Title: TBA

Date: Oct 02, 2015 11:00 AM

URL: http://pirsa.org/15100059

Abstract:

Vacua for quantum gravity

Marc Geiller

PI

Renormalization in Background Independent Theories: Foundations and Techniques
PI, October 2nd 2015

Based on:
Dittrich, MG, 1401.6441
Dittrich, MG, 1412.3752
Bahr, Dittrich, MG, 1506.08571

Introduction

Bottom-up approaches to quantum gravity

- ⋆ Gravity is space-time geometry, so quantum gravity could be quantum geometry
- ⋆ Description in terms of basic building blocks, i.e. quanta or atoms of geometry (loop quantum gravity, group field theory, causal dynamical triangulations, . . .)
- ⋆ How does the smooth diffeomorphism-invariant space-time around us emerge?
- ⋆ What are the phases of spin foam models and are there phase transitions?
- ⋆ How to extract physics and predictions?

Introduction

Bottom-up approaches to quantum gravity

- Gravity is space-time geometry, so quantum gravity could be quantum geometry
- Description in terms of basic building blocks, i.e. quanta or atoms of geometry (loop quantum gravity, group field theory, causal dynamical triangulations, ...)
- How does the smooth diffeomorphism-invariant space-time around us emerge?
- What are the phases of spin foam models and are there phase transitions?
- How to extract physics and predictions?

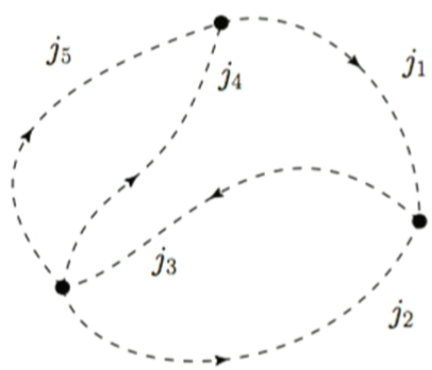
Loop quantum gravity and spin foams

- Holonomies and fluxes encoding extrinsic and intrinsic spatial geometry
- Diffeomorphism-invariant Hilbert space supporting the holonomy-flux algebra
- Derivation of quantum geometry (Ashtekar, Lewandowski, Rovelli, Smolin)
- Status of spin foam amplitudes: fundamental (what significance?) or auxiliary?

Motivations

AL representation (Ashtekar, Isham, Lewandowski)

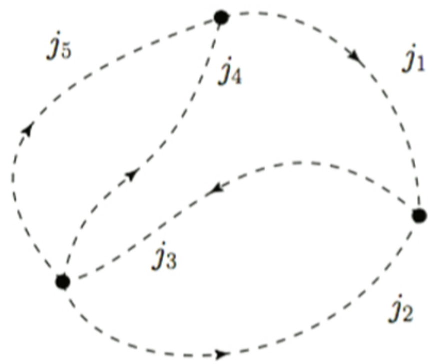
- Cyclic vacuum is a state with no excitations (no graph), $\langle 0|X_S(E)|0\rangle_{\text{AL}} = 0, \ \forall\ S$
- Holonomy operators (associated to links of graphs) are creation operators
- Spatial geometry vanishes in the vacuum: totally squeezed and degenerate state



Motivations

AL representation (Ashtekar, Isham, Lewandowski)

- Cyclic vacuum is a state with no excitations (no graph), $\langle 0|X_S(E)|0\rangle_{\rm AL} = 0,\ \forall\ S$
- Holonomy operators (associated to links of graphs) are creation operators
- Spatial geometry vanishes in the vacuum: totally squeezed and degenerate state
- Discrete basis of excitations labelled by dual graphs
- Embedding of Hilbert spaces based on embedding of dual graphs



Motivations

AL continuum structures

- ⋆ Graph Hilbert space $\mathcal{H}_\Gamma = L^2\big(\mathrm{SU}(2)^L/\mathrm{SU}(2)^N, \mathrm{d}\mu_{\mathrm{Haar}}\big)$
- ⋆ Inductive limit Hilbert space $\mathcal{H}_\infty = \cup_\Gamma \mathcal{H}_\Gamma / \sim\, = L^2\big(\overline{\mathcal{A}/\mathcal{G}}, \mathrm{d}\mu_{\mathrm{AL}}\big)$

Motivations

AL continuum structures

- ⋆ Graph Hilbert space $\mathcal{H}_\Gamma = L^2\big(\mathrm{SU}(2)^L/\mathrm{SU}(2)^N, \mathrm{d}\mu_{\mathrm{Haar}}\big)$
- ⋆ Inductive limit Hilbert space $\mathcal{H}_\infty = \cup_\Gamma \mathcal{H}_\Gamma / \sim\, = L^2\big(\overline{\mathcal{A}/\mathcal{G}}, \mathrm{d}\mu_{\mathrm{AL}}\big)$

Gravity as a topological theory with defects (Bianchi, Freidel, Ziprick, MG)

