Title: Gravitational Casimir Effect

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Abstract: I will derive the gravitonic Casimir effect with non-idealized boundary conditions. This allows the quantification of the gravitonic contribution to the Casimir effect from real bodies. I will show how to use this formula to calculate the meagre gravitonic Casimir effect in ordinary matter. I will also apply this formula to the speculated Heisenberg-Couloumb (HC) effect in superconductors, thereby providing a test for the validity of the HC theory, and, consequently, the existence of gravitons.

Gravitational Casimir Effect

JAMES Q. QUACH

Quach, PRL 114, 081104 (2015)

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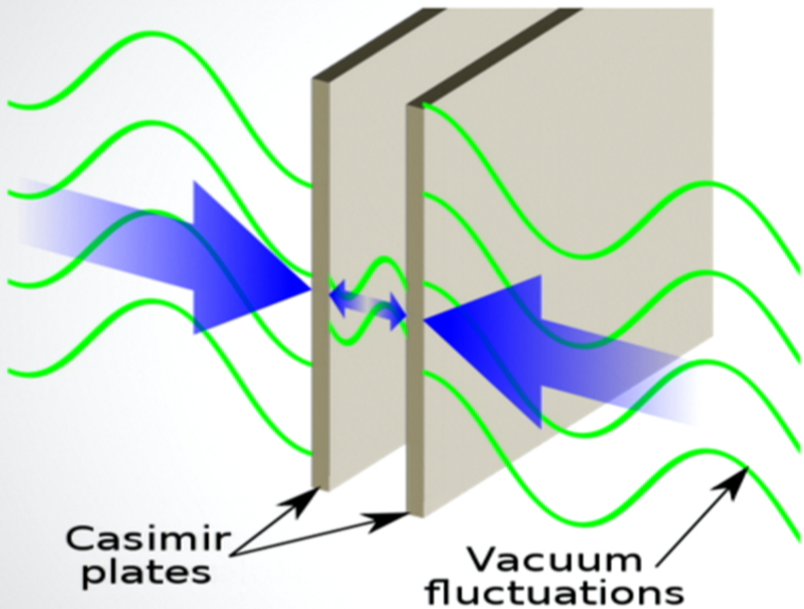
Casimir Effect



Hendrik Casimir (1948)

$$E_n = \hbar\omega(n + \frac{1}{2})$$

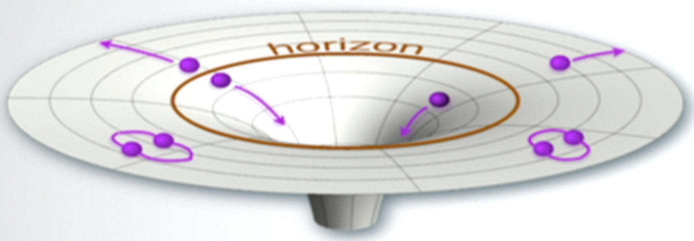
$$E_0 = \frac{\hbar\omega}{2}$$

$$P_I(a) = -\frac{\pi^2}{240}\frac{\hbar c}{a^4}$$

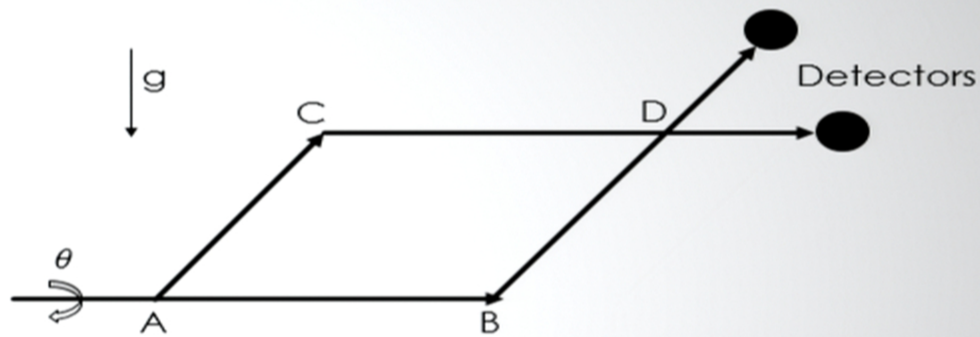
Outline

- Motivation
- Gravitational Casimir Effect
- Ordinary Materials
- Superconductors
- Dirac Hamiltonian in GW-background

