb{C}$
• If $t_C \equiv 0$, set $Z_i \equiv 0$
canonical slice $\mathcal{S}_0$ intersects all $\mathcal{H}_i$


$\sigma = \varphi = 0$
$m^R, m^C = 0$
$H = \left(\mu_C + t_C = 0,\ \mu_{\mathbb{R}} + t_{\mathbb{R}} = 0\right)/G \quad \left(Q\,Z + t^C = 0,\ Q\,z + t^R = 0\right)$
$\dim_H \mathcal{M}_H = N - \dim G$
$\mathcal{M}_H$
$\downarrow$
$\mathbb{R}^{3(N-r)}$
generic fiber = $(S^1)^{N-r}$ arguments of X, Y
moment maps for flavor sym G_H
$\mu_H^C = Q\,Z$
$\mu_H^R = Q\,z$
real codim. 3
S^1's degenerate along N hyperplanes in base
$H_i = \{X_i = Y_i = 0\}$
$= \{Z_i = z_i = 0\}$
Hol^C Lag slice $\mathcal{S}$: $\mu_H^C = \text{const}$ $(Z_i^C = \text{const})$
• generically $S^1 \to \mathcal{S} \to \mathbb{R}$, $\mathcal{S} \simeq (\mathbb{C}^*)^{N-r}$
• special slices $\cap$ $N-r$ hyperplanes, cut into orthants
$\mathbb{C}^2$ | $\mathbb{C}^2$
$\mathbb{C}^2$ | $\mathbb{C}^2$
C C_i
• If $t_C \equiv 0$, set $Z_i \equiv 0$
canonical slice $\mathcal{S}_0$ intersects all H_i
cut $\mathcal{S}_0$ into toric varieties

real codim-3

S^1's degenerate along N hyperplanes in base $\quad H_i = \{X_i = Y_i = 0\} = \{Z_i = z_i = 0\}$

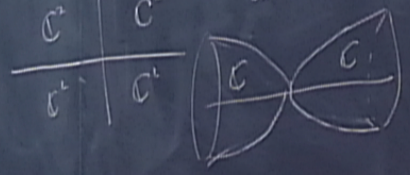
$$\begin{pmatrix} Q\cdot Z + t^{\mathbb{C}} = 0 \\ Q\cdot z + t^{\mathbb{R}} = 0 \end{pmatrix}$$

Hol$^{\mathbb{C}}$ Lag. slices of $\mathcal{S}$: $\mu_H^{\mathbb{C}} = \text{const}$ $\quad (Z_i^{\mathbb{C}} = \text{const})$

fiber is $(S^1)^{N-r}$ arguments of X, Y

for G_U $\quad \mu_H^{\mathbb{C}} = Q\cdot Z$, $\mu_H^{\mathbb{R}} = Q\cdot z$

- generically $\quad S^1 \to \mathcal{S} \to \mathbb{R} \simeq (\mathbb{C}^*)^{N-r}$
- special slices $\cap$ $N-r$ hyperplanes, cut into amoebas



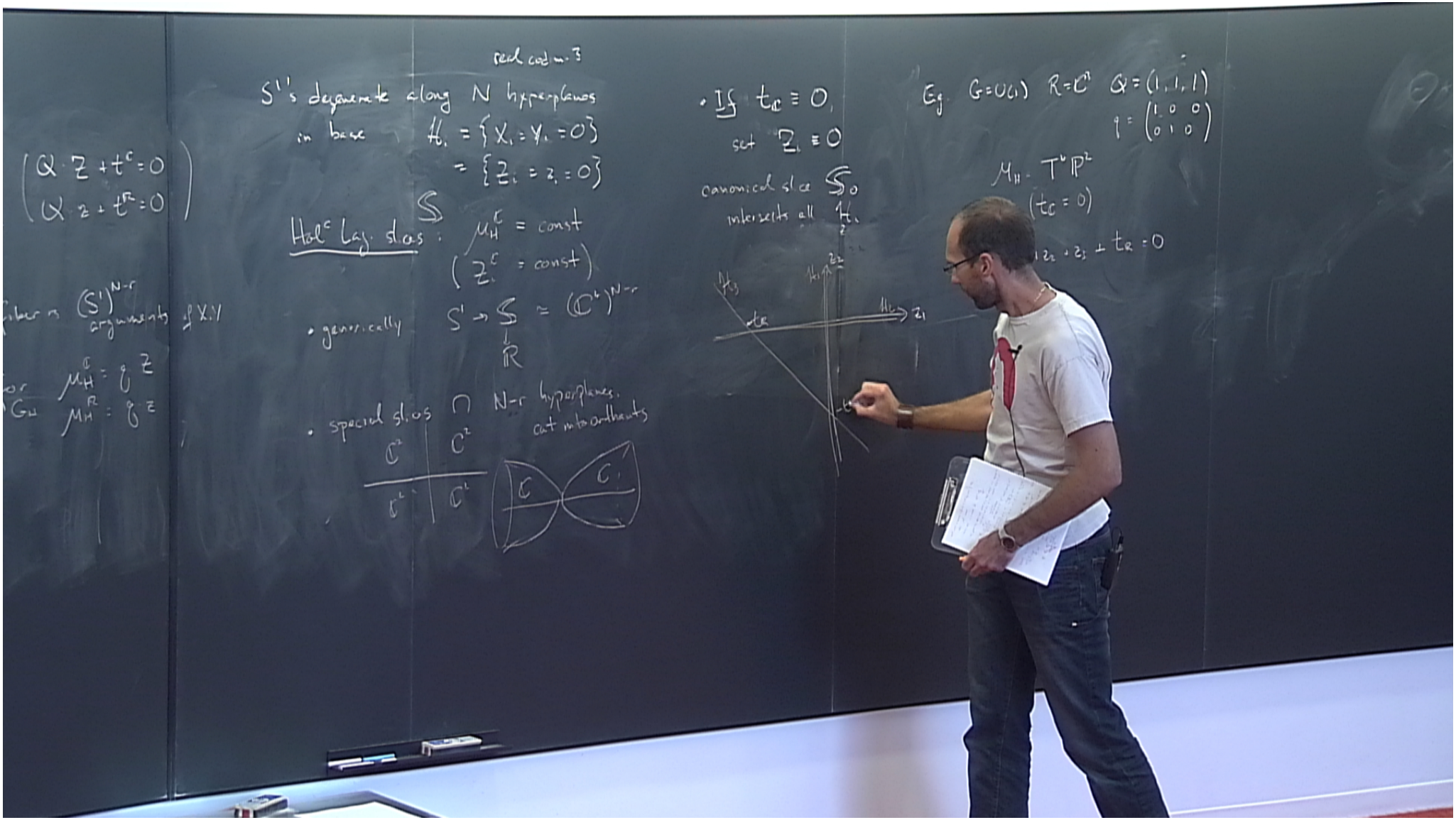
- If $t_{\mathbb{C}} \equiv 0$, set $Z_i \equiv 0$

canonical slice $\mathcal{S}_0$ intersects all H_i.



cut $\mathcal{S}_0$ into toric varieties.

Eg $G = U(1)$ $\quad R = \mathbb{C}^3$ $\quad Q = (1,1,1)$

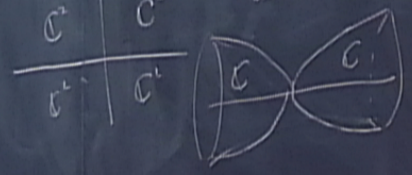

real codim 3
S^1's degenerate along N hyperplanes
in base $H_i = \{X_i = Y_i = 0\}$
$= \{z_i = z_i = 0\}$
Hol^c Lag. slices: $\mu_H^c = \text{const}$
$(z_i^c = \text{const})$
• generically $S^1 \to S \to \mathbb{R}$
$\simeq (\mathbb{C}^*)^{N-r}$
• special slices $\cap$ N-r hyperplanes,
cut into antheats
• If $t_R \equiv 0$,
set $z_i \equiv 0$
canonical slice S_0
intersects all H_i
Eg $G = U(1)$ $R = \mathbb{C}^3$ $Q = (1,1,1)$
$q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
$M_H = T^*\mathbb{P}^2$
$(t_C = 0)$
$|z_1|^2 + |z_2|^2 + |z_3|^2 - t_R = 0$
$Q \cdot z + t^c = 0$
$Q \cdot z + t^R = 0$
fiber = $(S^1)^{N-r}$ arguments $= |X_i|$
$\mu_H^c = q\,z$
$\mu_H^R = q\,|z|$

real codim-3

S^1's degenerate along N hyperplanes in base $H_i = \{X_i = Y_i = 0\} = \{Z_i = z_i = 0\}$

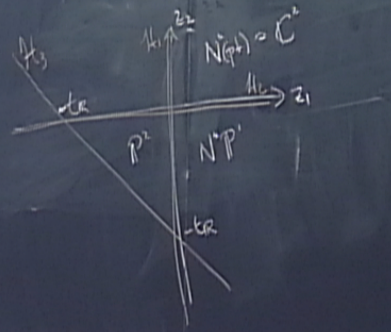
Hol^c Lag. slices $\mathbb{S}$: $\mu_H^c = \text{const}$ $(Z_i^c = \text{const})$

- generically $S^1 \to \mathbb{S} \to \mathbb{R} \simeq (\mathbb{C}^*)^{N-r}$
- special slices $\cap$ $N-r$ hyperplanes, cut into orthants



- If $t_{\mathbb{C}} \equiv 0$, set $Z_i \equiv 0$

canonical slice $\mathbb{S}_0$ intersects all H_i



Eg $G = U(1)$ $R = \mathbb{C}^3$ $Q = (1, 1, 1)$ $q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

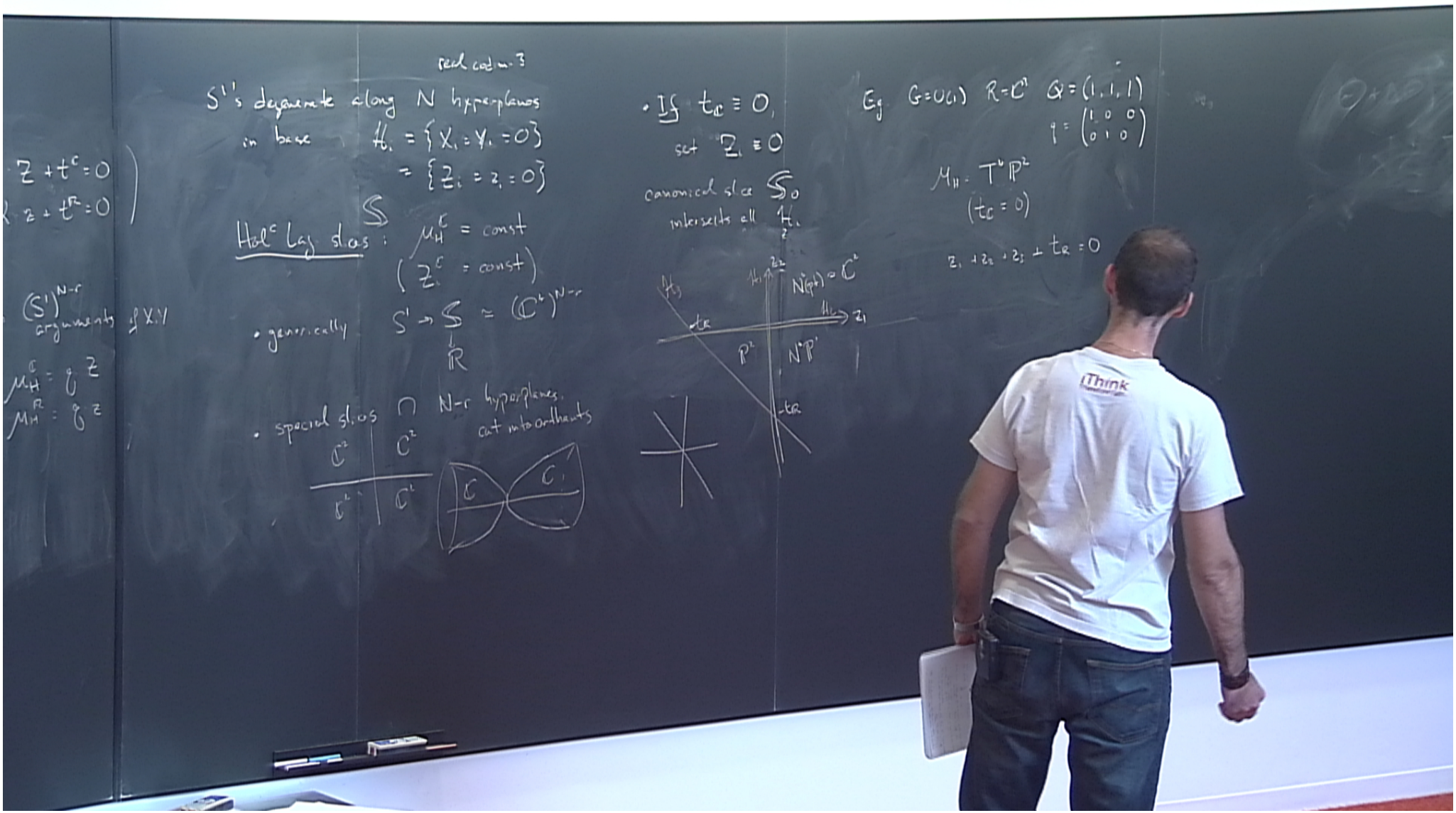
$\mathcal{M}_H = T^* \mathbb{P}^2$ $(t_{\mathbb{C}} = 0)$

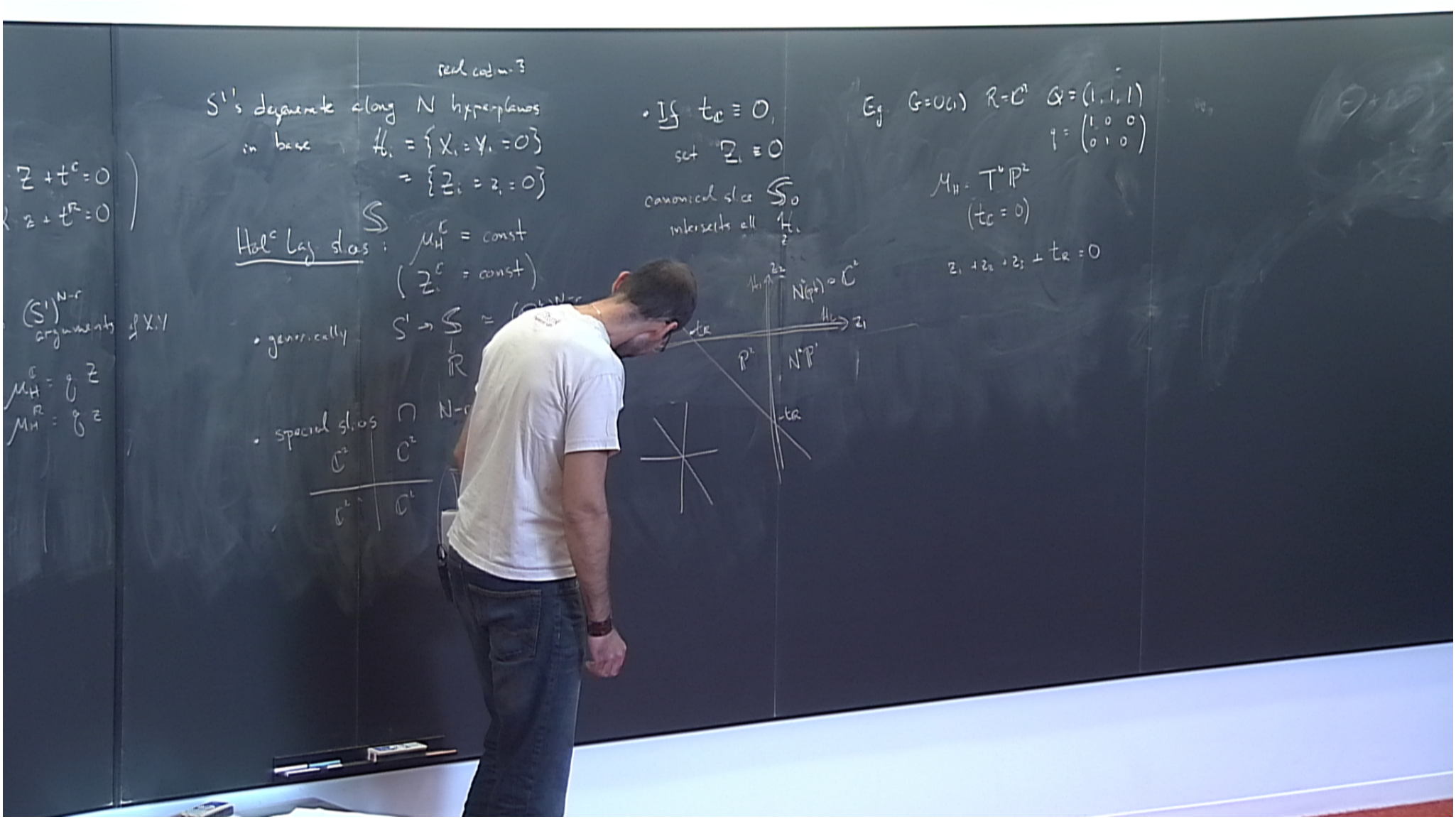
$z_1 + z_2 + z_3 \perp t_R = 0$

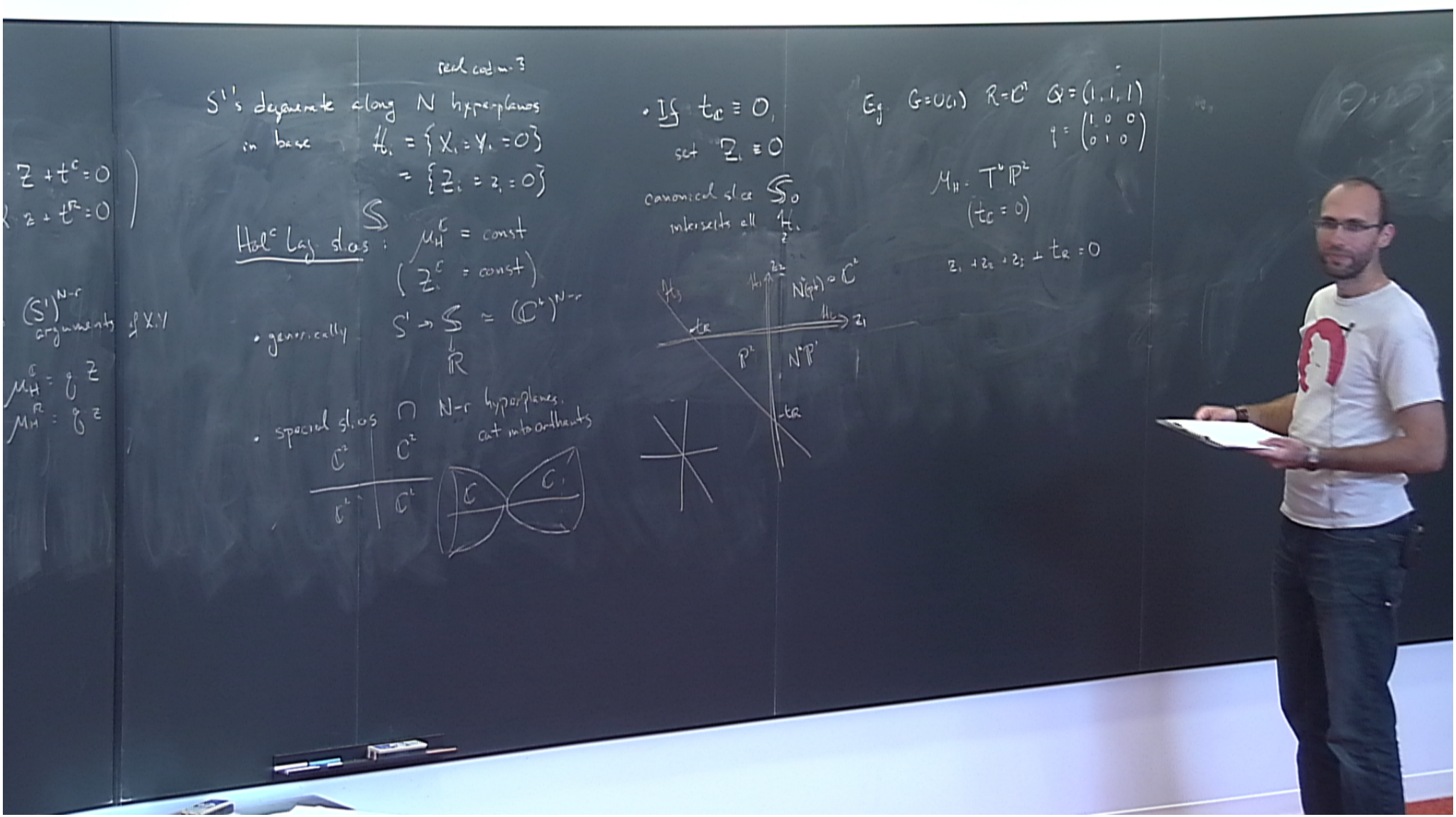
$\begin{pmatrix} Q \cdot Z + t^{\mathbb{C}} = 0 \\ Q \cdot z + t^R = 0 \end{pmatrix}$