- ⋆ Graph phase space $\mathcal{P}_\Gamma = \big(T^*\mathrm{SU}(2)\big)^L$
- ⋆ Isomorphic to the phase space of GR with almost-everywhere flat connections
- ⋆ Reconstruction of continuum fields from discrete holonomy-flux data
  - AL gauge $X(E)|0\rangle_{\mathrm{AL}} = 0$ and $E(x)$ almost-everywhere vanishing
  - BF gauge $F(A)|0\rangle_{\mathrm{BF}} = 0$ and $E(x)$ almost-everywhere flat


Motivations
AL continuum structures
Graph Hilbert space $\mathcal{H}_\Gamma = L^2(\mathrm{SU}(2)^L/\mathrm{SU}(2)^N, \mathrm{d}\mu_{\mathrm{Haar}})$
Inductive limit Hilbert space $\mathcal{H}_\infty = \cup_\Gamma \mathcal{H}_\Gamma/\sim\, = L^2(\overline{\mathcal{A}/\mathcal{G}}, \mathrm{d}\mu_{\mathrm{AL}})$
Gravity as a topological theory with defects (Bianchi, Freidel, Ziprick, MG)
Graph phase space $\mathcal{P}_\Gamma = (T^*\mathrm{SU}(2))^L$
Isomorphic to the phase space of GR with almost-everywhere flat connections
Reconstruction of continuum fields from discrete holonomy-flux data
AL gauge $X(E)|0\rangle_{\mathrm{AL}} = 0$ and $E(x)$ almost-everywhere vanishing
BF gauge $F(A)|0\rangle_{\mathrm{BF}} = 0$ and $E(x)$ almost-everywhere flat
In the AL representation the defects generate geometry

Motivations

AL continuum structures

- ⋆ Graph Hilbert space $\mathcal{H}_\Gamma = L^2\big(\mathrm{SU}(2)^L/\mathrm{SU}(2)^N, \mathrm{d}\mu_{\mathrm{Haar}}\big)$
- ⋆ Inductive limit Hilbert space $\mathcal{H}_\infty = \cup_\Gamma \mathcal{H}_\Gamma / \sim\, = L^2\big(\overline{\mathcal{A}/\mathcal{G}}, \mathrm{d}\mu_{\mathrm{AL}}\big)$

Gravity as a topological theory with defects (Bianchi, Freidel, Ziprick, MG)

- ⋆ Graph phase space $\mathcal{P}_\Gamma = \big(T^*\mathrm{SU}(2)\big)^L$
- ⋆ Isomorphic to the phase space of GR with almost-everywhere flat connections
- ⋆ Reconstruction of continuum fields from discrete holonomy-flux data
  - AL gauge $X(E)|0\rangle_{\mathrm{AL}} = 0$ and $E(x)$ almost-everywhere vanishing
  - BF gauge $F(A)|0\rangle_{\mathrm{BF}} = 0$ and $E(x)$ almost-everywhere flat
- ⋆ In the AL representation the defects generate geometry
- ⋆ Here we look for a representation where the defects generate curvature

Quantum gravity as a TQFT with defects

- ⋆ Need a Hilbert space supporting arbitrarily many excitations (defects) and the observable algebra generating them

Motivations

Uniqueness (Fleischhack, Lewandowski, Okołów, Sahlmann, Thiemann)

Motivations

Uniqueness (Fleischhack, Lewandowski, Okołów, Sahlmann, Thiemann)

- ⋆ Uniqueness of AL representation under technical assumptions, including
  - Diffeomorphism invariance
  - Weak continuity of exponentiated fluxes (i.e. the fluxes exist as operators)

Motivations

Uniqueness (Fleischhack, Lewandowski, Okołów, Sahlmann, Thiemann)

- ⋆ Uniqueness of AL representation under technical assumptions, including
  - Diffeomorphism invariance
  - Weak continuity of exponentiated fluxes (i.e. the fluxes exist as operators)
- ⋆ Very different from QFT, where there are many inequivalent representations

4 / 17

Motivations

Uniqueness (Fleischhack, Lewandowski, Okołów, Sahlmann, Thiemann)

- Uniqueness of AL representation under technical assumptions, including
  - Diffeomorphism invariance
  - Weak continuity of exponentiated fluxes (i.e. the fluxes exist as operators)
- Very different from QFT, where there are many inequivalent representations
- AL vacuum makes it difficult to construct semi-classical states: need condensates

Motivations

Uniqueness (Fleischhack, Lewandowski, Okołów, Sahlmann, Thiemann)

- ⋆ Uniqueness of AL representation under technical assumptions, including
  - Diffeomorphism invariance
  - Weak continuity of exponentiated fluxes (i.e. the fluxes exist as operators)
- ⋆ Very different from QFT, where there are many inequivalent representations
- ⋆ AL vacuum makes it difficult to construct semi-classical states: need condensates

Other possible representations

- ⋆ Introduce constant background geometry so that $\langle 0|X_S(E)|0\rangle_{\mathrm{KS}} = E_o$ (Koslowski, Sahlmann, Varadarajan, Campiglia)