QM & GR Interface

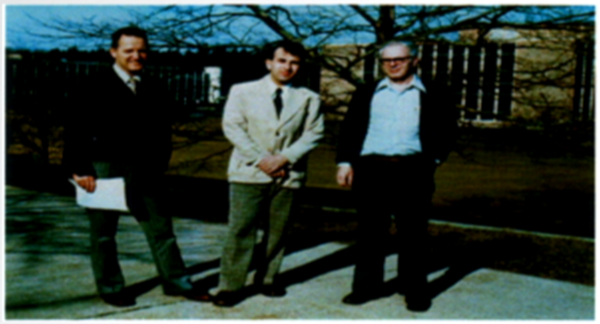


Hawking Radiation

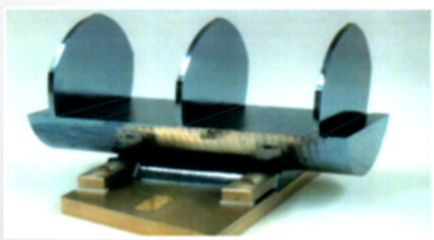


COW Experiment

QM & GR Interface

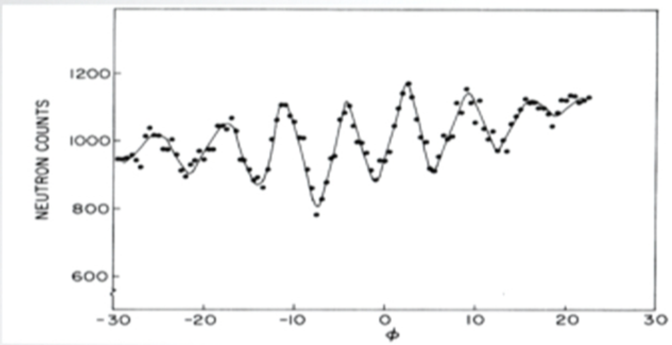


A. Overhauser, R. Colella, S. Werner

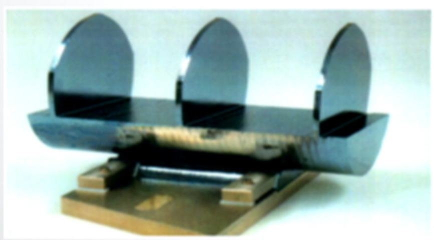


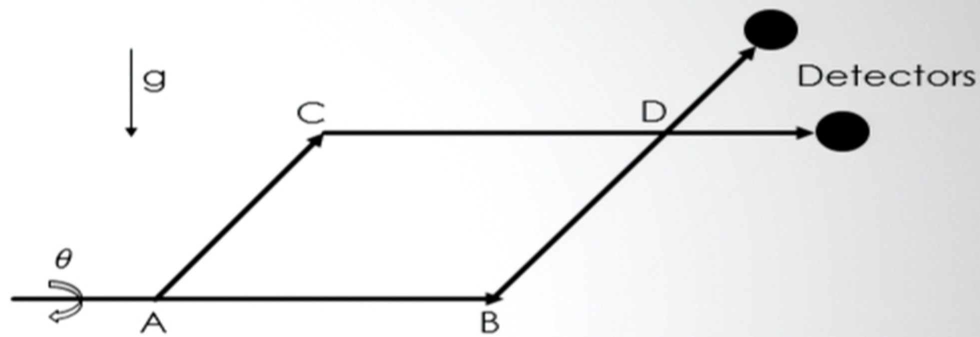
COW Experiment

QM & GR Interface



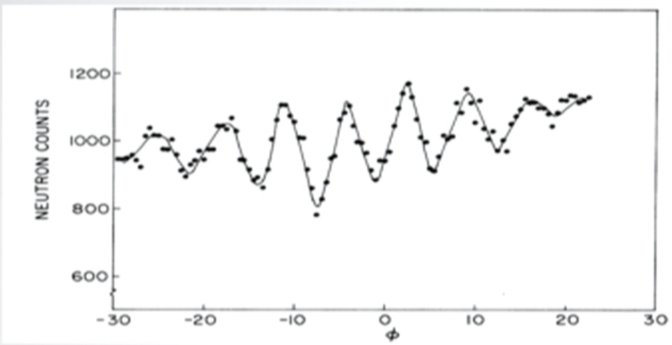
Colella, Overhauser, Werner, PRL 1975



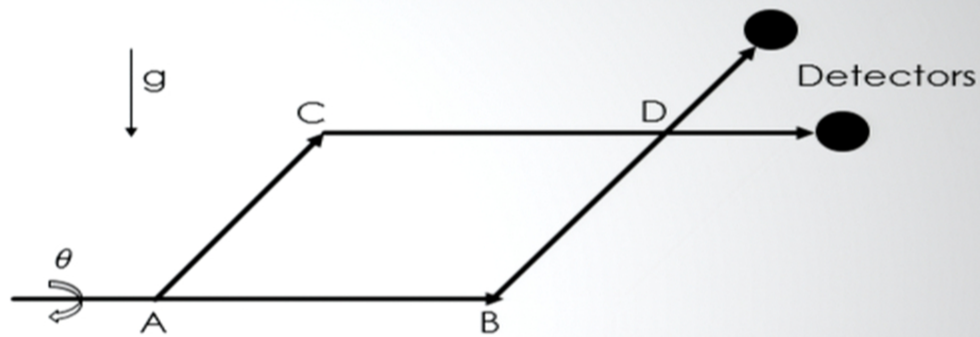


COW Experiment

QM & GR Interface