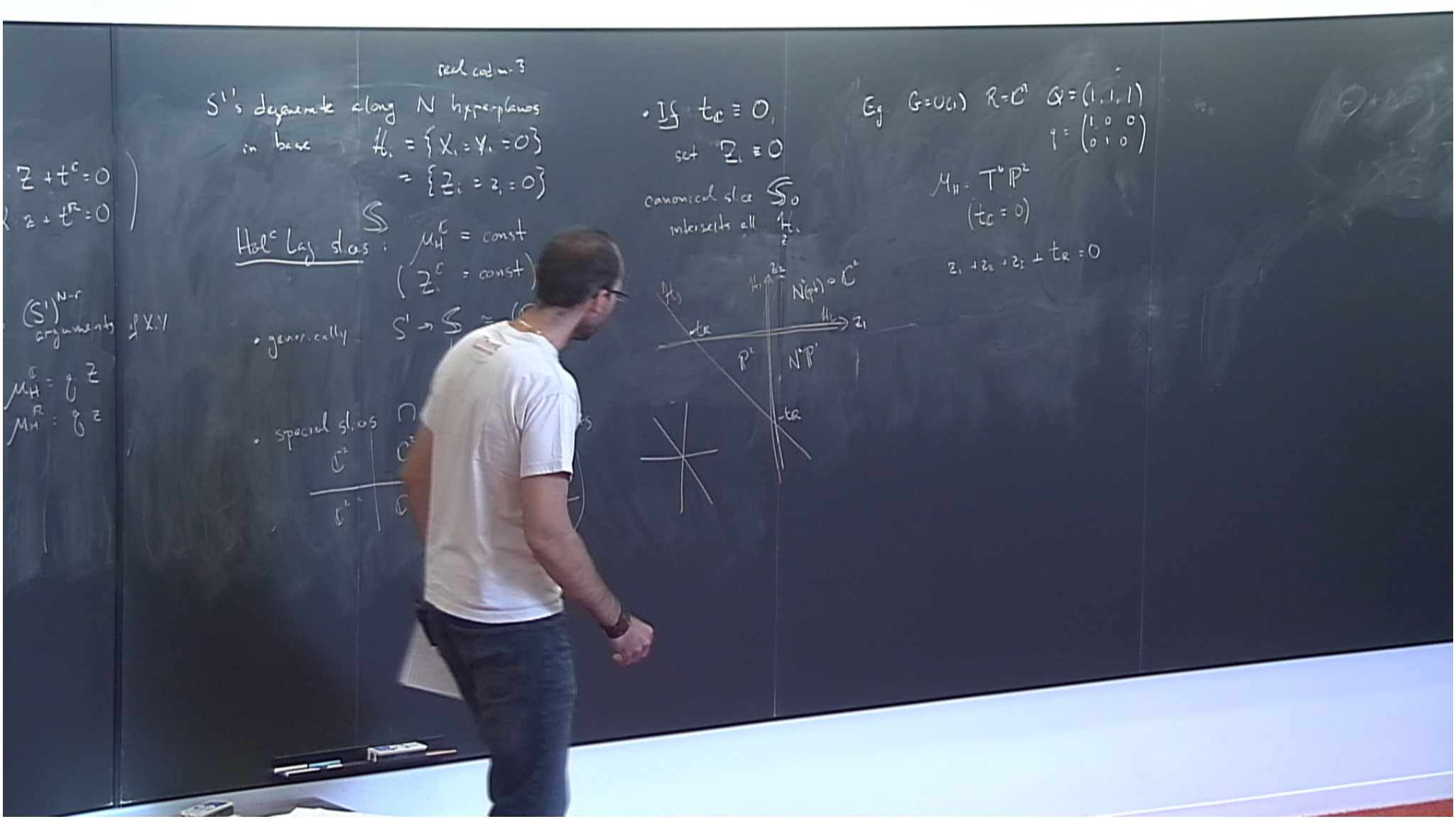
fibers $(S^1)^{N-r}$ arguments $|X_i|$

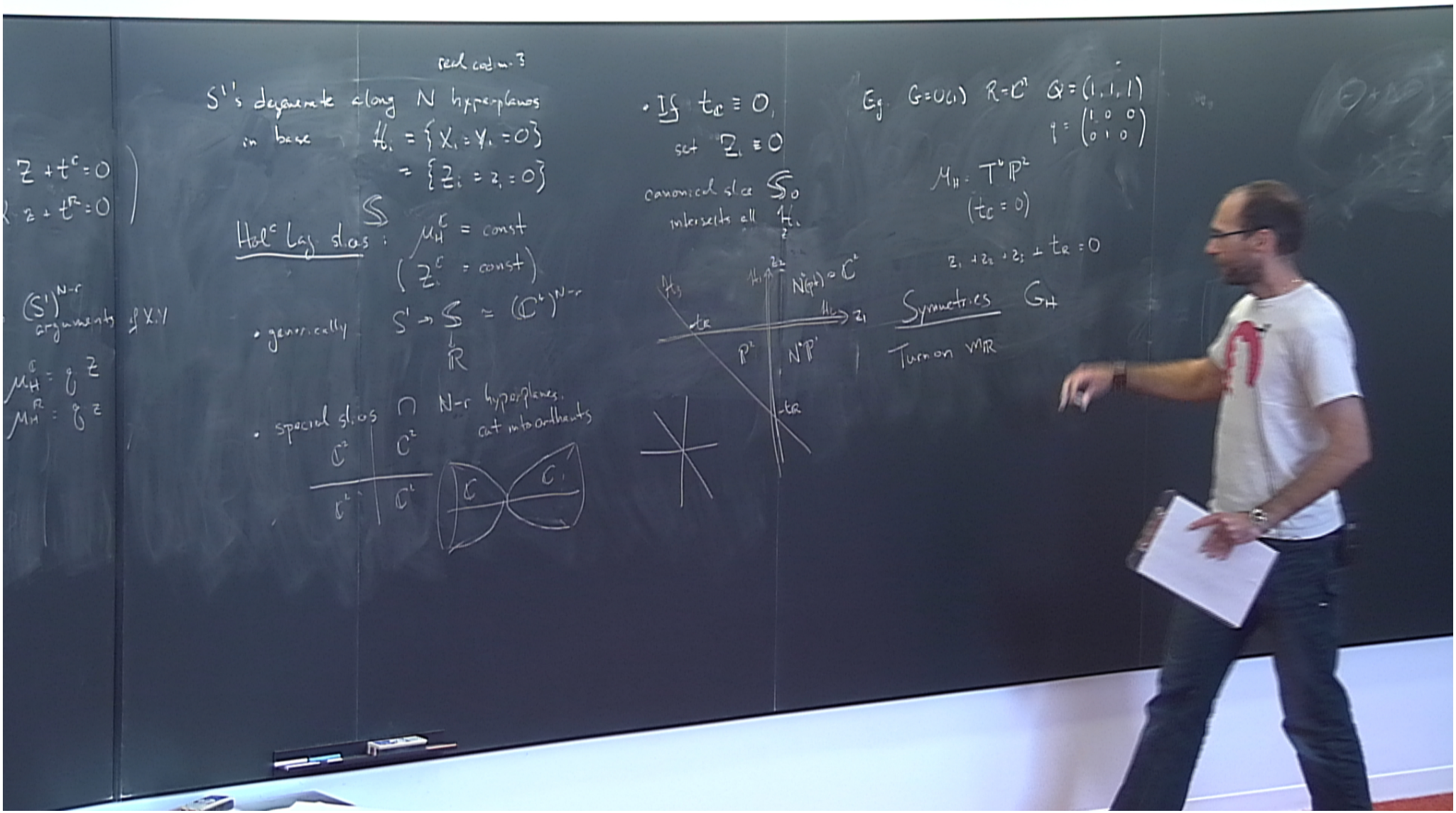
G_H $\mu_H^{\mathbb{C}} = q \cdot Z$ $\mu_H^R = q \cdot z$

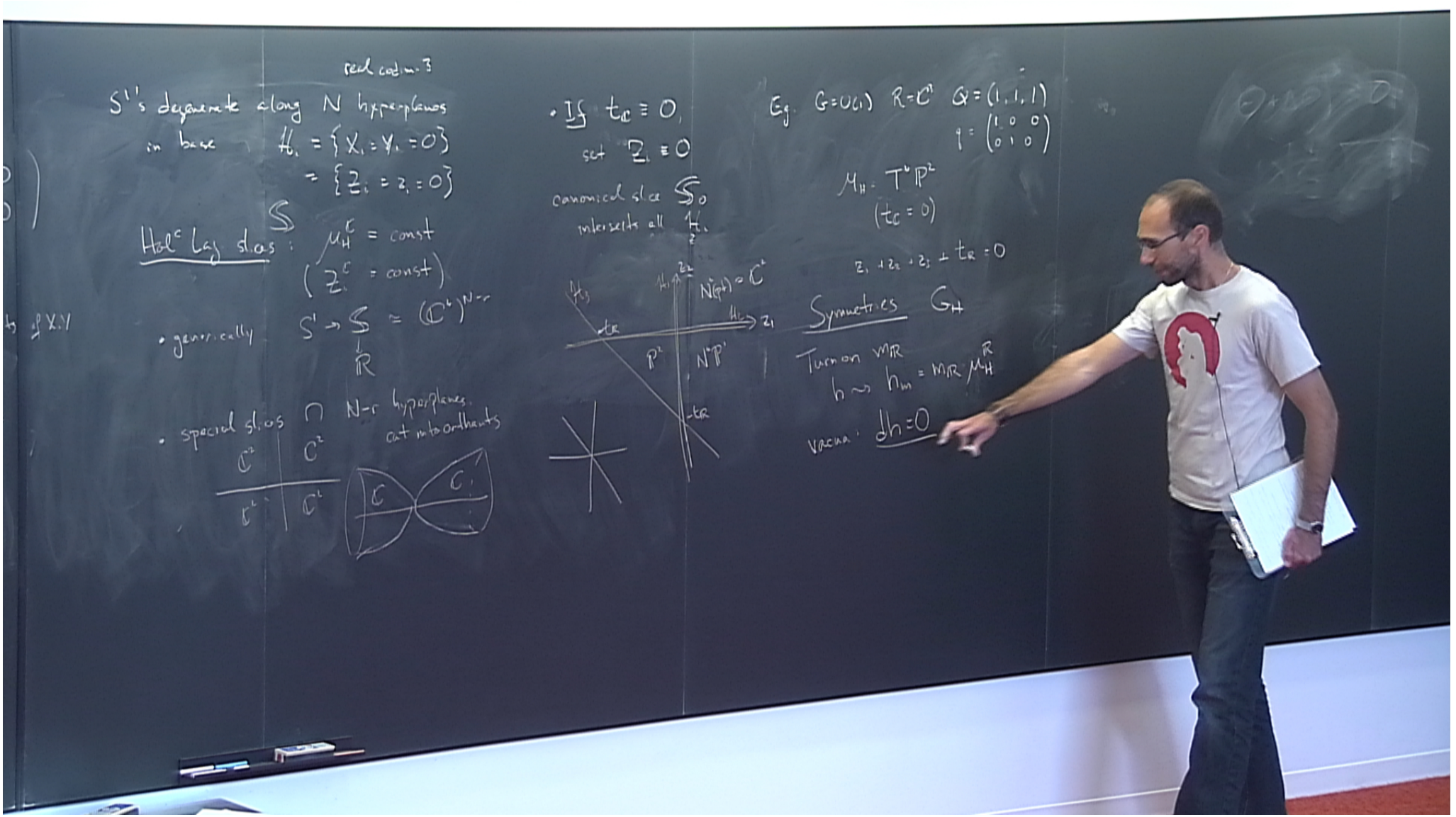

real codim 3
S^1's degenerate along N hyperplanes
in base $H_i = \{X_i = Y_i = 0\}$
$= \{\Sigma_i = z_i = 0\}$
Hal^c Lag slice S: μ_H^c = const
(Σ_i^c = const)
• generically $S^1 \to S \to \mathbb{R}$ $\simeq (\mathbb{C}^*)^{N-r}$
• special slices $\cap$ N-r hyperplanes, cut into onthants
$\mathbb{C}^2$ $\mathbb{C}^2$
$\mathbb{C}^2$ $\mathbb{C}^2$
C C'
• If $t_C = 0$,
set $\Sigma_i = 0$
canonical slice S_0
intersects all H_i
$N(p^1) = \mathbb{C}^2$
$-t_R$
z_1
$\mathbb{P}^2$
$N^*\mathbb{P}^1$
$-t_R$
Eg $G = U(1)$ $R = \mathbb{C}^3$ $Q = (1,1,1)$
$q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
$\mathcal{M}_H = T^*\mathbb{P}^2$
$(t_C = 0)$
$z_1 + z_2 + z_3 - t_R = 0$
$\Sigma + t^c = 0$
$\Sigma + t^R = 0$
$(S^1)^{N-r}$ arguments of X, Y
$\mu_H^c = q\,\Sigma$
$\mu_H^R = q\,z$

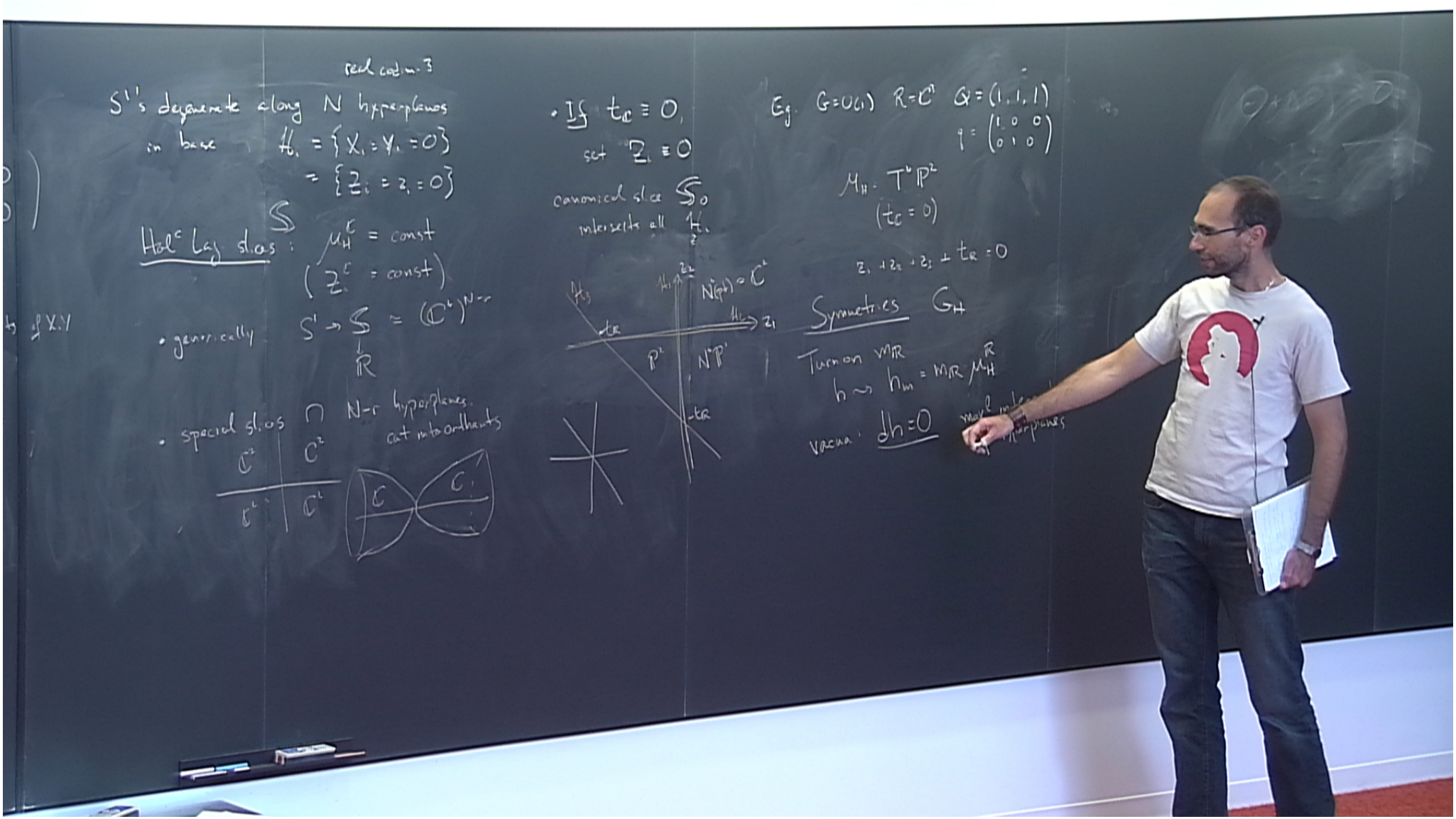

real codim-3
S^1's degenerate along N hyperplanes
in base $H_i = \{X_i = Y_i = 0\}$
$= \{z_i = 0\}$
Hal^c Lag slices: $\mu_H^c = \text{const}$
$(z_i^c = \text{const})$
• generically $S^1 \to S \to \mathbb{R}$
• special slices
• If $t_c = 0$,
set $\Sigma = 0$
canonical slice S_0
intersects all H_i
Eg $G = U(1)$ $R = \mathbb{C}$ $Q = (1,1,1)$
$\rho = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
$\mu_H = T^*\mathbb{P}^2$
$(t_c = 0)$
$z_1 + z_2 + z_3 - t_R = 0$
$N(\rho) = \mathbb{C}^2$
$\mathbb{P}^2$
$N^*\mathbb{P}^1$
$(S^1)^{N-r}$ arguments of X, Y
$\mu_H^c = g^c z$
$\mu_H^R = g^R z$