Motivations

Uniqueness (Fleischhack, Lewandowski, Okołów, Sahlmann, Thiemann)

- ⋆ Uniqueness of AL representation under technical assumptions, including
  - Diffeomorphism invariance
  - Weak continuity of exponentiated fluxes (i.e. the fluxes exist as operators)
- ⋆ Very different from QFT, where there are many inequivalent representations
- ⋆ AL vacuum makes it difficult to construct semi-classical states: need condensates

Other possible representations

Motivations

Uniqueness (Fleischhack, Lewandowski, Okołów, Sahlmann, Thiemann)

- ⋆ Uniqueness of AL representation under technical assumptions, including
  - Diffeomorphism invariance
  - Weak continuity of exponentiated fluxes (i.e. the fluxes exist as operators)
- ⋆ Very different from QFT, where there are many inequivalent representations
- ⋆ AL vacuum makes it difficult to construct semi-classical states: need condensates

Other possible representations

- ⋆ Introduce constant background geometry so that $\langle 0|X_S(E)|0\rangle_{\mathrm{KS}} = E_o$ (Koslowski, Sahlmann, Varadarajan, Campiglia)
- ⋆ Dualize all the ingredients of the AL representation → BF representation

Results in a nutshell

BF representation

- ⋆ Cyclic vacuum peaked on locally and globally flat connections, $\langle 0|F(A)|0\rangle_{\mathrm{BF}} = 0$

Results in a nutshell

BF representation

- ⋆ Cyclic vacuum peaked on locally and globally flat connections, $\langle 0|F(A)|0\rangle_{\mathrm{BF}} = 0$
- ⋆ Wilson surface operators (exponentiated fluxes) are creation operators

Results in a nutshell

BF representation

- ⋆ Cyclic vacuum peaked on locally and globally flat connections, $\langle 0|F(A)|0\rangle_{\mathrm{BF}} = 0$
- ⋆ Wilson surface operators (exponentiated fluxes) are creation operators
- ⋆ Vacuum is a physical state of topological BF theory

Results in a nutshell

BF representation

- ⋆ Cyclic vacuum peaked on locally and globally flat connections, $\langle 0|F(A)|0\rangle_{\mathrm{BF}} = 0$
- ⋆ Wilson surface operators (exponentiated fluxes) are creation operators
- ⋆ Vacuum is a physical state of topological BF theory
- ⋆ Discrete basis of excitations labelled by simplices of the triangulation

Results in a nutshell

BF representation

- ⋆ Cyclic vacuum peaked on locally and globally flat connections, $\langle 0|F(A)|0\rangle_{\mathrm{BF}} = 0$
- ⋆ Wilson surface operators (exponentiated fluxes) are creation operators
- ⋆ Vacuum is a physical state of topological BF theory
- ⋆ Discrete basis of excitations labelled by simplices of the triangulation
- ⋆ Embedding of Hilbert spaces based on embedding of triangulations

Results in a nutshell

BF representation

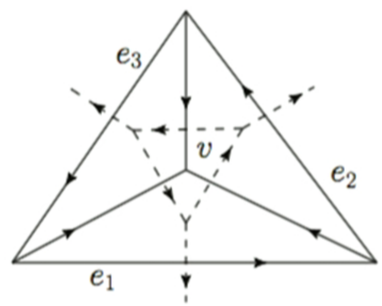
- ⋆ Cyclic vacuum peaked on locally and globally flat connections, $\langle 0|F(A)|0\rangle_{\text{BF}} = 0$
- ⋆ Wilson surface operators (exponentiated fluxes) are creation operators
- ⋆ Vacuum is a physical state of topological BF theory
- ⋆ Discrete basis of excitations labelled by simplices of the triangulation
- ⋆ Embedding of Hilbert spaces based on embedding of triangulations
- ⋆ Cylindrical consistency allows to define a continuum Hilbert space ...
- ⋆ ... and requires a discrete topology on the group (compactified flux space)
- ⋆ Provides a new realization of quantum geometry
- ⋆ Deals for the first time successfully with the gauge-covariant fluxes $X(E,h)$

Results in a nutshell

BF representation

- ⋆ Cyclic vacuum peaked on locally and globally flat connections, $\langle 0|F(A)|0\rangle_{\mathrm{BF}} = 0$
- ⋆ Wilson surface operators (exponentiated fluxes) are creation operators
- ⋆ Vacuum is a physical state of topological BF theory
- ⋆ Discrete basis of excitations labelled by simplices of the triangulation
- ⋆ Embedding of Hilbert spaces based on embedding of triangulations
- ⋆ Cylindrical consistency allows to define a continuum Hilbert space ...
- ⋆ ...and requires a discrete topology on the group (compactified flux space)
- ⋆ Provides a new realization of quantum geometry
- ⋆ Deals for the first time successfully with the gauge-covariant fluxes $X(E,h)$
- ⋆ Coarse-graining of the fluxes and curvature-induced torsion (violations of Gauss)

$$\mathbf{X}_{e_3\circ e_2\circ e_1} \neq 0 \quad \text{if} \quad g_\ell \neq \mathbb{1}$$