Colella, Overhauser, Werner, PRL 1975



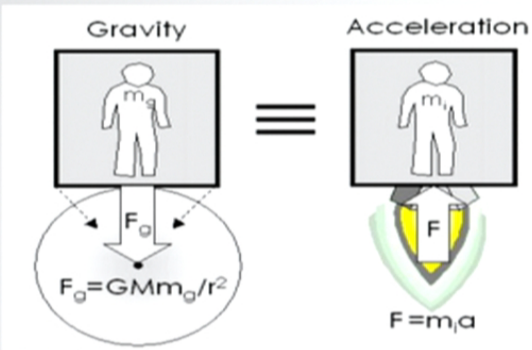
COW Experiment

$$H = \frac{\boldsymbol{p}^2}{2m_i} + m_g \boldsymbol{g}\boldsymbol{r} - \boldsymbol{\omega}\boldsymbol{L}$$

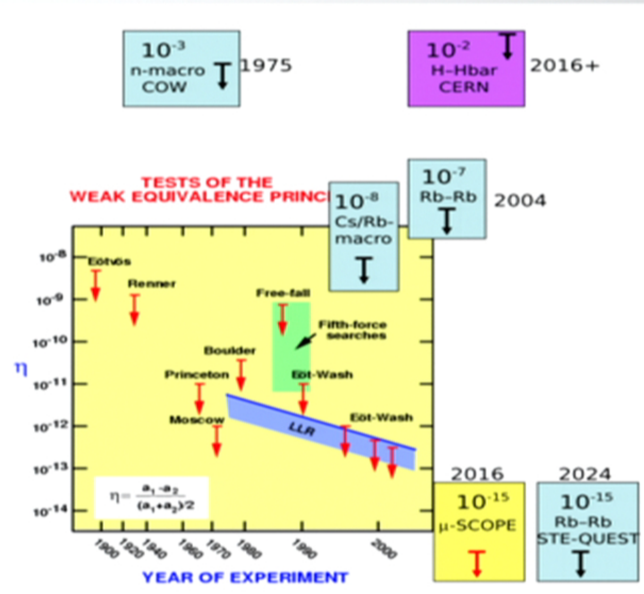
$$\Delta\beta_g \approx \frac{m_g}{m_i} \boldsymbol{g}\boldsymbol{K}TT'$$

Lammerzahl, Gen. Rel. Grav.1996

WEP violation in QM



There is no experiment that can be done, in a small confined space, which can detect the difference between a uniform gravitational field and an equivalent uniform acceleration.