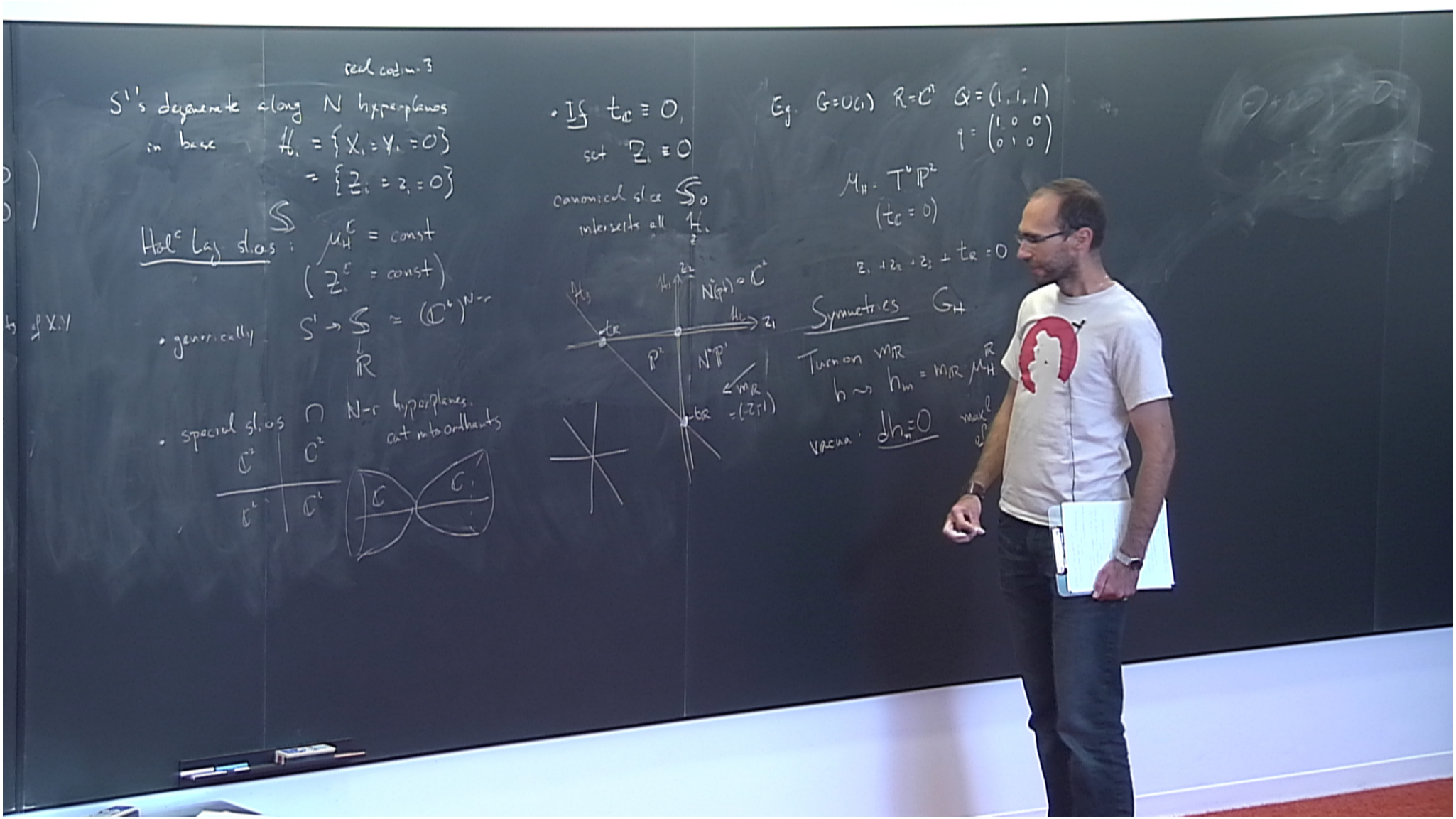

real codim 3
S^1's degenerate along N hyperplanes
in base $H_i = \{X_i = Y_i = 0\}$
$= \{Z_i = z_i = 0\}$
Hal^c Lag slices: $\mu_H^c = \text{const}$
$(Z_i^c = \text{const})$
• generically $S^1 \to S \to \mathbb{R} \simeq (\mathbb{C}^*)^{N-r}$
• special slices $\cap$ N-r hyperplanes, cut into orthants
• If $t_C = 0$, set $\Sigma_i = 0$
canonical slice S_0 intersects all H_i
$N(p) \simeq \mathbb{C}^2$
$-t_R$
$\mathbb{P}^2$
$N^*\mathbb{P}^1$
Eg $G = U(1)$ $R = \mathbb{C}^3$ $Q = (1,1,1)$
$\gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
$\mu_H : T^*\mathbb{P}^2$
$(t_C = 0)$
$z_1 + z_2 + z_3 + t_R = 0$
$Z + t^C = 0$
$z + t^R = 0$
$(S^1)^{N-r}$ arguments of X, Y
$\mu_H^C = \gamma Z$
$\mu_H^R = \gamma z$

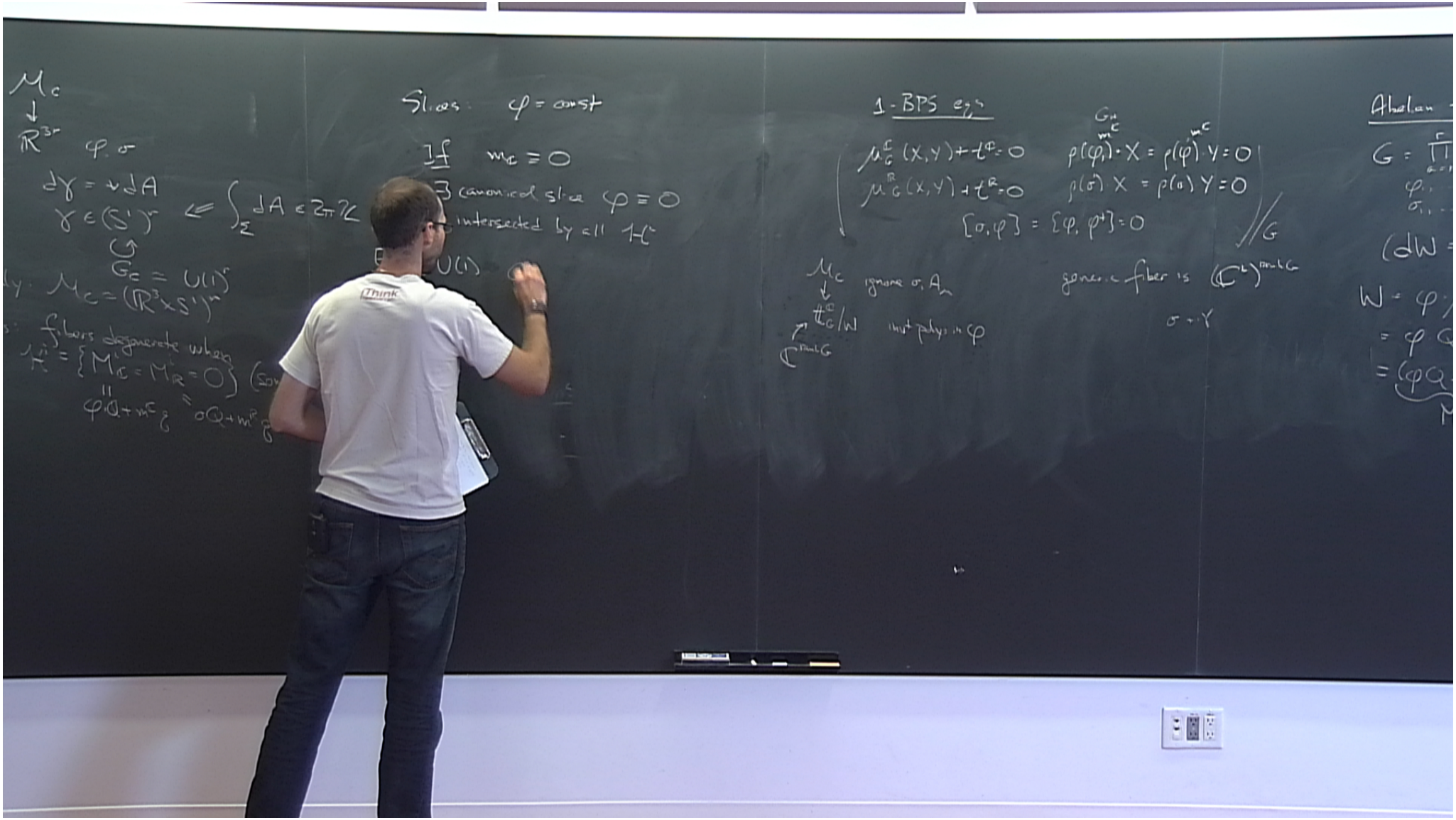

real codim-3
S^1's degenerate along N hyperplanes in base
$H_i = \{X_i = Y_i = 0\} = \{z_i = z_i = 0\}$
Hol^c Lag sl.cs: μ_H^C = const
z_i^C = const
• generically $S^1 \to S$
• special sl.cs
• If $t_C \equiv 0$, set $\Sigma_i \equiv 0$
canonical slice S_0 intersects all H_i
$N(pt) = \mathbb{C}^2$
$-t_R$
z_1
z_2
$\mathbb{P}^2$
$N^*\mathbb{P}^1$
Eg $G = U(1)$ $R = \mathbb{C}^3$ $Q = (1,1,1)$
$\gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
$\mathcal{M}_H = T^*\mathbb{P}^2$
$(t_C = 0)$
$z_1 + z_2 + z_3 - t_R = 0$
$(S^1)^{N-r}$ arguments of X, Y
$\mu_H^C = \sum_i Q_i |z_i|^2$
$\mu_H^R = \sum_i Q_i z_i$

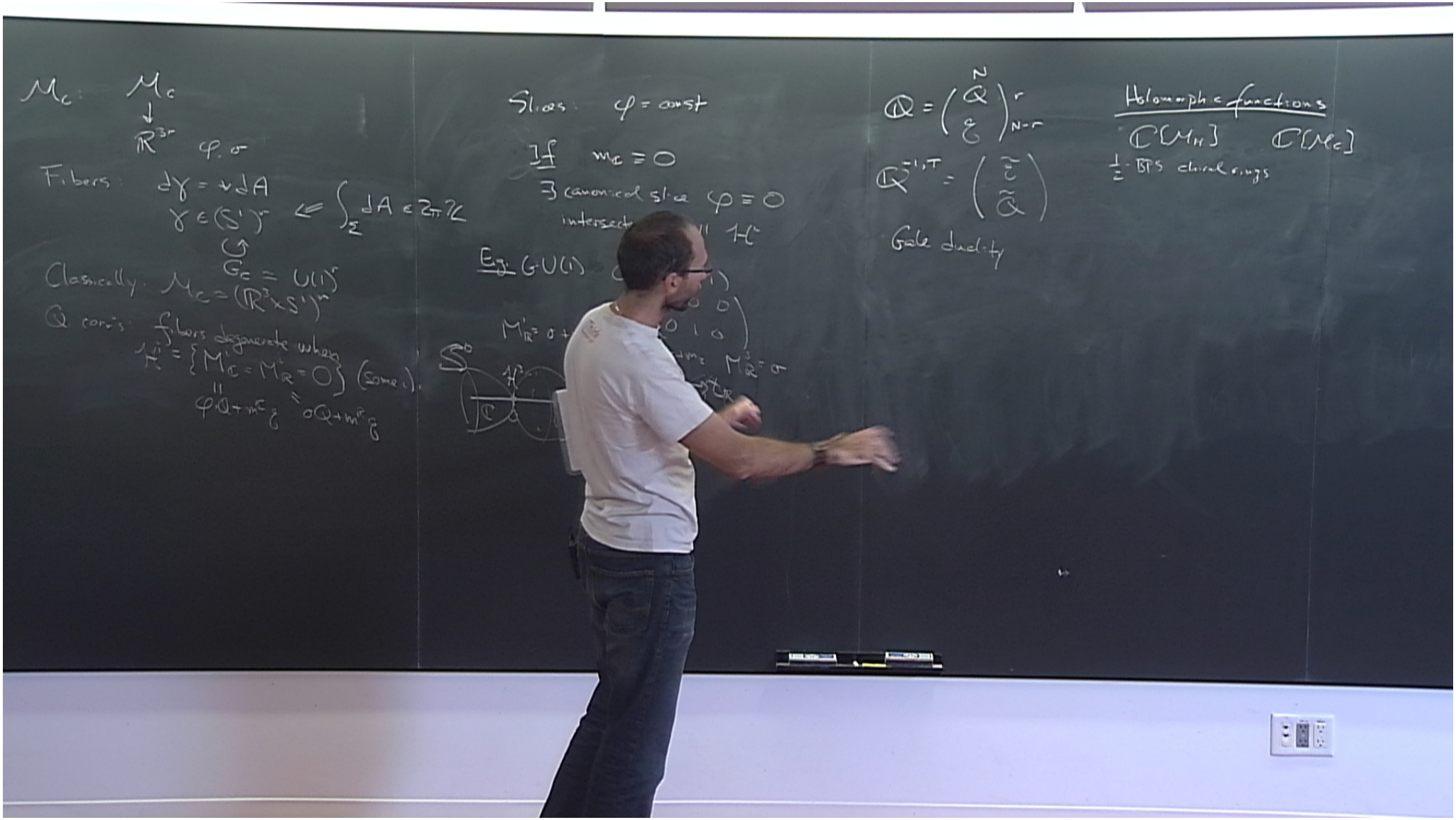

real codim-3
S^1's degenerate along N hyperplanes in base $H_i = \{X_i = Y_i = 0\} = \{Z_i = z_i = 0\}$
Hal^c Lag slice S: $\mu_H^c = \text{const}$ $(Z_i^c = \text{const})$
• generically $S^1 \to S \to \mathbb{R}$ $\simeq (\mathbb{C}^*)^{N-r}$
• special slices $\cap$ N-r hyperplanes, cut into annuli
$Z + t^c = 0$
$z + t^R = 0$
$(S^1)^{N-r}$ arguments of X_i
$\mu_H^c = \arg Z$
$\mu_H^R = |Z|$
• If $t_c = 0$, set $\Sigma = 0$
canonical slice S_0 intersects all H_i
$N(pt) = \mathbb{C}^2$
$\mathbb{P}^2$
$N^*\mathbb{P}^1$
$-t_R$
Eg $G = U(1)$ $R = \mathbb{C}$ $Q = (1,1,1)$
$\gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
$\mathcal{M}_H = T^*\mathbb{P}^2$
$(t_c = 0)$
$z_1 + z_2 + z_3 - t_R = 0$
Symmetries G_H
Turn on m_R

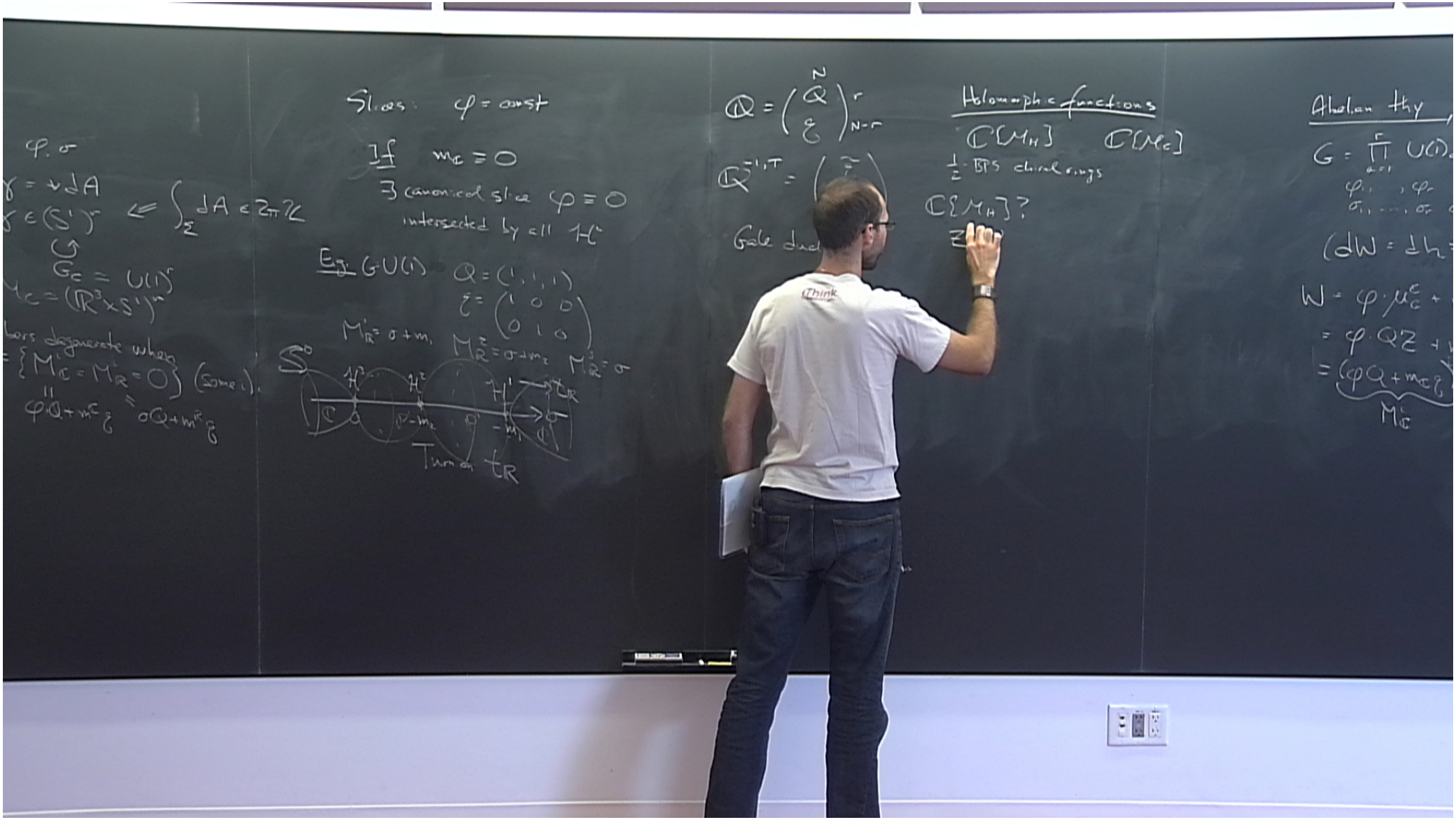

real codim-3
S^1's degenerate along N hyperplanes in base $H_i = \{X_i = Y_i = 0\} = \{Z_i = z_i = 0\}$
Hol^c Lag slices S: $\mu_H^C = \text{const}$ $(Z_i^C = \text{const})$
• generically $S^1 \to S \to \mathbb{R}$ $\simeq (\mathbb{C}^*)^{N-r}$
• special slices $\cap$ $N-r$ hyperplanes cut into orthants
$\mathbb{C}^2$ $\mathbb{C}^2$ $\mathbb{C}^2$ $\mathbb{C}^2$ C C'
• If $t_C \equiv 0$, set $Z_i \equiv 0$
canonical slice S_0 intersects all H_i
H_3 H_1 H_2 z_2 $N(\mathbb{P}^1) = \mathbb{C}^2$ z_1 t_R $\mathbb{P}^2$ $N^*\mathbb{P}^1$ $-t_R$
Eg. $G = U(1)$ $R = \mathbb{C}^3$ $Q = (1,1,1)$ $q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
$\mathcal{M}_H = T^*\mathbb{P}^2$ $(t_C = 0)$
$z_1 + z_2 + z_3 - t_R = 0$
Symmetries G_H
Turn on m_R $h \rightsquigarrow h_m = m_R \mu_H^R$
vacua: $dh = 0$