5 / 17

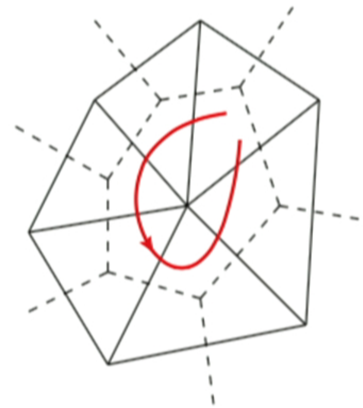
Configuration space

Simplicial LQG

- Triangulation Δ of d-dimensional spatial manifold Σ
- Graph Γ dual to Δ
- Simplicial (gauge-covariant) fluxes

Connection degrees of freedom

- Curvature encoded in holonomies around $(d-2)$-dimensional simplices (defects)



Configuration space

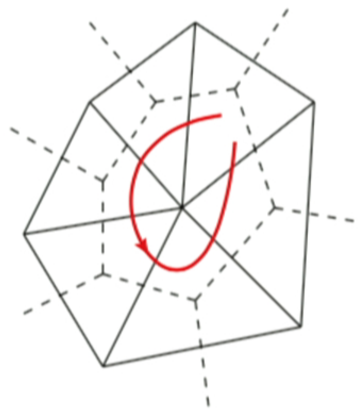
Simplicial LQG

- Triangulation Δ of d-dimensional spatial manifold Σ
- Graph Γ dual to Δ
- Simplicial (gauge-covariant) fluxes

Connection degrees of freedom

- Curvature encoded in holonomies around $(d-2)$-dimensional simplices (defects)
- Moduli space of flat connections

$$\begin{aligned} \mathcal{A}_0 &= \{A \in \mathcal{A} \mid F(A) = 0 \text{ on } \Sigma\backslash\Delta_{(d-2)}\}/\mathcal{G} \\ &= \mathrm{Hom}\big(\pi_1(\Sigma\backslash\Delta_{(d-2)}), G\big)/G \end{aligned}$$



Configuration space

Simplicial LQG

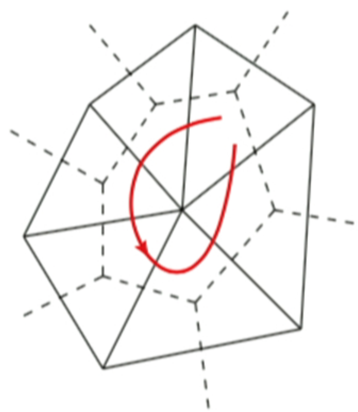
- Triangulation Δ of d-dimensional spatial manifold Σ
- Graph Γ dual to Δ
- Simplicial (gauge-covariant) fluxes

Connection degrees of freedom

- Curvature encoded in holonomies around $(d-2)$-dimensional simplices (defects)
- Moduli space of flat connections

$$\begin{aligned}\mathcal{A}_0 &= \{A \in \mathcal{A} \mid F(A) = 0 \text{ on } \Sigma\backslash\Delta_{(d-2)}\}/\mathcal{G} \\ &= \mathrm{Hom}\big(\pi_1(\Sigma\backslash\Delta_{(d-2)}), G\big)/G\end{aligned}$$

- Choose a tree in Γ, then $\mathcal{A}_0 \simeq G^{\#\text{ leaves } \ell}$, where ℓ labels the fundamental cycles



Quantum theory on a fixed triangulation

Refinement

- ⋆ Wave functions $\psi\{g_\ell\}$ of fundamental cycle holonomies

Quantum theory on a fixed triangulation

Refinement

- Wave functions $\psi\{g_\ell\}$ of fundamental cycle holonomies
- For a finer triangulation $\Delta' \succ \Delta$, the graph Γ' has more fundamental cycles
- States in $\mathcal{H}'_\Delta$ arising from embedding of states in $\mathcal{H}_\Delta$ must have finite norm

Quantum theory on a fixed triangulation

Refinement

- Wave functions $\psi\{g_\ell\}$ of fundamental cycle holonomies
- For a finer triangulation $\Delta' \succ \Delta$, the graph Γ' has more fundamental cycles
- States in $\mathcal{H}'_\Delta$ arising from embedding of states in $\mathcal{H}_\Delta$ must have finite norm

Inner product

- Choose auxiliary (e.g. Haar) inner product $\langle \cdot | \cdot \rangle_{\text{aux}}$

Quantum theory on a fixed triangulation

Refinement

- ⋆ Wave functions $\psi\{g_\ell\}$ of fundamental cycle holonomies
- ⋆ For a finer triangulation $\Delta' \succ \Delta$, the graph Γ' has more fundamental cycles
- ⋆ States in $\mathcal{H}'_\Delta$ arising from embedding of states in $\mathcal{H}_\Delta$ must have finite norm