Altschul, et al., Advances in Space Research (2015)

Enhanced gravitational interaction with quantum systems

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Gravitational Casimir Effect

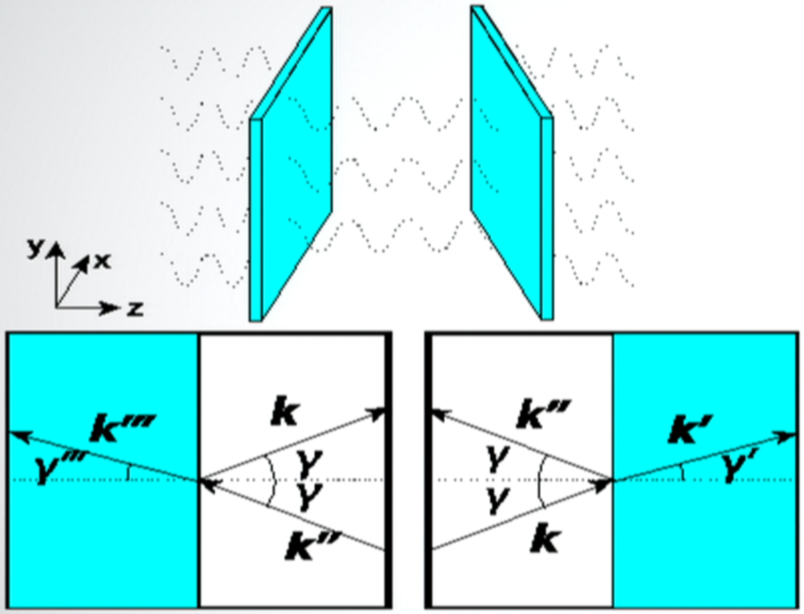
Gravitoelectromagnetism (GEM)

$$\nabla \cdot \boldsymbol{E} = \kappa \boldsymbol{\rho}^{(E)}$$
$$\nabla \cdot \boldsymbol{B} = \kappa \boldsymbol{\rho}^{(M)}$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} - \kappa \boldsymbol{J}^{(M)}$$
$$\nabla \times \boldsymbol{B} = \frac{\partial \boldsymbol{E}}{\partial t} + \kappa \boldsymbol{J}^{(E)}$$

$$\kappa = \frac{8\pi G}{c^4}$$
$$E_{ij} = C_{0i0j}$$
$$B_{ij} = \star\, C_{0i0j}$$

$$J_{\mu\nu\rho} = -T_{\rho[\mu,\nu]} + \frac{1}{3}\eta_{\rho[\mu}T_{,\nu]}$$
$$\rho_i^{(E)} = -J_{i00}$$
$$\rho_i^{(M)} = -\star J_{i00}$$
$$J_{ij}^{(E)} = J_{i0j}$$
$$J_{ij}^{(M)} = \star J_{i0j}$$

Gravitational Casimir Effect



$$E_0 = \frac{\hbar}{4\pi}\int_0^\infty k_\parallel dk_\parallel \sum_n (\omega_n^+ + \omega_n^\times)\sigma$$

$$\Delta\left[\left(1+\frac{\kappa\chi}{2}\right)\boldsymbol{E}^{TT}\right]=0$$

Ingraham, GRG 29, (1997)

$$\Delta\boldsymbol{B}^{TT}=0$$

$$\boldsymbol{E}^{TT} = \begin{bmatrix} \alpha(1-\frac{\sin^2\gamma}{2}) & \beta\cos\gamma \\ \beta\cos\gamma & -\alpha(1-\frac{\sin^2\gamma}{2}) \end{bmatrix} e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$$