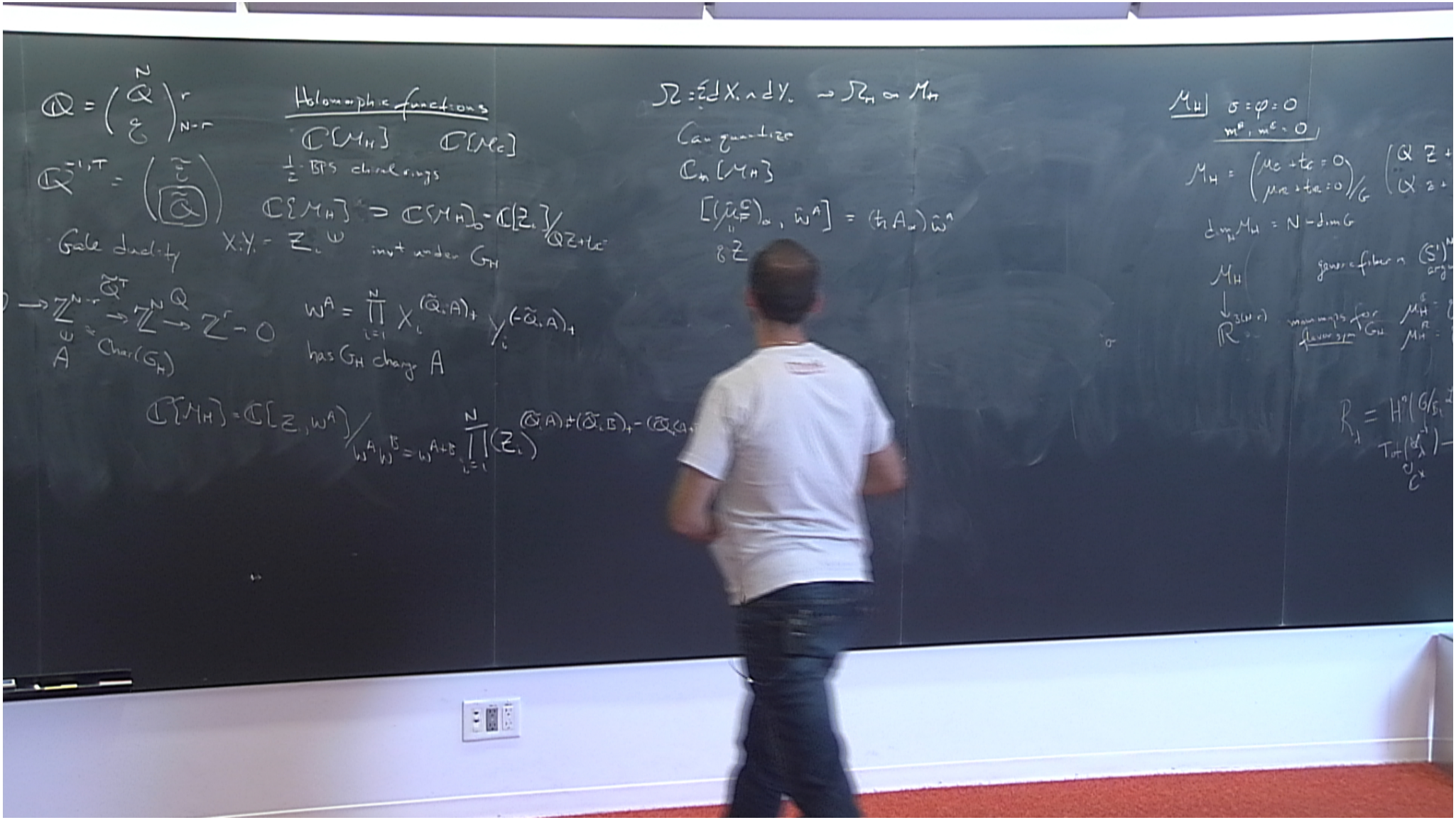

real codim-3
S^1's degenerate along N hyperplanes in base $H_i = \{X_i = Y_i = 0\} = \{Z_i = z_i = 0\}$
Hol^c Lag slices S: $\mu_H^c = \text{const}$ ($Z_i^c = \text{const}$)
• generically $S^1 \to S \simeq (\mathbb{C}^*)^{N-r}$ over $\mathbb{R}$
• special slices: $N-r$ hyperplanes cut into orthants
• If $t_C \equiv 0$, set $Z_i \equiv 0$
canonical slice S_0 intersects all H_i
$N(q) = \mathbb{C}^2$
$\mathbb{P}^2$
$N^*\mathbb{P}^1$
Eg. $G = U(1)$ $R = \mathbb{C}^3$ $Q = (1,1,1)$ $q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
$\mathcal{M}_H = T^*\mathbb{P}^2$ $(t_C = 0)$
$z_1 + z_2 + z_3 - t_R = 0$
Symmetries G_H
Turn on m_R
$h \rightsquigarrow h_m = m_R \mu_H^R$
vacua: $dh = 0$

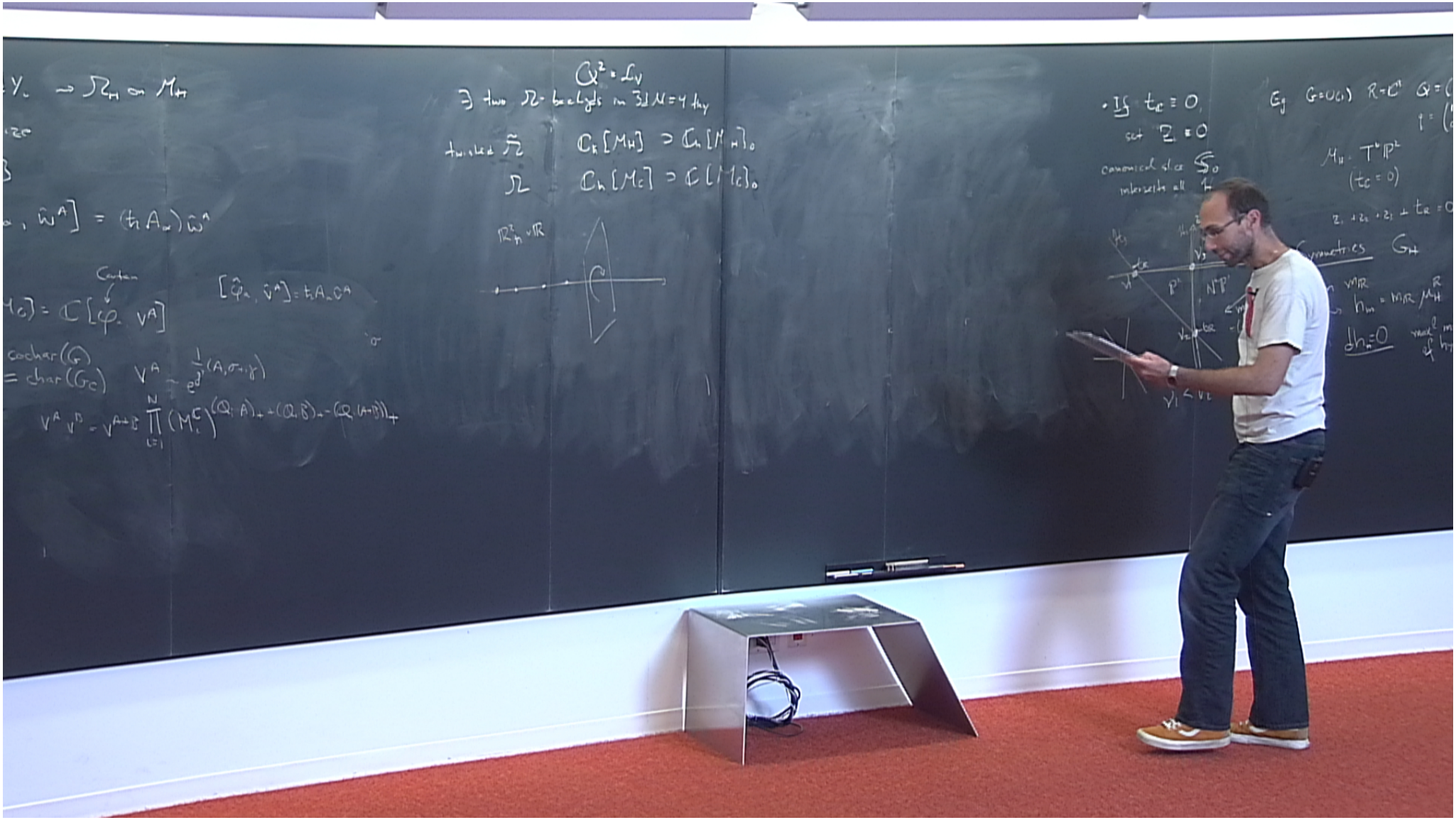

real codim-3
S^1's degenerate along N hyperplanes in base $H_i = \{X_i = Y_i = 0\} = \{Z_i = z_i = 0\}$
Hol^c Lag slices S: $\mu_H^C = \text{const}$ $(Z_i^C = \text{const})$
• generically $S^1 \to S \simeq (\mathbb{C}^*)^{N-r}$
$\downarrow \mathbb{R}$
• special slices $\cap$ N-r hyperplanes cut into ordinaries
• If $t_C \equiv 0$, set $Z_i \equiv 0$
canonical slice S_0 intersects all H_i
$N(\mathbb{P}^1) = \mathbb{C}^2$
$\mathbb{P}^2$
$N^*\mathbb{P}^1$
$m_R = (-Z:1)$
Eg. $G = U(1)$ $R = \mathbb{C}^3$ $Q = (1,1,1)$ $q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
$\mathcal{M}_H = T^*\mathbb{P}^2$ $(t_C = 0)$
$z_1 + z_2 + z_3 - t_R = 0$
Symmetries G_H
Twisted m_R
$h \rightsquigarrow h_m = m_R/\mu_H^R$
Vacua: $dh_m = 0$

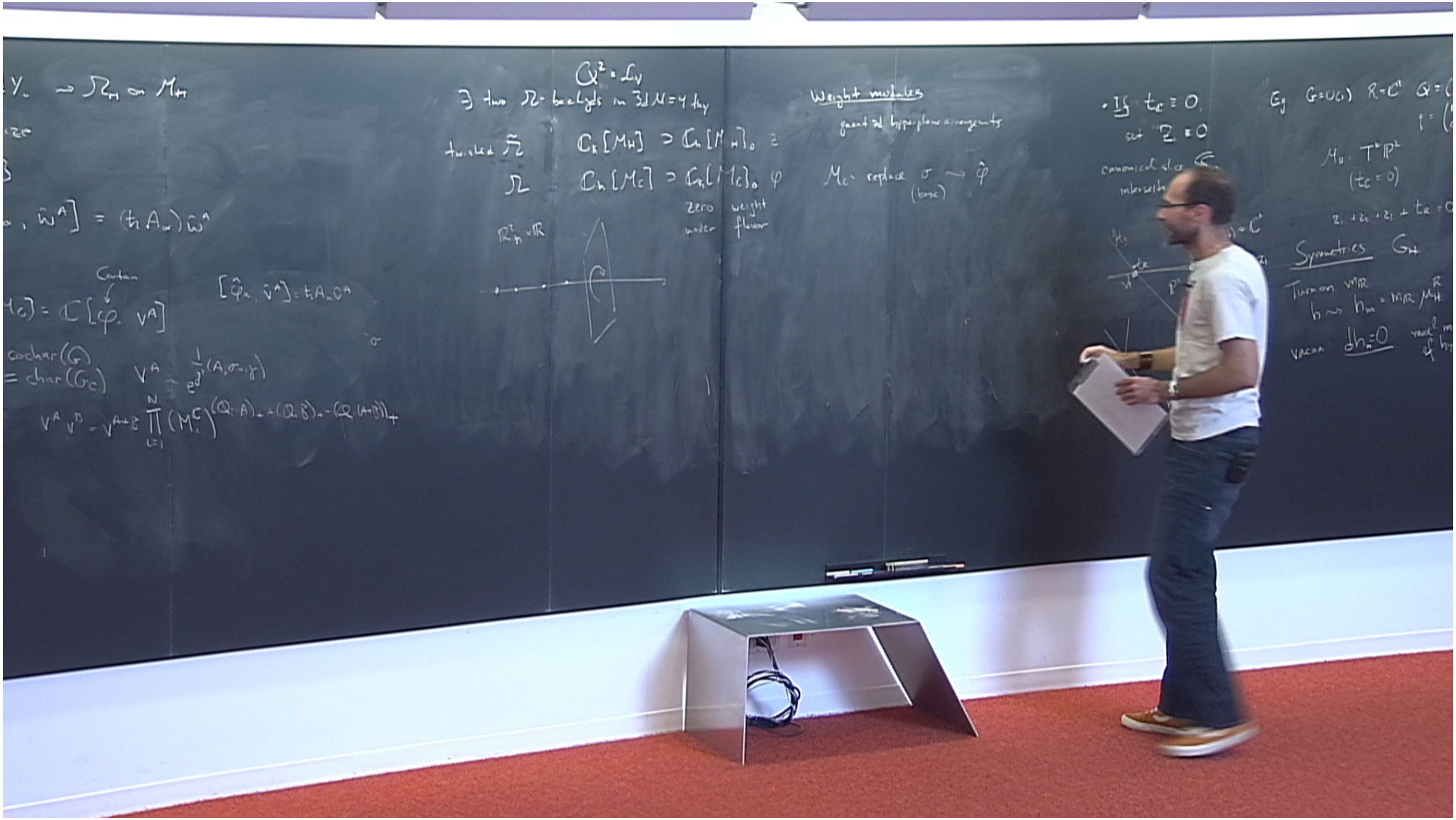

Slices φ = const
If
1-BPS eq's
Abelian
ignore σ, A
generic fiber is
intersected by
canonical slice

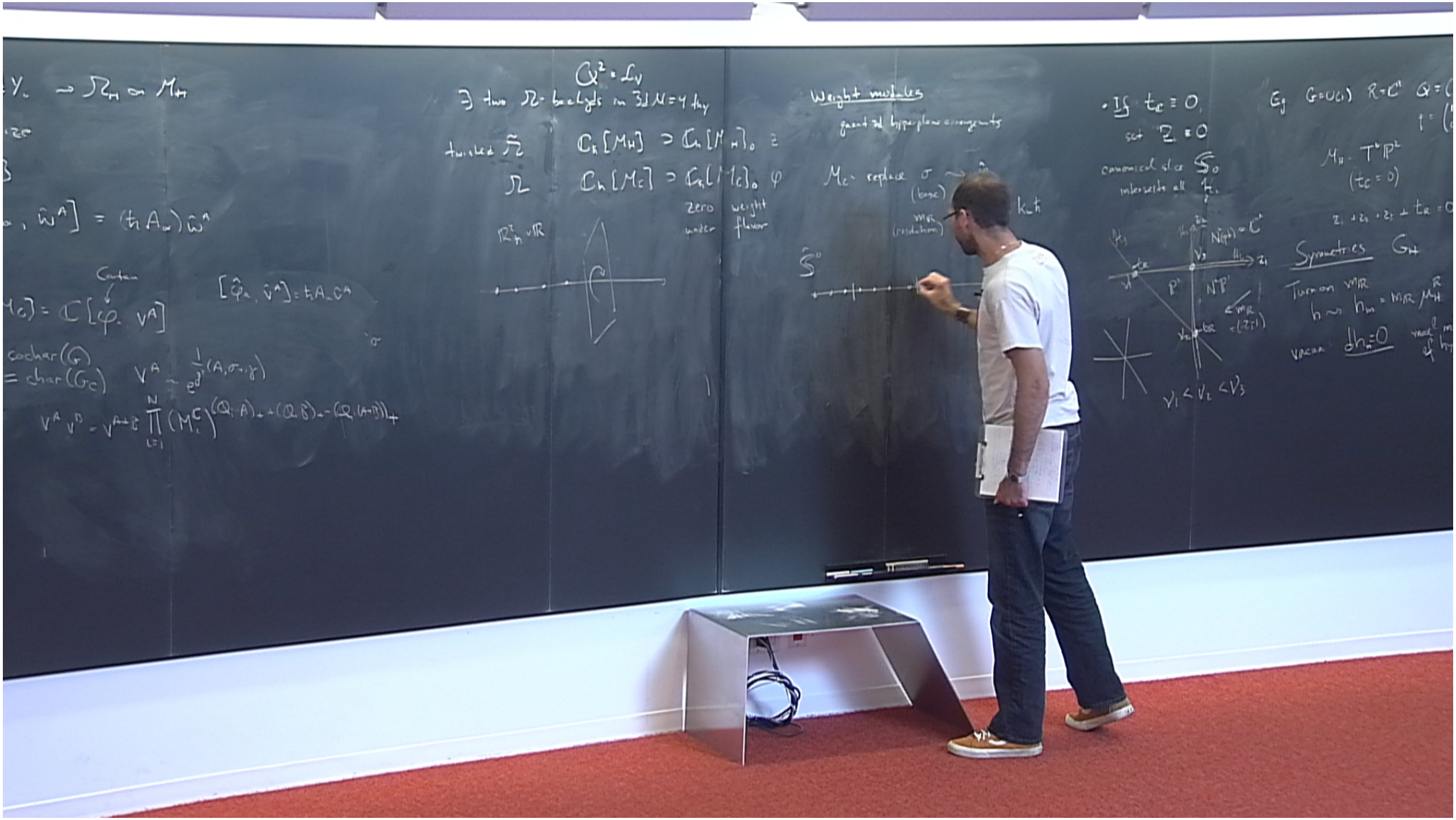

$\mathcal{M}_C$: $\mathcal{M}_C$
$\downarrow$
$\mathbb{R}^{3r}$ φ, σ
Fibers: $d\gamma = *dA$
$\gamma \in (S^1)^r$ $\Leftarrow$ $\int_\Sigma dA \in 2\pi\mathbb{Z}$
$\circlearrowleft$
$G_C = U(1)^r$
Classically: $\mathcal{M}_C = (\mathbb{R}^3 \times S^1)^r$
Q corr's: fibers degenerate when
$\mathcal{H}^i = \{M_C^i = M_R^i = 0\}$ (some i).
$\varphi \cdot Q + m_C \zeta = \sigma Q + m_R \zeta$
Slices: φ = const
If $m_C = 0$
$\exists$ canonical slice $\varphi = 0$
$\mathcal{Q} = \begin{pmatrix} N \\ Q \\ \mathcal{E} \end{pmatrix}^r_{N-r}$
$\mathcal{Q}^{-1,T} = \begin{pmatrix} \tilde{\mathcal{E}} \\ \tilde{Q} \end{pmatrix}$
Gale duality
Holomorphic functions
$\mathbb{C}[\mathcal{M}_H]$ $\mathbb{C}[\mathcal{M}_C]$
$\frac{1}{2}$-BPS chiral rings

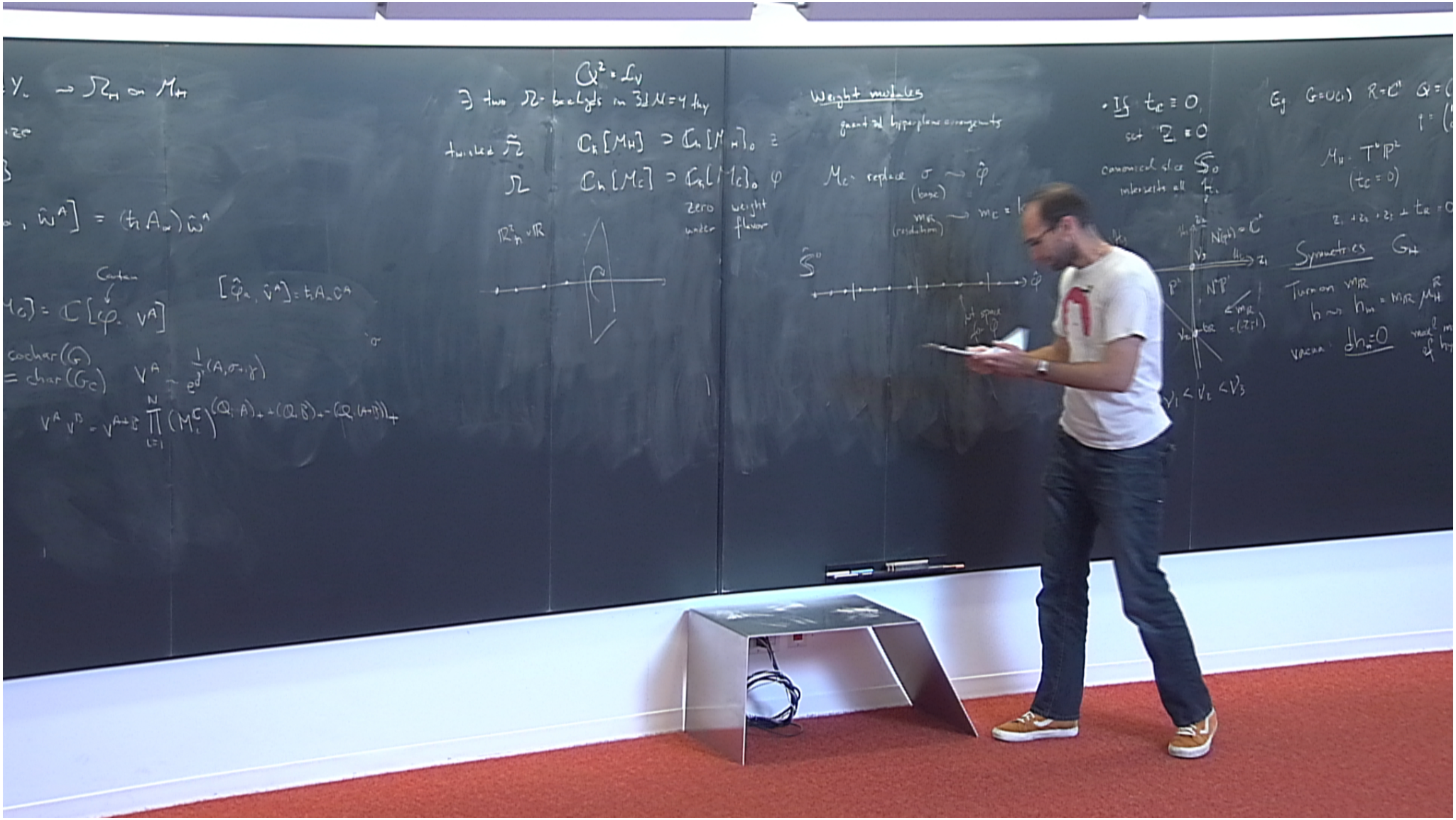

Slices $\varphi = const$
If $m_C = 0$
$\exists$ canonical slice $\varphi = 0$
intersected by all $\mathcal{H}^i$
Holomorphic functions
$\mathbb{C}[\mathcal{M}_H]$ $\mathbb{C}[\mathcal{M}_C]$
Abelian thy
Turn on t_R