Inner product

- ⋆ Choose auxiliary (e.g. Haar) inner product $\langle\cdot|\cdot\rangle_{\mathrm{aux}}$
- ⋆ Choose regulated states ψ^ε of finite norm in this auxiliary product
- ⋆ Use vacuum as reference state to define

$$\langle\psi_1|\psi_2\rangle = \lim_{\varepsilon\to 0} \frac{\langle\psi_1^\varepsilon|\psi_2^\varepsilon\rangle_{\mathrm{aux}}}{\langle\psi_1^\varepsilon|\psi_1^\varepsilon\rangle_{\mathrm{aux}}}$$

- ⋆ For heat kernel regulated delta function states peaked on $\alpha, \beta \in G$

$$\langle\psi_\alpha|\psi_\beta\rangle \overset{\varepsilon\to 0}{\longrightarrow} \delta(\alpha,\beta)$$

Quantum theory on a fixed triangulation

Refinement

- ⋆ Wave functions $\psi\{g_\ell\}$ of fundamental cycle holonomies
- ⋆ For a finer triangulation $\Delta' \succ \Delta$, the graph Γ' has more fundamental cycles
- ⋆ States in $\mathcal{H}'_\Delta$ arising from embedding of states in $\mathcal{H}_\Delta$ must have finite norm

Inner product

- ⋆ Choose auxiliary (e.g. Haar) inner product $\langle \cdot | \cdot \rangle_{\text{aux}}$
- ⋆ Choose regulated states ψ^ε of finite norm in this auxiliary product
- ⋆ Use vacuum as reference state to define

$$\langle \psi_1 | \psi_2 \rangle = \lim_{\varepsilon \to 0} \frac{\langle \psi_1^\varepsilon | \psi_2^\varepsilon \rangle_{\text{aux}}}{\langle \psi_{\mathbb{1}}^\varepsilon | \psi_{\mathbb{1}}^\varepsilon \rangle_{\text{aux}}}$$

- ⋆ For heat kernel regulated delta function states peaked on $\alpha, \beta \in G$

$$\langle \psi_\alpha | \psi_\beta \rangle \xrightarrow{\varepsilon \to 0} \delta(\alpha, \beta)$$

(non-separable) Hilbert space

- ⋆ Orthonormal basis $\psi_{\{\alpha\}}\{g_\ell\} = \prod_\ell \delta(\alpha_\ell, g_\ell)$ with $\langle \psi_{\{\alpha\}} | \psi_{\{\beta\}} \rangle = \delta(\{\alpha\}, \{\beta\})$

$$\mathcal{H}_\Delta = L^2 \left(\text{Hom} \left(\pi_1 \left(\Sigma \backslash \Delta_{(d-2)} \right), G \right), \mathrm{d}\mu_{\text{discrete}} \right)$$

Quantum theory on a fixed triangulation

Refinement

- ⋆ Wave functions $\psi\{g_\ell\}$ of fundamental cycle holonomies
- ⋆ For a finer triangulation $\Delta' \succ \Delta$, the graph Γ' has more fundamental cycles
- ⋆ States in $\mathcal{H}'_\Delta$ arising from embedding of states in $\mathcal{H}_\Delta$ must have finite norm

Inner product

- ⋆ Choose auxiliary (e.g. Haar) inner product $\langle \cdot | \cdot \rangle_{\text{aux}}$
- ⋆ Choose regulated states ψ^ε of finite norm in this auxiliary product
- ⋆ Use vacuum as reference state to define

$$\langle \psi_1 | \psi_2 \rangle = \lim_{\varepsilon \to 0} \frac{\langle \psi_1^\varepsilon | \psi_2^\varepsilon \rangle_{\text{aux}}}{\langle \psi_1^\varepsilon | \psi_1^\varepsilon \rangle_{\text{aux}}}$$

- ⋆ For heat kernel regulated delta function states peaked on $\alpha, \beta \in G$

$$\langle \psi_\alpha | \psi_\beta \rangle \overset{\varepsilon \to 0}{\longrightarrow} \delta(\alpha, \beta)$$

(non-separable) Hilbert space

- ⋆ Orthonormal basis $\psi_{\{\alpha\}}\{g_\ell\} = \prod_\ell \delta(\alpha_\ell, g_\ell)$ with $\langle \psi_{\{\alpha\}} | \psi_{\{\beta\}} \rangle = \delta\big(\{\alpha\}, \{\beta\}\big)$

$$\mathcal{H}_\Delta = L^2 \left(\text{Hom}\left(\pi_1\left(\Sigma \backslash \Delta_{(d-2)}\right), G\right), \text{d}\mu_{\text{discrete}} \right)$$