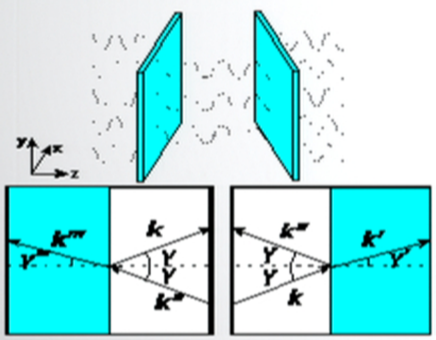
$$\boldsymbol{B}^{TT} = \begin{bmatrix} -\beta(1-\frac{\sin^2\gamma}{2}) & \alpha\cos\gamma \\ \alpha\cos\gamma & \beta(1-\frac{\sin^2\gamma}{2}) \end{bmatrix} e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$$

Gravitational Casimir Effect

$$\alpha'\left(1+\frac{\kappa\chi}{2}\right)\left(1-\frac{S'}{2}\right)e^{-q'a/2} = \alpha\left(1-\frac{S^2}{2}\right)e^{-qa/2} + \alpha''\left(1-\frac{S^2}{2}\right)e^{qa/2}$$

$$S = \sin\gamma$$
$$C = \cos\gamma$$

$$\alpha'''\left(1+\frac{\kappa\chi}{2}\right)\left(1-\frac{S'}{2}\right)e^{-q'a/2} = \alpha\left(1-\frac{S^2}{2}\right)e^{qa/2} + \alpha''\left(1-\frac{S^2}{2}\right)e^{-qa/2}$$



$$\alpha' C' e^{-\frac{q'a}{2}} = \alpha C e^{-\frac{qa}{2}} - \alpha'' C e^{qa/2}$$

$$-\alpha''' C' e^{-\frac{q'a}{2}} = \alpha C e^{-\frac{qa}{2}} - \alpha'' C e^{-qa/2}$$

$$q^2 = -k_z^2 = k_\parallel^2 - \frac{\omega^2}{c^2}$$
$$q'^2 = -k_z'^2 = k_\parallel^2 - (1+\kappa\chi)\frac{\omega^2}{c^2}$$

Gravitational Casimir Effect

$$\sum_n \omega_n(q) = \frac{1}{2\pi i}\left[\int_{i\infty}^{-i\infty} \omega(q)\, d\ln\Delta(\mathrm{q}) + \int_{C^+} \omega(q) d\ln\Delta(\mathrm{q})\right]$$

$$E_0 = \frac{\hbar}{4\pi}\int_0^\infty k_\parallel dk_\parallel \sum_n (\omega_n^+ + \omega_n^\times)\sigma$$

$$E(a) = \frac{E_0}{\sigma} - \lim_{a\to\infty}\frac{E_0}{a}$$

$$E(a) = \frac{\hbar}{4\pi^2}\int_0^\infty k_\parallel dk_\parallel \int_0^\infty [\ln(1 - r_+^2 e^{-2qa}) + \ln(1 - r_\times^2 e^{-2qa})] d\xi$$

$$r_+ = \frac{C'(S^2-2) - \left(1+\frac{\kappa\chi}{2}\right)C(S'^2-2)}{C'(S^2-2) + \left(1+\frac{\kappa\chi}{2}\right)C(S'^2-2)},\quad r_\times = \frac{\left(1+\frac{\kappa\chi}{2}\right)C'(S^2-2) - C(S'^2-2)}{\left(1+\frac{\kappa\chi}{2}\right)C'(S^2-2) + C(S'^2-2)}$$

Gravitational Casimir Effect

$$E(a) = \frac{\hbar}{4\pi^2}\int_0^\infty k_\parallel dk_\parallel \int_0^\infty [\ln(1 - r_+^2 e^{-2qa}) + \ln(1 - r_\times^2 e^{-2qa})]d\xi$$

$$r_+ = \frac{C'(S^2-2) - \left(1+\frac{\kappa\chi}{2}\right)C(S'^2-2)}{C'(S^2-2) + \left(1+\frac{\kappa\chi}{2}\right)C(S'^2-2)}, \quad r_\times = \frac{\left(1+\frac{\kappa\chi}{2}\right)C'(S^2-2) - C(S'^2-2)}{\left(1+\frac{\kappa\chi}{2}\right)C'(S^2-2) + C(S'^2-2)}$$