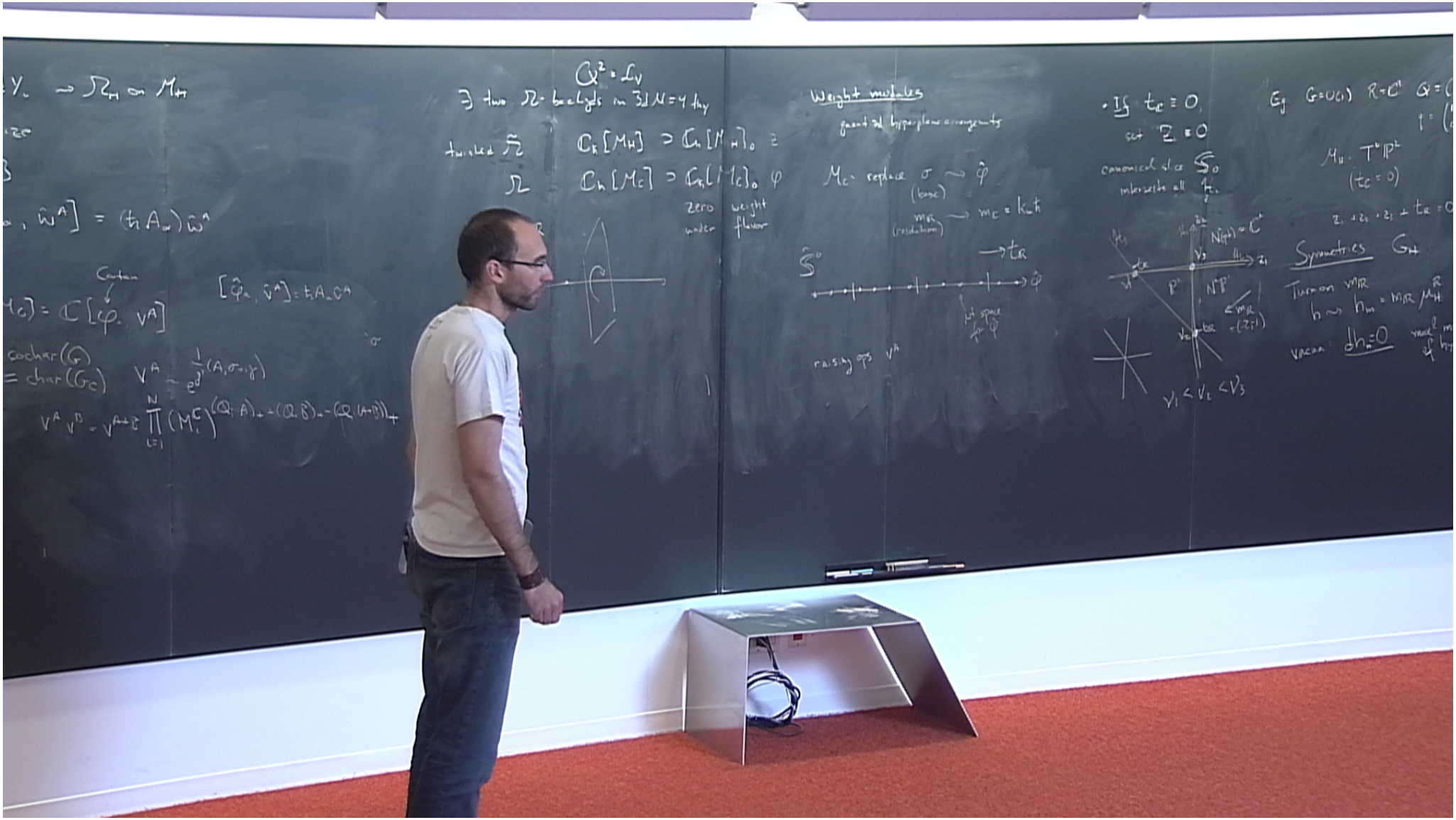

Holomorphic functions
Gale duality
has G_H charge A

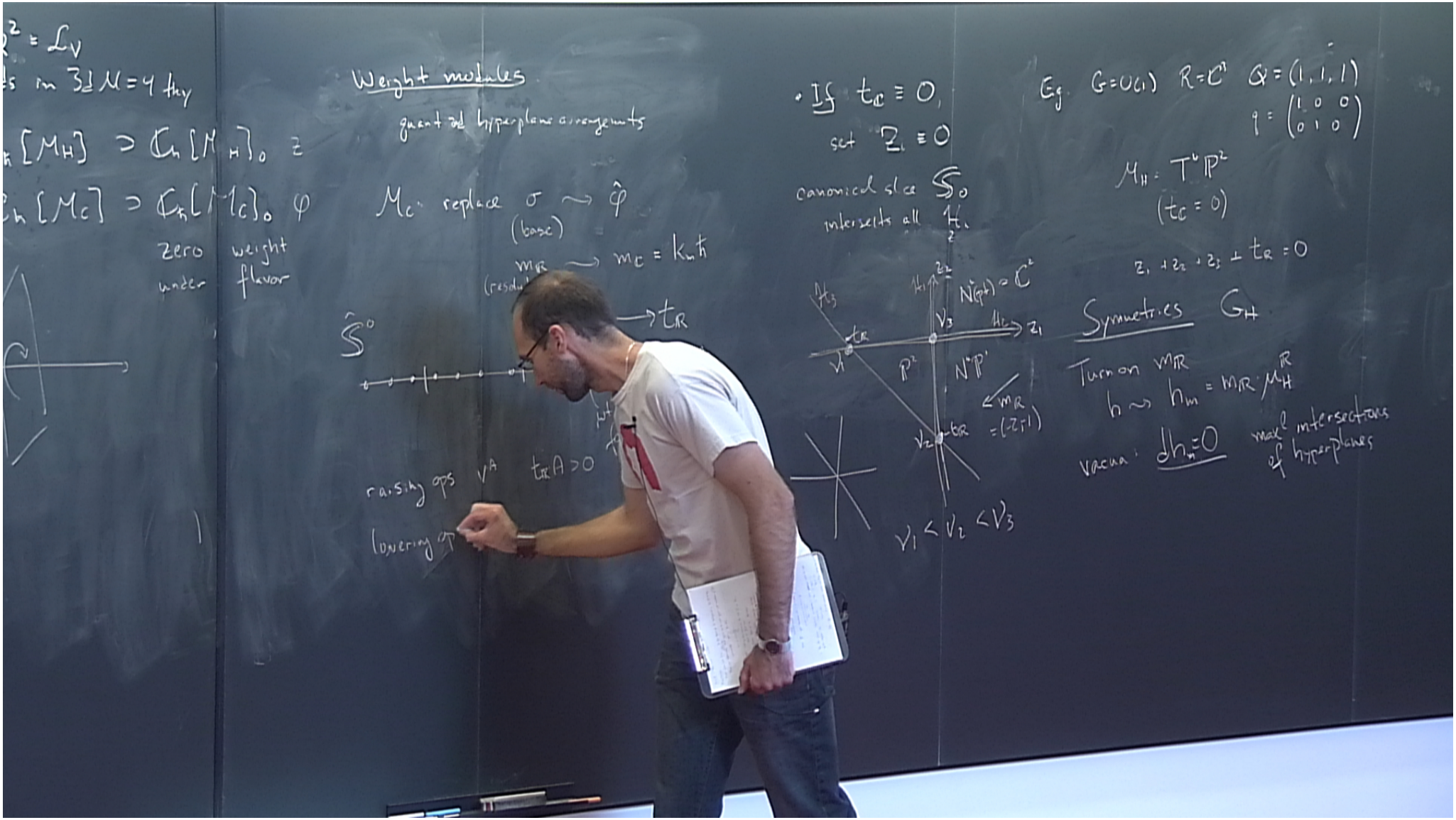

$Q^2 = \mathcal{L}_V$
$\exists$ two Ω-backgds in 3d $\mathcal{N}=4$ thy
twisted $\widetilde{\Omega}$ $\quad \mathbb{C}_R[M_H] \supset \mathbb{C}_R[M_H]_0$
Ω $\quad \mathbb{C}_R[M_C] \supset \mathbb{C}[M_C]_0$
$[\hat{\varphi}_a, \hat{v}^A] = t_i A_a{}^A$
$V^A V^B = V^{A+B} \prod_{i=1}^{N} (M_i^C)^{(Q_i A)_+ + (Q_i B)_+ - (Q_i (A+B))_+}$
If $t_{\mathbb{C}} = 0$,
set $\Sigma = 0$
canonical slice $\mathcal{S}_0$
intersects all
M_H: $T^*\mathbb{P}^1$
$(t_{\mathbb{C}} = 0)$
$z_1 + z_2 + z_3 + t_{\mathbb{R}} = 0$
symmetries G_H
$h_m = \mu_R / M_H$
$dh_m = 0$

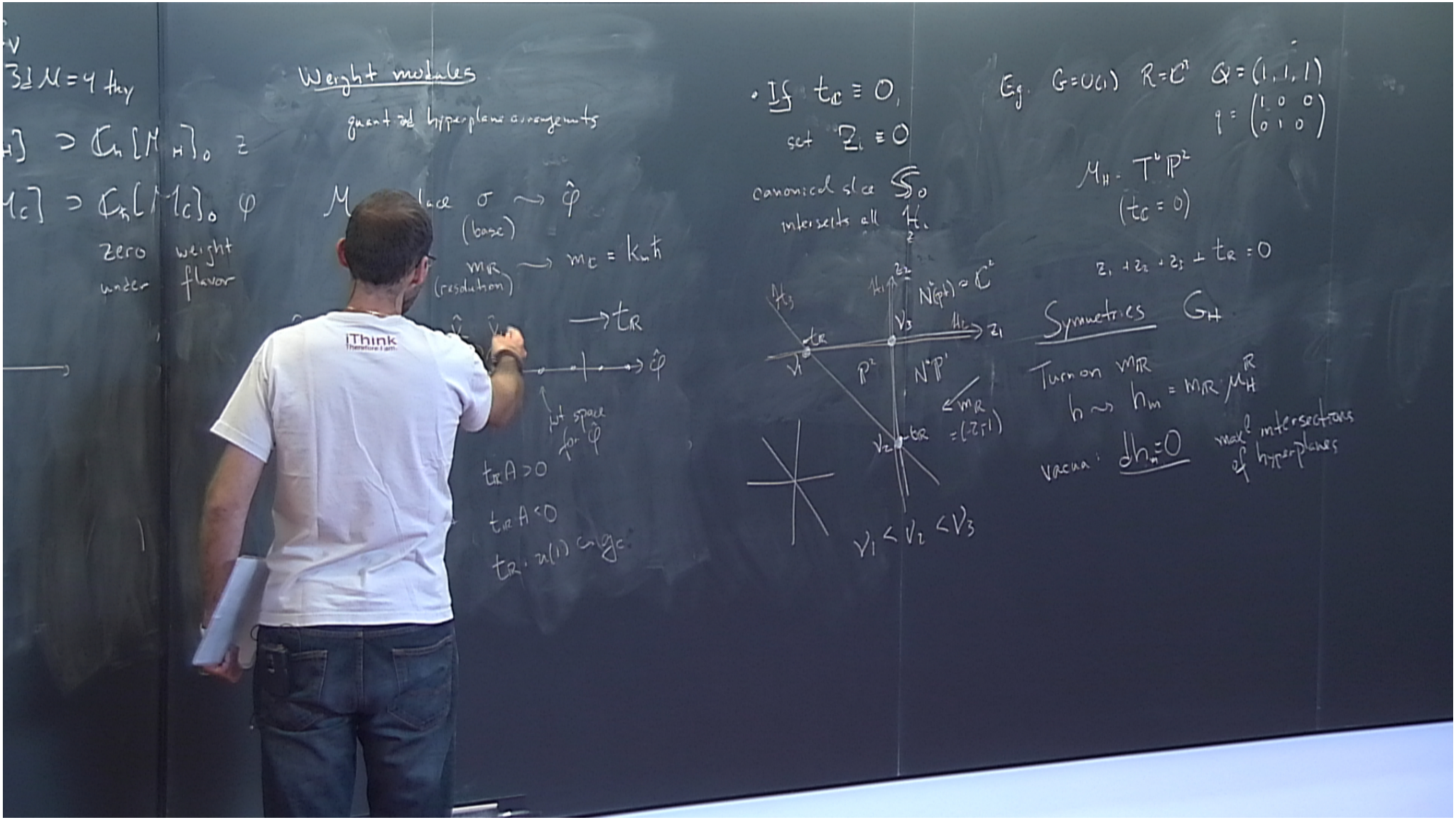

$Q^2 = \mathcal{L}_V$
∃ two Ω-backgrounds in 3d N=4 thy
twisted Ω
Ω
Weight modules
zero
weight
flavour
Symmetries
Coulomb
Cartan
vacua

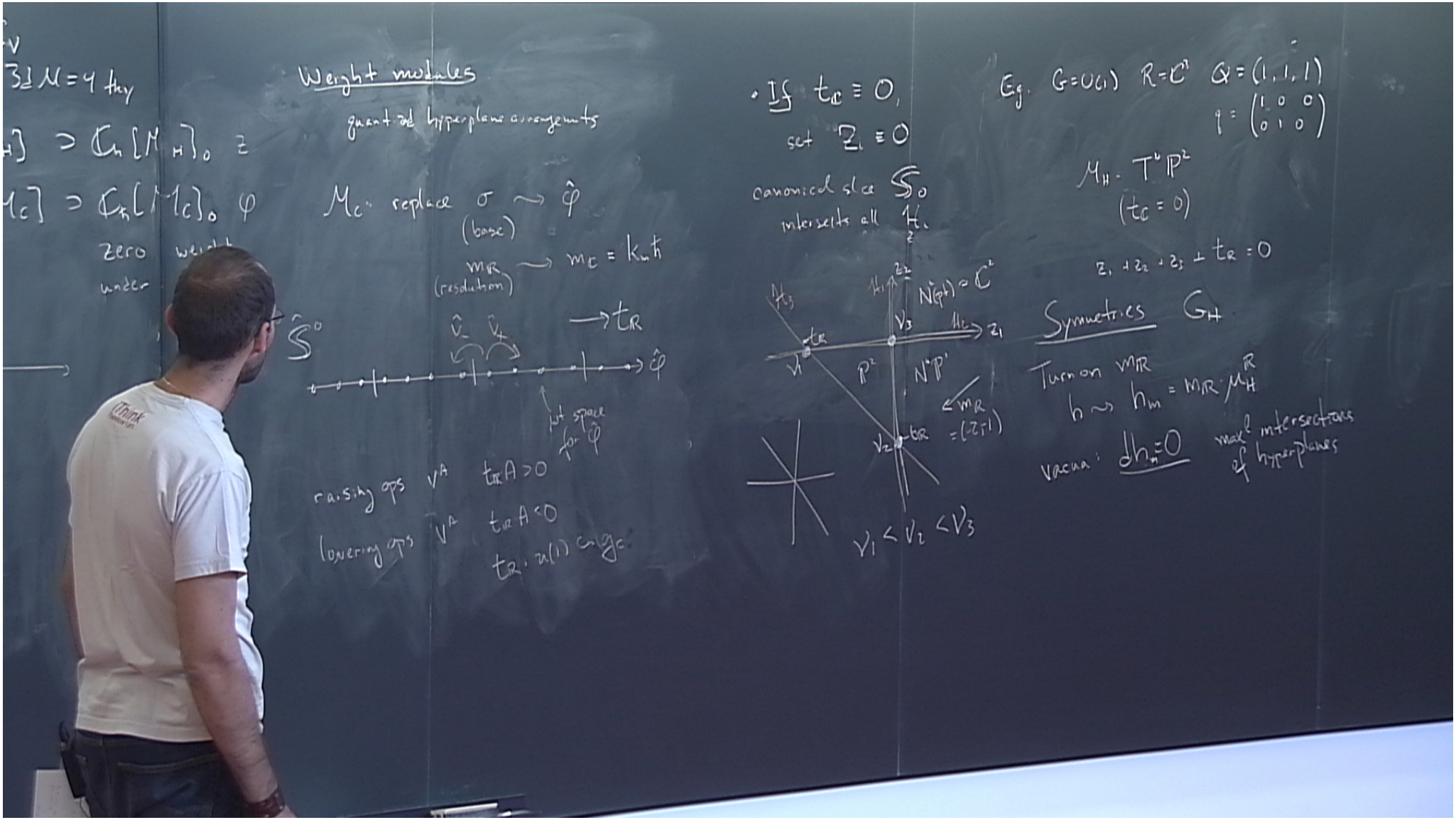

Weight modules
If $t_{\mathbb{C}} = 0$,
set $\Sigma = 0$
Symmetries
Turn on vMR
$h \rightsquigarrow h_m = m_{MR}$
vacua: $dh_m = 0$