Quantum theory on a fixed triangulation

Quantization of the fluxes

- ⋆ Discrete Hilbert space topology to accommodate flat vacuum and inductive limit
- ⋆ Fluxes don't exist as operators (cf. LQC and Bohr compactification)
- ⋆ Exponentiated symplectic flow of the fluxes (Wilson surface operators)

$$R_i^\alpha \psi\{g_\ell\} = \psi(g_1, \ldots, g_i\alpha, \ldots, g_{|\ell|})$$

Quantum theory on a fixed triangulation

Quantization of the fluxes

- Discrete Hilbert space topology to accommodate flat vacuum and inductive limit
- Fluxes don't exist as operators (cf. LQC and Bohr compactification)
- Exponentiated symplectic flow of the fluxes (Wilson surface operators)

$$R_i^{\alpha}\psi\{g_\ell\} = \psi(g_1, \ldots, g_i\alpha, \ldots, g_{|\ell|})$$

- Spectrum of translation operator $R^{\phi}\psi_{\alpha} = \psi_{\alpha-\phi}$ on $L^2(\mathrm{U}(1), \mathrm{d}\mu_{\text{discrete}})$
  - Discrete if ϕ rational

$$v_{\alpha,\kappa} = \frac{1}{\sqrt{q}}\sum_{n=0}^{q-1} e^{in\kappa\phi}\psi_{\alpha+n\phi} \qquad \mathrm{spec}(R^{\phi}) = \{e^{i\kappa\phi} \,|\, \kappa = 0, \ldots, q-1\}$$

  - Continuous if ϕ irrational

$$w_{\alpha,\rho} = \sum_{n\in\mathbb{Z}} e^{in\rho}\psi_{\alpha+n\phi} \qquad \mathrm{spec}(R^{\phi}) = \{e^{i\rho} \,|\, \rho \in [0, 2\pi)\}$$

Continuum Hilbert space

Inductive limit Hilbert space

- ⋆ We have Hilbert spaces $\mathcal{H}_\Delta$ on triangulations, and a partial order "$\prec$"
- ⋆ We can consider

inductive limit with refinement maps

$$\xrightarrow{\hspace{6cm}}$$

$$\text{coarser} \;\dots\; \prec \; \Delta_{i-2} \; \prec \; \Delta_{i-1} \; \prec \; \Delta_i \; \prec \; \Delta_{i+1} \; \prec \; \Delta_{i+2} \; \prec \; \dots \;\text{finer}$$

$$\xleftarrow{\hspace{6cm}}$$

projective limit with projection maps

- ⋆ The refinement maps satisfy all the required properties to define

$$\mathcal{H}_\infty = \cup_\Delta \mathcal{H}_\Delta / \sim$$

with a cylindrically-consistent inner product inherited from $\mathcal{H}_\Delta$

10 / 17

Philosophy

Why different vacua and representations?

- ⋆ Needed in order to describe phase transitions and condensation in QFT
- ⋆ Physical states of quantum GR will not be in the initial $\mathcal{H}_{\text{kin}}$ Hilbert space ...
- ⋆ ... but might be easier to reach starting from certain vacua
- ⋆ Path integral dynamics is a projector onto physical states
- ⋆ The path integral dynamics of LQG is spin foams, which is built upon BF theory

11 / 17

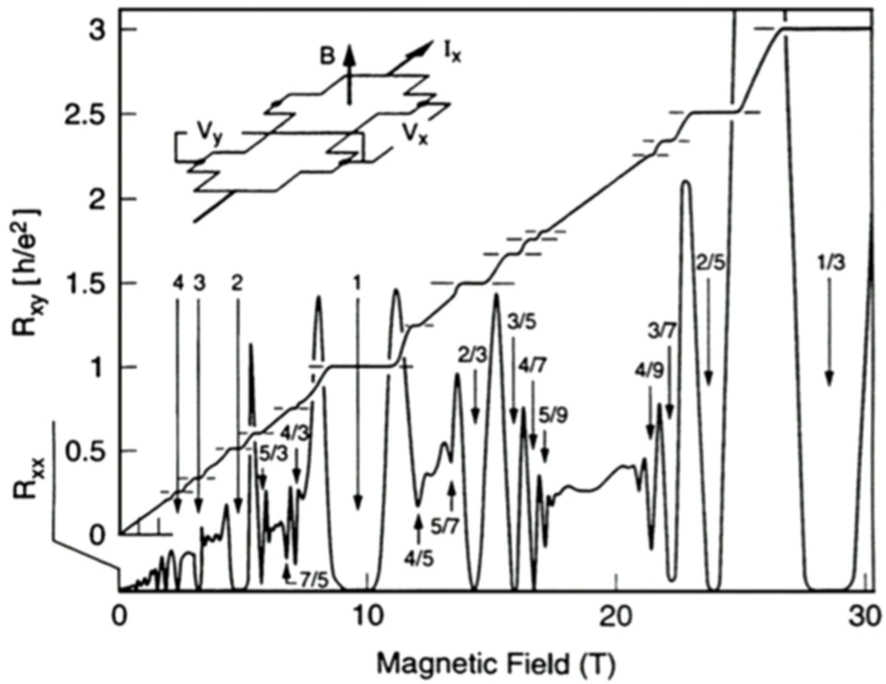
Turaev–Viro vacuum

3d quantum gravity with $\Lambda \neq 0$

- ⋆ Classical groups get deformed (also in $\Lambda = 0$ Chern–Simons): no group picture