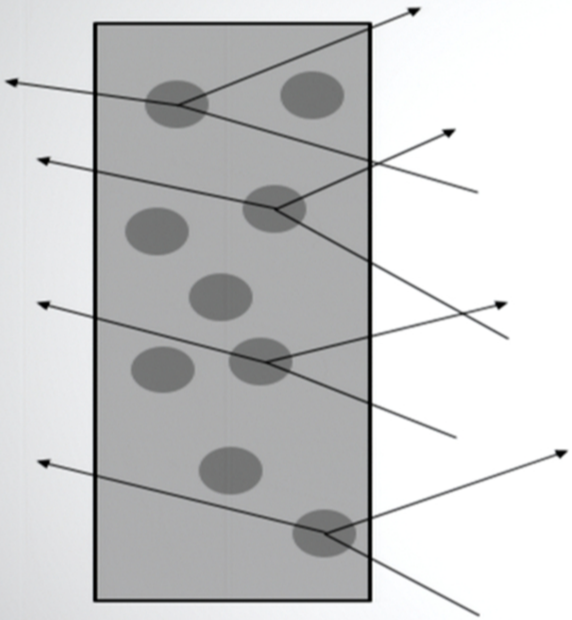
$$\alpha'\left(1+\frac{\kappa\chi}{2}\right)\left(1-\frac{S'}{2}\right)e^{-q'a/2} = \alpha\left(1-\frac{S^2}{2}\right)e^{-qa/2} + \alpha''\left(1-\frac{S^2}{2}\right)e^{qa/2}$$

$$\alpha' C'^{e^{-\frac{q'a}{2}}} = \alpha C e^{-\frac{qa}{2}} - \alpha'' C e^{qa/2}$$

$$\frac{\alpha''}{\alpha} = \frac{-C'(S^2-2) + \left(1+\frac{\kappa\chi}{2}\right)C(S'^2-2)}{C'(S^2-2) + \left(1+\frac{\kappa\chi}{2}\right)C(S'^2-2)}e^{qa}$$

Ordinary material

$$G = 6.67 \times 10^{-11}\mathrm{m^3kg^{-1}s^{-2}}$$



$$n = 1 + \frac{2\pi G\rho}{\omega^2}$$

$$\chi = \frac{1 - n(\omega)^2}{\kappa c^2}$$

Peters, PRD, 1974

$$P(a) = -\frac{\partial E(a)}{\partial a}$$

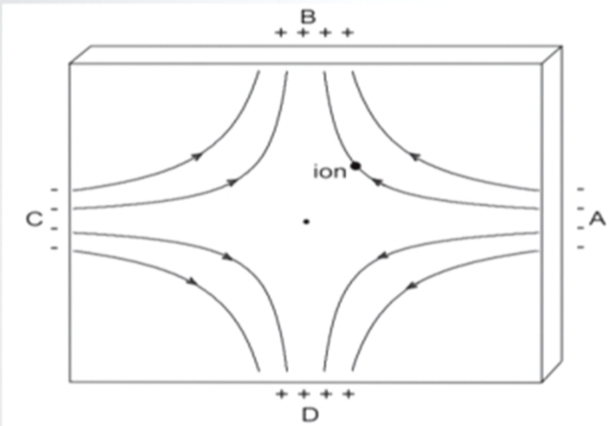
$$O[\rho] = 10^4\mathrm{kg\,m^{-3}}$$

$$P(10^{-6}) \approx 10^{-21}\mathrm{nPa}$$

$$O[\rho] = O[\frac{c^2}{G}] \approx 10^{27}\mathrm{kg/m^3}$$

Superconductor

DeWitt, PRL 16 , 1092 (1966)