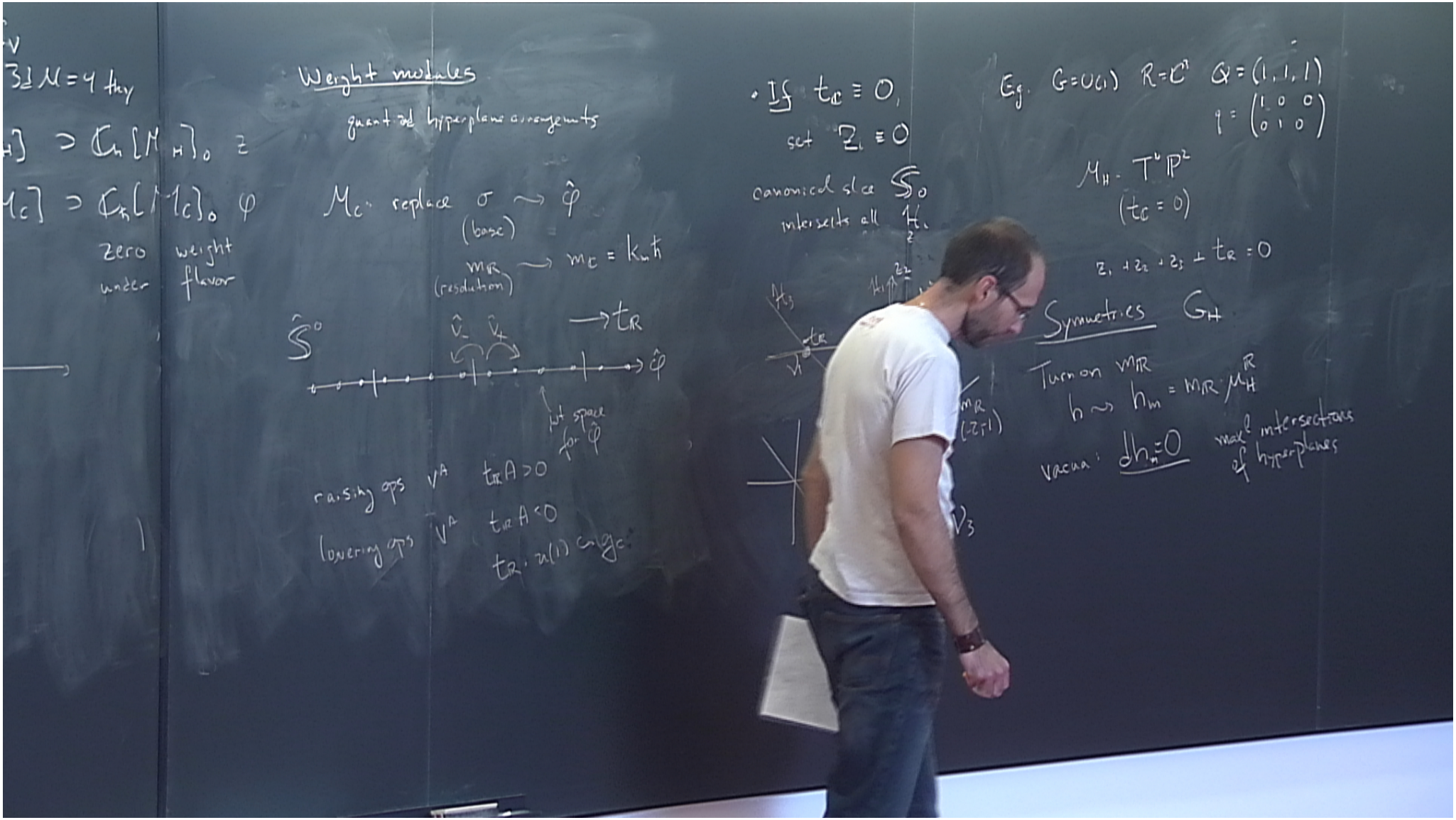

Weight modules
Symmetries

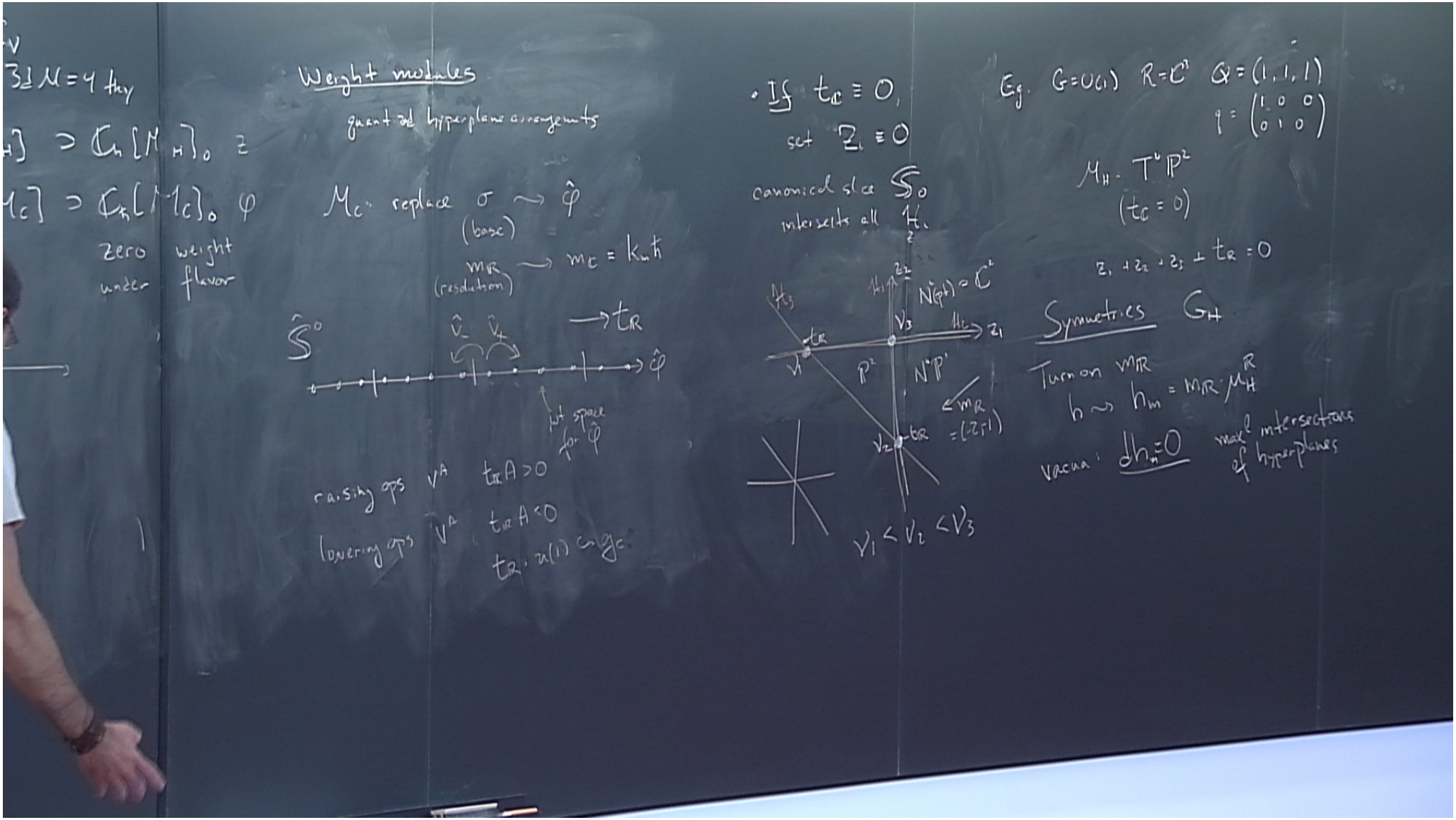

Weight modules
If $t_C = 0$,
set $\Sigma = 0$
Symmetries
Pirsa: 15100066
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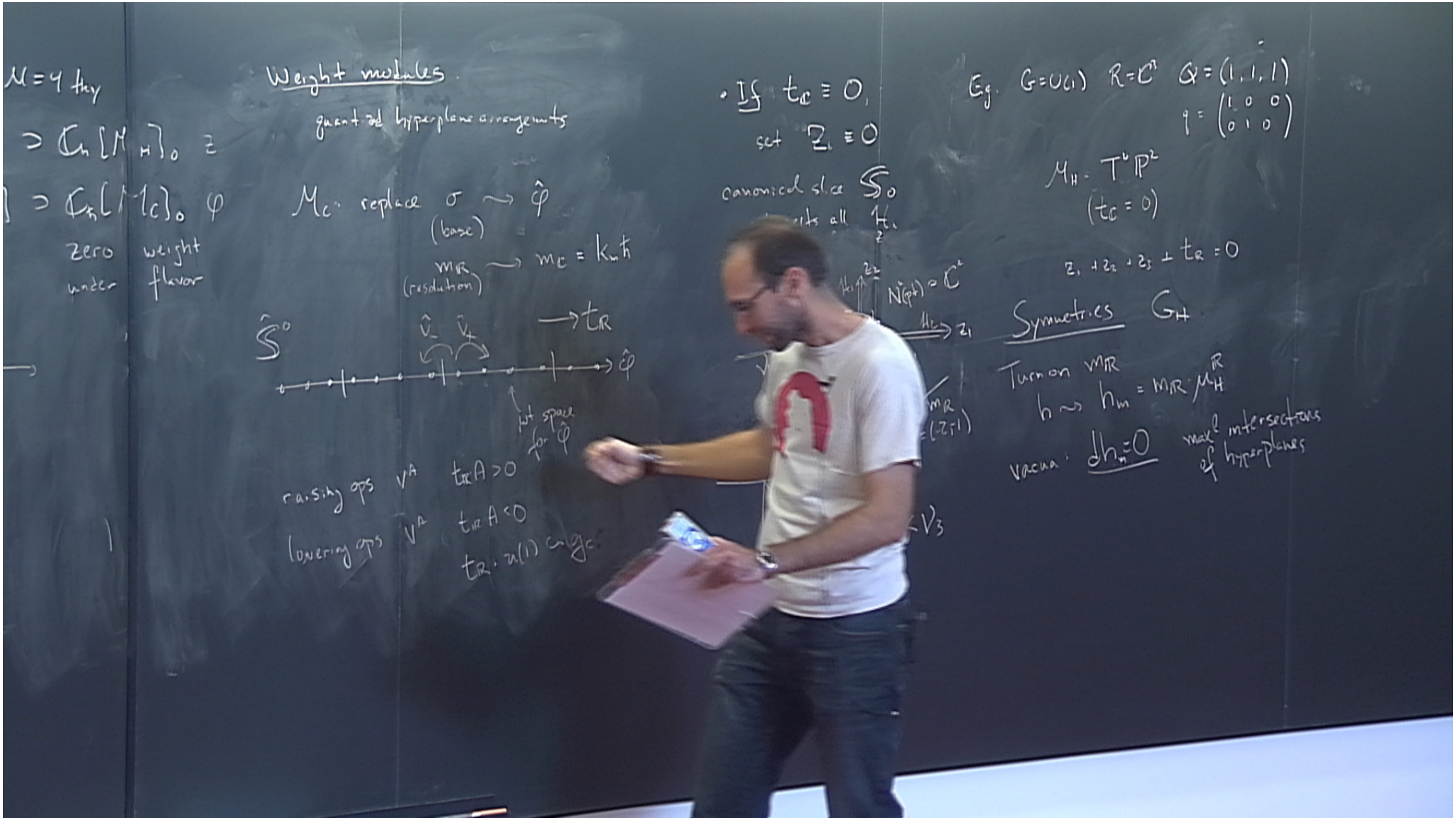

Weight modules
quantized hyperplane arrangements
Symmetries
Turn on
Vacua:
maximal intersections of hyperplanes

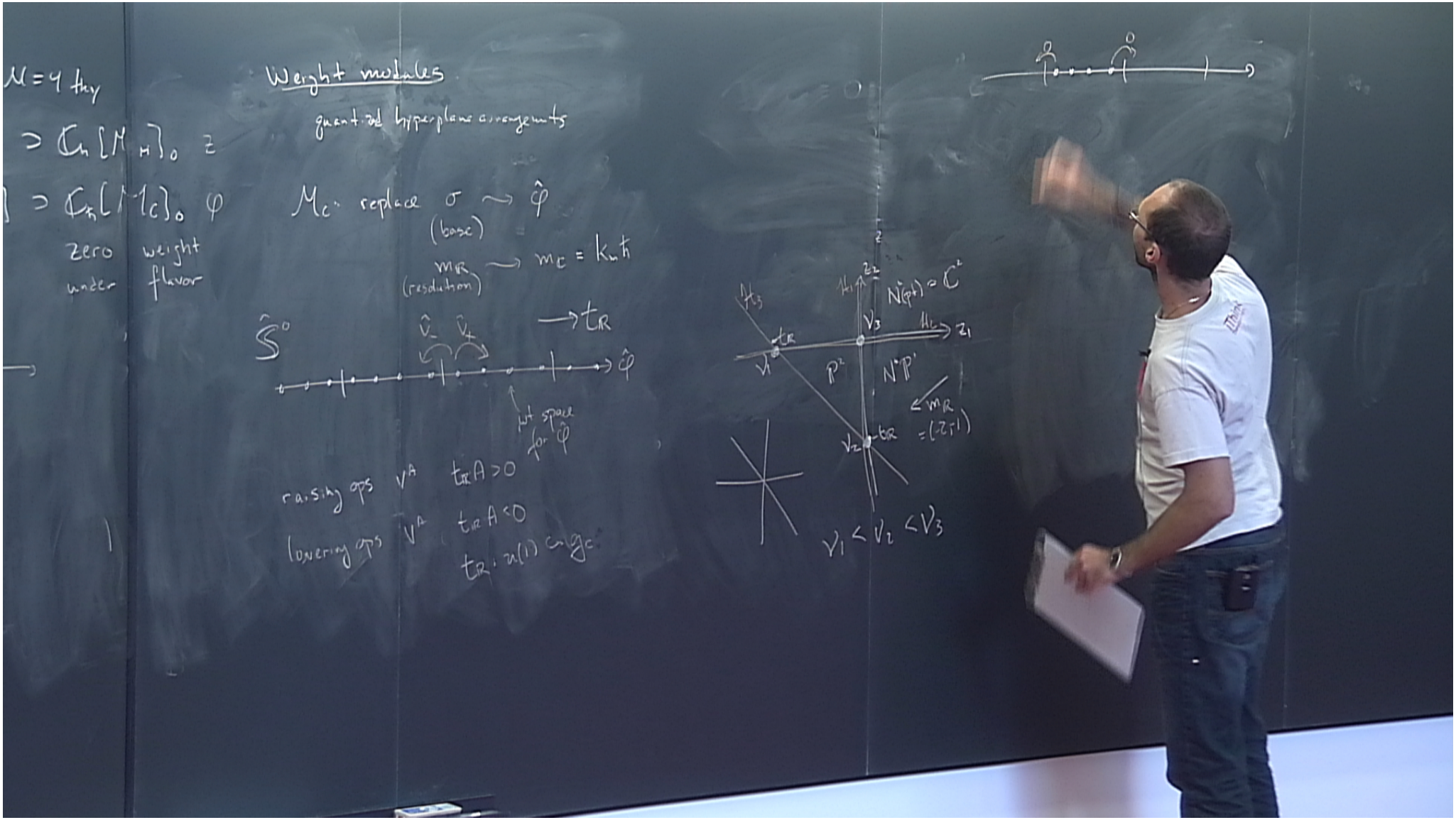

Weight modules
quantized hyperplane arrangements
zero weight
under flavor
(base)
(resolution)
Eg. G=U(1) R=C² Q=(1,1,1)
Symmetries G_H
Turn on m_R
Vacua: dh_n=0
maximal intersections of hyperplanes
P²
N*P¹
V1 < V2 < V3

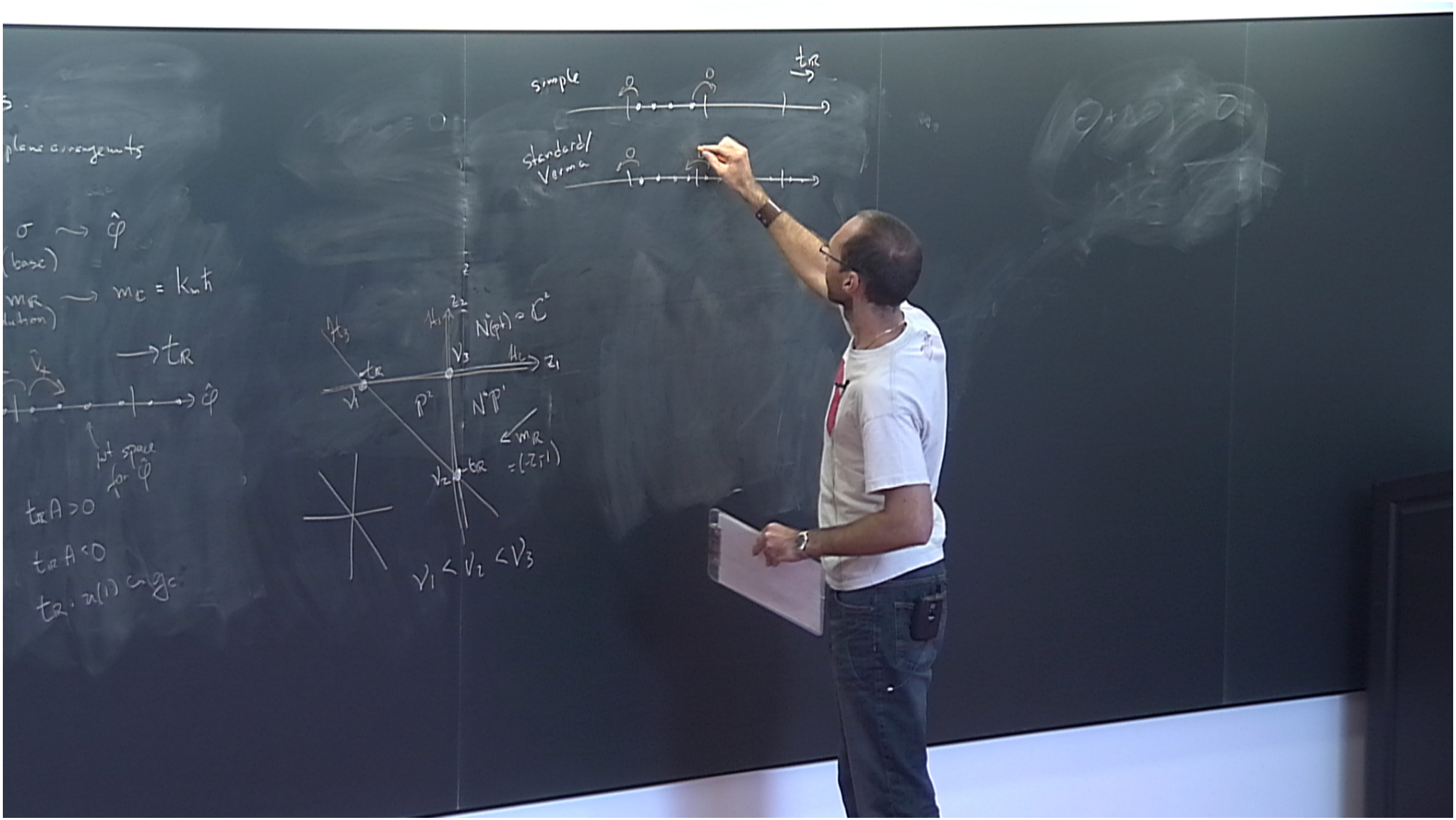

Weight modules
quantized hyperplane arrangements
M_C: replace $\sigma \rightsquigarrow \hat{\varphi}$
(base)
$m_R \rightsquigarrow m_C = k_m \hbar$
(resolution)
t_R
raising ops V^A $t_R A > 0$
lowering ops V^A $t_R A < 0$
$t_R \cdot u(1)$ charge
wt space for $\hat{\varphi}$
• If $t_C \equiv 0$,
set $Z_i \equiv 0$
canonical slice S_0
intersects all H_i
$N(p+1) \simeq \mathbb{C}^2$
$N^* P^1$
$m_R = (-Z-1)$
$V_1 < V_2 < V_3$
Eg. $G = U(1)$ $R = \mathbb{C}^3$ $Q = (1,1,1)$
$q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
$M_H = T^* \mathbb{P}^2$
$(t_C = 0)$
$Z_1 + Z_2 + Z_3 \perp t_R = 0$
Symmetries G_H
Turn on m_R
$h \rightsquigarrow h_m = m_R / M_H^R$
Vacua: $dh_m = 0$
max'l intersections of hyperplanes