Focus on TQFT with defects

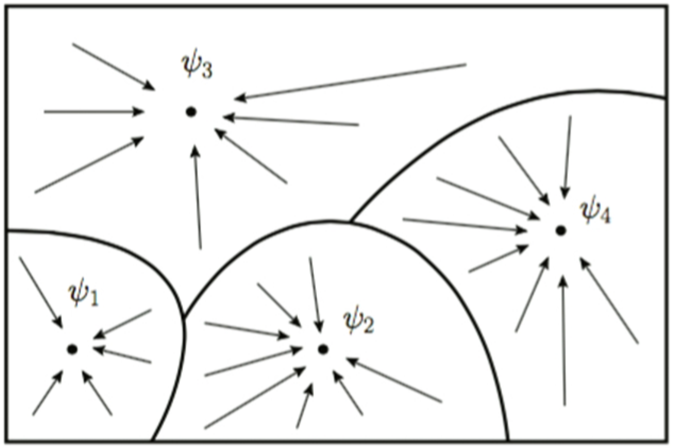
- ⋆ Fractional quantum Hall effect and topological order: beyond Landau symmetry breaking, long-range entanglement, ground state degeneracy, anyonic statistics



Turaev–Viro vacuum

String-net condensation (Levin, Wen)

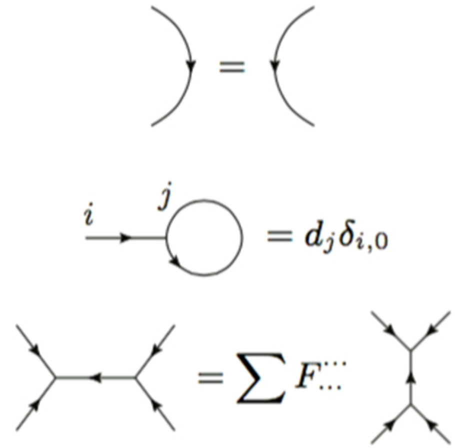
- ⋆ Fixed-point wave functions satisfy axioms of unitary tensor categories



Turaev–Viro vacuum

Closed Ribbon graph Hilbert space

- ⋆ Take a closed $2d$ surface Σ
- ⋆ Define $\mathcal{H}_\Sigma$ as span of **colored** fusion-compatible trivalent graphs in Σ modulo



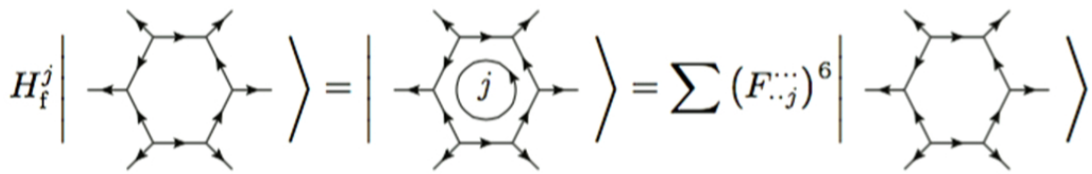
Turaev–Viro vacuum

Levin–Wen Hamiltonian

- ⋆ Hamiltonian for lattice 3d TQFT
- ⋆ Typically defined on honeycomb lattice as

$$H = -\sum_{\text{nodes}} H_{\text{n}} - \sum_{\text{faces}} H_{\text{f}} \qquad H_{\text{f}} = \sum_{j=0}^{\text{k}/2} \frac{d_j}{\sum_i d_i^2} H_{\text{f}}^j$$

with $H_{\text{n}} \equiv \mathcal{N}_{ijk}$ the fusion coefficients for three irreps



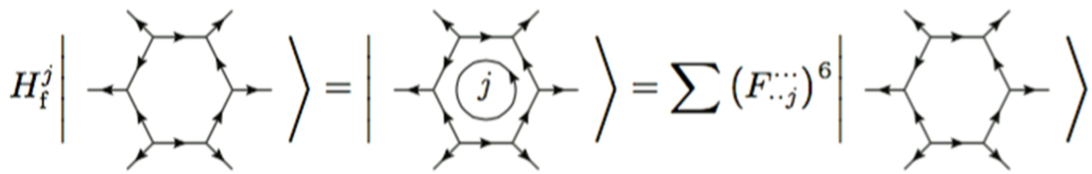
Turaev–Viro vacuum

Levin–Wen Hamiltonian

- ⋆ Hamiltonian for lattice 3d TQFT
- ⋆ Typically defined on honeycomb lattice as

$$H = -\sum_{\text{nodes}} H_{\text{n}} - \sum_{\text{faces}} H_{\text{f}} \qquad H_{\text{f}} = \sum_{j=0}^{\text{k}/2} \frac{d_j}{\sum_i d_i^2} H_{\text{f}}^j$$

with $H_{\text{n}} \equiv \mathcal{N}_{ijk}$ the fusion coefficients for three irreps