Heisenberg-Coulomb effect

Minter, Wegter-McNelly, Chiao, Physica E, 2010

$$H = \frac{(\boldsymbol{p} - q\boldsymbol{A} - m\boldsymbol{h})^2}{2m} + V$$

$$\boldsymbol{j}_G = \frac{1}{m}\mathrm{Re}\left[\psi^*\left(\frac{\hbar}{i}\nabla - q\boldsymbol{A} - m\boldsymbol{h}\right)\psi\right]$$
$$= \frac{1}{m}(-q\boldsymbol{A} - m\boldsymbol{h})\psi^*\psi$$

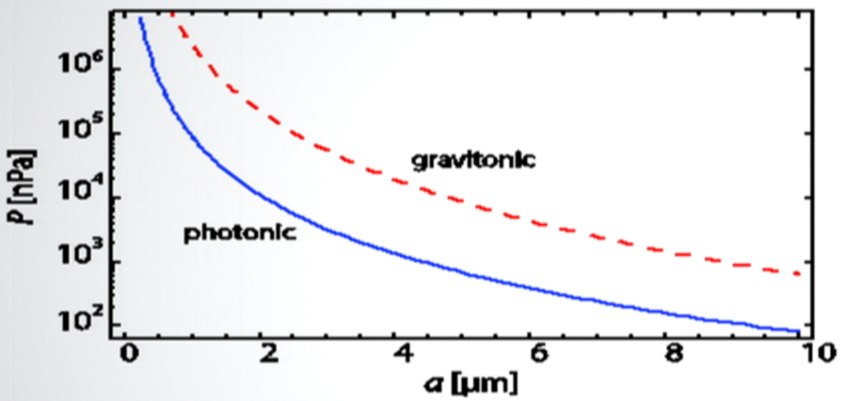
$$\boldsymbol{j}_G = \sigma_G \boldsymbol{E}_G$$

$$r_G = \left(1 + \frac{2\delta^2}{cd}\xi\right)^{-1}$$

$$r_E = \left(1 + \frac{2\lambda\delta^2}{cd^2}\xi\right)^{-1}$$

Superconductor

DeWitt, PRL 16 , 1092 (1966)



$d = 2\ \mathrm{nm}$ $\delta = 37\ \mathrm{nm}$ $\lambda = 83\ \mathrm{nm}$

$$H = \frac{(\boldsymbol{p} - q\boldsymbol{A} - m\boldsymbol{h})^2}{2m} + V$$

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Generalised Dirac Equation in GW background

$$i\hbar\gamma^a e_a^\mu(\partial_\mu - \Gamma_\mu - \frac{ie}{\hbar}A_\mu)\psi = mc\psi$$

$$ds^2 = c^2dt^2 - dx^2 + (1+2f)dy^2 - (1-2f)dz^2$$

$$f(t-x)$$

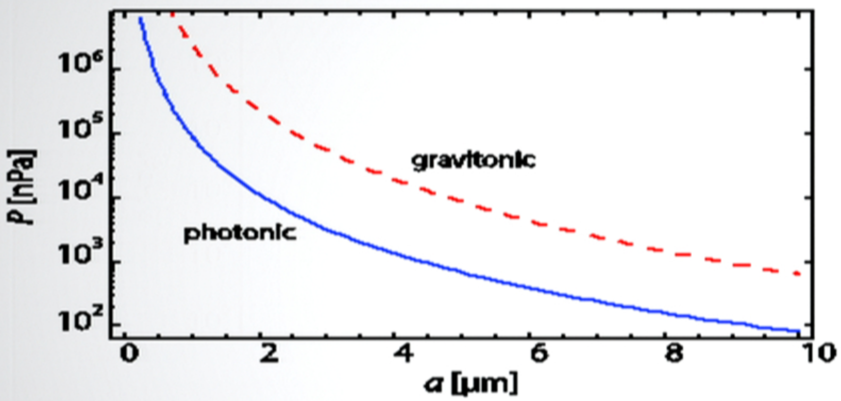
$$i\hbar\,\partial_t\psi = H\psi$$

$$H = c\alpha^i(p_i - eA_i) - cf\alpha^2(p_2 - eA_2) + cf^3(p_3 - eA_3) + \beta mc^2$$

$$(\boldsymbol{\alpha} \equiv \gamma^0\boldsymbol{\gamma}, \beta \equiv \gamma^0, \boldsymbol{p} \equiv -\mathrm{i}\hbar\nabla)$$