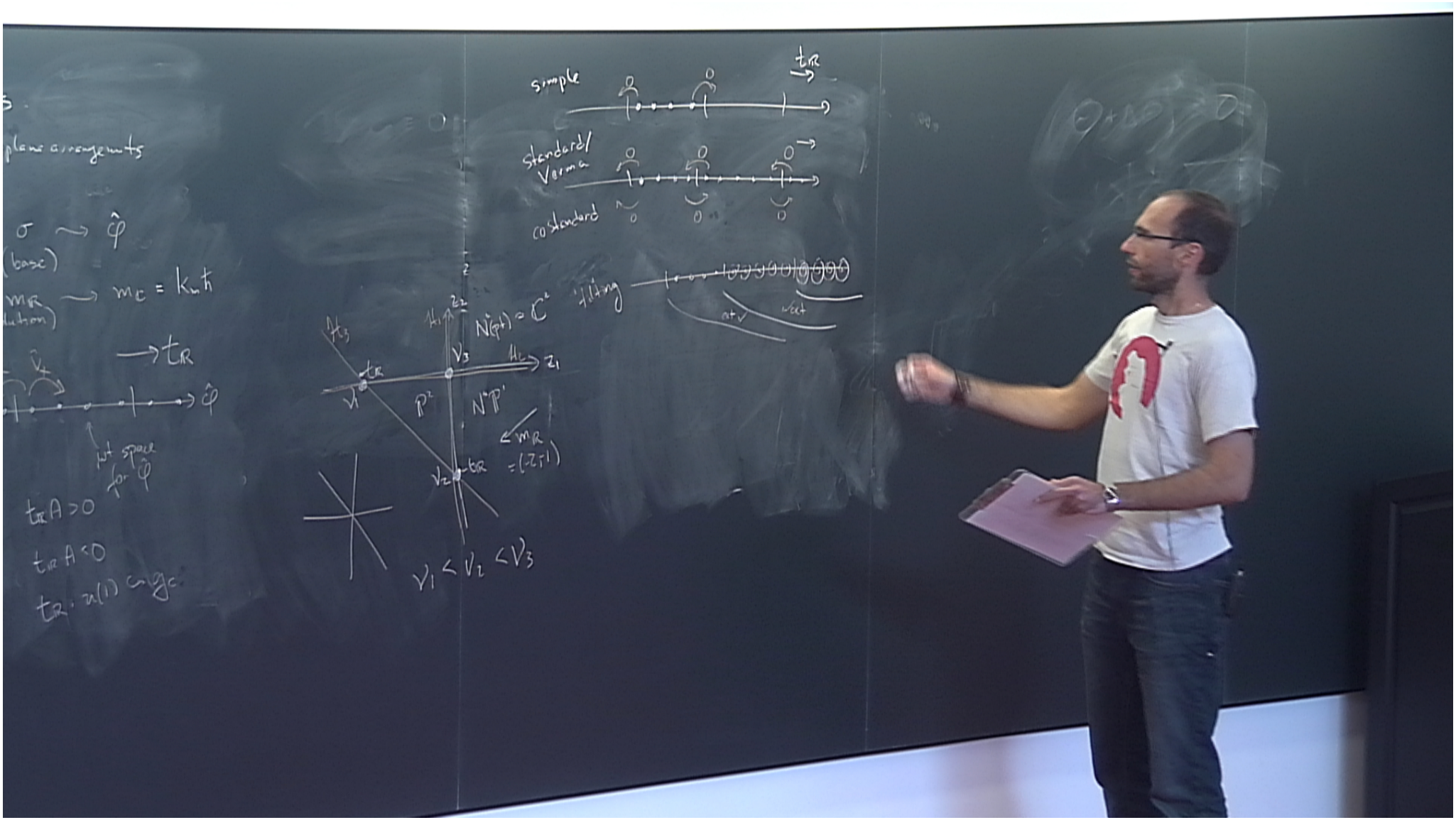

Weight modules
quantized hyperplane arrangements
M_C: replace $\sigma \rightsquigarrow \hat{\varphi}$
(base)
$m_R \rightsquigarrow m_C = k_m \hbar$
(resolution)
$\hat{S}^0$
t_R
$\hat{\varphi}$
$\hat{V}_-$
$\hat{V}_+$
Mt space for $\hat{\varphi}$
raising ops V^A $t_R A > 0$
lowering ops V^A $t_R A < 0$
$t_R \cdot u(1)$ charge
If $t_C \equiv 0$,
set $Z_i \equiv 0$
canonical slice S_0
intersects all H_i
Eg. $G = U(1)$ $R = \mathbb{C}^3$ $Q = (1,1,1)$
$q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
$M_H = T^*\mathbb{P}^2$
$(t_C = 0)$
$z_1 + z_2 + z_3 \perp t_R = 0$
Symmetries G_H
Turn on m_R
$h \rightsquigarrow h_m = m_R / M_H^R$
Vacua: $dh_m = 0$
max'l intersections of hyperplanes


Weight modules
quantized hyperplane arrangements
M_C: replace $\sigma \rightsquigarrow \hat{\varphi}$
(base)
m_R (resolution) $\rightsquigarrow m_C = k_m \hbar$
$\hat{S}^0$
t_R
$\hat{\varphi}$
wt space for $\hat{\varphi}$
raising ops V^A $t_R A > 0$
lowering ops V^A $t_R A < 0$
$t_R \cdot u(1)$ charge
• If $t_C \equiv 0$,
set $Z_i \equiv 0$
canonical slice S_0
intersects all H_i
$N(p+1) = \mathbb{C}^2$
$N^1 P^1$
$m_R = (-2, 1)$
$V_1 < V_2 < V_3$
Eg. $G = U(1)$ $R = \mathbb{C}^3$ $Q = (1, 1, 1)$
$q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
$M_H = T^* \mathbb{P}^2$
$(t_C = 0)$
$z_1 + z_2 + z_3 \perp t_R = 0$
Symmetries G_H
Turn on m_R
$h \rightsquigarrow h_m = m_R \cdot \mu_H^R$
Vacua: $dh_m = 0$
max'l intersections of hyperplanes

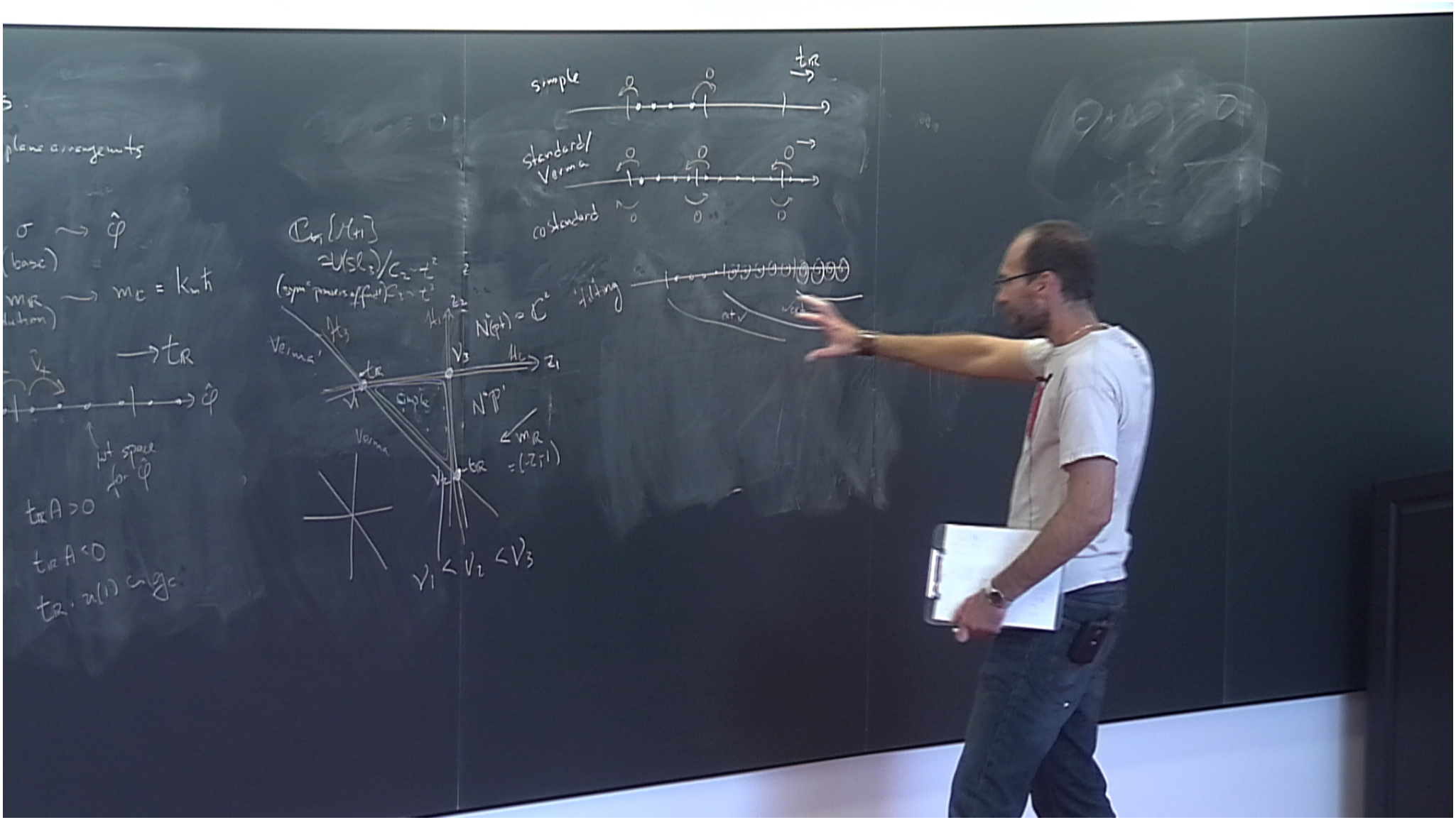

Weight modules
quantized hyperplane arrangements
Symmetries
Turn on
vacua
maximal intersections of hyperplanes

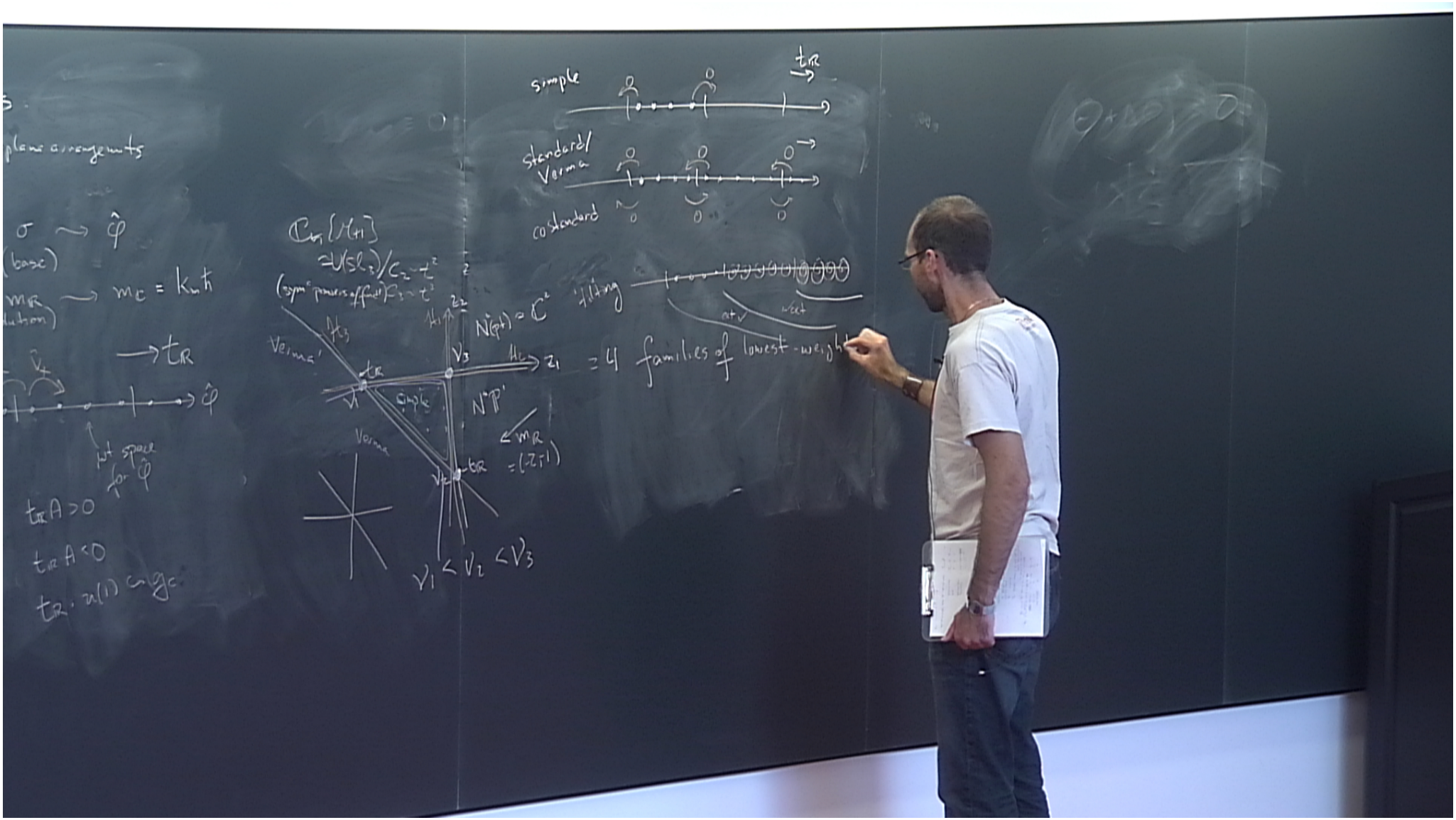

M = 4 thy
Weight modules
quantized hyperplane arrangements
zero weight
under flavor
replace σ ⇝ φ̂
(base)
m_R ⇝ m_C = k_m ħ
(resolution)
t_R
raising ops V^A t_R A > 0
lowering ops V^A t_R A < 0
t_R : u(1) charge
N(φ+) = C^2
N^P^2
N^P^1
m_R = (z_2, z_1)
V_1 < V_2 < V_3

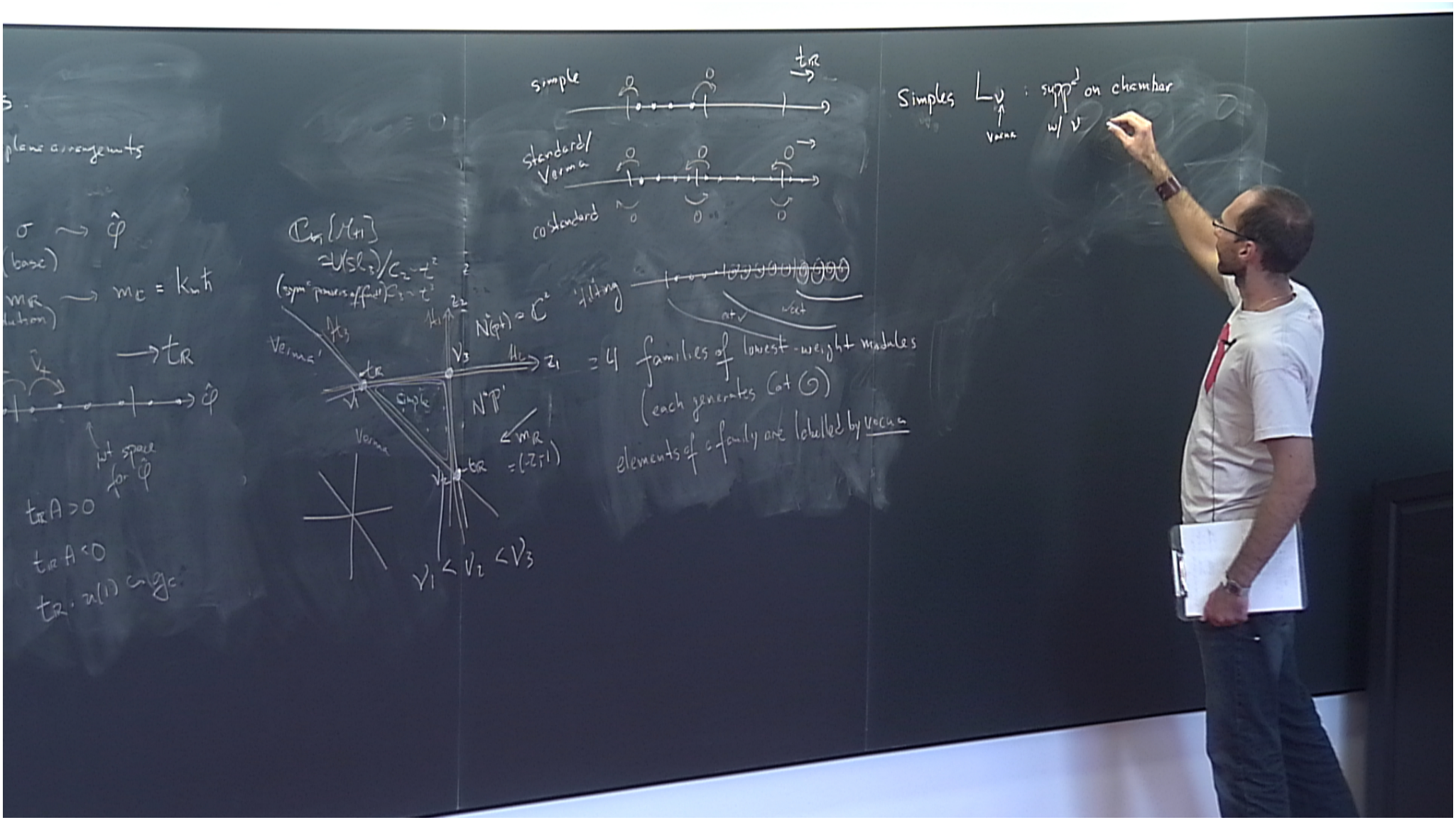

simple
t_R
Standard/
Verma
plane arrangements
$\sigma \rightsquigarrow \hat{\varphi}$
(base)
$m_R \rightsquigarrow m_C = k_m t$
$\rightarrow t_R$
$\hat{\varphi}$
$t_R A > 0$
$t_R A < 0$
$t_R \cdot u(1)$ conjugate
$N(\mathbb{P}^1) \simeq \mathbb{C}^2$
$\mathbb{P}^2$
$N^*\mathbb{P}^1$
$m_R = (-2,-1)$
$v_1 < v_2 < v_3$