- ⋆ Theorem (Kirillov Jr.): $\langle \Gamma | \prod_{\text{faces}} H_{\text{f}} | \Gamma' \rangle = \mathcal{Z}_{\text{TV}}(\Sigma \times [-1,1]; \Gamma, \Gamma')$
- ⋆ Torus ground state degeneracy $= \sum_{\vec{j}}$

$= (\text{k}+1)^2$

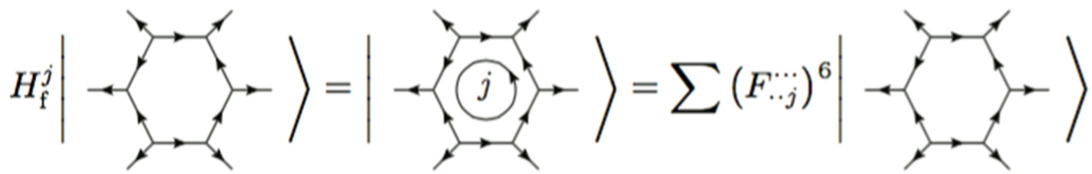
Turaev–Viro vacuum

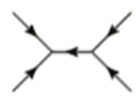
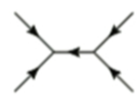
Levin–Wen Hamiltonian

- ⋆ Hamiltonian for lattice 3d TQFT
- ⋆ Typically defined on honeycomb lattice as

$$H = -\sum_{\text{nodes}} H_{\text{n}} - \sum_{\text{faces}} H_{\text{f}} \qquad\qquad H_{\text{f}} = \sum_{j=0}^{\text{k}/2} \frac{d_j}{\sum_i d_i^2} H_{\text{f}}^j$$

with $H_{\text{n}} \equiv \mathcal{N}_{ijk}$ the fusion coefficients for three irreps



- ⋆ Theorem (Kirillov Jr.): $\langle \Gamma | \prod_{\text{faces}} H_{\text{f}} | \Gamma' \rangle = \mathcal{Z}_{\text{TV}}(\Sigma \times [-1,1]; \Gamma, \Gamma')$
- ⋆ Torus ground state degeneracy $= \sum_{\vec{j}} \Big\langle$  $\Big| \prod_{\text{faces}} H_{\text{f}} \Big|$  $\Big\rangle = (\text{k}+1)^2$
- ⋆ Quasiparticle excitations located at defects (e.g. vertices) and labelled by irreps of Drinfeld center of category

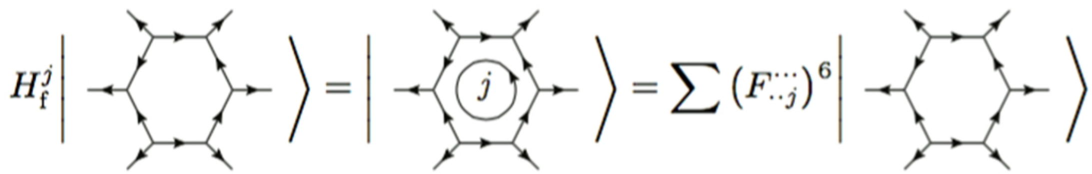
Turaev–Viro vacuum

Levin–Wen Hamiltonian

- ⋆ Hamiltonian for lattice 3d TQFT
- ⋆ Typically defined on honeycomb lattice as

$$H = -\sum_{\text{nodes}} H_{\mathrm{n}} - \sum_{\text{faces}} H_{\mathrm{f}} \qquad\qquad H_{\mathrm{f}} = \sum_{j=0}^{\mathrm{k}/2} \frac{d_j}{\sum_i d_i^2} H_{\mathrm{f}}^j$$

with $H_{\mathrm{n}} \equiv \mathcal{N}_{ijk}$ the fusion coefficients for three irreps



- ⋆ Theorem (Kirillov Jr.): $\langle \Gamma | \prod_{\text{faces}} H_{\mathrm{f}} | \Gamma' \rangle = \mathcal{Z}_{\mathrm{TV}}(\Sigma \times [-1,1]; \Gamma, \Gamma')$
- ⋆ Torus ground state degeneracy $= \sum_{\vec{j}}$  $= (\mathrm{k}+1)^2$

- ⋆ Quasiparticle excitations located at defects (e.g. vertices) and labelled by irreps of Drinfeld center of category

Conclusion

New framework and results

- ⋆ Full continuum Hilbert space supporting curvature excitations
- ⋆ Allows for geometrical coarse graining of the fluxes (intrinsic geometry)
- ⋆ Diffeomorphisms as vertex displacement (in $d = 2$)
- ⋆ New take on the dynamics and extraction of physics
- ⋆ New realization of quantum geometry
- ⋆ Same structures found in wider class of TQFTs (allows $\Lambda \neq 0$)

Generalizations and applications

- ⋆ Quantum groups and Λ
- ⋆ Non-commutative flux representation (space of generalized fluxes)
- ⋆ Non-compact gauge groups
- ⋆ Cosmology
- ⋆ Black holes