Superconductor

DeWitt, PRL 16 , 1092 (1966)



$d = 2\ \mathrm{nm} \qquad \delta = 37\ \mathrm{nm} \qquad \lambda = 83\ \mathrm{nm}$

$$H = \frac{(\boldsymbol{p} - q\boldsymbol{A} - m\boldsymbol{h})^2}{2m} + V$$

$$\begin{aligned} \boldsymbol{j}_G &= \frac{1}{m}\mathrm{Re}\left[\psi^*\left(\frac{\hbar}{i}\nabla - q\boldsymbol{A} - m\boldsymbol{h}\right)\psi\right] \\ &= \frac{1}{m}(-q\boldsymbol{A} - m\boldsymbol{h})\psi^*\psi \end{aligned}$$

$$\boldsymbol{j}_G = \sigma_G \boldsymbol{E}_G$$

$$r_G = \left(1 + \frac{2\delta^2}{cd}\xi\right)^{-1}$$

$$r_E = \left(1 + \frac{2\lambda\delta^2}{cd^2}\xi\right)^{-1}$$

Standard Foldy-Wouthuysen Transformation

$$O = \frac{1}{2}(H - \beta H \beta), \quad E = \frac{1}{2}(H + \beta H \beta)$$

$$U = e^{iS} \,, \qquad S = -\frac{i\beta}{2m} O$$

$$\begin{aligned} H' &= UHU^{+} = \exp(iS) H \exp(-iS) \\ &= H + i[S, H] + \frac{i^2}{2!}\left[S, [S, H]\right] + \cdots \end{aligned}$$

$$i[S, H] \approx -O$$

Exact Foldy-Wouthuysen Transformation

$$\{H, J\} = 0\,, \qquad J = i\gamma^5\beta$$

$$U_1 = \frac{1}{\sqrt{2}}(1 + J\Lambda), \quad U_2 = \frac{1}{\sqrt{2}}(1 + \beta J), \quad \Lambda \equiv H/\sqrt{H^2}$$

$$U = U_2 U_1$$

$$UHU^+ = \frac{1}{2}\beta\left(\sqrt{H^2} + \beta\sqrt{H^2}\beta\right) + \frac{1}{2}\left(\sqrt{H^2} - \beta\sqrt{H^2}\beta\right)J$$
$$= \left\{\sqrt{H^2}\right\}_{\text{even}}\beta + \left\{\sqrt{H^2}\right\}_{odd}J$$

Exact Foldy-Wouthuysen Transformation

$$\{H, J\} = 0\,, \qquad J = i\gamma^5\beta$$

$$U_1 = \frac{1}{\sqrt{2}}(1 + J\Lambda), \qquad U_2 = \frac{1}{\sqrt{2}}(1 + \beta \mathrm{J})$$

$$\Lambda = H/\sqrt{H^2}$$

$$\mathcal{H}_{FW} = UHU^{+} = \left\{\sqrt{H^2}\right\}_{\text{even}}\beta + \left\{\sqrt{H^2}\right\}_{odd}J$$

$$H_{FW} = \frac{1}{2m}\left(\delta^{ij} + 2fT^{ij}\right)\left[(p_i - eA_i)\left(p_j - eA_j\right) + e\hbar\epsilon_{jkl}\sigma^l\partial^k(A_i)\right] + \frac{\hbar}{2m}\partial^i(f)T^{jl}\epsilon_{ijk}\sigma^k(p_l - eA_l) + mc^2$$

$$T \equiv \mathrm{diag}(0, -1, 1)$$

Foldy-Wouthuysen Transformation

$$H = \frac{(\boldsymbol{p} - q\boldsymbol{A} - m\boldsymbol{h})^2}{2m} + V$$

$$H_{FW} = \frac{1}{2m}(\delta^{ij} + 2fT^{ij})[(p_i - eA_i)(p_j - eA_j)] + mc^2$$

arXiv:1506.08203 [hep-th]