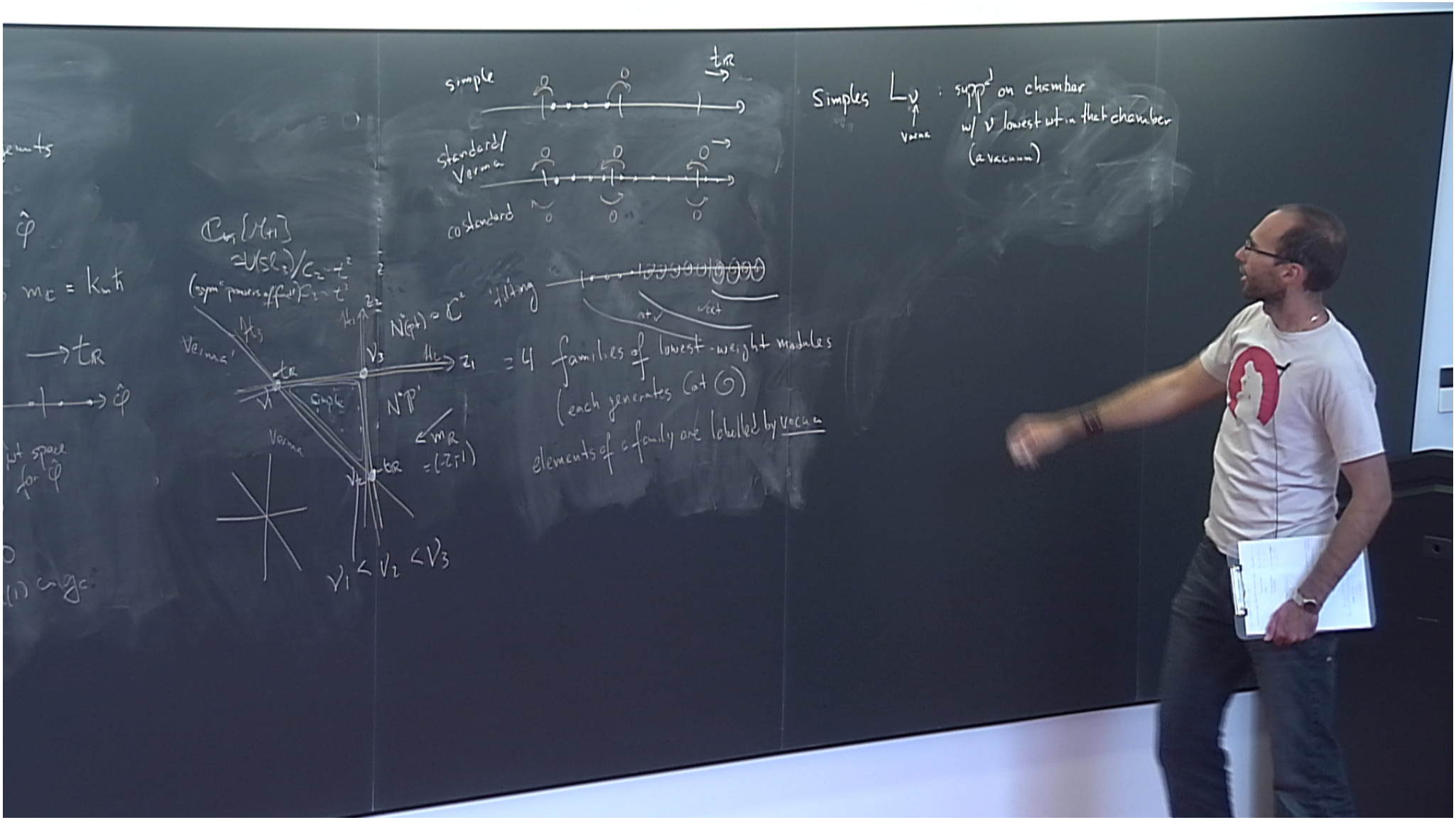

simple
standard/ Verma
costandard
fitting
$V_1 < V_2 < V_3$

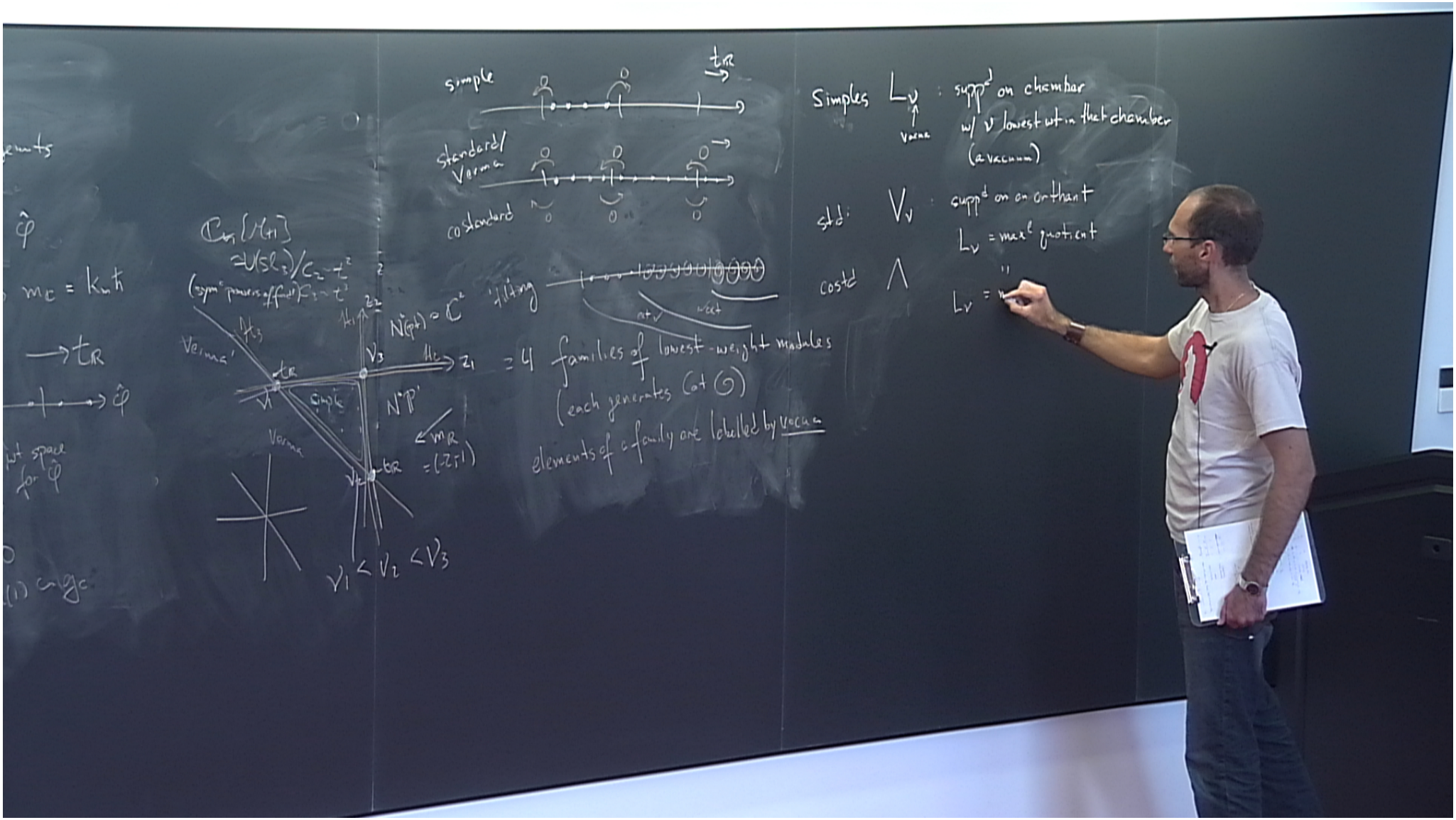

simple
Standard/ Verma
costandard
fitting
$V_1 < V_2 < V_3$

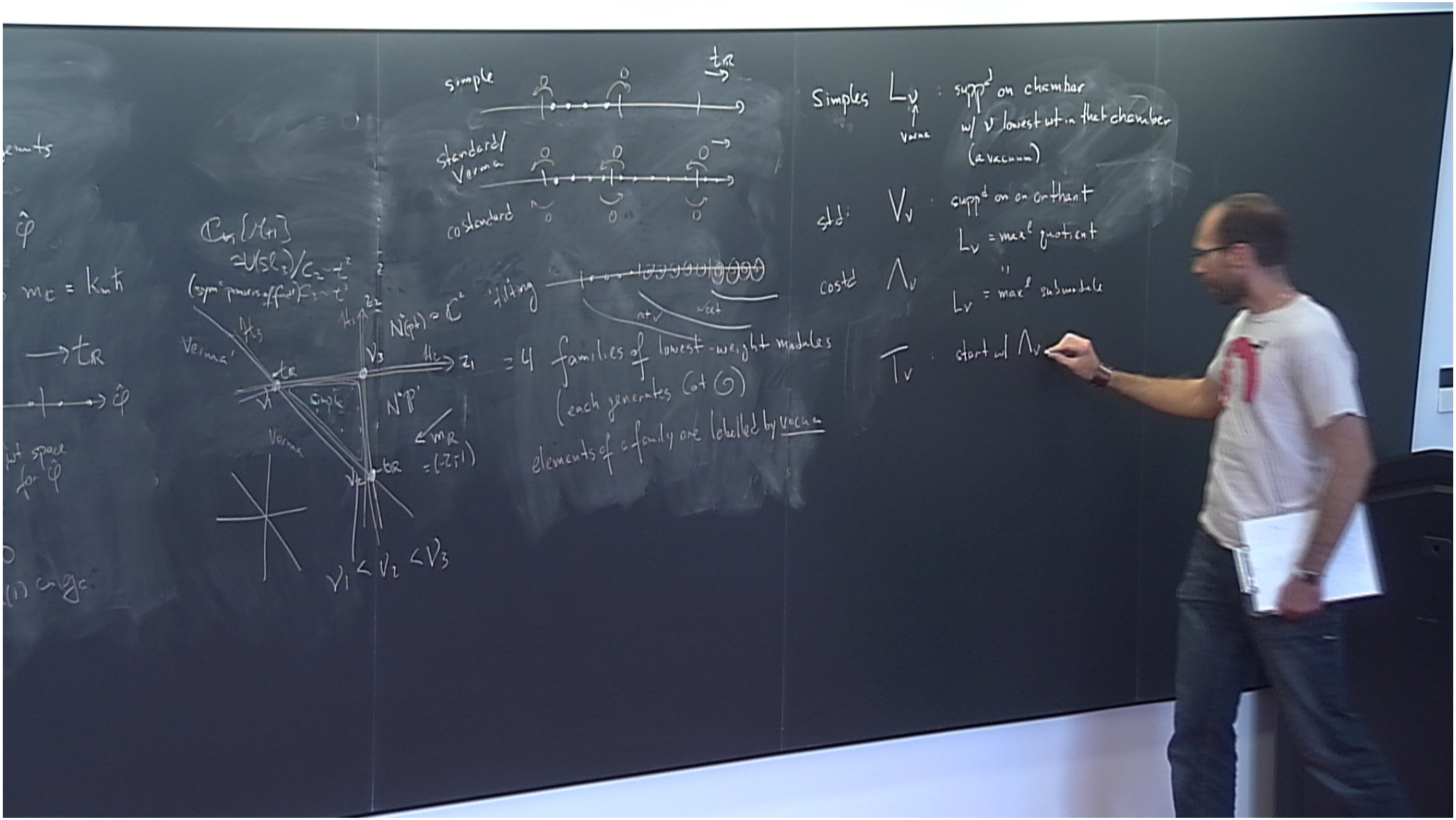

simple
standard/
Verma
costandard
fitting
Verma
simple
Verma
$V_1 < V_2 < V_3$
$N(p+) = C^2$
$t_R A > 0$
$t_R A < 0$

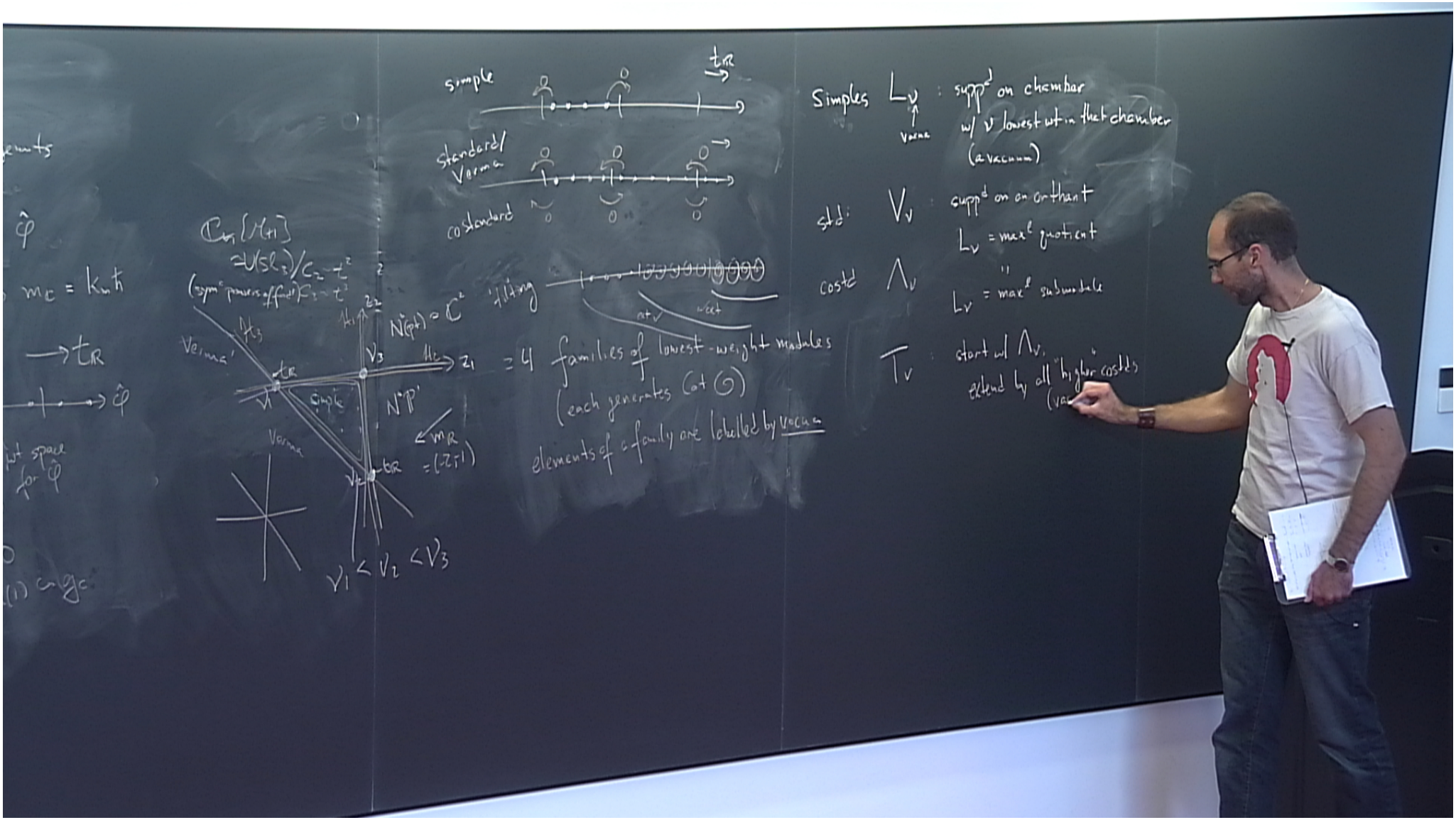

simple
standard/ Verma
costandard
fitting
= 4 families of lowest-weight
Verma
simple
Verma
$V_1 < V_2 < V_3$
$t_R A > 0$
$t_R A < 0$

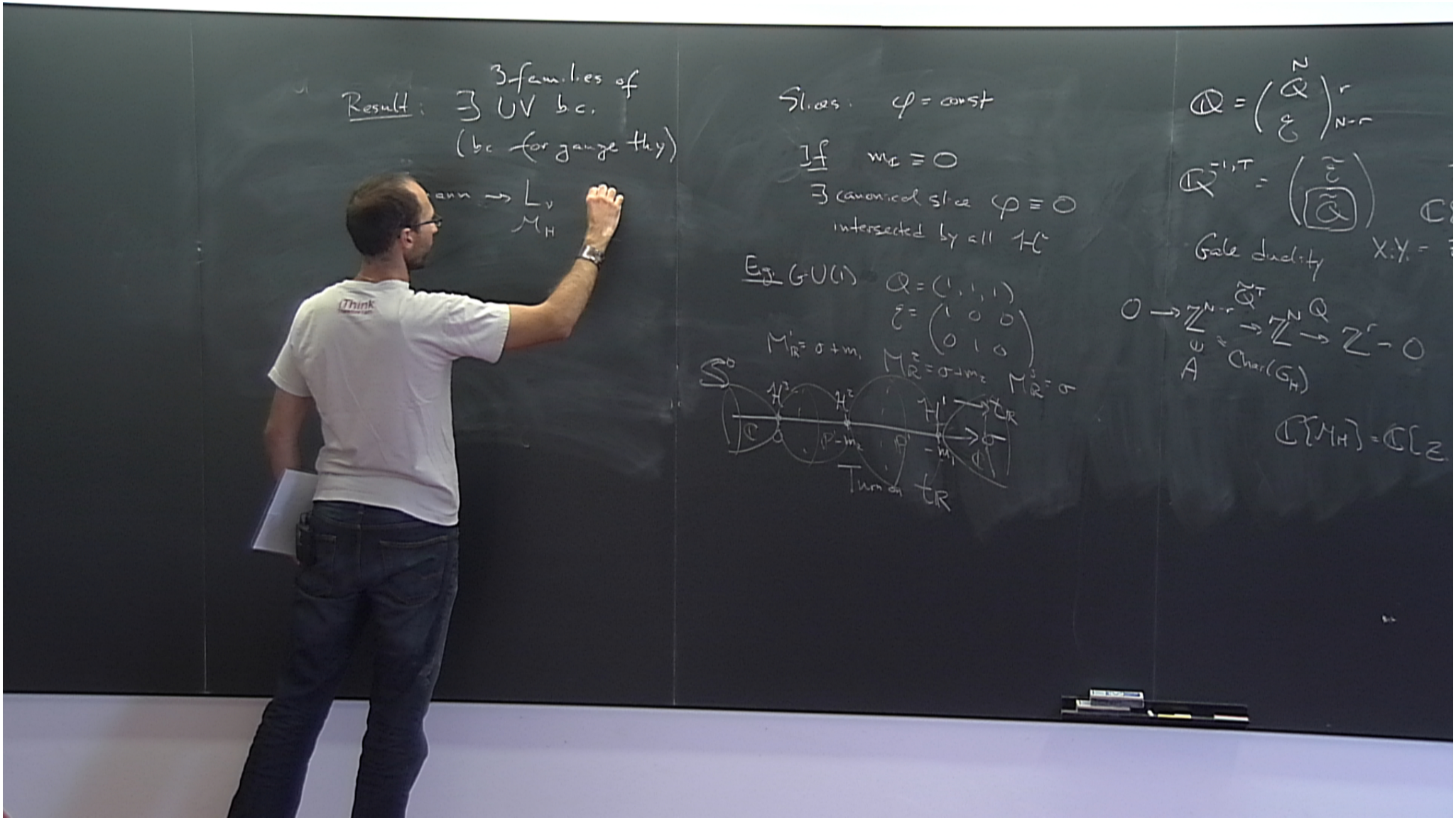

simple
standard/ Verma
costandard
Simples L_ν : supp^d on chamber w/ ν
Verma
t_R
$\hat{\varphi}$
$m_R = k_m t$
$t_R A > 0$
$t_R A < 0$
$t_R \cdot \alpha(1)$ sing.
wt space for $\hat{\varphi}$
$N(p^1) = C^2$
fitting
not ν
ν exact
$N^1 P^1$
$m_R = (-2,1)$
$V_1 < V_2 < V_3$
= 4 families of lowest-weight modules
(each generates Cat $\mathcal{O}$)
elements of a family are labelled by vacua


simple
standard/ Verma
costandard
Simples L_ν : supp'd on chamber w/ ν lowest wt in that chamber (a vacuum)
Verma
$m_L = k_m t$
$\to t_R$
$\to \hat{\varphi}$
$N(p+) = C^2$
fitting
not √
√ect
z_1 = 4 families of lowest-weight modules
(each generates (at 0)
elements of a family are labeled by vacua
$m_R = (-2, 1)$
$v_1 < v_2 < v_3$


simple
standard/ Verma
costandard
Simples L_ν : suppd on chamber w/ ν lowest wt in that chamber (a vacuum)
Verma
std: V_ν : suppd on an orthant
L_ν = max'l quotient
costd: $\wedge$
L_ν =
$N(p+1) = C^2$
= 4 families of lowest-weight modules
(each generates Cat O)
elements of a family are labelled by vacua
$V_1 < V_2 < V_3$
$m_R = (-2,-1)$
$m_L = k_m t$


simple
standard/ Verma
costandard
Simples L_ν : supp^d on chamber w/ ν lowest wt in that chamber (a vacuum)
std: V_ν : supp^d on an orthant
costd: Λ_ν
L_ν = max^l quotient
L_ν = max^l submodule
T_ν : start w/ Λ_ν
$N(p+1) = \mathbb{C}^2$ 'fitting'
= 4 families of lowest-weight modules
(each generates Cat O)
elements of a family are labelled by vacua
N^+P^+
$m_R = (-2,-1)$
$V_1 < V_2 < V_3$


simple
standard/ Verma
costandard
Simples L_ν : suppd on chamber w/ ν lowest wt in that chamber (a vacuum)
std: V_ν : suppd on an orthant
L_ν = max'l quotient
costd Λ_ν
L_ν = max'l submodule
T_ν : start w/ Λ_ν, extend by all "higher costds"
$N(p+1) = C^2$
= 4 families of lowest-weight modules
(each generates Cat O)
elements of a family are labelled by vacua
$V_1 < V_2 < V_3$


Result: ∃ UV b.c.
3 families of
(b.c. for gauge thy)
Slices: φ = const
If m_I = 0
∃ canonical slice φ ≡ 0
intersected by all H^i
Eg. G=U(1)
Q = (1, 1, 1)
Gale duality
X·Y = 0
Turn on
t_R